Appendix of supplementary materials for:

# RATE-AMPLIFYING DEMAND AND THE EXCESS SENSITIVITY OF LONG-TERM RATES

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## Contents

# A Additional empirical results

This Appendix collects several supplementary empirical results that are mentioned in the main text.

## A.1 Additional empirical results for Section 1

### A.1.1 Long-term private yields

Table A.1 shows that we obtain very similar results for the U.S. using a host of long-term private yields as the dependent variable in equation (1.1). We report results for both Aaa and Baa seasoned corporate bond yields from Moodys, the 10-year swap yield, and the yield on current coupon Fannie Mae MBS (FNCL). For of all these long-term private yields, the sensitivity to changes in 1-year Treasury rate is similar at high- and low- frequencies in the pre-2000 sample. Post-2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines significantly.

### A.1.2 Different short-term rates

Table A.2 shows that similar results for the U.S. hold using different proxies for the short rate in equation (1.1) of the main text—i.e., using changes in 3-month, 6-month, or 2-year Treasury yields as the independent variable in equation (1.1). (The results in Table 1 of main the text correspond to those reported in Table A.2 for the 1-year Treasury rate.) For all of these short rate proxies, the sensitivity of 10-year yields to changes in short rates was similar irrespective of frequency prior to 2000. After 2000, the sensitivity at high frequencies increases while the sensitivity at low frequencies declines.

**Table A.1: Regression of changes in corporate bond, swap and secondary mortgage market rates on short-term rates.** This table reports the estimated slope coefficients from equation (1.1) in the main text for each reported sample. Specifically, the dependent variables are long-term corporate bond yields with Moody's ratings of Baa and Aaa (labled BAA and AAA), the 10-year swap yield (SWAP10), and the yield on current-coupon Fannie Mae mortgage-backed-securities (FNCL). The independent variable in all regressions is the change in the 1-year nominal Treasury yield. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. We report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $1.5\times(h-1)$ (rounded to the nearest integer). Significance: $^{*}p < 0.1$, $^{**}$ $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

| | (1) BAA | (2) AAA | (3) SWAP10 | (4) FNCL | (5) BAA | (6) AAA | (7) SWAP10 | (8) FNCL |
|---|---|---|---|---|---|---|---|---|
| Daily | 0.48*** | 0.51*** | 0.73*** | 0.95*** | 0.56*** | 0.58*** | 0.95*** | 0.90*** |
| | [0.02] | [0.02] | [0.04] | [0.04] | [0.04] | [0.04] | [0.03] | [0.04] |
| Monthly | 0.52*** | 0.59*** | 0.82*** | 0.92*** | 0.24* | 0.37*** | 0.83*** | 0.75*** |
| | [0.05] | [0.05] | [0.07] | [0.06] | [0.13] | [0.10] | [0.10] | [0.10] |
| Quarterly | 0.49*** | 0.57*** | 0.80*** | 0.84*** | 0.08 | 0.24*** | 0.63*** | 0.54*** |
| | [0.06] | [0.06] | [0.07] | [0.07] | [0.12] | [0.07] | [0.09] | [0.09] |
| Yearly | 0.41*** | 0.55*** | 0.67*** | 0.72*** | -0.01 | 0.11 | 0.39*** | 0.34*** |
| | [0.08] | [0.09] | [0.07] | [0.10] | [0.12] | [0.06] | [0.05] | [0.07] |
| Sample | 1986-1999 | 1983-1999 | 1988-1999 | 1984-1999 | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 |

**Table A.2: Regressions of changes in long-term rates on short-term rates.** This table reports the estimated regression coefficients from equation (1.1) in the main text for each reported sample. The dependent variable is the change in the 10-year nominal U.S. Treasury yield. The independent variable is alternately the change in the 3-month, 6-month, 1-year, and 2-year nominal U.S. Treasury yield. Changes are considered with daily data, and with monthly data using monthly ($h = 1$) and annual ($h = 12$) horizons. In the 1971-1999 monthly sample, time $t$ runs from 1971m8 to 1999m12. In the 2000-2019 monthly sample, $t$ runs from 2000m1 to 2019m12. For $h > 1$, we report Newey-West (1987) standard errors are in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

| | Pre-2000 | | | Post-2000 | | |
|---|---|---|---|---|---|---|
| | (1) Daily | (2) Monthly | (3) Annual | (4) Daily | (5) Monthly | (6) Annual |
| Short rate = 3-month | 0.16*** | 0.26*** | 0.39*** | 0.27*** | 0.24** | 0.17*** |
| | [0.02] | [0.03] | [0.05] | [0.02] | [0.10] | [0.04] |
| Short rate = 6-month | 0.41*** | 0.37*** | 0.46*** | 0.65*** | 0.45*** | 0.18*** |
| | [0.02] | [0.03] | [0.06] | [0.03] | [0.11] | [0.05] |
| Short rate = 1-year | 0.56*** | 0.46*** | 0.56*** | 0.87*** | 0.66*** | 0.23*** |
| | [0.02] | [0.04] | [0.05] | [0.03] | [0.11] | [0.05] |
| Short rate = 2-year | 0.65*** | 0.57*** | 0.68*** | 0.87*** | 0.78*** | 0.34*** |
| | [0.01] | [0.04] | [0.05] | [0.02] | [0.09] | [0.07] |
| Observations | 7062 | 341 | 341 | 5002 | 239 | 228 |

### A.1.3 Hansen-Hodrick standard errors

One might be concerned about our use of overlapping changes in equations (1.1) and (1.2) when $h > 1$. In the main text, we computed Newey and West (1987) standard errors with a lag truncation parameter of $\lceil 1.5 \times h \rceil$. And, to address the tendency for statistical tests based on Newey and West (1987) standard errors to over-reject in finite samples, we compute $p$-values using the asymptotic theory of Kiefer and Vogelsang (2005) which gives more conservative $p$-values and has better finite-sample properties than traditional Gaussian asymptotic theory.

Table A.3 report the results from estimating equations (1.1) and (1.2) when $h > 1$, but instead using Hansen and Hodrick (1980) standard errors with a lag truncation parameter equal to $h$. Hansen and Hodrick (1980) standard errors are sometimes preferred when working with overlapping data, although they have the disadvantage that, unlike the Newey and West (1987) counterpart, the estimated variance-covariance matrix is not guaranteed to be positive-definite. The point estimates are identical by construction to those in Table 1 of the paper and the standard errors are nearly identical to those reported in the main text. Thus, we would draw almost identical inferences using Hansen and Hodrick (1980) standard errors instead of Newey and West (1987) standard errors.

**Table A.3: Regressions of changes in long-term rates on short-term rates with Hansen-Hodrick (1980) standard errors**. This table reports the estimated regression coefficients from equations (1.1) and (1.2) for $h > 1$. We report Hansen and Hodrick (1980) with lag truncation parameter of $h$. By construction the point estimates are the same as in Table 1. Results are not included for daily or monthly data, because there are no overlapping changes at those frequencies. Significance: $^{*}p < 0.1$, $^{**}$ $p < 0.05$, $^{***}p < 0.01$.

| Panel A. Ten-year zero coupon yields and IC | | | | |
|---|---|---|---|---|
| | Nominal | Nominal | Real | IC |
| Quarterly | 0.48*** | 0.44*** | 0.22** | 0.22 |
| | [0.04] | [0.07] | [0.11] | [0.14] |
| Semi-Annual | 0.50*** | 0.34*** | 0.21** | 0.13 |
| | [0.05] | [0.08] | [0.09] | [0.10] |
| Yearly | 0.56*** | 0.23*** | 0.15** | 0.08* |
| | [0.06] | [0.05] | [0.06] | [0.04] |
| **Panel B. Ten-year instantaneous forward yields and IC** | | | | |
| | Nominal | Nominal | Real | IC |
| Quarterly | 0.31*** | 0.06 | 0.09* | -0.03 |
| | [0.05] | [0.09] | [0.05] | [0.05] |
| Semi-Annual | 0.33*** | -0.02 | 0.04 | -0.06 |
| | [0.06] | [0.08] | [0.04] | [0.05] |
| Yearly | 0.39*** | -0.13 | -0.02 | -0.11** |
| | [0.07] | [0.06] | [0.05] | [0.05] |
| Sample | 1971-1999 | 2000-2019 | 2000-2019 | 2000-2019 |

### A.1.4 Estimates using non-overlapping data

Here we show the estimates and our inferences are quite similar if we simply use non-overlapping $h$-month changes. Consider our estimates of $\beta_{12}$—the coefficient from a regression involving 12-month changes—where concerns about the use of overlapping data are greatest. Table A.4 reports the results for estimating equations (1.1) and (1.2) with non-overlapping data for each possible month in the year—i.e., a regression in changes from month $m$ of one year to month $m$ of the next year—along with the averages across all 12 months. Naturally, the results differ slightly across the individual months. However, averaging across all 12 months, in the pre-2000 data, the regression has an average $\beta_{12} = 0.561$ with an average robust standard error of 0.076; in the post-2000 data, the regression has an average point estimate of $\beta_{12} = 0.222$ with an average standard error of 0.088. Thus, as one would expect, the average point estimate across these 12 separate estimators that each use non-overlapping changes are almost identical to our baseline estimator that use overlapping changes reported in Table 1 of the main text. (In Table 1, we have $\beta_{12} = 0.557$ in the pre-2000 period and $\beta_{12} = 0.230$ in the post-2000 period.) However, the average of the standard errors across these 12 estimators is larger than the Newey-West standard error from our baseline estimates which are 0.052 in the pre-2000 period and 0.053 post-2000. We attribute the larger average standard error to the loss of efficiency from discarding some of the data. Specifically, the standard error of the average of the 12 estimates will be less than average of the 12 standard errors to the extent the 12 estimates are imperfectly correlated.

Nonetheless, we draw similar conclusions even using this highly conservative approach that deliberately discards data. The difference between the resulting pre-2000 and post-2000 estimates of $\beta_{12}$ in yields are significant at the 5% level for 10 of the 12 monthly estimators using non-overlapping changes. And the difference between the two estimates of $\beta_{12}$ in forward rates are significant at the 5% level for all 12 of the monthly estimators using non-overlapping changes.

### A.1.5 U.S. evidence prior to the Great Inflation

Our baseline findings on the sensitivity of long-term U.S. Treasury yields to short-term Treasury yields use data beginning in 1971. Data on the Treasury term structure prior to the 1970s is far more limited because the U.S. Treasury did not regularly issue large quantities of debt at fixed, long-term maturities (Gürkaynak et al., 2007). Nevertheless, it is useful to examine the sensitivity of long-term yields to short-term yields prior to the Great Inflation, which ran from the late-1960s to the mid-1980s. Specifically, one plausible explanation for the strong sensitivity of long-term nominal yields during the 1971-1999 sample is that this was a period when long-run inflation expectations became unanchored and were continuously being revised in response to news (Gürkaynak et al., 2005). Since inflation-expectations have become firmly anchored in the past two decades, it is useful to compare the patterns we see in the post-2000 data to the those witnessed prior to the Great Inflation—another period when inflationary expectations were also more firmly anchored.

To examine the sensitivity of long-term yields prior to the Great Inflation, we use data on 10-year and 1-year Constant Maturity Treasury (CMT) yields from the Federal Reserve's H.15 statistical release, which are available on a monthly basis dating back to Apr 1953. As in equation (1.1), we use these data to estimate the coefficient $\beta_h$ from a regression of $h$-month changes in the 10-year CMT yield on $h$-month changes in the 1-year CMT yield. Our pre-Great Inflation sample ends in Aug-1968, since this is when standard measures of long-term inflation expectations began to drift up in the U.S.[1] For comparison, we also show the analogous $\beta_h$ coefficients obtained from regressions using 1- and 10-year CMT yields for the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples we considered in the main text.

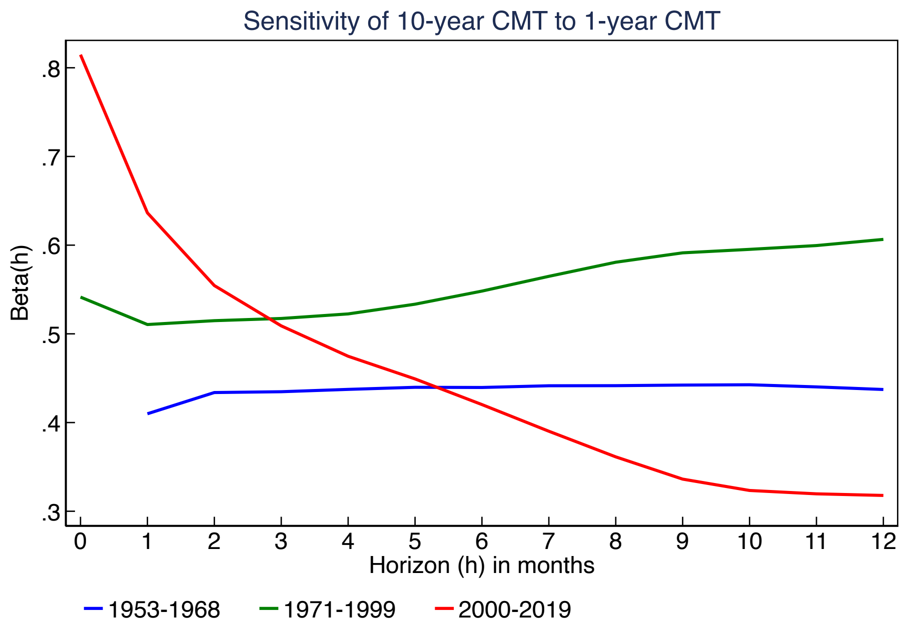
The results are shown in Figure A.1. The $\beta_h$ coefficients for the 1971-1999 and 2000-2019 subsamples using CMT yields are very similar to those shown in Figure 1 for zero-coupon yields. The $\beta_h$ coefficients for the 1953-1968 sample are around 0.44 and largely independent of the horizon $h$. The pre-1968 coefficients are lower than those in the 1971-2000 subsample, which hover around

[1]The results are similar if we end the analysis in July 1971, the month before our baseline analysis in Table 1 begins.

**Table A.4: Regressions of 12-month changes in long rates on 12-month changes in short-rate using non-overlapping data** This Table reports the estimate of $\beta_{12}$ from estimating equations (1.1) and (1.2) at the annual frequency ($h = 12$) but using non-overlapping 12-month changes from each possible month of the year to the same month in the following year. Heteroskedasticity robust standard errors are show in brackets. We also report the $p$-value from a test of the hypothesis that the pre-2000 and post-2000 estimates of $\beta_{12}$ are equal. Significance: $^{*}p < 0.1$, $^{**}$ $p < 0.05$, $^{***}p < 0.01$.

| | Yields | | | Forwards | | |
|---|---|---|---|---|---|---|
| Month | Pre-2000 | Post-2000 | $p-$val | Pre-2000 | Post-2000 | $p-$val |
| Jan | 0.57*** | 0.22** | 0.00 | 0.38*** | -0.14* | 0.00 |
| | [0.07] | [0.09] | | [0.10] | [0.07] | |
| Feb | 0.56*** | 0.21** | 0.00 | 0.43*** | -0.19** | 0.00 |
| | [0.08] | [0.09] | | [0.12] | [0.07] | |
| Mar | 0.48*** | 0.29*** | 0.21 | 0.38** | -0.14* | 0.00 |
| | [0.10] | [0.08] | | [0.14] | [0.07] | |
| Apr | 0.57*** | 0.23** | 0.01 | 0.39*** | -0.15 | 0.00 |
| | [0.09] | [0.09] | | [0.12] | [0.10] | |
| May | 0.48*** | 0.27** | 0.16 | 0.29** | -0.11 | 0.02 |
| | [0.08] | [0.12] | | [0.11] | [0.14] | |
| Jun | 0.59*** | 0.30** | 0.03 | 0.40*** | -0.04 | 0.01 |
| | [0.06] | [0.12] | | [0.08] | [0.15] | |
| Jul | 0.47*** | 0.21** | 0.01 | 0.24*** | -0.17 | 0.00 |
| | [0.06] | [0.08] | | [0.07] | [0.10] | |
| Aug | 0.56*** | 0.22** | 0.00 | 0.41*** | -0.13 | 0.00 |
| | [0.07] | [0.08] | | [0.07] | [0.11] | |
| Sep | 0.65*** | 0.26*** | 0.00 | 0.49*** | -0.08 | 0.00 |
| | [0.06] | [0.06] | | [0.09] | [0.08] | |
| Oct | 0.65*** | 0.22** | 0.00 | 0.49*** | -0.12 | 0.00 |
| | [0.07] | [0.09] | | [0.09] | [0.14] | |
| Nov | 0.53*** | 0.14** | 0.00 | 0.32** | -0.16** | 0.00 |
| | [0.11] | [0.06] | | [0.13] | [0.07] | |
| Dec | 0.62*** | 0.19 | 0.00 | 0.45*** | -0.08 | 0.00 |
| | [0.07] | [0.13] | | [0.10] | [0.13] | |
| Average | 0.56 | 0.22 | 0.03 | 0.39 | -0.14 | 0.00 |
| | [0.08] | [0.09] | | [0.10] | [0.10] | |
| Sample | 1971-1999 | 2000-2019 | | 1971-1999 | 2000-2019 | |

**Figure A.1: Pre-Great Inflation regressions of changes in long-term yields on short-term rates.** This figure plots the estimated regression coefficients $\beta_h$ from equation (1.1) versus horizon ($h$) for the Apr1953/Aug1968, Aug1971/Dec-1999, and Jan-2000/Dec201 9 subsamples: $y^{(10)}_{t+h} - y^{(10)}_t = \alpha_h + \beta_h(y^{(1)}_{t+h} - y^{(1)}_t) + \varepsilon_{t,t+h}$. The dependent variable is the $h$-month change in the 10-year nominal Constant Maturity Treasury yield and the independent variable is the $h$-month change in the 1-year nominal Constant Maturity Treasury yield. Changes are considered with daily data (plotted as $h = 0$ in the figure) and with monthly data using $h = 1$, ..., 12-month changes. Daily data on Constant Maturity Treasury yields is not available until 1962, so $\beta_{day}$ is missing for the Apr1953/Aug1968 subsample.




0.55. This lower level is consistent with the view that inflation expectations were better-anchored in the pre-1968 sample, although perhaps not as stable as in the post-2000 sample. However, while the level of the $\beta_h$ coefficients is lower in the pre-1968 sample, we also do not observe the strong dependence on horizon that is so evident in the post-2000 data. In summary, while the unanchoring and then reanchoring of long-run inflation expectations may help explain shifts in the *level* of $\beta_h$ over time, the strongly *frequency-dependent* sensitivity of long-term rates that we see in the post-2000 subsample appears to be something new under the sun.

### A.1.6 International evidence

Our focus is on the U.S., but it is useful to consider whether these same patterns are also observed in other large, highly-developed economies. In Table A.5, we briefly explore evidence for the U.K., Germany, and Canada. Panel A of Table A.5 shows estimates of equation (1.1) for the U.K., where data is available beginning in 1985. For the U.K., the estimates are broken out into real yields and inflation compensation. The evidence for the U.K. is remarkably similar to the U.S. evidence in Table 1. Before 2000, the daily coefficient ($\beta_{day} = 0.44$) and the yearly coefficient ($\beta_{12} = 0.38$) are similar in the U.K. After 2000, the daily sensitivity increases ($\beta_{day} = 0.89$), and the yearly sensitivity declines ($\beta_{12} = 0.29$). Because we have data on real yields prior to 2000 in the U.K., we can decompose the change in $\beta_h$ into its real and inflation compensation components. As shown in Table A.5, the inflation compensation component of $\beta_h$ is stable across sample periods and frequency $h$. Thus, most of the changes in $\beta_h$ are accounted for by changes in the real component of nominal yields.

Panel B of Table A.5 shows estimates of equation (1.1) for Germany and Canada. For Germany, monthly data is available beginning in 1972 and daily data is available starting in 2000. For Canada, monthly and daily data are available beginning in 1986. Again, we observe similar patterns to those in the U.S. In the pre-2000 sample, $\beta_h$ is stable across frequencies in Germany and Canada. After 2000, we observe greater sensitivity at high frequencies and less sensitivity at lower frequencies.

We also consider estimates of equation (1.1) for emerging market countries. Specifically, we look at changes in long-term and short-term yields for Mexico, South Africa, and the Philippines. The results are shown in Table A.6. The sample period is 2000-2019 because there was very little local-currency long-term debt in emerging markets before 2000. The results show a fairly flat coefficient $\beta_h$ and one that, if anything, rise with $h$. At long horizons, the coefficient is bigger than in advanced economies. This suggests that these emerging countries have higher long-run inflation uncertainty but that their bond yields are not highly affected by temporary rate-amplifying demand shocks.

#### A.1.7 Rolling estimation of $\beta_{12} - \beta_1$

Here we use an alternate procedure to date the break in the sensitivity of long-term rates to movements in short-term rates. As opposed to focusing simply on the break the low-frequency sensitivity, $\beta_{12}$, here we seek to date the emergence of the *frequency-dependent* sensitivity of long-term rates by examining $\beta_{12} - \beta_1$. To do so, we estimate

$$y_{t+1}^{(10)} - y_t^{(10)} = \alpha_1 + \beta_1 (y_{t+12}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+1} \quad \text{(A.1)}$$

$$y_{t+12}^{(10)} - y_t^{(10)} = \alpha_{12} + \beta_{12} (y_{t+12}^{(1)} - y_t^{(1)}) + \varepsilon_{t,t+12} \quad \text{(A.2)}$$

as a joint system using 10-year rolling windows and report $\beta_{12} - \beta_1$. These rolling estimates of $\beta_{12} - \beta_1$ are shown in Figure (A.2) below. As shown, $\beta_{12} - \beta_1$ is positive is earlier windows, hovers just below zero from the late 1980s and to the late 1990s, before turning significantly negative around 2000.

**Table A.5: Regressions of changes in long-term international rates on short-term rates.** This table reports the estimated regression coefficients from equation (1.1) for the United Kingdom (UK), Germany (DE), and Canada (CAN) on each reported sample. We obtain data on each country's zero-coupon government bond yield curve from each country's central bank website. The dependent variable is the change in the 10-year zero-coupon yield, either nominal, real, or their difference—i.e., inflation compensation (IC). The independent variable is the change in the 1-year nominal yield in all cases. Changes are considered with daily data, and with monthly data using monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. For $h > 1$, we report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, $^{**}\ p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

**Panel A:** UK 10-year zero-coupon yields

| | (1) Nominal | (2) Nominal | (3) Real | (4) Real | (5) IC | (6) IC |
|---|---|---|---|---|---|---|
| Daily | 0.44*** | 0.89*** | 0.14*** | 0.67*** | 0.29*** | 0.22*** |
| | [0.04] | [0.03] | [0.01] | [0.03] | [0.04] | [0.02] |
| Monthly | 0.47*** | 0.56*** | 0.19*** | 0.15 | 0.28*** | 0.41*** |
| | [0.06] | [0.13] | [0.04] | [0.23] | [0.08] | [0.14] |
| Quarterly | 0.49*** | 0.43*** | 0.23*** | 0.06 | 0.26*** | 0.38*** |
| | [0.08] | [0.11] | [0.04] | [0.17] | [0.10] | [0.09] |
| Semi-annual | 0.45*** | 0.39*** | 0.22*** | 0.07 | 0.23** | 0.32*** |
| | [0.09] | [0.08] | [0.05] | [0.11] | [0.11] | [0.06] |
| Yearly | 0.38*** | 0.29*** | 0.16** | 0.06 | 0.22*** | 0.23*** |
| | [0.06] | [0.07] | [0.06] | [0.08] | [0.08] | [0.03] |
| Sample | 1985-1999 | 2000-2019 | 1985-1999 | 2000-2019 | 1985-1999 | 2000-2019 |

**Panel B:** German and Canadian 10-year nominal zero-coupon yields

| | (1) DE | (2) DE | (3) CAN | (4) CAN |
|---|---|---|---|---|
| Daily | | 0.66*** | 0.42*** | 0.71*** |
| | | [0.03] | [0.03] | [0.03] |
| Monthly | 0.34*** | 0.50*** | 0.46*** | 0.53*** |
| | [0.05] | [0.10] | [0.05] | [0.08] |
| Quarterly | 0.41*** | 0.45*** | 0.51*** | 0.39*** |
| | [0.04] | [0.07] | [0.05] | [0.05] |
| Semi-annual | 0.41*** | 0.41*** | 0.50*** | 0.29*** |
| | [0.04] | [0.08] | [0.07] | [0.06] |
| Yearly | 0.43*** | 0.33*** | 0.43*** | 0.16** |
| | [0.04] | [0.10] | [0.08] | [0.07] |
| Sample | 1972-1999 | 2000-2019 | 1986-1999 | 2000-2019 |

**Figure A.2: Rolling regression estimates of** $\beta_{12} - \beta_1$ This figure plots rolling estimates of $\beta_{12} - \beta_1$ using 10-year rolling windows for estimation. Results are plotted against the midpoint of the 10-year rolling window. 95% confidence intervals are included (shaded areas). To conduct inference on $\beta_{12} - \beta_1$, we estimate these two regression equations jointly as a system using the Generalized Method of Moments (GMM). Standard errors for $\beta_{12} - \beta_1$ are formed using Newey-West standard errors with a lag truncation parameter of 18 and 95% critical values from the asymptotic theory of Kiefer and Vogelsang (2005). Specifically, the 95% confidence interval is ±2.41 times the estimated standard errors as opposed to ±1.96 under traditional asymptotic theory.

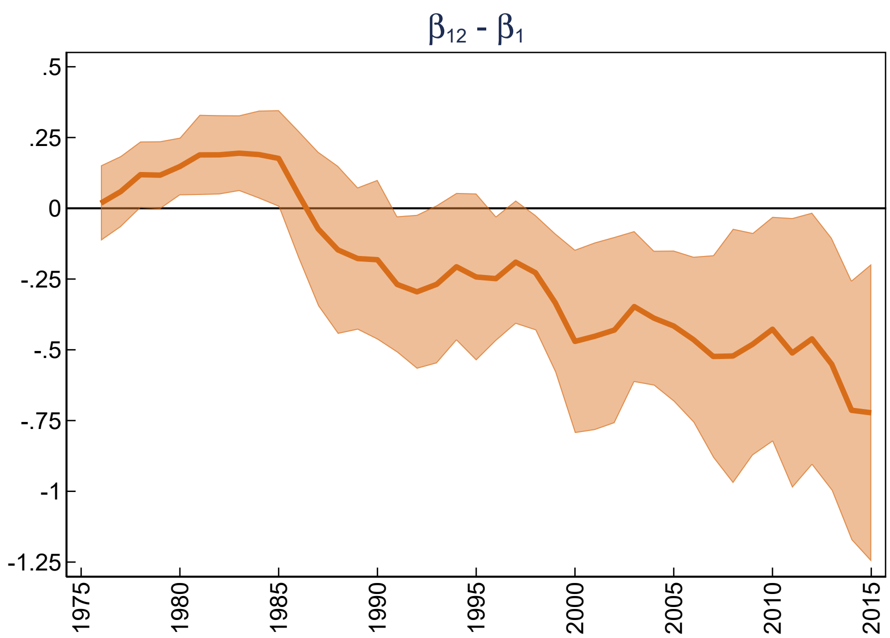


**Table A.6: Regressions of changes in long-term emerging market rates on short-term rates.** This table reports the estimated regression coefficients from equation (1.1) for Mexico, South Africa and the Philippines. These data come from the IMF International Financial Statistics and are not zero coupon yield curve data but rather use the long-term bond and short-term bill rates as reported in International Financial Statistics which are monthly. The sample period is 2000-2019. Changes are considered at monthly ($h = 1$), quarterly ($h = 3$), semi-annual ($h = 6$) and annual ($h = 12$) horizons. For $h > 1$, we report Newey-West (1987) standard errors in brackets, using a lag truncation parameter of $\lceil 1.5 \times h \rceil$; for $h = 1$, we report heteroskedasticity robust standard errors. Significance: $^{*}p < 0.1$, $^{**}$ $p < 0.05$, $^{***}p < 0.01$. Significance is computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

| | Mexico | South Africa | Philippines |
|---|---|---|---|
| Monthly | 0.28*** | 0.20*** | 0.44* |
| | [0.06] | [0.07] | [0.23] |
| Quarterly | 0.33*** | 0.21** | 0.59*** |
| | [0.05] | [0.09] | [0.20] |
| Semi-Annual | 0.34*** | 0.24** | 0.64*** |
| | [0.04] | [0.12] | [0.19] |
| Annual | 0.44*** | 0.40** | 0.62*** |
| | [0.04] | [0.17] | [0.20] |

## A.2 Additional empirical results for Section 2

### A.2.1 Predicting the returns on "level-mimicking" and "slope-mimicking" portfolios

As in Table 3 in main text, we first forecast the $k$-month excess return on $n = 10$-year zero-coupon bonds using level, slope, and the 6-month past changes in these two yield-curve factors:

$$rx^{(10)}_{t\to t+k} = \delta_0 + \delta_1 L_t + \delta_2 S_t + \delta_3(L_t - L_{t-6}) + \delta_4(S_t - S_{t-6}) + \varepsilon_{t\to t+k}. \tag{A.3}$$

In Table A.7, we report the results from estimating these predictive regressions for $k = 1$, 3, and 6-month returns. Panel A reports the results for the pre-2000 sample and Panel B shows the post-2000 results.[2,3]

In the post-2000 data, Table A.7 shows the past change in the level of rates is a robust predictor of the excess returns on long-term bonds. However, there is no such predictability in the pre-2000 data. For instance, in column (6) of Panel B, we see that, all else equal, a 100 bps increase in short-term rates over the prior 6 months is associated with a $\delta_3 = 166$ bps ($p$-val $< 0.01$) increase in expected 3-month bond returns and the difference between $\delta_3$ in the pre- and post-2000

[2]The yield on $k$-month Treasury bills, $y_t^{(k/12)}$, is from the yield curve estimates in Gürkaynak et al. (2007). However, this curve is based on coupon securities with at least three months to maturity and does not fit the very short end of the curve well in the pre-2000 data. Therefore, we take the 1-month bill yield from Ken French's website for the pre-2000 sample.

[3]We obtain broadly similar results in Table A.7 if we forecast returns using 3- or 12-month past changes in level and slope. And, the return predictability associated with past changes in level remains similar if, instead of controlling for level and slope, we control for the first five forward rates as in Cochrane and Piazzesi (2005).

data is statistically significant ($p$-val $< 0.01$). In untabulated results, we find that the post-2000 return predictability associated with past increases in the level of rates is short-lived and generally dissipates after $k = 6$ months. In other words, past increases in the level of rates lead to a temporary increase in the risk premia on long-term bonds.[4]

To draw out the connection to the predictable curve flattening shown in Table 2 of the main text, we show that these results for 10-year returns are related to predictability of the returns on what we refer to as "level-mimicking" and "slope-mimicking" portfolios. Specifically, we follow Joslin et al. (2014) and construct bond portfolios that locally mimic changes in the level and slope factors. Consider a factor-mimicking portfolio that places weight $w_n$ on zero-coupon bonds with $n$ years to maturity. The $k$-month excess return on this portfolio from $t$ to $t+k$ is $rx^P_{t\to t+k} = (\sum_n w_n \cdot rx^{(n)}_{t\to t+k})/|\sum_n w_n|$. The level-mimicking portfolio has a weight of $-1$ on 1-year bonds and no weight on any other bonds. For small $k$, we have $rx^{(10)}_{t\to t+k} \approx -10 \cdot (\Delta_k L_{t+k} + \Delta_k S_{t+k})$ and $rx^{(1)}_{t\to t+k} \approx -1 \cdot \Delta_k L_{t+k}$. Thus, the level- mimicking portfolio has a $k$-month excess return of $rx^{LEVEL}_{t\to t+k} = -1 \cdot rx^{(1)}_{t\to t+k} \approx \Delta_k L_{t+k}$. The slope-mimicking portfolio has a weight of 1 on 1-year bonds and of $-0.1$ on 10-year bonds, so $rx^{SLOPE}_{t\to t+k} = (1 \cdot rx^{(1)}_{t\to t+k} - 0.1 \cdot rx^{(10)}_{t\to t+k})/0.9 \approx \Delta_k S_{t+k}/0.9$. Finally, we note that the excess returns on 10-year bonds are just a linear combination of the excess returns on the level- and slope-mimicking portfolios: $rx^{(10)}_{t\to t+k} = -9 \cdot rx^{SLOPE}_{t\to t+k} - 10 \cdot rx^{LEVEL}_{t\to t+k}$.

In the two bottom blocks of Table A.7, we estimate equation (A.3) using $rx^{LEVEL}_{t\to t+k}$ and $rx^{SLOPE}_{t\to t+k}$ as the dependent variable. In the post-2000 sample, the excess returns on the slope-mimicking portfolio depend negatively on $L_t - L_{t-6}$, but the excess returns on the level-mimicking portfolio depends positively on $L_t - L_{t-6}$.[5] While the two effects partially offset when predicting 10-year excess returns, the net effect is positive and statistically significant in the post-2000 data. Furthermore, the results in Table A.7 where we forecast $rx^{LEVEL}_{t\to t+k}$ and $rx^{SLOPE}_{t\to t+k}$ are entirely consistent with those in Table 2 in the main text.

### A.2.2 Unspanned macroeconomic factors

In Table 4 of the paper (and Table A.7 above), we showed that lagged changes in levels can help predict excess returns in the post-2000 period. As detailed below, in standard affine models, if the true model is known, one can obtain the full set of state variables by inverting an appropriate set of yields—i.e., the state variables are spanned by current yields. An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve—i.e., it is not " spanned" by current yields—and cannot be recovered in this

[4]Consistent with the vast literature on lower-frequency movements in bond risk premia initiated by Fama and Bliss (1987) and Campbell and Shiller (1991), we find $\delta_2 > 0$—i.e., expected bond returns are high when the yield curve is steep.

[5]The latter fact is consistent with Piazzesi et al. (2015) and Cieslak (2018), who account for it either with expectational errors or time-varying risk premia. Brooks et al. (2019) also show that the federal funds rate displays short-term momentum.

**Table A.7: Estimates of predictive equations for bond excess returns.** This table reports the estimated regression coefficients in equation (A.3) using monthly data from the Aug-1971 to Dec-1999 and Jan-2000 to Dec-2019 samples. We report results various return forecast horizon ($k$). Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. For $k = 1$-month returns, we report heteroskedasticity robust standard errors are in brackets. For $k = 3$ and 6-month returns, we report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 5 and 9 months, respectively. In this case, $p$-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

**Panel A: Pre-2000 sample**

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| Dep. Var for k | 1 | 1 | 1 | 3 | 3 | 3 | 6 | 6 | 6 |
| Dependent Variable: $rx^{(10)}_{t\to t+k}$ | | | | | | | | | |
| $L_t$ | 0.17 | 0.18 | 0.22* | 0.53* | 0.54 | 0.65* | 0.99* | 1.01 | 1.25** |
| | [0.13] | [0.13] | [0.12] | [0.31] | [0.33] | [0.34] | [0.57] | [0.62] | [0.60] |
| $S_t$ | 0.55** | 0.61** | 0.66*** | 1.57** | 1.91*** | 2.07*** | 3.17*** | 3.62*** | 3.98*** |
| | [0.24] | [0.24] | [0.24] | [0.64] | [0.65] | [0.64] | [1.04] | [1.13] | [1.09] |
| $L_t - L_{t-6}$ | | 0.10 | -0.18 | | 0.59 | -0.22 | | 0.79 | -1.00 |
| | | [0.19] | [0.29] | | [0.49] | [0.54] | | [0.69] | [0.90] |
| $S_t - S_{t-6}$ | | | -0.57 | | | -1.67** | | | -3.67*** |
| | | | [0.44] | | | [0.81] | | | [1.30] |
| Adj.$R^2$ | 0.02 | 0.01 | 0.02 | 0.05 | 0.06 | 0.07 | 0.11 | 0.12 | 0.17 |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Dependent Variable: $rx^{LEVEL}_{t\to t+k}$ | | | | | | | | | |
| $L_t$ | -0.04* | -0.04* | -0.04** | -0.08* | -0.09* | -0.10** | -0.11** | -0.11** | -0.13** |
| | [0.02] | [0.02] | [0.02] | [0.04] | [0.05] | [0.05] | [0.05] | [0.05] | [0.05] |
| $S_t$ | -0.06 | -0.07* | -0.08** | -0.15* | -0.21** | -0.24*** | -0.23*** | -0.26*** | -0.29*** |
| | [0.04] | [0.03] | [0.04] | [0.09] | [0.08] | [0.08] | [0.09] | [0.09] | [0.09] |
| $L_t - L_{t-6}$ | | -0.01 | 0.04 | | -0.10 | 0.03 | | -0.04 | 0.09 |
| | | [0.03] | [0.04] | | [0.08] | [0.08] | | [0.09] | [0.09] |
| $S_t - S_{t-6}$ | | | 0.11* | | | 0.26** | | | 0.27* |
| | | | [0.06] | | | [0.11] | | | [0.14] |
| Adj.$R^2$ | 0.01 | 0.01 | 0.02 | 0.04 | 0.06 | 0.08 | 0.09 | 0.10 | 0.12 |
| Dependent Variable: $rx^{SLOPE}_{t\to t+k}$ | | | | | | | | | |
| $L_t$ | 0.02 | 0.02 | 0.03** | 0.03 | 0.04 | 0.04 | 0.01 | 0.01 | 0.01 |
| | [0.01] | [0.01] | [0.01] | [0.02] | [0.03] | [0.03] | [0.03] | [0.03] | [0.03] |
| $S_t$ | 0.00 | 0.01 | 0.01 | -0.01 | 0.02 | 0.03 | -0.09 | -0.11 | -0.12 |
| | [0.03] | [0.02] | [0.03] | [0.06] | [0.05] | [0.06] | [0.08] | [0.08] | [0.08] |
| $L_t - L_{t-6}$ | | 0.00 | -0.03 | | 0.04 | -0.00 | | -0.04 | 0.01 |
| | | [0.02] | [0.03] | | [0.05] | [0.06] | | [0.04] | [0.05] |
| $S_t - S_{t-6}$ | | | -0.06 | | | -0.10 | | | 0.11 |
| | | | [0.05] | | | [0.09] | | | [0.10] |
| Adj.$R^2$ | 0.01 | 0.01 | 0.01 | 0.02 | 0.03 | 0.03 | 0.04 | 0.05 | 0.06 |
| N | 341 | 335 | 335 | 341 | 335 | 335 | 341 | 335 | 335 |
| Sample | 1971-1999 | 1972-1999 | 1972-1999 | 1971-1999 | 1972-1999 | 1972-1999 | 1971-1999 | 1972-1999 | 1972-1999 |

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|---|
| Dep. Var for k | 1 | 1 | 1 | 3 | 3 | 3 | 6 | 6 | 6 |
| Dependent Variable: $rx^{(10)}_{t \to t+k}$ | | | | | | | | | |
| $L_t$ | 0.30*** | 0.32*** | 0.28** | 0.73*** | 0.81*** | 0.71** | 1.37*** | 1.50*** | 1.34** |
| | [0.11] | [0.11] | [0.11] | [0.27] | [0.28] | [0.30] | [0.44] | [0.47] | [0.51] |
| $S_t$ | 0.56*** | 0.63*** | 0.53** | 1.41*** | 1.65*** | 1.42** | 2.71*** | 3.07*** | 2.73*** |
| | [0.21] | [0.20] | [0.21] | [0.48] | [0.52] | [0.58] | [0.83] | [0.93] | [0.95] |
| $L_t - L_{t-6}$ | | 0.33 | 0.62* | | 0.98** | 1.66*** | | 1.33* | 2.39** |
| | | [0.24] | [0.33] | | [0.46] | [0.61] | | [0.73] | [1.07] |
| $S_t - S_{t-6}$ | | | 0.48 | | | 1.12 | | | 1.76 |
| | | | [0.35] | | | [0.72] | | | [1.22] |
| Adj.$R^2$ | 0.03 | 0.03 | 0.03 | 0.08 | 0.10 | 0.12 | 0.16 | 0.18 | 0.20 |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Dependent Variable: $rx^{LEVEL}_{t \to t+k}$ | | | | | | | | | |
| $L_t$ | -0.03*** | -0.03*** | -0.03*** | -0.09*** | -0.08*** | -0.07*** | -0.12*** | -0.11*** | -0.10*** |
| | [0.01] | [0.01] | [0.01] | [0.03] | [0.02] | [0.02] | [0.04] | [0.03] | [0.03] |
| $S_t$ | -0.04*** | -0.03** | -0.03** | -0.10*** | -0.07** | -0.05* | -0.13*** | -0.08* | -0.05 |
| | [0.01] | [0.01] | [0.01] | [0.03] | [0.03] | [0.03] | [0.04] | [0.04] | [0.04] |
| $L_t - L_{t-6}$ | | 0.05** | 0.03 | | 0.12** | 0.06 | | 0.19*** | 0.12 |
| | | [0.02] | [0.02] | | [0.06] | [0.06] | | [0.07] | [0.07] |
| $S_t - S_{t-6}$ | | | -0.04** | | | -0.09** | | | -0.11** |
| | | | [0.02] | | | [0.04] | | | [0.05] |
| Adj.$R^2$ | 0.06 | 0.10 | 0.11 | 0.13 | 0.21 | 0.23 | 0.21 | 0.37 | 0.39 |
| Dependent Variable: $rx^{SLOPE}_{t \to t+k}$ | | | | | | | | | |
| $L_t$ | 0.01 | -0.00 | 0.00 | 0.02 | -0.00 | 0.00 | -0.01 | -0.05 | -0.04 |
| | [0.01] | [0.01] | [0.01] | [0.03] | [0.03] | [0.03] | [0.06] | [0.04] | [0.05] |
| $S_t$ | -0.01 | -0.03* | -0.03 | -0.05 | -0.10** | -0.10* | -0.16* | -0.26*** | -0.24*** |
| | [0.02] | [0.02] | [0.02] | [0.05] | [0.05] | [0.05] | [0.08] | [0.08] | [0.08] |
| $L_t - L_{t-6}$ | | -0.09*** | -0.10*** | | -0.24*** | -0.25*** | | -0.36*** | -0.40*** |
| | | [0.02] | [0.03] | | [0.05] | [0.06] | | [0.07] | [0.11] |
| $S_t - S_{t-6}$ | | | -0.01 | | | -0.02 | | | -0.07 |
| | | | [0.03] | | | [0.07] | | | [0.13] |
| Adj.$R^2$ | 0.00 | 0.06 | 0.06 | 0.03 | 0.19 | 0.18 | 0.08 | 0.28 | 0.28 |
| N | 239 | 239 | 239 | 237 | 237 | 237 | 234 | 234 | 234 |
| Sample | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 | 2000-2019 |

way. Lagged changes in level and slope are unspanned factors, as in Joslin et al. (2013).

Other authors have considered macroeconomic variables as unspanned factors, and this is the more typical choice of unspanned factors. Thus, following Joslin et al. (2014) , we augment equation (A.3) with growth (measured by the 3 month moving average of the Chicago Fed National Activity Index) and inflation (measured by the Blue Chip forecast of CPI inflation over the next four quarters). Because of the availability of the inflation forecast data, the sample only starts in March 1980. The results are shown in Table A.8. In the post-2000 sample, we find that lagged changes in level are significant predictors of excess returns on 10-year bonds and are also significant predictors of returns on the slope-mimicking portfolio. Inflation is also a significant predictor of 10-year excess bond returns in this post-2000 sample, though it is not a significant predictor of returns on the slope-mimicking portfolio. Overall, our finding that lagged changes in level predict future bond returns in the post-2000 data is robust to the inclusion of the standard unspanned macroeconomic variables.

### A.2.3 Predicting the returns on bonds with different maturities

In this section, we examine the predictability for bond maturities other than $n = 10$ years. If, as we argue, past increases in short rates temporarily raise the net supply of long-term bonds that investors must hold, thereby raising the compensation investors require for bearing interest-rate risk, this should have a larger impact on the expected returns of long-term bonds than intermediate bonds. This is because the returns on long-term bonds are more sensitive to shifts in yields than those on intermediate bonds (Vayanos and Vila, 2020; Greenwood and Vayanos, 2014). We explore this prediction in Figure A.3. We separately forecast the 3-month returns on bonds with different maturities $n$, estimating:

$$rx_{t\to t+3}^{(n)} = \delta_0^{(n)} + \delta_1^{(n)} L_t + \delta_2^{(n)} S_t + \delta_3^{(n)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t\to t+3}^{(n)} \tag{A.4}$$

separately for $n = 1, ....20$-year bonds. We then plot the coefficients $\delta_3^{(n)}$ on the past change in level from estimating equation (A.4) versus bond maturity $n$ for the pre-2000 and post-2000 samples. (For the pre-2000 sample, the longest available maturity is $n = 15$ years). Consistent with the idea that past increases in short rates temporarily raise the compensation for bearing interest-rate risk, the coefficients $\delta_3^{(n)}$ are monotonically increasing in bond maturity $n$ in the post-2000 sample. By contrast, there is no such predictability in the pre-2000 sample.

### A.2.4 Predictable changes in the shape of the yield and forward rate curves

This temporary rise in the compensation for bearing interest rate risk impacts the yield and forward rate curves. As explained in Greenwood and Vayanos (2014), a short-lived rise in the compensation for bearing interest rate risk may have relatively constant or even a hump-shaped effect on the yield

**Table A.8: Estimates of predictive equations for bond excess returns including macro factors** This table reports the estimation of equation (2.6) augmented with growth (measured by the 3 month moving average of the Chicago Fed National Activity Index) and inflation (measured by the Blue Chip forecast of CPI inflation over the next four quarters) using monthly data from the subsample from March 1980 to December 1999 and the subample from January 2000 to December 2019. We report results various return forecast horizon (k). Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$. For k = 1-month returns, we report heteroskedasticity robust standard errors are in brackets. For k = 3 and 6-month returns, we report Newey and West (1987) standard errors in brackets, using a lag truncation parameter of 5 and 9 months, respectively. In this case, p-values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Dep. Var for k | 1 | 3 | 6 | 1 | 3 | 6 |
| Dependent Variable: $rx^{(10)}_{t\to t+k}$ | | | | | | |
| $L_t$ | 0.46** | 1.77*** | 3.47*** | 0.28** | 0.51 | 1.01* |
| | (0.22) | (0.38) | (0.48) | (0.14) | (0.32) | (0.53) |
| $S_t$ | 0.76** | 2.37*** | 4.55*** | 0.61*** | 1.65*** | 2.93*** |
| | (0.30) | (0.73) | (1.09) | (0.23) | (0.57) | (0.95) |
| $L_t - L_{t-6}$ | 0.31 | 1.01* | 0.98 | 0.73* | 1.81*** | 2.25** |
| | (0.34) | (0.55) | (0.87) | (0.42) | (0.63) | (1.00) |
| $S_t - S_{t-6}$ | -0.64 | -1.32 | -2.78** | 0.43 | 0.72 | 1.21 |
| | (0.51) | (0.93) | (1.30) | (0.35) | (0.69) | (1.17) |
| Growth | -2.34*** | -6.17*** | -9.68*** | -0.32 | -1.37 | -1.13 |
| | (0.77) | (1.85) | (2.72) | (0.88) | (1.00) | (0.82) |
| Inflation | -0.56* | -2.45*** | -4.69*** | 0.26 | 2.34** | 3.24*** |
| | (0.31) | (0.52) | (0.58) | (0.79) | (0.96) | (1.11) |
| Dependent Variable: $rx^{LEVEL}_{t\to t+k}$ | | | | | | |
| $L_t$ | -0.07* | -0.26*** | -0.36*** | -0.02** | -0.05*** | -0.08*** |
| | (0.04) | (0.06) | (0.03) | (0.01) | (0.02) | (0.03) |
| $S_t$ | -0.07 | -0.25*** | -0.29*** | -0.03** | -0.06* | -0.06 |
| | (0.05) | (0.08) | (0.08) | (0.01) | (0.03) | (0.04) |
| $L_t - L_{t-6}$ | -0.06 | -0.15** | -0.06 | 0.01 | 0.02 | 0.08 |
| | (0.05) | (0.06) | (0.07) | (0.03) | (0.07) | (0.08) |
| $S_t - S_{t-6}$ | 0.12** | 0.20** | 0.15 | -0.02 | -0.06 | -0.08 |
| | (0.06) | (0.09) | (0.10) | (0.02) | (0.04) | (0.05) |
| Growth | 0.43*** | 0.86*** | 0.69*** | 0.09** | 0.22*** | 0.20*** |
| | (0.13) | (0.17) | (0.19) | (0.04) | (0.08) | (0.07) |
| Inflation | 0.07 | 0.36*** | 0.48*** | -0.06* | -0.18** | -0.14 |
| | (0.07) | (0.10) | (0.05) | (0.03) | (0.09) | (0.09) |
| Dependent Variable: $rx^{SLOPE}_{t\to t+k}$ | | | | | | |
| $L_t$ | 0.03 | 0.09** | 0.01 | -0.01 | 0.00 | -0.02 |
| | (0.03) | (0.04) | (0.05) | (0.01) | (0.03) | (0.05) |
| $S_t$ | -0.00 | 0.02 | -0.18* | -0.04* | -0.11** | -0.26*** |
| | (0.03) | (0.07) | (0.10) | (0.02) | (0.05) | (0.09) |
| $L_t - L_{t-6}$ | 0.03 | 0.06 | -0.04 | -0.09** | -0.22*** | -0.34*** |
| | (0.03) | (0.06) | (0.05) | (0.04) | (0.07) | (0.10) |
| $S_t - S_{t-6}$ | -0.06 | -0.08 | 0.15 | -0.02 | -0.01 | -0.04 |
| | (0.04) | (0.09) | (0.11) | (0.03) | (0.07) | (0.13) |
| Growth | -0.21** | -0.27** | 0.30* | -0.06 | -0.09 | -0.10 |
| | (0.09) | (0.13) | (0.17) | (0.10) | (0.10) | (0.08) |
| Inflation | -0.02 | -0.13* | -0.02 | 0.04 | -0.06 | -0.20 |
| | (0.05) | (0.07) | (0.06) | (0.09) | (0.13) | (0.12) |
| Sample | 1971-1999 | 1971-1999 | 1972-1999 | 2000-2019 | 2000-2019 | 2000-2019 |

and forward curves as opposed to the monotonically increasing effect shown above for returns. The intuition is that the impact on bond yields equals the effect on a bond's average expected returns over its lifetime. As a result, a *temporary* rise in the compensation for bearing interest rate risk can have a greater impact intermediate-term yields than on long-term yields. Thus, we plot the slope coefficients $\delta_3^{(n)}$ versus maturity $n$ from estimating:

$$y_{t+3}^{(n-3/12)} - y_t^{(n)} = \delta_0^{(n)} + \delta_1^{(1)} L_t + \delta_2^{(1)} S_t + \delta_3^{(3)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t \to t+3}^{(n)}. \qquad \text{(A.5)}$$

and

$$f_{t+3}^{(n-3/12)} - f_t^{(n)} = \delta_0^{(n)} + \delta_1^{(n)} L_t + \delta_2^{(n)} S_t + \delta_3^{(n)} (L_t - L_{t-6}) + \delta_4^{(n)} (S_t - S_{t-6}) + \varepsilon_{t \to t+3}^{(n)}, \qquad \text{(A.6)}$$

for $n = 1, 2, ..., 20$ years for both the pre-2000 and post-2000 samples.[6] After 2000, the second plot in Figure A.3 shows that while past increases in the level of short rates forecast a future flattening of the yield curve in post-2000 data, the expected changes in long-term yields are relatively constant beyond 5 years. Turning to forward rates, the bottom plot in Figure A.3 shows that, post 2000, a past increase in short-term rates has a slight humped-shaped effect on the evolution of the forward curve with the peak impact at 4 years.

In summary, the results for different maturities support the view that past increases in short-term rates *temporarily* raise the compensation that investors earn for bearing interest-rate risk.

### A.2.5 Trading strategies

As another way of assessing the resulting return predictability documented in Section 2.2 of the paper, we consider simple market-timing strategies in which an investor decides to take either a long or short position in the slope-mimicking portfolio—i.e., in a "curve steepener" trade—every month. Specifically, we consider strategies that take a long (short) position in the slope-mimicking portfolio from month $t$ to month $t+1$ if $L_t < L_{t-h}$ ($L_t > L_{t-h}$). Alternatively, we consider strategies that take a position in the slope-mimicking portfolio from month $t$ to month $t+1$ that is proportional to $-(L_t - L_{t-h})$. Table A.9 computes the annualized Sharpe ratios of these two trading strategies for different choices of $h$, in the pre- and post-2000 samples. As shown in Table A.9 the implied annualized Sharpe ratios for these strategies range between about 0.35 to 0.65 in the post-2000 sample but were negligible in the pre-2000 sample.

[6] In equation (A.6), $f_t^{(n)} \equiv n y_t^{(n)} - (n-1)\, y_t^{(n)}$ is the 1-year rate $(n-1)$ years forward as opposed to the instantaneous forward $n$ years forward. Since $f_{t+k}^{(n-k/12)} - f_t^{(n)} = -(rx_{t \to t+k}^{(n)} - rx_{t \to t+k}^{(n-1)})$, there is a tight connection between the coefficients in equations (A.6) and (A.4). Specifically, defining $f_{t+k}^{(1-k/12)} - f_t^{(1)} = -rx_{t \to t+k}^{(1)}$, we have $rx_{t \to t+k}^{(n)} = \sum_{m=1}^{n} (f_t^{(m)} - f_{t+k}^{(m-k/12)})$. Thus, the coefficients in equation (A.4) for maturity $n$ can be recovered by summing up the $-1$ times coefficients in equation (A.6) for all maturities $m \leq n$. Similarly, the coefficients from equation (A.5) for maturity $n$ are approximately the average of the coefficients in equation (A.6) for all maturities $m \leq n$.

**Figure A.3: Predicting returns, the changes in forwards, and the change in yields for various bond maturities $n$.** This figure plots the coefficients $\delta_3^{(n)}$ on the past 6-month change in the level factor versus bond maturity $n$ from estimating equation (A.4) for returns, equation (A.5) for the change in yields, and equation (A.6) for the change in forward rates. The results are shown for $k = 3$-month returns or future changes. Due to the use of overlapping data, we plot 95% confidence intervals using Newey-West (1987) standard errors with a lag truncation parameter of 5. The critical values are computed using the asymptotic theory of Kiefer and Vogelsang (2005) which has better finite sample properties than traditional asymptotic theory.

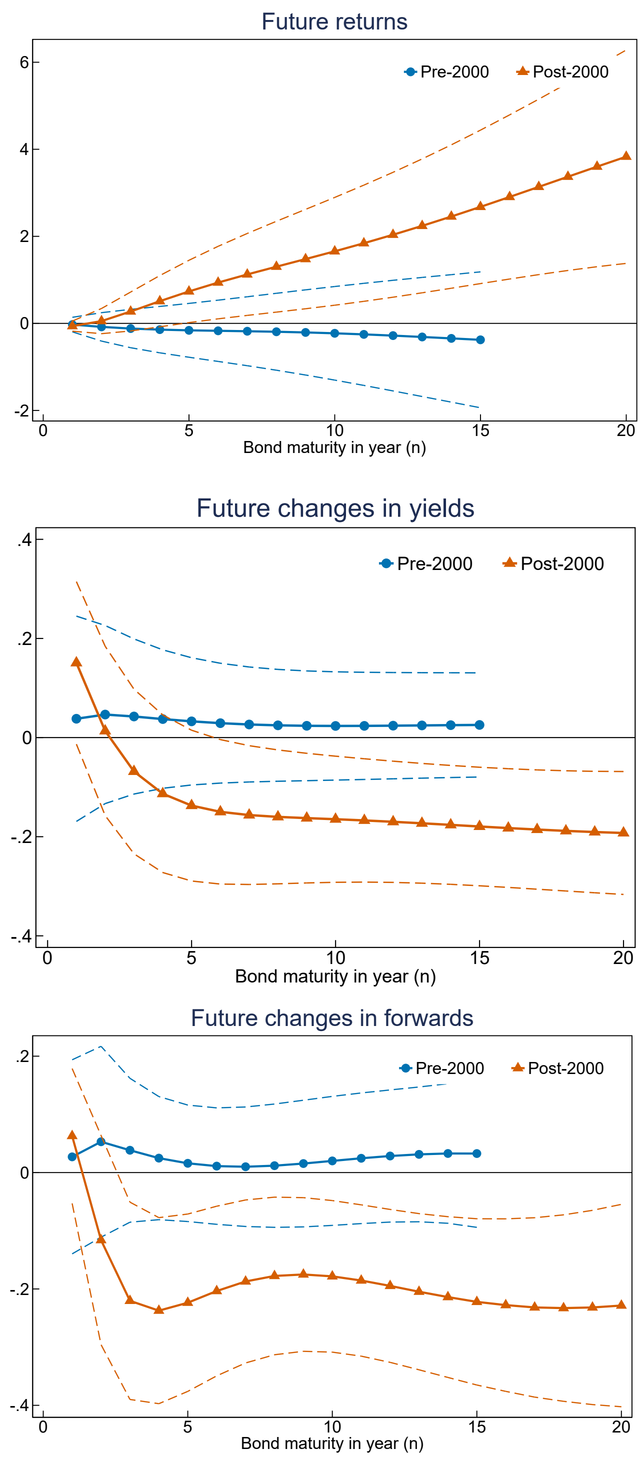


**Table A.9: Sharpe ratios for slope-mimicking portfolios** This table reports the annualized Sharpe ratios since 2000 of the strategy of going long (short) the slope-mimicking portfolio if the level fell (rose) over the previous $h$ months and also the strategy of taking a position in the slope-mimicking portfolio that is proportional to $-(L_t - L_{t-h})$, and holding the position from $t$ to $t+1$. The position is rebalanced each month. Annualized Sharpe ratios are computed as the sample average monthly excess returns multiplied by $\sqrt{12}$ and divided by the standard deviation of those monthly excess returns.

| Strategy with $h$: | 1 | 3 | 6 | 12 |
|---|---|---|---|---|
| Pre-2000 | | | | |
| $2 \times I(L_t - L_{t-h} < 0) - 1$ | 0.22 | 0.03 | -0.09 | -0.00 |
| $-(L_t - L_{t-h})$ | 0.08 | -0.01 | 0.12 | 0.09 |
| Post-2000 | | | | |
| $2 \times I(L_t - L_{t-h} < 0) - 1$ | 0.43 | 0.53 | 0.63 | 0.58 |
| $-(L_t - L_{t-h})$ | 0.36 | 0.56 | 0.43 | 0.42 |

## A.3 Additional empirical results for Section 4

### A.3.1 Reaching for yield

To empirically assess this reaching-for-yield explanation for our findings, we use quarterly data from the Federal Reserve's Financial Accounts on the aggregate net bond acquisitions by insurers (life plus property-casualty), pension funds (private plus state and local), and banks to construct empirical proxies for the bond demand of yield-seeking investors, $h_t$. We focus on these highly-regulated financial intermediaries since prior research has argued that they are most likely to be concerned about the current yield on their portfolios and, therefore, to reach for yield when interest rates decline.[7] For intermediaries in sector $i$, we compute the percentage bond flows in quarter $t$ as $\%FLOW_{i,t} = FLOW_{i,t}/HOLD_{i,t-1}$, where $FLOW_{i,t}$ denotes net bond acquisitions by intermediaries in sector $i$ during quarter $t$ and $HOLD_{i,t-1}$ is bond holdings at the end of quarter $t-1$. Bonds here include the sum of U.S. Treasury securities, agency debt and GSE-guaranteed mortgage-backed securities, and corporate bonds. Thus, our construction of these sector-level flows roughly mimics the construction of bond mutual fund flows above.

In Table A.10, we then estimate quarterly regressions that are analogous to equations (4.2) and (4.3) in the main text using these sector-level bond flows $\%FLOW_{i,t}$ as $X_t$. We report the results separately for the pre-2000 and post-2000 samples. As shown in Panel A of Table A.10, in the post-2000 data, we find little evidence that recent increase in short-term rates lead to a reduction in bond purchases by insurers, pensions, and banks. Furthermore, in Panels B and C, we find

[7] Insurers and banks are generally not required to include changes in the mark-to-market value of their portfolios in their reported earnings, which may give way to yield-seeking behavior. For prior work on reaching-for-yield by insurers, see Becker and Ivashina (2015). For pension funds, see Lu et al. (2019). For banks, see Maddaloni and Peydró (2011), Hanson and Stein (2015), and Drechsler et al. (2018).

little evidence that bond purchases by these intermediaries predict low excess returns on long-term bonds in the following quarter or subsequent yield-curve steepening as would be suggested by a reaching-for-yield explanation for our findings.

In summary, we find little evidence that reaching-for-yield plays a major role in driving the kind of short-lived overreaction of long-term yields to changes in short rates that we see since 2000. To be sure, this lack of evidence does not imply that reaching-for-yield plays an unimportant role in determining financial market risk premia more generally, especially at lower frequencies. This negative conclusion only applies to the ability of reaching-for-yield to explain the sorts of transitory fluctuations in bond risk premia that underpin the horizon-dependent excess sensitivity observed in recent decades.

## A.4 Additional empirical results for Section 5

### A.4.1 Bond market "conundrums"

Our findings can help explain the rising prevalence of bond market episodes like the one that former Federal Reserve Chairman Greenspan famously called the "conundrum"—the period after June 2004 when the Fed raised short-term rates, but longer-term yields declined. This "conundrum" was first noted in Greenspan (2005) and has been explored in many papers, including Backus and Wright (2007).

Consistent with the weaker low-frequency sensitivity of long-term rates in recent years, "conundrum" episodes—defined as 6-month periods where short- and long-term rates move in *opposite* directions—have grown increasingly common. Specifically, since 2000, 1- and 10-year nominal Treasury yields have moved in the *opposite* direction in 37% of all 6-month periods. By contrast, from 1971 to 1999, the corresponding figure was 18%, and the difference is statistically significant ($p$-val $< 0.001$).

Here we show that the non-Markovian dynamics documented in Section 2—the fact that past changes in the level of rates increasingly predict a future flattening of the yield curve—help explain several noteworthy "conundrums." Figure A.4 plots 1-year and 10-year Treasury rates around three widely discussed "conundrums": Greenspan's original 2004 "conundrum," 2008 which was a "conundrum in reverse," and the 2017 "conundrum." In all three cases, 1-year and 10-year yields moved in opposite directions.

Consider Greenspan's original 2004 "conundrum." To draw the link between non-Markovian yield-curve dynamics and this "conundrum," we use the system of predictive equations for level and slope from Table 2 in the main text. Starting in May 2004, we simulate the counterfactual path of 10-year yields that would have prevailed if, in the post-2000 sample, the slope of the yield curve had not responded to past changes in the level. To do so, we take the unrestricted estimates of the predictive equation (2.1b) for slope from column (6) in Table 2 and the restricted estimates from

**Table A.10: The role of reaching-for-yield: Evidence from sectoral bond market flows.** Data on sectoral-level bond market flows are from the Federal Reserve's Financial Accounts. Bond holdings include the sum of Treasury Securities (Table 210), Agency and GSE-Backed Securities (Instrument Table 211), and Corporate Bonds (Table 213). Our series for "Insurers" combines together data for Property-Casualty Insurance Companies (Table 115) and Life Insurance Companies (Table 116); "Pensions" combines together data for Private Pension Funds (Table 118) and State and Local Government Employee Retirement Funds (Table 120); and "Banks" uses data for U.S.-chartered depository institutions (Table 111). For intermediary sector $i$, we then compute the percentage bond flow in quarter $t$ as $\%FLOW_{i,t} = FLOW_{i,t}/HOLD_{i,t-1}$, where $FLOW_{i,t}$ denotes net bond acquisitions by intermediaries in sector $i$ during quarter $t$ and $HOLD_{i,t-1}$ is bond holdings at the end of quarter $t-1$. Panel A reports the estimated regression coefficients for equation (4.2) using $X_t = \%FLOW_{i,t}$ for each sector $i$. We estimate these regressions using quarterly data for the 1971Q3-1999Q4 and 2000Q1-2019Q4 samples. Panels B and C report the estimated coefficients for equation (4.3). We report heteroskedasticity robust standard errors in brackets. Significance: $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

| | Pre-2000 | | | Post-2000 | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Sector ($i$): | Insurance | Pensions | Banks | Insurance | Pensions | Banks |
| Dependent Variable: $FLOW_{i,t}$ | | | | | | |
| $L_t$ | 0.24*** | 0.27*** | 0.33*** | 0.23*** | -0.17 | 0.06 |
| | [0.05] | [0.07] | [0.12] | [0.07] | [0.19] | [0.16] |
| $S_t$ | 0.71*** | -0.25* | 0.67** | 0.47*** | -0.00 | 0.42 |
| | [0.11] | [0.15] | [0.28] | [0.11] | [0.29] | [0.29] |
| $L_t - L_{t-2}$ | -0.04 | -0.28** | -0.53** | 0.16 | 1.44*** | 0.05 |
| | [0.09] | [0.11] | [0.26] | [0.17] | [0.42] | [0.41] |
| Adj.$R^2$ | 0.25 | 0.24 | 0.12 | 0.18 | 0.16 | 0.01 |
| N | 112 | 112 | 112 | 80 | 80 | 80 |
| Dependent Variable: $rx^{(10)}_{t\to t+1}$ | | | | | | |
| $L_t$ | 0.39 | 0.68* | 0.68* | 0.95** | 0.90** | 0.85** |
| | [0.46] | [0.41] | [0.39] | [0.39] | [0.35] | [0.36] |
| $S_t$ | 0.99 | 1.34* | 1.86** | 1.83** | 1.66*** | 1.68** |
| | [0.92] | [0.78] | [0.83] | [0.74] | [0.61] | [0.64] |
| $FLOW_{t-1\to t}$ | 0.54 | -0.57 | -0.50*** | -0.56 | 0.24 | -0.20 |
| | [0.59] | [0.40] | [0.15] | [0.73] | [0.18] | [0.32] |
| Adj.$R^2$ | 0.02 | 0.03 | 0.06 | 0.06 | 0.07 | 0.06 |
| N | 114 | 114 | 114 | 79 | 79 | 79 |
| Dependent Variable: $rx^{SLOPE}_{t\to t+1}$ | | | | | | |
| $L_t$ | 0.03 | 0.04 | 0.03 | 0.02 | 0.01 | 0.01 |
| | [0.03] | [0.03] | [0.03] | [0.04] | [0.04] | [0.04] |
| $S_t$ | -0.04 | -0.03 | -0.04 | -0.05 | -0.06 | -0.06 |
| | [0.09] | [0.08] | [0.08] | [0.07] | [0.06] | [0.06] |
| $FLOW_{t-1\to t}$ | 0.02 | -0.03 | 0.02 | -0.00 | -0.03* | 0.03 |
| | [0.05] | [0.03] | [0.02] | [0.07] | [0.02] | [0.03] |
| Adj.$R^2$ | 0.01 | 0.01 | 0.02 | 0.00 | 0.04 | 0.02 |
| N | 114 | 114 | 114 | 79 | 79 | 79 |
| Sample | 1971-1999 | 1971-1999 | 1971-1999 | 2000-2019 | 2000-2019 | 2000-2019 |

column (4) which constrain past changes to have no effect ($\delta_{3S} = \delta_{4S} = 0$). Starting in May 2004, we generate the counterfactual path of 10-year yields that would have obtained if $\delta_{3S} = \delta_{4S} = 0$. We hold the level factor at its actual value and use the residuals from the unrestricted regression in column (6), but set the parameters for the slope equation to their estimated values from the *restricted* regression in column (4).

The top panel of Figure A.4 plots the actual 1- and 10-year yields over this 2004 conundrum period along with the 10-year yield under this counterfactual scenario. Had the slope not responded to lagged changes in the level of the yield curve, Figure A.4 shows that, instead of falling, 10-year yields would have risen in 2004. The next two panels repeat this exercise for the 2008 "conundrum in reverse" (starting in December 2007) and the 2017 "conundrum" (starting in November 2016). If the slope had not responded to past changes in level, 10- and 1-year yields would have moved in the same direction in both cases.

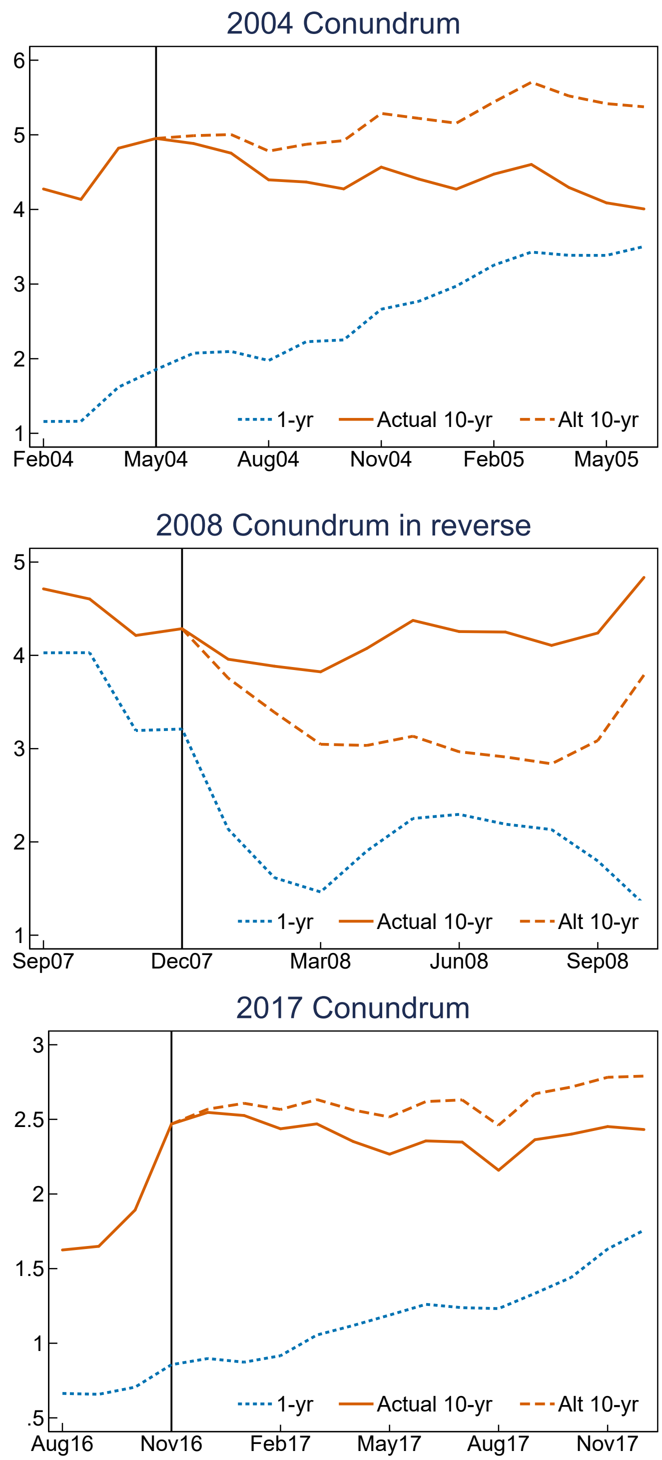
### A.4.2 Implications for affine term-structure models

**Summary** We explore the implications of our results for affine term structure models which are a widely-used, reduced-form tools for understanding the term structure of bond yields (Duffee, 2002; Duffie and Kan, 1996). In these models, the $n$-year zero coupon yield, $y_t^{(n)}$, takes the affine form: $y_t^{(n)} = \alpha_{0(n)} + \boldsymbol{\alpha}'_{1(n)}\mathbf{x}_t$, where $\mathbf{x}_t$ is a vector of state variables and the $\alpha_{0(n)}$ and $\boldsymbol{\alpha}_{1(n)}$ satisfy a set of recursive equations. Below, we apply the estimation methodology of Adrian et al. (2013) and fit affine term structure models using the first $K$ principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We show that standard affine models—models that are Markovian with respect to these current yield-curve factors—cannot fit our key finding that the sensitivity of long rates to short rates $\beta_h$ declines so strongly with horizon $h$ in the post-2000 data. Furthermore, we show that this remains so even if we estimate models that include many (e.g., $K = 5$) current yield-curve factors as state variables.

However, we show that our key finding is consistent with non-Markovian term structure models in which past lags of the yield-curve factors are treated as "unspanned state variables." In standard affine models, if the true model is known, one can recover the full set of state variables $\mathbf{x}_t$ by inverting an appropriate set of yields—i.e., the state variables are "spanned" by current yields. An unspanned state variable is a variable that is useful for forecasting future bond yields and returns but that has no impact on the current yield curve. This non-Markovian model allows us to parsimoniously capture our result that past changes in the level of rates are useful for forecasting future bond yields and returns. And, similar models have been considered in Joslin et al. (2013). To be clear, we do not argue that the past increase in the level of rates is *literally* unspanned. Instead, we think this variable is *close* to being unspanned.

Finally, we use a bootstrap procedure to test the hypothesis that each affine model is correctly

**Figure A.4: Counterfactual paths of ten-year yields in selected "conundrum" episodes.** This figure plots 1- and 10-year yields in the original 2004 "conundrum" episode, the 2008 "conundrum in reverse" episode and the "2017 conundrum." As described in the text, we also plot counterfactual 10-year yields (Alt 10-yr) generated from restricting the slope to depend on lags of level and slope, but not also on lagged changes in level and slope.



specified, using the ratio of yearly to monthly coefficients from equation (1.1) as the test statistic. The test rejects if the observed value of $\beta_{12}/\beta_1$ is too high or low to have been generated by that model. This test is in the spirit of Giglio and Kelly (2018), who test the hypothesis an affine model is correctly specified by checking whether the comovement of yields at different points on the curve is consistent with the model. Using this bootstrap procedure, we conclude that, in the post-2000 sample, the Markovian models are decisively rejected: if these standard models were correctly specified it would be highly unlikely to observe a value of $\beta_{12}/\beta_1$ as small as we do in the data. However, the non-Markovian models are not rejected post-2000. Thus, we conclude that affine models need to include lagged yield-curve factors to match the fact that the sensitivity of long rates declines so sharply with horizon post-2000.

**Affine term-structure models** A standard discrete-time affine term-structure model (Duffee, 2002; Duffie and Kan, 1996) starts from the assumption that there is a $m \times 1$ state vector $\mathbf{x}_t$ that follows a VAR(1) under the physical or P-measure:

$$\mathbf{x}_t = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t, \tag{A.7}$$

where the error term is Gaussian with mean zero and identity variance-covariance matrix. The short-term riskless interest rate between time $t$ and $t+1$ is an affine function of the state vector: $i_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$. Meanwhile, the pricing kernel or stochastic discount factor is

$$M_{t+1} = \exp(-i_t - \boldsymbol{\lambda}_t'\boldsymbol{\varepsilon}_{t+1} - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t), \tag{A.8}$$

where the prices of factor risk, $\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t$, are also an affine function of the state vector. The price of an $n$-period zero-coupon bond, $P_t^{(n)}$, satisfies the recursion

$$P_t^{(n)} = E_t^{\mathrm{P}}[M_{t+1}P_{t+1}^{(n-1)}] = \exp(-i_t)E_t^{\mathrm{Q}}[P_{t+1}^{(n-1)}]. \tag{A.9}$$

Here $E_t^{\mathrm{P}}[\cdot]$ denotes expectations under the physical measure or P-measure and $E_t^{\mathrm{Q}}[\cdot]$ denotes expectations under the risk-neutral pricing measure or Q-measure. (For any random variable $X_{t+1}$, $E_t^{\mathrm{Q}}[X_{t+1}] = E_t^{\mathrm{P}}[M_{t+1}X_{t+1}]/E_t^{\mathrm{P}}[M_{t+1}]$.) Under the Q-measure, the state variables evolve according to

$$\mathbf{x}_t = \boldsymbol{\mu}^* + \boldsymbol{\Phi}^*\mathbf{x}_{t-1} + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_t, \tag{A.10}$$

where $\boldsymbol{\mu}^* = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0$ and $\boldsymbol{\Phi}^* = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1$.

After extensive, but well-known algebra, it follows that

$$P_t^{(n)} = \exp(a_{(n)} + \mathbf{b}_{(n)}'\mathbf{x}_t), \tag{A.11}$$

where $a_{(n)}$ is a scalar and $\mathbf{b}_{(n)}$ is an $m \times 1$ vector that satisfy the recursions:

$$a_{(n+1)} = -\delta_0 + a_{(n)} + \mathbf{b}_{(n)}'\boldsymbol{\mu}^* + \frac{1}{2}\mathbf{b}_{(n)}'\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathbf{b}_{(n)} \tag{A.12}$$

$$\mathbf{b}_{(n+1)} = \boldsymbol{\Phi}^{*\prime}\mathbf{b}_{(n)} - \boldsymbol{\delta}_1, \tag{A.13}$$

starting from $a_{(1)} = -\delta_0$ and $\mathbf{b}_1 = -\boldsymbol{\delta}_{(1)}$. The continuously compounded yield on an $n$-period zero-coupon bond, $y_t^{(n)}$, is in turn given by

$$y_t^{(n)} = -n^{-1}\log(P_t^{(n)}) = -n^{-1}a_{(n)} - n^{-1}\mathbf{b}'_{(n)}\mathbf{x}_t. \tag{A.14}$$

**Markovian models** We now show that standard affine models—models that are Markovian with respect to these current yield-curve factors—cannot fit our key finding that the sensitivity of long rates to short rates $\beta_h$ declines so strongly with horizon $h$ in the post-2000 data. Applying the estimation methodology of Adrian et al. (2013), we fit affine term-structure models with monthly data using the first $K$ principal components of 1- to 10-year yields as the state variables $\mathbf{x}_t$. We do this in the pre-2000 and post-2000 samples separately for $K = 2$ to 5. We then take the estimated model parameters and work out the model-implied $\beta_h$ regression coefficients. Specifically, let $\boldsymbol{\Gamma}(j) = E[(\mathbf{x}_{t+j} - E[\mathbf{x}_{t+j}])(\mathbf{x}_t - E[\mathbf{x}_t])']$ denote the autocovariance function of the state vector, which can be obtained from the equations $vec(\boldsymbol{\Gamma}(0)) = (\mathbf{I} - \boldsymbol{\Phi}\otimes\boldsymbol{\Phi})^{-1}vec(\boldsymbol{\Sigma}\boldsymbol{\Sigma}')$ and $\boldsymbol{\Gamma}(j) = \boldsymbol{\Phi}^j\boldsymbol{\Gamma}(0)$ for $j \geq 1$. The population coefficient in a regression of $h$-month changes in 120-month yields on $h$-month changes in 12-month yields is then

$$\beta_h = \frac{E[(y_{t+h}^{(120)} - y_t^{(120)})(y_{t+h}^{(12)} - y_t^{(12)})]}{E[(y_{t+h}^{(12)} - y_t^{(12)})^2]} = \frac{1}{10}\frac{\mathbf{b}'_{(120)}[2\boldsymbol{\Gamma}(0) - \boldsymbol{\Gamma}(h) - \boldsymbol{\Gamma}(h)']\mathbf{b}_{(12)}}{\mathbf{b}'_{(12)}[2\boldsymbol{\Gamma}(0) - \boldsymbol{\Gamma}(h) - \boldsymbol{\Gamma}(h)']\mathbf{b}_{(12)}}. \tag{A.15}$$

These model-implied regression coefficients are shown in Panel A of Table A.11. Even if we include a large number of factors as state variables, these standard affine models fail to match the low-frequency decoupling between short- and long-term yields that we observe in the post-2000 data.

**Non-Markovian models** We next consider an alternate affine model that augments the state vector $\mathbf{x}_t$ to include not just $K$ principal components of yields, but also $L-1$ additional lags of these principal components. Thus, by construction, the model is non-Markovian with respect to the filtration given by the current principal components. We furthermore treat these lagged principal components as unspanned state variables, as described earlier. Formally, this means that if the first $K$ elements of the state vector $\mathbf{x}_t$ are the current principal components, all but the first $K$ elements of $\boldsymbol{\delta}_1$ are zero, and the upper right $K \times K(L-1)$ block of $\boldsymbol{\Phi}^*$ is a matrix of zeros. As Duffee (2011) explains, a state variable will be unspanned if it has *perfectly* offsetting effects on the evolution of future short rates and future term premia.

Economically speaking, this is a rather unusual model. However, it allows us to parsimoniously capture our key finding that past changes in the level of rates are useful for forecasting future yields. In addition, similar models have been considered in Joslin et al. (2013). To be clear, we do not believe that the past increase in the level of rates is *literally* unspanned—i.e., that it has no effect on the current yield curve. Instead, we would argue that this variable is *close* to being unspanned. Specifically, like any factor that has a short-lived impact on bond risk premia, past increases in the level of rates should have only a small effect on the yield curve. Thus, in practice it will likely be quite difficult to recover information about this variable from current yields alone—e.g., because yields are measured with a tiny amount of error or because the true data-generating model evolves over time—so conditioning on this variable will add information beyond that revealed by current yields.[8]

Again, the parameters of this augmented model can be estimated, imposing the restriction that the lagged principal components are unspanned factors as in Adrian et al. (2013). The model-implied $\beta_h$ regression coefficients can again be derived from equation (A.15). These model-implied coefficients are shown in Panel B of Table A.11. The augmented model is able to get reasonably close to matching the empirical regression coefficients at both high- and low-frequencies and in both samples.

**Table A.11: Affine Term Structure Model-Implied coefficients in regression of monthly/yearly changes in 10-year yields on changes in 1-year yields** This table reports the slope coefficients in equation (A.15) corresponding to the parameters in an affine term structure model estimated as proposed by Adrian et al. (2013) over August 1971-December 2000 and January 2001-December 2019 subsamples at monthly ($h = 1$) and yearly ($h = 12$) frequencies. The term structure model uses $K$ principal components of yields as state variables in panel A, and adds $L-1$ additional lags of these principal components (for a total of $LK$ state variables) in panel B. $p$-values are also reported; these are two-sided bootstrap $p$-values comparing the sample value of $\beta_{12}/\beta_1$ with the bootstrap distribution using that affine model. As memo items the results of the regressions using actual yields are included—these are simply transcribed from Table 1.

| | | Pre-2000 | | | Post-2000 | | |
|---|---|---|---|---|---|---|---|
| | | $\beta_1$ | $\beta_{12}$ | $p$-value | $\beta_1$ | $\beta_{12}$ | $p$-value |
| Panel A: ATSM with $K$ principal components of yields as factors | | | | | | | |
| $K = 2$ | | 0.42 | 0.49 | 0.89 | 0.74 | 0.66 | 0.000 |
| $K = 3$ | | 0.46 | 0.52 | 0.65 | 0.73 | 0.57 | 0.003 |
| $K = 4$ | | 0.47 | 0.52 | 0.62 | 0.72 | 0.52 | 0.006 |
| $K = 5$ | | 0.47 | 0.52 | 0.52 | 0.69 | 0.50 | 0.007 |
| Panel B: ATSM with $L-1$ lags as additional unspanned factors | | | | | | | |
| $K = 2$ | $L = 6$ | 0.42 | 0.50 | 0.84 | 0.74 | 0.27 | 0.65 |
| $K = 3$ | $L = 6$ | 0.46 | 0.54 | 1.00 | 0.74 | 0.31 | 0.41 |
| $K = 2$ | $L = 12$ | 0.42 | 0.51 | 0.76 | 0.77 | 0.24 | 0.89 |
| $K = 3$ | $L = 12$ | 0.46 | 0.55 | 0.90 | 0.77 | 0.29 | 0.60 |
| Memo: Estimates in data (from Table 1) | | 0.46 | 0.56 | | 0.64 | 0.20 | |

[8] Indeed the model in Section 3, would not feature unspanned variables if we recast the model to have a set of zero-coupon bonds with different maturities. Using the resulting affine model, one could recover the full $(k+2)$-dimensional state vector $\mathbf{x}_t$ from any set of $(k+2)$ yields. However, many of these variables would be close to unspanned—they would have only minimal effects on yields—and, in practice, it would be difficult to extract them from yields.

**A bootstrap-based test of model misspecification** As another way of looking at this, we use a bootstrap to test the hypothesis that each estimated affine term structure model is correctly specified. Our test uses the ratio of yearly to monthly coefficients ($\beta_{12}/\beta_1$) as the test statistic; the test rejects if this ratio is too low or too high to have been generated by the estimated Q-measure model. This is a test in the spirit of Giglio and Kelly (2018), who test the hypothesis that a given affine model is correctly specified by checking whether the comovement of rates at different points on the term structure is consistent with the estimated Q-measure dynamics.

To implement this test, we simulate the bootstrap distribution of the ratio $\beta_{12}/\beta_1$ for each estimated affine term-structure model. To do this for a given model, we first generate bootstrapped time series of the state vector $\mathbf{x}_t$ using a residual-based bootstrap based on our estimates of the P-measure dynamics in equation (A.7). Using our estimates of $a_{(n)}$ and $\mathbf{b}_{(n)}$ for that same model—which reflect the estimated dynamics under the Q-measure—in combination with a set of bootstrapped draws of the corresponding yield measurement errors, we obtain a bootstrapped time series of yields $y_t^{(n)}$ for $n = 12$ and 120 months. We then compute $\beta_{12}$, $\beta_1$, and the ratio $\beta_{12}/\beta_1$ for each bootstrapped time series. Repeating this exercise many times, we obtain a bootstrap distribution for the ratio $\beta_{12}/\beta_1$.

For each model, we then compute the two-sided $p$-value of the observed ratio $\beta_{12}/\beta_1$ with respect to this bootstrap distribution. These $p$-values are also reported in Table A.11. We find that in the pre-2000 sample, none of the models are rejected. In the post-2000 sample, the Markovian models are decisively rejected: if these standard models were correctly specified it would be highly unlikely to observe a value of $\beta_{12}/\beta_1$ as small as we do in the data. However, the non-Markovian models are not rejected in the post-2000 sample. In this way, we again conclude that a non-Markovian term structure model is required to match the yield-curve dynamics in the post-2000 data.

**Conclusion** To summarize, our conclusion is that affine term-structure models need to include lagged yield-curve factors to match the frequency-dependent sensitivity of long-term rates we observe in recent years. A large number of static yield curve factors will not do the job.

# B Rate-amplification mechanisms: Microfoundations

In this Appendix, we formalize the three supply-and-demand driven channels of rate amplification discussed in the main text: the mortgage refinancing channel, the investor extrapolation channel, and the reaching-for-yield channel. For each channel, we first show how it can be used to microfound rate-amplifying shocks to the net supply of long-term bonds similar to those we introduced in reduced-form in Section 3 of the main text. We then embed each channel our general modelling framework and provide an illustrative calibration.

## B.1 The mortgage refinancing channel

The model setup follows Malkhozov et al. (2016).[9] There is a constant face value $M$ of outstanding long-term, *fixed-rate* mortgages with an embedded prepayment option. The primary mortgage rate, denoted $y_t^M$, equals the long-term bond yield, $y_t$, plus a constant spread, $\lambda$: $y_t^M = y_t + \lambda$. (This constant spread play no role in the resulting analysis and can be set to zero without loss of generality.) Let $c_t^M$ denote the average coupon on outstanding mortgages at the beginning of time $t$. We assume that $c_t^M$ evolves according to the following law of motion:

$$c_{t+1}^M - c_t^M = -\eta \cdot (c_t^M - y_t^M), \tag{A.16}$$

where $\eta \in [0, 1]$. The difference between the beginning-of-period average mortgage coupon $c_t^M$ and the current primary mortgage rate $y_t^M$ is called the "refinancing incentive." Thus, according to equation (A.16), when the refinancing incentive is higher at time $t$, more households refinance their existing high-coupon mortgages at time $t$, leading the average mortgage coupon to fall from $t$ to $t+1$. Iterating on equation (A.16) and making use of the fact that $y_t^M = y_t + \lambda$, we obtain:

$$c_t^M = \sum_{j=0}^{\infty} \eta \, (1-\eta)^j \, y_{t-1-j}^M = \sum_{j=0}^{\infty} \eta \, (1-\eta)^j \, y_{t-1-j} + \lambda. \tag{A.17}$$

Thus, the average mortgage coupon is just a backward-looking, geometric average of past long-term yields plus a constant. While clearly a simplification, this is a good empirical description of the average coupon on outstanding mortgages.[10]

We assume the *effective* gross supply of long-bonds that bond investors must hold at time $t$ is

$$s_t = M \cdot DUR_t^M, \tag{A.18}$$

where $M$ is face value of outstanding mortgages and $DUR_t^M$ is the average "duration" or effective maturity of outstanding mortgages at time $t$.[11] When $s_t$ is high, bond investors must collectively bear greater interest rate risk in equilibrium. We assume that average mortgage duration at time $t$ is

$$DUR_t^M = \overline{DUR}^M - N \cdot \left(c_t^M - y_t^M\right), \tag{A.19}$$

where $N > 0$ is the so-called "negative convexity" of the average mortgage. Intuitively, when the refinancing incentive $(c_t^M - y_t^M)$ is high, many households are likely to refinance their mortgages in the near-term, implying that the average mortgage behaves more like a short-term bond—i.e.,

[9] Hanson (2014) explores the mortgage refinancing channel in a two period model. We follow the modelling approach in Malkhozov et al. (2016) since this allows us to speak to the dynamics which are our primary focus here.

[10] A realistic elaboration would incorporate state-dependence in the elasticity of refinancing with respect to the incentive. For instance, one might assume $c_{t+1}^M - c_t^M = -\eta \left[c_t^M - y_t^M\right] \cdot (c_t^M - y_t^M)$ where $\eta[\cdot] > 0$ and $\eta'[\cdot] > 0$, implying that the average coupon falls more when $c_t^M > y_t^M$ than when $c_t^M < y_t^M$—see e.g., Berger et al. (2018); Eichenbaum et al. (2018)

[11] Formally, duration is the semi-elasticity of a bond's price with respect to its yield. Thus, the longer a bond's duration, the greater is its exposure to movements in interest rates.

$DUR_t^M$ is low and bond investors must bear less interest rate risk. By contrast, when the refinancing incentive is low, households are less likely to refinance and the typical mortgage behaves more like a long-term bond. Again, this is a good empirical description of $DUR_t^M$ (Hanson, 2014; Malkhozov et al., 2016).[12]

Combining equations (A.17), (A.18), and (A.19), the *effective* supply of long-bonds at time $t$ is

$$s_t = M \cdot \overline{DUR}^M + MN \cdot (y_t - \textstyle\sum_{j=0}^{\infty} \eta\,(1-\eta)^j\, y_{t-1-j}). \tag{A.20}$$

In other words, bond investors must bear greater interest rate risk when the long-term yield is currently high relative to its backward-looking, geometric average—i.e., when interest rates have recently risen.

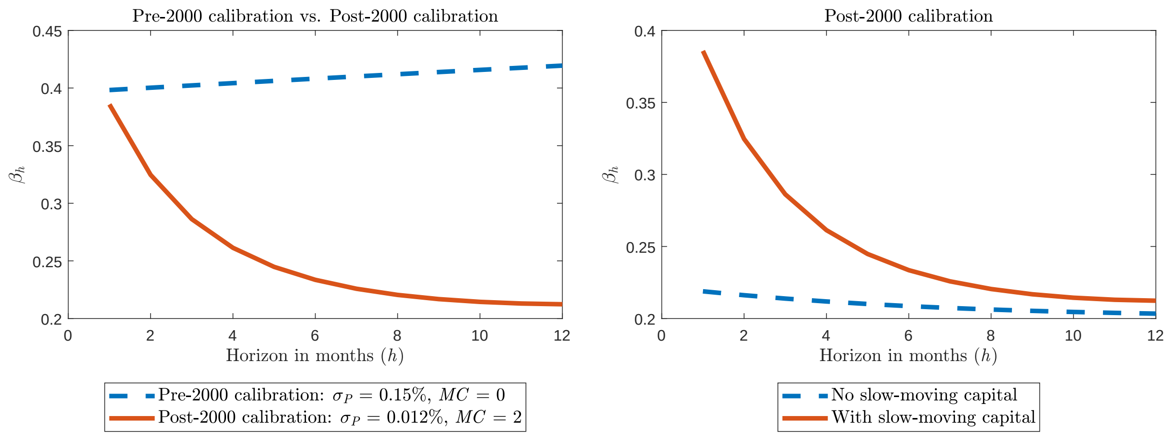
In Internet Appendix C, we solve and calibrate this model of the mortgage refinancing channel. In this version of the model, there are two reasons why shocks to short-term interest rates give rise to transitory movements in the term premium component of long-term yields. First, when $\eta > 0$, mortgage refinancing waves trigger *temporary* shifts in the effective supply of long-term bonds—i.e., these effective supply shocks are less persistent than the underlying shocks to short-term interest rates. Second, these supply shocks are met by a slow-moving arbitrage response. This combination of transitory supply shocks and a slow-moving arbitrage response creates short-lived imbalances in the market for long-term bonds, leading long-term yields to temporarily overreact to short-term rates.[13]

Naturally, this version of the model can match the key stylized fact we have documented, namely that $\beta_h = \beta_h = Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]/Var\left[i_{t+h} - i_t\right]$ is a sharply declining function of $h$ in the post-2000 data but not in the pre-2000 data. As an illustrative calibration, we assume that $MN = 2$ in the post-2000 data and $MN = 0$ in the pre-2000 calibration. In other words, we assume that the mortgage refinancing channel is operative in the post-2000 period, but was not operative in the

[12] As detailed in Hanson (2014), there are two key reasons why movements in expected mortgage refinancing temporarily alters the aggregate amount of interest rate risk that specialized bond investors must bear. First, households only *gradually* refinance their mortgages following a decline in primary mortgage rates. Second, household borrowers do not alter their asset-side holdings of long-term bonds to hedge the time-varying interest rate risk they are bearing on the liability side. In combination, these features mean that households are effectively borrowing shorter term during refinancing waves when the refinancing incentive, $(c_t^M - y_t^M)$, is high. As a result, households bear greater interest rate risk during refinancing waves, while bond investors bear less risk. In summary, refinancing waves function like shocks to the effective supply of long-term bonds because risk sharing between households and specialized bond investors is imperfect and varies over time.

[13] One simplification of this model is that all bond investors hold mortgage-backed securities (MBS) and, thus, bear a time-varying amount interest rate risk. In practice, two different kinds of investors own MBS. One set of MBS investors—e.g., mortgage banks and the government sponsored enterprises—"delta-hedge" the embedded prepayment option and, thus, bear a (relatively) constant amount of interest rate risk over time. Other MBS investors do not delta-hedge and bear a time-varying amount of risk. As discussed in Hanson (2014), in the first instance, it does not matter whether some MBS holders delta-hedge the prepayment option since the relevant hedging flows correspond one-for-one with changes in the aggregate quantity of duration risk. However, a slow-moving arbitrage response to refinancing waves arguably becomes more relevant to the extent that some MBS investors delta-hedge their time-varying interest rate exposure.

**Figure A.5: Illustrative calibration of mortgage refinancing model.**




pre-2000 period. We assume $\eta = 0.15$ in both periods. The values of all other model parameters, including $q = 0.30$ and $k = 12$, are the same as those in the calibrations in Section 3.

## B.2 Investor extrapolation channel

Recalling that $i_t = i_{P,t} + i_{T,t}$, we assume that diagnostic investors make biased forecasts of the persistent and transitory components of short-term interest rates. Following Maxted (2020), we assume the expectations of diagnostic investors are given by:

$$E_t^D [i_{P,t+1}] = \bar{\imath} + \rho_P (i_{P,t} - \bar{\imath}) + \theta \cdot m_{P,t}, \quad \text{(A.21a)}$$

$$E_t^D [i_{T,t+1}] = \rho_T i_{T,t} + \theta \cdot m_{T,t}, \quad \text{(A.21b)}$$

where

$$m_{P,t} = \kappa_P m_{P,t-1} + \varepsilon_{P,t} = (i_{P,t} - \bar{\imath}) - (\rho_P - \kappa_P) \sum\nolimits_{j=0}^{\infty} \kappa_P^j (i_{P,t-j-1} - \bar{\imath}), \quad \text{(A.22a)}$$

$$m_{T,t} = \kappa_T m_{T,t-1} + \varepsilon_{T,t} = i_{T,t} - (\rho_T - \kappa_T) \sum\nolimits_{j=0}^{\infty} \kappa_T^j i_{T,t-j-1}, \quad \text{(A.22b)}$$

$\theta \geq 0$, $\kappa_P \in [0, \rho_P]$, and $\kappa_T \in [0, \rho_T]$. When $\theta = 0$, diagnostic expectations coincide with rational expectations, which we continue to denote using $E_t [\cdot]$. When $\theta > 0$, equations (A.21) and (A.22) imply that diagnostic investors tend to overestimate future short-term rates when short rates have recently risen. And, the $\kappa_P$ and $\kappa_T$ parameters govern the persistence of their mistaken beliefs about short rates.[14] While diagnostic investors make biased forecasts of short rates, we assume for simplicity that they form rational forecasts of all other relevant state variables.

[14] As shown in Maxted (2020), these $\kappa$ parameters are a simple way of parameterizing the "background context" that diagnostic investors use to assess the "representativeness" of incoming data for future states. Specifically, in the limit where $\kappa_P$ and $\kappa_T \to 0$, the background context when making forecasts at time $t$ is what diagnostic investors knew at time $t-1$ as in Bordalo et al. (2017) and the resulting expectational errors are very short-lived. In the opposite limit where $\kappa_P \to \rho_P$ and $\kappa_T \to \rho_T$, the background context at time $t$ is the unconditional distribution of short rates as in D'Arienzo (2020) and the resulting expectational errors are far more persistent.

A mass $f$ of bond investors have diagnostic expectations and their demand for long-term bonds is:

$$h_t = \tau \frac{E_t^D [rx_{t+1}]}{Var_t^D [rx_{t+1}]} = \tau \frac{E_t^D [rx_{t+1}]}{Var_t [rx_{t+1}]}, \tag{A.23}$$

where $E_t^D [rx_{t+1}]$ denotes diagnostic investors' biased expectation of bond excess returns.[15] There is a mass $(1-f)$ of a bond investors with rational expectations. Of these rational investors, fraction $q$ are fast-moving with demands $b_t = \tau (E_t [rx_{t+1}] / Var_t [rx_{t+1}])$ and fraction $(1-q)$ are slow-moving and only rebalance the portfolios every $k$ periods. The demand for long-term bonds from the subset of slow-moving investors who are active at time $t$ is $d_t = \tau(E_t[\sum_{j=1}^k rx_{t+j}]/Var_t[\sum_{j=1}^k rx_{t+j}])$. We assume the *gross supply* of long-term bonds is *constant* over time and equal to $\bar{s}$. Thus, the market clearing condition for long-term bonds at time $t$ is:

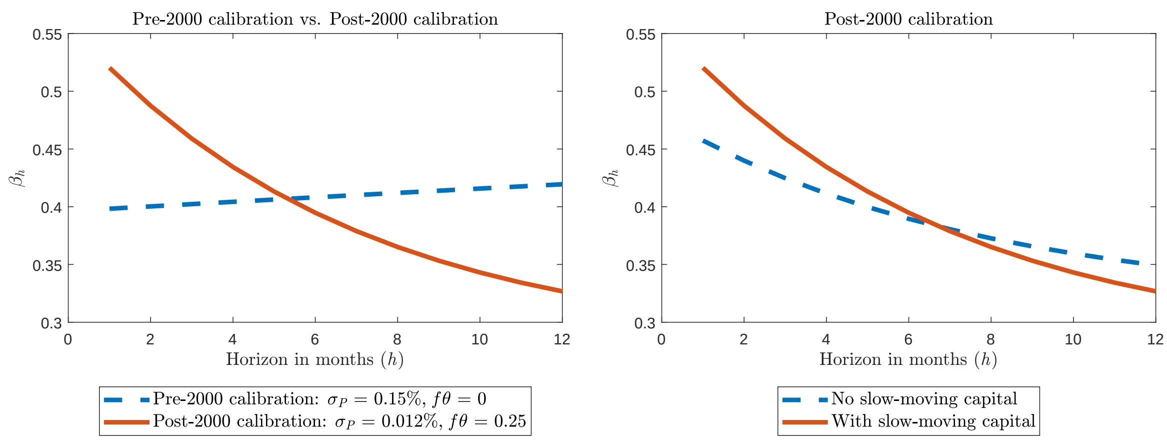
$$\overbrace{fh_t + (1-f)\, qb_t + (1-f)\,(1-q)\, k^{-1} d_t}^{\text{Active demand}} = \overbrace{\bar{s} - (1-f)\,(1-q) k^{-1} \sum\nolimits_{i=1}^{k-1} d_{t-i}}^{\text{Active supply}}. \tag{A.24}$$

In Internet Appendix C, we solve for equilibrium in this setting. When short rates have recently fallen, diagnostic investors underestimate future short-term interest rates and, as a result, want to hold more long-term bonds. To accommodate this induced demand shock, rational investors must reduce their holdings of long-term bonds, pushing down the term premium compensation required by rational investors. Since the biases of diagnostic investors are tied to recent changes in short rates, this model is nearly isomorphic to our reduced-form specification where the short-rate-driven shocks to bond supply are more transient than short rates. In particular, this investor extrapolation channel leads long-term rates to *temporarily* overreact to movements in short rates due to the combination of (i) transitory shifts in non-fundamental demand for long-term bonds that are triggered by short rate shocks and (ii) a slow-moving arbitrage response to these non-fundamental demand shifts.

As shown in the illustrative calibration below, this model in which investors extrapolate changes in short-term interest rates can match the key stylized facts we document. In the post-2000 period, we assume that $f = 50\%$ of investors have diagnostic expectations with parameter $\theta = 0.5$ (we set $\kappa_P = \kappa_T = 0.8$) and that $q = 0.15$ and $k = 18$, so there is a fairly slow-moving arbitrage response to the resulting non-fundamental shifts in demand for long-term bonds. In the pre-2000 period, we assume that $f = 0$. As discussed in the main text, the rise in $f$ is meant to capture the growing importance of extrapolation-prone bond fund investors in the U.S. bond market in recent decades. The values of all other model parameters are the same as those in the calibrations in Section 3.

[15] We have $Var_t^D [rx_{t+1}] = Var_t [rx_{t+1}]$ since, as shown by Maxted (2020), diagnostic investors perceive the same conditional variance of future short-term interest rates as rational investors.

**Figure A.6: Illustrative calibration of the investor extrapolation model.**




## B.3 Reaching-for-yield channel

We assume that fraction $f$ of bond investors are "yield-seeking" and have non-standard preferences as in Hanson and Stein (2015). The idea is that, for either frictional or behavioral reasons, these investors care about the *current yield* on their portfolios over and above expected portfolio returns. Specifically, yield-seeking investors' demand for long-term bonds is:

$$h_t = \tau \frac{y_t - i_i}{V^{(1)}}. \tag{A.25}$$

Since $E_t[rx_{t+1}] = (y_t - i_i) - (\phi/(1-\phi)) \cdot E_t[y_{t+1} - y_t]$, equation (A.25) implies that yield-seeking investors are only concerned with the current income or carry from holding long-term bonds and neglect any expected capital gains and losses from holding long-term bonds. A mass $(1-f)$ of a bond investors are expected-return-oriented and have standard mean-variance preferences. Of these expected-return-oriented investors, fraction $q$ are fast-moving investors with demands $b_t = \tau(E_t[rx_{t+1}]/Var_t[rx_{t+1}])$ and fraction $(1-q)$ are slow-moving. The demand for long-term bonds from the subset of slow-moving investors who are active at time $t$ is $d_t = \tau(E_t[\sum_{j=1}^{k} rx_{t+j}]/Var_t[\sum_{j=1}^{k} rx_{t+j}])$. We assume the *gross supply* of long-term bonds is *constant* over time and equal to $\bar{s}$. Thus, the market clearing condition for long-term bonds at time $t$ is the same as in equation (A.24).

In Internet Appendix C, we solve for equilibrium in this setting. To build intuition, first consider the case where there is no slow-moving capital. In this case where $q = 1$, our model is simply an infinite-horizon version of the 2-period model in Hanson and Stein (2015). Because expected mean reversion in short rates implies that the yield curve is steep when short rates are low, yield-seeking investors' demand for long-term bonds is higher when short rates are lower.[16] To accommodate this induced demand shock, expected-return-oriented investors must reduce their holdings of long-term bonds when short rates are low, pushing down the term premium compensation they require.

[16] Formally, $E_t[y_{t+1} - y_t] > 0$ when $i_t$ is low, so $(y_t - i_i) > E_t[rx_{t+1}]$ when $i_t$ is low. As a result, $h_t = \tau((y_t - i_i)/Var_t[rx_{t+1}]) > \tau(E_t[rx_{t+1}]/Var_t[rx_{t+1}]) = b_t$ when $i_t$ is low. Conversely, $h_t < b_t$ when $i_t$ is high.

Thus, in the absence of slow-moving capital, long-term rates are excessively sensitive to short rates because short rates and term premium move in the same direction. However, without slow-moving capital, this excess sensitivity is *not* greater at short horizons—i.e., changes in short rates do not create temporary market imbalances.

When $q < 1$, our model adds a slow-moving arbitrage response to the price-pressure created by yield-seeking investors. This means that the excess sensitivity of long-term rates to short-term rates will be greatest at short horizons. The intuition is simple. Suppose there is an decline in short rates which steepens the yield curve, thereby boosting yield-seeking investors' demand for long-term bonds. In the short-run, the only expected-return-oriented investors can absorb this induced demand shock for long-term bonds are the fast-moving ones and the slow-moving ones who initially happen to be active. However, the mass of slow-moving investors who can absorb this induced demand shock grows over time. As a result, the excess sensitivity of long-term rates to movements in short-term rates is greatest at high frequencies and diminishes at lower frequencies.

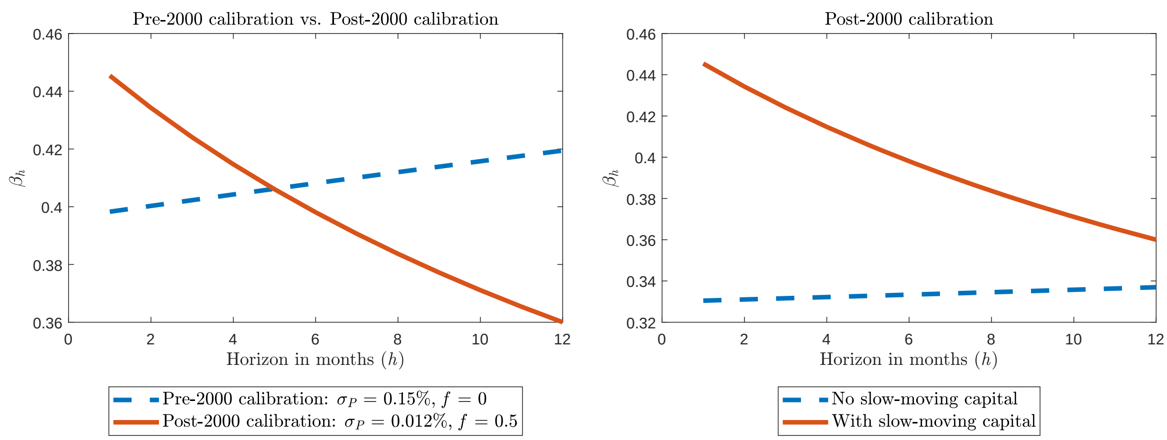
This model of the reaching-for-yield channel can qualitatively match the key stylized facts we have documented. Specifically, in the post-2000 period, we assume that $f = 0.5$, $q = 0.1$, and $k = 24$—i.e., we assume a good deal of reaching for yield and a fairly sluggish arbitrage response. By contrast, we assume that $f = 0$ in the pre-2000 period. The rise in $f$ is consistent with the idea that yield-seeking investor behavior has become stronger in recent decades. The values of all other model parameters are the same as those in the calibrations in Section 3. As shown below, we see that $\beta_h$ is decreasing in $h$ in the post-2000 calibration, but is increasing in $h$—and far less variable—in the pre-2000 calibration.

While the combination of reaching-for-yield and slow-moving capital generates horizon-dependent sensitivity, our calibrations struggle to *quantitatively* match the profile of $\beta_h$ seen in the post-2000 data. Specifically, comparing the calibration of the reaching-for-yield model in Figure A.7 with those for the mortgage refinancing and investor extrapolation models in Figures A.5 and A.6, respectively, we see that the former model struggles generate quantitatively match the steep post-2000 profile of $\beta_h$. This is because the reaching-for-yield channel generates highly persistent shifts rate-amplifying net supply, whereas the refinancing and extrapolation channels generate transitory shifts in net supply. And, as we have emphasized throughout, strongly horizon-dependent excess sensitivity is most likely to arise when transitory rate-amplifying supply-and-demand shocks are met by a slow-moving arbitrage response.

# C Solution of the baseline model

This Appendix provides additional details on our economic model. In particular, we provide additional details on how we solve for the rational expectations equilibrium of our model.

**Figure A.7: Illustrative calibration of the investor reaching-for-yield model.**




## C.1 Long-term nominal bonds

In this subsection, we derive the Campbell and Shiller (1988) approximation to the return on a default-free perpetuity.

Consider a perpetual default-free nominal bond which pay a nominal coupon of $K$ each period. Let $P_t$ denote the nominal price of the long-term bond at time $t$. Thus, the nominal return on the long-term bond from $t$ to $t+1$ is

$$1 + R_{t+1} = \frac{P_{t+1} + K}{P_t}. \tag{A.26}$$

Defining $\phi \equiv 1/(1+K) < 1$, the one-period log return on the bond from time $t$ to $t+1$ is approximately

$$r_{t+1} \equiv \ln(1 + R_{t+1}) \approx \overbrace{\frac{1}{1-\phi}}^{D} y_t - \overbrace{\frac{\phi}{1-\phi}}^{D-1} y_{t+1}, \tag{A.27}$$

where $y_t$ is the log yield-to-maturity at time $t$ and

$$D = \frac{1}{1-\phi} = \frac{K+1}{K} \tag{A.28}$$

is the Macaulay duration when the bond is trading at par. The log-linear approximation for default-free coupon-bearing bonds in equation (A.27) appears in Chapter 10 of Campbell et al. (1996).

To derive this approximation, note that the Campbell-Shiller (1988) approximation of the 1-period log return on the long-term bond is

$$r_{t+1} = \ln(P_{t+1} + K) - p_t \approx \varphi + \phi p_{t+1} + (1-\phi) k - p_t, \tag{A.29}$$

where $p_t = \log(P_t)$ is the log price, $k = \log(K)$ is the log coupon, and where $\phi = 1/(1 + \exp(k - \overline{p}))$ and $\varphi = -\log(\phi) - (1-\phi)\log(\phi^{-1} - 1)$ are parameters of the log-linearization. Iterating equation

(A.29) forward, we find that the log bond price is

$$p_t = (1-\phi)^{-1}\varphi + k - \sum_{i=0}^{\infty}\phi^i E_t\left[r_{t+i+1}\right]. \tag{A.30}$$

Applying this approximation to the *yield-to-maturity*, defined as the *constant return* that equates bond price and the discounted value of promised cashflows, we obtain

$$p_t = (1-\phi)^{-1}\varphi + k - (1-\phi)^{-1} y_t. \tag{A.31}$$

Equation (A.27) then follows by substituting the expression for $p_t$ in equation (A.31) into the Campbell-Shiller return approximation in equation (A.29).

Assuming the steady-state price of the bonds is par ($\overline{p}=0$), we have $\phi = 1/(1+K)$. Thus, bond duration is $D = -\partial p_t/\partial y_t = (1-\phi)^{-1} = (1+K)/K$. Since $-\partial p_t/\partial y_t = -(\partial P_t/\partial Y_t)((1+Y_t)/P_t) = (Y_t^{\cdot}+1)/Y_t$ this corresponds to Macaulay duration when the bonds are trading at par ($Y_t = K$).

Let $i_t$ denote the interest rate on short-term nominal bonds from $t$ to $t{+}1$ and let $rx_{t+1} \equiv r_{t+1} - i_t$ denote the excess return on long-term nominal bonds over short-term nominal bonds from $t$ to $t{+}1$. Then, iterating equation (A.27) forward and taking expectations, the yield on long-term nominal bonds is given by:

$$y_t = (1-\phi)\sum_{j=0}^{\infty}\phi^j E_t\left[i_{t+j} + rx_{t+j+1}\right]. \tag{A.32}$$

## C.2 Bond market participants

There are two types of risk-averse arbitrageurs in the model, each with identical risk tolerance $\tau$, who differ solely in the frequency with which they can rebalance their bond portfolios.

The first group of arbitrageurs are fast-moving arbitrageurs who are free to adjust their holdings of long-term and short-term bonds each period. Fast-moving arbitrageurs are present in mass $q$ and we denote their demand for long-term bonds at time $t$ by $b_t$. Fast-moving arbitrageurs have mean-variance preferences over 1-period portfolio log returns. Thus, their demand for long-term bonds at time $t$ is given by

$$b_t = \tau \frac{E_t\left[rx_{t+1}\right]}{Var_t\left[rx_{t+1}\right]}, \tag{A.33}$$

where

$$rx_{t+1} \equiv r_{t+1} - i_t = \frac{1}{1-\phi} y_t - \frac{\phi}{1-\phi} y_{t+1} - i_t \tag{A.34}$$

is the excess return on long-term bonds from $t$ to $t+1$.

The second group of arbitrageurs is a set of slow-moving arbitrageurs who can only adjust their holdings of long-term and short-term bonds every $k$ periods. Slow-moving arbitrageurs are present in mass $1-q$. A fraction $1/k$ of these slow-moving arbitrageurs is active each period and can reallocate their portfolios. However, they must then maintain this same portfolio allocation for the next $k$ periods. As in Duffie (2010), this is a reduced-form way to model the frictions that limit the speed of capital flows. Since they only rebalance their portfolios every $k$ periods, slow-moving arbitrageurs have mean-variance preferences over their $k$-period *cumulative* portfolio excess return. Thus, the demand for long-term bonds from the subset of slow-moving arbitrageurs who are active at time $t$ is given by

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{A.35}$$

### C.3 Risk factors

**Short-term nominal interest rates:** Short-term nominal bonds are available in perfectly elastic supply. At time $t$, arbitrageurs learn that short-term bonds will earn a riskless log return of $i_t$ in nominal terms between time $t$ and $t+1$. We assume that the short-term nominal interest rate is the sum of a highly persistent component $i_{P,t}$ and a more transient component $i_{T,t}$:

$$i_t = i_{P,t} + i_{T,t}. \tag{A.36}$$

We assume that the persistent component $i_{P,t}$ follows an exogenous AR(1) process:

$$i_{P,t+1} = \bar{\imath} + \rho_P \left(i_{P,t} - \bar{\imath}\right) + \varepsilon_{P,t+1}, \tag{A.37}$$

where $0 < \rho_P < 1$ and $Var_t\left[\varepsilon_{P,t+1}\right] = \sigma_P^2$. Similarly, we assume that the transient component $i_{T,t}$ follows an exogenous AR(1) process:

$$i_{T,t+1} = \rho_T i_{T,t} + \varepsilon_{T,t+1}, \tag{A.38}$$

where $0 < \rho_T \leq \rho_P < 1$ and $Var_t\left[\varepsilon_{T,t+1}\right] = \sigma_T^2$.

**Supply of long-term bonds:** We assume that the long-term nominal bond is available in an exogenous, time-varying *net supply* $s_t$ that must be held in equilibrium by fast arbitrageurs and slow-moving arbitrageurs. This net supply equals the *gross supply* of long-term bonds *minus the demand* for long-term bonds from any unmodeled agents who have inelastic demand for these bonds. Formally, we assume that $s_t$ follows an AR(1) process:

$$s_{t+1} = \bar{s} + \rho_s \left(s_t - \bar{s}\right) + \varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}, \tag{A.39}$$

where $0 < \rho_s \leq \rho_T$, $C > 0$, and $Var_t\left[\varepsilon_{s,t+1}\right] = \sigma_s^2$. The $\varepsilon_{s,t+1}$ shocks in equation (A.39) capture other forces that are unrelated to short rates which also impact the net supply of long-term bonds. While the model can be solved for any arbitrary correlation structure between the $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$ shocks, we assume, for simplicity, that these three underlying shocks are mutually orthogonal.

To understand the implied process for net bond supply, let $L$ denote the time-series lag operator and note

$$\begin{aligned}(1-\rho_s L)(s_t-\overline{s}) &= \varepsilon_{s,t}+C\varepsilon_{P,t}+C\varepsilon_{T,t} \\ &= \varepsilon_{s,t}+C(1-\rho_P L)(i_{P,t}-\overline{\imath})+C(1-\rho_T L)\,i_{T,t}.\end{aligned} \tag{A.40}$$

Working out the lag polynomial, we see that

$$\begin{aligned}s_t = \overline{s} &+ C[(i_{P,t}-\overline{\imath})-(\rho_P-\rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j(i_{P,t-j-1}-\overline{\imath})] \\ &+ C[i_{T,t}-(\rho_T-\rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j i_{T,t-j-1}]+[\textstyle\sum_{j=0}^{\infty}\rho_s^j\varepsilon_{s,t-j}].\end{aligned} \tag{A.41}$$

which follows from the fact that

$$\begin{aligned}(1-\rho_s L)^{-1}(1-\rho_x L)\,x_t &= \textstyle\sum_{j=0}^{\infty}\rho_s^j L^j(1-\rho_x L)\,x_t \\ &= x_t-\rho_x x_{t-1}+\rho_s x_{t-1}-\rho_s\rho_x x_{t-2}+\rho_s^2 x_{t-2}-\rho_s^3\rho_x x_{t-2}+\cdots \\ &= x_t-(\rho_x-\rho_s)\textstyle\sum_{j=0}^{\infty}\rho_s^j x_{t-j-1}.\end{aligned} \tag{A.42}$$

### C.4 Equilibrium Conjecture

For the sake of concreteness, suppose that $k=4$. We conjecture that equilibrium yields take the form

$$y_t=\alpha_0+\boldsymbol{\alpha}_1'\mathbf{x}_t, \tag{A.43}$$

and that the demands of active slow-moving arbitrageurs are of the form

$$d_t=\delta_0+\boldsymbol{\delta}_1'\mathbf{x}_t, \tag{A.44}$$

where the $k+2$ dimensional state vector is

$$\mathbf{x}_t=\begin{bmatrix} i_{P,t}-\overline{i} \\ i_{T,t} \\ s_t-\overline{s} \\ d_{t-1}-\delta_0 \\ d_{t-2}-\delta_0 \\ d_{t-3}-\delta_0 \end{bmatrix}. \tag{A.45}$$

These assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix $\Gamma$ is a function of the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageur demand so we write $\Gamma=\Gamma(\boldsymbol{\delta}_1)$. Specifically, we have

$$\begin{aligned}\mathbf{x}_{t+1} &= \Gamma(\delta)\,\mathbf{x}_t+\boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_s & 0 & 0 & 0 \\ \delta_P & \delta_T & \delta_s & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_{P,t}-\overline{i} \\ i_{T,t} \\ s_t-\overline{s} \\ d_{t-1}-\delta_0 \\ d_{t-2}-\delta_0 \\ d_{t-3}-\delta_0 \end{bmatrix}+\begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ \varepsilon_{s,t+1}+C\varepsilon_{P,t+1}+C\varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}\end{aligned} \tag{A.46}$$

where $\Sigma \equiv Var_t[\boldsymbol{\epsilon}_{t+1}]$. Since we have assumed that $\varepsilon_{P,t+1}$, $\varepsilon_{T,t+1}$, and $\varepsilon_{s,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & C\sigma_P^2 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & C\sigma_\pi^2 & 0 & 0 & 0 \\ C\sigma_P^2 & C\sigma_T^2 & C^2\sigma_P^2 + C^2\sigma_T^2 + \sigma_s^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.47}$$

We adopt the convention that $\mathbf{e}$ is the vector with a 1 corresponding to $i_{P,t} - \bar{i}$ and $i_{T,t}$ and 0s elsewhere, i.e., $\mathbf{e} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}'$; $\mathbf{e}_s$ is the basis vector with a 1 corresponding to $s_t - \bar{s}$ and 0s elsewhere, i.e., $\mathbf{e}_s = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}'$; and $\mathbf{e}_d$ is a 1 corresponding to $d_{t-1} - \delta_0$, $d_{t-2} - \delta_0$, $\cdots$, $d_{t-(k-1)} - \delta_0$ and 0s elsewhere, i.e., $\mathbf{e}_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}'$.

Finally, we denote $\mathbf{C}_{[t+i,t+j]} = Cov[\mathbf{x}_{t+i}, \mathbf{x}_{t+j} | \mathbf{x}_t]$ and note that

$$\mathbf{C}_{[t+i,t+j]} = \sum_{s=1}^{\min\{i,j\}} [\mathbf{\Gamma}^{i-s}] \mathbf{\Sigma} [\mathbf{\Gamma}^{j-s}]', \tag{A.48}$$

so $\mathbf{C}_{[t+j,t+i]} = \mathbf{C}'_{[t+i,t+j]}$.

## C.5 Arbitrageur demands

**Fast-moving arbitrageurs' demand:** Given this conjecture, we work out fast-moving arbitrageurs' demand for long-term bonds. Given the conjectured form of equilibrium yields, the realized 1-period excess returns on long bonds from $t$ to $t+1$ is

$$\begin{aligned} rx_{t+1} &= \frac{1}{1-\phi} y_t - \frac{\phi}{1-\phi} y_{t+1} - i_t \\ &= \left(\alpha_0 - \bar{i}\right) + \left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' \mathbf{x}_t - \left(\frac{\phi}{1-\phi}\boldsymbol{\alpha}_1\right)' \mathbf{x}_{t+1}, \end{aligned} \tag{A.49}$$

which implies

$$E_t[rx_{t+1}] = \left(\alpha_0 - \bar{i}\right) + \left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' \mathbf{x}_t - \left(\frac{\phi}{1-\phi}\boldsymbol{\alpha}_1\right)' \mathbf{\Gamma} \mathbf{x}_t \tag{A.50}$$

and

$$Var_t[rx_{t+1}] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \mathbf{\Sigma} \boldsymbol{\alpha}_1 \tag{A.51}$$

Thus, fast-moving arbitrageurs' demand for long-term bonds is

$$b_t = \tau \frac{E_t[rx_{t+1}]}{Var_t[rx_{t+1}]} = \left[\tau \frac{\alpha_0 - \bar{i}}{\left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \mathbf{\Sigma} \boldsymbol{\alpha}_1}\right] + \left[\tau \frac{\left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' - \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1' \mathbf{\Gamma}}{\left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \mathbf{\Sigma} \boldsymbol{\alpha}_1}\right] \mathbf{x}_t. \tag{A.52}$$

**Slow-moving arbitrageurs' demand:** We next work out slow-moving arbitrageurs' demand for long-term bonds. Given our conjecture, the realized $k$-period cumulative excess returns on long bonds from $t$ to $t+k$ is

$$\begin{aligned}\textstyle\sum_{j=1}^{k} rx_{t+j} &= \textstyle\sum_{j=0}^{k-1} (y_{t+j} - i_{t+j}) - \frac{\phi}{1-\phi} (y_{t+k} - y_t) \\ &= k\left(\alpha_0 - \bar{i}\right) + (\boldsymbol{\alpha}_1 - \mathbf{e})' \left(\textstyle\sum_{j=0}^{k-1} \mathbf{x}_{t+j}\right) - \frac{\phi}{1-\phi} \boldsymbol{\alpha}_1' (\mathbf{x}_{t+k} - \mathbf{x}_t)\end{aligned} \tag{A.53}$$

Thus, expected $k$-period cumulative returns are

$$E_t[\textstyle\sum_{j=1}^{k} rx_{t+j}] = k\left(\alpha_0 - \bar{i}\right) + \left((\boldsymbol{\alpha}_1 - \mathbf{e})' (\mathbf{I} - \boldsymbol{\Gamma})^{-1} + \frac{\phi}{1-\phi} \boldsymbol{\alpha}_1'\right) \left(\mathbf{I} - \boldsymbol{\Gamma}^k\right) \mathbf{x}_t, \tag{A.54}$$

and the variance of $k$-period cumulative excess returns is

$$\begin{aligned}Var_t[\textstyle\sum_{j=1}^{k} rx_{t+j}] &= Var_t[(\boldsymbol{\alpha}_1 - \mathbf{e})' \left(\textstyle\sum_{j=1}^{k-1} \mathbf{x}_{t+j}\right) - \left(\frac{\phi}{1-\phi}\right) \boldsymbol{\alpha}_1' \mathbf{x}_{t+k}] \\ &= (\boldsymbol{\alpha}_1 - \mathbf{e})' \left(\textstyle\sum_{l=1}^{k-1} \sum_{j=1}^{k-1} \mathbf{C}_{[t+l,t+j]}\right) (\boldsymbol{\alpha}_1 - \mathbf{e}) + \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \mathbf{C}_{[t+k,t+k]} \boldsymbol{\alpha}_1 \\ &- 2\left(\frac{\phi}{1-\phi}\right) (\boldsymbol{\alpha}_1 - \mathbf{e})' \textstyle\sum_{j=1}^{k-1} \mathbf{C}_{[t+j,t+k]} \boldsymbol{\alpha}_1.\end{aligned} \tag{A.55}$$

Slow-moving arbitrageurs' demand long long-term bonds is

$$d_t = \tau \frac{E_t[\sum_{j=1}^{k} rx_{t+j}]}{Var_t[\sum_{j=1}^{k} rx_{t+j}]}. \tag{A.56}$$

Thus, given our conjectures, slow-moving arbitrageurs demands will indeed take a linear form. Specifically, we have

$$\delta_0 = \tau \frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}} \tag{A.57}$$

where $V^{(k)} = Var_t[\sum_{j=1}^{k} rx_{t+j}]$ and

$$\boldsymbol{\delta}_1' = \tau \frac{\left((\boldsymbol{\alpha}_1 - \mathbf{e})' (\mathbf{I} - \boldsymbol{\Gamma})^{-1} + \frac{\phi}{1-\phi} \boldsymbol{\alpha}_1'\right)}{V^{(k)}} \left(\mathbf{I} - \boldsymbol{\Gamma}^k\right) \tag{A.58}$$

## C.6 Equilibrium solution

To solve for the equilibrium, we need to clear the market for bonds in a way that is consistent with optimization on the part of fast-moving arbitrageurs and slow-moving arbitrageurs. The market-clearing condition is

$$\overbrace{(1-q)k^{-1} d_t + q b_t}^{\text{Active demand}} = \overbrace{s_t - (1-q)(k^{-1} \textstyle\sum_{i=1}^{k-1} d_{t-i})}^{\text{Active supply}}. \tag{A.59}$$

Letting $V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1'\boldsymbol{\Sigma}\boldsymbol{\alpha}_1$, denote the variance of 1-period excess returns, active demand is

$$\begin{aligned}
&(1-q)k^{-1}d_t + qb_t \\
&= \left[(1-q)k^{-1}\delta_0 + q\tau\frac{(\alpha_0 - \bar{i})}{V^{(1)}}\right] + \left[(1-q)k^{-1}\boldsymbol{\delta}_1' + q\tau\frac{\left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' - \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\boldsymbol{\Gamma}}{V^{(1)}}\right]\mathbf{x}_t
\end{aligned} \tag{A.60}$$

Active supply is

$$\begin{aligned}
&s_t - (1-q)k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i} \\
&= \left[\bar{s} - (1-q)\frac{(k-1)}{k}\delta_0\right] + \left[\left(\mathbf{e}_s - (1-q)k^{-1}\mathbf{e}_d\right)'\right]\mathbf{x}_t.
\end{aligned} \tag{A.61}$$

Matching constants terms, we obtain

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \tag{A.62}$$

Matching slope coefficients, we have

$$\begin{aligned}
\boldsymbol{\alpha}_1 &= (1-\phi)\left[\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\mathbf{e} + (1-\phi)\frac{V^{(1)}}{\tau q}\left[\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left[\mathbf{e}_s - k^{-1}(1-q)\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right] \\
&= \frac{1-\phi}{1-\phi\rho_P}\mathbf{e}_P + \frac{1-\phi}{1-\phi\rho_T}\mathbf{e}_T + \frac{V^{(1)}}{\tau q}\left[\frac{1-\phi}{1-\phi\rho_s}\mathbf{e}_s - k^{-1}(1-q)(1-\phi)\left[\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right]
\end{aligned} \tag{A.63}$$

where $\mathbf{e}_P = \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right]'$ and $\mathbf{e}_T = \left[\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right]'$.

Thus, equilibrium yields take the form

$$\begin{aligned}
y_t &= \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t \\
&= \overbrace{\bar{i} + \frac{1-\phi}{1-\phi\rho_P}\left(i_{P,t} - \bar{i}\right) + \frac{1-\phi}{1-\phi\rho_T}i_{P,t}}^{\text{Expected future short real rates}} \\
&\quad + \overbrace{\frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right)}^{\text{Unconditional term premia}} \\
&\quad + \overbrace{\left[\frac{V^{(1)}}{\tau q}\left(\frac{1-\phi}{1-\phi\rho_s}\left(s_t - \bar{s}\right) - (1-\phi)(1-q)k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)'\left[\mathbf{I} - \phi\boldsymbol{\Gamma}\right]^{-1}\mathbf{x}_t\right)\right]}^{\text{Conditional term premia}}.
\end{aligned} \tag{A.64}$$

Equilibrium excess returns are given by

$$E_t\left[rx_{t+1}\right] = \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) + \frac{V^{(1)}}{\tau q}\left[\left(s_t - \bar{s}\right) - (1-q)k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)'\mathbf{x}_t\right] \tag{A.65}$$

### C.7 Equilibrium existence and uniqueness

A rational expectations equilibrium of our model is a fixed point of a specific operator involving the "price-impact" coefficients, $(\boldsymbol{\alpha}_1')$, which show how bond supply and inactive slow-moving arbitrageur demand impact bond yields, and the "demand-impact" coefficients, $(\boldsymbol{\delta}_1')$, which show how bond supply and inactive demand impact the demand of active slow-moving arbitrageurs. Specifically, let $\boldsymbol{\omega} = (\boldsymbol{\alpha}_1', \boldsymbol{\delta}_1')'$ and consider the operator $\mathbf{f}(\boldsymbol{\omega}_0)$ which gives (i) the price-impact coefficients that will clear the market for long-term bonds and (ii) the demand-impact coefficients consistent with optimization on the part of active slow-moving arbitrageurs when arbitrageurs conjecture that $\boldsymbol{\omega} = \boldsymbol{\omega}_0$ at all future dates. Thus, a rational expectations equilibrium of our model is a fixed point $\boldsymbol{\omega}^* = \mathbf{f}(\boldsymbol{\omega}^*)$.

Specifically, an equilibrium solves the following system of equations

$$\boldsymbol{\alpha}_1 = (1-\phi)\left[\mathbf{I} - \phi\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)'\right]^{-1}\left[\mathbf{e} + \frac{V^{(1)}(\boldsymbol{\alpha}_1)}{\tau q}\left(\mathbf{e}_s - k^{-1}(1-q)(\mathbf{e}_d + \boldsymbol{\delta}_1)\right)\right] \tag{A.66}$$

and

$$\boldsymbol{\delta}_1' = \tau\frac{\left((\boldsymbol{\alpha}_1 - \mathbf{e})'(\mathbf{I} - \Gamma(\boldsymbol{\delta}_1))^{-1} + \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)}\left(\mathbf{I} - \boldsymbol{\Gamma}(\boldsymbol{\delta}_1)^k\right) \tag{A.67}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\Gamma(\boldsymbol{\delta}_1)$ to emphasize that the transition matrix depends on $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)$ to emphasize that the $k$-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. We can write this system of non-linear equations more compactly as

$$\boldsymbol{\alpha}_1 = \mathbf{f}_{\boldsymbol{\alpha}_{A1}}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1) \text{ and } \boldsymbol{\delta}_1 = \mathbf{f}_{\boldsymbol{\delta}_1}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1) \tag{A.68}$$

or simply as $\boldsymbol{\omega} = \mathbf{f}(\boldsymbol{\omega})$ where $\boldsymbol{\omega} = (\boldsymbol{\alpha}_1', \boldsymbol{\delta}_1')'$.

This is a system of $2(k+1)$ equations in $2(k+1)$ unknowns. However, in any rational expectations equilibrium of our model, bond yields always reflect the expected path of future short rates. As a result, equilibrium bond holdings do not depend directly on short rates. Formally, it is easy to see that, in any equilibrium, active slow-moving arbitrageur demand does not depend on $i_{P,t}$ and $i_{T,t}$, so the first two elements of $\boldsymbol{\delta}_1'$ are zeros and the first two elements are $\boldsymbol{\alpha}_1'$ are $(1-\phi)/(1-\phi\rho_P)$ and $(1-\phi)/(1-\phi\rho_T)$, respectively. This implies that an equilibrium of our model is a solution to a system of $2k$ nonlinear equations in $2k$ unknowns. Specifically, we need to determine how equilibrium yields and active slow-moving demand respond to shifts in the supply of bonds: this generates 2 unknowns and 2 corresponding equations. We also need to determine how equilibrium yields and active slow-moving demand respond to the holdings of inactive slow-moving arbitrageurs: this generates $2(k-1)$ unknowns and $2(k-1)$ corresponding equations.

We solve the relevant system of $2k$ nonlinear equations numerically using the Powell hybrid algorithm which performs a quasi-Newton search to find solutions to a system of nonlinear equations starting from an initial guess. To find all of the solutions, we apply this algorithm by sampling over $10,000$ different initial guesses. Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for $\alpha_0$ and $\delta_0$ using

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \text{ and } \delta_0 = \tau \frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}}, \tag{A.69}$$

which yields

$$\alpha_0 = \bar{i} + \frac{\bar{s}}{\tau\left[q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}\right]} \text{ and } \delta_0 = \frac{\frac{k}{V^{(k)}}}{q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}} \times \bar{s}. \tag{A.70}$$

When asset supply is stochastic, an equilibrium solution only exists if arbitrageurs are sufficiently risk tolerant (i.e., for $\tau$ sufficiently large). When an equilibrium exists, there are multiple equilibrium solutions. Equilibrium non-existence and multiplicity of this sort arise in overlapping-generations, rational-expectations models such as ours where risk-averse arbitrageurs with finite investment horizons trade an infinitely-lived asset that is subject to supply shocks.[17] Different equilibria correspond to different self-fulfilling beliefs that arbitrageurs can hold about the price-impact of supply shocks and, hence, the risks associated with holding long-term bonds. See Greenwood et al. (2018) for an extensive discussion of these issues.

The intuition for equilibrium multiplicity can be understood most clearly in the simple case when there are only fast-moving arbitrageurs. If arbitrageurs are sufficiently risk tolerant there are two equilibria in this special case: a low price impact (or low return volatility) equilibrium and a high price impact (or high return volatility) equilibrium. If arbitrageurs believe that supply shocks will have a large impact on long-term bonds prices, they will perceive bonds as being highly risky. As a result, arbitrageurs will only absorb a positive supply shock if they are compensated by a large decline in bond prices, making the initial belief self-fulfilling. However, if arbitrageurs believe that bond prices will be less sensitive to supply shocks, they will perceive bond as being less risky and will absorb a supply shock even if they are only compensated by a modest decline in bond prices.

Things are slightly more complicated in our general model with slow-moving capital. Specifically, the introduction of slow-moving capital can give rise to additional unstable equilibria. However, we always find a unique equilibrium that is stable in the sense that equilibrium is robust to a small perturbation in arbitrageurs' beliefs regarding the equilibrium that will prevail in the future. Formally, letting $\boldsymbol{\omega}^{(1)} = \boldsymbol{\omega}^* + \boldsymbol{\xi}$ for some small $\boldsymbol{\xi}$ and defining $\boldsymbol{\omega}^{(n)} = \mathbf{f}(\boldsymbol{\omega}^{(n-1)})$, an equilibrium $\boldsymbol{\omega}^*$ is stable if $\lim_{n\to\infty} \boldsymbol{\omega}^{(n)} = \boldsymbol{\omega}^*$ and is unstable if $\lim_{n\to\infty} \boldsymbol{\omega}^{(n)} \neq \boldsymbol{\omega}^*$. Let $\{\lambda_i\}$ denote the eigenvalues of the Jacobian $\mathbf{D}_{\boldsymbol{\omega}}\mathbf{f}\left(\boldsymbol{\omega}^*\right)$. If $\max_i |\lambda_i| < 1$, then $\boldsymbol{\omega}^*$ is stable; if $\max_i |\lambda_i| > 1$, then $\boldsymbol{\omega}^*$ is unstable.

Consistent with Samuelson's (1947) "correspondence principle," which says that the comparative statics of stable equilibria have certain properties, this unique stable equilibrium has com-

[17] For previous treatments of these issues, see Spiegel (1998), Bacchetta and van Wincoop (2003), Watanabe (2008), Banerjee (2011), Greenwood and Vayanos (2014), Albagli (2015), and Greenwood, Hanson, and Liao (forthcoming).

parative statics that accord with standard economic intuition. By contrast, the unstable equilibria have comparative statics that run contrary to standard intuition.[18] We focus on this unique stable equilibrium in our numerical illustrations.

### C.8 Behavior of $\beta_h$ in the baseline model

Consider the model-implied counterpart of the empirical regression coefficient in equation (1.1). In the model, the coefficient $\beta_h$ from a regression of $y_{t+h} - y_t$ on $i_{t+h} - i_t$ is:

$$\beta_h = \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}, \tag{A.71}$$

where $\mathbf{V} = Var\left[\mathbf{x}_t\right]$ denotes the variance of the state vector $\mathbf{x}_t$ and $\mathbf{e}$ denotes the $(k+2)\times 1$ vector with ones in the first and second positions and zeros elsewhere.

To derive this expression, note that $y_{t+h} - y_t = \boldsymbol{\alpha}_1'\left(\mathbf{x}_{t+h} - \mathbf{x}_t\right)$ and $i_{t+h} - i_t = \mathbf{e}'\left(\mathbf{x}_{t+h} - \mathbf{x}_t\right)$. Since the state-vector $\mathbf{x}_t$ follows a VAR(1) process $\mathbf{x}_{t+1} = \boldsymbol{\Gamma}\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1}$ with $\Sigma = Var\left[\boldsymbol{\epsilon}_{t+1}\right]$, we have $vec(\mathbf{V}) = (I - \boldsymbol{\Gamma}\otimes\boldsymbol{\Gamma})^{-1}vec(\boldsymbol{\Sigma})$. Noting that $Cov\left[\mathbf{x}_{t+j}, \mathbf{x}_t'\right] = \boldsymbol{\Gamma}^j\mathbf{V}$ and $Cov[\mathbf{x}_t, \mathbf{x}_{t+j}'] = \mathbf{V}\left(\boldsymbol{\Gamma}'\right)^j$, we have $Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] = 2\mathbf{V} - \boldsymbol{\Gamma}^h\mathbf{V} - \mathbf{V}(\boldsymbol{\Gamma}')^h$ and the result follows.

We can then demonstrate the following result:

**Proposition 1.** *The dependence of the coefficient $\beta_h$ on time horizon $h$ is governed by (i) the persistence $\rho_x$ of the three shocks $x \in \{s, T, P\}$, (ii) the volatilities of the two short-rate shocks, $\sigma_T$ and $\sigma_P$, (iii) the strength of the rate-amplification mechanisms $C$, and (iv) the degree to which capital is slow moving $q$.*

1. *When there are no rate-amplifying net supply shocks ($C = 0$), changes in term premia are unrelated to shifts in short rates and long-term yields do not exhibit excess sensitivity. Furthermore,*
   - *(a) if $\rho_T = \rho_P$, $\beta_h$ is independent of $h$, $\sigma_T$, and $\sigma_P$.*
   - *(b) if $\rho_T < \rho_P$, $\beta_h$ is* increasing *in $h$; the level of $\beta_h$ falls with $\sigma_T$ and rises with $\sigma_P$ for all $h$.*
2. *When there are rate-amplifying net supply shocks ($C > 0$), changes in term premia are positively correlated with changes in short rates and long-term yields exhibit excess sensitivity. Furthermore,*
   - *(a) if $\rho_s = \rho_T = \rho_P$, and all capital is fast-moving ($q = 1$), then $\beta_h$ is independent of $h$;*
   - *(b) if $\rho_s \leq \rho_T = \rho_P$ and either (i) supply shocks are transient ($\rho_s < \rho_T$) or (ii) capital is slow-moving ($q < 1$), then $\beta_h$ is decreasing in $h$;*
   - *(c) if $\rho_s \leq \rho_T < \rho_P$, $\beta_h$ can be non-monotonic in $h$.*

[18] For instance, in the special case where there is no slow-moving capital, the low price-impact equilibrium is stable and the high price-impact equilibrium is unstable. At the stable equilibrium, an increase in the volatility of short rates or the volatility of supply shocks is associated with an increase in the price-impact coefficient and an increase in the volatility of returns. By contrast, these comparitive statics take the opposite sign at the unstable equilibrium.

**Proof:** To demonstrate this result, it suffices to consider two special cases. First, we consider the case where there is no slow-moving capital ($q = 1$). We use this case to establish part 1., part 2.(a), and part 2.(c) of the Proposition. To establish part 2.(b), we study the case where $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$. The arguments given in this special case generalize naturally to the case where $k > 2$.

### C.8.1 Solution in the special case without slow-moving capital ($q = 1$)

We first solve the model the special case in which there is no slow-moving capital ($q = 1$). In this special case, the equilibrium yield on long-term bonds is

$$y_t = \overbrace{\left(\bar{\imath} + \tau^{-1} V^{(1)} \bar{s}\right)}^{\alpha_0} + \overbrace{\frac{1-\phi}{1-\rho_P \phi}}^{\alpha_P} (i_{P,t} - \bar{\imath}) + \overbrace{\frac{1-\phi}{1-\rho_T \phi}}^{\alpha_T} i_{T,t} + \overbrace{\tau^{-1} V^{(1)} \frac{1-\phi}{1-\rho_s \phi}}^{\alpha_s} (s_t - \bar{s}), \tag{A.72}$$

where

$$\begin{aligned} V^{(1)} &= Var_t\left[\frac{\phi}{1-\phi} y_{t+1}\right] \qquad (A.73)\\ &= Var_t\left[\frac{\phi}{1-\rho_T\phi}\varepsilon_{P,t+1} + \frac{\phi}{1-\rho_T\phi}\varepsilon_{T,t+1} + \tau^{-1}V^{(1)}\frac{\phi}{1-\rho_s\phi}\left(\varepsilon_{s,t+1} + C\varepsilon_{P,t+1} + C\varepsilon_{T,t+1}\right)\right] \end{aligned}$$

is the smaller root of the following quadratic equation:

$$\begin{aligned} 0 = &\left[\left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}\sigma_s\right)^2 + \left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_P\right)^2 + \left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_T\right)^2\right] \times \left(V^{(1)}\right)^2 \qquad (A.74)\\ &+ \left[2\left(\frac{\phi}{1-\rho_P\phi}\sigma_P\right)\left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_P\right) + 2\left(\frac{\phi}{1-\rho_T\phi}\sigma_T\right)\left(\tau^{-1}\frac{\phi}{1-\rho_s\phi}C\sigma_T\right) - 1\right] \times V^{(1)}\\ &+ \left[\left(\frac{\phi}{1-\rho_P\phi}\sigma_P\right)^2 + \left(\frac{\phi}{1-\rho_T\phi}\sigma_T\right)^2\right]. \end{aligned}$$

In this case, the model-implied regression coefficient is

$$\begin{aligned} \beta_h &= \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} \qquad (A.75)\\ &= \frac{\alpha_P Var\left[\Delta_h i_{P,t}\right] + \alpha_T Var\left[\Delta_h i_{T,t}\right] + \alpha_s\left(Cov\left[\Delta_h i_{P,t}, \Delta_h s_t\right] + Cov\left[\Delta_h i_{T,t}, \Delta_h s_t\right]\right)}{Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]}. \end{aligned}$$

where for $X \in \{P, T\}$ we have $Var\left[\Delta_h i_{X,t}\right] = 2\left[(1-\rho_X^h)/\left(1-\rho_X^2\right)\right]\sigma_X^2$ and $Cov\left[\Delta_h i_{X,t}, \Delta_h s_t\right] = C\left[\left(2-\rho_s^h-\rho_X^h\right)/\left(1-\rho_s\rho_X\right)\right]\sigma_X^2$.

We first consider the *level* of $\beta_h$ irrespective of horizon $h$. Inspecting equation (A.75), it is easy to see that:

- **When $C = 0$ and $\rho_T < \rho_P$, the *level* of $\beta_h$ is increasing in $\sigma_P$ for all $h$.** An increase in $\sigma_P$ raises the fraction of total short-rate variation at all horizons that is due to movements in the more persistent component (i.e., raises $Var\left[\Delta_h i_{P,t+h}\right] / \left(Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]\right)$ for all $h$). Since shocks to the more persistent component of short rates have larger impact on long-term yields via a straightforward expectations hypothesis channel (i.e., since $\alpha_P > \alpha_T$), an increase in $\sigma_P$ raises the level of $\beta_h$ at all horizons.

We next consider the way $\beta_h$ behaves as a function of horizon $h$. Again, using equation (A.75), it is easy to show that:

- **When $C = 0$ and $\rho_T = \rho_P$, $\beta_h$ is a constant that is independent of $h$.** These assumptions imply that the expectations hypothesis holds—i.e., there is no excess sensitivity—and that all shocks to short rates have the same persistence. In this benchmark case, $\beta_h = \alpha_P = \alpha_T$ for all $h$—i.e., the sensitivity of long rates to short rates is the same at all horizons.[19] Furthermore, $\beta_h$ is independent of $\sigma_T$ and $\sigma_P$.
- **When $C = 0$ and $\rho_T < \rho_P$, $\beta_h$ is an increasing function of $h$.** These assumptions imply that the expectations hypothesis holds, but there are now transient and persistent shocks to short rates. In this case, $\beta_h$ rises with $h$ since (i) movements in the more persistent component of short rates are associated with larger movement in long-term yields (i.e., $\alpha_P > \alpha_T$) and (ii) because the persistent component dominates changes in short rates at longer horizons (i.e., $Var\left[\Delta_h i_{P,t+h}\right] / \left(Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]\right)$ rises with $h$ when $\rho_T < \rho_P$).
- **When $C > 0$ and $\rho_s = \rho_T = \rho_P$, $\beta_h$ is a constant that is independent of $h$.** In this case, there is excess sensitivity—shifts in short rates lead to shifts in the term premium on long-term bonds—but the excess sensitivity is the same irrespective of horizon $h$. This is because $\Delta_h s_{t+h} = C\Delta_h i_{P,t+h} + C\Delta_h i_{T,t+h}$ when $\rho_s = \rho_T = \rho_P$ (see equation (A.41)) and $Var\left[\Delta_h i_{P,t+h}\right] / \left(Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]\right) = \sigma_P^2 / \left(\sigma_P^2 + \sigma_T^2\right)$ when $\rho_T = \rho_P$.
- **When $C > 0$ and $\rho_s < \rho_T = \rho_P$, $\beta_h$ is a decreasing function of $h$.** In this case, long-term interest rates exhibit excess sensitivity to movements in short rates that declines with horizon $h$. Intuitively, if the supply shocks induced by shocks to short rates are more transient than

[19] To see this, note that (treating $h$ as continuous), we have

$$\frac{\partial}{\partial h}\left(\frac{Var\left[\Delta_h i_{P,t+h}\right]}{Var\left[\Delta_h i_{P,t+h}\right] + Var\left[\Delta_h i_{T,t+h}\right]}\right) = \frac{\partial}{\partial h}\left(\frac{\frac{1-\rho_P^h}{1-\rho_P^2}\sigma_P^2}{\frac{1-\rho_T^h}{1-\rho_T^2}\sigma_T^2 + \frac{1-\rho_P^h}{1-\rho_P^2}\sigma_P^2}\right) = \sigma_P^2\sigma_T^2 \frac{\left(1-\rho_T^h\right)\left(1-\rho_P^h\right)}{\left(1-\rho_P^2\right)\left(1-\rho_T^2\right)} \frac{-\frac{\ln(\rho_P)\rho_P^h}{1-\rho_P^h} + \frac{\ln(\rho_T)\rho_T^h}{1-\rho_T^h}}{\left(\frac{1-\rho_T^h}{1-\rho_T^2}\sigma_T^2 + \frac{1-\rho_P^h}{1-\rho_P^2}\sigma_P^2\right)^2}.$$

Since $\rho_T < \rho_P$, then result then follows from the fact that $-\frac{\ln(\rho)\rho^h}{1-\rho^h}$ is increasing in $\rho$ for $\rho \in [0,1)$. To see this last fact note that

$$-\frac{\partial}{\partial \rho}\frac{\ln(\rho)\rho^h}{1-\rho^h} = -\frac{1-\rho^h+\ln(\rho)h}{\left(1-\rho^h\right)^2}\rho^{h-1} \propto -\left(1-\rho^h+\ln(\rho)h\right) \equiv f(\rho).$$

We have $f(1) = 0$ and $\lim_{\rho\to 0} f(\rho) = \infty$ and $f'(\rho) = -h\left(1-\rho^h\right)/\rho < 0$. Thus, we have $f(\rho) > 0$ for all $\rho \in [0,1)$.

the underlying shocks to short rates, then term premia will react more in the short run than in the long run. Thus, there will be greater excess sensitivity in the short run.[20]

- **When $C > 0$ and $\rho_s < \rho_T < \rho_P$, $\beta_h$ can be a globally non-monotonic function of $h$.** Decompose the long-term yield into an expectations hypothesis component $eh_t$ that reflects expected future short rates and a term premium piece$tp_t$ that reflects expected future bond risk premium: $y_t = eh_t + tp_t$. Thus, we have $\beta_h = \beta_h^{eh} + \beta_h^{tp}$ where $\beta_h^{eh} = Cov\left[eh_{t+h} - eh_t, i_{t+h} - i_t\right]/Var\left[i_{t+h} - i_t\right]$ and $\beta_h^{tp} = Cov\left[tp_{t+h} - tp_t, i_{t+h} - i_t\right]/Var\left[i_{t+h} - i_t\right]$. In this case, $\beta_h^{eh}$ is an increasing an concave function of $h$ (the logic here is the same as $C = 0$ and $\rho_T < \rho_P$). And, $\beta_h^{tp}$ is a decreasing and convex function of $h$ (the logic here is the same as the case where $C > 0$ and $\rho_s < \rho_T = \rho_P$). Depending on which effect dominates, $\beta_t = \beta_h^{eh} + \beta_h^{tp}$ can either be monotonically increasing (e.g., if $\rho_P$ is much larger than $\rho_T$ and $\rho_s$ is near $\rho_T$), monotonically decreasing (e.g., if $\rho_P$ is near $\rho_T$ and $\rho_s$ is much smaller than $\rho_T$), U-shaped, or inverse-U-shaped.

### C.8.2 Special case where $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$

**Summary:** Suppose that $C > 0$, $q < 1$, $k = 2$, and $\rho_s \leq \rho_T = \rho_P \equiv \rho_i$. Then we always have $\beta_h - \beta_{h-1} < 0$ for $h \leq 2$. Furthermore, $\beta_h$ is globally increasing in the sense that $\beta_1 > \lim_{h\to\infty} \beta_h$. However, it is possible for $\beta_h$ to oscillate non-monotonically for $h > 2$.

**The solution:** Here we have $y_t = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t$ and $d_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$. The state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_t - \bar{i} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \end{bmatrix}, \tag{A.76}$$

and its dynamics are given by

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\delta)\,\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \qquad\qquad (A.77) \\ &= \begin{bmatrix} \rho_i & 0 & 0 \\ 0 & \rho_s & 0 \\ 0 & \delta_s & \delta_d \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ C\varepsilon_{i,t+1} + \varepsilon_{s,t+1} \\ 0 \end{bmatrix}, \end{aligned}$$

where $\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right]$, $\delta_s > 0$, and $\delta_d < 0$. Assuming for simplicity that $\varepsilon_{i,t+1}$ and $\varepsilon_{s,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_i^2 & C\sigma_i^2 & 0 \\ C\sigma_i^2 & C^2\sigma_i^2 + \sigma_s^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A.78}$$

[20] To show this, it suffices to show that $\left(2 - \rho_s^h - \rho_T^h\right)/\left(2 - \rho_T^h - \rho_T^h\right) > 0$ is decreasing in $h$ when $\rho_s < \rho_T$. To see this note that

$$\frac{\partial}{\partial h}\left(\frac{2 - \rho_s^h - \rho_T^h}{2 - \rho_T^h - \rho_T^h}\right) = \frac{2\left(1 - \rho_s^h\right)\left(1 - \rho_T^h\right)\left(-\frac{\rho_s^h \ln \rho_s}{1 - \rho_s^h} + \frac{\rho_T^h \ln \rho_T}{1 - \rho_T^h}\right)}{\left(2 - \rho_T^h - \rho_T^h\right)^2}.$$

Since $\rho_s < \rho_T$, then result then follows from the fact that $-\frac{\ln(\rho)\rho^h}{1-\rho^h}$ is increasing in $\rho$ for $\rho \in [0, 1)$.

Using the face that $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we can show that

$$\mathbf{V} = Var \begin{bmatrix} i_t - \bar{i} \\ s_t - \bar{s} \\ d_{t-1} - \delta_0 \end{bmatrix} = \begin{bmatrix} \frac{\sigma_i^2}{1-\rho_i^2} & \frac{C\sigma_i^2}{1-\rho_i\rho_s} & \frac{\rho_i\delta_s}{1-\delta_d\rho_i}\frac{C\sigma_i^2}{1-\rho_i\rho_s} \\ \frac{C\sigma_i^2}{1-\rho_i\rho_s} & \frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} & \frac{\delta_s\rho_s}{1-\delta_d\rho_s}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} \\ \frac{\rho_i\delta_s}{1-\delta_d\rho_i}\frac{C\sigma_i^2}{1-\rho_i\rho_s} & \frac{\delta_s\rho_s}{1-\delta_d\rho_s}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} & \frac{\delta_s^2(1+\delta_d\rho_s)}{(1-\delta_d^2)(1-\delta_d\rho_s)}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} \end{bmatrix}. \quad \text{(A.79)}$$

Equilibrium excess returns are given by

$$E_t[rx_{t+1}] = \frac{V^{(1)}}{\tau q}(\bar{s} - (1-q)\delta_0) + \frac{V^{(1)}}{\tau q}\left[(s_t - \bar{s}) - \frac{1}{2}(1-q)(\delta_s(s_t - \bar{s}) + (1+\delta_d)(d_{t-1} - \delta_0))\right] \quad \text{(A.80)}$$

Equilibrium yields are given by

$$y_t = \overbrace{\bar{i} + \frac{1-\phi}{1-\phi\rho_i}\left(i_{i,t} - \bar{i}\right)}^{\text{Expected future short real rates}} + \overbrace{\frac{V^{(1)}}{\tau q}(\bar{s} - (1-q)\delta_0)}^{\text{Unconditional term premia}} \quad \text{(A.81)}$$

$$+ \overbrace{\left[\frac{V^{(1)}}{\tau q}\left(\frac{1-\phi}{1-\phi\rho_s}(s_t - \bar{s}) - \frac{1}{2}(1-q)\left(\frac{1-\phi}{1-\phi\rho_s}\frac{1+\phi}{1-\phi\delta_d}\delta_s(s_t - \bar{s}) + \frac{1-\phi}{1-\phi\delta_d}(1+\delta_d)(d_{t-1} - \delta_0)\right)\right)\right]}^{\text{Conditional term premia}}.$$

**Characterizing the solution:** Here we show that:

1. $\alpha_s > 0$ (an increase in supply $s_t$ is associated with an increase in long-term yields $y_t$);
2. $\delta_s > 0$ (an increase in supply $s_t$ is associated with an increase in bond purchases $d_t$ by active slow investors);
3. $\alpha_d < 0$ (an increase in the holdings of inactive slow investors $d_{t-1}$ is associated with a decline in long-term yields $y_t$); and
4. $-1 < \delta_d < 0$ (an increase in the holdings of inactive slow investors $d_{t-1}$ is associated with a decline in bond purchases $d_t$ by active slow investors).

To begin, note that the market-clearing condition is this case is:

$$s_t - (1-q)\frac{1}{2}d_{t-1} = qb_t + (1-q)\frac{1}{2}d_t. \quad \text{(A.82)}$$

Plugging in the relevant expressions for $b_t = \left(\tau/V^{(1)}\right) \times E_t[(y_t - i_t) - (\phi/(1-\phi))(y_{t+1} - y_t)]$ and $d_t$, we have

$$\begin{aligned} & s_t - (1-q)\frac{1}{2}d_{t-1} \quad \text{(A.83)} \\ = \; & q\frac{\tau}{V^{(1)}}\begin{pmatrix} \left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\left(\alpha_s(s_t - \bar{s}) + \alpha_d(d_{t-1} - \delta_0)\right) \\ -\frac{\phi}{1-\phi}\left[\alpha_s\rho_s(s_t - \bar{s}) + \alpha_d\left[\delta_s(s_t - \bar{s}) + \delta_d(d_{t-1} - \delta_0)\right]\right] \end{pmatrix} \\ & +(1-q)\frac{1}{2}\left(\delta_0 + \delta_s(s_t - \bar{s}) + \delta_d(d_{t-1} - \delta_0)\right). \end{aligned}$$

(Here we have use the solution for $\alpha_i$ and the fact that $\delta_i = 0$). Similarly, we have

$$\begin{aligned} d_t - \delta_0 &= \delta_s (s_t - \overline{s}) + \delta_d (d_{t-1} - \delta_0) \qquad \text{(A.84)} \\ &= \frac{\tau}{V^{(2)}} \left( \begin{array}{c} E_t[(y_t - i_t) + (y_{t+1} - i_{t+1}) - (\phi/(1-\phi))(y_{t+2} - y_t)] \\ -E[(y_t - i_t) + (y_{t+1} - i_{t+1}) - (\phi/(1-\phi))(y_{t+2} - y_t)] \end{array} \right) \\ &= \frac{\tau}{V^{(2)}} \left( \begin{array}{c} [\alpha_s (s_t - \overline{s}) + \alpha_d (d_{t-1} - \delta_0)] \\ + [\alpha_s \rho_s (s_t - \overline{s}) + \alpha_d [\delta_s (s_t - \overline{s}) + \delta_d (d_{t-1} - \delta_0)]] \\ -\frac{\phi}{1-\phi} \left( \begin{array}{c} [\alpha_s \rho_s^2 (s_t - \overline{s}) + \alpha_d [(\delta_s (\delta_d + \rho_s))(s_t - \overline{s}) + \delta_d^2 (d_{t-1} - \delta_0)]] \\ - [\alpha_s (s_t - \overline{s}) + \alpha_d (d_{t-1} - \delta_0)] \end{array} \right) \end{array} \right) \end{aligned}$$

Differentiating these two equations with respect to $s_t$ and $d_{t-1}$, we obtain the following four conditions in four unknowns $(\alpha_s, \alpha_d, \delta_s, \delta_d)$:

$$\begin{aligned} 1 &= q \frac{\tau}{V^{(1)}[\alpha_s]} \left( \frac{1 - \phi \rho_s}{1 - \phi} \alpha_s - \frac{\phi}{1 - \phi} \alpha_d \delta_s \right) + (1 - q) \frac{1}{2} \delta_s, \qquad \text{(A.85a)} \\ -(1-q)\frac{1}{2} &= q \frac{\tau}{V^{(1)}[\alpha_s]} \left( \frac{1 - \phi \delta_d}{1 - \phi} \alpha_d \right) + (1 - q) \frac{1}{2} \delta_d, \qquad \text{(A.85b)} \\ \delta_s &= \frac{\tau}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]} \left( \frac{1 - \phi \rho_s}{1 - \phi} ((\rho_s + 1)\alpha_s + \alpha_d \delta_s) - \frac{\phi (1 + \delta_d)}{1 - \phi} \alpha_d \delta_s \right), \qquad \text{(A.85c)} \\ \delta_d &= \frac{\tau}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]} \frac{1 - \phi \delta_d}{1 - \phi} (\delta_d + 1) \alpha_d, \qquad \text{(A.85d)} \end{aligned}$$

where we write $V^{(1)}[\alpha_s]$ to emphasize that $V^{(1)} = Var_t[rx_{t+1}] > 0$ depends on $\alpha_s$ and $V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]$ to emphasize that $V^{(2)} = Var_t[rx_{t+1} + rx_{t+2}] > 0$ depends on $(\alpha_s, \alpha_d, \delta_s, \delta_d)$.

- First, we show that $\delta_d < 0$. Combining (A.85b) and (A.85d), we have

$$\delta_d = \frac{\tau}{V^{(2)}} (\delta_d + 1) \frac{1 - \phi \delta_d}{1 - \phi} \alpha_d = -\frac{1}{2} \frac{1 - q}{q} \frac{V^{(1)}}{V^{(2)}} (\delta_d + 1)^2 < 0. \qquad \text{(A.86)}$$

- Next we want to show that $-1 < \delta_d$. We have

$$(1 + \delta_d) = 1 - \frac{1}{2} \frac{1 - q}{q} \frac{V^{(1)}[\alpha_s]}{V^{(2)}[\boldsymbol{\alpha}, \boldsymbol{\delta}]} (1 + \delta_d)^2. \qquad \text{(A.87)}$$

  We have $(1 + \delta_d) > 0$ or $\delta_d > -1$ in the stable solution.

- Next, since $-1 < \delta_d < 0$, equation (A.85d) implies that $\alpha_d < 0$.

- To show that $\delta_s > 0$, we combine (A.85c), (A.85d), and (A.85a) to obtain

$$\begin{aligned} \delta_s &= \frac{\tau}{V^{(2)}} \left( \frac{1 - \phi \rho_s}{1 - \phi} ((\rho_s + 1)\alpha_s + \alpha_d \delta_s) - \phi \delta_s \left( \frac{1 + \delta_d}{1 - \phi} \alpha_d \right) \right) \qquad \text{(A.88)} \\ &= \tau \frac{(\rho_s + 1)}{V^{(2)}} \frac{1 - \phi \rho_s}{1 - \phi} \alpha_s + \frac{\tau}{V^{(2)}} \frac{1 - \phi \rho_s}{1 - \phi} \alpha_d \delta_s - \delta_s \frac{\phi \delta_d}{1 - \phi \delta_d} \text{ [using (A.85d)]} \\ &= \frac{(\rho_s + 1)}{\tau V^{(2)}} \left( \frac{\tau V^{(1)}}{q} \left( 1 - (1 - q) \frac{1}{2} \delta_s \right) + \frac{\phi}{1 - \phi} \alpha_d \delta_s \right) \\ &\quad + \frac{\tau}{V^{(2)}} \frac{1 - \phi \rho_s}{1 - \phi} \alpha_d \delta_s - \delta_s \frac{\phi \delta_d}{1 - \phi \delta_d} \text{ [using (A.85a)]} \\ &= \frac{(\rho_s + 1)}{q} \frac{V^{(1)}}{V^{(2)}} \left( 1 - (1 - q) \frac{1}{2} \delta_s \right) + \tau \frac{\alpha_d \delta_s}{V^{(2)}} \frac{1 + \phi}{1 - \phi} - \delta_s \frac{\phi \delta_d}{1 - \phi \delta_d}. \end{aligned}$$

Rearranging, we obtain

$$\delta_s = (1-\phi\delta_d)\left[\frac{(\rho_s+1)}{q}\frac{V^{(1)}[\alpha_s]}{V^{(2)}[\boldsymbol{\alpha},\boldsymbol{\delta}]}\left(1-(1-q)\frac{1}{2}\delta_s\right)+\tau\frac{\alpha_d\delta_s}{V^{(2)}}\frac{1+\phi}{1-\phi}\right]. \tag{A.89}$$

Since $\delta_d < 0$ and $\alpha_d < 0$, this equation implies that we must have $\delta_s > 0$ in equilibrium. Specifically, if $\delta_s < 0$, then the right-hand-side of this equation is positive and the left-hand-side is negative. Thus, we cannot have $\delta_d < 0$ in equilibrium and must instead have $\delta_s > 0$ in equilibrium.

- Finally, we want to show that $\alpha_s > 0$. Combining (A.85a) and (A.85b), we obtain

$$\alpha_s = \frac{\tau^{-1}V^{(1)}}{q}\frac{1-\phi}{1-\phi\rho_s}\left(1-\frac{1}{2}(1-q)\,\delta_s\frac{1+\phi}{1-\phi\delta_d}\right). \tag{A.90}$$

Since $\delta_d < 0$, we have $(1+\phi)/(1-\phi\delta_d) < 1$ and it suffices to show that $2 \geq (1-q)\,\delta_s$ for all $q \in [0,1]$. When $q = 1$, we have $(1-q)\,\delta_s = 0$. When $q = 0$, we have $(1-q)\,\delta_s = 2$. Since $(1-q)\,\delta_s$ is monotonically increasing in $q$ in the model's stable equilibrium, it follows that $2 \geq (1-q)\,\delta_s$ for all $q \in [0,1]$. Therefore, we have $2 > (1-q)\,\delta_s\,[(1+\phi)/(1-\phi\delta_d)]$ and hence $\alpha_s > 0$ for all $q \in [0,1]$.

**Computing $\beta_h$:** We now compute and characterize $\beta_h$. We have

$$\beta_h = \frac{Cov[y_{t+h}-y_t, i_{t+h}-i_t]}{Var[i_{t+h}-i_t]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}. \tag{A.91}$$

Using the fact that

$$\boldsymbol{\Gamma}^h = \begin{bmatrix} \rho_i^h & 0 & 0 \\ 0 & \rho_s^h & 0 \\ 0 & \delta_s\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d} & \delta_d^h \end{bmatrix}, \tag{A.92}$$

we have

$$Var[\mathbf{x}_{t+h}-\mathbf{x}_t] = 2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h \tag{A.93}$$

$$= \begin{bmatrix} 2\frac{1-\rho_i^h}{1-\rho_i^2}\sigma_i^2 & C\frac{2-\rho_s^h-\rho_i^h}{1-\rho_s\rho_i}\sigma_i^2 & \frac{C\sigma_i^2}{1-\rho_i\rho_s}\frac{\rho_i\delta_s\left(2-\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)}{1-\rho_i\delta_d} \\ C\frac{2-\rho_s^h-\rho_i^h}{1-\rho_s\rho_i}\sigma_i^2 & \frac{2\left(1-\rho_s^h\right)\left(C^2\sigma_i^2+\sigma_s^2\right)}{1-\rho_s^2} & \frac{\delta_s\left(\rho_s\frac{2-\rho_s^h-\delta_d^h}{1-\delta_d\rho_s}-\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)\left(C^2\sigma_i^2+\sigma_s^2\right)}{1-\rho_s^2} \\ \frac{C\sigma_i^2}{1-\rho_i\rho_s}\frac{\rho_i\delta_s\left(2-\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)}{1-\rho_i\delta_d} & \frac{\delta_s\left(\rho_s\frac{2-\rho_s^h-\delta_d^h}{1-\delta_d\rho_s}-\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)\left(C^2\sigma_i^2+\sigma_s^2\right)}{1-\rho_s^2} & \frac{2\delta_s^2\left(\left(1-\delta_d^h\right)\frac{1+\delta_d\rho_s}{(1-\delta_d^2}-\rho_s\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}\right)}{(1-\delta_d\rho_s)}\frac{C^2\sigma_i^2+\sigma_s^2}{1-\rho_s^2} \end{bmatrix}$$

Thus, we have

$$\beta_h = \alpha_i + \alpha_s \times R_{i,s}(h) + \alpha_d \times R_{i,d}(h), \tag{A.94}$$

where $\alpha_i > 0$, $\alpha_s > 0$, $\alpha_d < 0$, and

$$R_{i,s}(h) = \frac{Cov[i_{t+h}-i_t, s_{t+h}-s_t]}{Var[i_{t+h}-i_t]} = C\frac{1-\rho_i^2}{1-\rho_s\rho_i}\frac{2-\rho_s^h-\rho_i^h}{2\left(1-\rho_i^h\right)}, \tag{A.95}$$

$$R_{i,d}(h) = \frac{Cov[i_{t+h}-i_t, d_{t+h-1}-d_{t-1}]}{Var[i_{t+h}-i_t]} = C\frac{1-\rho_i^2}{1-\rho_s\rho_i}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\frac{2-\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}}{2\left(1-\rho_i^h\right)}.$$

**Proof that $\beta_2 < \beta_1$:** We know that $R_{i,s}(h)$ is strictly decreasing in $h$ when $\rho_s < \rho_i$ and is constant when $\rho_s = \rho_i$. Thus, since $\alpha_s > 0$, it follows that the second term $(\alpha_{1,s} \times R_{i,s}(h))$ is weakly decreasing in $h$ and is strictly decreasing when $\rho_s < \rho_i$. Since $\alpha_d < 0$ when $q < 1$, if we can show that $R_{i,d}(2) > R_{i,d}(1)$, then we have $\alpha_d \times R_{i,d}(2) < \alpha_d \times R_{i,d}(1)$ and we are done. We have

$$R_{i,d}(h) = C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\left(1+\frac{\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}}{2\left(1-\rho_i^h\right)}\right) = C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\left(1+F\left(h,\rho_s\right)\right) \tag{A.96}$$

where

$$F\left(h,\rho_s\right) = \frac{\rho_i^h-\delta_d^h-\frac{1-\delta_d\rho_i}{\rho_i}\frac{\rho_s^h-\delta_d^h}{\rho_s-\delta_d}}{2\left(1-\rho_i^h\right)}. \tag{A.97}$$

We want to show that $F(2,\rho_s) > F(1,\rho_s)$. We have

$$F\left(2,\rho_s\right)-F\left(1,\rho_s\right) = \frac{1}{2\rho_i}\left(1+\rho_i-\frac{\delta_d\left(1-\rho_s\rho_i\right)}{\left(1-\rho_i^2\right)}-\frac{\rho_s-\rho_i^3}{1-\rho_i^2}\right). \tag{A.98}$$

Note that $F(2,\rho_s)-F(1,\rho_s)$ is decreasing in $\rho_s$ when $\rho_s \in [0,\rho_i]$. Thus, setting $\rho_s = \rho_i$, we have

$$F\left(2,\rho_s\right)-F\left(1,\rho_s\right) \geq F\left(2,\rho_i\right)-F\left(1,\rho_i\right) = \frac{1}{2\rho_i}\left(1-\delta_d\right) > 0. \tag{A.99}$$

Thus, we have $F(2,\rho_s) > F(1,\rho_s)$ and thus $R_{i,d}(2) > R_{i,d}(1)$.

**Proof that $\beta_h$ is globally increasing in the sense that $\beta_1 > \lim_{h\to\infty}\beta_h$:** To show that is globally increasing, it suffices to show that $\lim_{h\to\infty} R_{i,d}(h) > R_{i,d}(1)$. We have

$$\lim_{h\to\infty} R_{i,d}(h) = C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d} > 0 \tag{A.100}$$

since $\rho_i \in (0,1)$, $\rho_s \in (0,1)$ and $\delta_d \in (-1,0)$. We also have

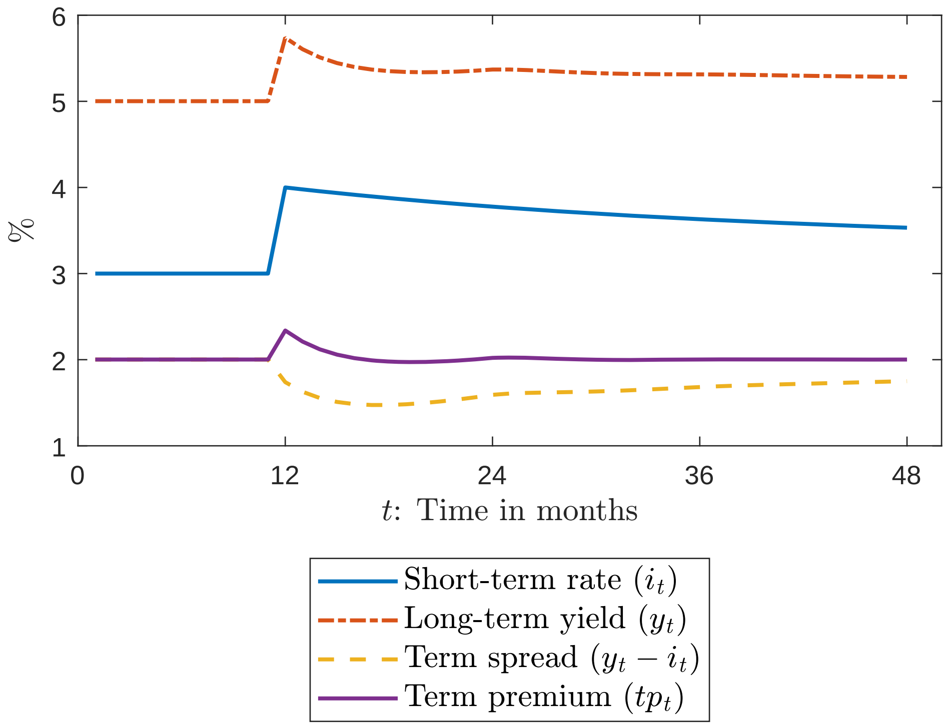
$$R_{i,d}(1) = -C\frac{\left(1-\rho_i^2\right)}{1-\rho_i\rho_s}\frac{\rho_i\delta_s}{1-\rho_i\delta_d}\frac{1}{2\rho_i}\left(1-\rho_i\right) < 0. \tag{A.101}$$

Thus, we conclude that $\beta_1 > \lim_{h\to\infty}\beta_h$.

## C.9 Model-implied impulse response to a short-rate shock

Figure A.8 shows the model-implied impulse response functions in the post-2000 calibration following a 100 bp shock to short rates that lands in month $t = 12$. (We assume there is a 50 bp shock to both the persistent and transient components of the short rate.) The long-term yield is the sum of an expectations-hypothesis component and a term premium component: $y_t = eh_t + tp_t$. Thus, the term spread is $y_t - i_t = (eh_t - i_t) + tp_t$. The figure shows impulse responses for short-term rates $(i_t)$, long-term yields $(y_t)$, the term spread $(y_t - i_t)$, and the term premium $(tp_t)$.

**Figure A.8: Model-implied impulse response functions for the post-2000 calibration.** For the post-2000 calibration, we show the response of short-term and long-term interest rates following a one-time shock to short-term interest rates. We plot short-term nominal interest rates ($i_t$), long-term nominal yields ($y_t$), the term spread ($y_t - i_t$), and the term premium ($tp_t$). Initially, short-term nominal rates are at their steady-state level of $\bar{\imath} = 3\%$ and the term premium on long-term nominal bonds is at a steady-level of 2%. We then assume there is a 50 bp shock to both the persistent and transient components of the short rate that lands at $t = 12$, leading short-term nominal rates to jump from 3% to 4%.




The initial shock to short rates leads to a rise in term premia. Thus, relative to the expectations-hypothesis, long-term rates are excessively sensitive to short rates. However, the rise in term premia wears off quickly, explaining our key finding that $\beta_h$ declines sharply with horizon $h$. Nonetheless, the impulse to short rates causes the yield curve to flatten on impact as in the data. This is because $(eh_t - i_t)$ falls on impact and this flattening due to the expectations hypothesis outweighs the steepening due to the rise in term premia. However, the initial rise in short rates predicts additional yield curve flattening—and predictable reversals in long-term yields—over the following months.

# D Modelling rate-amplifying mechanisms

## D.1 The mortgage refinancing channel

### D.1.1 General model solution

We now we drop $s_t - \overline{s}$ from the state vector $\mathbf{x}_t$ and add $c_t^M - \overline{c}^M = c_t^M - (\alpha_0 + \lambda)$ to the state vector. As always, we conjecture that $y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$ and $d_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t$. Let $\mathbf{e}_c' \mathbf{x}_t = c_t^M - \overline{c}^M$, we then have

$$s_t = M \times \overline{DUR}^M + MN \times \left(y_t^M - c_t^M\right) = M \times \overline{DUR}^M + MN \times (\boldsymbol{\alpha}_1 - \mathbf{e}_c)' \mathbf{x}_t. \tag{A.102}$$

The law of motion for $c_t^M$ is

$$c_{t+1}^M - \overline{c}^M = (1-\eta)\left(c_t^M - \overline{c}^M\right) + \eta\left(y_t^M - \overline{y}^M\right). \tag{A.103}$$

Making use of the fact that $\left(y_t^M - \overline{y}^M\right) = (y_t - \overline{y}) = \boldsymbol{\alpha}_1' \mathbf{x}_t$, we can write

$$\mathbf{e}_c' \mathbf{x}_{t+1} = (1-\eta)\,\mathbf{e}_c' \mathbf{x}_t + \eta \boldsymbol{\alpha}_1' \mathbf{x}_t. \tag{A.104}$$

Thus, our assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix $\Gamma$ is a function of (i) the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageur demand and (ii) the parameters $\boldsymbol{\alpha}_1$ governing yields, so we write $\Gamma = \Gamma(\boldsymbol{\delta}_1, \boldsymbol{\alpha}_1)$. Specifically, for $k = 4$, we have

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\boldsymbol{\delta}_1, \boldsymbol{\alpha}_1)\,\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 \\ \eta\alpha_P & \eta\alpha_T & (1-\eta)+\eta\alpha_c & \eta\alpha_{d_1} & \eta\alpha_{d_2} & \eta\alpha_{d_3} \\ \delta_P & \delta_T & \delta_c & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{P,t} - \overline{i} \\ i_{T,t} \\ c_t^M - \overline{c}^M \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{A.105}$$

where $\Sigma \equiv Var_t[\boldsymbol{\epsilon}_{t+1}]$. Since $\varepsilon_{P,t+1}$ and $\varepsilon_{T,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.106}$$

To solve for the equilibrium, we need to clear the market for bonds in a way that is consistent with optimization on the part of fast-moving arbitrageurs and slow-moving arbitrageurs. The market-clearing condition is

$$\overbrace{(1-q)k^{-1}d_t + qb_t}^{\text{Active demand}} = \overbrace{M \times [\overline{DUR}^M + N \times \left(y_t^M - c_t^M\right)] - (1-q)(k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i})}^{\text{Active supply}}. \tag{A.107}$$

Letting $V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1$, denote the variance of 1-period excess returns, active demand is

$$\begin{aligned} &(1-q)k^{-1}d_t + qb_t \qquad \text{(A.108)} \\ &= \left[(1-q)k^{-1}\delta_0 + q\tau\frac{(\alpha_0 - \bar{i})}{V^{(1)}}\right] + \left[(1-q)k^{-1}\boldsymbol{\delta}_1' + q\tau\frac{\left(\frac{1}{1-\phi}\boldsymbol{\alpha}_1 - \mathbf{e}\right)' - \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\boldsymbol{\Gamma}}{V^{(1)}}\right]\mathbf{x}_t \end{aligned}$$

Active supply is

$$M \times [\overline{DUR}^M + N \times \left(y_t^M - c_t^M\right)] - (1-q)k^{-1}\textstyle\sum_{i=1}^{k-1} d_{t-i} \qquad \text{(A.109)}$$

$$= \left[M \times \overline{DUR}^M - (1-q)\frac{(k-1)}{k}\delta_0\right] + \left[MN \times (\boldsymbol{\alpha}_1 - \mathbf{e}_c) - (1-q)k^{-1}\mathbf{e}_d\right]' \mathbf{x}_t. \qquad \text{(A.110)}$$

Matching constant terms, we obtain

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}(M \times \overline{DUR}^M - (1-q)\delta_0). \qquad \text{(A.111)}$$

Matching slope coefficients and solving we find

$$\boldsymbol{\alpha}_1 = (1-\phi)\left[\left(1 - (1-\phi)\frac{MN}{q\tau}V^{(1)}\right)\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left[\mathbf{e} - \frac{V^{(1)}}{q\tau}\left(MN\mathbf{e}_c + (1-q)\,k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right)\right]. \qquad \text{(A.112)}$$

In summary, an equilibrium in this model extension solves the following system of equations

$$\boldsymbol{\alpha}_1 = (1-\phi)\left[\left(1 - (1-\phi)\frac{MN}{q\tau}V^{(1)}(\boldsymbol{\alpha}_1)\right)\mathbf{I} - \phi\left[\boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)\right]'\right]^{-1}\left[\mathbf{e} - \frac{V^{(1)}(\boldsymbol{\alpha}_1)}{q\tau}\left(MN\mathbf{e}_c + (1-q)\,k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right)\right. \qquad \text{(A.113)}$$

and

$$\boldsymbol{\delta}_1' = \tau\frac{\left((\boldsymbol{\alpha}_1 - \mathbf{e})'\left(\mathbf{I} - \boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)\right)^{-1} + \frac{\phi}{1-\phi}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)}\left(\mathbf{I} - \boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)^k\right), \qquad \text{(A.114)}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\boldsymbol{\Gamma}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)$ to emphasize that the transition matrix depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1, \boldsymbol{\delta}_1)$ to emphasize that the $k$-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. Unlike in our baseline model, the first two elements of $\boldsymbol{\delta}_1$ are going to be positive, since slow-moving arbitrageurs will buy more long-term bonds when interest rates rise.[21] Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for $\alpha_0$ and $\delta_0$ using

$$\alpha_0 = \bar{i} + \frac{V^{(1)}}{\tau q}\left(\bar{s} - (1-q)\delta_0\right) \text{ and } \delta_0 = \tau\frac{k\left(\alpha_0 - \bar{i}\right)}{V^{(k)}}, \qquad \text{(A.115)}$$

[21] If we were to add independent shocks to the supply of long-term bonds to the model, then the mortgage refinancing channel would also lead long-term yields to temporarily over-react to those bond supply shocks.

which yields

$$\alpha_0 = \bar{i} + \frac{\bar{s}}{\tau\left[q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}\right]} \text{ and } \delta_0 = \frac{\frac{k}{V^{(k)}}}{q\frac{1}{V^{(1)}} + (1-q)\frac{k}{V^{(k)}}} \times \bar{s}. \tag{A.116}$$

### D.1.2 Proof of Proposition 2

**Proposition 2.** ***Mortgage refinancing model.*** *For simplicity suppose $\rho_T = \rho_P$. When $MN > 0$, long-term yields are excessively sensitive to short rates. When $MN > 0$ and $\eta = 0$, this excess sensitivity is only horizon-dependent—i.e., the model-implied regression coefficients $\beta_h$ in equation (A.71) only decline with horizon $h$—when arbitrage capital is slow moving ($q < 1$). By contrast, when $MN > 0$ and $\eta > 0$, $\beta_h$ declines with horizon $h$ even if all arbitrage capital is fast-moving ($q = 1$).*

**Proof:** To demonstrate this result, it suffices to consider two special cases. First, we consider the case where there is no slow-moving capital ($q = 1$) and where and $\rho_T = \rho_P \equiv \rho_i$. Next, we study the case where $q < 1$, $k = 2$, and $\rho_T = \rho_P \equiv \rho_i$. The arguments given in this special case generalize naturally to the case where $k > 2$.

**Case #1: $q = 1$ and $\rho_T = \rho_P \equiv \rho_i$:** In this case, we have

$$y_t = \alpha_0 + \alpha_1'\mathbf{x}_t = \alpha_0 + \alpha_i\left(i_t - \bar{i}\right) + \alpha_c\left(c_t^M - \bar{c}^M\right). \tag{A.117}$$

The law of motion for the state vector is

$$\mathbf{x}_{t+1} = \Gamma(\boldsymbol{\alpha})\,\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} = \begin{bmatrix} \rho_i & 0 \\ \eta\alpha_i & (1-\eta)+\eta\alpha_c \end{bmatrix}\begin{bmatrix} i_t - \bar{i} \\ c_t^M - \bar{c}^M \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \end{bmatrix}, \tag{A.118}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_i^2 & 0 \\ 0 & 0 \end{bmatrix}. \tag{A.119}$$

Thus, we have

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 (\alpha_i)^2\,\sigma_i^2. \tag{A.120}$$

Letting

$$Z = MN\tau^{-1}V^{(1)} = MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2 (\alpha_i)^2\,\sigma_i^2, \tag{A.121}$$

we have:

$$\begin{aligned}
\begin{bmatrix} \alpha_i \\ \alpha_c \end{bmatrix} &= (1-\phi)\left(\left((1-(1-\phi)\,MN\tau^{-1}V^{(1)}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \phi\begin{bmatrix} \rho_i & \alpha_i\eta \\ 0 & (1-\eta)+\eta\alpha_c \end{bmatrix}\right)^{-1}\begin{bmatrix} 1 \\ -(\tau)^{-1}V^{(1)}MN \end{bmatrix} \\
&= \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i-(1-\phi)Z}\left(1 - \frac{\frac{\phi}{1-\phi}Z\alpha_i\eta}{1-Z+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\right) \\ -\frac{Z}{1-Z+\frac{\phi}{1-\phi}\eta(1-\alpha_c)} \end{bmatrix}.
\end{aligned} \tag{A.122}$$

Thus, the full fixed point problem is:

$$\left[\begin{array}{c}\alpha_i\\ \alpha_c\end{array}\right]=\left[\begin{array}{c}\frac{1-\phi}{1-\phi\rho_i-(1-\phi)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}\left(1-\frac{\eta\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^3(\alpha_i)^3\sigma_i^2}{1-\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\right)\\ -\frac{MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}{1-MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\end{array}\right]. \tag{A.123}$$

We now characterize the solution and $\beta_h$ in two special limiting cases. We then characterize the solution in general.

**Limit where $\tau^{-1}MN \to 0$:** In the limit, where the mortgage financing channel disappears—i.e., where $\tau^{-1}MN \to 0$, we have $\alpha_i \to (1-\phi)/(1-\phi\rho_i)$ and $\alpha_c \to 0$. Thus, we have $\beta_h = (1-\phi)/(1-\phi\rho_i)$ for all $h$—i.e., the expectations hypothesis holds and there is no excess sensitivity.

**Limit where $\tau^{-1}MN > 0$ and $\eta = 0$:** In the limit where $\tau^{-1}MN > 0$ but where $\eta = 0$, the fixed point problem simplifies to

$$\left[\begin{array}{c}\alpha_i\\ \alpha_c\end{array}\right]=\left[\begin{array}{c}\frac{1-\phi}{1-\phi\rho_i-(1-\phi)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}\\ -\frac{MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}{1-MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}\end{array}\right]. \tag{A.124}$$

The condition for $\alpha_i$ then implies that

$$\alpha_i = g(\alpha_i) = \frac{1-\phi}{1-\phi\rho_i}\left(1+\tau^{-1}\sigma_i^2 MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^3\right) \tag{A.125}$$

There are at most 3 solutions, $\alpha_i^* = g(\alpha_i^*)$, to this cubic equation. Since $g(0) > 0$ and $g'(\alpha_i) > 0$ for all $\alpha_i$, there will always be a single negative solution $\alpha_i^* < 0$ that is unstable in the sense that $g'(\alpha_i^*) > 1$. When $\tau^{-1}\sigma_i^2 MN(\phi/(1-\phi))^2$ is large there will be no positive solutions. When $\tau^{-1}\sigma_i^2 MN(\phi/(1-\phi))^2$ is small, there are two positive solutions, the smaller of which is stable in the sense that $g'(\alpha_i^*) < 1$. We are always interested in the stable solution of this fixed point problem. Focusing on this stable solution, it is then trivially the case that $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$. When $\eta = 0$, we have $c_t^M = \bar{c}^M$ and thus $\alpha_c\left(c_t^M - \bar{c}\right) = 0$. In other words, $\alpha_c$ is not relevant in equilibrium. Thus, when $q = 1$, $\tau^{-1}MN > 0$, and $\eta = 0$, we have $\beta_h = \alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ for all $h$. In other words, the expectations hypothesis fails and there is excess sensitivity, but this excess sensitivity is not horizon dependent.

**General case where $\tau^{-1}MN > 0$ and $\eta > 0$:** Here we have

$$\left[\begin{array}{c}\alpha_i\\ \alpha_c\end{array}\right]=\left[\begin{array}{c}\frac{1-\phi}{1-\phi\rho_i-(1-\phi)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}\left(1-\frac{\eta\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^3(\alpha_i)^3\sigma_i^2}{1-\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\right)\\ -\frac{MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2}{1-MN\tau^{-1}\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2+\frac{\phi}{1-\phi}\eta(1-\alpha_c)}\end{array}\right]. \tag{A.126}$$

Relative to the case where $\eta = 0$, $\alpha_i$ is reduced because mortgage refinancing dynamics—i.e., the fact that the average mortgage coupon evolves over time—mean that the induced duration supply shocks are expected to be less persistent. Analogously, $\alpha_c$ is reduced in absolute value for the same reason.

*Characterizing the solution*: We first show that $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ and $\alpha_c^* < 0$ in any stable solution. We can rewrite the first condition for $\alpha_i$ to obtain

$$\alpha_i = \frac{1-\phi}{1-\phi\rho_i - \eta\phi\alpha_c}\left(1 + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2 (\alpha_i)^3 \sigma_i^2\right) \tag{A.127}$$

Thus, for a fixed $\alpha_c < 0$ the solution when $\eta > 0$ is lower than the solution when $\eta = 0$. We want to show that we still have $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ in any stable solution. It suffices to show that the fixed point operator returns $\alpha_i > (1-\phi)/(1-\phi\rho_i)$ if we initially suppose that $\alpha_i = (1-\phi)/(1-\phi\rho_i)$. Concretely, suppose that a stable solution exists and guess that the solution is $\alpha_i = (1-\phi)/(1-\phi\rho_i)$. Then, using the condition for $\alpha_i$, we have

$$\alpha_c = -\frac{1}{\eta}\tau^{-1}MN\frac{\phi(1-\phi)}{(1-\phi\rho_i)^2}\sigma_i^2. \tag{A.128}$$

Now plug this into the condition for $\alpha_c$ and solve for $\alpha_i$ to obtain

$$(\alpha_i)^2 = \frac{1 + \frac{\phi}{1-\phi}\eta + \tau^{-1}MN\frac{\phi^2}{(1-\phi\rho_i)^2}\sigma_i^2}{\eta\frac{\phi}{1-\phi}\frac{(1-\phi\rho_i)^2}{(\phi-1)^2} + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2\sigma_i^2} \tag{A.129}$$

Since

$$\begin{aligned}&\frac{1 + \frac{\phi}{1-\phi}\eta + \tau^{-1}MN\frac{\phi^2}{(1-\phi\rho_i)^2}\sigma_i^2}{\eta\frac{\phi}{1-\phi}\frac{(1-\phi\rho_i)^2}{(\phi-1)^2} + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2\sigma_i^2} - \left(\frac{1-\phi}{1-\phi\rho_i}\right)^2 \\ = \;& \frac{\tau}{\phi}\frac{(1-\phi)^3}{MN\phi(1-\phi)\sigma_i^2 + \tau\eta(1-\phi\rho_i)^2} > 0,\end{aligned} \tag{A.130}$$

this implies that we must have $\alpha_i^* > (1-\phi)/(1-\phi\rho_i)$ in any stable solution.

We want to show that $\alpha_c < 0$ in any stable solution. It suffices to show that the fixed point operator returns $\alpha_c < 0$ if we initially suppose that $\alpha_c = 0$. Suppose $\alpha_c = 0$. Then, $\alpha_i$ solves

$$\alpha_i = \frac{1-\phi}{1-\phi\rho_i}\left(1 + \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^3\sigma_i^2\right) > \frac{1-\phi}{1-\phi\rho_i}. \tag{A.131}$$

Assuming this solution is stable, we have

$$Z = \tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2 < 3\left(\frac{1-\phi}{1-\phi\rho_i}\right)\tau^{-1}MN\left(\frac{\phi}{1-\phi}\right)^2(\alpha_i)^2\sigma_i^2 < 1. \tag{A.132}$$

Then $\alpha_c$ solves

$$-\frac{\phi}{1-\phi}\eta\left[\alpha_c^2\right] + \left[(1-Z) + \frac{\phi}{1-\phi}\eta\right]\alpha_c + Z = 0. \tag{A.133}$$

The relevant stable solution is the smaller solution. And the smaller solution here is negative. Thus, we have $\alpha_c^* < 0$ in any stable solution.

*Calculating* $\beta_h$: To calculate $\beta_h$ we make the following observations. First, using the fact that $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we have

$$\mathbf{V} = \begin{bmatrix} \frac{\sigma_i^2}{1-\rho_i^2} & \frac{\eta\alpha_i\sigma_i^2\rho_i}{\left(1-\rho_i^2\right)(1-\rho_i+\eta\rho_i(1-\alpha_c))} \\ \frac{\eta\alpha_i\sigma_i^2\rho_i}{\left(1-\rho_i^2\right)(1-\rho_i+\eta\rho_i(1-\alpha_c))} & \frac{\eta\alpha_i^2\sigma_i^2(1+\rho_i-\eta\rho_i(1-\alpha_c))}{(1-\alpha_c)\left(1-\rho_i^2\right)(1+\eta\alpha_c+1-\eta)(1-\rho_i+\eta\rho_i(1-\alpha_c))} \end{bmatrix} \quad (A.134)$$

Next, using the fact that

$$\mathbf{\Gamma}^h = \begin{bmatrix} (\rho_i)^h & 0 \\ \eta\alpha_i \frac{(\rho_i)^h - ((1-\eta)+\eta\alpha_c)^h}{\rho_i - ((1-\eta)+\eta\alpha_c)} & ((1-\eta)+\eta\alpha_c)^h \end{bmatrix} \quad (A.135)$$

we have

$$Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] = 2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h \quad (A.136)$$

$$= \begin{bmatrix} 2\left(1-\rho_i^h\right)Var\left[i_t\right] & \begin{array}{c}\left(2-\rho_i^h-((1-\eta)+\eta\alpha_c)^h\right)Cov\left[i_t,c_t\right] \\ -\eta\alpha_i\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)}Var\left[i_t\right]\end{array} \\ \begin{array}{c}\left(2-\rho_i^h-((1-\eta)+\eta\alpha_c)^h\right)Cov\left[i_t,c_t\right] \\ -\eta\alpha_i\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)}Var\left[i_t\right]\end{array} & 2\left(\begin{array}{c}\left(1-((1-\eta)+\eta\alpha_c)^h\right)Var\left[c_t\right] \\ -\eta\alpha_i\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)}Cov\left[i_t,c_t\right]\end{array}\right) \end{bmatrix}$$

Thus, we have

$$\begin{aligned} \beta_h &= \alpha_i + \alpha_c \frac{\left(2-2\rho_i^h+\rho_i^h-((1-\eta)+\eta\alpha_c)^h\right)Cov\left[i_t,c_t\right] - \eta\alpha_i\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{\rho_i-((1-\eta)+\eta\alpha_c)}Var\left[i_t\right]}{2\left(1-\rho_i^h\right)Var\left[i_t\right]} \quad (A.137) \\ &= \alpha_i + \alpha_c\frac{Cov\left[i_t,c_t\right]}{Var\left[i_t\right]} + \alpha_c\left(\frac{Cov\left[i_t,c_t\right]}{Var\left[i_t\right]} - \frac{\eta\alpha_i}{\rho_i-((1-\eta)+\eta\alpha_c)}\right)\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{2\left(1-\rho_i^h\right)} \\ &= \alpha_i\left(\frac{1-\rho_i(1-\eta)}{1-\rho_i+\eta\rho_i(1-\alpha_c)}\right) \\ &\quad + \left[-\frac{1-\rho_i^2}{(\rho_i-((1-\eta)+\eta\alpha_c))(1-\rho_i((1-\eta)+\eta\alpha_c))}\alpha_c\alpha_i\eta\right]\left\{\frac{\rho_i^h-((1-\eta)+\eta\alpha_c)^h}{2\left(1-\rho_i^h\right)}\right\}. \end{aligned}$$

There are two cases to consider:

1. If $\rho_i > (1-\eta) + \eta\alpha_c$, the term in square brackets is positive. And, since $\rho_i > (1-\eta) + \eta\alpha_c$, the term in curly braces is decreasing in $h$. Thus, $\beta_h$ is decreasing in $h$.[22]

2. Alternately, if $\rho_i < (1-\eta) + \eta\alpha_c$, the term in square brackets is negative. And, since $\rho_i < (1-\eta) + \eta\alpha_c$, the term in curly braces is increasing in $h$. Thus, $\beta_h$ is decreasing in $h$.

Thus, $\beta_h$ is decreasing in $h$ when $\eta > 0$, $\tau^{-1}MN > 0$, and $q = 1$.

[22] To show that the term in curly braces is decreasing in $h$ when $\rho_i > \rho_c \equiv (1-\eta) + \eta\alpha_c$, recall that we showed above that $\left(2-\rho_i^h-\rho_c^h\right)/\left(2-\rho_i^h-\rho_i^h\right)$ exceeds 1 and is decreasing in $h$ when $\rho_i > \rho_c$. However, we have $\left(\rho_i^h-\rho_c^h\right)/\left(2\left(1-\rho_i^h\right)\right) = \left(2-\rho_i^h-\rho_c^h\right)/\left(2-\rho_i^h-\rho_i^h\right) - 1$. Thus, the term in curly brackets is positive and decreasing in $h$ when $\rho_i > \rho_c$. Conversely, the term in curly brackets is negative and increasing in $h$ when $\rho_i < \rho_c$.

**Summary:** When there is no slow-moving capital ($q = 1$), the mortgage refinancing model generate excess sensitivity when $\tau^{-1}MN > 0$. Furthermore, this excess sensitivity is horizon dependent—i.e., it is most pronounced at high frequencies—when $\eta > 0$.

**Case #2:** $q < 1$**,** $k = 2$**, and** $\rho_T = \rho_P \equiv \rho_i$

**Solution:** In this case, we have

$$y_t = \alpha_0 + \alpha_1' \mathbf{x}_t = \alpha_0 + \alpha_i \left(i_t - \bar{i}\right) + \alpha_c \left(c_t^M - \bar{c}^M\right) + \alpha_d \left(d_{t-1} - \delta_0\right). \tag{A.138}$$

The law of motion for the state vector is

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma\left(\boldsymbol{\alpha}\right) \mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_i & 0 & 0 \\ \eta\alpha_i & (1-\eta) + \eta\alpha_c & \eta\alpha_d \\ \delta_i & \delta_c & \delta_d \end{bmatrix} \begin{bmatrix} i_t - \bar{i} \\ c_t^M - \bar{c}^M \\ d_{t-1} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \\ 0 \end{bmatrix}, \end{aligned} \tag{A.139}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_i^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{A.140}$$

Thus, we have

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \left(\alpha_i\right)^2 \sigma_i^2. \tag{A.141}$$

Defining

$$Z = \frac{MN}{q\tau} V^{(1)} = \frac{MN}{q\tau} \left(\frac{\phi}{1-\phi}\right)^2 \left(\alpha_i\right)^2 \sigma_i^2 \tag{A.142}$$

(we have defined $Z$ to depend on $q$), the solution is

$$\begin{aligned} \boldsymbol{\alpha}_1 &= (1-\phi)\left[\left(1-(1-\phi)Z\right)\mathbf{I} - \phi\boldsymbol{\Gamma}'\right]^{-1}\left[\mathbf{e} - \left(Z\mathbf{e}_c + \frac{Z}{MN}(1-q)\,k^{-1}\left(\mathbf{e}_d + \boldsymbol{\delta}_1\right)\right)\right] \\ &= (1-\phi)\left[\left(1-(1-\phi)Z\right)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \phi \begin{bmatrix} \rho_i & \eta\alpha_i & \delta_i \\ 0 & (1-\eta)+\eta\alpha_c & \delta_c \\ 0 & \eta\alpha_d & \delta_d \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 - \frac{Z}{MN}\frac{\delta_i(1-q)}{2} \\ -Z - \frac{Z}{MN}\delta_c\frac{1}{2}(1-q) \\ -\frac{Z}{MN}\left(\delta_d+1\right)\frac{1}{2}(1-q) \end{bmatrix} \end{aligned} \tag{A.143}$$

The solution does not readily simplify because $\Gamma$ is no longer lower triangular. In all other cases, $\Gamma$ is effectively lower triangular, which makes the expressions much nicer.

**Characterizing the solution:** As above, one can show that we have $\alpha_i > (1-\phi)/(1-\phi\rho_i)$, $\alpha_c < 0$, and $\alpha_d < 0$ in any stable solution. Similarly, we have $\delta_i > 0$, $\delta_c < 0$, and $-1 < \delta_d < 0$ in any stable solution.

**Computing $\beta_h$:** Using $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we have

$$\mathbf{V}= \begin{bmatrix} V_i & C_{i,c} & C_{i,d} \\ C_{i,c} & V_c & C_{c,d} \\ C_{i,d} & C_{c,d} & V_d \end{bmatrix} \tag{A.144}$$

where

$$V_i = \frac{\sigma_i^2}{1-\rho_i^2}, \tag{A.145}$$

$$C_{i,c} = \frac{\sigma_i^2}{1-\rho_i^2}\rho_i \frac{\eta\alpha_i\left(1-\delta_d\rho_i\right)+\eta\alpha_d\delta_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \tag{A.146}$$

$$C_{i,d} = \frac{\sigma_i^2}{1-\rho_i^2}\rho_i \frac{\delta_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)+\delta_c\eta\alpha_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \tag{A.147}$$

Since $C_{i,c} \equiv Cov\left[i_t, c_t^M\right] > 0$ and $C_{i,d} \equiv Cov\left[i_t, d_{t-1}\right] > 0$ in any stable equilibrium when $\eta > 0$ and $q < 1$, we have

$$\alpha_i\left(1-\delta_d\rho_i\right)+\alpha_d\delta_i\rho_i > 0, \tag{A.148}$$

$$\delta_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)+\delta_c\eta\alpha_i\rho_i > 0, \tag{A.149}$$

$$1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2 > 0. \tag{A.150}$$

Next, letting

$$\mathbf{\Gamma}^h = \begin{bmatrix} \rho_i & 0 & 0 \\ \gamma_i & \gamma_c & \gamma_d \\ \delta_i & \delta_c & \delta_d \end{bmatrix}^h \equiv \begin{bmatrix} a_h & 0 & 0 \\ b_h & d_h & f_h \\ c_h & e_h & g_h \end{bmatrix}, \tag{A.151}$$

we have

$$\begin{aligned} & Var\left[\mathbf{x}_{t+h}-\mathbf{x}_t\right] \\ = \; & 2\mathbf{V} - \mathbf{\Gamma}^h\mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h \\ = \; & \left[ \begin{matrix} 2\left(1-a_h\right)V_i & \left(2-d_h-a_h\right)C_{c,i}-f_hC_{i,d}-b_hV_i & \left(2-a_h-\right. \\ \left(2-d_h-a_h\right)C_{c,i}-f_hC_{i,d}-b_hV_i & 2\left(1-d_h\right)V_c-2f_hC_{c,d}-2b_hC_{c,i} & \left(2-g_h-d_h\right)C_{c,d} \\ \left(2-a_h-g_h\right)C_{i,d}-e_hC_{i,c}-c_hV_i & \left(2-g_h-d_h\right)C_{c,d}-b_hC_{d,i}-c_hC_{c,i}-e_hV_c-V_df_h & 2\left(1-g_h\right. \end{matrix} \right. \end{aligned}$$

Combing these elements, we have

$$\begin{aligned} \beta_h &= \alpha_i + \alpha_c\frac{\left(2-a_h-d_h\right)C_{i,c}-f_hC_{i,d}-b_hV_i}{2\left(1-a_h\right)V_i} + \alpha_d\frac{\left(2-a_h-g_h\right)C_{i,d}-e_hC_{i,c}-c_hV_i}{2\left(1-a_h\right)V_i} \\ &= \left[\alpha_i + \alpha_c\frac{C_{i,c}}{V_i} + \alpha_d\frac{C_{i,d}}{V_i}\right] \\ &\quad + \left[\alpha_c\left\{\frac{\left(a_h-d_h\right)C_{i,c}-f_hC_{i,d}-b_hV_i}{2\left(1-a_h\right)V_i}\right\} + \alpha_d\left\{\frac{\left(a_h-g_h\right)C_{i,d}-e_hC_{i,c}-c_hV_i}{2\left(1-a_h\right)V_i}\right\}\right] \end{aligned} \tag{A.152}$$

Since $\lim_{h\to\infty}\mathbf{\Gamma}^h=\mathbf{0}$, we have

$$\begin{aligned}
\lim_{h\to\infty}\beta_h &= \alpha_i+\alpha_c\frac{C_{i,c}}{V_i}+\alpha_d\frac{C_{i,d}}{V_i} \\
&= \alpha_i+\alpha_c\rho_i\frac{\eta\alpha_i\left(1-\delta_d\rho_i\right)+\eta\alpha_d\delta_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&\quad +\alpha_d\rho_i\frac{\delta_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)+\delta_c\eta\alpha_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&= \frac{\left(\alpha_i-\delta_d\alpha_i\rho_i+\alpha_d\delta_i\rho_i\right)\left(1-\rho_i\left(1-\eta\right)\right)}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&= \frac{\left(1-\rho_i\left(1-\eta\right)\right)}{\rho_i\eta}\rho_i\frac{\eta\alpha_i\left(1-\delta_d\rho_i\right)+\eta\alpha_d\delta_i\rho_i}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
&= \frac{\left(1-\rho_i\left(1-\eta\right)\right)}{\rho_i\eta}\frac{C_{i,c}}{V_i}>0.
\end{aligned} \tag{A.153}$$

We also have

$$\beta_h=\lim_{h\to\infty}\beta_h+\alpha_c\mathcal{R}_{i,c}\left(h\right)+\alpha_d\mathcal{R}_{i,d}\left(h\right) \tag{A.154}$$

where

$$\begin{aligned}
\mathcal{R}_{i,c}\left(h\right) &= \frac{Cov\left[i_{t+h}-i_t,c_{t+h}^M-c_t^M\right]}{Var\left[i_{t+h}-i_t\right]}-\frac{Cov\left[i_t,c_t^M\right]}{Var\left[i_t\right]} \\
&= \frac{\left(a_h-d_h\right)C_{ci}-f_hC_{id}-b_hV_i}{2\left(1-a_h\right)V_i} \\
&= \frac{\begin{array}{c}\left(a_h-d_h\right)\rho_i\frac{\eta\alpha_i(1-\delta_d\rho_i)+\eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -f_h\rho_i\frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i)+\delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2}-b_h\end{array}}{2\left(1-a_h\right)}
\end{aligned} \tag{A.155}$$

and

$$\begin{aligned}
\mathcal{R}_{i,d}\left(h\right) &= \frac{Cov\left[i_{t+h}-i_t,d_{t+h-1}-d_{t-1}\right]}{Var\left[i_{t+h}-i_t\right]}-\frac{Cov\left[i_t,d_{t-1}\right]}{Var\left[i_t\right]} \\
&= \frac{\left(a_h-g_h\right)C_{i,d}-e_hC_{i,c}-c_hV_i}{2\left(1-a_h\right)V_i} \\
&= \frac{\begin{array}{c}\left(a_h-g_h\right)\rho_i\frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i)+\delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -e_h\rho_i\frac{\eta\alpha_i(1-\delta_d\rho_i)+\eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2}-c_h\end{array}}{2\left(1-a_h\right)}
\end{aligned} \tag{A.156}$$

**Proof that** $\lim_{h\to\infty}\beta_h<\beta_1$**:** We have

$$\begin{aligned}
&\beta_1-\lim_{h\to\infty}\beta_h \\
= &\ \alpha_c\mathcal{R}_{i,c}\left(1\right)+\alpha_d\mathcal{R}_{i,d}\left(1\right) \\
= &\ \frac{1}{2}\left(\rho_i+1\right)\frac{\left[-\alpha_d\right]\left(\delta_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)+\delta_c\eta\alpha_i\rho_i\right)+\left[-\alpha_c\right]\eta\left(\alpha_i\left(1-\delta_d\rho_i\right)+\alpha_d\delta_i\rho_i\right)}{1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i-\delta_d\rho_i\left(1-\left(\left(1-\eta\right)+\eta\alpha_c\right)\rho_i\right)-\eta\alpha_d\delta_c\rho_i^2} \\
> &\ 0.
\end{aligned} \tag{A.157}$$

We know that the denominator is positive. Furthermore, since $\alpha_d < 0$ and $(\delta_i (1 - ((1-\eta) + \eta\alpha_c) \rho_i) + \delta_c \eta \alpha_i \rho_i) > 0$ and $\alpha_c < 0$ and $(\alpha_i (1 - \delta_d \rho_i) + \alpha_d \delta_i \rho_i) > 0$, the numerator is also positive.

**Proof that $\beta_2 < \beta_1$:** We want to show that

$$\beta_2 - \lim_{h\to\infty} \beta_h < \beta_1 - \lim_{h\to\infty} \beta_h.$$

We have

$$\begin{bmatrix} a_2 & 0 & 0 \\ b_2 & d_2 & f_2 \\ c_2 & e_2 & g_2 \end{bmatrix} = \begin{bmatrix} \rho_i & 0 & 0 \\ \eta\alpha_i & (1-\eta)+\eta\alpha_c & \eta\alpha_d \\ \delta_i & \delta_c & \delta_d \end{bmatrix}^2 \tag{A.158}$$

$$= \begin{bmatrix} \rho_i^2 & 0 & 0 \\ \eta\alpha_i (1-\eta+\eta\alpha_c) + \eta\alpha_i\rho_i + \eta\alpha_d\delta_i & ((1-\eta)+\eta\alpha_c)^2 + \eta\alpha_d\delta_c & \eta\alpha_d ((1-\eta)+\eta\alpha_c+\delta_d) \\ \delta_d\delta_i + \delta_i\rho_i + \eta\delta_c\alpha_i & \delta_c ((1-\eta)+\eta\alpha_c+\delta_d) & \delta_d^2 + \eta\alpha_d\delta_c \end{bmatrix}$$

We have

$$\begin{aligned} \beta_2 - \lim_{h\to\infty} \beta_h &= \alpha_c \frac{\begin{array}{c} \left(\rho_i^2 - \left(((1-\eta)+\eta\alpha_c)^2 + \eta\alpha_d\delta_c\right)\right) \rho_i \frac{\eta\alpha_i(1-\delta_d\rho_i)+\eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -\eta\alpha_d ((1-\eta)+\eta\alpha_c+\delta_d) \rho_i \frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i)+\delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -(\eta\alpha_i (1-\eta+\eta\alpha_c) + \eta\alpha_i\rho_i + \eta\alpha_d\delta_i) \end{array}}{2\left(1-\rho_i^2\right)} \\ &+\alpha_d \frac{\begin{array}{c} \left(\rho_i^2 - \left(\delta_d^2 + \eta\alpha_d\delta_c\right)\right) \rho_i \frac{\delta_i(1-((1-\eta)+\eta\alpha_c)\rho_i)+\delta_c\eta\alpha_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -\delta_c ((1-\eta)+\eta\alpha_c+\delta_d) \rho_i \frac{\eta\alpha_i(1-\delta_d\rho_i)+\eta\alpha_d\delta_i\rho_i}{1-((1-\eta)+\eta\alpha_c)\rho_i-\delta_d\rho_i(1-((1-\eta)+\eta\alpha_c)\rho_i)-\eta\alpha_d\delta_c\rho_i^2} \\ -(\delta_d\delta_i + \delta_i\rho_i + \eta\delta_c\alpha_i) \end{array}}{2\left(1-\rho_i^2\right)} \end{aligned} \tag{A.159}$$

Comparing this with $\beta_1 - \lim_{h\to\infty} \beta_h$, we have

$$\begin{aligned} \beta_1 - \beta_2 &= \frac{1}{2} \left\{ \frac{(1-\rho_i)\delta_i (\alpha_d\delta_d - \alpha_d)}{1 - ((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \right\} \\ &+ \frac{\eta}{2} \left\{ \frac{\begin{array}{c} \eta\alpha_i \left[\alpha_c^2 - \alpha_c - \delta_d\alpha_c^2\rho_i + \alpha_c (\delta_d + \alpha_d\delta_c)\rho_i\right] \\ + \left[\alpha_c\alpha_d\delta_i + \alpha_d\delta_c\alpha_i(1-\rho_i) - \alpha_d\delta_i\rho_i - \alpha_c\alpha_d\delta_d\delta_i\rho_i + \alpha_d(\delta_d+\alpha_d\delta_c)\delta_i\rho_i\right] \end{array}}{1 - ((1-\eta)+\eta\alpha_c)\rho_i - \delta_d\rho_i (1 - ((1-\eta)+\eta\alpha_c)\rho_i) - \eta\alpha_d\delta_c\rho_i^2} \right\} \\ &> 0. \end{aligned} \tag{A.160}$$

The first term in curly braces is positive. We just need to consider the second term in curly braces. The denominator is positive. Finally, since $(\delta_d + \alpha_d\delta_c) < 0$, all the terms in the numerator are positive. Thus, the second term in curly braces is also positive and we conclude that $\beta_1 > \beta_2$.

## D.2 The investor extrapolation channel

### D.2.1 General model solution

We we drop $s_t - \overline{s}$ from the state vector $\mathbf{x}_t$ and now add $m_{P,t}$ and $m_{T,t}$ to the state vector. For the sake of concreteness, suppose that $k = 4$. We conjecture that equilibrium yields take the form $y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$, and that the demands of active slow-moving arbitrageurs are of the form $d_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t$, where the $k+3$ dimensional state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \\ m_{P,t} \\ m_{T,t} \end{bmatrix}. \tag{A.161}$$

Rational investors believe the law motion for the state vector is:

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma\left(\boldsymbol{\delta}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 & 0 \\ \delta_P & \delta_T & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} & \delta_{m_P} & \delta_{m_P} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_T \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \\ m_{P,t} \\ m_{T,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \\ \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \end{bmatrix}, \end{aligned} \tag{A.162}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_P^2 & 0 & 0 & 0 & 0 & \sigma_P^2 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 & 0 & \sigma_T^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_P^2 & 0 & 0 & 0 & 0 & \sigma_P^2 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 & 0 & \sigma_T^2 \end{bmatrix}. \tag{A.163}$$

By contrast, diagnostic investors believe the law of motion is:

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma_D\left(\boldsymbol{\delta}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 & \theta & 0 \\ 0 & \rho_T & 0 & 0 & 0 & 0 & \theta \\ \delta_P & \delta_T & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} & \delta_{m_P} & \delta_{m_T} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa_P & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_T \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \\ m_{P,t} \\ m_{T,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \\ \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \end{bmatrix}. \end{aligned} \tag{A.164}$$

Thus, both rational investors and diagnostic investors properly forecast the evolution of diagnostic investors' time-varying biases $m_{P,t}$ and $m_{T,t}$. However, diagnostic investors' forecasts of future short rates depend on $m_{P,t}$ and $m_{T,t}$, while rational investors' short rate forecasts do not depend on $m_{P,t}$ and $m_{T,t}$. Intuitively, the biases of diagnostic investors ($m_{P,t}$ and $m_{T,t}$) are like time-varying "signals" about future short rates. Diagnostic investors incorrectly believe that these signals are informative about future short rates. Rational investors know that these signals are irrelevant for forecasting future short rates (given what they already know), but they still need to keep track of these signals and forecast their evolution because these signals influence the future demands of diagnostic investors.

Rational investors and diagnostic investors agree about the risk of holding long-term bonds until the next period, but disagree about expected returns. Specifically, both rational and diagnostic investors believe the variance of 1-period excess returns is

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1. \tag{A.165}$$

Rational investors form unbiased forecasts of expected returns given by:

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1' \left(\mathbf{I} - \phi\boldsymbol{\Gamma}\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t. \tag{A.166}$$

Thus, the demands of fast-moving rational investors are:

$$b_t = \tau\frac{E_t\left[rx_{t+1}\right]}{V^{(1)}} = \tau\frac{\left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1' \left(\mathbf{I} - \phi\boldsymbol{\Gamma}\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t}{V^{(1)}}. \tag{A.167}$$

By contrast, diagnostic investors form biased forecasts of expected returns given by:

$$E_t^D\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1' \left(\mathbf{I} - \phi\boldsymbol{\Gamma}_D\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t.$$

Thus, the demands of diagnostic investors are:

$$h_t = \tau\frac{E_t^D\left[rx_{t+1}\right]}{V^{(1)}} = \tau\frac{\left(\alpha_0 - \bar{i}\right) + \frac{1}{1-\phi}\boldsymbol{\alpha}_1' \left(\mathbf{I} - \phi\boldsymbol{\Gamma}_D\right)\mathbf{x}_t - \mathbf{e}'\mathbf{x}_t}{V^{(1)}}. \tag{A.168}$$

It is easy to see that

$$E_t^D\left[rx_{t+1}\right] = E_t\left[rx_{t+1}\right] - \frac{\phi}{1-\phi}\theta\left(\alpha_P m_{P,t} + \alpha_T m_{T,t}\right), \tag{A.169}$$

where $\alpha_P = (1-\phi)/(1-\phi\rho_P)$ and $\alpha_T = (1-\phi)/(1-\phi\rho_T)$ are first and second elements of $\boldsymbol{\alpha}_1$, respectively. Thus, diagnostic investors underestimate the expected return to holding long-term bonds when short rates have recently rise—i.e., when $m_{P,t}$ and $m_{T,t}$ are positive—and therefore demand fewer long-term bonds that fast-moving rational investors.

Active demand at time $t$ is

$$\begin{aligned} & f h_t + (1-f)\,q b_t + (1-f)(1-q)\,k^{-1} d_t \\ & = \left[\tau\left(f + (1-f)\,q\right)\frac{\left(\alpha_0 - \bar{i}\right)}{V^{(1)}} + (1-f)(1-q)\,k^{-1}\delta_0\right] \\ & + \left[\tau f\frac{\frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}_D\right) - \mathbf{e}'}{V^{(1)}} + \tau(1-f)\,q\frac{\frac{1}{1-\phi}\boldsymbol{\alpha}_1'\left(\mathbf{I} - \phi\boldsymbol{\Gamma}\right) - \mathbf{e}'}{V^{(1)}} + (1-f)(1-q)\,k^{-1}\boldsymbol{\delta}_1'\right]\mathbf{x}_t \end{aligned} \tag{A.170}$$

and active supply is

$$\begin{aligned}
& \bar{s}-(1-f)(1-q)k^{-1}\sum\nolimits_{i=1}^{k-1}d_{t-i} && \text{(A.171)} \\
= & \left[\bar{s}-(1-f)(1-q)\frac{(k-1)}{k}\delta_0\right]+\left[-(1-f)(1-q)k^{-1}\mathbf{e}_d'\right]\mathbf{x}_t. &&
\end{aligned}$$

Matching constants terms, we obtain

$$\alpha_0=\bar{i}+\frac{V^{(1)}}{\tau}\frac{\bar{s}-(1-f)(1-q)\delta_0}{f+(1-f)q} \tag{A.172}$$

Matching slope coefficients on $\mathbf{x}_t$, we obtain

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\mathbf{I}-\phi\left(\frac{f\boldsymbol{\Gamma}_D+(1-f)q\boldsymbol{\Gamma}}{f+(1-f)q}\right)'\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)k^{-1}(\mathbf{e}_d+\boldsymbol{\delta}_1)}{f+(1-f)q}\right] \tag{A.173}$$

In summary, an equilibrium in this model extension solves the following system of equations

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\mathbf{I}-\phi\left(\frac{f\boldsymbol{\Gamma}_D(\boldsymbol{\delta}_1)+(1-f)q\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)}{f+(1-f)q}\right)'\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}(\boldsymbol{\alpha}_1)}{\tau}\frac{(1-f)(1-q)k^{-1}(\mathbf{e}_d+\boldsymbol{\delta}_1)}{f+(1-f)q}\right] \tag{A.174}$$

and

$$\boldsymbol{\delta}_1'=\tau\frac{\left((\boldsymbol{\alpha}_1-\mathbf{e})'(\mathbf{I}-\boldsymbol{\Gamma}(\boldsymbol{\delta}_1))^{-1}+\frac{\theta}{1-\theta}\boldsymbol{\alpha}_1'\right)}{V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)}\left(\mathbf{I}-\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)^k\right), \tag{A.175}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)$ and $\boldsymbol{\Gamma}_D(\boldsymbol{\delta}_1)$ emphasize that the true and perceived transition matrices depend on $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)$ to emphasize that the $k$-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. As in our baseline model, the first two elements of $\boldsymbol{\delta}_1$ are going to be zero, since, in the absence of mistakes by diagnostic investors, rational investors don't want to adjust their holdings as a function of short rates. With diagnostic expectations, it is not the level of short rates that governs investors' biased perceptions. Instead, what matters is the recent changes in short rates as summarized by $m_{P,t}$ and $m_{T,t}$. Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for $\alpha_0$ and $\delta_0$ using

$$\alpha_0=\bar{i}+\frac{V^{(1)}}{\tau}\frac{\bar{s}-(1-f)(1-q)\delta_0}{f+(1-f)q}\text{ and }\delta_0=\tau\frac{k\left(\alpha_0-\bar{i}\right)}{V^{(k)}}, \tag{A.176}$$

which yields

$$\alpha_0 = \bar{i}+\frac{\bar{s}/\tau}{(f+(1-f)q)\frac{1}{V^{(1)}}+(1-f)(1-q)\frac{k}{V^{(k)}}}\text{ and} \tag{A.177}$$

$$\delta_0 = \frac{\frac{k}{V^{(k)}}}{(f+(1-f)q)\frac{1}{V^{(1)}}+(1-f)(1-q)\frac{k}{V^{(k)}}}\times\bar{s}. \tag{A.178}$$

#### D.2.2 Proof of Proposition 3

**Proposition 3.** ***Investor extrapolation model.*** *For simplicity suppose $\rho_T = \rho_P$ and $\kappa_T = \kappa_P$. When $f\theta > 0$, long rates are excessively sensitive to short rates. When $f\theta > 0$ and $\kappa_T = \rho_T$, this excess sensitivity is only horizon-dependent—i.e., the regression coefficients $\beta_h$ only decline with horizon $h$—when unbiased arbitrage capital is slow moving ($q < 1$). By contrast, when $f\theta > 0$ and $\kappa_P < \rho_T$, $\beta_h$ declines with horizon $h$ even if all arbitrage capital is fast-moving ($q = 1$).*

**Proof:** To demonstrate this result, it suffices to consider two special cases. We suppose throughout that $\rho_T = \rho_P \equiv \rho_i$ and $\kappa_T = \kappa_P \equiv \kappa_i$. First, we consider the case where there is no slow-moving capital ($q = 1$). Next, we study the case where $q < 1$ and $k = 2$. The arguments given in this special $k = 2$ case generalize naturally to the case where $k > 2$.

**Case #1: No slow-moving capital ($q = 1$), $\rho_T = \rho_P \equiv \rho_i$, and $\kappa_T = \kappa_P \equiv \kappa_i$** We assume that $\rho_T = \rho_P \equiv \rho_i$, and $\kappa_T = \kappa_P \equiv \kappa_i$. In the absence of slow-moving capital ($q = 1$), the model with extrapolative investors can be solved analytically. Specifically, when $q = 1$, we have

$$\boldsymbol{\alpha}_1 = \begin{bmatrix} \alpha_i \\ \alpha_{m_i} \end{bmatrix} = (1-\phi)\left[\mathbf{I} - \phi\left(f\boldsymbol{\Gamma}_D + (1-f)\boldsymbol{\Gamma}\right)'\right]^{-1}\mathbf{e} = \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i} \\ f\theta\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} \end{bmatrix}. \tag{A.179}$$

In this case, the model-implied coefficient $\beta_h$ from regression of $y_{t+h} - y_t$ on $i_{t+h} - i_t$ is

$$\begin{aligned} \beta_h &= \frac{Cov\left[y_{t+h} - y_t, i_{t+h} - i_t\right]}{Var\left[i_{t+h} - i_t\right]} \\ &= \frac{\alpha_i Var\left[\Delta_h i_t\right] + \alpha_{m_i} Cov\left[\Delta_h m_{i,t}, \Delta_h i_t\right]}{Var\left[\Delta_h i_{t+h}\right]}, \end{aligned} \tag{A.180}$$

where $Var\left[\Delta_h i_t\right] = 2\left[(1-\rho_i^h)/\left(1-\rho_i^2\right)\right]\sigma_i^2$ and $Cov\left[\Delta_h m_{i,t}, \Delta_h i_t\right] = \left[\left(2-\kappa_i^h-\rho_i^h\right)/\left(1-\kappa_i\rho_i\right)\right]\sigma_i^2$.

We then can demonstrate the following results:

- When $f\theta = 0$, then $\beta_h$ is a constant that is independent of $h$..
- When $f\theta > 0$ and $\kappa_i = \rho_i$, then $\beta_h$ is a constant that is independent of $h$. Specifically, we have
$$\beta_h = \frac{1-\phi}{1-\phi\rho_i}\left(1 + f\theta\frac{\phi}{1-\phi\rho_i}\right) > \frac{1-\phi}{1-\phi\rho_i}. \tag{A.181}$$
Thus, there is excess sensitivity relative to the expectations hypothesis, but this sensitivity is not horizon dependent.
- When $f\theta > 0$ and $\kappa_i < \rho_i$, $\beta_h$ is a decreasing function of $h$. In other words, there is horizon-dependent excess sensitivity.

- When $f\theta > 0$, we have

$$E_t\left[rx_{t+1}\right] = \frac{\tau^{-1}V^{(1)}}{1-f}\left(\overline{s} - fh_t\right). \tag{A.182}$$

Thus, we have $\partial E_t\left[rx_{t+1}\right]/\partial h_t < 0$—i.e., the expected returns on long-term bonds are low when the demand of extrapolative agents is high.

**Case #2:** $q < 1$**,** $k = 2$**,** $\rho_T = \rho_P \equiv \rho_i$**, and** $\kappa_T = \kappa_P \equiv \kappa_i$

**Solution:** We conjecture that equilibrium yields take the form $y_t = \alpha_0 + \boldsymbol{\alpha}_1'\mathbf{x}_t$, and that the demands of active slow-moving arbitrageurs are of the form $d_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$, where the state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_t - \overline{i} \\ d_{t-1} - \delta_0 \\ m_t \end{bmatrix}. \tag{A.183}$$

Rational investors believe the law motion for the state vector is:

$$\begin{aligned}\mathbf{x}_{t+1} &= \Gamma\left(\boldsymbol{\delta}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_i & 0 & 0 \\ 0 & \delta_d & \delta_m \\ 0 & 0 & \kappa \end{bmatrix}\begin{bmatrix} i_t - \overline{i} \\ d_{t-1} - \delta_0 \\ m_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \\ \varepsilon_{i,t+1} \end{bmatrix},\end{aligned} \tag{A.184}$$

where

$$\Sigma \equiv Var_t\left[\boldsymbol{\epsilon}_{t+1}\right] = \begin{bmatrix} \sigma_i^2 & 0 & \sigma_i^2 \\ 0 & 0 & 0 \\ \sigma_i^2 & 0 & \sigma_i^2 \end{bmatrix}. \tag{A.185}$$

By contrast, diagnostic investors believe the law of motion is:

$$\begin{aligned}\mathbf{x}_{t+1} &= \Gamma_D\left(\boldsymbol{\delta}\right)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_i & 0 & \theta \\ 0 & \delta_d & \delta_m \\ 0 & 0 & \kappa_i \end{bmatrix}\begin{bmatrix} i_t - \overline{i} \\ d_{t-1} - \delta_0 \\ m_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \\ \varepsilon_{i,t+1} \end{bmatrix}.\end{aligned} \tag{A.186}$$

The solution for $\boldsymbol{\alpha}_1$ takes the form:

$$\begin{aligned}\boldsymbol{\alpha}_1 &= \begin{bmatrix} \frac{1-\phi}{1-\phi\rho_i} \\ 0 \\ \frac{\theta f}{f+(1-f)q}\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} \end{bmatrix} \\ &\quad -\frac{1}{2}\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)}{f+(1-f)q}\begin{bmatrix} 0 \\ \frac{1-\phi}{1-\phi\delta_d}\left(1+\delta_d\right) \\ \frac{1-\phi}{(1-\kappa_i\phi)(1-\phi\delta_d)}\left(\delta_d\left(1-\phi\delta_d\right) + \phi\delta_m\left(1+\delta_d\right)\right) \end{bmatrix}\end{aligned} \tag{A.187}$$

Thus, we have

$$\alpha_i = \frac{1-\phi}{1-\phi\rho_i} > 0 \tag{A.188}$$

$$\alpha_d = -\frac{1}{2}\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)}{f+(1-f)q}\frac{1-\phi}{1-\phi\delta_d}(1+\delta_d) < 0 \tag{A.189}$$

$$\begin{aligned}\alpha_m &= \frac{\theta f}{f+(1-f)q}\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} \\ &+ \left[-\frac{1}{2}\frac{V^{(1)}}{\tau}\frac{(1-f)(1-q)}{f+(1-f)q}\frac{1-\phi}{(1-\phi\delta_d)}(1+\delta_d)\right]\frac{\delta_d(1-\phi\delta_d)+\phi\delta_m(1+\delta_d)}{(1-\kappa_i\phi)(1+\delta_d)} \\ &= \frac{\theta f}{f+(1-f)q}\frac{\phi}{1-\phi\kappa_i}\frac{1-\phi}{1-\phi\rho_i} + \alpha_d\frac{\delta_d(1-\phi\delta_d)+\phi\delta_m(1+\delta_d)}{(1-\kappa_i\phi)(1+\delta_d)} > 0.\end{aligned} \tag{A.190}$$

In this case, we have

$$E_t[rx_{t+1}] = \frac{\tau^{-1}V^{(1)}}{(1-f)q}\left[\overline{s}-(1-f)(1-q)(d_t+d_{t-1})/2 - fh_t\right]. \tag{A.191}$$

Thus, large purchases by extrapolative investors are associated with low expected excess returns on long-term bonds in the time series

$$\frac{Cov[rx_{t+1},h_t]}{Var[h_t]} = -\frac{\tau^{-1}V^{(1)}}{(1-f)q}\left[f+(1-f)(1-q)\frac{Cov[(d_t+d_{t-1})/2,h_t]}{Var[h_t]}\right] < 0. \tag{A.192}$$

The inequality follows because the term in square brackets is positive—i.e., since slow-moving investors do not fully offset the demand shocks from extrapolative investors.

**Characterizing the solution:** As above, we can show that $\alpha_i > 0$, $\delta_i = 0$, $\alpha_m > 0$, $\delta_m > 0$, $\alpha_d < 0$, and $-1 < \delta_d < 0$.

**Computing $\beta_h$:** Now we compute

$$\beta_h = \frac{Cov[y_{t+h}-y_t, i_{t+h}-i_t]}{Var[i_{t+h}-i_t]} = \frac{\boldsymbol{\alpha}_1'(2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}{\mathbf{e}'(2\mathbf{V}-\boldsymbol{\Gamma}^h\mathbf{V}-\mathbf{V}(\boldsymbol{\Gamma}')^h)\mathbf{e}}. \tag{A.193}$$

Using the fact that $vec(\mathbf{V}) = (\mathbf{I}-\boldsymbol{\Gamma}\otimes\boldsymbol{\Gamma})^{-1}vec(\boldsymbol{\Sigma})$, we have

$$\mathbf{V} = Var[\mathbf{x}_t] = \begin{bmatrix} \frac{\sigma_i^2}{1-\rho_i^2} & \sigma_i^2\delta_m\frac{\rho_i}{(1-\kappa_i\rho_i)(1-\delta_d\rho_i)} & \frac{\sigma_i^2}{1-\kappa_i\rho_i} \\ \sigma_i^2\delta_m\frac{\rho_i}{(1-\kappa_i\rho_i)(1-\delta_d\rho_i)} & \sigma_i^2\frac{\delta_m^2(\kappa_i\delta_d+1)}{(1-\kappa_i^2)(1-\delta_d^2)(1-\kappa_i\delta_d)} & \kappa_i\sigma_i^2\frac{\delta_m}{(1-\kappa_i^2)(1-\kappa_i\delta_d)} \\ \frac{\sigma_i^2}{1-\kappa_i\rho_i} & \kappa_i\sigma_i^2\frac{\delta_m}{(1-\kappa_i^2)(1-\kappa_i\delta_d)} & \frac{\sigma_i^2}{1-\kappa_i^2} \end{bmatrix}. \tag{A.194}$$

Since

$$\boldsymbol{\Gamma}^h = \begin{bmatrix} \rho_i^h & 0 & 0 \\ 0 & \delta_d^h & \delta_m\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d} \\ 0 & 0 & \kappa_i^h \end{bmatrix}, \tag{A.195}$$

we have

$$
\begin{aligned}
& Var\left[\mathbf{x}_{t+h}-\mathbf{x}_t\right] \qquad (A.196)\\
= \;& 2\mathbf{V}-\mathbf{\Gamma}^h\mathbf{V}-\mathbf{V}(\mathbf{\Gamma}')^h \\
= \;& \begin{bmatrix}
2\sigma_i^2\frac{1-\rho_i^h}{1-\rho_i^2} & \delta_m\frac{\sigma_i^2\left(\rho_i\frac{2-\delta_d^h-\rho_i^h}{1-\delta_d\rho_i}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i\rho_i} & \frac{2-\kappa_i^h-\rho_i^h}{1-\kappa_i\rho_i}\sigma_i^2 \\
\delta_m\frac{\sigma_i^2\left(\rho_i\frac{2-\delta_d^h-\rho_i^h}{1-\delta_d\rho_i}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i\rho_i} & 2\sigma_i^2\frac{\delta_m^2\left(\frac{(\kappa_i-\delta_d)(1-\delta_d^h)}{\kappa_i-\delta_d}\frac{(\kappa_i\delta_d+1)}{1-\delta_d^2}-\kappa_i\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{\left(1-\kappa_i^2\right)(1-\kappa_i\delta_d)} & \delta_m\frac{\sigma_i^2\left(\kappa_i\frac{2-\kappa_i^h-\delta_d^h}{1-\kappa_i\delta_d}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i^2} \\
\frac{2-\kappa_i^h-\rho_i^h}{1-\kappa_i\rho_i}\sigma_i^2 & \delta_m\frac{\sigma_i^2\left(\kappa_i\frac{2-\kappa_i^h-\delta_d^h}{1-\kappa_i\delta_d}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}\right)}{1-\kappa_i^2} & 2\sigma_i^2\frac{1-\kappa_i^h}{1-\kappa_i^2}
\end{bmatrix}
\end{aligned}
$$

Thus, we have

$$\beta_h=\alpha_i+\alpha_m\times R_{i,m}\left(h\right)+\alpha_d\times R_{i,d}\left(h\right) \qquad (A.197)$$

where $\alpha_i>0$, $\alpha_m>0$, $\alpha_d<0$, and

$$R_{i,m}\left(h\right) = \frac{Cov\left[i_{t+h}-i_t,m_{t+h}-m_t\right]}{Var\left[i_{t+h}-i_t\right]}=\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{2-\kappa_i^h-\rho_i^h}{2\left(1-\rho_i^h\right)}, \qquad (A.198)$$

$$R_{i,d}\left(h\right) = \frac{Cov\left[i_{t+h}-i_t,d_{t+h-1}-d_{t-1}\right]}{Var\left[i_{t+h}-i_t\right]}=\delta_m\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{\rho_i\frac{2-\delta_d^h-\rho_i^h}{1-\delta_d\rho_i}-\frac{\kappa_i^h-\delta_d^h}{\kappa_i-\delta_d}}{2\left(1-\rho_i^h\right)}. \qquad (A.199)$$

**Proof that $\beta_1>\beta_2$ and that $\beta_1>\lim_{h\to\infty}\beta_h$:** Since $\alpha_m>0$ and $R_{i,m}\left(h\right)$ is decreasing in $h$ when $\kappa_i<\rho_i$, it follows that $\alpha_m\times R_{i,m}\left(h\right)$ is decreasing in $h$ when $\kappa_i<\rho_i$.

Since $\alpha_d<0$, it then suffices to show that $R_{i,d}\left(1\right)<R_{i,d}\left(2\right)$ and that $R_{i,d}\left(1\right)<\lim_{h\to\infty}R_{i,d}\left(h\right)$. We have

$$R_{i,d}\left(1\right) = -\frac{\delta_m}{2}\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{1}{1-\delta_d\rho_i}<0. \qquad (A.200)$$

$$R_{i,d}\left(2\right) = \frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{\kappa_i^2-\delta_d^2}{\kappa_i-\delta_d}\right) \qquad (A.201)$$

$$\lim_{h\to\infty}R_{i,d}\left(h\right) = \delta_m\frac{1-\rho_i^2}{1-\kappa_i\rho_i}\frac{\rho_i}{1-\delta_d\rho_i}>0. \qquad (A.202)$$

We have

$$
\begin{aligned}
R_{i,d}\left(2\right)-R_{i,d}\left(1\right) &= \frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{\kappa_i^2-\delta_d^2}{\kappa_i-\delta_d}\right)+\frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\frac{1-\rho_i^2}{1-\delta_d\rho_i} \qquad (A.203)\\
&= \frac{\delta_m}{2}\frac{1}{1-\kappa_i\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{\kappa_i^2-\delta_d^2}{\kappa_i-\delta_d}+\frac{1-\rho_i^2}{1-\delta_d\rho_i}\right)\\
&> \frac{\delta_m}{2}\frac{1}{1-\rho_i}\left(\rho_i\frac{2-\delta_d^2-\rho_i^2}{1-\delta_d\rho_i}-\frac{1-\delta_d^2}{1-\delta_d}+\frac{1-\rho_i^2}{1-\delta_d\rho_i}\right)\\
&= \frac{\delta_m}{2}\frac{\rho_i^2+2\rho_i-\delta_d}{1-\delta_d\rho_i}>0
\end{aligned}
$$

where the inequality uses the fact that

$$\frac{\partial}{\partial \kappa_i} \frac{\kappa_i^2 - \delta_d^2}{\kappa_i - \delta_d} = \frac{2\kappa_i (\kappa_i - \delta_d) - (\kappa_i^2 - \delta_d^2)}{(\kappa_i - \delta_d)^2} = 1 \tag{A.204}$$

and the fact that $\kappa_i < 1$.

## D.3 Reaching for yield channel

### D.3.1 General model solution

We we drop $s_t - \bar{s}$ from the state vector $\mathbf{x}_t$. For the sake of concreteness, suppose that $k = 4$. We conjecture that equilibrium yields take the form $y_t = \alpha_0 + \boldsymbol{\alpha}_1' \mathbf{x}_t$, and that the demands of active slow-moving arbitrageurs are of the form $d_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t$, where the $k+1$ dimensional state vector is

$$\mathbf{x}_t = \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix}. \tag{A.205}$$

These assumptions imply that the state vector follows an AR(1) process. Critically, the transition matrix $\Gamma$ is a function of the parameters $\boldsymbol{\delta}_1$ governing slow-moving arbitrageur demand so we write $\Gamma = \Gamma(\boldsymbol{\delta}_1)$. Specifically, we have

$$\begin{aligned} \mathbf{x}_{t+1} &= \Gamma(\delta)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+1} \\ &= \begin{bmatrix} \rho_P & 0 & 0 & 0 & 0 \\ 0 & \rho_T & 0 & 0 & 0 \\ \delta_P & \delta_T & \delta_{d_1} & \delta_{d_2} & \delta_{d_3} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{P,t} - \bar{i} \\ i_{T,t} \\ d_{t-1} - \delta_0 \\ d_{t-2} - \delta_0 \\ d_{t-3} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{P,t+1} \\ \varepsilon_{T,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \tag{A.206}$$

where $\Sigma \equiv Var_t[\boldsymbol{\epsilon}_{t+1}]$. Assuming for simplicity that $\varepsilon_{P,t+1}$ and $\varepsilon_{T,t+1}$ are mutually orthogonal, we have

$$\Sigma = \begin{bmatrix} \sigma_P^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_T^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.207}$$

Letting $V^{(1)} = Var_t[rx_{t+1}] = \left(\frac{\phi}{1-\phi}\right)^2 \boldsymbol{\alpha}_1' \boldsymbol{\Sigma} \boldsymbol{\alpha}_1$, denote the variance of 1-period excess returns, active demand at time $t$ is

$$\begin{aligned} & fh_t + (1-f)qb_t + (1-f)(1-q)k^{-1}d_t \\ &= \left[ (f + (1-f)q)\tau \frac{(\alpha_0 - \bar{i})}{V^{(1)}} + (1-f)(1-q)k^{-1}\delta_0 \right] \\ &+ \left[ f\tau \frac{(\boldsymbol{\alpha}_1 - \mathbf{e})'}{V^{(1)}} + (1-f)q\tau \frac{\frac{1}{1-\phi}\boldsymbol{\alpha}_1'(\mathbf{I} - \phi\boldsymbol{\Gamma}) - \mathbf{e}'}{V^{(1)}} + (1-f)(1-q)k^{-1}\boldsymbol{\delta}_1' \right] \mathbf{x}_t. \end{aligned} \tag{A.208}$$

Active supply is

$$\begin{aligned}&\overline{s}-(1-f)\,(1-q)k^{-1}\sum\nolimits_{i=1}^{k-1}d_{t-i} \\ =\ &\left[\overline{s}-(1-f)\,(1-q)\frac{(k-1)}{k}\delta_0\right]+\left[-(1-f)\,(1-q)k^{-1}\mathbf{e}_d'\right]\mathbf{x}_t.\end{aligned} \tag{A.209}$$

Matching constants terms, we obtain

$$\alpha_0=\overline{i}+\frac{V^{(1)}}{\tau}\frac{\overline{s}-(1-f)\,(1-q)\delta_0}{f+(1-f)\,q} \tag{A.210}$$

which is the same in the investor extrapolation model. Matching slope coefficients, we obtain:

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\frac{f\,(1-\phi)\,\mathbf{I}+(1-f)\,q\,(\mathbf{I}-\phi\boldsymbol{\Gamma}')}{f+(1-f)\,q}\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}}{\tau}\frac{(1-f)\,(1-q)k^{-1}\,(\mathbf{e}_d+\boldsymbol{\delta}_1)}{(f+(1-f)\,q)}\right] \tag{A.211}$$

In summary, an equilibrium in this model extension solves the following system of equations

$$\boldsymbol{\alpha}_1=(1-\phi)\left[\frac{f\,(1-\phi)\,\mathbf{I}+(1-f)\,q\left(\mathbf{I}-\phi\boldsymbol{\Gamma}\,(\boldsymbol{\delta}_1)'\right)}{f+(1-f)\,q}\right]^{-1}\left[\mathbf{e}-\frac{V^{(1)}\,(\boldsymbol{\alpha}_1)}{\tau}\frac{(1-f)\,(1-q)k^{-1}\,(\mathbf{e}_d+\boldsymbol{\delta}_1)}{(f+(1-f)\,q)}\right] \tag{A.212}$$

and

$$\boldsymbol{\delta}_1'=\tau\frac{\left((\boldsymbol{\alpha}_1-\mathbf{e})'\,(\mathbf{I}-\boldsymbol{\Gamma}\,(\boldsymbol{\delta}_1))^{-1}+\frac{\theta}{1-\theta}\boldsymbol{\alpha}_1'\right)}{V^{(k)}\,(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)}\left(\mathbf{I}-\boldsymbol{\Gamma}\,(\boldsymbol{\delta}_1)^k\right), \tag{A.213}$$

where we write $V^{(1)}(\boldsymbol{\alpha}_1)$ to emphasize that the 1-period return variance depends on $\boldsymbol{\alpha}_1$; $\boldsymbol{\Gamma}(\boldsymbol{\delta}_1)$ to emphasize that the transition matrix depends on $\boldsymbol{\delta}_1$; and $V^{(k)}(\boldsymbol{\alpha}_1,\boldsymbol{\delta}_1)$ to emphasize that the $k$-period return variance depends on $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$. Unlike in our baseline model, the first two elements of $\boldsymbol{\delta}_1$ are going to be positive, since slow-moving arbitrageurs will buy more long-term bonds when interest rates rise. Once a solution for $\boldsymbol{\alpha}_1$ and $\boldsymbol{\delta}_1$ is in hand, we can compute $V^{(1)}$ and $V^{(k)}$ and can then solve for $\alpha_0$ and $\delta_0$ using

$$\alpha_0=\overline{i}+\frac{V^{(1)}}{\tau}\frac{\overline{s}-(1-f)\,(1-q)\delta_0}{f+(1-f)\,q}\text{ and }\delta_0=\tau\frac{k\left(\alpha_0-\overline{i}\right)}{V^{(k)}}, \tag{A.214}$$

which yields

$$\begin{aligned}\alpha_0 &= \overline{i}+\frac{\overline{s}/\tau}{(f+(1-f)\,q)\,\frac{1}{V^{(1)}}+(1-f)\,(1-q)\frac{k}{V^{(k)}}}\text{ and} \\ \delta_0 &= \frac{\frac{k}{V^{(k)}}}{(f+(1-f)\,q)\,\frac{1}{V^{(1)}}+(1-f)\,(1-q)\frac{k}{V^{(k)}}}\times\overline{s}.\end{aligned} \tag{A.215}$$

#### D.3.2 Proof of Proposition 4

**Proposition 4.** ***Investor reaching for yield model.*** *Suppose $\rho_T=\rho_P$. When $f>0$, long rates are excessively sensitive to short rates. However, this excess sensitivity is only horizon-dependent when arbitrage capital is slow moving ($q<1$).*

**Proof:** To demonstrate this result, it suffices to consider two special cases. We suppose throughout that $\rho_T = \rho_P \equiv \rho_i$. First, we consider the case where there is no slow-moving capital ($q = 1$). Next, we study the case where $q < 1$ and $k = 2$. The arguments given in this special $k = 2$ case generalize naturally to the case where $k > 2$.

**Special case #1:** $q = 1$ **and** $\rho_T = \rho_P \equiv \rho_i$ In this case, we have

$$i_{t+1} - \bar{i} = \rho_i \left(i_t - \bar{i}\right) + \varepsilon_{i,t+1}, \tag{A.216}$$

where $Var_t\left[\varepsilon_{i,t+1}\right] = \sigma_i^2$. Yields take the form $y_t = \alpha_0 + \alpha_{1,i}\left(i_t - \bar{i}\right)$. Thus, we have

$$E_t\left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \left(\frac{1 - \phi\rho_i}{1 - \phi}\alpha_{1,i} - 1\right)\left(i_t - \bar{i}\right) \tag{A.217}$$

and

$$V^{(1)} = Var_t\left[rx_{t+1}\right] = \left(\frac{\phi}{1 - \phi}\right)^2 \alpha_i^2 \sigma_i^2. \tag{A.218}$$

Market clearing condition is:

$$\begin{aligned} \bar{s} &= fh_t + (1 - f)\, b_t \qquad \text{(A.219)} \\ &= f\tau\frac{y_t - i_i}{V^{(1)}} + (1 - f)\,\tau\frac{E_t\left[rx_{t+1}\right]}{V^{(1)}} \\ &= f\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\alpha_{1,i} - 1\right)\left(i_t - \bar{i}\right)}{V^{(1)}} + (1 - f)\,\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\frac{1 - \phi\rho_i}{1 - \phi}\alpha_{1,i} - 1\right)\left(i_t - \bar{i}\right)}{V^{(1)}}. \end{aligned}$$

Thus, we have $\alpha_0 = \bar{i} + \tau^{-1}V^{(1)}\bar{s}$ and

$$\alpha_i = \frac{1 - \phi}{f\,(1 - \phi) + (1 - f)\,(1 - \phi\rho_i)} \geq \frac{1 - \phi}{1 - \phi\rho_i}, \tag{A.220}$$

with strict inequality when $f > 0$. In other words, we have $\partial\alpha_i/\partial f > 0$, so there is excess sensitivity when $f > 0$. We also have

$$E_t\left[rx_{t+1}\right] = \tau^{-1}V^{(1)}\bar{s} + \frac{f\phi\,(1 - \rho_i)}{f\,(1 - \phi) + (1 - f)\,(1 - \phi\rho_i)}\left(i_t - \bar{i}\right).$$

Thus, we have $\partial E_t\left[rx_{t+1}\right]/\partial i_t > 0$. Thus, in this case where $q = 1$, we have

$$\beta_h = \alpha_i^* = \frac{1 - \phi}{f\,(1 - \phi) + (1 - f)\,(1 - \phi\rho_i)}, \tag{A.221}$$

which is independent of horizon $h$.

**Special case #2:** $q < 1$, $k = 2$**, and** $\rho_T = \rho_P \equiv \rho_i$

**Solution:** In this case, we have $i_{t+1} - \bar{i} = \rho_i \left(i_t - \bar{i}\right) + \varepsilon_{i,t+1}$ where $Var_t \left[\varepsilon_{i,t+1}\right] = \sigma_i^2$. Yields take the form

$$y_t = \alpha_0 + \alpha_i \left(i_t - \bar{i}\right) + \alpha_d \left(d_{t-1} - \delta_0\right) \tag{A.222}$$

Demands of active slow-moving investors are given by

$$d_t = \delta_0 + \delta_i \left(i_t - \bar{i}\right) + \delta_d \left(d_{t-1} - \delta_0\right). \tag{A.223}$$

The state variable dynamics are given by

$$\mathbf{x}_{t+1} = \begin{bmatrix} \rho_i & 0 \\ \delta_i & \delta_d \end{bmatrix} \begin{bmatrix} i_t - \bar{i} \\ d_{t-1} - \delta_0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{i,t+1} \\ 0 \end{bmatrix} \tag{A.224}$$

Thus, we have

$$E_t \left[rx_{t+1}\right] = \left(\alpha_0 - \bar{i}\right) + \left(\frac{1 - \phi\rho_i}{1 - \phi}\alpha_i - \frac{\phi}{1 - \phi}\alpha_d\delta_i - 1\right)\left(i_t - \bar{i}\right) + \frac{1 - \phi\delta_d}{1 - \phi}\alpha_d \left(d_{t-1} - \delta_0\right). \tag{A.225}$$

and

$$V^{(1)} = Var_t \left[rx_{t+1}\right] = \left(\frac{\phi}{1 - \phi}\right)^2 \alpha_i^2 \sigma_i^2. \tag{A.226}$$

The market clearing condition is:

$$\begin{aligned} & \bar{s} - (1 - f)(1 - q)\frac{1}{2}d_{t-1} \qquad \text{(A.227)} \\ = \; & f\tau\frac{y_t - i_i}{V^{(1)}} + (1 - f) q\tau\frac{E_t \left[rx_{t+1}\right]}{V^{(1)}} + (1 - f)(1 - q)\frac{1}{2}d_t \\ = \; & f\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\alpha_i - 1\right)\left(i_t - \bar{i}\right) + \alpha_d \left(d_{t-1} - \delta_0\right)}{V^{(1)}} \\ & + (1 - f) q\tau\frac{\left(\alpha_0 - \bar{i}\right) + \left(\frac{1 - \phi\rho_i}{1 - \phi}\alpha_i - \frac{\phi}{1 - \phi}\alpha_d\delta_i - 1\right)\left(i_t - \bar{i}\right) + \frac{1 - \phi\delta_d}{1 - \phi}\alpha_d \left(d_{t-1} - \delta_0\right)}{V^{(1)}} \\ & + (1 - f)(1 - q)\frac{1}{2}\left(\delta_0 + \delta_i \left(i_t - \bar{i}\right) + \delta_d \left(d_{t-1} - \delta_0\right)\right) \end{aligned}$$

Matching constant and slope terms, we obtain

$$\alpha_0 = \bar{i} + \frac{1}{\tau}\frac{V^{(1)}}{f + (1 - f) q}\left[\bar{s} - (1 - f)(1 - q)\delta_0\right] > 0 \tag{A.228}$$

$$\alpha_i = \frac{\left[f + (1 - f) q\right](1 - \phi) + (1 - f) q\phi\alpha_d\delta_i - (1 - \phi)(1 - f)(1 - q)\frac{1}{2}\tau^{-1}V^{(1)}\delta_i}{f(1 - \phi) + (1 - f) q (1 - \phi\rho_i)} > 0 \tag{A.229}$$

$$\alpha_d = -\tau^{-1}V^{(1)}\frac{(1 - f)(1 - q)\frac{1}{2}(1 + \delta_d)}{f + (1 - f) q\frac{1 - \phi\delta_d}{1 - \phi}} < 0. \tag{A.230}$$

**Characterizing the solution:** As above, we can show that

$$0 < \frac{1 - \phi}{1 - \phi\rho_i} < \alpha_i < 1, \tag{A.231}$$

$0 < \delta_i$, $\alpha_d < 0$, and $-1 < \delta_d < 0$. To see this, note that

$$\begin{aligned} d_t - \delta_0 &= \delta_i \left(i_t - \bar{i}\right) + \delta_d \left(d_{t-1} - \delta_0\right) \quad \text{(A.232)} \\ &= \frac{\tau}{V^{(2)}} \left( \begin{array}{c} \left[\alpha_i \left(i_t - \bar{i}\right) + \alpha_d \left(d_{t-1} - \delta_0\right)\right] \\ + \left[\alpha_i \rho_i \left(i_t - \bar{i}\right) + \alpha_d \left[\delta_i \left(i_t - \bar{i}\right) + \delta_d \left(d_{t-1} - \delta_0\right)\right]\right] \\ -\frac{\phi}{1-\phi} \left( \begin{array}{c} \left[\alpha_i \rho_i^2 \left(i_t - \bar{i}\right) + \alpha_d \left[\delta_i \left(\delta_d + \rho_i\right) \left(i_t - \bar{i}\right) + \delta_d^2 \left(d_{t-1} - \delta_0\right)\right]\right] \\ - \left[\alpha_i \left(i_t - \bar{i}\right) + \alpha_d \left(d_{t-1} - \delta_0\right)\right] \\ - \left(i_t - \bar{i}\right) - \rho_i \left(i_t - \bar{i}\right) \end{array} \right) \end{array} \right) \end{aligned}$$

Thus, we have the four conditions in four unknowns:

$$\alpha_i = \frac{\left[f + (1-f) q\right](1-\phi) + (1-f) q \phi \alpha_d \delta_i - (1-\phi)(1-f)(1-q) \frac{1}{2} \tau^{-1} V^{(1)} \delta_i}{f(1-\phi) + (1-f) q (1-\phi \rho_i)} \quad \text{(A.233a)}$$

$$\alpha_d = -\tau^{-1} V^{(1)} \frac{(1-f)(1-q) \frac{1}{2} (1+\delta_d)}{f + (1-f) q \frac{1-\phi\delta_d}{1-\phi}} \quad \text{(A.233b)}$$

$$\delta_i = \frac{\tau}{V^{(2)}} (1+\rho_i) \left(\alpha_i \frac{1-\phi\rho_i}{1-\phi} - 1\right) + \frac{\tau}{V^{(2)}} \frac{\alpha_d}{1-\phi} \left((1-\phi)\delta_i - \phi\left(\delta_i(\delta_d + \rho_i) + \delta_d^2\right)\right) \quad \text{(A.233c)}$$

$$\delta_d = \frac{\tau}{V^{(2)}} \frac{\alpha_d}{1-\phi} (\delta_d + 1)(1 - \phi\delta_d) \quad \text{(A.233d)}$$

- Combining (A.233b) and (A.233d) we have

$$\delta_d = -\frac{1}{2} \frac{V^{(1)}}{V^{(2)}} \frac{(1-f)(1-q)}{f + (1-f) q \frac{1-\phi\delta_d}{1-\phi}} \frac{1-\phi\delta_d}{1-\phi} (1+\delta_d)^2 . \quad \text{(A.234)}$$

  Thus, we have $-1 < \delta_d < 0$.

- Using (A.233b) and the fact that $(1 + \delta_d) > 0$, we then have $\alpha_d < 0$.

- From (A.233a), we have

$$0 = f \frac{\tau}{V^{(1)}} (\alpha_i - 1) + (1-f) q \frac{\tau}{V^{(1)}} \left(\frac{1-\phi\rho_i}{1-\phi} \alpha_i - 1\right) + (1-f) \left((1-q) \frac{1}{2} - q \frac{\tau}{V^{(1)}} \frac{\phi}{1-\phi} \alpha_d\right) \delta_i . \quad \text{(A.235)}$$

  Thus, in the stable equilibrium, we have $\delta_i > 0$ and $\frac{1-\phi}{1-\phi\rho_i} < \alpha_i < 1$. Thus, an increase in short-term rates leads (i) yield-seeking investors to sell bonds and leads both fast- and slow-moving expected-return oriented investors to buy bonds.

**Computing $\beta_h$:** Using the fact that $vec(\mathbf{V}) = (\mathbf{I} - \mathbf{\Gamma} \otimes \mathbf{\Gamma})^{-1} vec(\mathbf{\Sigma})$, we have

$$\mathbf{V} = \begin{bmatrix} \sigma_i^2 \frac{1}{1-\rho_i^2} & \sigma_i^2 \frac{\rho_i \delta_i}{(1-\rho_i^2)(1-\delta_d\rho_i)} \\ \sigma_i^2 \frac{\rho_i \delta_i}{(1-\rho_i^2)(1-\delta_d\rho_i)} & \sigma_i^2 \frac{\delta_i^2 (1+\delta_d\rho_i)}{(1-\delta_d^2)(1-\rho_i^2)(1-\delta_d\rho_i)} \end{bmatrix} \quad \text{(A.236)}$$

Using the fact that

$$\mathbf{\Gamma}^h = \begin{bmatrix} \rho_i^h & 0 \\ \delta_i \frac{\rho_i^h - \delta_d^h}{\rho_i - \delta_d} & \delta_d^h \end{bmatrix} \tag{A.237}$$

we have

$$\begin{aligned} & Var\left[\mathbf{x}_{t+h} - \mathbf{x}_t\right] \\ = \; & 2\mathbf{V} - \mathbf{\Gamma}^h \mathbf{V} - \mathbf{V}(\mathbf{\Gamma}')^h \\ = \; & \begin{bmatrix} 2Var\left[i_t\right]\left(1-\rho_i^h\right) & \left(2-\rho_i^h-\delta_d^h\right) Cov\left[i_t, d_{t-1}\right] - \delta_i \frac{\rho_i^h-\delta_d^h}{\rho_i-\delta_d} Var\left[i_t\right] \\ \left(2-\rho_i^h-\delta_d^h\right) Cov\left[i_t, d_{t-1}\right] - \delta_i \frac{\rho_i^h-\delta_d^h}{\rho_i-\delta_d} Var\left[i_t\right] & 2Var\left[d_{t-1}\right]\left(1-\delta_d^h\right) - 2\delta_i \frac{\rho_i^h-\delta_d^h}{\rho_i-\delta_d} Cov\left[i_t, d_{t-1}\right] \end{bmatrix} \end{aligned} \tag{A.238}$$

Thus, we have

$$\beta_h = \frac{Cov\left[i_{t+h} - i_t, y_{t+h} - y_t\right]}{Var\left[i_{t+h} - i_t\right]} = \alpha_i + \alpha_d \times R_{i,d}\left(h\right) \tag{A.239}$$

where $\alpha_i > 0$, $\alpha_d < 0$, and

$$\begin{aligned} R_{i,d}\left(h\right) &= \frac{Cov\left[i_{t+h} - i_t, d_{t+h-1} - d_{t-1}\right]}{Var\left[i_{t+h} - i_t\right]} \\ &= \frac{\left(2-\rho_i^h-\delta_d^h\right)\sigma_i^2 \frac{\rho_i \delta_i}{(1-\rho_i^2)(1-\delta_d \rho_i)} - \delta_i \frac{\rho_i^h - \delta_d^h}{\rho_i - \delta_d} \sigma_i^2 \frac{1}{1-\rho_i^2}}{2 \frac{1-\rho_i^h}{1-\rho_i^2} \sigma_i^2} \\ &= \frac{\rho_i \delta_i}{1 - \delta_d \rho_i} - \left[\frac{1}{2}\delta_i \frac{1-\rho_i^2}{\rho_i + \delta_d^2 \rho_i - \delta_d \rho_i^2 - \delta_d}\right] \frac{\rho_i^h - \delta_d^h}{1 - \rho_i^h} \end{aligned} \tag{A.240}$$

**Proof that $\beta_2 < \beta_1$ and that $\lim_{h\to\infty} \beta_h < \beta_1$.** Since $\alpha_d < 0$, it suffices to show that $R_{i,d}\left(2\right) > R_{i,d}\left(1\right)$ and that $\lim_{h\to\infty} R_{i,d}\left(h\right) > R_{i,d}\left(1\right)$. These results follow from the facts that

$$R_{i,d}\left(1\right) = -\frac{1}{2}\delta_i \frac{1-\rho_i}{1-\delta_d \rho_i} < 0, \tag{A.241}$$

$$R_{i,d}\left(2\right) = \frac{1}{2}\delta_i \frac{\rho_i - \delta_d}{1-\delta_d \rho_i} > 0, \tag{A.242}$$

$$\lim_{h\to\infty} R_{i,d}\left(h\right) = \frac{\rho_i \delta_i}{1-\delta_d \rho_i} > 0. \tag{A.243}$$

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