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(Noisy) communication

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Abstract Communication is central to many settings in marketing and economics. A focal attribute of communication is miscommunication. We model this key characteristic as a noise in the messages communicated, so that the sender of a message is uncertain about its perception by the receiver, and then identify the strategic consequences of miscommunication. We study a model where competing senders (of different types) can invest in improving the precision of the informative but noisy message they send to a receiver, and find that there exists a separating equilibrium where senders' types are completely revealed. Thus, although communication is noisy it delivers perfect results in equilibrium. This result stems from the fact that a sender's willingness to invest in improving the precision of their messages can itself serve as a signal. Interestingly, the content of the messages is ignored by the receiver in such a signaling equilibrium, but plays a central role by shaping her beliefs off the equilibrium path (and thus, enables separation between the types). This result also illustrates the uniqueness of the signaling model presented here. Unlike other signaling models, the suggested model does not require that the costs and benefits of the senders will be correlated with their types to achieve separation.

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The model's results have implications for various marketing communication tools such as advertising and sales forces.

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1 Introduction

In 2000 the presidential candidate, Al Gore, demanded to have three debates with his competitor, George Bush. Gore, who was perceived to be the better debater preferred three debates over one since he knew that a debate, like any other form of communication, is noisy. In other words, although he was likely to perform better in a debate, there was a positive probability that Bush would 'win' any specific debate. For the same reason Bush preferred one debate over three. The subsequent 'debate about the debates' was protracted, and disturbed people in the Republican party. Pundits claimed that Bush's reluctance to have three debates would hurt his public image. Indeed, in order to avoid such damage Bush eventually agreed to three debates.[1]

Competing firms invariably face a similar situation when communicating with consumers about their products. Consider the case where one firm offers a better product than its competitor(s), either because of its quality or because its non-quality attributes fit the preferences of the specific consumer better. Advertisements are one way to inform consumers who are uncertain about these product attributes. However, like in the example above, ads are noisy means of communication—so that it is possible that exposure to ads may still not guarantee that the better firm is perceived as such. A way to improve communication is by increasing the number of ads. Of course, the competing firms have different incentives in doing so. As a result, like in the example above, communication can occur not only through ad content but also via the number of ads.

These examples illustrate features common to several settings where a sender communicates information to a receiver. First, senders compete in doing so. Second, communication is noisy (i.e., the senders are uncertain about the receivers' perception of their messages). Third, information can be conveyed both through the message content of the senders (the debate) and through the senders' willingness to invest in such messages (the number of debates). These features are usually absent in work on communication or strategic information transmission. Instead, the typical model encompasses

[1] See the following articles in the New York Times: "Bush, Facing Criticism, Abandons Debate Stance" (September 9, 2000); "One Debate Down, Three to Go" (September 10, 2000); "Dropping All of His Objections, Bush Agrees to Panel's Debates" (September 15, 2000); and, "Candidates Agree on Formats for Three Debates" (September 17, 2000).

one sender, messages that are not noisy, and, in most cases, information that is revealed *either* indirectly (i.e., through signaling) *or* directly (i.e., message content). Each of these basic real-world features affects the mechanism by which information is communicated between senders and receivers. Therefore, one might question how robust are the insights of prior work to the inclusion of these features.

We present a model with these features, and use it to study the nature and effectiveness of communication in equilibrium. Our model can shed new light on several applications in marketing, for example an advertisement appearing on television, or a salesperson speaking with a potential client.

Communication is central to marketing. Invariably, however, communicators face a fundamental problem—they do not have full control over the perception of the message (they send) by the receivers. In other words, the message might be misunderstood. Miscommunication in general might be due to various reasons such as misreading a message, misinterpreting it, not paying enough attention, or vague language. We refer to the uncertainty over receivers' perception as "noisy communication."[2]

The model presented here aims to capture the strategic consequences of noisy communication. The basic ingredients of the model are as follows. Two senders (of different types) court a receiver. For example, in the political case, "senders" are the presidential candidates; the "receiver" is the public; and the differences in "types" correspond to the differences between the candidates in their ability to run the country. The competing senders can send a message that noisily conveys information about their type. Each sender can determine the precision of his message. The cost of sending a message is increasing in its precision. The receiver prefers one type over the other, but does not know the types associated with each sender. In all other respects, senders are symmetric: the cost function of the message's precision and the revenues from being chosen by the receiver are the same for both senders. Senders simultaneously decide about the precision of their messages, and the receiver then selects one of them.

We find that a separating equilibrium—in which senders' types are completely revealed by their actions—exists for any set of parameters. Specifically, while one sender invests in improving the precision of his message, his competitor does not, and the receiver (optimally) selects the one that sends the more precise message. In other words, although communication is noisy it delivers perfect results.

Interestingly, the content of the message is ignored by the receiver in equilibrium. This means that even if the content of the (noisy) message indicates that the sender is not the type that the receiver prefers, she still selects him. At the same time, the content of the message is not superfluous: it affects

[2]For example, in their text book on advertising Belch and Belch (2007) write: "Throughout the communication process, the message is subject to extraneous factors that can distort or interfere with its reception. This unplanned distortion or interference is known as **noise**."

the receiver's beliefs off the equilibrium path and, as a result, supports the separating equilibrium.

This result illustrates one of the main differences between the signaling model presented here and the standard signaling model. Specifically, since the separation is enabled by the reliance of the off-equilibrium beliefs on the content of the message, we do not need to assume that the cost or the benefits of the competing senders is correlated with their type. In other words, some of the requirements of standard signaling models can be relaxed in this setting. Another departure from most signaling models is that, in our setting, there is competition among communicators. This feature is not only natural to examine many applications in marketing, but it also has substantive behavioral effects in the signaling framework. Specifically, the existence of competition provides another case in which the receiver knows that one of the senders has deviated—when the actions of both senders are identical. In the standard one-sender, two-type model, this channel would not be present and, as a result, there would be no separating equilibrium.[3] In other words, these two seemingly close models (two senders with two possible types versus one sender with two possible types) lead to very different predictions. Competition thus results, in our case, in more information being conveyed through a message than what it physically contains.

The results of this model have implications for marketing scholars and practitioners. As mentioned earlier, marketers face a fundamental problem—they do not have full control over the perception of their message by the consumers. Indeed, it is fair to say that marketing communication tools such as advertising and sales forces are noisy. For example, Jacoby and Hoyer (1982, 1989) demonstrated that about one-third of all ads are miscomprehended by consumers. The presented model implies that communication can deliver perfect results even when it is noisy, since senders' types are completely revealed in equilibrium. An application of this is that although ads are noisy, advertisers' choice of media can ensure that consumers are fully informed (Anand and Shachar 2006).[4]

A second implication for marketing relates to the role of the senders' willingness to provide information. Specifically, a firm's willingness to provide information itself provides information. In other words, firms can overcome the noisiness of the communication tools by their mere readiness to provide information about their products. An application of this is a retailer's choice to adopt a salesformat that allows buyers to ask "everything about everything"

[3]Put differently, in any signaling model there is more than one sender's type. However, in a standard signaling model the number of senders is limited to one. Thus, these models do not accommodate the possibility that there is more than one player whose actions are observed by the receiver.

[4]While the model here explores communication in general, Anand and Shachar (2006) study the specific application of advertising. As a result, that paper focuses on a quite different mechanism of information transmission. Specifically, while the study here, message precision serves as the signal, in the other study the signaling is done via media selection (i.e., targeting).

rather than only to provide information that benefits the seller (Bhardwaj et al. 2005).[5] Similarly, firms might commit not to edit consumer comments on their websites.

1.1 Related literature

This study is related to previous work on signaling and on communication. These two lines of work explore different avenues of information transmission. Signaling games focus on *indirect* information transmission in which the receiver infers the sender's private information from her actions.[6] Studies of communication focus on *direct* information transmission. In these settings a message with information content is being sent to a receiver who uses it in a simple Bayesian fashion.[7] The model presented here allows for both direct and indirect avenues of information transmission.

The major distinction between this study and previous work on communication lies in the nature of communication. Previous studies recognize that communication is far from perfect and offer various approaches to model it. They assume, for example, that messages need not be fully verifiable, that senders may be able only to refute claims, that senders may deceive, etc. For example, Okuno-Fujiwara et al. (1990) and Fishman and Hagerty (1990) examine settings where fully verifiable disclosures are not feasible; Lipman and Seppi (1995) and Glazer and Rubinstein (2000) examine settings where senders can refute, but not prove, claims; Sanchirico (2001), Deneckere and Severinov (2003) and Bull and Watson (2004) allow for deception; and, Bhardwaj et al. (2005) assume that the buyer and the seller can only discuss one attribute of the product. However, the common feature in these studies is that senders have full control over what information the receiver will perceive. We depart from this assumption and study "noisy communication"–a feature that would appear to be especially salient in several arenas in economics and marketing. As a result, while previous studies of communication usually focus on what the sender says (or does not say), we focus on how much the sender invests in improving the precision of his message.[8]

Several of these studies examine communication through debates. Although this type of communication may not apply well to most marketing applications,

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[5] Bhardwaj et al. (2005) show that a monopolist who offers a high quality product prefers to give up control over the message and allow the buyer to ask whatever she wants about the product. The revelation format facilitates the use of price as a signaling mechanism.

[6] An important strand of this literature, going back to Crawford and Sobel (1982), examines how information is strategically transmitted through messages in cheap talk games.

[7] For example, the information systems theory of communication (Shannon 1948) and subsequent papers in economics that draw on it, notably the theory of teams (Marshack and Radner 1972). In all those studies agents are non-strategic.

[8] The setup of our model is different in other aspects such as the cost of sending a message. Exceptions are those that focus on deception in which the cost of a non-truthful messsage is higher than the cost of a truthful one; see, for example, Sanchirico (2001) and Deneckere and Severinov (2003).

it is worth noting that the result in one such study (Lipman and Seppi 1995) is consistent with our finding about the effectiveness of communication. Specifically, Lipman and Seppi show that the receiver makes full, correct inferences even when the ability of senders to refute others' claims is rather weak. In other words, communication can be effective even when it is quite limited. In addition to the difference in the mechanism for communication, there are various other differences between the settings. In Lipman and Seppi, communicated is limited but not noisy, there is no uncertainty about how the receiver perceives a message, messages are costless, and senders move sequentially.

The difference between our modeling of communication and the standard approach in signaling models expresses itself in other ways as well. The most important differences relate to the informative content of the messages and competition between senders. Specifically, since the costs and benefits are identical for both senders here, their actions cannot serve as a signal unless the messages contain *some* information. Furthermore, information revelation here results, ultimately, from competition between the senders (that is absent in standard signaling games).[9] For both reasons, information transmission through signals works quite differently here than in the standard case.

The next section presents a simple model in which the senders cannot determine the precision of their messages but only whether to send a message or not. This section provides the initial intuition for the more general model, which is presented in Section 3. Section 4 discusses the applications, and Section 5 concludes.

2 A simple model

We start by presenting a simple model that provides a preliminary formal examination of (a) the consequences of noisy communication, (b) the amount of information revealed under different scenarios, and (c) the basic idea that willingness to provide information can itself provide information. The next section studies a more general version of this model.

This section starts with a description of the model setup and then proceeds by characterizing the equilibrium under two scenarios. In the first case, the receiver bases her decision only on the content of the messages. In the second case, her decision also depends on the actions of the senders.

2.1 Model setup

Players and types Two senders (denoted by s where $s = \{1, 2\}$) court a receiver, who will select one of them. There are two types of senders, denoted by H and L. Nature selects, with probability 0.5, one of the senders to be of type H and the other of type L.

[9] Exceptions are Hertzendorf and Overgaard (2001) and Daughety and Reinganum (2000).

Payoffs The receiver's payoff depends on the selected sender's type, which is unknown to her. Specifically, her payoff from sender H is 1, and from sender L is 0. The distinction between L and H only means that the two senders are not identical from the receiver's point of view: one of them is better than the other.[10]

The payoff of a sender is 1 if he is selected and 0 otherwise. These payoffs are not type-specific.

Message technology Each sender may send a (noisy) message to the receiver indicating his type. The probability that the perception of the message is correct is q. Thus, for example, if L sends a message, there is a q probability that the receiver will perceive it as "I am L" and a $1-q$ probability that she will perceive it as "I am H". Notice that since it is common knowledge that one of the senders is L and the other is H, the message "I am L" is equivalent to "I am L and the other sender is H".[11]

We assume that $q \neq 0.5$, and for simplicity $q > 0.5$. Notice that any $q \neq \frac{1}{2}$ is informative. In other words, only when $q = \frac{1}{2}$ is it the case that the message does not distinguish between the two types. Here, we consider the natural case of $q > \frac{1}{2}$, which means that "correct messages are more likely than incorrect messages". Another interpretation of this technology is of a "test" that the senders can voluntarily take, where the precision of the test in determining the senders' type is q.

The cost of sending a message is $c > 0$.

In this simple model the senders are allowed to send at most one message with a predetermined precision. This assumption is relaxed in the next section.

Sequence of events Senders act first: each simultaneously decides whether to send a message or not. After receiving the messages, the receiver selects one of the senders.

There are various marketing applications that can fit such a setting. Consider, for example, advertising. In that case, the two senders are firms that are pursuing a consumer. One firm offers a product that better fits the consumer's taste either because it is of high quality (vertical differentiation) or because it matches her preferences better (horizontal differentiation). The messages are ads which are both costly (to produce and air) and noisy (as discussed above).

Section 2.3 presents the signaling properties of this model. It is shown that when the receiver incorporates information about the senders' *actions* in her

[10] In other words, by ignoring the cases that both senders are of the same type (either L or H), we focus on the interesting scenario in which there is some difference between the two senders and thus one of them is better than the other from the receiver's point of view.

[11] (a) Of course, when the message is so simple, the probability of miscommunication is minuscule. However, in reality messages are rarely so simple.

(b) Like other signaling models, L can try and deceive the receiver by imitating H's actions. However, he is not allowed to deceive in the message content. Still, as pointed out, the perception of the message is random and thus there is still a chance that the receiver will interpret a message from L as saying "I am H."

decision, these actions completely reveal, in equilibrium, the senders' type. In order to provide initial insight into this model, however, we first solve it under the assumption that the receiver selects a sender based only on the content of the messages. We refer to this solution as the "non-strategic" one.

Throughout, the analysis focuses only on pure strategies.

2.2 Non strategic solution

We start by presenting the decision rule of the receiver and then describe the senders' strategies.

2.2.1 Receiver's decision rule

When the receiver bases her expectations only on the content of the messages, it is easy to show, using Bayes rule, that from her point of view the probability that sender s is of type H (denoted by μ_s^0) is:

$$\mu_s^0 = \left[1 + \left[\frac{1-q}{q}\right]^{r_s}\right]^{-1} \tag{1}$$

where r_s is equal to the number of messages indicating that s is of type H minus the number of messages indicating otherwise. For example, if s sent a message that was perceived as "I am L " and the other sender sent a message that was perceived as "I am H", $r_s = -2$. Notice that $r_1 = -r_2$.

The receiver's decision rule turns out to be quite simple, she selects the sender for which r_s is positive. If r_s is equal to zero (which occurs either when neither sender sends a message, or when the messages contradict each other and thus the receiver cannot discriminate between senders anyway), she selects one of the senders randomly with probability 0.5.

Notice that, according to this decision rule, the sender that does *not* send a message may still be chosen: this follows from the fact that messages are noisy and the receiver is non-strategic. For example, if only one player sends a message and it is read by the receiver as "I am L", the receiver will select the player that did not send a message.

2.2.2 Senders' equilibrium strategies

From the senders' point of view the perceptions of the messages are uncertain and thus r_s and μ_s^0 are random variables. The objective function of the senders is the difference between the expected payoff and the cost (which is c if he sent a message and zero otherwise). Notice that the expected payoff is equal to the probability that s will be selected, viz $\Pr(r_s > 0) + \frac{1}{2}\Pr(r_s = 0)$, and that this probability is equal to q if s is H and $(1-q)$ if s is L. [12]

[12]The last statement is true when at least one player sends a message. When neither one of them sends a message the probability of being selected is, obviously, $\frac{1}{2}$ for each of the senders.

Since $q > 0.5$, it is easy to show that, in equilibrium, L does not send a message. And, H sends a message if (and only if) $c < q - 0.5$. It is not optimal for L to send a message since correct messages are more likely than incorrect messages (and messages are costly). On the other hand, H sends a message as long as the increase in his expected payoff $q - 0.5$ is bigger than the cost of sending a message, c.

The probability that the receiver selects H is q which is lower than 1.

This result clarifies our interest in the signaling aspect of this game. The receiver might want to base her decision on the senders' action (rather than just the content of the message), since these actions may convey information about the sender's type and thereby resolve the uncertainty that she faces. In other words, while the content of the messages is noisy, the actions of the players are not. Thus, if the actions can differentiate between the two senders, the indirect (inferred) information from sending a message can be more valuable than the direct information in the content of the messages.

2.3 Strategic solution (a separating equilibrium)

Here, we examine the case where the receiver incorporates both the *content* of the messages and the senders' *actions* when forming her beliefs on the sender's type. We (a) show that there exists a separating equilibrium for this game, (b) characterize this equilibrium, and (c) demonstrate that it is unique. Furthermore, we also show that for the equilibrium to exist, the message needs to be informative (i.e., q can be equal to $\frac{1}{2} + \varepsilon$, but it cannot be equal to $\frac{1}{2}$). Finally, we discuss how the model findings can shed light on a topical phenomenon, spam, and some suggested solutions to that problem.

2.3.1 Existence

Here, we show that there exists a sequential equilibrium, in which (a) only H sends a message, and (b) the receiver is not uncertain about the type of each sender.

To begin with, we specify the beliefs of the receiver at each of her four information sets.

$$\mu_s = \left\{ \begin{array}{ll} 1 & \text{if } s \text{ sends a message and his competitor does not} \\ 0 & \text{if } s \text{ does not send a message and his competitor does} \\ \mu_s^0(r_s) & \text{otherwise} \end{array} \right\} \tag{B}$$

That is, if *only* one player sends a message, the receiver believes that the sender is H (irrespective of the content of the message). In other cases, the receiver relies on the information content in the messages (where relevant) to form her beliefs, as given in Eq. 1. The off-the-equilibrium beliefs (presented in the last line of B) seem reasonable behaviorally. Specifically, when both senders send a message, their actions cannot distinguish between them, and thus it seems sensible for the receiver to rely on the content of the messages. Accordingly, when neither of them sends a message, the receiver's beliefs reflect her prior.

Given these beliefs, the following table represents the expected net payoffs of both types of senders.

		H	
		does not send a message	sends a message
L	does not send a message	0.5 0.5	$1-c$ 0
	sends a message	0 $1-c$	$q-c$ $(1-q)-c$

The following proposition characterizes the equilibrium of this game.

Proposition 1 *When* $0.5 > c > 1-q$ *, there exists a sequential equilibrium where beliefs are given by* B*,* H *sends a message and* L *does not.*

Proof Given beliefs B, sending a message is a dominant strategy for H ($1-c > 0.5$ because $0.5 > c$ and $q-c > 0$ since $q > 0.5 > c$). Given that H sends a message, not sending a message yields a higher payoff for L than sending one ($(1-q)-c < 0$ because $c > 1-q$).

It is straightforward to show that beliefs are consistent, given these strategies of the senders. □

The intuition behind this result is simple. Notice that the expected net payoffs are type specific only when both players send a message. For all other cells the expected net payoffs are not type specific. This is due to the assumptions that the costs, the benefits and the prior probability of being H are the same for both senders. However, when both senders send a message the off-the-equilibrium beliefs are provoked and the receiver uses the content of the messages, which is more likely to favor H over L. Specifically, the probability of being selected is q for H and $(1-q)$ for L. Although $q > (1-q)$, when H sends a message, L might still like to mimic him because otherwise his profit is zero. In order to deter L from mimicking, c or q should be high enough.[13] This leads to the condition $c > 1-q$. The condition $0.5 > c$ is trivial—it just requires that the cost is not too high (otherwise, the game would be uninteresting).[14]

Interestingly, the beliefs in B are not only sensible behaviorally (as discussed above), but are also consistent and thus are part of a sequential equilibrium.

[13]Recall that a high q implies that the content of the message is more likely to indicate that this player is L.

[14](a) Another way to think about the limits of the interval for c is the following. They are based on the probability that L is selected when any sender deviate from his equilibrium strategy (which is 0.5 if H deviates and $1-q$ if L deviates). This reflects the fact that both senders in this game are competing over the "market share" of L.

(b) This interval is at a higher cost level than the interval for the non-strategic equilibrium. This is not surprising: in the non-strategic equilibrium the cost should be low enough for H to send a message. In the separating equilibrium, it should be high enough to deter L from mimicking.

When communication is noisy it is reasonable to expect that its result would be uncertain. Indeed, in the previous subsection we found that the effectiveness of communication depends on the precision of the message technology. Specifically, the receiver selected H with probability q. Put differently, the probability that the receiver would select L was not zero, but rather $1 - q$. This result reflects the imperfection of noisy communication.

In contrast, in this section we find that although communication is noisy, it can deliver perfect results. Specifically, in equilibrium the receiver selects H with certainty. The difference between the two cases is based on the strategic considerations of the receiver and the resulting strategic actions of the senders. When the receiver acts strategically and recognizes that she can infer the senders' identity from their actions, she does not make mistakes in her selection.

In other words, the way to overcome the problem of noisy communication is based on the senders' willingness to provide information about their identity. In equilibrium, only H is willing to provide such information, and as a result, this willingness serves the receiver as a signal.

An interesting feature of this game is that although the content of the messages is not used in equilibrium, it is not superfluous. Before explaining why the content of the messages is not superfluous, we clarify why it is not used in equilibrium. In equilibrium the receiver gets only one message. This message indicates that the sender of the message is L with probability $1 - q$. Even in such a case, however, the receiver will select this sender. In other words, the sender is selected by the receiver even if the realization of the message was negative from his point of view. Notice that such behavior is not only grounded formally but also quite sensible. The reason is that the receiver has, in such a case, two pieces of information. One is noisy and the other isn't. Thus, it is reasonable to expect that she will base her decision solely on the non-noisy source of information.

However, the finding that the content of the message is ignored in equilibrium does not imply that it is redundant: unless the receiver is using the content of the messages off-the-equilibrium there is no way to deter L from imitating H. In other words, the content of messages supports the particular equilibrium by shaping off-equilibrium beliefs.

The results of this simple model can shed some light on a topical phenomenon—spam—as well as suggested solutions to this problem. In the non-strategic case, L does not have any incentive to send a message even when the cost parameter, c, is very small. However, in the strategic case, L has an incentive to send a message, and c should be high enough to deter him. Interestingly, in environments in which the cost of sending a message is fairly small, such as e-mails and telemarketing, receivers face a problem of receiving too many messages that are not well-linked to their tastes or interests. This is, obviously, consistent with the strategic solution of the model. The model predicts that in such a setting, increasing c can deter L from sending messages and thus improve the matching between senders and receivers. This idea underlies several mechanisms that some scholars have recently suggested as ways to reduce irrelevant telemarketing messages (see, for example, Ayres and Funk 2002). And, a recent business model (Vanquish.com) that aims to eliminate online spam is also based on a similar principle: it "tags individual emails with electronic vouchers showing that the message is backed by money the sender is willing to lose if the recipient decides the email is spam".[15]

2.3.2 Uniqueness

So far we have shown that a separating equilibrium exists. The following proposition states that for the relevant parameter values this equilibrium is unique.

Proposition 2 *When $0.5 > c > 1 - q$, there exists a unique sequential equilibrium where beliefs are given by B, H sends a message and L does not.*

Proof See Appendix A. □

2.3.3 Non-informative messages

In appendix B we show that a separating sequential equilibrium does not exist when $q = 0.5$. In other words, while for $q = 0.5 + \varepsilon$ this separating equilibrium exists, for $q = 0.5$, it does not. This means that sending a message can serve as a signal on senders' identity even when the messages are quite noisy—but as long as they have *some* information content. When messages have no information content, sending a message cannot serve as a signal on senders' identity.

3 General model

In the simple model of the previous section, senders' actions are limited. Each can send at most one message of pre-determined precision. In this section, senders have more flexibility: each can decide on the amount of information they transfer to the receiver. A sender can do so either by endogenously choosing the precision of their message and/or by sending multiple messages. This section extends the simple model to incorporate the possibility of endogenous precision. (It can be shown that the results presented below hold also for the case of predetermined precision with multiple messages.) These extensions are well-suited for various applications, for example to analyze competition in the political arena, where candidates can decide how much private information about themselves they are willing to reveal; or, competition in advertising, where firms are not restricted to sending at most one ad with a predetermined precision.

[15] See http://www.vanquish.com/press/ps_clean_email.shtml.

3.1 Endogenous precision

3.1.1 Model Setup

The setting of the model is the same as of the simple model with the following exceptions. Each sender can determine the precision of his message, $q_s \in [0.5, 1]$. Thus, for example, each sender can choose to send a non-informative message ($q_s = 0.5$), a fully-informative message ($q_s = 1$), or a noisy message at any level of precision between 0.5 and 1. The cost of producing a message is a function of its precision, $C(q)$, with $C(0.5) = 0$. We assume that the cost function is increasing in q ($C' > 0$) and convex ($C'' > 0$). In other words, the marginal cost of improving the precision of a message is higher for messages that are already quite precise.[16]

The setting of the model is common knowledge. The receiver observes the q_s chosen by each sender but is uncertain about nature's selection (i.e., the type of each sender). This assumption reflects a receiver's ability to identify the amount of information revealed in a message. For example, when one of the candidates in a presidential debate gives a vague answer, voters can identify his reluctance to provide information. The same argument holds for sales force.

The net payoff of sender s is $\pi_s(q_s) = d_s - C(q_s)$, where d_s is a binary variable which is equal to 1 if the receiver selected s and 0 otherwise.

As in the previous section, we start with the non-strategic equilibrium and then demonstrate the existence of a separating equilibrium where the senders' decisions on the precision of their messages serves the receiver as a signal about their type.

3.1.2 Non strategic equilibrium

We start by demonstrating that in the non-strategic equilibrium H sends an informative message and its competitor does not ($q_s > 0.5$ if s is H, and $q_s = 0.5$ if s is L).

The individual's decision A non-strategic receiver updates her belief that s is H using only the content of the message (and not the potential signaling aspects of the choice of precision by senders). The probability that s is H, denoted by μ_s^0), is (using Bayes rule):

$$\mu_s^0 = \left[1 + \left[\frac{1 - q_s}{q_s}\right]^{m_s} \left[\frac{1 - q_{s'}}{q_{s'}}\right]^{-m_{s'}}\right]^{-1}$$

where m_s, which represents the perceived content of the message sent by s, is a binary variable that is equal to 1 if the message sent by s indicates that he is H and -1 otherwise; and $s' = 3 - s$ (i.e., s' is the competitor of s).

[16]For example, in many situations ad agencies find it difficult to increase the precision of their ads, because they need to ensure that the ad is memorable, attractive, etc. As the ad become more precise, essential aspects of memorability and attractiveness are likely to be blemished.

As before, the receiver's decision rule is quite simple. If $q_s \neq q_{s'}$ her decision is based on one message only—the more precise one. Specifically, she selects the sender that the more precise message identifies as H. Notice that the selected sender does not have to be the one that sent the more precise message. If $q_s = q_{s'}$, her decision is based, as before, on r_s (i.e., $r_s = m_s - m_{s'}$). She selects s if $r_s > 0$, and his competitor if $r_s < 0$. And if r_s is equal to zero, she selects one of them randomly with probability 0.5.

Senders' strategies From the senders' point of view the perceived content of the messages is uncertain (i.e., m_s and $m_{s'}$ are random variables).

Let $E[\pi_s(q_s); q_{s'}]$ denote the expected net payoff function for s if he sends a message with precision q_s and his competitor sends a message with precision $q_{s'}$. This expected net payoff function is

$$E[\pi_s(q_s); q_{s'}] = E(d_s|q_s, q_{s'}) - C(q_s). \tag{2}$$

The following proposition characterizes the non-strategic equilibrium.[17]

Proposition 3 (Non strategic equilibrium) *In any non-strategic equilibrium, L sends an uninformative message ($q_s = 0.5$ if s is L), and H sends a message with precision strictly greater than* 0.5 *as long as* $C'(0.5) < 1$.

Proof In Appendix C. □

The intuition of this result is quite simple. Since an informative message cannot increase the probability that the receiver will choose L and sending a message is costly, it is optimal for L to send a non-informative message. The other side of this story is that as the message become more precise, the probability that the receiver will select H increases. Thus, it is optimal for H to send an informative message with precision level that equalizes the marginal probability of being selected with the marginal cost. Let q° denote this precision level. Specifically, $C'(q^\circ) = 1$.

3.1.3 Strategic equilibrium

Here, we examine the case where the receiver incorporates in her decision both the statistical information that is revealed in the content of the messages, and the signaling information that is revealed by senders' choice of precision. We show that in equilibrium only one sender sends an informative message. In addition, however, in equilibrium the receiver has no uncertainty about nature's selection.

We start by specifying the receiver's beliefs, and then show that these beliefs are part of a separating equilibrium. In this equilibrium, the precision of the messages chosen by the senders are, $q_s = q^* > 0.5$ if s is H and $q_s = 0.5$ if s is L.

[17] Notice, for example, that if $C(q) = c(q - 0.5)^2$, then $C'(0.5) = 0$ for any cost parameter.

Denote as $\mu_s^1(\bullet)$ the posterior probability function that s is H. Then, the receiver's beliefs (denoted by $B1$) are:

$$\mu_s^1(q_s, q_{s'}, m_s, m_{s'}) = \left\{ \begin{array}{c} 1 \text{ if } q_s = q^* \text{ and } q_{s'} \neq q^* \\ 0 \text{ if } q_{s'} = q^* \text{ and } q_s \neq q^* \\ \mu_s^0(m_s, m_{s'}) \text{ otherwise} \end{array} \right\} \quad \text{(B1)}$$

where q^* satisfies the conditions:

$$1 - C(q^*) \geq q^\circ - C(q^\circ) \quad (3)$$

$$\text{and } 0 \geq (1 - q^*) - C(q^*) \quad (4)$$

The logic behind these beliefs can be stated as follows: if the actions are consistent with equilibrium strategies $\{q^*, 0.5\}$, then the identities of the senders are revealed perfectly. The same holds true if only one of them follows the equilibrium strategy of H (i.e., $q_s = q^*$ for only one sender). When neither or both senders choose q^*, then the receiver bases her expectation only on the statistical information revealed via the messages, and not on the strategic choices of the senders.

In other words, if only one sender is willing to bear the cost associated with q^*, then the actions of the senders distinguish between them. Otherwise, the choice of precision cannot differentiate between the two senders and the receiver resorts to the content of the messages.

The precision of the message sent by H in equilibrium should satisfy the two inequalities: (3) and (4). The first inequality ensures that H would not like to deviate from his choice of precision in equilibrium, and the second ensures that L would not like to imitate H.

Consider Eq. 3 first. In equilibrium the net payoff of H is $1 - C(q^*)$. However, if H deviates, it provokes the off the equilibrium beliefs and the receiver bases her decision on the perceived content of the message that she receives. In this case, H's expected net payoff is $\max_{0.5 \leq q \leq 1} q - C(q)$, which is equal to $q^\circ - C(q^\circ)$.[18] Thus, if q^* satisfies the condition $1 - C(q^*) > q^\circ - C(q^\circ)$, it is not optimal for H to deviate.

Now, consider Eq. 4. In equilibrium the payoff of L is 0. If L imitates H, he provokes the off equilibrium beliefs and his expected net payoff is $(1 - q^*) - C(q^*)$. Thus, if q^* satisfies the condition $0 > (1 - q^*) - C(q^*)$, it is not optimal for L to mimic H.

Therefore, a necessary condition for the existence of a separating equilibrium is that there exists a q^* that satisfies both inequalities. Define $\overline{q}$ and $\underline{q}$ as

[18] Recall that q° is the precision selected by H in the non-strategic equilibrium and that it satisfies the following equation $C'(q^\circ) = 1$.

the q's that satisfy the two conditions (3) and (4), respectively, with equality. Lemmas 7 to 9 in Appendix D show that the interval $[\underline{q}, \overline{q}]$ is interior and non-empty; that is, $0.5 < \underline{q} < 1$, and $\overline{q} > \underline{q}$. These ensure that any q such that $\underline{q} < q < \overline{q}$, can be supported in a perfect Bayesian equilibrium. This yields the main result:

Proposition 4 (PBE) *There exists a perfect Bayesian equilibrium where beliefs are given by* $B1$ *and senders' pure strategies are* $q_s = q^* \in [\underline{q}, \overline{q}]$ *if* s *is* H *and* $q_s = 0.5$ *if* s *is* L.

Proof In Appendix D. □

Thus, in equilibrium H sends an informative message and his competitor does not, all uncertainty about nature's selection is resolved, and the receiver chooses sender H with certainty.

The logic of the proof is quite simple. When the precision is endogenous, q can be set high enough in order to deter L from imitating H. Indeed, this is the case for any $q > \underline{q}$. The main challenge is to show that such a precision is not too high for H. (Recall that when H deviates, he optimally chooses q°). In other words, one needs to show that $\left[1 - C(\underline{q})\right] > \left[q^\circ - C(q^\circ)\right]$.

When $\underline{q} < q^\circ$ this is trivial. In such a case, H is required, in equilibrium (i.e., $q = \underline{q}$), to pay a lower cost ($C(\underline{q}) < C(q^\circ)$) and get in return a higher probability of being selected ($1 > q^\circ$).

When $\underline{q} > q^\circ$, things are a bit more interesting. Notice that H's net payoff in equilibrium (i.e., when $q = \underline{q}$) is $\underline{q}$ (since at $\underline{q}$, $\left[1 - C(q)\right] = q$). Thus, the equilibrium strategy is optimal for H as long as $\underline{q} > \left[q^\circ - C(q^\circ)\right]$, and this is immediate from $\underline{q} > q^\circ$.

This proposition illustrates the robustness of the separating result. In other words, the result still obtains when senders are allowed to determine the precision of their messages. Furthermore, the conditions for such a solution are less restrictive than before. Whereas in the simple model, a separating equilibrium existed only for a specific interval of the cost function, here the signaling equilibrium exists for any parameter value. The difference between the cases is that in the general model, H can always increase the precision to the point that imitation is too costly for L.

3.1.4 Discussion

The results here imply that when senders can invest in improving the precision of communication (as in many real-world applications), uncertainty is completely resolved and communication is perfect. This result holds even though communication remains noisy in equilibrium (i.e., $q^* < 1$).

Communication is perfect despite being noisy because (a) messages have some information content and (b) a sender's investment in improving the informativeness of the message (i.e., increasing its precision) serves as a signal that enables the receiver to distinguish between the two senders. In other words, the willingness to provide information itself provides information. Furthermore, this result holds for any set of parameters.

As before, an interesting feature of the separating equilibrium is that, in equilibrium, the receiver ignores the content of the message. Furthermore, doing so leads her to make the best choice. The following example is illustrative. Consider a case where only s sends an informative message and he is perceived by the receiver to be L (i.e., $m_s = -1$). Although the message content does not favor the sender, the receiver (knowing that her perception might be wrong) chooses this sender, and her choice is optimal with certainty. Thus, in the perfect Bayesian equilibrium, the receiver always chooses the sender that gives her the highest utility. In contrast, recall that when the receiver does not behave strategically, she occasionally chooses sender L.

4 Applications

This section discusses a few marketing applications in which messages are informative but noisy. In these settings, (a) marketers are frequently aware of the signaling aspects of their choice of precision, and (b) consumers tend to strategically respond to these actions (i.e., choice of precision).

Advertisements are probably amongst the most significant messages sent by firms. Ads fit the setting of the suggested model nicely: (a) the messages are noisy, (b) firms can increase the message precision (either directly or indirectly by increasing the number of ads), and (c) firms compete. It seems that the model can shed new light on an interesting aspect of informative ads—the effectiveness that is due to repetition. Specifically, it was shown in prior empirical work that consumers' tendencies to purchase the promoted product are an increasing function in the number of ads, and that such tendencies cannot be fully explained by Bayesian learning (Anand and Shachar 2005). The model presented here can offer an explanation for this empirical regularity. The logic is that since sending multiple ads is obviously a costly way to improve the precision of a message, as a result it can serve as a signal. Thus, consumers' tendencies to purchase the promoted product might be due to the signaling aspect of multiple ads.[19]

[19]Notice that the suggested explanation differs from the signaling theory of advertising presented by Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986) in various significant ways. For example, while the standard theory assumes that the content of the ads is empty, the suggested model focuses on informative advertising. Furthermore, while in the standard model the separation is enabled by repeated purchases, here we do not require repeated purchases and the separation is based on the usefulness of ads' content off the equilibrium path.

Interestingly, Zhao (2000) shows that under a certain condition the result of the studies mentioned above is reversed and higher advertising is associated with a lower quality firm. The condition is that advertising spending does not serve only as a signal but also as a determinant of the size of the market (via raising awareness). Furthermore, Desai (2000) demonstrates that advertising spending can serve as a signal of quality to the retailer.

A firm's salesforce is another noisy way through which it communicates with consumers. A question that has received attention recently is how active a firm's salespersons should be. Some firms, such as Apple, Sony, NikeTown, and Ford Motors, have recently adopted a more passive role for their salespeople.[20] Instead of hovering around the costumer, the salespeople are available for questions and offer the consumers the option of self-service. Unlike the traditional approach ("hovering"), the new format ("self-service") allows the consumer to get more precise information from her point of view, since she can ask everything about everything. According to the model presented here, the new approach taken by these firms might be due to their desire to differentiate themselves from lower-quality firms. In other words, since the new format demonstrates a greater willingness to provide information, one would predict that low-quality firms should be less inclined to adopt such a format even in the future. As a result, one can also expect that the choice of information-revelation format should signal the quality of the firm.

In addition to advertising and salesforces, there are many other ways in which firms communicate information to their constituents—through their websites, annual reports, public announcements, by engaging in word-of-mouth activities, etc. The idea that noisiness of communication can be overcome by a firm's willingness to provide information finds application in these settings as well. For example, firms may declare that, in addition to allowing customer reviews of its products on its website, they will not editorialize negative reviews as well.[21]

Beyond these, there are other applications as well. A political candidate may choose to reveal his prior military record even though such disclosure is not necessary, in order to allow voters to discriminate between him and his rival. A suspect, by refusing to take a lie detector test, may increase suspicion of guilt even though the test results are known to be noisy. Another appealing example was described to us by a colleague who was searching for a new house. The landlord stepped out of earshot when this colleague talked to the previous

[20]See discussion in Bhardwaj et al. (2005).

[21]For example, according to Forrester Research, 26% of online retailers currently allow individuals to input product reviews on the firm's website. Until recently, many firms reviewed these reviews and rejected those that were negative about the firm's product. Some firms are now changing their policy. For example, web retailer Overstock.com "had been relying on its merchandising group—the employees responsible for deciding which products to sell on the site—to monitor reviews submitted by customers, but found that the group tended to approve only positive reviews. In January, the Salt Lake City-based company changed the monitoring responsibilities to its marketing team. The company now says it posts both positive and negative comments. "We learned that customers won't trust the site if there are only positive reviews," says Tad Martin, senior vice president of merchandising and operations at Overstock.' " ("Giving Reviews the Thumbs Down", *Wall Street Journal*, August 4, 2005).

tenants about the apartment, and by doing so demonstrated his willingness to provide them with precise information. That colleague rented the apartment.[22]

5 Conclusion

This study formulates a central aspect of communication—miscommunication. We model it as a noise in the messages sent and received. This means that the sender of a message is uncertain about its perception by the receiver. We study a model where competing senders can send an informative but noisy message to a receiver, and find that there exists a separating equilibrium where senders' identity is completely revealed. Thus, although communication is noisy it delivers perfect results in equilibrium, and matches senders and receivers well. Interestingly, the information content of the messages is ignored by the receiver in such a signaling equilibrium; however, it plays a central role by shaping her beliefs off the equilibrium path.

In equilibrium senders overcome the noisiness of the messages by showing their willingness to invest in improving the precision of messages. This investment in information distinguishes between the sender whose product better matches the receiver's taste and that of his competitor. The intuition is, quite simply, that a sender's willingness to provide information itself provides information.

The basic idea of this general model can be applied to various forms of marketing communication. For example, elsewhere (Anand and Shachar 2006), we study an application of this idea to advertising. We show that even though advertising messages themselves are noisy, a firm's media selection (i.e., targeting) decision serves as a signal of product attributes and resolve consumer uncertainty, thereby improving the matching between consumers and products. There are, obviously, many additional forms in which firms communicate with consumers and other constituents, such as through salespeople, websites, announcements, annual reports, etc. Applying the basic idea presented here to such avenues of communication may lead to additional theoretical and empirical findings.

There are certain natural directions in which to extend the model. Such extensions can relax some of the assumptions either with respect to the information set of the receiver or with respect to the competition that the senders face. For example, the relevant assumptions about the information set are that (a) the receiver is not uncertain about the actions of the firms, and (b) she does not have any other sources of information (other than the messages of the senders). It might be interesting to examine the effect of relaxing the first assumption on the role of the message content on the equilibrium path, and

[22] We thank Dina Mayzlin for this example.

to analyze the interaction between the messages from the senders and other sources of information.

This study brings attention to a primitive of marketing—noisy communication (and its strategic consequences)—that has not received adequate attention so far. It seems that exploring aspects of noisy communication beyond those studied here can be fruitful. For example, while in our model noisy communication is an obstacle, in certain applications it might considered an advantage.[23] Future work could identify and analyze the different incentives behind noisy communication, and characterize these empirically.

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Appendix

Appendix A: Uniqueness of the simple model

There are two types of separating equilibria: (1) sender's strategies depend on their type, and (2) sender's strategies do not depend on their type.

In Appendix A.1 we show that for the first case there is a unique sequential equilibrium (the one presented in Section 2) in which $c > 1 - q$.

In Appendix A.2 we show that for the second case there is a unique sequential equilibrium in which $c < 1 - q$.

Thus, for $c > 1 - q$ there is a unique separating (sequential) equilibrium as stated in Proposition 2.

A.1 Strategies depend on senders' types

Here, we study a separating equilibrium in which sender's strategies depend on their types.

We start by characterizing the only set of consistent beliefs in such a separating equilibrium. Then we show that for these beliefs and the given parameter values, there is a unique separating equilibrium.

In any separating equilibrium, either H sends a message and L not, or the reverse is true. We consider each case in turn.

Case 1 H sends a message, and L does not.

[23] It is well known that political candidates occasionally increase the nosiness of their messages in order to maximize their winning probability. The rationale behind such a strategy is related to the "median voter theory" which suggests minimum differentiation between the candidates.

Let H send a message with probability $1-\varepsilon_H$, and L send a message with probability ε_L (both ε_H and ε_L are greater than 0). Recall that the prior probability that player s is H is $\mu_s^0(r_s)$. Then (using Bayes rule) the receiver's beliefs at each of her four information sets are:

$$\mu_s^\varepsilon = \left\{ \begin{array}{ll} \frac{(1-\varepsilon_H)(1-\varepsilon_L)\mu_s^0(r_s)}{(1-\varepsilon_H)(1-\varepsilon_L)\mu_s^0(r_s)+\varepsilon_H\varepsilon_L(1-\mu_s^0(r_s))} & \text{if } A_s=(1,0) \\ \frac{\varepsilon_H\varepsilon_L\mu_s^0(r_s)}{\varepsilon_H\varepsilon_L\mu_s^0(r_s)+(1-\varepsilon_H)(1-\varepsilon_L)(1-\mu_s^0(r_s))} & \text{if } A_s=(0,1) \\ \frac{(1-\varepsilon_H)\varepsilon_L\mu_s^0(r_s)}{(1-\varepsilon_H)\varepsilon_L\mu_s^0(r_s)+(1-\varepsilon_H)\varepsilon_L(1-\mu_s^0(r_s))} & \text{if } A_s=(1,1) \\ \frac{\varepsilon_H(1-\varepsilon_L)\mu_s^0(r_s)}{\varepsilon_H(1-\varepsilon_L)\mu_s^0(r_s)+(1-\varepsilon_L)\varepsilon_H(1-\mu_s^0(r_s))} & \text{if } A_s=(0,0) \end{array} \right\} \tag{5}$$

where A_s is an indicator vector in which the first variable is equal to 1 if s sends a message and zero otherwise, and the second variable is equal to 1 if s's competitor sends a message and zero otherwise.

It is straightforward to show that the limit of μ^ε as $\varepsilon_H, \varepsilon_L \to 0$ is:

$$\mu_s = \left\{ \begin{array}{rl} 1 & \text{if } A_s=(1,0) \\ 0 & \text{if } A_s=(0,1) \\ \mu_s^0(r_s) & \text{if } A_s=(1,1) \\ \mu_s^0(r_s) & \text{if } A_s=(0,0) \end{array} \right\}$$

which are exactly the beliefs in (B).

Recall that under these beliefs, the following table represents the net payoff functions of both senders.

		H: does not send a message	H: sends a message
L	does not send a message	0.5 (H); 0.5 (L)	$1-c$ (H); 0 (L)
	sends a message	0 (H); $1-c$ (L)	$q-c$ (H); $1-q-c$ (L)

Now, one can see that when c is not in the interval $[1-q, 0.5]$, there is no separating equilibrium in which H sends a message and L does not: (a) when $c < 1-q$, sender L finds it profitable to imitate H; (b) when $c > 0.5$, sender H deviates.

Case 2 L sends a message, and H does not.

In this case, it is easy to show that the only consistent beliefs are:

$$\mu_s = \left\{ \begin{array}{rl} 0 & \text{if } A_s=(1,0) \\ 1 & \text{if } A_s=(0,1) \\ \mu_s^0(r_s) & \text{if } A_s=(1,1) \\ \mu_s^0(r_s) & \text{if } A_s=(0,0) \end{array} \right\}$$

The following table represents the net payoff functions of both senders.

		H: does not send a message	*H*: sends a message
L	does not send a message	0.5 0.5	$-c$ 1
	sends a message	1 $-c$	$q-c$ $1-q-c$

It is clear that in this case, it is not optimal for either sender to send a message.

A.2 Strategies do not depend on senders' types

Lemma 5 *A separating equilibrium in which sender's strategies do not depend on their type, exists if and only if* $1-q>c$.

Proof Without loss of generality, consider the case where sender 1 sends a message and sender 2 does not.

First, we characterize beliefs. To obtain consistent beliefs, we describe the players strategies.

Player 1 sends a message with probability $1-\varepsilon_{1H}$ if he is H and $1-\varepsilon_{1L}$ if he is L. Player 2 sends a message with probability ε_{2H} if he is H and ε_{2L} if he is L. Denote the prior probability that player 1 is H by p. Then (using Bayes rule) the beliefs that player 1 is H are:

$$\mu_1^\varepsilon = \left\{ \begin{array}{ll} \frac{(1-\varepsilon_{1H})(1-\varepsilon_{2L})p}{(1-\varepsilon_{1H})(1-\varepsilon_{2L})p+(1-\varepsilon_{1L})(1-\varepsilon_{2H})(1-p)} & \text{if } A_1=(1,0) \\ \frac{\varepsilon_{1H}\varepsilon_{2L}p}{\varepsilon_{1H}\varepsilon_{2L}p+\varepsilon_{1L}\varepsilon_{2H}(1-p)} & \text{if } A_1=(0,1) \\ \frac{(1-\varepsilon_{1H})\varepsilon_{2L}p}{(1-\varepsilon_{1H})\varepsilon_{2L}p+(1-\varepsilon_{1L})\varepsilon_{2H}(1-p)} & \text{if } A_1=(1,1) \\ \frac{\varepsilon_{1H}(1-\varepsilon_{2L})p}{\varepsilon_{1H}(1-\varepsilon_{2L})p+\varepsilon_{1L}(1-\varepsilon_{2H})(1-p)} & \text{if } A_1=(0,0) \end{array} \right\} \tag{6}$$

This can be rewritten as:

$$\mu_1^\varepsilon = \left\{ \begin{array}{ll} \frac{p}{p+\frac{(1-\varepsilon_{1L})(1-\varepsilon_{2H})}{(1-\varepsilon_{1H})(1-\varepsilon_{2L})}(1-p)} & \text{if } A_1=(1,0) \\ \frac{p}{p+\frac{\varepsilon_{1L}\varepsilon_{2H}}{\varepsilon_{1H}\varepsilon_{2L}}(1-p)} & \text{if } A_1=(0,1) \\ \frac{p}{p+\frac{(1-\varepsilon_{1L})\varepsilon_{2H}}{(1-\varepsilon_{1H})\varepsilon_{2L}}(1-p)} & \text{if } A_1=(1,1) \\ \frac{p}{p+\frac{\varepsilon_{1L}(1-\varepsilon_{2H})}{\varepsilon_{1H}(1-\varepsilon_{2L})}(1-p)} & \text{if } A_1=(0,0) \end{array} \right\} \tag{7}$$

It is clear that the limit of μ_1^ε for $A_1=(1,0)$ is p. The μ_1^ε of the other elements can be either 0 or 1 (depending on the the ratios $\frac{\varepsilon_{1L}}{\varepsilon_{1H}}$ and $\frac{\varepsilon_{2H}}{\varepsilon_{2L}}$.)

Using these beliefs, we can now check for optimality of sender strategies. Note that we can ignore the case where the limit of μ_1^ε for $A_1=(0,0)$ is 1, since in this case, it is optimal for any type of player 1 to deviate. Thus, we only focus on cases where the limit of μ_1^ε for $A_1=(0,0)$ is 0.

Thus, we are interested in two cases [notice also that the limit of μ_1^ε for $A_1 = (0, 1)$ is irrelevant for the Nash equilibrium].

Case 1 μ is given by:

$$\mu = \left\{ \begin{array}{rll} p & \text{if} & A_1 = (1,0) \\ \text{Not relevant} & \text{if} & A_1 = (0,1) \\ 0 & \text{if} & A_1 = (1,1) \\ 0 & \text{if} & A_1 = (0,0) \end{array} \right\}$$

In this case, the expected net payoffs to senders are:

2 \ 1	does not send a message	sends a message
does not send a message	0 1	$p-c$ $1-p$
sends a message		$-c$ $1-c$

Irrespective of the choice by nature, there is no (separating) equilibrium in this case: (a) If $p > c$, then it is optimal for 2 to deviate. (b) But if $p < c$, then it is optimal for 1 to deviate.

Case 2 μ is given by:

$$\mu = \left\{ \begin{array}{rll} p & \text{if} & A_1 = (1,0) \\ \text{Not relevant} & \text{if} & A_1 = (0,1) \\ 1 & \text{if} & A_1 = (1,1) \\ 0 & \text{if} & A_1 = (0,0) \end{array} \right\}$$

In this case, the expected net payoffs to senders are:

2 \ 1	does not send a message	sends a message
does not send a message	0 1	$p-c$ $1-p$
sends a message		$1-c$ $-c$

In this case, 2 has no incentive to deviate. The conditions that assure that 1 will not deviate are: (a) if he is of type H, it must be that $q > c$ and if he is of type L it must be that $1 - q > c$. Thus, a necessary condition to sustain a (separating) equilibrium is that $1 - q > c$. □

Appendix B: Non-informative messages

Here, we show that when the messages are non-informative (i.e., $q = 0.5$), there is no separating (sequential) equilibrium.

Lemma 6 *When $q = 0.5$ a separating sequential equilibrium does not exist.*

Proof There are two types of potential separating equilibria: (1) H sends a message and L does not, and (b) the other way around.

Case 1 H send a message with probability $1 - \varepsilon_H$ and L sends a message with probability ε_L. It is easy to show that the consistent beliefs are:

$$\mu_s = \begin{Bmatrix} 1 & \text{if } A_s = (1, 0) \\ 0 & \text{if } A_s = (0, 1) \\ 0.5 & \text{otherwise} \end{Bmatrix} \tag{8}$$

Thus, the expected net payoffs are:

		H: does not send a message	H: sends a message
L	does not send a message	0.5 0.5	$1 - c$ 0
	sends a message	0 $1 - c$	$0.5 - c$ $1 - 0.5 - c$

When $c < 0.5$, then the only equilibrium of this game is (1, 1) and when $c > 0.5$, then the only equilibrium of this game is (0, 0). Thus, there is no separating equilibrium that is consistent with these beliefs.

Case 2 L sends a message with probability $1 - \varepsilon_L$ and H sends a message with probability ε_H. It is easy to show that the consistent beliefs are:

$$\mu_s = \begin{Bmatrix} 0 & \text{if } A_s = (1, 0) \\ 1 & \text{if } A_s = (0, 1) \\ 0.5 & \text{otherwise} \end{Bmatrix} \tag{9}$$

and the only equilibrium is (0, 0) irrespective of the cost. □

Appendix C: Endogenous precision: non-strategic equilibrium

Proposition 3 (Non strategic equilibrium)

Proof We start by demonstrating that sending a non-informative message ($q = 0.5$) is a dominant strategy for L. Then we show that as a result, it is optimal for H to send an informative message ($q > 0.5$). To simplify the presentation, assume (without loss of generality) that player 1 is H.

If $q_2 \leq q_1$ (i.e., the precision of the message sent by L is at most as precise as the message sent by H), the probability that L is selected (irrespective of q_2) is $1 - q_1$.[24]

And, if $q_2 > q_1$, the probability that L is selected is $1 - q_2$, which is lower than $1 - q_1$.

Thus, the probability that L is selected is a non-increasing function in q_2, while his cost is an increasing function in q_2. It follows that $q_2 = 0.5$ is a dominant strategy for L.

Next, we show that if $C'(0.5) < 1$, it is optimal for H to send an informative message ($q > 0.5$). It is easy to show that $E(d_1|q_1, q_2 = 0.5) = q_1$ and thus:

$$\frac{\partial E\left[\pi_1(q_1); q_2 = 0.5\right]}{\partial q_1} = 1 - C'(q_1)$$

Let q^0 denote the optimal precision for H. Specifically, q^0 satisfies the following condition $1 - C'(q^0) = 0$. Since $C'' > 0$ and $C'(0.5) < 1$, it immediately follows that $q^0 > 0.5$. □

Appendix D: Endogenous precision: strategic equilibrium

Lemma 7 *There exists a* $\underline{q}$ *where* $0.5 < \underline{q} < 1$ *that satisfies the condition:* $(1 - \underline{q}) - C(\underline{q}) = 0$*. Furthermore, for any* $q > \underline{q}$*,* $(1 - q) - C(q) < 0$.

Proof The function $(1 - q) - C(q)$ is decreasing in q, and is positive at $q = 0.5$ and negative at $q = 1$. □

Lemma 8 *There exists a* $\overline{q}$ *where* $\overline{q} > 0.5$ *that satisfies the condition:* $1 - C(\overline{q}) = \max_{0.5 \leq q \leq 1} q - C(q)$*. Furthermore, for any* $q < \overline{q}$*,* $1 - C(q) > \max_{0.5 \leq q \leq 1} q - C(q)$.

Proof The function $1 - C(q) - \left[q^0 - C(q^0)\right]$ is decreasing in q and is positive at $q = 0.5$. □

Lemma 9 $\overline{q} > \underline{q}$.

Proof The function $1 - C(q) - \left[q^0 - C(q^0)\right]$ is decreasing in q. Next, we show that it is positive at $\underline{q}$.

If $\underline{q} < q^0$, $1 - C(\underline{q}) - \left[q^0 - C(q^0)\right] = \left[1 - q^0\right] + \left[C(q^0) - C(\underline{q})\right]$ where both elements are positive.

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[24]If $q_2 < q_1$, the receiver bases her decision only on m_1 (i.e., the perceived content of the more precise message). The probability that $m_1 = -1$ (i.e., indicating that 2 is H) is $1 - q_1$. Thus, $E(d_2|q_2, q_1) = 1 - q_1$.

If $q_2 = q_1$, the probability that $m_1 = -1$ and $m_2 = 1$ (i.e., both messages indicate that 2 is H and the receiver selects 2) is equal to $(1 - q_1)^2$ and the probability that the messages contradict one another (in this case the receiver selects one of the senders randomly) is $2q_1(1 - q_1)$. Thus, $E(d_2|q_2, q_1) = (1 - q_1)^2 + \frac{1}{2}2q_1(1 - q_1) = (1 - q_1)$.

If $\underline{q} > q^0$, $1 - C(\underline{q}) - \left[q^0 - C(q^0)\right] > \left[1 - \underline{q}\right] - C(\underline{q}) + C(q^0) > \left[1 - \underline{q}\right] - C(\underline{q}) = 0$. □

We are now ready to prove Proposition 4:

Proof (Proposition 4) To simplify the presentation, assume (without loss of generality) that player 1 is H. Given the beliefs and $q_1 = q^*$, where $\underline{q} < q^* < \overline{q}$, L will optimally choose $q_2 = 0.5$ since: (a) choosing any q such that $0.5 < q < q^*$ or $q > q^*$ involves a cost without any revenues, and (b) choosing $q_2 = q^*$ leads to losses from Lemma 8.

Given the beliefs and $q_2 = 0.5$, H will optimally choose $q_1 = q^*$ since the highest payoff from any $q \neq q^*$ is $q^0 - C(q^0)$ which is smaller than the equilibrium payoff $1 - C(q^*)$ as illustrated by Lemma 9.

It is trivial to show that beliefs agree with senders' strategies. □

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