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Benjamin N. Roth

Working Paper 20-078

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Funding for this research was provided in part by Harvard Business School.

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Benjamin N. Roth†

June, 2021

Abstract

I present a model of financing social enterprises to delineate the role of impact investors relative to "pure" philanthropists. I characterize the optimal scale and structure of a social enterprise when financed by grants, and when financed by investments. Impact investing creates value by inducing firms to shift towards profitability and away from their social mission. I draw implications for the choice of debt versus equity. I show that impact investing is more valuable when firms face less competition in their product markets. I operationalize these insights for the case of Husk Power, a social enterprise that received impact investment.

Firms that create considerable social or environmental value without also generating positive discounted cash flows are often financed through government grants and philanthropic donations. However over the last several decades a growing class of *impact investors* has utilized the traditional tools of entrepreneurial finance to support some of these social enterprises. Recent estimates indicate that the various types of private debt and equity investors with the dual objectives of profit and social or environmental impact manage upwards of $500 billion.[1] By comparison, total charitable giving in the United States was just over $410 billion in 2017.[2] Moreover, about 41% of large foundations in the United States now report using investments as a partial alternative to traditional grant-making activities to achieve their programatic goals.[3]

*I would like to thank Abhijit Banerjee, Vivek Bhattacharya, Eric Budish, Paul Brest, Shawn Cole, Ray Fisman, Vikram Gandhi, Bob Gibbons, Dan Green, Ravi Jagannathan, Dean Karlan, Scott Kominers, Larry Kramer, Ernest Liu, Adair Morse, Ramana Nanda, Paul Niehaus, Andrei Shleifer, Brian Trelstad, Mitch Weiss, and Mark Wolfson for helpful discussions and feedback. Any remaining errors are my own.

†Harvard Business School. Email: broth@hbs.edu

[1] See Mudaliar et al. (2019).

[2] See Giving USA (2017).

[3] See Buchanan et al. (2015).

This paper is primarily concerned with the question of when a philanthropist can have more impact as an investor than as a donor. Moreover, how do the optimal size and business model of a social enterprise depend on whether it receives an investment or grant? In considering these questions, impact investors often provide two answers.

1. Impact investors can help social enterprises reach financial sustainability and no longer rely on a constant stream of philanthropic subsidies.

2. Impact investors can help social enterprises reach a larger scale and increase their impact.[4]

I argue that an analysis from first principles should recognize that scale and financial sustainability are choices about a firm's business model, which can be determined independently from choices about a firm's financing; even for-profit businesses sometimes receive grant financing. Using a formal model I examine the logic underlying these two assertions. I demonstrate that the choice to make an investment rather than a grant, and organizational sustainability are indeed linked, though perhaps not as Assertion 1 suggests. Regarding Assertion 2, I find that relative to donors, impact investors sometimes, but do not always, increase the scale of a social enterprise.

The simple model in this paper isolates the relationship between a single financier and entrepreneur and allows for the separate determination of business model and mode of financing. In particular I study a social entrepreneur who is distinguished from a "traditional" entrepreneur by the fact that he values not only profits but also the consumer welfare and aggregate externalities deriving from his production. I refer to these latter two objectives as the entrepreneur's mission. The entrepreneur chooses two factors: invested capital and the price charged for the good he sells. In general, "price" is a stand-in for any business model decision that directly influences both the profitability and social impact of the organization. A financier must determine how much capital to funnel to, or extract from, the enterprise in each period.

At baseline I assume that actions are fully contractible, so that the financier maximizes her own objectives subject to ensuring the entrepreneur achieves at least his outside option utility.[5] I consider the following thought experiment. First, I characterize the optimal scale and structure of the social enterprise when the financier is constrained to only offer grants – i.e. the financier can subsidize the firm's operation but cannot extract profits. Second, as a function of the enterprise's solution under this *grant financing regime*, I ask when allowing the financier to make investments, and freely extract profits from the firm, leads to a strictly better outcome. Because at baseline actions are contractible, this latter *impact investing regime* can be understood as the first-best arrangement. The primary question of interest is then when does grant financing achieve the first-best outcome, and when not,

[4] See eg. Cohen (2014), Hopkins and Olvera (2013), and Huppé and Silva (2013).

[5] For ease of distinction, I arbitrarily refer to the entrepreneur as "he" and the financier as "she."

how does investment financing differ in terms of its consequences on the social enterprise?

The first contribution of the model is a definition and characterization of sustainable organizations. An organization has reached sustainability when it is a net distributor to its financiers rather than a net receiver. Utilizing this definition, I demonstrate that it is organizational sustainability that leads to impact investing, rather than the other way around as Assertion 1 suggests. Namely, a philanthropist can have more impact with an investment than with a grant for organizations that would be sustainable under the grant financing regime. Whereas, for organizations that are not sustainable under the grant financing regime, grants achieve the first-best outcome. The logic for this result is as follows. When grant financed, the scale of sustainable organizations is not disciplined by the donor's cost of capital, as their operations are wholly reliant on their own revenues. In contrast, because an investor can extract profits out of her portfolio firms, the scale of her portfolio firms is always disciplined by her cost of capital. Therefore, for firms that would be sustainable under the grant financing regime, as an investor, the financier always chooses a different scale of operation than she would as a donor.

Regarding Assertion 2 above, I demonstrate that relative to donors, impact investors sometimes increase and sometimes decrease the scale of a social enterprise. Whether the impact investor pushes her portfolio firm to increase or decrease its scale depends on two factors. First, a financier who makes an investment can withdraw profits from her portfolio firm if and when the firm reaches profitability. This is appealing in situations where the financier anticipates that the social entrepreneur will build a financially sustainable business but will exhaust his most impactful opportunities for investment or will extract a large fraction of his firm's profit through the distribution of retained earnings or overhead. Counter to Assertion 2, this consideration pushes the impact investor to reduce the long-run scale of her portfolio firm's operations, relative to if she had been a donor.

The second consideration pushes in the opposite direction. Because she is a partial owner of the firm and its cash flows, an investor places higher value on the firm's profits than would a donor. Therefore consistent with Assertion 2, relative to firms financed through donations alone, firms financed by impact investors have stronger incentive to exploit opportunities to expand their scale, when doing so also increases their profitability.

Critically, however, relative to grant financiers, the value that impact investors provide is to help firms exploit new opportunities for profit that are somewhat, but not extremely misaligned with the firm's primary mission. Even under grant financing a firm should exploit all opportunities that increase both profit and mission. Therefore, in partial tension with Assertion 2, the situations in which impact investors increase the scale of their portfolio firms arise from a motive to increase their portfolio firms' profits (potentially to be reallocated to another socially productive use) and comes at the expense of the portfolio firms' missions.

Returning to the grant financing regime, the model also yields a distinction between two kinds of sustainable organizations. *Satiated firms* can finance their socially efficient scale through revenue generated from the sale of their goods and services. In contrast, *hungry firms* are constrained by their customers' willingness to pay for their goods and services, and operate below their socially efficient scale. In both cases, the socially efficient scale of a sustainable firm is judged according to the value of its customers' capital, rather than the financier's, as by definition the firm is sustained by payments from its customers (i.e. a sustainable firm supported by grant financing has reached its optimal scale when the marginal social value of its output is equalized with the cost of its *customers'* capital).

I demonstrate that relative to donors, impact investors may expand the scale of satiated firms precisely when doing so helps them exploit new opportunities for profit. In contrast, even though hungry firms operate below their socially efficient scale – judged according to their customers' cost of capital – impact investors always reduce their scale relative to if they were grant financed. By definition, hungry firms have exhausted all of their opportunities to generate profit by increasing their scale, and hence an investor reduces their scale until the marginal social value of their output is equated with the financier's, rather than the customers' cost of capital.

After characterizing the baseline model I analyze two extensions. In the first, the entrepreneur's actions are no longer contractible, so that the choices of scale and price must be incentive compatible. I demonstrate that when the entrepreneur places more weight on mission (relative to profits) than does the financier, the intuitions outlined above are virtually unchanged. In contrast, when the entrepreneur values profits more highly than does the financier, there is no longer a channel by which impact investment can induce an expansion of the firm's scale relative to grant financing. Instead, the primary way by which investments deliver additional value over grants is by reallocating some of the firm's profits away from the private consumption of the entrepreneur.

This setting introduces a distinction between debt and equity. Debt allows the financier to place a lower bound on how much profit the firm must produce (Jensen, 1986), and equity diminishes the weight that the entrepreneur places on profits (eg. Ross, 1973, Holmstrom, 1979). While this is typically viewed as a drawback of equity, my analysis highlights that when the entrepreneur overvalues profit relative to the firm's social mission, muting the firm's desire for profits may be an advantage.[6]

The *non-distribution* constraint on nonprofits may be the most common mechanism by which social entrepreneurs are induced to prioritize mission over profit. However, social enterprises seeking impact investment may sometimes be unable to incorporate as nonprofits, especially in settings where (for reasons outside of the model) equity is the most attractive form of financing. My analysis

[6]Models with non-pecuniary benefits in corporate finance typically posit that the manager overvalues them relative to the financier. The merit of equity in this setting arises from the possibility that the financier weighs non-pecuniary social benefit more highly than the firm's management.

highlights that in such settings equity may nevertheless be an effective substitute in inducing social entrepreneurs to prioritize their mission over profits. This merit of equity is closely related to the analysis of Glaeser and Shleifer (2001) and Chowdhry et al. (2019), although neither of these papers studies the distinction between debt and equity. I discuss this connection further in the following section.

In the second extension I analyze how competition in the firm's product market influences the value-add of impact investing over grant financing. I demonstrate that impact investors provide more value when the firms they support operate in less competitive environments. The core value that impact investors provide is to push firms toward profitability when doing so comes at only small cost to the firms' social missions. When firms face less competition in their product markets, marginally raising the price they charge is less distortionary to demand, thereby increasing their profits by more and harming their social missions by less.

Finally as a demonstration of how to operationalize the ideas discussed above, I examine the case of Husk Power, a rural electrification company operating mainly in Bihar, India. Utilizing data on its costs of operation as well as estimates of elasticity of demand that it faces from its customers, drawn from Burgess et al. (2017) and Lassiter et al. (2018), I quantify the tradeoff that Husk Power faces between its profit and its social mission. I discuss how this informs whether Husk Power should be optimally supported through grants or investments.

This paper relates to several literatures. A large literature within economics explores the consequences of intrinsic motivation in principal-agent problems. Specifically, these papers posit that employees are motivated not only by the possibility of profit but also by the intrinsic value of their output (eg. Bénabou and Tirole, 2003, 2006, Besley and Ghatak, 2005, Prendergast, 2007). Rather than focusing on optimal incentive provision, I (primarily) abstract from any misalignment of interests. Instead, under the premise of intrinsic motivation, I study a classic question in finance – what is the optimal mode of financing? While that literature primarily focuses on the distinction between various claims on a firm's assets (eg. Tirole, 2010), that financiers place intrinsic value on a firm's output introduces the possibility that grants are the optimal form of financing. Therefore at baseline I abstract from the distinction between claims on a firm's assets and instead ask when it is optimal to take any claim on the firm's assets versus offering a simple grant.

This paper also relates to the literature on corporate social responsibility, which explores when and why private companies engage in prosocial behavior (eg. Bénabou and Tirole, 2010). Rather than exploring when engaging in corporate social responsibility improves a firm's profitability (eg. Shleifer and Summers, 1988, Besley and Ghatak, 2007, Margolis et al., 2007), I posit that the firm directly values social goals in addition to profits, and provide a tractable framework to study its operational decisions. In this sense my paper is related to Nilsson and Robinson (2017).

My analysis of a firm operating under the grant financing regime relates to the theoretical literature studying the operation of non-profits. Like in my analysis, Newhouse (1970) and Weisbrod (1988) characterize the behavior of a firm that places intrinsic value on its output, and for whom revenue is a means to expand production. My analysis differs from these in several ways. Most critically among these is that I compare a grant-financed firm (which could, but need not be, interpreted as a non-profit) to a firm financed by impact investment. As mentioned above, my analysis also relates to Glaeser and Shleifer (2001), which considers the non-profit organization of a firm as a means to ameliorate an manager's interest to over-prioritize profits. I further discuss the connection of my analysis to this work in Section 4, where I consider a misalignment of interests between the entrepreneur and financier.

There is also a growing literature on impact investing, discussion of which is deferred to the following section.

1 A Brief Primer on Impact Investing

The term *impact investing* has evolved to encompass a wide range of investing activities, both in public markets and in privately held companies.[7] Impact investing sometimes refers to efforts to align public equity holdings with an investor's values, such as when investors prioritize holding shares of firms that meet certain environmental, social, and governance standards. In contrast, this paper is concerned with the segment of investors who allocate their capital with the explicit intention of creating social value in addition to private financial returns. The 2019 Annual Impact Investor Survey orchestrated by the Global Impact Investing Network estimates this market to comprise upwards of $500 billion in assets under management, about 69% of which are targeted toward privately held companies.[8]

Though there are no universal criteria for what constitutes an impact investment, the sectors representing the largest share of impact investing dollars are affordable housing, microfinance, energy, financial services, food and agriculture, water and sanitation, and healthcare. Together these seven sectors receive more than 65% of impact investments.

A variety of asset managers engage in impact investing, including venture capital and private equity firms, development finance institutions, high net worth individuals, family offices, and foundations. Foundations often utilize impact investments – in the form of program related investments (PRIs) – alongside their traditional grant making efforts, with the explicit intention, and requirement, of redeploying any proceeds generated from the investments. PRIs count towards the five percent of the endowment that must be spent on distribution to charitable causes and operating costs, required of foundations to maintain their tax-exempt status. About 41% of large foundations report making

[7] For more comprehensive reviews of the field and its history, see Cole et al. (2018) and Trelstad (2016).

[8] Unless otherwise stated, all statistics about the impact investing field are drawn from Mudaliar et al. (2019).

investments explicitly to achieve their programatic goals.[9] Since 2009 The Bill and Melinda Gates Foundation, in particular, has allocated more than $1.5 billion to a variety of for-profit and nonprofit firms through PRIs.[10]

Given the range of asset managers involved, it is unsurprising that impact investors also vary based on whether they explicitly accept investments with below-market risk-adjusted expected returns. The Global Impact Investing Network estimates that 34% of impact investors target below-market returns while the remainder aim to generate market-rate returns while also pursuing social or environmental impact. Regardless of their anticipated financial returns, many impact investors primarily support firms that they hope will one day be able to attract commercial investments.

The body of academic research on impact investing is small but growing. Several papers investigate whether, as a matter of practice, impact investors earn lower returns than their commercial counterparts. Examining financial data from 53 private equity impact funds, Gray et al. (2015) argues that impact investors may achieve competitive financial returns. In contrast, using Preqin data, Barber et al. (2017) finds that self proclaimed impact funds earn on average 4.7% lower internal rate of return than their commercial counterparts, and that the limited partners who support impact funds are often willing to accept below market returns on their investments. Kovner and Lerner (2015) finds that firms supported by community development venture capital firms (CDVCs) are less likely to reach a successful exit than those supported by traditional venture capitalists, though it also concludes that CDVC firms in a region tend to attract traditional venture capitalists. Complicating the debate is that none of these studies have a measure of the impact achieved by each investment. Taking an approach based on first principles, Brest et al. (2016) argues that for impact investors to achieve real impact they must provide money or support to social enterprises that would not have been provided in the investors' absence. On these grounds, that paper argues that many impact investments *must* yield below market return, as those that offer market rate returns are likely to have been financed even in the absence of impact investors.

A growing body of theoretical work examines how investors with social preferences allocate their capital in the presence of commercial capital markets. Oehmke and Opp (2019), Broccardo et al. (2020), and Landier and Lovo (2020) study activist impact investors whose goal is to induce firms to take greener actions, while Heinkel et al. (2001), Green and Roth (2020), Pastor et al. (2020), and Pedersen et al. (2019) study passive socially motivated investors. Hart and Zingales (2017) offers a theoretical treatment of the contractual arrangements that social investors might employ to induce firms to take greener actions, and Geczy et al. (2019) provides an empirical characterization of the various contractual terms employed by impact investors to monitor and facilitate impact.

[9] See Buchanan et al. (2015).
[10] See Brest (2016).

In contrast to the aforementioned work, this paper investigates when it is better to support social entrepreneurship through an investment rather than a grant. The answer turns out to be related to the degree to which a socially-oriented organization is positioned to generate a profit. In this sense my work is related to Nilsson and Robinson (2017) and Morgan and Tumlinson (2019), which investigate when profit oriented firms should engage in corporate social responsibility, and when socially oriented firms should generate profits.

Perhaps most closely related is Chowdhry et al. (2019), which also analyzes a social financier's decision of whether to make an investment or a donation. The authors of that paper focus on an agency problem between commercial and socially oriented investors and note that by diminishing the commercial investor's stake in a firm, the impact investor can discipline the commercial investor to pursue socially desirable objectives. In contrast, I primarily abstract from contracting failures. Instead this paper can be thought of as an analysis of the first-best relationship between a socially oriented financier and a social entrepreneur. In doing so I identify situations in which the financier wishes to make an investment, thereby only partially subsidizing the firm's operations, rather than fully subsidizing the firm's operations through a donation.[11] In Section 4 I introduce an agency problem similar to that of Chowdhry et al. (2019). In contrast to that setting, debt and equity are not equivalent in the environment I study, and I identify situations in which each may be the optimal claim on the firm.

2 A Baseline Model of Social Entrepreneurship

Players and Technology. A social entrepreneur (alternatively referred to as a firm) and financier aim to serve a continuum of customers with mass 1. The social entrepreneur has a project that transforms capital into output such that he can serve y customers at cost $c(y)$, which is assumed to be continuously differentiable with $c'(y) > 0$ and $c''(y) > 0$. Each customer i has a privately-known value v_i for the firm's product, with $v_i \sim U[0,1]$.

The entrepreneur chooses output, or scale, y and price p. Each customer's best alternative use of funds yields a marginal utility of r_c. Thus, at a price of p, each customer i would like to purchase the firm's goods if $v_i \geq pr_c$. Therefore, the social entrepreneur faces a downward sloping demand curve $D(p) = \max\{1 - pr_c, 0\}$.

In addition to the private value v_i that each consumer enjoys, I assume there is a positive externality from the firm's output, represented in aggregate by $E(y) > 0$, which is assumed to be continuously

[11] The terms "donation" and "investment" are also used differently in Chowdhry et al. (2019) than in the present paper. In Chowdhry et al. (2019), a socially oriented investor and commercial investor can co-invest to support a social enterprise. Donations in Chowdhry et al. (2019) occur when the optimal outcome can be supported by a partial donation from the social investor, with the residual being supported by the commercial investor. In contrast, in my setting that arrangement would map to an investment. I study a single financier and ask when she would fully subsidize a firm's operations, which I refer to as a grant or donation, and when she would only partially subsidize a firm's operations, which I refer to as an investment.

differentiable with $E'' \leq 0$. That is, when y customers are served, the total positive externality is $E(y)$.

Timing. The model unfolds over two periods. At time $t = 0$ the entrepreneur must raise 1 unit of capital. If he succeeds then at time $t = 1$ he has access to the project described above. Though events within the period unfold simultaneously, they can be understood as happening in sequence. First the firm sets a price for its goods. Then the firm's customers prepay for its goods – the firm need not meet the full demand for its goods at its chosen price. The firm then uses that revenue, potentially in conjunction with additional capital from the financier, to produce and deliver the promised goods to its customers.

Contracts. The deep-pocketed financier can offer the entrepreneur up-front financing 1 at time 0 in exchange for a repayment (or continued subsidy) of R at time 1. To highlight that the main results do not hinge on a misalignment of interests, at baseline I assume that the business-model choices of p and y are contractible – an assumption I subsequently relax in Section 4. At baseline there is no distinction between debt and equity, so a contract can be summarized by $\langle p, y, R \rangle$.

Preferences. The entrepreneur is motivated both by his own share of profits, and by total social impact. Let $\pi(y,p) \equiv py - c(y)$ be the firm's profit. The firm's impact, or *mission* is represented by $M(y,p)$, and is the sum of aggregate consumer surplus and the positive externality. That is $M(y,p) \equiv y\left(\frac{1+pr_c}{2}\right) - pyr_c + E(y)$, where the first term reflects that when y customers are served at price p, their average value for the firm's product is $\frac{1+pr_c}{2}$.[12] The entrepreneur's objective function is represented by

$$M(y,p) + r_e(\pi(y,p) - R)$$

where r_e represents the weight he places on his own profit relative to the firm's mission. The entrepreneurs' outside option is normalized to 0.

The financier also values both profits and mission. The financier's value of profits is normalized to be 1 (representing the financier's next best use of capital, and contrasting the marginal utility of customers' and the entrepreneur's consumptions, r_c, and r_e). Therefore the financier chooses p, y, and R to solve

$$\max_{y,p,R} \{M(y,p) + R\} \tag{1}$$

such that

$$y \leq D(p)$$

and

$$R \leq \pi(y,p)$$

The first constraint dictates that the firm's choice of output cannot exceed demand at its chosen price.

[12] When $y < D(p)$ the firm rations output and allocates it uniformly at random amongst customers with value $v_i \geq pr_c$.

The second constraint is the entrepreneur's limited liability constraint; because price and output are contractible, the investor need only leave the entrepreneur with positive utility and a weakly positive share of the profits. In Section 4 I drop the assumption that p and y are contractible and instead of the limited liability constraint, the financier's choices of p and y must be incentive compatible.

Grant Financing and Impact Investing. To identify the circumstances when an investment is superior to a grant, I separately consider two financing regimes. In the ***grant financing regime*** the financier is restricted to only give grants, and therefore she can offer continued subsidy, but cannot extract profits from the firm. Therefore, in the grant financing regime, the financier solves the maximization problem represented by (1) subject to the additional constraint that $R \leq 0$. I denote by p^{GF}, y^{GF}, R^{GF} the optimal price, output, and transfer under the grant financing regime.

In the ***impact investing regime*****,** the financier can demand a claim on the firm's cash flows in exchange for up-front financing at time 0. Therefore the financier solves the maximization problem represented by (1) without additional constraints. I denote by p^{II} , y^{II}, and R^{II} the optimal price, output, and transfer under the impact investing regime.

The distinction between the two regimes is that in the impact investing regime the financier has the option to "recycle" the firm's profits by utilizing them at the same value as the rest of her capital. Though unmodeled, I assume that the financier can either reallocate the firm's profits to a new opportunity, or consume them. By contrast, in the grant financing regime, because the financier cannot extract the firm's profits, she is restricted to either allocate profits for the entrepreneurs' consumption (an activity for which she places 0 value), or to invest all revenue into the firm's production. In both regimes the firm is permitted to earn negative profits if it receives continued subsidy from the financier.

Clearly the impact investing regime always performs weakly better than the grant financing regime, as one can replicate the grant outcome within the impact financing regime. The central question of this paper is when the impact investing regime performs strictly better than the grant financing regime, and how the optimal business-model choices differ under each regime. This analysis can therefore be understood as illuminating the potential benefits of impact investing, which would need to be compared to the unmodeled costs of establishing the relevant capabilities to assess and contract on such deals.

2.1 A Running Example: Husk Power

I will periodically refer to Husk Power to ground the ideas in this paper. Husk Power is a for-profit social enterprise that has received financing from several impact investors. It provides affordable electricity in rural areas, operating primarily in Bihar, India. Husk produces electricity using solar power and by burning rice husk, both of which are cleaner than the most common alternatives – coal and diesel.

Husk charges a fee for its electricity but because it serves a poor population it has not yet succeeded in attracting commercial financing. However, that Husk generates consumer surplus for an impoverished population and that it produces a positive environmental externality are both reasons that an impact investor might like to subsidize its operation. These are the two components of the firm's mission outlined in the previous section. Whether the optimal subsidy takes the form of a grant (full subsidy) or investment (partial subsidy) is the central question of this paper.

2.2 Further Discussion of Modeling Assumptions

A few of the model's features warrant further discussion. First, I assume the firm's output produces a positive externality. I assume that the aggregate externality is weakly concave and deliberately allow for the possibility that the aggregate positive externality is *decreasing* in the firm's output y. In the context of Husk Power this assumption might be satisfied if increasing its scale requires it to increase its reliance on rice husk and reduce its reliance on solar power, as the former is relatively more abundant but produces more carbon emissions than the latter. Alternatively Husk's increased scale might reduce the managers' ability to monitor product quality.

If instead the aggregate externality was restricted to be strictly increasing, all of the analysis to follow would be unchanged except for that of Proposition 3. In that proposition I demonstrate that for an impact investor to increase the scale of a firm relative to its optimal scale under grant financing, there must be a non-trivial tradeoff between the firm's scale and its mission.

Second, I assume that customers pay for the service prior to receiving it. This assumption is necessary for the firm to use its revenue as a substitute for outside capital from the financier. It could alternatively be replaced by granting the firm access to short-term receivables financing to meet its demand each period.

Third, I have assumed that the financier places no weight on the entrepreneur's profits in her objective function. This can be understood in two ways. First, the financier's preferences can be understood as an approximation of the social welfare function. According to this understanding, the entrepreneur's marginal utility of consumption is small relative to aggregate consumer welfare, the total externality, and the financier's next most socially productive use of capital. Alternatively, the financier can be understood as maximizing preferences that diverge from the social welfare function, in which case she values her own capital (either to finance her own consumption or an alternative socially productive activity), and the firm's mission.

Finally, I have ruled out the possibility of government taxes and subsidies. In this model a government subsidy would incentivize a profit-maximizing firm to produce the first-best level of output. Ruling out the possibility of government intervention allows for the study of private-sector intervention in situations where the government may be constrained, either by the preferences of their electorate or by bureaucratic limitations.

3 An Analysis of the Baseline Model

I analyze the model in two stages. First I study the optimal choices of price and output when the financier is constrained to offer grants, paying special attention to when the firm is sustainable under the grant financing regime. I subsequently relax this constraint to identify the circumstances in which a financier would strictly prefer to offer an investment, and the resulting consequences on the firm's structure relative to the grant financing regime.

3.1 Sustainability Under the Grant financing Regime

Core to the analysis is the definition of a sustainable organization. Define a firm to be sustainable if after time 0, it is fully financed by its own revenue.

Definition 1. A firm is *sustainable under the grant financing regime* if $p^{GF}y^{GF} \geq c\left(y^{GF}\right)$. A firm is *sustainable under the impact investing regime* if $p^{II}y^{II} \geq c\left(y^{II}\right)$.

I first characterize the price that a sustainable firm charges, as a function of its scale, and then characterize its scale. Finally I characterize when a firm is optimally sustainable under the grant financing regime.

Recalling that the firm's mission

$$M(y,p) \equiv y\left(\frac{1+pr_c}{2}\right) - pyr_c + E(y),$$

it is apparent that there are two forces governing the influence of the firm's price on its mission. First, setting a higher price improves allocative efficiency by screening out lower value consumers. Second, increasing the price reduces consumer welfare by inducing a higher transfer from consumers to the entrepreneur. Because demand is uniformly distributed the latter force dominates. A sustainable firm charges the lowest price that allows it to cover its costs. Therefore we write

$$p^{sustainable}(y) = \frac{c(y)}{y}$$

Next I characterize a sustainable firm's scale. Let $p^{max}(y) \equiv \frac{1-y}{r_c}$ be the inverse demand curve – the maximum price the firm can charge while still having demand $D\left(p^{max}(y)\right) = y$. Define the firm's maximal revenue curve to be $R^{max}(y) \equiv p^{max}(y)\,y$ – the revenue that the firm can generate by producing y and charging its customers their maximum willingness to pay.

Figures 1a and 1b plot the firm's maximal revenue curve and cost curve.

There are several points of interest in Figure 1. First, define the "break even" level of output y^{BE} to be the non-zero point at which $p^{max}\left(y^{BE}\right)y^{BE} = c\left(y^{BE}\right)$. That is, when the firm produces y^{BE} and charges the maximal price $p^{max}\left(y^{BE}\right)$, it generates just enough revenue to cover its costs, $c\left(y^{BE}\right)$.

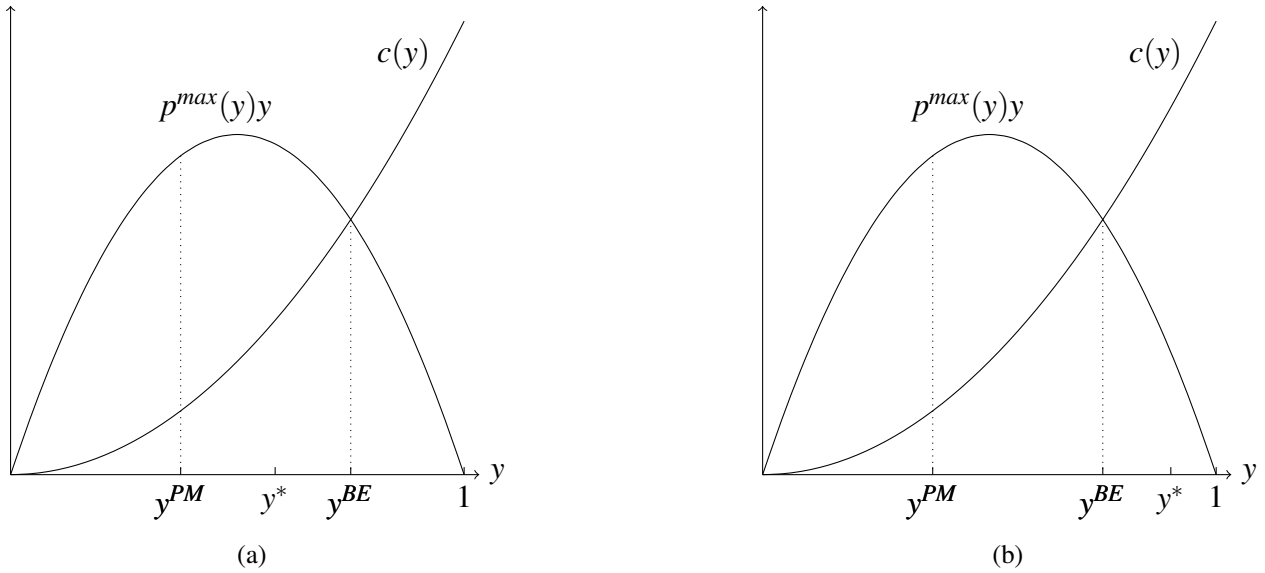

Figure 1

When no such point exists, I define $y^{BE} \equiv 0$.

Next, define y^* and p^* to be the solution to the relaxed variant of the grant financier's optimization problem:

$$\max_{y,p} \{M(y,p) + R\}$$

such that

$$R \leq 0,$$

which, for sustainable firms, reduces to

$$\max_{y} M\left(y, p^{sustainable}(y)\right). \tag{2}$$

That is, y^* is the level of output the firm would provide if it was unconstrained by its customers' willingness to pay. This is a concave maximization problem, and y^* solves

$$\frac{1}{2} + E'(y^*) = c'(y^*)\frac{r_c}{2}$$

The first result summarizes the above analysis and characterizes the firm's choice of p and y when it is sustainable under the grant financing regime.

Lemma 1. *If the firm is sustainable under the grant financing regime it operates at*

$$y^{Sustainable} \equiv \min\left\{y^*, y^{BE}\right\}.$$

The corresponding price is

$$p^{sustainable}(y)$$

The break even level of output y^{BE} is the maximum scale at which the firm can operate without outside capital and y^* is the firm's optimal scale when financed exclusively through the firm's own revenue and unconstrained by the customers' demand curve. If the firm is sustainable, it operates at the lesser of these two points.

Definition 2. When a sustainable firm operates at y^* I say the firm is *satiated.* When a sustainable firm operates at y^{BE} I say the firm is *hungry.*

A satiated firm could expand its scale while remaining sustainable but doing so would harm its mission. The extra revenue extracted from the firm's customers would more than offset the increase in positive externality and consumption utility from expanding the firm's scale. In contrast a hungry firm would always improve its mission by expanding its scale, but is constrained by its customers willingness to pay for its goods.

Now that I have established where the firm operates when it is optimally sustainable, the remaining question is when the firm chooses to be sustainable versus when it relies on continued subsidy.

First, the firm is never sustainable under the grant financing regime if the financier judges the customers' capital to be more valuable than her own.

Lemma 2. *If $r_c > 1$, the firm is not sustainable under the grant financing regime.*

Charging a fee for the firm's goods reduces demand. So when the financier judges money held by the firm's customers to be more valuable than money held herself, the firm optimally charges a price of 0 and is fully supported by the financier at all times t.

In contrast, when $r_c < 1$, the firm charges a positive price and may or may not be sustainable. The condition that $r_c < 1$ does not necessarily imply that the financier values the marginal unit of her own consumption more highly than that of the firm's customers. Rather, it could imply that the financier has an alternative, potentially philanthropic use of capital that has higher marginal social value than the marginal unit of consumption of the firm's customers. While this lemma is mathematically trivial, it highlights that the goal of organizational sustainability requires that the firm's owners must deem their own use of capital to be of more value than a direct transfer to the firm's customers. As I have already established that the firm charges a price of 0 when $r_c > 1$, from here onwards I assume that

financier deems her own capital to be more valuable than the customers': $r_c < 1$.

Next, If $y^* < y^{BE}$ the firm is optimally sustainable under the grant financing regime, as y^* is the firm's optimal scale under the grant financing regime, when unconstrained by customer demand, and when $y^* < y^{BE}$, it is also feasible.

Lemma 3. *If $y^{sustainable} = y^*$, the firm is sustainable under the grant financing regime.*

If $y^* > y^{BE}$ then the firm cannot self-finance at y^*. The following result characterizes when the firm's optimum is y^{BE} and it is financially sustainable, versus when it relies on continued subsidy.

Lemma 4. *If $y^{sustainable} = y^{BE}$, the firm is sustainable under the grant financing regime if and only if*

$$\frac{d}{dy}M\left(y^{BE}, p^{max}\left(y^{BE}\right)\right) + \frac{d}{dy}\pi\left(y^{BE}, p^{max}\left(y^{BE}\right)\right) < 0$$

The decision of whether to operate at y^{BE} or beyond it is a determination of whether the benefit to the firm's mission from expanding its scale outweighs the cost of the subsidy. The cost of the subsidy has two components. First, the financier must pay for the additional output and second, as increasing the supply results in moving down the customers' demand curve, the financier must also cover the resulting loss in revenue. When $y^{BE} < y^*$ and the cost of this subsidy outweighs the benefit, the firm is sustainable and operates at y^{BE}.

3.2 A Graphical Analysis of the Grant Financing Problem

The preceding discussion can be summarized and enriched via the following graphical analysis. Let

$$V^{GF}(y) \equiv \max_{p,R}\{M(y,p) + R\}$$

such that

$$y \leq D(p)$$

$$R \leq \pi(y,p)$$

$$R \leq 0$$

be the financier's value as a function of output under the grant financing problem. Holding fixed $y \leq y^{BE}$, the firm optimally charges the average cost of production, $p^{sustainable}(y) = \frac{c(y)}{y}$, is fully financed by revenues, earns 0 profits, and rations its goods. Holding fixed $y > y^{BE}$, by definition the firm cannot finance production through revenue alone. In this case it charges $p^{max}(y)$ and the financier covers the shortfall $-\pi(y, p^{max}(y)) = c(y) - p^{max}(y)$.

Further, note that for any p, the objective function $M(y,p) + \pi(y,p)$ can be rewritten as $M\left(y, p^{sustainable}(y)\right) + (1 - r_c)\pi(y,p)$, where the first term reflects the firm's mission charging the full cost of production to

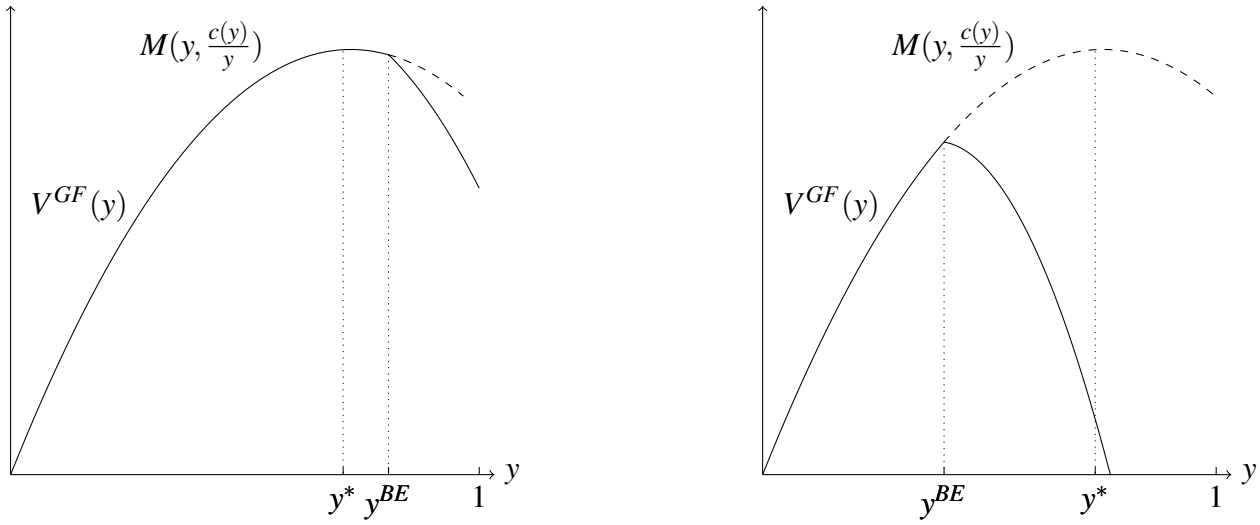

Figure 2

its customers, and the second term reflects the value of the financier's subsidy (or, in the next section, the value of her profits).

Putting these two insights together I write the grant financing value function as

$$V^{GF}(y) = M\left(y, p^{sustainable}(y)\right) + \mathbb{I}\left(y \geq y^{BE}\right)(1 - r_c)\,\pi\left(y, p^{max}(y)\right)$$

where $\mathbb{I}(\cdot)$ is an indicator function. V^{GF} is plotted in Figure 2, for the case where $y^* < y^{BE}$ (2a), and for the case where $y^{BE} < y^*$ (2b).

A few features of Figure 2 are of note. First, V^{GF} is kinked at y^{BE}. Below y^{BE} the firm is sustained through revenue alone and hence is fully supported by the relatively cheap cost of capital r_c belonging to its customers. Beyond y^{BE} the firm requires subsidy from the financier even when charging customers' full willingness to pay, $p^{max}(y)$, and hence its marginal cost of capital jumps discontinuously to $r_f \equiv 1$.

Next, as y^* is the peak of $M\left(y, p^{sustainable}(y)\right)$, when $y^* < y^{BE}$ it coincides with the peak of $V^{GF}(y)$, as is the case in Figure 2a. Therefore, when feasible, y^* is the firm's optimal point of operation in the grant financing regime. Lemma 3 dictates that in this case the firm is sustainable and satiated, as $\frac{d}{dy}V^{GF}(y^*) = \frac{d}{dy}M\left(y^*, p^{sustainable}(y^*)\right) = 0$.

Finally, consider the case where $y^* < y^{BE}$, as in Figure 2b. The firm operates at y^{BE} when

$$\lim_{y \to y^{BE}-} \frac{d}{dy}V^{GF}(y) = \frac{d}{dy}M\left(y^{BE}, p^{sustainable}\left(y^{BE}\right)\right) > 0$$

and

$$\lim_{y \to y^{BE}+} \frac{d}{dy} V^{GF}(y) = \frac{d}{dy} M\left(y^{BE}, p^{sustainable}\left(y^{BE}\right)\right) + \frac{d}{dy}(1-r_c)\,\pi\left(y^{BE}, p^{max}\left(y^{BE}\right)\right) < 0.$$

The first condition dictates that at its customers' cost of capital r_c, the firm finds it worthwhile to marginally expand production. The second condition dictates that at the financier's cost of capital $r_f \equiv 1$ it is not worth the necessary subsidy to expand production. The first condition is always met when $y^{BE} < y^*$, as $M\left(y, p^{sustainable}(y)\right)$ is concave in y. Lemma 4 therefore dictates that when the second condition is also met, the firm is sustainable and hungry.

Figure 2b depicts the case where the firm is sustainable and hungry (i.e. it operates at y^{BE}). If instead the slope of V^{GF} were positive directly to the right of y^{BE}, the figure would depict a firm whose optimal choice of operation would be beyond y^{BE}. In that case, the financier would find that the marginal value of additional output exceeded the necessary subsidy and the firm would not be sustainable under the grant financing regime.

3.3 Suitability for Impact Investing

In this section I characterize the firms for which the impact investing regime is strictly superior to the grant financing regime. This can be understood as studying the first-best problem, asking when its solution coincides with grant financing, and when not, how the solution differs. The first result is that impact investing is superior for all organizations that are sustainable under the grant financing regime. In this sense, sustainability is a precondition for impact investing, rather than the other way around. In all cases in which the impact investor operates the firm at a different scale than a grant financier would have, the impact investor expands the total social impact of the firm, measured according to the financier's objective function. However, this comes from increasing the firm's profitability $\pi(y,p)$ and *reducing* its social mission $M(y,p)$.

Proposition 1. *The impact investing regime is generically strictly superior to the grant financing regime if and only if the firm is sustainable under the grant financing regime.*

Moreover, in all such cases, $\pi\left(y^{II}, p^{II}\right) > \pi\left(y^{GF}, p^{GF}\right)$ *and* $M\left(y^{II}, p^{II}\right) < M\left(y^{GF}, p^{GF}\right)$.

Under the grant financing regime all sustainable firms operate without the financier's capital, so their optimal scale is not disciplined by the financier's cost of capital. Because impact investors can extract profits from their portfolio firms, the size of every firm in the impact investing regime is governed by the financier's cost of capital. Therefore, impact investors change the size of all firms that are sustainable under the grant financing regime.

Importantly, any time an impact investor makes a firm more profitable, it always comes at the expense of harming the firm's mission. Any opportunity to increase both profits and mission would

have been exploited even under the grant financing regime.

I next aim to understand this result graphically. Define $V^{II}(y)$ to be the financier's value under the impact investing regime. That is,

$$V^{II}(y) \equiv \max_{p,R} \{M(y,p) + R\}$$

such that

$$y \leq D(p)$$

$$R \leq \pi(y,p)$$

Now that the financier can pull profit out of the firm, and because $r_c < r_f \equiv 1$, the firm always charges $p^{max}(y)$. Therefore, following Section 3.2 I rewrite the value function as

$$V^{II}(y) = M\left(y, p^{sustainable}(y)\right) + (1-r_c)\,\pi\left(y, p^{max}(y)\right) + \mathbb{I}\left(y \leq y^{BE}\right) y \left\{ \frac{1 + p^{max}(y)\, r_c}{2} - \frac{1 + p^{sustainable}(y)\, r_c}{2} \right\}. \tag{3}$$

As in the grant financing regime, the first term in the impact investor's objective function reflects the firm's mission, charging the full cost of output to the firm's customers. The second term reflects the value of the firm's profit, which the impact investor enjoys at all points of the firm's operation. The final term reflects the benefit to allocative efficiency arising from raising the price from $p^{sustainable}(y)$ to $p^{max}(y)$. When the firm operates below its break even level y^{BE} in the grant financing regime, it only charges enough to cover its costs and therefore rations its output. In the impact investing regime, the firm charges its highest feasible price even below its break-even scale, ensuring that the highest value consumers receive its products.

The impact investor's objective $V^{II}(y)$ is depicted in Figure 3 , for the case where $y^* < y^{BE}$ (3a), and for the case where $y^{BE} < y^*$ (3b).

Compared to Figure 2, one can see that $V^{GF}(y)$ and $V^{II}(y)$ coincide to the right of y^{BE}. This is the region in which the firm receives ongoing subsidy from the financier and hence earns strictly negative return. Firms that optimally operate in this region under the grant financing regime continue to do so in the impact investing regime, as the constraint that $R \leq 0$ is non-binding. In contrast, to the left of y^{BE}, $V^{II}(y)$ is everywhere greater than $V^{GF}(y)$. In this region the financier in the impact investing regime withdraws profits from the firm.

In some cases the impact investor reduces the firm's scale relative to that in the grant financing regime and in other cases the impact investor increases the scale and profitability of her portfolio firm. As I will show in Propositions 2 and 3, whether the impact investor increases or reduces the

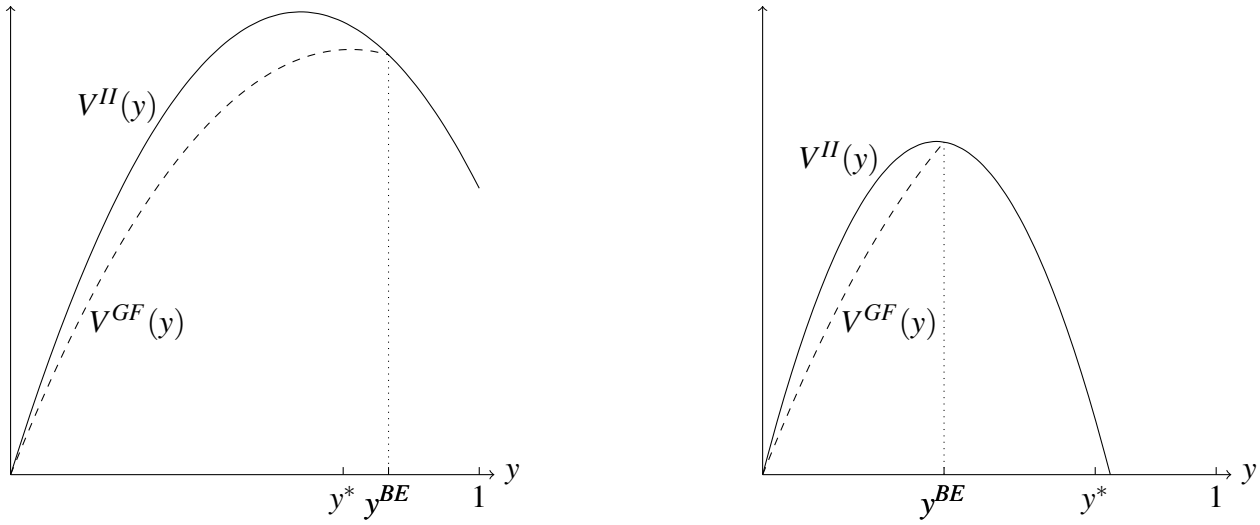

Figure 3

scale of the firm depends on the nature of its sustainability.

Proposition 2. *Relative to the grant financing regime, impact investors always reduce the scale of hungry firms.*

Recall that under the grant financing regime, hungry firms – those that operate at y^{BE} – are constrained by their customers' willingness to pay and therefore operate below their socially efficient scale, judged according to their customers' cost of capital. Nevertheless, the impact investor always reduces the scale of these firms. As discussed in Section 3.2, the fact that these firms are sustainable under the grant financing regime implies that the marginal social benefit of expanding the firm's scale is less than the financier's cost of capital, else the financier would have perpetually subsidized the firm under the grant financing regime. Therefore, as the impact investor is able to reduce the size of the firm and withdraw the resulting profit, this is always worthwhile on the margin.

Referring back to figure 3b, to the right of y^{BE}, the derivative of both value functions $V^{GF}(y)$ and $V^{II}(y)$ are equal. In contrast to $V^{GF}(y)$, $V^{II}(y)$ has a continuous derivative as the financier can always pull profits from the firm, and hence the fact that the firm was not worth expanding beyond y^{BE} under the grant financing regime implies that the impact investor optimally reduces its scale.

In effect, hungry firms create enough social value to justify their upfront investment but will have exhausted their most impactful opportunities for investment before they have exhausted their operating profit. Therefore impact investors optimally tax and reallocate the profit of these firms rather than allowing them to invest it into their operations (or, as described in Section 4, distribute it to their owners).

Proposition 3. *Relative to the grant financing regime, impact investors increase (decrease) the scale of a satiated firm if and only if* $\frac{d}{dy}\pi\left(y^*, p^{max}\left(y^*\right)\right) > 0$ *(*$\frac{d}{dy}\pi\left(y^*, p^{max}\left(y^*\right)\right) < 0$*).*

Figure 3a depicts a firm that is satiated under the grant financing regime. Though in the grant financing regime the firm has already maximized its mission $M\left(y^*, p^{sustainable}\left(y^*\right)\right)$, in the example depicted in the figure, the firm can further increase its profitability by reducing its scale. Proposition 3 dictates that the firm's scale always moves in the direction of increased profitability in the impact investing regime.

To see this, refer to Equation 3. At its satiated scale y^*, the firm has already maximized its mission $M\left(y, p^{sustainable}\left(y\right)\right)$, and hence the derivative of the first term is 0 with respect to scale. The magnitude of the benefit to allocative efficiency is proportional to the firm's profits and hence the impact investor increases the firms scale if and only if doing so also increases its profits.

At first this result may seem counterintuitive. In the grant financing regime, hungry firms would like to increase their mission by increasing their scale were they not constrained by their customers' willingness to pay, whereas satiated firms have maximized their mission. Yet impact investors always contract the scale of hungry firms, and may expand the scale of satiated firms.

This result derives from the fact that the impact investor's comparative advantage is helping firms increase their impact through utilizing profits, not elevating their mission. Because the impact investor can pull profit out of the firm, marginal profits are judged according to the impact investor's cost of capital, rather than at the marginal value of additional output. Firms financed by investors rather than donors therefore place higher value on profits. This is the mechanism that drives satiated firms to increase their scale when doing so increases profits. In contrast, by definition hungry firms have already exploited all of their opportunities to generate profit through increasing their scale, and so there is no scope for impact investors to increase their scale and profitability.

Underlying Proposition 3 is that impact investing unlocks new opportunities to create value and profit. Essentially, impact investors facilitate the exploitation of profitable opportunities whose feasibility arises from the operation of a social enterprise. The opportunity may not be sufficiently lucrative that a commercial investor would finance the entire capital outlay required to start the enterprise, but once the capital outlay has been justified by its social value the new business opportunity becomes commercially viable. To take advantage of the new opportunity, the financier must be able to extract profit from the firm – therefore these opportunities may go unexploited when social enterprises are financed by donations alone.

3.4 Two Heuristics to Guide Impact Investors

The preceding analysis identifies two heuristics for judging when impact investing may be superior to grant financing. The first heuristic is an interpretation Proposition 2 in plain English: impact

investors can discipline the spending of their portfolio firms. All firms that can attract support under the grant financing regime generate enough social value to justify their fixed cost, judged according to the financier's cost of capital. But firms that generate sufficient revenue to be sustainable may allocate the marginal dollar they generate toward unproductive use. By making an investment rather than a grant, impact investors can extract and reallocate that dollar.

The second heuristic is a plain English interpretation of Proposition 3: by taking an ownership stake in the firm, a financier can empower the entrepreneur to exploit a profitable opportunity that arises as a result of investments made on the basis of their social value. Impact investors increase the value of a firm's profit by providing an additional avenue through which it can be used. Hence marginal opportunities to generate profit which also harm the firm's mission may only be worthwhile when the financier is a partial owner of the firm. In effect, impact investing enables profitable businesses to be built on the groundwork laid by social enterprises.

A Second Look at Husk Power

While the first heuristic is straightforward, it may be instructive to consider the second heuristic in the context Husk Power, introduced in Section 2.1.

Had Husk instead been financed exclusively by donations, its donors may have insisted it distribute its electricity for a very low fee, just to cover its variable cost of operation. Once it had incurred the cost of setting up the power plants and distribution networks (justified on the basis of their social value), Husk may have encountered an opportunity to generate a profit by on-boarding additional customers onto each of its electricity grids. If doing so would reduce the reliability of service for inframarginal customers, or would necessitate increasing Husk's carbon footprint at an outsized rate, and therefore would hamper Husk's mission of providing clean energy in rural areas, its donors may have discouraged it from exploiting this opportunity since it already covered its cost of operations.

Because Husk's financiers are investors, not donors, they have taken a claim on its future profits. They may therefore not only empower Husk to capitalize on this opportunity, but may also offer Husk more capital than donors would have, as they are positioned to benefit from the profits that capital would now generate. Notably, even if Husk could never generate enough revenue to attract commercial financing, impact investment may still improve upon donation by inducing Husk to generate additional profits on the margin.

4 Extension: Price and Scale are Non-Contractible

In this section I extend the model such that price and scale are no longer contractible. A financial contract now only specifies the initial transfer of capital and subsequent repayment, and price and scale are now chosen by the entrepreneur. Therefore, even when grant financed, the firm can generate positive profits, which are consumed by the entrepreneur. The divergence of interests between the financier and entrepreneur now introduces an agency problem, so within the impact investing regime

I separately consider debt and equity and their consequences on the entrepreneur's behavior.

I analyze the problem in two stages. First, the financier offers the entrepreneur a contract that specifies a transfer of 1 capital from the financier to the entrepreneur at time 0, and a transfer from the entrepreneur to the financier at time 1. I separately consider debt contracts, which specify a transfer $\mathscr{R}$ in each period independent of the firm's profit, and equity contracts, which specify a transfer $\lambda\pi(y,p)$ in each period for $\lambda \in [0,1]$. The transfer $\mathscr{R}$ in debt contracts can be either positive or negative, reflecting the possibilities that the financier either continues to support the firm or extracts its revenues. I write $\mathscr{T}(\pi)$ to accommodate the possibility of either debt of equity (but I do not allow for transfers that are an arbitrary function of profits).

At all time $t = 1$, the entrepreneur then chooses y and p to solve

$$\max_{y,p} \{M(y,p) + r_e(\pi(y,p) - \mathscr{T}(\pi(y,p)))\} \tag{4}$$

s.t.

$$y \leq D(p)$$

$$\pi(y,p) \geq \mathscr{T}(\pi(y,p))$$

That is, given the transfer $\mathscr{T}(\pi)$, the entrepreneur chooses output and price to maximize the sum of his social mission $M(y,p)$, and the value of profits he extracts from the firm, measured by the firm's profits $\pi(y,p)$ minus the transfer the entrepreneur makes to the financier $\mathscr{T}(\pi)$. The entrepreneur values his own profits at rate r_e and I continue to normalize the value of the financier's profits to $r_f \equiv 1$. The entrepreneur is constrained by the demand curve $D(p)$ and the financing constraint which specifies that the firm's profit must exceed the transfer to the financier.

The financier's objective function is

$$V(y,p,\mathscr{T}(\pi)) \equiv M(y,p) + \mathscr{T}(\pi(y,p)).$$

Denoting the entrepreneur's solution to problem (4) by $(y(\mathscr{T}(\pi)), p(\mathscr{T}(\pi)))$, at time 0 the financier chooses $\mathscr{T}(\pi)$ to solve

$$\max_{\mathscr{T}(\pi)} V(y(\mathscr{T}(\pi)), p(\mathscr{T}(\pi)), \mathscr{T}(\pi)). \tag{5}$$

In the grant financing regime, the financier solves (5) subject to the constraint that $\lambda = 0$ in the case of equity and $\mathscr{R} \leq 0$ in the case of debt (i.e. the financier can continue to support the firm in period 1 but cannot extract profits from the firm), and in the impact investing regime the financier solves (5) without additional constraints.

In the analysis to follow, I assume $r_e > r_c$ so that the entrepreneur values his own use of profits more highly than his customers'. I separately analyze the cases where $r_e < 1$ so that the entrepreneurs' weight on profits (relative to mission) is less than the financier's, and $r_e > 1$ so that he values his profits more highly than does the financier.

4.1 The Entrepreneur Values His Profits Less Than The Financier's: $r_e < 1$

I first analyze the case for which $r_e < 1$. Because the entrepreneur values his own profits less highly than does the financier the model and analysis are very similar to the preceding sections. In particular, Lemma 2, and Propositions 1 and 2 continue to hold. Redefining satiated firms to be any firm that is optimally sustainable under the grant financing regime and that operates at $y < y^{BE}$ (rather than operating precisely at y^*), Lemmas 1 and 4, and Proposition 3 continue to hold as well. Because the impact investor values profits more than does the entrepreneur, the firm will prioritize profits more highly under the impact investing regime. Therefore, the two primary advantages of impact investing highlighted in the previous section are qualitatively unchanged. Further, debt is the optimal financial instrument.

Proposition 4. *When $r_e < 1$ debt is the optimal financial instrument.*

Debt allows the financier to ensure the first-best outcome. This is so because debt allows the financier to put a lower bound on the profits the firm must return each period (Jensen, 1986). Since when $r_e < 1$ the entrepreneur's inclination is to produce less profit than the financier would like, debt is always optimal.

4.2 The Entrepreneur Values His Profits More Than The Financier's: $r_e > 1$

I next analyze the case for which $r_e > 1$. I first consider an impact investor using debt financing. As above, after suitably redefining satiated firms, Lemmas 2, 1, and 4 continue to hold.

Proposition 1 can now be strengthened.

Proposition 5. *When $r_e > 1$, the impact investing regime is generically strictly superior to the grant financing regime if and only if the firm is sustainable under the grant financing regime. Under the optimal debt contract $\left(y^{GF}, p^{GF}\right) = \left(y^{II}, p^{II}\right)$.*

An impact investor using debt never changes the size of the portfolio firm relative to grant financing. This follows from the fact that, so long as the entrepreneur generates enough profit to repay $\mathscr{R}$ to the financier, his optimization is independent of $\mathscr{R}$. So if the financier sets $\mathscr{R} \leq \pi\left(y^{GF}, p^{GF}\right)$, the entrepreneur will choose the same scale of operation. The financier can change the scale of operation relative to the grant financing regime by demanding $\mathscr{R} > \pi\left(y^{GF}, p^{GF}\right)$, but as the entrepreneur already prioritizes profits more highly than the financier would like, this is never optimal. Therefore, in the impact investing regime, the financier sets $\mathscr{R} = \pi\left(y^{GF}, p^{GF}\right)$ and extracts the firm's full profits without changing its scale of operation.

Recall, in Section 3 I highlighted two ways for an impact investor to create value relative to a grant financier. The first is that the impact investor can extract profits from her portfolio firm when they are they are no longer being put to good use on the margin, either because the firm has run out of impactful investment opportunities, or, in this case, because the entrepreneur consumes the firms marginal profits rather than reinvesting them. The second manner in which impact investors create value is by inducing the entrepreneur to exploit new opportunities for profit, as the impact investor can allocate those profits to other high value opportunities. When $r_e > 1$, the entrepreneur is already over-exploiting opportunities for profit relative to the impact investor's preferences, and therefore impact investors only create value by taxing and reallocating the profits the firm would have made even under the grant financing regime.

Moreover, equity may be the optimal financial instrument.

Proposition 6. *For r_e sufficiently large, equity is the optimal financial instrument.*

Unlike in the case where $r_e < 1$, the financier now faces a non-trivial tradeoff between debt and equity. While the debt contract allows the financier to extract all of the firm's profit, it does not change the entrepreneur's incentive to pursue profit on the margin. In contrast, equity depresses the entrepreneur's incentive to pursue profit and results in him placing a relatively higher weight on the firm's mission. While depressing the entrepreneur's incentive to pursue profit is typically viewed as a drawback of equity, in this context it is a merit as both the financier and the entrepreneur value a non-pecuniary benefit of the firm's output. The insight that the structure of financing can discipline an entrepreneur to pursue social goals is closely connected to the results of Glaeser and Shleifer (2001) and Chowdhry et al. (2019), though neither of those papers studies the distinction between debt and equity.

Stepping outside of the model, the non-distribution constraint on nonprofits may be the most common way that social entrepreneurs are encouraged to pursue their firm's mission over profits. Often, however, attracting financing from private investors requires the ability to sell equity and therefore precludes firms from incorporating as nonprofits. Proposition 6 suggests that in such settings, equity financing may impose a similar discipline on the entrepreneur.

5 Extension: Competition in the Product Market

I next examine how competition in the firm's product market influences the value of impact investing relative to grant financing, and demonstrate that impact investing provides more value in less competitive markets.

I modify the environment in Section 2 by introducing a second firm. There is now an *entrant,* which corresponds to the firm in the prior analysis, and an *incumbent.* The incumbent inelastically supplies

a good at price $\bar{p}$.[13] The two firms provide goods that are substitutes in the sense that consumers can purchase at most one of the two goods. Value for the entrant's good is still distributed uniformly on the unit interval, however now each consumer who has value v_i for the entrant's good also has value θv_i for the incumbent's good, for $\theta \in [0,1]$. The parameter θ is therefore a measure of how competitive the environment is – for $\theta = 1$ the goods are perfect substitutes and for $\theta = 0$ the entrant faces no competition from the incumbent.

While I continue to assume that the entrant produces positive externality $E(y)$ from distributing y goods, I assume that the incumbent produces no externality at all. The entrant can therefore be understood to provide a cleaner alternative to the incumbent's product. A financier who supports the entrant creates social value by providing a positive externality and increasing consumer surplus.

For a given level of competition θ, define

$$\Delta^{II}(\theta) \equiv M\left(y^{II}, p^{II}\right) + \pi\left(y^{II}, p^{II}\right) - \left(M\left(y^{GF}, p^{GF}\right) + \pi\left(y^{GF}, p^{GF}\right)\right)$$

to be the difference in the financier's value under the impact investing regime and the financier's value under the grant financing regime. Therefore $\Delta^{II}(\theta)$ can be understood as the value of impact investing, and may be of interest in an environment where developing the capabilities to make investments – rather than pure donations – carries some costs. In such settings, the higher is $\Delta^{II}(\theta)$, the more likely it is to be worthwhile to incur these costs. The following result is the key insight of this section.

Proposition 7. *Impact investing is more valuable when there is less competition in the product market:* $\frac{d\Delta^{II}(\theta)}{d\theta} < 0$.

To understand the logic underlying Proposition 7 it is necessary to first understand the entrant's demand curve. At a price of p, a consumer i will demand the entrant's good if and only if $v_i \geq pr_c$, and $v_i - pr_c \geq \theta v_i - \bar{p}r_c$. The former inequality dictates that the entrant's good must be worth at least p to the consumer, and the latter inequality dictates that at a price of p, the entrant's good must create more consumer surplus for i than does the incumbent's good at price $\bar{p}$. Therefore the entrant faces a demand curve

$$D(p,\theta) = \min\left\{1 - pr_c, 1 - \frac{p-\bar{p}}{(1-\theta)}r_c\right\}$$

When $p < \tilde{p}(\theta) \equiv \frac{\bar{p}}{\theta}$, no one who derives positive consumer surplus from the entrant's good would prefer to buy the incumbent's good. At these low prices, the entrant's demand curve is $D(p,\theta) = 1 - pr_c$. When $p > \tilde{p}$, everyone who derives positive consumer surplus from the entrant's good, also does so from the incumbents good. Therefore when $p > \tilde{p}(\theta)$, competition binds and the demand curve is $D(p,\theta) = 1 - \frac{p-\bar{p}}{(1-\theta)}r_c$. This kinked demand curve is depicted in Figure 4 for two levels of

[13] The incumbent's price $\bar{p}$ is exogenous. The incumbent can be understood as a large player whose calculus is unaffected by the presence of an entrant.

competition $\theta_1 < \theta_2$.

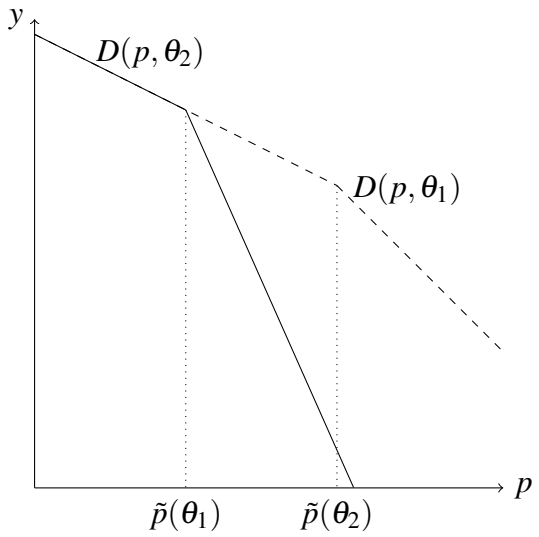

Figure 4

To understand the logic of Proposition 7, recall that the value of impact investing comes from altering the entrant's scale and price to increase its profitability at the cost of its mission. The core insight underlying Proposition 7 is that the required harm to a firm's social mission resulting from a given increase in its profitability is smaller in less competitive environments.

To see this, note that at all prices, the entrant's demand curve is steeper in the more competitive environment. For any increase in the entrant's price, the entrant suffers a larger reduction in its demand in the more competitive environment. This implies that the price increase confers a smaller benefit to the entrant's profits and a larger harm to its mission in the more competitive environment. Therefore relative to donors, impact investors create less value for firms in more competitive markets.

6 A Case Study in Assessing the Suitability of Impact Investment: Husk Power

In this section I apply the heuristics developed formally in Section 3.3 and stated informally in Section 3.4 to the case of Husk Power. Husk Power, described in Section 2.1, is a rural electrification company that has received impact investment. I provide a quantitative analysis of their cost and revenue curves and use them to assess whether 1) an investor could productively tax the firms profits and reduce its scale or 2) justify an increase in the firm's scale and profitability.

In general, quantifying the forces in the above model requires three ingredients: An assessment of the social value of the firm's output, its cost of production, and the demand curve it faces. The first of these is often subjective and is required in the context of any philanthropic decision making. The latter

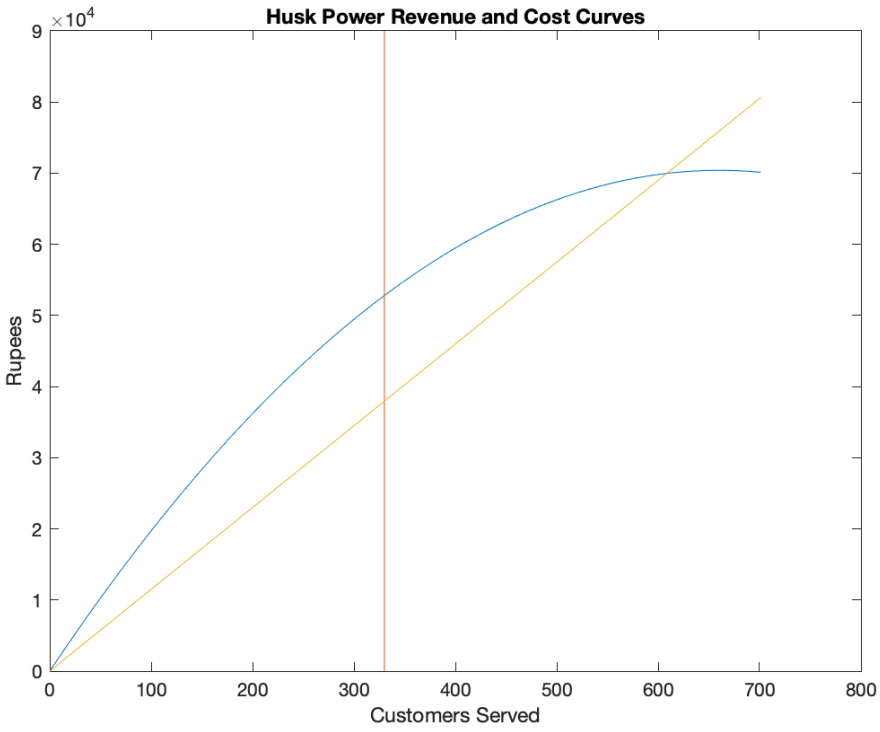

Figure 5

two are straightforward to measure, and for the case of Husk Power, relevant estimates are provided by Burgess et al. (2017). The authors conducted an experiment in 2013 in collaboration with Husk Power in which they randomized the price of Husk's electricity and measured the resulting demand. In doing so they determined that reducing the price from 200 rupees to 100 rupees increased demand from 2 percent of eligible households up to 17 percent, where demand is measured by whether a household ever paid for a connection. The authors also note that the variable cost of production is 115 rupees per served household.

Extrapolating a linear curve through their estimates implies that demand is

$$D(p) = 32 - .15p$$

where demand is measured in percent of the population served. Company reports suggest that at the prevailing price of 160 rupees per connection, each power plant serves around 330 customers across several villages (Lassiter et al., 2018). Therefore one can equivalently write the demand curve on a per household basis as

$$D^{HH}(p) = 1320 - 6.1875p$$

Combining this estimated demand curve with the aforementioned variable cost of production, Figure 5 presents the empirical analogue of Figure 1: the revenue and cost curves of a single power plant operated by Husk Power.

The vertical line plots Husk Power's status quo point of operation. At the time of these estimates Husk operated very close to profit maximization (the profit maximizing point of operation is at a price of 164.2 rupees per household, reaching about 304 households per plant). This may not be a surprise, as Husk is supported primarily by impact investors rather than donors, who are thus positioned to benefit from the profits that Husk generates.

At the time that Husk Power was raising capital, its financiers could use figures like this to assess whether an investment or donation is more suitable. To assess the importance of the first heuristic, would-be donors could use Husk's business model projections to ask themselves if they are comfortable with the amount of money being disbursed to the firm's owners. If not, they may prefer to make an investment and reduce the profits available for the firms current owners.

The second heuristic – whether Husk is positioned to generate additional profits at sufficiently small cost to its social mission – is less relevant given its status quo plan of charging the profit-maximizing price. But suppose instead Husk planned to charge a price of, say, 125 rupees, reach about 545 households per plant, and just barely break even. In that case, Husk's would-be donors could use Figure 5 to assess the tradeoff between generating additional profits by raising the price, and the resulting reduction in the number of households that received access to affordable electricity. If the reduction in Husk's social mission were deemed to be smaller than the value that Husk's financiers could generate using the additional profits, they may decide to invest in Husk rather than making a donation, and encourage it to raise its prices.

7 Discussion

This article has focused on the circumstances in which philanthropists can have more impact with an investment than with a grant. I critically examine two common explanations for the merits of impact investing. First, impact investors can help firms reach financial sustainability, and second, impact investors can help firms scale. I utilize a simple model to uncover the nuance underlying these two assertions regarding the consequences of impact investing.

First I provide a formal definition of organizational sustainability and characterize the situations in which a social enterprise should be sustainable. I demonstrate that impact investing is justified when an organization would have been sustainable even if grant financed. In this sense, organizational sustainability leads to impact investing, rather than the other way around. I draw a distinction between sustainable firms that can self-finance their optimal scale, termed *satiated firms*, and sustainable firms whose scale is constrained by their customers' willingness or ability to pay for goods and services, termed *hungry firms*.

Impact investors may optimally increase or decrease the scale of a firm relative to if it were grant-financed. By providing firms an additional channel to productively allocate their profits, impact in-

vestors can help firms exploit new opportunities to generate profits and increase their scale. Doing so may be optimal when capitalizing on these opportunities is not in strong tension with a firm's mission. In contrast, when no such opportunities exist, impact investors may reduce the scale of sustainable organizations. These are the firms for which a financier has determined the marginal impact of an additional dollar invested does not justify its cost, either because the firm has run out of impactful opportunities or because marginal profits are channeled to the firm's owners rather than invested in the firm's mission. I demonstrate that whether impact investors expand or reduce the scale of the organization depends on the nature of its sustainability. By definition, hungry firms have exhausted their opportunities to generate profit through increased scale, and hence impact investors always reduce their scale. However, impact investors may increase the scale of satiated firms, which under grant financing have maximized their mission and so may not have exploited all opportunities for profit.

Put in other words, philanthropic impact investors should hold two considerations when determining whether to offer a grant or investment to a particular organization. First, is this an organization that would be worth subsidizing into the indefinite future? If yes, a grant may be superior, as an investment will result in withdrawing capital from organizations that continue to have high impact. If no, an investment enables a financier to discipline relatively low-value spending of the organizations they support. Second, does this organization have opportunities to generate profits that would not come at significant cost to their primary mission? If yes, investing provides a channel to redirect those profits and increase their impact.

Next I consider the possibility that the entrepreneur's choices are non-contractible. I demonstrate that most insights from the earlier analysis are robust to this extension, and highlight a new distinction between debt and equity. Equity dampens the entrepreneur's marginal incentive to pursue profits. While this is typically viewed as a drawback of equity financing, in the context of social entrepreneurship it may be a merit when the entrepreneur values profits too highly relative to the firm's social mission.

I then demonstrate that impact investors create more value – relative to donors – when their firms face less competition in the product market. The core value that impact investors provide is to help firms increase their profits when doing so comes at only small cost to their social mission. I demonstrate that there are more such opportunities when firms face less competition in the product market.

Finally, I provide a simple demonstration of how financiers might quantify the considerations in this paper in the context of Husk Power. Quantifying these considerations only requires basic cost and revenue projections, an estimate of the firm's demand curve, and an assessment of the social value of the firm's output.

I close this section by considering several other common justifications for impact investing. Impact

investors enjoy control rights over their portfolio firms that traditional donors typically lack; Impact investors can help to professionalize their portfolio firms by ensuring they meet the accounting and transparency standards necessary for commercial investment; Impact investors can help signal a firm's ability to repay its debts, thereby enabling it to attract commercial capital. I argue each of these is fundamentally separable from the distinction between a grant and an investment. In principle, grant financiers could contract on board seats and other control rights. Grants could be accompanied by consulting services to help professionalize grantees. And the signal of a firm's accountability could be derived from its balance sheet and income statement rather than by returning capital to its financiers. Therefore, I argue that the conditions outlined in this article should remain the primary justification for an investment rather than a grant.

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Omitted Proofs

Proof of Proposition 1

First I show that when impact investing is superior to grant financing, the firm is sustainable under the grant financing regime. By concavity of the firm's objective function it must be that $\frac{d}{dy}M\left(y,p^{max}\left(y\right)\right)+\frac{d}{dy}\pi\left(y,p^{max}\left(y\right)\right)<0$ for all $y>y^{II}$. If impact investing is superior to the grant financing regime then $\pi\left(y^{II},p^{II}\right)>0$. Therefore $y^{II}<y^{BE}$. Hence by Lemma 4 the firm is sustainable under the grant financing regime.

If the firm is sustainable under the grant financing regime then $y^{GF}=\min\left\{y^{BE},y^{*}\right\}$. If $y^{GF}=y^{BE}$ then by Lemma 4, generically

$$\frac{d}{dy}\pi\left(y^{GF},p^{max}\left(y^{GF}\right)\right)+\frac{d}{dy}M\left(y^{GF},p^{max}\left(y^{GF}\right)\right)<0.$$

The left hand side of the above inequality is the derivative to the unconstrained objective function

$$M\left(y,p^{max}\left(y\right)\right)+\pi\left(y,p^{max}\left(y\right)\right)$$

corresponding to the maximization problem under the impact investing regime. Thus the firm's objective function can be improved via a local decrease in its scale under the impact investing regime.

If $y^{GF}=y^{*}$ then, generically, $p^{GF}<p^{max}\left(y^{*}\right)$. Therefore the firm can make a local improvement by raising p^{GF} to $p^{max}\left(y^{*}\right)$ under the impact investing regime.

In all such cases, $\left(y^{II},p^{II}\right)\neq\left(y^{GF},p^{GF}\right)$. Therefore, by revealed preference, $\pi\left(y^{II},p^{II}\right)+M\left(y^{II},p^{II}\right)>\pi\left(y^{GF},p^{GF}\right)+M\left(y^{GF},p^{GF}\right)$. Further, as the only difference between the impact investing regime and the grant financing regime is the relaxation of the constraint that $\pi\left(y,p\right)\leq 0$, it is immediate that $\pi\left(y^{II},p^{II}\right)>0\geq\pi\left(y^{GF},p^{GF}\right)$. Finally, because $\pi\left(y^{II},p^{II}\right)>0$, $\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)$ is a feasible point of operation under the grant financing regime, with $\frac{y^{II}}{c\left(y^{II}\right)}<p^{max}\left(y^{II}\right)=p^{II}$. Moreover, $\pi\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)=0$. Therefore,

$$\begin{aligned}M\left(y^{GF},p^{GF}\right)&\geq M\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)\\&>M\left(y^{II},p^{max}\left(y^{II}\right)\right)\\&=M\left(y^{II},p^{II}\right)\end{aligned}$$

The first inequality follows because $\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)$ is an achievable point of operation under the grant financing regime that results in 0 profits. If the grant financing regime operates at a point with negative

profits, it must achieve more of its mission. The second inequality follows from the fact that the firm's mission is declining in the price it charges, and the third equality follows from the fact that under the impact investing regime the firm always operates on the maximal revenue curve. □

Proof of Proposition 2

The difference between the impact investing regime and the grant financing regime is that the impact investing regime does not have the constraint $R \leq 0$. Therefore, when $\left(y^{II}, p^{II}\right) \neq \left(y^{GF}, p^{GF}\right)$, it must be that $\pi\left(y^{II}, p^{II}\right) > 0$. All y such that there exists a p with $\pi(y,p) > 0$ satisfy $y < y^{BE}$. □

Proof of Proposition 3

Replicating Equation 3 we have

$$V^{II}(y) = M\left(y, p^{sustainable}(y)\right) + (1-r_c)\,\pi\left(y, p^{max}(y)\right) + \mathbb{I}\left(y \leq y^{BE}\right) y \left\{ \frac{1+p^{max}(y)\,r_c}{2} - \frac{1+p^{sustainable} r_c}{2} \right\}$$

By definition of y^* we have that $M\left(y^*, p^{sustainable}(y^*)\right) = 0$. Moreover, the third term can be rewritten as

$$y \left\{ \frac{1+p^{max}(y)\,r_c}{2} - \frac{1+p^{sustainable} r_c}{2} \right\} = \frac{1}{2r_c}\pi\left(p^{max}(y), y\right)$$

Hence

$$\frac{d}{dy}V^{II}(y^*) = \left(1 - r_c + \mathbb{I}\left(y \leq y^{BE}\right)\frac{1}{2r_c}\right)\frac{d}{dy}\pi\left(p^{max}(y^*), y^*\right)$$

□

Proof of Proposition 4

Let $\left(y^{II}, p^{II}\right) = \underset{y,p}{\operatorname{argmax}}\, M(y,p) + \pi(y,p)$. When $r_e < 1$, the debt contract that requires $\mathscr{R} = \pi\left(y^{II}, p^{II}\right)$ induces the first-best outcome. (In contrast, any equity contract reduces the entrepreneur's (already too low) incentive to prioritize profit.) □

Proof of Proposition 5

Let $r_e > 1$. Suppose the firm is sustainable under the grant financing regime, and generates $\pi\left(y^{GF}, p^{GF}\right)$ profit. Then under the grant financing regime the financier's per period utility is $M\left(y^{GF}, p^{GF}\right)$. In contrast, the debt contract that requires payment $\mathscr{R} = \pi\left(y^{GF}, p^{GF}\right)$ generates per period utility of $\pi\left(y^{GF}, p^{GF}\right) + M\left(y^{GF}, p^{GF}\right)$.

Now suppose that the firm is not sustainable under the grant financing regime. Then recalling that

$$\left(y^{GF}, p^{GF}\right) \equiv \underset{y,p}{\operatorname{argmax}}\, M(y,p) + r_e \pi(y,p),$$

$\pi\left(y^{GF}, p^{GF}\right) < 0$. Therefore the solution to $\max_{y,p} M(y,p) + \pi(y,p)$ also has negative profit, and impact investing will therefore induce the same outcome as grant financing. □

Proof of Proposition 6

Recall that $\left(y^{II}, p^{II}\right) \equiv \underset{y,p}{\text{argmax}}\, M(y,p) + \pi(y,p)$, and that $\left(y^{PM}, p^{PM}\right) \equiv \underset{y,p}{\text{argmax}}\, \pi(y,p)$. Fix any example where these two solutions do not coincide, and where $\pi\left(y^{II}, p^{II}\right) \geq 0$. Now take r_e to be finite but large. As r_e becomes arbitrarily large, the entrepreneur's choice of operation becomes arbitrarily close to $\left(y^{PM}, p^{PM}\right)$. The optimal debt contract extracts the full profits of the firm, but, for $r_e > 1$, it does not change the entrepreneur's point of operation, as the entrepreneur's marginal utility of profits is unchanged. In contrast, the equity contract which extracts a fraction $\frac{r_e - 1}{r_e}$ of the profits induces the entrepreneur to choose $\left(y^{II}, p^{II}\right)$. As $r_e \to \infty$, the investor's utility from this contract converges to $M\left(y^{II}, p^{II}\right) + \pi\left(y^{II}, p^{II}\right)$. Therefore, for sufficiently large r_e, the optimal equity contract is superior to the optimal debt contract.

Now consider the case where $\pi\left(y^{II}, p^{II}\right) < 0$. It is no longer the case that the entrepreneur will choose $\left(y^{II}, p^{II}\right)$ under the equity contract that extracts a fraction $\frac{r_e - 1}{r_e}$ of the firm's profits, as doing so would violate his budget constraint. However consider the optimal point of operation

$$\left(y^{II'}, p^{II'}\right) \equiv \underset{y,p}{\text{argmax}}\, M(y,p) + \pi(y,p)$$

such that

$$\pi(y,p) \geq 0$$

There exists some equity share $s(r_e)$ such that if the financier takes $s(r_e)$ the entrepreneur will select $\left(y^{II'}, p^{II'}\right)$. As $r_e \to \infty$, $s(r_e) \to 1$, and the equity contract that takes $s(r_e)$ will dominate the debt contract which extracts the full profitability of the firm. □

Proof of Proposition 7

Suppose that $\theta_1 < \theta_2$. I will demonstrate that $\Delta^{II}(\theta_1) \geq \Delta^{II}(\theta_2)$. Let $y^{GF}(\theta_i)$ and $p^{GF}(\theta_i)$ denote the grant financing scale and price when the competition parameter is θ_i.

Lemma 5. *The grant financing price is decreasing in the level of competition and the grant financing scale is increasing in the level of competition.* $y^{GF}(\theta_1) \leq y^{GF}(\theta_2)$ *and* $p^{GF}(\theta_1) \geq p^{GF}(\theta_2)$

The basic logic underlying this lemma is that the firm has more monopoly power in lower competition environments and therefore chooses a weakly higher price and weakly lower scale. To see the logic more precisely, suppose that the opposite conclusion held: $p^{GF}(\theta_1) < p^{GF}(\theta_2)$. By concavity of the firm's objective function, when the competition parameter is θ_2, the objective function is increasing in p at $p^{GF}(\theta_1)$. But the firm faces a weakly steeper demand curve in θ_2 than it does in θ_1, which implies that its objective function is increasing in p at $p^{GF}(\theta_1)$ when the competition parameter is θ_1 . This is a contradiction. The argument is analogous for y.

Lemma 6. *If the firm is sustainable in the grant financing regime at* θ_2*, it is also sustainable in the grant financing regime at* θ_1*. Moreover,* $y^{GF}(\theta_1) = y^{GF}(\theta_2)$ *and* $p^{GF}(\theta_1) = p^{GF}(\theta_2)$.

By Lemma 5 we know that $y^{GF}(\theta_1) \leq y^{GF}(\theta_2)$. We also know that $y^{BE}(\theta_1) \geq y^{BE}(\theta_2)$ because the demand curve is weakly less constraining when the competition parameter is θ_1. Further note that $y^*(\theta_1) = y^*(\theta_2) \equiv y^*$, because by definition of y^* the demand curve is not binding.

Now, if $y^{GF}(\theta_2) = y^*$, then $y^{GF}(\theta_1) = y^*$ because $y^* \leq y^{BE}(\theta_2) \leq y^{BE}(\theta_1)$ by virtue of the fact that $y^{GF}(\theta_2) = y^*$, so $y^{GF}(\theta_1) = y^*$ by Lemma 3. The conclusion imediately fllows.

Otherwise we have $y^{GF}(\theta_2) = y^{BE}(\theta_2)$. Then we have $y^{GF}(\theta_1) \leq y^{BE}(\theta_2) \leq y^{BE}(\theta_1)$, which implies that either $y^{GF}(\theta_1) = y^{BE}(\theta_1) = y^{BE}(\theta_2)$ and the conclusion immediately follows, or $y^{GF}(\theta_1) < y^{BE}(\theta_1)$ in which case $y^{GF}(\theta_1) = y^*$ by Lemma 3, and therefore $y^{GF}(\theta_1) = y^{GF}(\theta_2) = y^*$ and the conclusion immediately follows.

The proposition follows immediately from Lemma 6. If impact investing has positive value in θ_2 (i.e. if the firm is sustainable under the grant financing regime in θ_2), then it also has positive value in θ_1. Moreover, the two firms operate at the same point in the grant financing regime. The impact investing regime must then create weakly more value in θ_1 because the impact investor is weakly less constrained.