Structural GARCH: The Volatility-Leverage Connection

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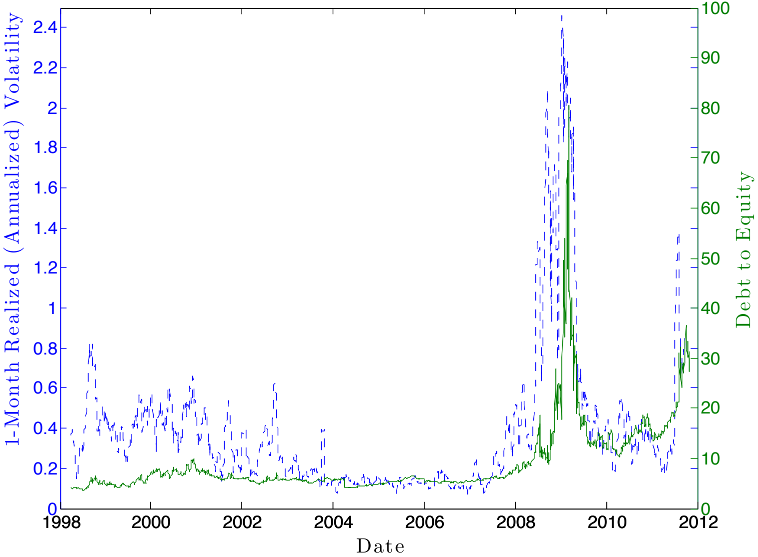
Introduction

BAC Leverage and Realized Volatility



Leverage and Equity Volatility

- Crisis highlighted how leverage and equity volatility are tightly linked
- "Leverage Effect" has been around - e.g. Black (1976), Christie (1982) - but...
- A dynamic volatility model that incorporates leverage directly has remained elusive

This Paper

- GARCH-type model where equity volatility is amplified (non-linearly) by leverage as in structural models of credit
- **Asset** return series from observed equity series
- **Assets** have time-varying volatility at high frequencies
- Statistical test of how leverage affects volatility
- Two applications:
  1. Systemic Risk: SRISK and Precautionary Capital (today)
  2. Leverage Effect (in the paper)

Theoretical Foundation

Structural Models of Credit

- Under relatively weak assumptions on the vol process, structural models say $E_t = f\left(A_t, D_t, \sigma_{A,t}, \tau, r_t\right)$
  - A_t = market value of assets
  - D_t = book value of debt
  - $\sigma_{A,t}$ = stochastic asset volatility
- Generic dynamics for assets and asset variance (allow for jumps later):

$$\frac{dA_t}{A_t} = \mu_A(t)dt + \sigma_{A,t}dB_A(t)$$
$$d\sigma^2_{A,t} = \mu_v(t, \sigma_{A,t})dt + \sigma_v(t, \sigma_{A,t})dB_v(t)$$

- $B_A(t)$ and $B_v(t)$ potentially correlated

Equity Returns and Equity Volatility

Introducing the Leverage Multiplier

- Apply Itō Lemma and ignore drift (our model is daily, and daily equity returns ≈ 0):

$$\frac{dE_t}{E_t} = LM_t \sigma_{A,t} dB_A(t) + \frac{v_t}{E_t} \frac{\sigma_v(t, \sigma_{A,t})}{2\sigma_{A,t}} dB_v(t)$$

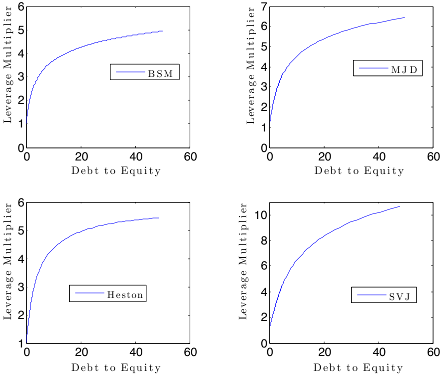
$$\approx LM_t \times \sigma_{A,t} \times dB_A(t)$$

$$vol_t\left(\frac{dE_t}{E_t}\right) \approx LM_t \times \sigma_{A,t}$$

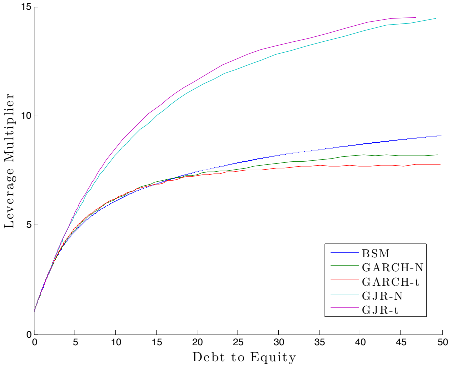
- $LM_t = LM\left(E_t/D_t, 1, \sigma_{A,t}, \tau, r_t\right)$ is the "leverage multiplier"
- LM_t amplifies asset shocks and volatility
- Two questions:
  1. How much does the higher order term contribute? Not Much
  2. What does LM_t look like? Robust shape across models

The Leverage Multiplier: Three Basic Properties

Popular Continuous Time Option Pricing Models



Discrete Time: GARCH Option Pricing



1. $LM(0) = 1$. Mechanical, since assets = equity
2. Monotonically increasing. More leverage means more risk
3. Concave. Reducing leverage more powerful than increasing leverage

Structural GARCH

Our Specification

- The challenge is choosing the right functional form for LM_t
- We use simple transformations of Black-Scholes-Merton (BSM) functions:

$$LM_t(D_t/E_t, \sigma^f_{A,t}, \tau) = \left[\triangle_t^{BSM} \times g^{BSM}\left(E_t/D_t, 1, \sigma^f_{A,t}, \tau\right) \times \frac{D_t}{E_t} \right]^{\phi}$$

  $g^{BSM}(\cdot)$ is inverse BSM call function. Δ_t^{BSM} is BSM delta
- $\phi \neq$ specific option pricing model
- Our parametrization preserves necessary properties of LM, but still allows us to change its scale

The Full Recursive Model

Structural GARCH

$$
\begin{aligned}
r_{E,t} &= LM_{t-1} \times r_{A,t} \\
&= LM_{t-1} \times \sqrt{h_{A,t}} \times \varepsilon_{A,t} \\
h_{A,t} &\sim GJR(\omega, \alpha, \gamma, \beta) \\
LM_{t-1} &= \left[\triangle_{t-1}^{BSM} \times g^{BSM} \left(E_{t-1}/D_{t-1}, 1, \sigma_{A,t-1}^{f}, \tau \right) \times \frac{D_{t-1}}{E_{t-1}} \right]^{\phi} \\
\sigma_{A,t-1}^{f} &= \sqrt{\mathbb{E}_{t-1} \left[\sum_{s=t}^{t+\tau} h_{A,s} \right]}
\end{aligned}
$$

So parameter set is $\Theta = (\omega, \alpha, \gamma, \beta, \phi)$

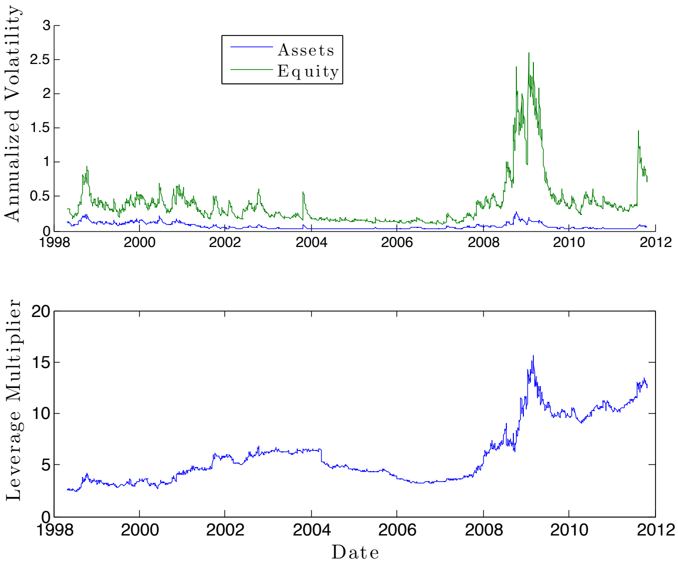
Estimation Results

Estimation Details

- Estimate for 82 financials via QMLE; iterate over $\tau \in [1, 30]$
- Equity returns and balance sheet information from Bloomberg
- D_t is exponentially smoothed book value of debt
  - smoothing parameter = 0.01, so half-life of weights ≈ 70 days
- We estimate the model using two approaches for $\sigma^f_{A,t-1}$, then use the highest likelihood:
  1. A dynamic forecast for asset volatility over life of the option
  2. The unconditional volatility of the asset GJR process

Bank of America: Structural GARCH Estimation

$\phi = 1.4 \quad (t = 11.4)$



Parameter Values

Cross-Sectional Summary of Estimated Parameters

Parameter	**Mean**	**Mean t-stat**	**% with $\|t\| > 1.64$**
ω	2.7e-06	1.70	47.2
α	0.0458	3.07	86.1
γ	0.0721	2.91	80.6
β	0.9024	80.08	100
ϕ	0.9834	4.00	73.6

- $(\omega, \alpha, \gamma, \beta)$ are standard GJR parameters - for assets, not equity
- Average $\tau = 8.34$
- Leverage matters

Application: SRISK

SRISK

How much would a financial firm need to function normally in another crisis?

- Acharya et. al (2012) and Brownlees and Engle (2012)
- Three steps:
  1. GJR-DCC model using firm equity and market index returns
  2. Expected firm equity return if market falls by 40% over 6 months $\equiv$ LRMES
  3. Combine LRMES with book value of debt to determine capital shortfall in a crisis
- The crisis in this case is a 40% drop in the stock market index over 6 months

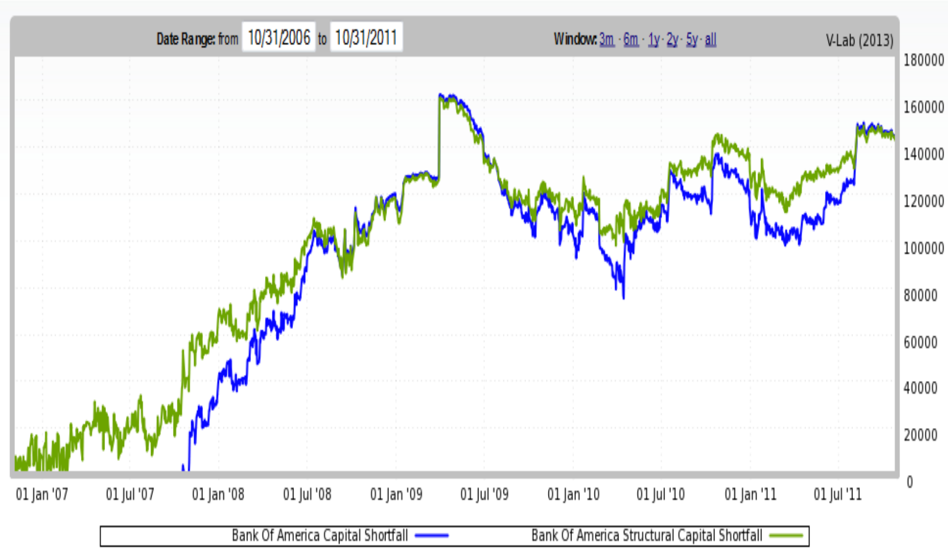
The Role of Leverage?

Thought Experiment with Structural GARCH

- Firm experiences sequence of negative equity (asset) shocks
- Level of leverage goes up rapidly
- Leverage multiplier increases, equity vol amplification higher
- Painfully obvious in the crisis, so build into SRISK

Bank of America

Capital Shortfall: 2006-2011



Precautionary Capital

Defining Precautionary Capital

e.g. How much additional equity would a bank need, today, to be 90% sure they won't need bailout money in a future crisis?

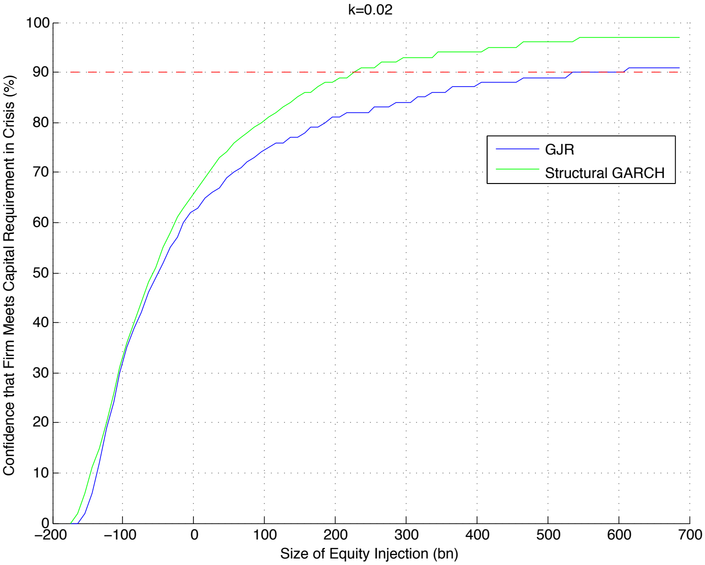
- SRISK: how much capital would a firm need in a financial crisis to return to a equity/asset ratio of k%?
- Precautionary Capital: How much capital do we have to add to the firm *today* so that we can have a level of certainty, α, that the firm meets a capital requirement of k% in a crisis?
- Uses the quantiles of the future return distribution
- We set $k = 2\%$ and vary α

Primary Takeaway in a Nutshell

- Standard volatility models don't have a channel for leverage, so adding equity to the firm today won't reduce future risk
- Structural GARCH: reducing leverage today reduces future risk
  - The effect is further enhanced by the concavity of the LM
- Engle and Siriwardane (2014) use this idea to suggest a risk-based total leverage capital requirement

Precautionary Capital: BAC

BAC on 10/1/2008: $E_0 = 173.9$ bn; $D_0 = 1,670.1$ bn



What’s Next

Other Applications

- Endogenous Crisis Probability with Structural GARCH
- Estimation of Distance to Crisis
- Endogenous Capital Structure and Leverage Cycles
- Counter-cyclical Capital Regulation
- Model of CDS Volatility

Appendix

Ignore Higher Order Terms

$$\frac{dE_t}{E_t} = LM_t \sigma_{A,t} dB_A(t) + \frac{\nu_t}{E_t} \frac{\sigma_v(t, \sigma_{A,t})}{2\sigma_{A,t}} dB_v(t)$$

How much do the higher order terms contribute?

- Not much. Simple intuition...
- Volatility mean reversion speed ≪ typical debt maturities, so ...
- Total volatility over option is effectively constant
- We verify in paper for variety of option pricing models

Back

Dynamic Forecast vs Constant Forecast

- Estimate two types of models:
  1. Using a dynamic forecast for asset volatility over life of the option
  2. Using unconditional volatility of GJR process
- Then take the model that delivers the highest likelihood
- A few outliers where ϕ hits lower bound (exclude from subsequent analysis):
  - SCHW, JNS, LM, BK, BLK, NTRS, CME, CINF, TMK, UNH