Appendix Proofs of Propositions

A.1 Generalization of Proposition 2 to case of non-zero rate-setting noise

Society's ex ante problem is to choose a central banker with concern about market volatility θ_c. In the absence of noise, this central banker will implement the rational-expectations equilibrium $k(\theta_c)$ given by Equation (12) in the main text replacing θ with θ_c. In the absence of noise, society's ex ante loss function is given by

$$\left((1-k(\theta_c))^2+\theta\right)\sigma_\varepsilon^2,$$

which is minimized by setting $\theta_c = 0$ so that $k(\theta_c) = 1$.

In the presence of noise, the appointed central banker will implement the rational-expectations equilibrium $k(\theta_c)$ given by Equation (9) in the main text replacing θ with θ_c. Differentiating $k(\theta_c)$ with respect to θ_c yields

$$\frac{\partial k}{\partial \theta_c} = -\left(1-\frac{2\theta_c k\tau_u^2\left(\tau_\varepsilon+k^2\tau_u\right)\left(k^2\tau_u-\tau_\varepsilon\right)}{\left(\left(\tau_\varepsilon+k^2\tau_u\right)^2+\theta_c\left(k\tau_u\right)^2\right)^2}\right)^{-1}\frac{\left(\tau_\varepsilon+k^2\tau_u\right)^2\left(k\tau_u\right)^2}{\left(\left(\tau_\varepsilon+k^2\tau_u\right)^2+\theta_c\left(k\tau_u\right)^2\right)^2}<0.$$

In the presence of noise, society's ex ante loss function is given by

$$L = E\left[\left((1-k)\varepsilon_t+u_t\right)^2+\theta\left(\chi\left(k\varepsilon+u_t\right)\right)^2\right] = (1-k)^2\sigma_\varepsilon^2+\sigma_u^2+\theta\chi^2\left(k^2\sigma_\varepsilon^2+\sigma_u^2\right).$$

Substituting in for the rational-expectations definition of χ and differentiating with respect to θ_c gives the first order condition

$$\frac{\partial L}{\partial \theta_c} = \left[-2(1-k)\sigma_\varepsilon^2+\frac{2k\theta\tau_u}{\left(\tau_\varepsilon+k^2\tau_u\right)^3}\left(k^2\tau_u+\tau_\varepsilon\right)\right]\frac{\partial k}{\partial \theta_c}.$$

At the optimum θ_c in the presence of noise, the term in square brackets will be zero. When we discuss the optimal θ_c in the presence of noise in footnote 14 of the main text, we numerically solve for the value of θ_c and therefore k that sets the term in square brackets to zero.

A.2 Proof of Proposition 3

We begin by characterizing the behavior of an appointed central banker who care about a finite-horizon yield, which can be represented as a weighted average of the short rate and the infinite-horizon rate:

$$i_t^{finite} = \alpha i_t + (1-\alpha)\, i_t^{\infty}.$$

The degree of concern about the volatility of the finite-horizon yield is parameterized by θ_c. The central banker follows a rule of the form

$$i_t = i_{t-1} + k(\theta_c)\cdot\varepsilon_t,$$

while the market conjectures that the central banker is following a rule of the form

$$i_t = i_{t-1} + \kappa(\theta_c)\cdot\varepsilon_t.$$

Given this, the market's conjecture about ε_t is

$$\widetilde{\varepsilon}_t = \frac{k}{\kappa}\varepsilon_t$$

and it sets the infinite-horizon forward rate to

$$i_t^{\infty} = i_{t-1} + \widetilde{\varepsilon}_t.$$

Thus, the change in the finite-horizon yield is

$$\Delta i_t^{finite} = \alpha \left(i_t - i_{t-1}\right) + (1-\alpha)\,\widetilde{\varepsilon}_t.$$

The central banker picks $k\left(\theta_c\right)$ to minimize the loss function

$$\begin{aligned} E_t\left[\left(i_t^* - i_t\right)^2 + \theta_c\left(\Delta i_t^{finite}\right)^2\right] &= (1-k)^2\,\sigma_\varepsilon^2 + \theta_c\alpha^2 k^2\sigma_\varepsilon^2 \\ &\quad + \theta_c\frac{(1-\alpha)^2}{\kappa^2}k^2\sigma_\varepsilon^2 + 2\theta_c\alpha\,(1-\alpha)\,\frac{k^2}{\kappa}\sigma_\varepsilon^2. \end{aligned}$$

Differentiating with respect to k yields the first order condition

$$0 = -\,(1-k)\,\sigma_\varepsilon^2 + \theta_c\alpha^2 k\sigma_\varepsilon^2 + \theta_c\frac{(1-\alpha)^2}{\kappa^2}k\sigma_\varepsilon^2 + 2\theta_c\alpha\,(1-\alpha)\,\frac{k}{\kappa}\sigma_\varepsilon^2.$$

In rational expectations we have $\kappa = k$, so this reduces to

$$0 = -\,(1-k)\,\sigma_\varepsilon^2 + \underbrace{\theta_c\alpha^2 k\sigma_\varepsilon^2}_{\text{A}} + \underbrace{\theta_c\frac{(1-\alpha)^2}{k}\sigma_\varepsilon^2}_{\text{B}} + \underbrace{2\theta_c\alpha\,(1-\alpha)\,\sigma_\varepsilon^2}_{\text{C}}. \tag{A.1}$$

Term A reflects the direct effect of changing k on the short-rate component of our representation of the finite-horizon yield. Term B reflects the effect of changing k on the inferred shock $\widetilde{\varepsilon}_t$, and term C reflects the interaction of the two effects. Note that (A.1) implies that

$$\frac{\partial k}{\partial\theta_c} = \left[\begin{array}{c} -\left(\sigma_\varepsilon^2\left(1+\theta_c\alpha^2 k\right) - \theta_c\frac{(1-\alpha)^2}{k^2}\sigma_\varepsilon^2\right)^{-1} \\ \times\left(\alpha^2 k\sigma_\varepsilon^2 + \frac{(1-\alpha)^2}{k}\sigma_\varepsilon^2 + 2\theta_c\alpha\,(1-\alpha)\,\sigma_\varepsilon^2\right) \end{array}\right] < 0.$$

Society's ex ante loss is given by

$$E_t\left[\left(i_t^* - i_t\right)^2 + \theta\left(\Delta i^{finite}\right)^2\right] = \left(1-k\left(\theta_c\right)\right)^2\sigma_\varepsilon^2 + \theta\left(\begin{array}{c} \alpha^2 k\left(\theta_c\right)^2\sigma_\varepsilon^2 + (1-\alpha)^2\sigma_\varepsilon^2 \\ +2\alpha\,(1-\alpha)\,k\left(\theta_c\right)\sigma_\varepsilon^2 \end{array}\right).$$

Differentiating with respect to θ_c yields the first order condition

$$\left[-\,(1-k)\,\sigma_\varepsilon^2 + \theta\alpha k\sigma_\varepsilon^2 + \theta\alpha\,(1-\alpha)\,\sigma_\varepsilon^2\right]\frac{\partial k}{\partial\theta_c} = 0.$$

Evaluating at $\theta_c = \theta$ and using the appointed central banker's first order condition (A.1), we have that the term in the square brackets is

$$-\,(1-k)\,\sigma_\varepsilon^2 + \underbrace{\theta\alpha k\sigma_\varepsilon^2}_{\text{A}} + \underbrace{\theta\alpha\,(1-\alpha)\,\sigma_\varepsilon^2}_{\text{C/2}} = \underbrace{-\theta\frac{(1-\alpha)^2}{k}\sigma_\varepsilon^2}_{\text{-B}} - \underbrace{\theta\alpha\,(1-\alpha)\,\sigma_\varepsilon^2}_{\text{-(C/2)}} < 0.$$

Since $\partial k/\partial\theta_c < 0$, this implies that society's first order condition is positive at $\theta_c = \theta$ and its ex ante loss can be reduced by setting $\theta_c < \theta$.

A.3 Proof of Proposition 4

Given its conjecture about the rule the Fed is following, $\phi(\varepsilon_t; \nu_t)$, the market's conjecture about ε_t is

$$\widetilde{\varepsilon_t} = \phi^{-1}\left(f\left(\varepsilon_t; \nu_t\right); \nu_t\right)$$

and the Fed's loss function is

$$\left(\varepsilon_t + \nu_t - f\left(\varepsilon_t; \nu_t\right)\right)^2 + \theta\left(\phi^{-1}\left(f\left(\varepsilon_t; \nu_t\right); \nu_t\right) + \nu_t\right)^2.$$

Consider the effect on the value of the loss function of a small perturbation in the value of $f(\varepsilon_t; \nu_t)$, df. The effect of this perturbation is zero at the optimal $f(\cdot;\cdot)$ so we have

$$-\left(\varepsilon_t + \nu_t - f\left(\varepsilon_t; \nu_t\right)\right) + \theta\left(\phi^{-1}\left(f\left(\varepsilon_t; \nu_t\right); \nu_t\right) + \nu_t\right)\frac{\partial\phi^{-1}}{\partial i}|_{i=f(\varepsilon_t;\nu_t)} = 0. \tag{A.2}$$

Since $\phi\left(\phi^{-1}(x)\right) = x$, we have

$$\frac{\partial\phi^{-1}}{\partial i}|_{i=f(\varepsilon_t;\nu_t)} = \frac{1}{\frac{\partial\phi}{\partial i}|_{\varepsilon=\phi^{-1}(f(\varepsilon_t;\nu_t);\nu_t)}}.$$

Substituting into (A.2) gives

$$-\left(\varepsilon_t + \nu_t - f\left(\varepsilon_t; \nu_t\right)\right) + \theta\left(\phi^{-1}\left(f\left(\varepsilon_t; \nu_t\right); \nu_t\right) + \nu_t\right)\frac{1}{\frac{\partial\phi}{\partial i}|_{\varepsilon=\phi^{-1}(f(\varepsilon_t;\nu_t);\nu_t)}} = 0.$$

Imposing rational expectations, we have $\phi = f$ so that this reduces to the differential equation

$$\frac{\partial f}{\partial\varepsilon}|_{\varepsilon=\varepsilon_t}\left(\varepsilon_t + \nu_t - f\left(\varepsilon_t; \nu_t\right)\right) = \theta\left(\varepsilon_t + \nu_t\right),$$

which the optimal $f(\cdot;\cdot)$ must satisfy.

Now conjecture that $f = k_\varepsilon\varepsilon_t + c$. In this case the differential equation reduces to

$$k_\varepsilon\left(\varepsilon_t + \nu_t - k_\varepsilon\varepsilon_t - c\right) = \theta\left(\varepsilon_t + \nu_t\right)$$

or

$$k_\varepsilon\left(1 - k_\varepsilon\right)\varepsilon_t + k_\varepsilon\left(\nu_t - c\right) = \theta\varepsilon_t + \theta\nu_t$$

Matching coefficients yields

$$\begin{aligned} k_\varepsilon\left(1 - k_\varepsilon\right) &= \theta \quad \text{and} \\ c &= \nu_t\left(1 - \frac{\theta}{k_\varepsilon}\right) \end{aligned}$$

Thus, we can write the optimal $f(\cdot;\cdot)$ as

$$f = k_\varepsilon\varepsilon_t + k_\nu\nu_t$$

where $k_\varepsilon(1 - k_\varepsilon) = \theta$ and $k_\nu = 1 - \theta/k_\varepsilon$. From the definition of k_ε we have

$$\begin{aligned} k_\nu &= 1 - \frac{\theta}{k_\varepsilon} \\ &= 1 - \left(1 - k_\varepsilon\right) = k_\varepsilon. \end{aligned}$$

A.4 Proof of Proposition 5

We begin by solving the time t problem, taking the forward guidance i^f_{t-1} as given. Given its conjecture about the rule the Fed is following, $\phi\left(\varepsilon_t; i^f_{t-1}\right)$, the market's conjecture about ε_t is

$$\widetilde{\varepsilon}_t = \phi^{-1}\left(f\left(\varepsilon_t; i^f_{t-1}\right); i^f_{t-1}\right)$$

and the Fed's loss function is

$$\left(\varepsilon_t - f\left(\varepsilon_t; i^f_{t-1}\right)\right)^2 + \theta\left(\phi^{-1}\left(f\left(\varepsilon_t; i^f_{t-1}\right); i^f_{t-1}\right)\right)^2 + \gamma\left(i^f_{t-1} - i_{t-1} - f\left(\varepsilon_t; i^f_{t-1}\right)\right)^2.$$

Consider the effect on the value of the loss function of a small perturbation in the value of $f\left(\varepsilon_t; i^f_{t-1}\right)$, df. The effect of this perturbation is zero at the optimal $f(\cdot;\cdot)$ so we have

$$-\left(\varepsilon_t - f\left(\varepsilon_t; i^f_{t-1}\right)\right) + \theta\left(\phi^{-1}\left(f\left(\varepsilon_t; i^f_{t-1}\right); i^f_{t-1}\right)\right)\frac{\partial\phi^{-1}}{\partial i}\Big|_{i=f\left(\varepsilon_t; i^f_{t-1}\right)} - \gamma\left(i^f_{t-1} - i_{t-1} - f\left(\varepsilon_t; i^f_{t-1}\right)\right) = 0.$$

Since $\phi\left(\phi^{-1}(x)\right) = x$, we have

$$\frac{\partial\phi^{-1}}{\partial i}\Big|_{i=f\left(\varepsilon_t; i^f_{t-1}\right)} = \frac{1}{\frac{\partial\phi}{\partial i}\Big|_{\varepsilon=\phi^{-1}\left(f\left(\varepsilon_t; i^f_{t-1}\right); i^f_{t-1}\right)}}.$$

So the optimal $f(\cdot;\cdot)$ satisfies:

$$-\left(\varepsilon_t - f\left(\varepsilon_t; i^f_{t-1}\right)\right) + \theta\left(\phi^{-1}\left(f\left(\varepsilon_t; i^f_{t-1}\right); i^f_{t-1}\right)\right)\frac{1}{\frac{\partial\phi}{\partial i}\Big|_{\varepsilon=\phi^{-1}\left(f\left(\varepsilon_t; i^f_{t-1}\right); i^f_{t-1}\right)}} - \gamma\left(i^f_{t-1} - i_{t-1} - f\left(\varepsilon_t; i^f_{t-1}\right)\right) = 0.$$

Imposing rational expectations, we have $\phi = f$ so that this reduces to the differential equation

$$\frac{\partial f}{\partial\varepsilon}\Big|_{\varepsilon=\varepsilon_t}\left(\varepsilon_t - f\left(\varepsilon_t; i^f_{t-1}\right) + \gamma\left(i^f_{t-1} - i_{t-1} - f\left(\varepsilon_t; i^f_{t-1}\right)\right)\right) = \theta\varepsilon_t,$$

which the optimal $f(\cdot;\cdot)$ must satisfy.

Now conjecture that $f = k\varepsilon_t + c$. In this case the differential equation reduces to

$$k\left(\varepsilon_t - k\varepsilon_t - c + \gamma\left(i^f_{t-1} - i_{t-1} - k\varepsilon_t - c\right)\right) = \theta\varepsilon_t$$

or

$$k(1 - k - \gamma k)\varepsilon_t - kc + k\gamma\left(i^f_{t-1} - i_{t-1} - c\right) = \theta\varepsilon_t$$

Matching coefficients implies that

$$\begin{aligned} k(1 - k - \gamma k) &= \theta \quad \text{and} \\ -kc + k\gamma\left(i^f_{t-1} - i_{t-1} - c\right) &= 0. \end{aligned} \tag{A.3}$$

Given the rule $f\left(\varepsilon_t; i^f_{t-1}\right)$ the Fed will follow at time t, we can now solve for the optimal forward guidance at $t-1$. The loss function at time t is given by

$$\begin{aligned} & ((1-k)\varepsilon_t - c)^2 + \theta\varepsilon_t^2 + \gamma\left(i^f_{t-1} - i_{t-1} - k\varepsilon_t - c\right)^2 \\ = \; & ((1-k)\varepsilon_t - c)^2 + \theta\varepsilon_t^2 + \gamma\left(k\varepsilon_t + \frac{1}{\gamma}c\right)^2 \end{aligned}$$

which in expectation at $t-1$ is equal to

$$\left((1-k)^2+\gamma k^2+\theta\right)\sigma_\varepsilon^2+\frac{\gamma}{1+\gamma}\left(i_{t-1}^f-i_{t-1}\right)^2. \tag{A.4}$$

This is minimized by setting $i_{t-1}^f = i_{t-1}$.

Finally we solve for the optimal γ. Differentiating (A.3) with respect to γ, we have

$$\frac{\partial k}{\partial \gamma}=\frac{-k^2}{2k\left(1+\gamma\right)-1}<0$$

so long as $k > 1/2$, which it will be in the parameter space we study. Setting $i_{t-1}^f = i_{t-1}$ and differentiating the ex ante loss function (A.4) with respect to γ yields

$$\left(-2\left(1-k\right)\frac{\partial k}{\partial \gamma}+k^2\left(1-\frac{2k\gamma}{2k\left(1+\gamma\right)-1}\right)\right)\sigma_\varepsilon^2>0$$

since $\partial k/\partial\gamma < 0$ and $k > 1/2$. Since the ex ante loss is increasing in γ, it is optimal to set $\gamma = 0$.

A.5 Proof of Proposition 6

Given its conjecture about the rule the Fed is following, $\phi\left(\varepsilon_t;\eta_t\right)$, the market's conjecture about ε_t is

$$\widetilde{\varepsilon}_t=\phi^{-1}\left(f\left(\varepsilon_t;\eta_t\right);\eta_t\right)$$

and the Fed's loss function is

$$\left(\varepsilon_t-f\left(\varepsilon_t;\eta_t\right)\right)^2+\theta\left(\phi^{-1}\left(f\left(\varepsilon_t;\eta_t\right);\eta_t\right)+\eta_t\right)^2.$$

Consider the effect on the value of the loss function of a small perturbation in the value of $f\left(\varepsilon_t;\eta_t\right)$, df. The effect of this perturbation is zero at the optimal $f\left(\cdot;\cdot\right)$ so we have

$$-\left(\varepsilon_t-f\left(\varepsilon_t;\eta_t\right)\right)+\theta\left(\phi^{-1}\left(f\left(\varepsilon_t;\eta_t\right);\eta_t\right)+\eta_t\right)\frac{\partial\phi^{-1}}{\partial i}|_{i=f(\varepsilon_t;\eta_t)}=0. \tag{A.5}$$

Since $\phi\left(\phi^{-1}\left(x\right)\right)=x$, we have

$$\frac{\partial\phi^{-1}}{\partial i}|_{i=f(\varepsilon_t;\eta_t)}=\frac{1}{\frac{\partial\phi}{\partial i}|_{\varepsilon=\phi^{-1}(f(\varepsilon_t;\eta_t);\eta_t)}}.$$

Substituting into (A.5) gives

$$-\left(\varepsilon_t-f\left(\varepsilon_t;\eta_t\right)\right)+\theta\left(\phi^{-1}\left(f\left(\varepsilon_t;\eta_t\right);\eta_t\right)+\eta_t\right)\frac{1}{\frac{\partial\phi}{\partial i}|_{\varepsilon=\phi^{-1}(f(\varepsilon_t;\eta_t);\eta_t)}}=0.$$

Imposing rational expectations, we have $\phi = f$ so that this reduces to the differential equation

$$\frac{\partial f}{\partial\varepsilon}|_{\varepsilon=\varepsilon_t}\left(\varepsilon_t-f\left(\varepsilon_t;\eta_t\right)\right)=\theta\left(\varepsilon_t+\eta_t\right),$$

which the optimal $f\left(\cdot;\cdot\right)$ must satisfy.

Now conjecture that $f = k_\varepsilon \varepsilon_t + c$. In this case the differential equation reduces to

$$k_\varepsilon \left(\varepsilon_t - k_\varepsilon \varepsilon_t - c\right) = \theta \left(\varepsilon_t + \eta_t\right)$$

or

$$k_\varepsilon \left(1 - k_\varepsilon\right) \varepsilon_t - k_\varepsilon c = \theta \varepsilon_t + \theta \eta_t$$

Matching coefficients yields

$$\begin{aligned} k_\varepsilon \left(1 - k_\varepsilon\right) &= \theta \quad \text{and} \\ c &= -\frac{\theta}{k_\varepsilon} \eta_t. \end{aligned}$$

Thus, we can write the optimal $f\left(\cdot;\cdot\right)$ as

$$f = k_\varepsilon \varepsilon_t + k_\eta \eta_t$$

where $k_\varepsilon \left(1 - k_\varepsilon\right) = \theta$ and $k_\eta = -\theta / k_\varepsilon$.

A.6 Proof of Proposition 7

Given ε_t (new private information about the target i_t^*) and X_{t-1} (the existing gap between i_{t-1}^* and i_{t-1}, which is public information), the central banker follows a rule of the form

$$i_t = i_{t-1} + k_X \cdot X_{t-1} + k_\varepsilon \cdot \varepsilon_t.$$

Rational investors understand that the Fed treats X_{t-1} and ε_t differently and conjecture it is following a rule of the form

$$i_t = i_{t-1} + \kappa_X \cdot X_{t-1} + \kappa_\varepsilon \cdot \varepsilon_t.$$

Given the conjecture and i_t, the investors back out

$$E^R\left[\varepsilon_t\right] = \frac{i_t - i_{t-1} - \kappa_X X_{t-1}}{\kappa_\varepsilon}.$$

To further simplify the problem, we also assume the following timing convention. First, based on its knowledge of X_{t-1}, the Fed decides on the value of the adjustment parameters k_X and k_ε it will use for the time t FOMC meeting. After making this decision, it deliberates further, and in so doing discovers the committee's consensus value of ε_t. Thus, the Fed picks k_X and k_ε, taking as given X_{t-1}, but before knowing the realization of ε_t. This timing convention is purely a technical trick that makes the problem more tractable without really changing anything of economic substance. Without it, the Fed's adjustment rule would turn out to depend on the realization of the product $X_{t-1}\varepsilon_t$. With the timing trick, what matters instead is the expectation of the product, which is zero. This simplification maintains the linearity of the Fed's optimal adjustment rule. We should emphasize that even with this somewhat strained intra-meeting timing, the Fed still behaves on a discretionary basis from one meeting to the next. Thus, while it agrees to values of k_X and k_ε in the first part of the time-t meeting, it has no ability to bind itself to those values across meetings. Hence the basic commitment problem remains.

Thus, the central banker picks k_X and k_ε to minimize the loss function

$$\begin{aligned} E_t\left[\left(i_t^* - i_t\right)^2 + \theta\left(\Delta i_t^\infty\right)^2\right] &= E_t\left[\left(\left(1 - k_X\right) X_{t-1} + \left(1 - k_\varepsilon\right)\varepsilon_t\right)^2 + \theta\left(E^R\left[\varepsilon_t\right]\right)^2\right] \\ &= \left(1 - k_X\right)^2 X_{t-1}^2 + \left(1 - k_\varepsilon\right)^2 \sigma_\varepsilon^2 + \theta\left(\left(\frac{1}{\kappa_\varepsilon}\left(k_X - \kappa_X\right)\right)^2 X_{t-1}^2 + \left(\frac{k_\varepsilon}{\kappa_\varepsilon}\right)^2 \sigma_\varepsilon^2\right). \end{aligned}$$

Differentiating with respect to k_X yields

$$-(1-k_X)+\theta\left(\frac{1}{\kappa_\varepsilon}(k_X-\kappa_X)\right)=0.$$

Imposing rational expectations we have $k_X=\kappa_X$ which implies that $k_X=1$.

Differentiating with respect to k_ε yields

$$0=-(1-k_\varepsilon)+\theta\left(\frac{k_\varepsilon}{\kappa_\varepsilon^2}\right).$$

Imposing rational expectations we have $k_\varepsilon=\kappa_\varepsilon$ which implies that

$$k_\varepsilon=\frac{k_\varepsilon^2}{k_\varepsilon^2+\theta}$$

as before.

A.7 Proof of Proposition 8

We now assume that investors do not perfectly observe X_{t-1}. Instead they observe

$$s_{t-1}=X_{t-1}+z_{t-1}$$

where $z_{t-1}\sim N\left(0,\sigma_z^2\right)$. We assume that investors observe s_{t-1} before i_t, so that the expectation of X_{t-1} given s_{t-1},

$$E\left[X_{t-1}|s_{t-1}\right]=\frac{Cov\left(X_{t-1},s_{t-1}\right)}{Var\left(s_{t-1}\right)}s_{t-1}=\frac{\sigma_X^2}{\sigma_X^2+\sigma_z^2}s_{t-1},$$

is already impounded into the infinite horizon forward rate before the Fed picks i_t.Thus, the change in the infinite horizon forward rate at time t is the revision in the market's expectations about the target rate given the change in the federal funds rate:

$$\begin{aligned}\Delta i_t^\infty &= E\left[i_t^*|\Delta i_t,s_{t-1}\right]-E\left[i_{t-1}^*|s_{t-1}\right]\\ &= E\left[X_{t-1}+\varepsilon_t|\Delta i_t,s_{t-1}\right]-E\left[X_{t-1}|s_{t-1}\right]=\chi_i\Delta i_t-\chi_s s_{t-1}\end{aligned}$$

where computation of the conditional expectations shows that

$$\begin{aligned}\chi_i &= \frac{\kappa_\varepsilon\sigma_\varepsilon^2\left(\sigma_X^2+\sigma_z^2\right)+\kappa_X\sigma_X^2\sigma_z^2}{\kappa_\varepsilon^2\sigma_\varepsilon^2\left(\sigma_X^2+\sigma_z^2\right)+\kappa_X^2\sigma_X^2\sigma_z^2}\\ \chi_s &= \frac{\kappa_\varepsilon\left(\kappa_X-\kappa_\varepsilon\right)\sigma_\varepsilon^2\sigma_X^2}{\kappa_\varepsilon^2\sigma_\varepsilon^2\left(\sigma_X^2+\sigma_z^2\right)+\kappa_X^2\sigma_X^2\sigma_z^2}+\frac{\sigma_X^2}{\sigma_X^2+\sigma_z^2}.\end{aligned}$$

Note that when $\sigma_z^2=0$, we have

$$\begin{aligned}\chi_i &= \frac{1}{\kappa_\varepsilon}\\ \chi_s &= \frac{\kappa_X}{\kappa_\varepsilon}\end{aligned}$$

so that

$$\chi_i\left(k_\varepsilon\varepsilon_t+k_X X_{t-1}\right)-\chi_s X_{t-1}=\varepsilon_t$$

in rational expectations and we are back to the expressions we had in Proposition 7 where investors observed X_{t-1} with no noise. In addition, note that when $\sigma_z \to \infty$, we have

$$\begin{aligned} \chi_i &\to \frac{\kappa_\varepsilon \sigma_\varepsilon^2 + \kappa_X \sigma_X^2}{\kappa_\varepsilon^2 \sigma_\varepsilon^2 + \kappa_X^2 \sigma_X^2} \\ \chi_s &\to 0 \end{aligned}$$

so investors put no weight on the signal s_{t-1}.

We retain the timing convention used in Proposition 7, so that the Fed picks k_X and k_ε and then ε_t and z_t are realized. The loss function is then

$$\begin{aligned} E_t\left[(i_t^* - i_t)^2 + \theta\left(\Delta i_t^\infty\right)^2\right] &= E_t\left[\left(i_{t-1}^* + \varepsilon_t - i_{t-1} - k_X \cdot X_{t-1} - k_\varepsilon \cdot \varepsilon_t\right)^2 + \theta\left(\Delta i_t^\infty\right)^2\right] \\ &= \left(1 - k_X\right)^2 X_{t-1}^2 + \left(1 - k_\varepsilon\right)^2 \sigma_\varepsilon^2 \\ &\quad + \theta\left(\left(k_X \chi_i - \chi_s\right)^2 X_{t-1}^2 + \chi_i^2 k_\varepsilon^2 \sigma_\varepsilon^2 + \chi_s^2 \sigma_z^2\right) \end{aligned}$$

Differentiating with respect to k_X yields

$$k_X = \frac{1 + \theta \chi_i \chi_s}{1 + \theta \chi_i^2}$$

For $\theta > 0$, this implies $k_X < 1$ since $\chi_i > \chi_s$.

Differentiating with respect to k_ε yields

$$k_\varepsilon = \frac{1}{1 + \theta \chi_i^2}$$

This implies that $k_X > k_\varepsilon$ and $k_X = k_\varepsilon$ when $\chi_s = 0$, which happens as $\sigma_z \to \infty$.

When $\sigma_z^2 \to \infty$, we have $k_X = k_\varepsilon$ so that

$$\begin{aligned} k_X &= \frac{\left(k_\varepsilon^2 \sigma_\varepsilon^2 + k_X^2 \sigma_X^2\right)^2}{\left(k_\varepsilon^2 \sigma_\varepsilon^2 + k_X^2 \sigma_X^2\right)^2 + \theta\left(k_\varepsilon \sigma_\varepsilon^2 + k_X \sigma_X^2\right)^2} \\ &= \frac{1}{1 + \theta\left(\frac{1}{k_X}\right)^2} \end{aligned}$$

which is our baseline expression in the static model. Thus, when $\sigma_z^2 \to \infty$ the Fed moves equally slowly over X_{t-1} and ε_t and the speed is given by the same expression as in the static model with no noise.