

The Message Supports the Medium*

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Abstract

This paper examines the simple idea that willingness to provide information provides information. We study a model where competing senders can send a noisy message to a receiver, and find that there exists a separating equilibrium where senders' identity is revealed by their willingness to send the noisy message. Interestingly, the content of the messages is ignored by the receiver in such a signaling equilibrium; however, it plays a central role by shaping her beliefs off the equilibrium path. This separating equilibrium exists even when the senders face the same costs and revenues, and the messages are only slightly informative (i.e., their precision is low). When the senders are allowed to determine the precision of the information (either by affecting the precision of the message or by sending multiple messages) the signaling solution holds for any set of parameters. The theory helps shed new light on some long-standing puzzles concerning advertising.

Key words: Information, signaling, advertising.

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1. Introduction

In many economic settings, agents attempt to communicate information about themselves to others using messages that are not precise (i.e., communication is noisy). For example, this paper is one form of communication – it represents an academic study. Obviously, the paper is a noisy message of the study. A presidential debate is also a noisy message about the identity of the candidates. And, advertising by firms may contain some information, but is widely considered to be quite noisy.

As long as the messages are not completely noisy (i.e., their precision is not zero), a sender’s willingness to send a message might serve as a signal on her identity. For example, in 2000 when George Bush was reluctant to accept the format of the debate suggested by the Commission on Presidential Debates, prominent Republicans were worried that his unwillingness would hurt his chances in the elections (*New York Times*, September 10, 2000).

One of the main aims of this study is to formally examine the basic idea that willingness to provide information can provide information, and to explore its implications. We present three models. The simple model that we study in section 2 does not fit neatly into any of the real-world examples above, yet. In this model, two senders (of different types) court a receiver. The senders can send a message that noisily conveys information about their type. The receiver prefers one type over the other, but does not know the types associated with each sender. In all other respects, senders are symmetric: the cost of sending a message, and the revenues from being chosen by the receiver, are the same for both senders. Senders simultaneously decide whether or not to send messages, and the receiver then selects one of the senders.

We find that if the cost parameter falls in a specific interval, there exists a unique separating sequential equilibrium where senders’ types are completely revealed by their actions. Specifically, one sender sends a message and his competitor does not, and the receiver (optimally) selects the one that sent the message. In other words, the content of the message is ignored by the receiver in equilibrium. This means that even if the content of the (noisy) message indicates that the sender is *not* the type that the receiver prefers, she still selects him. At the same time, the content of the message is not superfluous: it affects the receiver’s beliefs off the equilibrium path and, as a result, supports the separating equilibrium. We show that senders’ actions (willingness to provide information) can serve as a signal only if the messages are not completely noisy (i.e., their precision is not zero).

An interesting feature is that a separating equilibrium exists despite the fact that, unlike canonical games of signaling and cheap talk, senders face the same costs and revenues. The reason that the conditions required to support such an equilibrium appear to be weaker than in these other settings can be traced to the informative nature of the messages themselves.

While in the simple model of Section 2, senders are allowed to send at most one

message with predetermined precision, we extend the model in section 3 to allow senders to determine the precision of their message or to choose any non-negative number of messages.

These extensions are intended to enrich the model with more realistic assumptions and to examine the robustness of the results. In real-world communications senders are usually not restricted to send at most one message with predetermined precision. In the presidential debates example, a candidate might prefer a more flexible form in order to allow the voters to get more precise information about him. On the other hand, a candidate might give vague answers in order to keep voters (relatively) uninformed. In the advertising example, firms can decide both on the precision of their ads and on the number of exposures.

These extensions demonstrate the robustness and generality of the basic idea that willingness to provide information is a signal. We find that a separating equilibrium – in which senders’ types are completely revealed by their actions – exists for any set of parameters. Furthermore, in the multiple messages model we find that the separating equilibrium exists even when the predetermined precision is very low. However, when the messages are too noisy to be informative, this separating equilibrium collapses. This result implies that messages with “little information” have sharply different effects than those with “no information”. This finding has important implications for the application studied in section 4 – the advertising market.

Section 4 focuses attention on two interesting issues about advertising (1) how advertising works, and (2) the effectiveness of ads with “little information”. Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986a) addressed these issues by demonstrating that, for experience goods, in equilibrium ad spending signals product quality. The solution offered by Becker and Murphy (1993) is based on the assumption that ads and products are complements.

We show that the theory presented here can shed new light on these two phenomena. The ingredients of such a theory are as follows: there are two consumers, two products, and two media channels. Consumer preferences over product attributes are correlated with their choice of media channel. Consumers are uncertain about product attributes. And, firms can send ads through each media channel, where ads are noisy signals on product attributes. In this setting, there exists a separating equilibrium where each firm sends ads only through the media channel consumed by the individual that prefers its product. The key point is that the media selection by firms for their ads (i.e., targeting) serves as a signal of their product attributes.

This result immediately addresses the first issue – that is, consumers should respond positively to advertising exposures because these serve as a signal about the positive match between their tastes and the attributes of the promoted product. Furthermore, it also sheds new light on the role of ads with “little information”. Interestingly, previous theories of

non-informative advertising were motivated by the fact that “a nontrivial amount of advertising (especially on television) has little or no obvious informational content” (Milgrom and Roberts, 1986a). These studies interpret “little information” as “no information.” However, we show that even when ads are very noisy (“little information”) the separating equilibrium exists while when ads are too noisy to be informative (“no information”) the equilibrium collapses.

In this section we also describe the differences between the models in Kilhstrom and Riordan (1984) and Milgrom and Roberts (1986a; hereafter we refer to these papers as KM) and that suggested here. One of them is that while in KM ad spending is a signal of product quality, in the model here the media selection is a signal of product (horizontal) attributes. Furthermore, while in KM, ad content is irrelevant, in our setting the effectiveness of messages as a signal depends on their content.

The model can also shed some light on a recent trend among advertisers – personalization of ads. The distinction between targeting and personalization is simple. Personalization means that the firm informs the recipient that she is being targeted. For example, Google and Amazon make an effort to ensure that the recipient of an ad knows that the ad was designed for people like her. Indeed, the model demonstrates that the effectiveness of advertising is stronger when the receiver is aware that the firms (a) know her taste and (b) base their targeting strategies on this.

This study is related to previous work on signaling and on strategic communication. These two lines of work explore different avenues of information transmission. Signaling games focus on *indirect* information transmission in which the receiver infers the sender’s private information from her actions. Studies of communication focus on *direct* information transmission. They examine how different mechanisms of communication affect the amount of information revealed in equilibrium. Like some studies of strategic communication, the model presented here allows for both direct and indirect avenues of information transmission.¹

The major distinction between this study and previous work on communication lies in the nature of communication. Previous studies recognize that communication is far from perfect and offer various approaches to model it. They assume, for example, that messages need not be fully verifiable, that senders may be able only to refute claims, that senders may deceive, etc.² However, the common feature in these studies is that senders have full control over what information the receiver will *perceive*. We depart from this assumption

¹The literature on strategic communication goes back to Milgrom and Roberts (1986b), where receivers take into account the sender’s strategic incentives of what to report. As a result, receivers extract more information in equilibrium than that conveyed directly through the messages.

²For example, Okuno-Fujiwara, Postelwaite, and Suzumura (1990) and Fishman and Hagerty (1990) examine settings where fully verifiable disclosures are not feasible; Lipman and Seppi (1995) and Glazer and Rubinstein (2000) examine settings where senders can refute, but not prove, claims; and Sanichirico (2001), Deneckere and Severinov (2003) and Bull and Watson (2004) allow for deception.

and study what we believe is a central attribute of communication—miscommunication. In reality, the sender seldom knows how the information will be perceived by the receiver. For example, we can only guess how the reader understood the thirty-eight sentences prior to this one. Miscommunication might be due to various reasons such as misreading a message, misinterpreting it, not paying enough attention, or vague language.

As a result, while previous studies of communication usually focus on *what* the sender says (or does not say), we focus on how much the sender invests in improving the precision of his message. We refer to the investment in improving message precision as the “willingness to provide information.”³

Our focus on miscommunication yields interesting implications. For example, receivers ignore the content of the message in equilibrium; but, the content of the message is not redundant since it affects off-equilibrium beliefs. At the same time, the result that nature’s selection is fully revealed even when senders are restricted to low-precision messages is somewhat similar to Lipman and Seppi (1995). Their work launched the literature on communication games with “partial provability”—that is, where a sender “may be able to prove only some, but not all, of what he knows”. The setting of Lipman and Seppi (1995) is quite different than ours not only with respect to the method of information transmission: messages are costless and senders move sequentially. They show that the receiver makes full, correct inferences even when the ability of senders to refute others’ claims is rather weak.

As pointed above, this study is also related to work on signaling. Its focus on noisy communication leads to some important differences from the canonical signaling game. First, previous studies require that either the cost and benefit of senders be correlated with their type in order to induce separation; in our model, separation occurs although the cost and benefit are the same for both types. Second, standard signaling games focus attention on one sender.⁴ Of course, this does not mean that the receiver faces only one sender, but it does mean that senders do not compete. However, as the earlier examples illustrate, competition between senders is often important in practice.

2. A simple model

The purpose of the simple model is to offer a formal foundation for the basic idea – that “willingness to provide information provides information”.

³The setup of our model is different in other aspects such as the cost of sending a message. Previous studies typically assume that sending a message is costless. Exceptions are those that focus on deception in which the cost of a non-truthful message is higher than the cost of a truthful one; see, for example, Sanchirico 2001 and Deneckere and Severinov 2003. One similarity between our setup and that of some communication studies is the existence of multiple senders. A nice application is Daughety and Reinganum 2000.

⁴An exception is Hertzendorf and Overgaard (2001).

2.1. Model setup

Players and types: Two senders (denoted by s where $s = \{1, 2\}$) court a receiver, who will select one of them. There are two types of senders, denoted by H and L . Nature selects (with probability of 0.5) one of the senders to be of type H and the other of type L . Let s_H denote the sender who was selected by nature to be of type H and the other sender by s_L .

Payoffs: The receiver’s payoff depends on the selected sender’s type, which is unknown to her. Specifically, her payoff from sender H is 1, and from sender L is 0. The distinction between L and H only means that the two senders are not identical from the receiver’s point of view: one of them is better than the other. The payoff of the selected sender is 1 irrespective of its type.

Message technology: Each sender may send a (noisy) message to the receiver about nature’s choice. Let the binary variable $m_s = \{1, 2\}$ represent the content the message sent by s , where $m_s = s_H$ with probability q , and $m_s = s_L$ with probability $1 - q$. We assume that $q \neq 0.5$, and for simplicity $q > 0.5$. Notice that any $q \neq \frac{1}{2}$ is informative. Here, we consider the natural case of $q > \frac{1}{2}$, which means that “correct messages are more likely than incorrect messages”. Another interpretation of this technology is as a “test” that the senders can voluntarily take, where the precision of the test in determining the senders’ type is q . The cost of sending a message is $c > 0$.

In this simple model the senders are allowed to send at most one message with a predetermined precision. These assumptions are relaxed in the next section.

Sequence of events: Senders act first: each simultaneously decides whether to send a message or not. This choice by sender s is represented by the binary variable a_s . It is equal to one if sender s chooses to send a message and zero otherwise. After receiving the messages, the receiver selects one of the senders. The binary variable d_s represents her choice: it is equal to 1 if she chooses sender s , and zero otherwise.

Subsection 2.3 presents the signaling properties of this model. It is shown that when the receiver incorporates information about the senders’ *actions* in her decision, these actions completely reveal, in equilibrium, the senders’ type. In order to provide initial insight into this model, however, we first solve it under the assumption that the receiver selects a sender based only on the content of the messages, m_1 and m_2 . We refer to this solution as the “non-strategic” one.

Throughout, the analysis focuses only on pure strategies.

2.2. Non strategic solution

We start by presenting the decision rule of the receiver and then describe the senders’ strategies.

2.2.1. Receiver's decision rule

When the receiver bases her expectations only on the content of the messages, it is easy to show, using Bayes rule, that from her point of view the probability that $s_H = s$ (denoted by μ_s^0) is:

$$\mu_s^0 = \left[1 + \left[\frac{1-q}{q} \right]^{h_s} \right]^{-1} \quad (2.1)$$

where h_s represents the number of messages indicating that $s_H = s$ minus the number of messages indicating otherwise. Formally, $h_s = \sum_{i: a_i=1} (2I\{m_i = s\} - 1)$, where $I\{\cdot\}$ is the indicator function.⁵

The receiver's decision rule turns out to be quite simple: She selects the sender for which h_s is positive. If h_s is equal to zero (which occurs either when neither sender sends a message, or when $m_1 \neq m_2$ so that the receiver cannot discriminate between senders anyway), she selects one of the senders randomly with probability 0.5.

Notice that, according to this decision rule, the sender that does *not* send a message may still be chosen: this follows from the fact that messages are noisy and receivers are non-strategic.

2.2.2. Senders' equilibrium strategies

From the senders' point of view h_s is a random variable, denoted by \tilde{h}_s . The objective function of the senders is: $E(d_s|a_1, a_2) - ca_s$. Since $q > 0.5$, it is easy to show that, in equilibrium, s_L does not send a message. And, s_H sends a message if $c < q - 0.5$.

The probability that the receiver selects H is q which is lower than 1.

This result clarifies our interest in the signaling aspect of this game. The receiver might want to base her decision on the senders' action (rather than just the content of the message), since these actions may convey information about the sender's type and thereby resolve the uncertainty that she faces.

2.3. A separating equilibrium

2.3.1. Existence

Here, we examine the case where the receiver incorporates both the content of the messages m_s and the senders' actions a_s when forming her beliefs on the sender's type. We show that there exists a sequential equilibrium, in which (a) only H sends a message, and (b) the receiver is not uncertain about the type of each sender.

To begin with, we specify the beliefs of the receiver at each of her four information sets.

⁵ $I\{\cdot\}$ equals one if the expression in brackets is true, and zero otherwise.

$$\mu_s = \begin{cases} 1 & \text{if } A_s = (1, 0) \\ 0 & \text{if } A_s = (0, 1) \\ \mu_s^0(h_s) & \text{otherwise} \end{cases} \quad (\text{B})$$

where $A_s = (a_s, a_{3-s})$. That is, if **only** one sender sends a message, the receiver believes that the sender is H . In other cases, the receiver relies on the information content in the messages (where relevant) to form her beliefs, as given in (2.1).

Given these beliefs, the following table represents the payoffs of both types of senders.

		H	
		$a = 0$	$a = 1$
L	$a = 0$	0.5	$1 - c$
	$a = 1$	0	$q - c$
		$a = 0$	$a = 1$
		0.5	0
		$1 - c$	$(1 - q) - c$

Proposition 2.1. *When $0.5 > c > 1 - q$, there exists a sequential equilibrium where beliefs are given by B , $a_{sH} = 1$, and $a_{sL} = 0$.*

Proof. Given beliefs B , $a_{sH} = 1$ is a dominant strategy for H ($1 - c > 0.5$ because $0.5 > c$ and $q - c > 0$ since $q > 0.5 > c$). Given $a_{sH} = 1$, $a_{sL} = 0$ yields a higher payoff than $a_{sL} = 1$ ($(1 - q) - c < 0$ because $c > 1 - q$).

It is straightforward to show that beliefs are consistent, given these strategies of the senders. ■

The intuition behind this result is simple. Because the benefit and cost are the same for both types of senders, if one of them does not send a message, it is optimal for its competitor to send one. However, if one of them sends a message, it is optimal for the other sender to do the same only if the other is H . The reason for this distinction between the senders lies in the fact that the off-the-equilibrium beliefs rely on the content of the messages. The content of the messages are more likely to favor H over L . For this reason when H sends a message, L prefers not to.

In equilibrium the cost parameter has to be between the lowest and the highest probability that L is selected when the receiver bases her decision on the content of the messages.⁶

2.3.2. Discussion: The role of informative content in messages

An interesting feature of this game is that although the content of the messages is not used in equilibrium, it is not superfluous. In equilibrium the receiver gets only one message.

⁶This interval is at a higher cost level than the interval for the non-strategic equilibrium. This is not surprising: in the non-strategic equilibrium the cost should be low enough for H to send a message. In the separating equilibrium, it should be high enough to deter L from mimicking.

This message will indicate that the sender of the message is L with probability $1 - q$. Even in such a case, the receiver will select this sender. In other words, the sender is selected by the receiver even if the realization of the message was negative from the sender’s point of view. However, this result does not imply that the content of the messages is redundant: unless the receiver is using the content of the messages off-the-equilibrium there is no way to deter L from imitating H . In other words, the content of messages supports the particular equilibrium by shaping off-equilibrium beliefs.

An additional interesting feature is that a separating equilibrium exists despite the fact that, unlike canonical games of signaling and cheap talk, senders face the same costs and revenues.⁷ The reason can be traced to the information content of the messages themselves, and, once again, their role off-the-equilibrium path. In this respect, this signaling model departs from the standard one.

2.3.3. Uniqueness

So far we have shown that a separating equilibrium exists. The following proposition states that for the relevant parameter values (i.e., $0.5 > c > 1 - q$) this equilibrium is unique.

Proposition 2.2. *When $0.5 > c > 1 - q$, there exists a unique sequential equilibrium where beliefs are given by B , $a_{s_H} = 1$, and $a_{s_L} = 0$.*

Proof. See Appendix A. ■

In appendix A we also show that a separating sequential equilibrium does not exist when $q = 0.5$. In other words, while for $q = 0.5 + \varepsilon$ this separating equilibrium exists, for $q = 0.5$, it does not. This means that even when the messages are quite noisy, sending a message can serve as a signal on senders’ identity (for specific values of c). However, when the messages are too noisy to be informative, sending a message cannot serve as a signal on senders’ identity.

3. General models

In the simple model of the previous section, senders’ actions are limited. Each can send at most one message of pre-determined precision. In this section, senders have more flexibility: each can decide on the amount of information they transfer to the receiver. A sender can do so either by endogenously choosing the precision of their message or by sending multiple messages. This section extends the simple model to integrate these possibilities in turn.

⁷In a signaling game, a necessary condition for a separating equilibrium is that the cost of sending a message (or, more generally, choosing an action) is correlated with the sender’s type (this is the Spence-Mirrlees “single crossing property”). In a cheap talk setting—where the cost of sending messages is costless and therefore the same across senders—a necessary condition for a separating equilibrium is that the senders have different preferences over the receiver’s actions (see, for example, Crawford and Sobel 1982).

These extensions are well-suited for various applications, for example to analyze competition in advertising, where firms are not restricted to sending at most one ad with a pre-determined precision.

3.1. Endogenous precision

3.1.1. Model Setup

The setting of the model is the same as of the simple model with the following exceptions. Each sender can determine the precision of his message, $q_s \in [0.5, 1]$. Thus, for example, each sender can choose to send a non-informative message ($q_s = 0.5$), a fully-informative message ($q_s = 1$), or a noisy message at any level of precision between 0.5 and 1. The cost of producing a message is a function of its precision, $C(q)$, with $C(0.5) = 0$. We assume that the cost function is increasing in q ($C' > 0$) and convex ($C'' > 0$). Thus, it is difficult to make an already informative message more precise.

The setting of the model is common knowledge. The receiver observes the q_s chosen by each sender but is uncertain about nature's selection. This assumption reflects a receiver's ability to identify the amount of information revealed in a message. For example, when one of the candidates in a presidential debate gives a vague answer, voters can identify his reluctance to provide information.

The profit of firm s is $\pi_s(q_s) = d_s - C(q_s)$.

As in the previous section, we start with the non-strategic equilibrium and then demonstrate the existence of a separating equilibrium where the senders' decisions on the precision of their messages serves the receiver as a signal about their type.

3.1.2. Non strategic equilibrium

We start by demonstrating that in the non-strategic equilibrium firm s_H sends an informative message ($q_{s_H} > 0.5$) and its competitor does not ($q_{s_L} = 0.5$).

The individual's decision A non-strategic receiver updates her belief that $s_H = s$ using only the content of the message (and not the choice of precision by senders). The probability that $s_H = s$, denoted by μ_s^0 , is given using Bayes rule as:

$$\mu_s^0 = \left[1 + \left[\frac{1 - q_s}{q_s} \right]^{(1-2|s-m_s|)} \left[\frac{1 - q_{3-s}}{q_{3-s}} \right]^{(1-2|s-m_{3-s}|)} \right]^{-1}$$

As before, the receiver's decision rule is quite simple. If $q_s \neq q_{3-s}$ her decision is based on one message only—the more precise one. Specifically, she selects the sender that the more precise message identifies as H . If $q_s = q_{3-s}$, her decision is based, as before, on h_s (i.e.,

$h_s = \sum_i (2I\{m_i = s\} - 1)$). She selects s if $h_s > 0$, and his competitor if $h_s < 0$. And if h_s is equal to zero, she selects one of them randomly with probability 0.5.

Senders' strategies > From the senders' point of view m_s and m_{3-s} are random variables.

Let $E[\pi_s(q_s); q_{3-s}]$ denote the expected profit function for s if it sends a message with precision q_s and its competitor sends a message with precision q_{3-s} . This expected profit function is

$$E[\pi_s(q_s); q_{3-s}] = E(d_s | q_s, q_{3-s}) - C(q_s). \quad (3.1)$$

Let $q_{s_L}^{ns}$ and $q_{s_H}^{ns}$ denote the senders' strategies in the non-strategic equilibrium. The following proposition characterizes this equilibrium.

Proposition 3.1 (Non strategic equilibrium). *In any non-strategic equilibrium, s_L sends an uninformative message ($q_{s_L}^{ns} = 0.5$), and s_H sends a message with precision strictly greater than 0.5 as long as $C'(0.5) < 1$.*

Proof. We start by demonstrating that $q_{s_L}^{ns} = 0.5$ in equilibrium. To see why, note that if s_L sends a message with precision $q_{s_L} \leq q_{s_H}$, his expected revenue is $1 - q_{s_H}$.⁸ And, if s_L sends a message with precision $q_{s_L} > q_{s_H}$, his expected revenue is $1 - q_{s_L}$, which is lower than $1 - q_{s_H}$. Thus, his expected revenue is a non-increasing function in q_{s_L} , while his cost is an increasing function in q_{s_L} . It follows that $q_{s_L} = 0.5$ is a dominant strategy for s_L .

Next, we show that if $C'(0.5) < 1$, $q_{s_H}^{ns} > 0.5$.⁹ Since $q_{s_L}^{ns} = 0.5$, it is easy to show that $E(d_{s_H} | q_{s_H}, 0.5) = q_{s_H}$ and thus:

$$\frac{\partial E[\pi_{s_H}(q_{s_H}); q_{s_L}]}{\partial q_{s_H}} = 1 - C'(q_{s_H})$$

Since $C'' > 0$ and $C'(0.5) < 1$, it immediately follows that $q_{s_H}^{ns} > 0.5$. Specifically, $q_{s_H}^{ns}$ satisfies the following condition:

$$1 - C'(q_{s_H}^{ns}) = 0$$

■

⁸If $q_{s_L} < q_{s_H}$, the receiver bases her decision only on m_{s_H} . The probability that $m_{s_H} = s_L$ is $1 - q_{s_H}$. Thus, $E(d_{s_L} | q_{s_L}, q_{s_H}) = 1 - q_{s_H}$.

If $q_{s_L} = q_{s_H}$, the probability that $m_{s_H} = m_{s_L} = s_L$ (in this case, the receiver selects s_L) is equal to $(1 - q_{s_H})^2$ and the probability that the messages contradict one another (in this case the receiver selects one of the products randomly) is $2q_{s_H}(1 - q_{s_H})$. Thus, $E(d_{s_L} | q_{s_L}, q_{s_H}) = (1 - q_{s_H})^2 + \frac{1}{2}2q_{s_H}(1 - q_{s_H}) = (1 - q_{s_H})$.

⁹Notice, for example, that if $C(q) = c(q - 0.5)^2$, then $C'(0.5) = 0$ for any cost parameter.

3.1.3. Strategic equilibrium

Here, we examine the case where the receiver incorporates in her decision both the statistical information that is revealed in the content of the messages, and the signaling information that is revealed by their precision. We show that in equilibrium, only one sender sends an informative message. In addition, however, in equilibrium the receiver has no uncertainty about nature's selection.

We start by specifying the receiver's beliefs and senders' strategies, and then show that these beliefs and strategies are part of a separating equilibrium. In this equilibrium, the precisions of the messages chosen by the senders are, respectively, $q_{s_H}^* > 0.5$ and $q_{s_L}^* = 0.5$.

Denote as $\mu_s^1(\bullet)$ the posterior probability function that $s_H = s$. Then, the receiver's beliefs (denoted by $B1$) are:

$$\mu_s^1(q_s, q_{3-s}, m_s, m_{3-s}) = \left\{ \begin{array}{l} 1 \text{ if } q_s = q_{s_H}^* \text{ and } q_{3-s} \neq q_{s_H}^* \\ 0 \text{ if } q_{3-s} = q_{s_H}^* \text{ and } q_s \neq q_{s_H}^* \\ \mu_s^0(m_s, m_{3-s}) \text{ otherwise} \end{array} \right\} \quad (B1)$$

where $q_{s_H}^*$ satisfies the conditions:

$$1 - C(q_{s_H}^*) \geq \max_{0.5 \leq q \leq 1} q - C(q) \quad (3.2)$$

$$\text{and } 0 \geq (1 - q_{s_H}^*) - C(q_{s_H}^*) \quad (3.3)$$

The logic behind these beliefs can be stated as follows: if the actions are consistent with equilibrium strategies $\{q_{s_H}^*, 0.5\}$, then the identities of the senders are revealed perfectly. The same holds true if only one of them follows the equilibrium strategy of H (i.e., $q_s = q_{s_H}^*$ for only one sender). When neither or both senders choose $q_{s_H}^*$, then the receiver bases her expectation only on the statistical information revealed via the messages, and not on senders' strategies.

The precision of the message sent by H in equilibrium should satisfy the two inequalities (3.2) and (3.3). The first inequality ensures that H would not like to deviate from his choice of precision in equilibrium, and the second ensures that L would not like to imitate H . Notice that in equilibrium the profit of s_H is $1 - C(q_{s_H}^*)$, and the payoff of s_L is 0. If H deviates, it provokes the off the equilibrium beliefs and the receiver bases her decision on the content of the message that is sent. In this case, s_H 's payoff is $\max_{0.5 \leq q \leq 1} q - C(q)$. If L imitates H , it provokes the off the equilibrium beliefs and his payoff is $(1 - q_{s_H}^*) - C(q_{s_H}^*)$.

Thus, a necessary condition for the existence of a separating equilibrium is that there exists a $q_{s_H}^*$ that satisfies both inequalities. Define \underline{q} and \bar{q} as the q 's that satisfy the two conditions (3.2) and (3.3), respectively, with equality. Lemmas B.1 to B.3 in Appendix B

show that the interval $[\underline{q}, \bar{q}]$ is interior and non-empty; that is, $0.5 < \underline{q} < 1$, and $\bar{q} > \underline{q}$. These ensure that any q^* such that $\underline{q} < q^* < \bar{q}$, can be supported in a perfect Bayesian equilibrium. This yields the main result:

Proposition 3.2 (PBE). *There exists a perfect Bayesian equilibrium where beliefs are given by B1 and senders' pure strategies are $q_{sH}^* \in [\underline{q}, \bar{q}]$, and $q_{sL}^* = 0.5$.*

Proof. In Appendix B. ■

Thus, in equilibrium H sends an informative message and his competitor does not, all uncertainty about nature selection is resolved, and the receiver chooses sender H with certainty.

This proposition illustrates the robustness of the result that a sender's willingness to provide information can serve as a signal. In other words, the result still obtains when senders are allowed to determine the precision of their messages. Furthermore, the conditions for such a solution are less restrictive than before. Whereas in the simple model, a separating equilibrium existed only for a specific interval of the cost function, here the signaling equilibrium exists for any parameter value.

The logic of the proof is quite simple. When the precision is endogenous, q can be set high enough in order to deter L from imitating H . Indeed, this is the case for any $q > \underline{q}$. The main challenge is to show that such a precision is not too high for H . (Recall that when H deviates, he optimally chooses q_{sH}^{ns}).

When $\underline{q} < q_{sH}^{ns}$ this is trivial. In such a case, H would have been willing to pay a higher cost ($C(q_{sH}^{ns}) > C(\underline{q})$) in order to get a lower probability of being selected ($q_{sH}^{ns} < 1$).

Figure 1 demonstrates the case where $\underline{q} > q_{sH}^{ns}$. Notice when H deviates he is selected by the receiver with probability q_{sH}^{ns} . Thus by deviating he is losing a payoff of $1 - q_{sH}^{ns}$ and saving some costs $- [C(\underline{q}) - C(q_{sH}^{ns})]$. The figure shows that it is always the case that the lost payoff is higher than the saved cost. The reason is that $[C(\underline{q}) - C(q_{sH}^{ns})] < C(\underline{q})$ and when $\underline{q} > q_{sH}^{ns}$, $(1 - q_{ns}) > (1 - \underline{q})$. Since, by definition of \underline{q} , $(1 - \underline{q}) = C(\underline{q})$, we immediately get that for H the lost payoff from deviation, $1 - q_{sH}^{ns}$, is bigger than the saved cost, $[C(\underline{q}) - C(q_{sH}^{ns})]$. Furthermore, the figure also demonstrates that \bar{q} is always higher than \underline{q} .

As before, an interesting feature of the separating equilibria is that, in equilibrium, the receiver ignores the content of the message. Moreover, doing so leads her to make the best choice. The following example is illustrative. Consider a case where only s sent a message and its content indicates that nature selected him to be L (i.e., $m_s = (3 - s)$). Although the message content does not favor the sender, the receiver chooses him and her choice is optimal with certainty. Thus, in the perfect Bayesian equilibrium, the receiver always chooses the sender that gives her the highest utility. In contrast, recall that when the receiver does not behave strategically, she occasionally chooses sender L .

3.2. Multiple messages

Even when the precision of messages is predetermined, senders can affect the informativeness of their communication by sending multiple messages. In other words, senders can de facto determine the precision of the information they provide to the receiver by increasing the “sample size.” For example, in many situations ad agencies find it difficult to increase the precision of their ads, because they need to ensure that the ad is memorable, attractive, etc. However, they can overcome this difficulty by sending multiple ads. Presidential debates, where the number of debates and their length vary, can serve as an additional example.

Appendix C describes the model and its solutions. This subsection outlines the main results.

The setting of the model is similar to the simple model in section 2, with the following exception: each sender can determine the number of messages $N_s \geq 0$. In other words, each sender can send any non-negative number of ads, and the integer N_s represents the number of messages sent by sender s . As before, the cost of sending a message is c , and thus the profit of s is $\pi_s(N_s) = d_s - cN_s$.

The results of this model are, as one might expect, very similar to the model with endogenous precision. Specifically, there exists a separating equilibrium in which H sends a positive number of messages and L does not. This is stated formally in the following Proposition.

Proposition 3.3 (PBE). *For $q > 0.5$, there exists a perfect Bayesian equilibrium in which L does not send any messages and H sends a strictly positive number of messages (i.e., $N_{s_H}^* > N_{s_L}^* = 0$).*

Proof. In Appendix C. ■

This model reveals an additional insight of interest: that is, “little information” in ads can have quite different consequences than “no information”. Specifically, we show in appendix C that the separating equilibrium exists even when the predetermined precision is arbitrarily small (i.e., $q = 0.5 + \varepsilon$). Intuitively, even when the available messages are very noisy, H can differentiate itself from L by sending enough messages. However, when the messages are completely uninformative (i.e., $q = 0.5$) the separating equilibrium collapses and H cannot differentiate itself from L . This discontinuity at $q = 0.5$ illustrates the difference between ads having “little information” and “no information”. This can shed light on an important application of this model, advertising, that we discuss in the next section.

4. Discussion: An application to advertising

The central idea of this paper—that a sender’s “willingness to provide information provides information”—can shed new light on two interesting phenomena about advertising. We discuss these here.

Two Phenomena How advertising works: The first phenomenon concerns “how advertising works.” Economists have long been interested in: “. . . why advertising might affect customers’ choices and thus of why firms might choose to advertise.”¹⁰ The common modeling approach is to include exposure to ads in the utility function.¹¹ However, if advertising simply has a direct effect on the utility, then ads can be claimed “. . . to create wants and to change and distort tastes.” (Becker and Murphy 1993). Thus, economists have suggested alternative explanations.

Becker and Murphy (1993) have justified this direct effect by suggesting that ads and the goods advertised can be complements. A different approach was presented by Nelson (1974), and formalized in Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986a), who suggested that in equilibrium ad intensity can signal product quality.

Although novel in their explanation, each approach has also come under criticism. The Becker-Murphy theory can be viewed as de facto assuming that ad intensity stimulates the tendency to buy the product. The signaling explanation has also disturbed commentators, for a different reason. According to this view, advertising is no different from any other activity—including, for example, burning money—that can also serve as a quality signal. Accordingly, critics note that that there would surely appear to be other, more productive, avenues to burn money. Furthermore, Becker and Murphy (1993) point out: “. . . the pure signaling interpretation implies that companies should advertise how much they spend on advertising, yet almost no companies do that.”

Ads with little information: A second, and related, phenomenon about advertising has to do with the perceived lack of information in advertisements. Interestingly, both the explanations described above—of “advertising as complements” and “advertising as signals”—was motivated by the observation that most ads do not appear to contain much information. Thus, Milgrom and Roberts (1986a) appeal to the fact that “a non-trivial amount of advertising . . . has little or no obvious informational content.” And, Becker and Murphy (1992) note as well that “it is ‘obvious’ that many ads provide essentially no information.”¹²

As said, the central idea presented in this paper—that a sender’s “willingness to provide

¹⁰ Milgrom and Roberts 1986.

¹¹ Indeed, empirical support for the direct effect of advertising is present even after one accounts for the informative content of advertising; see Anand and Shachar (2004).

¹² See also Bagwell and Ramey (1994).

information provides information”— can perhaps shed new light on these two phenomena. Previous sections demonstrated a simple but formal game-theoretic foundation for this idea. Applying the model to a real-world setting like the market for advertising, however, requires that it be extended and more thoroughly justified in specific ways.

Setup of an Augmented Model To do so, consider an augmented version of the model with multiple messages that has the following ingredients: (a) there are two firms, and two individuals, (b) there is product differentiation and consumer heterogeneity (specifically, the utility of one individual is positive from one product and negative from the other, and the tastes of the other individual are the opposite),¹³ (c) consumers are uncertain about products’ attributes, (d) ad content is a noisy message on products attributes, (e) firms know consumers’ preferences, (f) there are two media channels, and (g) each individual is exposed only to one media channel, and each media channel is consumed only by one individual. In other words, the two individuals differ in their preferences over products and media channels.

While assumptions (a)-(c) are quite standard, the other assumptions require additional justification for this particular application. Several stylized facts might be offered as evidence for each of these assumptions.

First, consider the assumption that ad content is a noisy message on product attributes. One method of measuring the amount of information in advertising is “content analysis”, and has been provided in Resnik and Stern (1977). This method involves determining what types of information are presented in an ad. Resnik and Stern presented 14 information categories or “cues”, such as price, quality, performance etc. (The relevant questions about performance, for example, were: “What does the product do and how well it do what it is designed to do in comparison to alternative purchases?”) Many studies followed this method and Abernethy and Franke (1996) summarize their results in a meta-analysis. They found that 84 percent of all the ads (91,438) have had at least one cue.¹⁴

Despite information content in ads, it is also fair to say that information in ads is noisy. In a series of studies, Jacoby and Hoyer (1982 and 1989) demonstrated that about one-third of all ads are miscomprehended by consumers. Consequently, it is natural to formulate advertising as a noisy signal on product attributes.¹⁵

¹³We also assume, for simplicity that: (1) The cost of producing a product is zero, (2) the price of the products is determined exogenously at 1, (3) the utility of the individual already accounts for the products’ prices, and (4) The consumer’s utility from the outside alternative is zero. Thus, she will always prefer to buy a product.

¹⁴Furthermore, 58 percent have had at least two ads and 33 percent have had at least 3. While newspapers appear to include more informative advertising (98 percent of them have had at least one cue), ads in television are also quite informative (71 percent). The product category that was the most informative was automobile (97 percent) and furniture/home furnishings/appliances (96 percent).

¹⁵In 1979, the Educational Foundation of the American Association of Advertising Agencies surveyed nearly 2,700 consumers about the content of 60 thirty-second televised communications—including ads,

Second, consider the assumption that firms know consumer preferences. Indeed, firms often spend large amounts of money analyzing the markets in which they operate, and the tastes of consumers. Furthermore, research firms such as “Mediamark Research Inc” collect individual level data on consumers that portraits their demographics, life style, consumption and media habits. Consequently, firms have a good assessment of these preferences based on individuals’ demographics, life styles and purchase histories.

Third, consider the assumptions about media channels. In practice, there are many classes of media—for example, television, radio, newspapers, magazines, billboards, and direct mail—and many categories within each media class. For example, on television there are morning shows, daytime shows, prime time shows, late night shows, etc. Even within each time slot category, there exist various options to advertisers. For example, within prime time, there are shows that appeal mostly to men (like *Monday Night Football*) or shows that appeal especially to women, such as romantic dramas with a feminine cast. Thus, media channels, within-channel categories, and within-category classes, differ in their audience composition. (For example, while Fox’s audience is relatively young, CBS’ audience is relatively old). Furthermore, as the number of media channels has grown in the last decade, and the audiences have become more fragmented, each of these channels has ended up serving more specific interests.

In addition to multiple media channels, firms have multiple data sources that characterize the audiences of the different media channels, and these characterizations are rather detailed. For example, such information includes not only the demographics of the audiences, but their lifestyles and consumption habits as well.¹⁶ These observations motivate the assumption that firms are aware of the media consumption of individuals.

Model Solution Now, it is easy to show that the main result of the model with multiple messages (section 3.2) immediately applies here. Specifically, there exists a separating equilibrium where each firm sends ads only through the media channel consumed by the individual that prefers its product. All uncertainty about product attributes is resolved through the media selection of the firms, and each individual chooses the product that is “right” for her with certainty. The specifics of this equilibrium are presented and discussed

public service communications, and editorial content. They found that about 29 percent of the communications (ads or other content) were miscomprehended by consumers, as measured by a particular series of true-false questions (Jacob Jacoby and Wayne Hoyer [1982]). A second study was conducted a few years later and included print (magazine) ads and editorial content. In this study, about 1,250 consumers were asked questions about fifty-four full-page magazine ads and another fifty-four editorial pages. Roughly 20 percent of the material was miscomprehended, with another 15 percent being simply “not understood” (Jacob Jacoby and Wayne Hoyer [1989]).

¹⁶Such data sets are offered, for example, by research firms like the Nielsen Research company, Information Resources, Inc, and Simmons Market Research Bureau.

in Anand and Shachar (2000).¹⁷ The key point is that the decision by firms to *target* their ads to particular media channels serves as a signal of their product attributes.

Discussion and Implications Why is this result of interest? First, consider the implications for the effectiveness of targeted advertising. The importance of precise targeting is not new. And, it has been previously shown that firms should (and do) target their ads to consumers who, a priori, have a higher tendency to consume the promoted product. The added value of the suggested model is that firms not only inform the appropriate consumers by targeting them with their ads, but also send a signal about their product attributes. Specifically, the consumer knows that in equilibrium it is optimal only for the firm who offers the products that best fits her taste to advertise in the media channels that she is exposed to. Thus, when she decides between two firms—one that advertised in the media channels that she is exposed to and the other that did not—she would have a higher tendency to purchase the product for which she had seen ads. The consumer’s logic can be stated as follows: “if this firm is willing to pay the cost to inform *me* about its product, the product’s attributes must suit *my* tastes better than those of the competitors.”¹⁸

Notice that this also implies that consumers should respond positively to advertising exposures because these serve as a signal about the positive match between their tastes and the attributes of the promoted product. According to this explanation, the empirical regularity describes equilibrium behavior rather than primitives of consumer preferences.

Second, the model can also shed some light on a recent trend among advertisers – personalization of ads. Consider the new mail service offered by google – Gmail. This service includes a memory of one gigabyte and “no spam”. Instead, google guarantees that commercial mail will be directly linked to the interest of the recipient. To achieve this, the firm scans all the personal mail received by the client and determines her area of interest. This strategy is different from simply targeting. Targeting does not guarantee that the recipient knows that she is targeted. Personalization, which is what Gmail does, makes sure that the recipient is aware that the ads were designed for people like her. In the model we have assumed that the receiver knows that the firms know her taste and base their targeting strategies on this. Personalization ensures that this is indeed the case.

Third, the model suggests why ads with “little information” can be effective. As discussed previously, ad agencies are required to make sure that their ads are memorable,

¹⁷Note that the model with multiple messages immediately applies here because the two markets are separable.

Anand and Shachar (2000) also study the case where prices are determined endogenously. In this case, the two markets are not independent. However, the separating result still holds.

¹⁸That is, following Marshall McLuhan’s known phrase, we find that the medium is more important than the message. But, it turns out that there is more than that. The information content in ads is not superfluous: *some* information content in ads is necessary for firms to reveal themselves through their decisions on which media channels to air their ads in. In other words, the message supports the medium.

attractive etc. But, making an ad memorable and attractive may often constrain the precision of each specific ad. Thus, it is likely that many ads would have only “little information”. Previous studies have interpreted “little information” as “no information”. But, the findings here indicate that ads with “little information” may have quite different effects than ads with “no information”. Specifically, when $q = 0.5$, there is no separating equilibrium and consumers cannot use ad as a signal; in contrast, when $q = 0.5 + \varepsilon$ a separating equilibrium exists, and consumers should respond positively to advertising exposures.

Fourth, the model is robust to each of the critiques of the standard signaling model of advertising described earlier. Recall that these critiques include the fact that in the model of “advertising as signals”, (a) the content of ads is irrelevant, (b) the effectiveness of ads does not depend on ad exposure, and (c) advertising is no different an activity than is the burning of money by firms. In contrast, in the model presented here, the effectiveness of a message as signals depends on the message content. As a result, none of these critiques applies any longer.

Finally, our model implies that advertising improves the matching between consumers and products and thus has positive welfare implications.¹⁹

5. Conclusion

This paper examines the simple idea that willingness to provide information is a signal. We study a model where competing senders can send a noisy message to a receiver, and find that there exists a unique separating equilibrium where senders’ identities are revealed by their willingness to send the noisy message. This result holds even when the senders face the *same* costs and revenues, and the messages are only slightly informative (i.e., their precision is low). Interestingly, the content in the message is not used by a receiver in such a signaling equilibrium; however, it shapes her beliefs off the equilibrium path.

When the senders are allowed to determine the precision of the information (either by affecting the precision of the message or by sending multiple messages) the signaling solution holds for any set of parameters.

When applied to the advertising market, the model suggests that the media selection of firms (i.e., targeting) can serve consumers as a signal of products’ attributes. Firms’ media selection, rather than the content in the ads, may itself reveal information about product attributes. That is, following Marshall McLuhan’s known phrase (“the medium is the message”), we find that the medium is more important than the message. But, it turns out that there is more than that. The information content in ads is not superfluous: *some* information content in ads is necessary for firms to reveal themselves through their

¹⁹The matching role of advertising is examined empirically in Mitra and Lynch (1996) and Anand and Shachar (2004).

decisions on which media channels to air their ads in. In other words, the message supports the medium.

This result can shed new light on two interesting phenomena about advertising. The first is the simple empirical regularity that the tendency to purchase a product is increasing in exposures to ads. The logic is that a person who is exposed to ads knows that the firm targeted her. Thus she correctly concludes that the promoted product suits her taste, and purchases it.

The second empirical regularity upon which the model can shed new light is that some ads contain only “little information”. This stylized fact motivated previous models of advertising which assumed that ads contain “no information”. But, the findings here indicate that ads with “little information” may have quite different effects than ads with “no information”. Specifically, when ads have “no information”, there is no separating equilibrium and consumers cannot use ad as a signal; in contrast, when ads have arbitrarily “little information” a separating equilibrium exists.

Although both previous papers (Kihlstrom and Riordan [1984] and Milgrom and Roberts [1986a]) and the model here imply that ads are signals, there are various significant differences between the two approaches. First, while in this prior literature, ad spending is a signal of product quality, in the model presented here the media selection is a signal of product (horizontal) attributes. Second, while in the traditional models of advertising as signals: (a) ad content is irrelevant, (b) the effectiveness of ads does not depend on ad exposure, and (c) advertising is no different than “burning money,” in the suggested model the effectiveness of messages as signals depends on the message content. Thus, in our setting, the type of message chosen is not arbitrary.

The model suggests an interesting distinction between targeting and personalization of ads. Personalization means that the firm informs the recipient that she is being targeted. The assumption that the consumer knows that firms know her tastes, and target their ads accordingly, is critical in the model — without it, targeting cannot serve as a signal. This might explain a recent trend among firms to personalize their ads. For example, Google and Amazon make an effort to ensure that the recipient of an ad knows that the ad was designed for people like her. The prediction that strategies of targeting versus personalization might have different effectiveness lends itself easily to experimental testing.

The model presented here might be relevant for additional applications. However, to achieve this some assumptions need to be relaxed. For example, in some cases, (e.g., testimony by defendants in trials), the assumption of competing senders may be less relevant. In other cases (e.g., presidential debates), the competing senders should agree together on the precision of the messages (i.e., the number and format of the debates). In this case, the disagreements that typically precede the final arrangement — the “debates about debates” — may convey valuable information through signaling.²⁰ A natural application of

²⁰In the 2000 presidential campaign, for example, Gore (considered to be the stronger debater ex-ante)

this model is, obviously, a person’s willingness to take a test – for example, drug tests for athletes, agreeing to a police search without a warrant, etc.

Finally, we assumed that senders can only send messages whose content is more likely to be correct than incorrect. In other words, we do not allow senders to intentionally deceive the receiver. An interesting extension of this model is to allow senders to manipulate the message technology.

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preferred more debates. The Bush campaign preferred settings (e.g., television studios) that might reduce the size of the live audience at the debates. Further, commercials were aired, particularly by the Gore campaign, that advertised the reluctance of the other side to debate. And, ultimately, the Bush campaign agreed to all the conditions initially laid out by Commission on Presidential Debates (that Gore had agreed to three months earlier), after criticism from fellow Republicans who feared that his opposition was quickly becoming “a distraction”. Each of these aspects fits the model nicely: Despite noisy messages, senders may have different incentives to send messages; differences between senders in how many messages they choose to send may itself reveal information about the sender’s type; and, the sender of the “right type” has an incentive to advertise these differences as well (in turn addressing the Becker-Murphy criticism of signaling models, namely “why don’t firms advertise how much they advertise?”).

For more information see the following articles in the *New York Times*: “Bush, Facing Criticism, Abandons Debate Stance” (September 9, 2000); “One Debate Down, Three to Go” (September 10, 2000); “Dropping All of His Objections, Bush Agrees to Panel’s Debates” (September 15, 2000); and, “Candidates Agree on Formats for Three Debates” (September 17, 2000).

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Appendix

A. Uniqueness of the simple model

There are two types of separating equilibria: (1) sender's strategies depend on their type, and (2) sender's strategies do not depend on their type.

In appendix A1 we show that for the first case there is a unique sequential equilibrium (the one presented in section 2) in which $c > 1 - q$.

In appendix A2 we show that for the second case there is a unique sequential equilibrium in which $c < 1 - q$.

Thus, for $c > 1 - q$ there is a unique separating (sequential) equilibrium as stated in Proposition 2.2.

A.1. Strategies depend on senders' types

Here, we study a separating equilibrium in which sender's strategies depend on their types.

We start by characterizing the only set of consistent beliefs in such a separating equilibrium. Then we show that for these beliefs and the given parameter values, there is a unique separating equilibrium.

In any separating equilibrium, either H sends a message and L not, or the reverse is true. We consider each case in turn.

CASE 1: H sends a message, and L does not.

Let H send a message with probability $1 - \varepsilon_H$, and L send a message with probability ε_L (both ε_H and ε_L are greater than 0). Recall that the prior probability that player s is H is $\mu_s^0(h_s)$. Then (using Bayes rule) the receiver's beliefs at each of her four information sets are:

$$\mu_s^\varepsilon = \left\{ \begin{array}{l} \frac{(1-\varepsilon_H)(1-\varepsilon_L)\mu_s^0(h_s)}{(1-\varepsilon_H)(1-\varepsilon_L)\mu_s^0(h_s)+\varepsilon_H\varepsilon_L(1-\mu_s^0(h_s))} \text{ if } A_s = (1, 0) \\ \frac{\varepsilon_H\varepsilon_L\mu_s^0(h_s)}{\varepsilon_H\varepsilon_L\mu_s^0(h_s)+(1-\varepsilon_H)(1-\varepsilon_L)(1-\mu_s^0(h_s))} \text{ if } A_s = (0, 1) \\ \frac{(1-\varepsilon_H)\varepsilon_L\mu_s^0(h_s)}{(1-\varepsilon_H)\varepsilon_L\mu_s^0(h_s)+(1-\varepsilon_H)\varepsilon_L(1-\mu_s^0(h_s))} \text{ if } A_s = (1, 1) \\ \frac{\varepsilon_H(1-\varepsilon_L)\mu_s^0(h_s)}{\varepsilon_H(1-\varepsilon_L)\mu_s^0(h_s)+(1-\varepsilon_L)\varepsilon_H(1-\mu_s^0(h_s))} \text{ if } A_s = (0, 0) \end{array} \right\} \quad (\text{A.1})$$

It is straightforward to show that the limit of μ_s^ε as $\varepsilon_H, \varepsilon_L \rightarrow 0$ is:

$$\mu_s = \left\{ \begin{array}{l} 1 \text{ if } A_s = (1, 0) \\ 0 \text{ if } A_s = (0, 1) \\ \mu_s^0(h_s) \text{ if } A_s = (1, 1) \\ \mu_s^0(h_s) \text{ if } A_s = (0, 0) \end{array} \right\}$$

which are exactly the beliefs in B.

Recall that under these beliefs, the following table represents the payoff functions of both senders.

		H	
		a = 0	a = 1
L	a = 0	0.5	1 - c
	a = 1	0	q - c
		1 - c	1 - q - c

Now, one can see that when c is not in the interval $[1 - q, 0.5]$, there is no separating equilibrium in which H sends a message and L does not: (a) when $c < 1 - q$, sender L finds it profitable to imitate H ; (b) when $c > 0.5$, sender H deviates.

CASE 2: L sends a message, and H does not.

In this case, it is easy to show that the only consistent beliefs are:

$$\mu_s = \begin{cases} 0 & \text{if } A_s = (1, 0) \\ 1 & \text{if } A_s = (0, 1) \\ \mu_s^0(h_s) & \text{if } A_s = (1, 1) \\ \mu_s^0(h_s) & \text{if } A_s = (0, 0) \end{cases}$$

The following table represents the payoff functions of both senders.

		H	
		a = 0	a = 1
L	a = 0	0.5	-c
	a = 1	1	q - c
		a = 0	a = 1
		0.5	1
		-c	1 - q - c

It is clear that in this case, it is not optimal for either sender to send a message.

A.2. Strategies do not depend on senders' types

Lemma A.1. *A separating equilibrium in which sender's strategies do not depend on their type, exists if and only if $1 - q > c$.*

Proof. Without loss of generality, consider the case where sender 1 sends a message and sender 2 does not.

First, we characterize beliefs. To obtain consistent beliefs, we describe the players strategies.

Player 1 sends a message with probability $1 - \varepsilon_{1H}$ if he is H and $1 - \varepsilon_{1L}$ if he is L . Player 2 sends a message with probability ε_{2H} if he is H and ε_{2L} if he is L . Denote the prior probability that player 1 is H by p . Then (using Bayes rule) the beliefs that player 1 is H are:

$$\mu_1^\varepsilon = \begin{cases} \frac{(1-\varepsilon_{1H})(1-\varepsilon_{2L})p}{(1-\varepsilon_{1H})(1-\varepsilon_{2L})p+(1-\varepsilon_{1L})(1-\varepsilon_{2H})(1-p)} & \text{if } A_1 = (1, 0) \\ \frac{\varepsilon_{1H}\varepsilon_{2L}p}{\varepsilon_{1H}\varepsilon_{2L}p+\varepsilon_{1L}\varepsilon_{2H}(1-p)} & \text{if } A_1 = (0, 1) \\ \frac{(1-\varepsilon_{1H})\varepsilon_{2L}p}{(1-\varepsilon_{1H})\varepsilon_{2L}p+(1-\varepsilon_{1L})\varepsilon_{2H}(1-p)} & \text{if } A_1 = (1, 1) \\ \frac{\varepsilon_{1H}(1-\varepsilon_{2L})p}{\varepsilon_{1H}(1-\varepsilon_{2L})p+\varepsilon_{1L}(1-\varepsilon_{2H})(1-p)} & \text{if } A_1 = (0, 0) \end{cases} \quad (\text{A.2})$$

This can be rewritten as:

$$\mu_1^\varepsilon = \begin{cases} \frac{p}{p+\frac{(1-\varepsilon_{1L})(1-\varepsilon_{2H})}{(1-\varepsilon_{1H})(1-\varepsilon_{2L})}(1-p)} & \text{if } A_1 = (1, 0) \\ \frac{p}{p+\frac{\varepsilon_{1L}\varepsilon_{2H}}{\varepsilon_{1H}\varepsilon_{2L}}(1-p)} & \text{if } A_1 = (0, 1) \\ \frac{p}{p+\frac{(1-\varepsilon_{1L})\varepsilon_{2H}}{(1-\varepsilon_{1H})\varepsilon_{2L}}(1-p)} & \text{if } A_1 = (1, 1) \\ \frac{p}{p+\frac{\varepsilon_{1L}(1-\varepsilon_{2H})}{\varepsilon_{1H}(1-\varepsilon_{2L})}(1-p)} & \text{if } A_1 = (0, 0) \end{cases} \quad (\text{A.3})$$

It is clear that the limit of μ_1^ε for $A_1 = (1, 0)$ is p . The μ_1^ε of the other elements can be either 0 or 1 (depending on the the ratios $\frac{\varepsilon_{1L}}{\varepsilon_{1H}}$ and $\frac{\varepsilon_{2H}}{\varepsilon_{2L}}$.)

Using these beliefs, we can now check for optimality of sender strategies. Note that we can ignore the case where the limit of μ_1^ε for $A_1 = (0, 0)$ is 1, since in this case, it is optimal to any type of player 1 to deviate. Thus, we only focus on cases where the limit of μ_1^ε for $A_1 = (0, 0)$ is 0.

Thus, we are interested in two cases (notice that the limit of μ_1^ε for $A_1 = (0, 1)$ is irrelevant for the Nash equilibrium).

CASE 1: μ is given by:

$$\mu = \begin{cases} p & \text{if } A_1 = (1, 0) \\ \text{Not relevant} & \text{if } A_1 = (0, 1) \\ 0 & \text{if } A_1 = (1, 1) \\ 0 & \text{if } A_1 = (0, 0) \end{cases}$$

In this case, the payoffs to senders are:

		1	
		$a = 0$	$a = 1$
2	$a = 0$	0 1	$p - c$ $1 - p$
	$a = 1$		$-c$ $1 - c$

Irrespective of the choice by nature, there is no (separating) equilibrium in this case: (a) If $p > c$, then it is optimal for 2 to deviate. (b) But if $p < c$, then it is optimal for 1 to deviate.

CASE 2: μ is given by:

$$\mu = \left\{ \begin{array}{l} p \text{ if } A_1 = (1, 0) \\ \text{Not relevant if } A_1 = (0, 1) \\ 1 \text{ if } A_1 = (1, 1) \\ 0 \text{ if } A_1 = (0, 0) \end{array} \right\}$$

In this case, the payoffs to senders are:

		1	
		$a = 0$	$a = 1$
2	$a = 0$	0 1	$p - c$ $1 - p$
	$a = 1$		$1 - c$ $-c$

In this case, 2 has no incentive to deviate. The conditions that assure that 1 will not deviate are: (a) if he is of type H , it must be that $q > c$ and if he is of type L it must be that $1 - q > c$. Thus, a necessary condition to sustain a (separating) equilibrium is that $1 - q > c$. ■

Next, we show that when the receiver ignores the content of the message, there is no separating (sequential) equilibrium.

Lemma A.2. *When $q = 0.5$ a separating sequential equilibrium does not exist.*

Proof. There are two types of potential separating equilibrium: (1) H sends a message and L does not, and (b) the other way around.

CASE 1: H send a message with probability $1 - \varepsilon_H$ and L sends a message with probability ε_L . It is east to show that the consistent beliefs are:

$$\mu_s = \left\{ \begin{array}{l} 1 \text{ if } A_s = (1, 0) \\ 0 \text{ if } A_s = (0, 1) \\ 0.5 \text{ otherwise} \end{array} \right\} \quad (\text{A.4})$$

Thus, the payoffs are:

		H	
		$a = 0$	$a = 1$
L	$a = 0$	0.5 0.5	$1 - c$ 0
	$a = 1$	0 $1 - c$	$0.5 - c$ $1 - 0.5 - c$

When $c < 0.5$, then the only equilibrium of this game is $(1, 1)$ and when $c > 0.5$, then the only equilibrium of this game is $(0, 0)$. Thus, there is no separating equilibrium that is consistent with these beliefs.

CASE 2: L send a message with probability $1 - \varepsilon_L$ and H sends a message with probability ε_H . It is easy to show that the consistent beliefs are:

$$\mu_s = \begin{cases} 0 & \text{if } A_s = (1, 0) \\ 1 & \text{if } A_s = (0, 1) \\ 0.5 & \text{otherwise} \end{cases} \quad (\text{A.5})$$

and the only equilibrium is $(0, 0)$ irrespective of the cost. ■

B. Endogenous precision

Lemma B.1. *There exists a \underline{q} where $0.5 < \underline{q} < 1$ that satisfies the condition: $(1 - \underline{q}) - C(\underline{q}) = 0$. Furthermore, for any $q > \underline{q}$, $(1 - q) - C(q) < 0$.*

Proof. The function $(1 - q) - C(q)$ is decreasing in q , and is positive at $q = 0.5$ and negative at $q = 1$. ■

Lemma B.2. *There exists a \bar{q} where $\bar{q} > 0.5$ that satisfies the condition: $1 - C(\bar{q}) = \max_{0.5 \leq q \leq 1} q - C(q)$. Furthermore, for any $q < \bar{q}$, $1 - C(q) > \max_{0.5 \leq q \leq 1} q - C(q)$.*

Proof. The function $1 - C(q) - [q_{s_H}^{ns} - C(q_{s_H}^{ns})]$ is decreasing in q and is positive at $q = 0.5$ ■

Lemma B.3. $\bar{q} > \underline{q}$.

Proof. The function $1 - C(q) - [q_{s_H}^{ns} - C(q_{s_H}^{ns})]$ is decreasing in q . Next, we show that, it is positive at \underline{q} .

If $\underline{q} < q_{s_H}^{ns}$, $1 - C(\underline{q}) - [q_{s_H}^{ns} - C(q_{s_H}^{ns})] = [1 - q_{s_H}^{ns}] + [C(q_{s_H}^{ns}) - C(\underline{q})]$ where both elements are positive.

If $\underline{q} > q_{s_H}^{ns}$, $1 - C(\underline{q}) - [q_{s_H}^{ns} - C(q_{s_H}^{ns})] > [1 - \underline{q}] - C(\underline{q}) + C(q_{s_H}^{ns}) > [1 - \underline{q}] - C(\underline{q}) = 0$. ■

We are now ready to prove Proposition 3.2:

Proof. [Proposition 3.2] Given the beliefs and $q_{s_H} = q_{s_H}^*$, sender s_L will optimally choose $q_{s_L} = 0.5$ since: (a) choosing any q such that $0.5 < q < q_{s_H}^*$ or $q > q_{s_H}^*$ involves a cost without any revenues, and (b) choosing $q_{s_L} = q_{s_H}^*$ leads to losses from Lemma B.1.

Given the beliefs and $q_{s_L} = 0.5$, sender s_H will optimally choose $q_{s_H} = q_{s_H}^*$ since the highest payoff from any $q \neq q_{s_H}^*$ is $q_{s_H}^{ns} - C(q_{s_H}^{ns})$ which is smaller than the equilibrium payoff $1 - C(q_{s_H}^*)$ as illustrated by Lemma B.2.

It is trivial to show that beliefs agree with senders' strategies. ■

C. Multiple Messages

The setup of the model is described in the text. We start by demonstrating that in the non-strategic equilibrium only s_H sends messages. Then, we solve for his optimal number of messages, $N_{s_H}^{ns}$.

Using Bayes rule, it is easy to show that the probability that $s_H = s$ (denoted by μ_s^0) based only on the content of the messages is:

$$\mu_s^0 = \left[1 + \left[\frac{1 - q}{q} \right]^{h_s} \right]^{-1}$$

where h_s represents the number of messages indicating that $s_H = s$ compared to the number of messages indicating otherwise. Formally, $h_s = \sum_{k=1}^2 \sum_{i=1}^{N_k} (2I\{m_{k,i} = s\} - 1)$, where $m_{k,i}$ is the i th message of

sender k . This means that h_s is a sufficient statistic in this case. Notice that since the messages provide information about nature's selection, the source of the message does not matter.

The receiver's decision rule is quite simple. She selects the sender for which h_s is positive. If h_s is equal to zero, she select one of the senders randomly with probability 0.5.

From the senders' point of view h_s is a random variable, denoted by \widetilde{h}_s . The expected payoff function of s (where N represents is the number of messages sent by both senders) is:

$$E(d_s|N) - cN_s = \left[\Pr(\widetilde{h}_s > 0|N) + 0.5 \Pr(\widetilde{h}_s = 0|N) \right] - cN_s \quad (\text{C.1})$$

The probability functions in (C.1) can be rewritten as

$$\begin{aligned} \Pr(\widetilde{h}_s > 0|N) &= \sum_{k > \frac{N}{2}} \binom{N}{k} (q_s)^k (1 - q_s)^{(N-k)} \\ \Pr(\widetilde{h}_s = 0|N) &= \begin{cases} \binom{N}{\frac{N}{2}} [q(1-q)]^{\frac{N}{2}} & \text{when } N \text{ is even} \\ 0 & \text{when } N \text{ is odd} \end{cases} \end{aligned} \quad (\text{C.2})$$

where $q_{s_H} = q$, and $q_{s_L} = 1 - q$. For s_H , $\Pr(\widetilde{h}_s > 0|N)$ depends on all the events in which the number of "correct" realizations of the messages is larger than the number of "incorrect" ones. For his competitor, obviously, the opposite holds. The probability of a "tie" (i.e., the number of correct and incorrect messages canceling each other out), $\Pr(\widetilde{h}_s = 0|N)$, is the same for both senders. It is a positive value when N is even, and zero otherwise. Notice that when N is even, there is a positive probability that the number of "correct" messages equals the number of "incorrect" ones. If N is odd, of course, this event can never occur.

Notice that when s_L sends a message there is a higher probability that the message would decrease the receiver's tendency to select him than increase it.

The following Lemmas are used in the subsequent proposition (C.3).

Lemma C.1. $E(d_{s_H}|N)$ is a non decreasing function in N .

Proof. $E(d_{s_H}|N)$ is equal to:

$$\Pr(\widetilde{h}_{s_H} > 1|N) + \Pr(\widetilde{h}_{s_H} = 1|N) + 0.5 \Pr(\widetilde{h}_{s_H} = 0|N)$$

Notice that:

$$\Pr(\widetilde{h}_{s_H} > 0|N+1) = \Pr(\widetilde{h}_{s_H} > 1|N) + q \Pr(\widetilde{h}_{s_H} = 1|N) + q \Pr(\widetilde{h}_{s_H} = 0|N)$$

and

$$\Pr(\widetilde{h}_{s_H} = 0|N+1) = (1-q) \Pr(\widetilde{h}_{s_H} = 1|N) + q \Pr(\widetilde{h}_{s_H} = -1|N)$$

Thus,

$$\begin{aligned} E(d_{s_H}|N+1) - E(d_{s_H}|N) &= \\ (q - 0.5) \Pr(\widetilde{h}_{s_H} = 0|N) + 0.5 \left[q \Pr(\widetilde{h}_{s_H} = -1|N) - (1-q) \Pr(\widetilde{h}_{s_H} = 1|N) \right] \end{aligned}$$

Notice that the first term is relevant only when N is even, and that the second term is relevant only when N is odd.

Furthermore, the second term is equal to zero, since:

$$\begin{aligned} \Pr(\widetilde{h}_{s_H} = 1|N) &= \binom{N}{\frac{N+1}{2}} q^{\frac{N+1}{2}} (1-q)^{\frac{N-1}{2}}, \\ \Pr(\widetilde{h}_{s_H} = -1|N) &= \binom{N}{\frac{N-1}{2}} q^{\frac{N-1}{2}} (1-q)^{\frac{N+1}{2}}, \text{ and } \binom{N}{\frac{N+1}{2}} = \binom{N}{\frac{N-1}{2}}. \end{aligned}$$

Thus, when N is odd $[E(d_{s_H}|N+1) - E(d_{s_H}|N)] = 0$ and when N is even it is equal to $(q-0.5) \Pr(\widetilde{h_{s_H}} = 0|N)$.

Since $q > 0.5$, $E(d_{s_H}|N)$ is a non decreasing function in N . ■

Since the receiver prefers H over L , it is not surprising that getting additional informative messages increases the probability that she will choose H . This also means that by sending messages to the receiver, s_L (weakly) decrease his expected payoff. Thus, one would expect that s_L would not send messages. Indeed, this is established in proposition C.3 below.

Furthermore, the proof of Lemma C.1 shows that the change in $E(d_{s_H}|N)$ when N increases by 1 is:

$$\begin{aligned} (q_s - 0.5) \Pr(\widetilde{h_s} = 0|N) & \quad \text{when } N \text{ is even} \\ 0 & \quad \text{when } N \text{ is odd} \end{aligned} \tag{C.3}$$

This means that the increase in the expected payoff (as a result of an additional message) depends on the probability that the realizations of the previous ads lead to a ‘‘tie’’.²¹ When N is odd, the change in the expected payoff with one more ad is zero.²²

The following Lemma shows that the probability of a tie, and therefore the marginal payoff of s_H , decreases in N .

Lemma C.2. $\Pr(\widetilde{h_{s_H}} = 0|N)$ is a non increasing function in N .

Proof. For even N , $\Pr(\widetilde{h_{s_H}} = 0|N) = \left(\frac{N}{2}\right) [q(1-q)]^{\frac{N}{2}}$.

It is easy to show that

$$\binom{N+2}{\frac{N+2}{2}} = 4 \binom{N+1}{N+2} \binom{N}{\frac{N}{2}}$$

Thus,

$$\frac{\Pr(\widetilde{h_{s_H}} = 0|N+2)}{\Pr(\widetilde{h_{s_H}} = 0|N)} = 4 \left(\frac{N+1}{N+2}\right) [q(1-q)]$$

Since $q > 0.5$, we get that $4[q(1-q)] < 1$. Obviously $\left(\frac{N+1}{N+2}\right) < 1$, and thus $\Pr(\widetilde{h_{s_H}} = 0|N+2) < \Pr(\widetilde{h_{s_H}} = 0|N)$. ■

The rationale behind this Lemma is the following. Since $q > 0.5$ the probability that the number of correct realizations would be equal to the number of correct realizations is diminishing in N .

Lemma C.2 coupled with the fact that the marginal cost of a message is constant, suggest that as long as c is not too large, s_H would send a positive (and finite) number of messages. Indeed, this is established in the following proposition. Let $N_{s_L}^{ns}$ and $N_{s_H}^{ns}$ represent the number of messages sent in equilibrium by s_L and s_H respectively.

Proposition C.3. *In any non-strategic equilibrium, $N_{s_L}^{ns} = 0$, and $N_{s_H}^{ns} > 0$, where $N_{s_H}^{ns}$ is finite.*

²¹This result is consistent with the political-economy literature. See, for example, Shachar and Nalebuff (1999).

²²To see this intuitively, note that a change in payoff can occur only when the number of correct messages thus far exceeds or is less than the number of incorrect messages by s_H . Let γ_1 denote the probability of any event where the sum of ‘‘correct’’ messages exceeds the number of incorrect ones by s_H ; in this case, the addition of one message will, with probability $(1-q)$, result in the total number of correct and incorrect messages canceling each other out. Conversely, let γ_2 denote the probability of any event where the sum of ‘‘incorrect’’ messages exceeds the number of correct messages by s_H ; in this case, the addition of one message will, with probability q , result in the total number of correct and incorrect messages canceling each other out. It is easy to show that $\gamma_1(1-q) = \gamma_2q$. The change in payoff when N is odd and one additional message is sent, is given by: $\frac{1}{2}\gamma_2q - \frac{1}{2}\gamma_1(1-q)$, which, after substituting for γ_1 in terms of γ_2 , equals 0.

Proof. We first show that $N_{s_L}^{ns} = 0$. Lemma C.1 states that $E(d_{s_H}|N)$ is non-decreasing in N . Thus, each message sent by s_L weakly decreases his expected payoff (which is $1 - E(d_{s_H}|N)$). Since messages are costly ($c > 0$) sending *any* message decreases the payoff of s_L . Therefore, $N_{s_L}^{ns} = 0$ is a dominant strategy.

Next, we show that $N_{s_H}^{ns} > 0$.

Notice that $N_{s_H}^{ns}$ must be an odd number. Recall, from Lemma C.1, that the change in $E(d_{s_H}|N)$ when N increases by 1, is equal to zero if N is odd. Thus, for an even N , by decreasing the number of messages s_H sends by 1, he does not change his expected revenues but lower it cost.

Furthermore, $N_{s_H}^{ns}$ should satisfy the following conditions:

$$\begin{aligned} \left[E(d_{s_H}|N_{s_H}^{ns}) - cN_{s_H}^{ns} \right] - \left[E(d_{s_H}|N_{s_H}^{ns} - 2) - c(N_{s_H}^{ns} - 2) \right] &\geq 0 \\ \left[E(d_{s_H}|N_{s_H}^{ns}) - cN_{s_H}^{ns} \right] - \left[E(d_{s_H}|N_{s_H}^{ns} + 2) - c(N_{s_H}^{ns} + 2) \right] &> 0 \end{aligned} \quad (C.4)$$

In other words, increasing or decreasing $N_{s_H}^{ns}$ by 2 results in negative marginal payoff for s_H . (Notice that we add (and subtract) 2 from $N_{s_H}^{ns}$ since we are only considering odd numbers).

Using (C.3), these conditions can be re-written as

$$\begin{aligned} \Pr(\widetilde{h_{s_H}} = 0 | N_{s_H}^{ns} - 1) &\geq \frac{2c}{(q - 0.5)} \\ \Pr(\widetilde{h_{s_H}} = 0 | N_{s_H}^{ns} + 1) &< \frac{2c}{(q - 0.5)} \end{aligned} \quad (C.5)$$

Since $\Pr(\widetilde{h_{s_H}} = 0 | N)$ is a weakly decreasing function in N and c is not too large, there exists an $N_{s_H}^{ns} > 0$ that satisfies these conditions. ■

This result means that while s_H sends messages, his competitor does not. Let $\pi_{s_H}^{ns}$ denote the expected payoff of H in this non-strategic equilibrium.

It is often thought that as messages (for example, ads) become more precise the tendency to send them increases. It turns out that this is not the case. The effect of q on $N_{s_H}^{ns}$ is not monotonic. Recall that the marginal expected revenue is represented by $(q - 0.5) \Pr(\widetilde{h_s} = 0 | N)$. Notice that for both $q = 0.5$ and $q = 1$ the expected marginal revenue is 0. When $q = 0.5$, the marginal expected revenue is equal to zero because messages are not informative and therefore do not change the receiver's behavior. When $q = 1$, messages are not noisy and the probability of a tie is zero. Once again, the marginal expected revenue is zero. When q increases from 0.5, the probability of a "tie" (i.e., $\Pr(\widetilde{h_s} = 0 | N)$) diminishes, but the effectiveness of messages in informing the receivers $(q - 0.5)$ increases. The first effect reduces the incentive to send more messages, while the second increases it. It is easy to show that the first effect is more significant for q close to 0.5, while the second dominates for q close to 1.

Next, we examine the case that the receiver acts strategically. We show that there exists a separating equilibrium in which H sends messages and L does not.

We start by specifying the receiver's beliefs in this equilibrium. The receiver's beliefs (denoted by $B2$ are) that $s_H = s$ are:

$$\mu_s^1(N_s, N_{3-s}, h_s) = \begin{cases} 1 & \text{if } N_s = N_{s_H}^* \text{ and } N_{3-s} \neq N_{s_H}^* \\ 0 & \text{if } N_s \neq N_{s_H}^* \text{ and } N_{3-s} = N_{s_H}^* \\ \mu_s^0(h_s) & \text{otherwise} \end{cases} \quad (B2)$$

where $N_{s_H}^*$ satisfies the conditions:

$$1 - cN_{s_H}^* \geq \max_{0 \leq N} E(d_{s_H}|N) - cN$$

$$\text{and } 0 \geq E(d_{s_L}|2N_{s_H}^*) - cN_{s_H}^*$$

These beliefs are, obviously, quite similar to the case of endogenous precision. Notice that $\max_{0 \leq N} E(d_{s_H}|N) - cN = \pi_{s_H}^{ns}$ which is the expected payoff of H in the non-strategic equilibrium.

The next three Lemmas are used in the main proposition C.7. These lemmas show that there exists an \bar{N} so that for any $N < \bar{N}$, $1 - cN \geq \pi_{s_H}^{ns}$ (i.e., H would not like to deviate from his equilibrium strategy)

and there exists an $\underline{N} > 0$ so that for any $N > \underline{N}$, $E(d_{s_L}|N) - cN < 0$ (i.e, L would lose if he imitates H). Finally, we show that when $q > 0.5$, $\overline{N} > \underline{N}$.

Lemma C.4 (\underline{N}). *There exists an $\underline{N} > 0$ that satisfies the conditions: $[E(d_{s_L}|2\underline{N}) - c\underline{N}] < 0$ and $[E(d_{s_L}|2(\underline{N} - 1)) - c(\underline{N} - 1)] \geq 0$. Furthermore, for any $N > \underline{N}$, $[E(d_{s_L}|2N) - cN] < 0$.*

Proof. The expected payoff function of L from imitating H ($E(d_{s_L}|2N) - cN$) is (a) equal to 0.5 at $N = 0$, (b) $-\infty$ at $N = \infty$, and (c) monotonically decreasing in N (because $E(d_{s_L}|2N)$ is decreasing in N and cN is increasing in N). ■

Lemma C.5 (\overline{N}). *There exists an \overline{N} that satisfies the conditions: $1 - c\overline{N} \geq \pi_{s_H}^{ns}$ and $1 - c(\overline{N} + 1) < \pi_{s_H}^{ns}$. Furthermore, for any $N < \overline{N}$, $1 - cN > \pi_{s_H}^{ns}$.*

Proof. $1 - cN - \pi_{s_H}^{ns}$ is (a) positive at $N = 0$ (since $\pi_{s_H}^{ns}$ must be smaller than 1), (b) $-\infty$ at $N = \infty$, and (c) monotonically decreasing in N . ■

Lemma C.6. *For $q > 0.5$, $\underline{N} \leq \overline{N}$.*

Proof. Since $1 - cN - \pi_{s_H}^{ns}$ is monotonically decreasing in N , we only need to show that $1 - c\underline{N} - \pi_{s_H}^{ns} \geq 0$. Recall that $\pi_{s_H}^{ns} = E(d_{s_H}|N_{s_H}^{ns}) - cN_{s_H}^{ns}$, and thus $1 - c\underline{N} - \pi_{s_H}^{ns} = [1 - E(d_{s_H}|N_{s_H}^{ns})] + c[N_{s_H}^{ns} - \underline{N}]$. If $N_{s_H}^{ns} \geq \underline{N}$, it is immediate that $[1 - E(d_{s_H}|N_{s_H}^{ns})] + c[N_{s_H}^{ns} - \underline{N}] > 0$. Next, consider the case where $N_{s_H}^{ns} < \underline{N}$. $[1 - E(d_{s_H}|N_{s_H}^{ns})] + c[N_{s_H}^{ns} - \underline{N}] = E(d_{s_L}|N_{s_H}^{ns}) - c[N_{s_H}^{ns} - \underline{N}] \geq E[d_{s_L}|(\underline{N} - 1)] - c[\underline{N} - 1] \geq E[d_{s_L}|2(\underline{N} - 1)] - c[\underline{N} - 1] \geq 0$ by Lemma C.4. ■
We are now ready to restate the main result (this is a more precise statement of proposition 3.3):

Proposition C.7 (PBE). *For $q > 0.5$, there exists a perfect Bayesian equilibrium where beliefs are given by $B2$ and senders' pure strategies are given by: $N_{s_H} = N_{s_H}^* \in [\underline{N}, \overline{N}]$, and $N_{s_L}^* = 0$.*

Proof. Given the beliefs and $N_{s_H} = N_{s_H}^* \in [\underline{N}, \overline{N}]$, s_L will choose not to deviate from $N_{s_L} = 0$ since: (a) any $N_{s_L} \neq N_{s_H}^*$ involves a cost without any revenues, and (b) $N_{s_L} = N_{s_H}^*$ leads to losses from Lemma C.4.

Given the beliefs and $N_{s_L} = 0$, s_H will optimally choose $N_{s_H} = N_{s_H}^* \in [\underline{N}, \overline{N}]$ since any $N_{s_H} \neq N_{s_H}^*$ leads to lower expected payoff from Lemma C.5.

It is trivial to show that the beliefs $B2$ are consistent with these strategies of firms. ■

Thus, in equilibrium s_H sends ads and his competitor does not. All uncertainty about product attributes is resolved and the individual chooses the product that is best for her with certainty.

The previous proposition studied the PBE when $q > 0.5$. The following Lemma and Proposition examine the case of $q = 0.5$.

Lemma C.8. *For $q = 0.5$, $\underline{N} > \overline{N}$.*

Proof. When $q = 0.5$, for any N , we get that $E(d_{s_H}|N) = 0.5$ and $E(d_{s_L}|N) = 0.5$. Thus, it is easy to show that $\underline{N} > \frac{1}{2c} > \overline{N}$. ■

Proposition C.9. *When $q = 0.5$ there does not exist any perfect Bayesian equilibrium in which beliefs are given by $B2$.*

Proof. Since $\underline{N} > \overline{N}$ there is no $N_{s_H}^*$ that can satisfy the conditions of beliefs $B2$. ■

This means that when $q = 0.5 + \varepsilon$ a separating equilibrium exists, but when $q = 0.5$ it does not hold any more. This results indicates that ‘‘little information’’ can sometimes be ‘‘enough information’’, and in any case, it is very different from ‘‘no information’’.

In Anand and Shachar (2000) we show that a pooling equilibrium does not exist when beliefs are $B2$, and that the separating equilibrium holds for a more general class of beliefs.

Figure 1

