Learning from Deregulation:

The Asymmetric Impact of Lockdown and Reopening on Risky Behavior During COVID-19*

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Abstract

During the COVID-19 pandemic, states issued and then rescinded stay-at-home orders that restricted mobility. We develop a model of learning by deregulation, which predicts that lifting stay-at-home orders can signal that going out has become safer. Using restaurant activity data, we find that the implementation of stay-at-home orders initially had a limited impact, but that activity rose quickly after states' reopenings. The results suggest that consumers inferred from reopening that it was safer to eat out. The rational, but mistaken inference that occurs in our model may explain why a sharp rise of COVID-19 cases followed reopening in some states.

Keywords: coronavirus, COVID-19, public health measures, mobility

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1. INTRODUCTION

Governments sometimes go so far as to ban an activity that is deemed sufficiently unsafe, at least until the perceived risk has sufficiently abated. Beaches are closed when sharks are spotted in the water, restaurants with unclean food practices are shuttered, and, in the spring of 2020, whole states and countries were essentially shut down because of fear of COVID-19.

Lifting these bans can have both a direct effect—people who always wanted to do the activity can do so—and an indirect effect—the lifting of the regulation sends a signal suggesting that the activity is now safe. We refer to this signaling effect as learning from deregulation. In this paper, we develop a model of learning from deregulation and test the model's predictions about restaurant visits before and after COVID-19 lockdown periods.

Our model is motivated by the basic course of COVID-19. Beginning in March 2020, states and cities around the United States issued stay-at-home orders that restricted mobility and lasted for months. Data from SafeGraph and Yelp shows that people had largely stopped eating in restaurants before the local lockdowns took effect. Indeed, once we control for nationwide trends, the onset of local regulation had almost no impact, which has led some to argue that the lockdowns had no effect on behavior (Goolsbee & Syverson, 2021; Gupta et al., 2020).

PLACE FIGURE 1 HERE

Extending the data through the end of the regulatory lockdown, we observe an *asymmetric* response to reopening, where restaurant visits spiked shortly after state governments allowed restaurants to reopen for dine-in service. As Figure 1 shows, the public radically reduced restaurant visits before lockdown orders were issued, but reopening was immediately followed by sharp increases in eating out. Why would the impact of ending a regulation be larger in magnitude than the impact of imposing the regulation?

Standard static models of regulation and behavior suggest a symmetry between the imposition and the elimination of a restriction, but that symmetry ends if restricting behavior also reduces the flow of information about that behavior. Section II presents a simple dynamic signaling model, where the public had information about safety before closing, but not before reopening. Consequently, there is no signaling impact of closing, but if the government has private information about risk then reopening provides a signal of safety.

In the model, consumers take an action, such as dining out, that is normally safe, but has temporarily become risky. Both the government and consumers learn about the onset of the risk simultaneously, which first leads consumers to reduce their activity and then leads the government to impose a lockdown. By assumption, consumers move first because dining in takes less time than crafting an executive order. Once the shutdown order occurs, consumers lose their direct source of information about the risk, but the government receives a private signal about safety and decides whether to reopen the economy.

We show that there exists a separating or semi-pooling equilibrium where the reopening decision signals that the world has become safer, which then triggers a positive consumer response to reopening. If politicians are risk-averse, perhaps because they have a relatively high probability of reelection, then the equilibrium is separating and reopening perfectly signals safety. If some governments are risk-loving, perhaps because their re-election is a long shot, then governments with an adverse signal may pool with governments that receive a positive signal. In that case, reopening can lead to harm because consumers interpret reopening to suggest greater safety than is the case.

After developing this model, we then explore the model's testable implications in the data. In Section III, we discuss our data sources from SafeGraph and Yelp. In Section IV, we present our results on the asymmetric impact of the lockdowns and reopenings. In this section, we also estimate the interaction between re-opening and the Republican vote share in the 2016 general election, which we interpret as an indication of optimism about safety. Additionally, we show the relationship between the timing of reopening and post-reopening COVID-19 rates, which suggests that consumers interpreted reopening to imply less risk than there was in reality. Section V concludes. Neither the model nor our empirical work suggests that reopening was necessarily a good or bad decision, but it does suggest consumers interpreted states' reopening decisions as a signal that the risk had abated, even in states where that does not appear to have been the case.

2. A SIMPLE MODEL OF BELIEFS AND REGULATORY REOPENING

This paper focuses on a simple, common hypothesis: consumers believe that the end of a regulatory lockdown signifies that they can safely return to their interactive activities. To make sense of this belief, and of why it might occasionally be incorrect, we investigate a simple, stylized model of a government that chooses whether or not to end a regulatory ban in the presence of private information. We treat this as a game between a unitary decision-maker (the government) and non-strategic consumers, who undertake an action if they believe that the risk associated with that action is less costly than the reward. We will describe the sequence of events before the reopening decision and explain why they are consistent with our assumptions about the actors' behaviors, and then characterize the Bayesian Nash equilibrium in the reopening game between government and consumers.

2.1 Model setup

Individuals can take a potentially risky action "A," such as going to a restaurant. The net flow benefit from the action is denoted $B_i > 0$ and we abstract away from non-health related costs. The distribution of benefits is characterized by a cumulative distribution function F(B) and related density f(B). Taking the action creates a risk of harm which is unknown, time changing and denoted

π_t. Consumers' expected value of π_t at the start of period t is $\tilde{\tilde{\pi}}_t$. The private cost of harm equals C and the social cost of harm equals $C + \Delta$. The share of individuals who will undertake the action, unless there is a public lockdown, equals $1 - F(\tilde{\tilde{\pi}}_t\, C)$. If there is a lockdown, then no one undertakes the action. The social costs of harm will also be spread across consumers, but those costs will not influence behavior.

While the action in this model could refer to going to a restaurant, and the harm could be catching a contagious disease, this model can also describe other settings where governments ban activities. For example, the government can ban swimming because of either rough waves or sharks, or the government can ban a restaurant because of food poisoning. We assume that the government's policy toolkit is particularly limited: it can either ban the action or not.

At the period of reopening, the government's objective function is the expected value of $G(Average\ Consumer\ Welfare - Social\ Cost\ of\ Harm)$, where $G(x) = x + \alpha|x|$. The function G(.) is concave around zero if $\alpha < 0$ and convex if $\alpha > 0$. We assume that $-1 < \alpha < 1$, so that the function is always monotonic. The α term can generate a divergence between the government's reopening decision and the socially optimal reopening decision, because the government behavior is either more risk averse or risk loving than the public at large. We assume that α is known to the public. We interpret this parameter as a reflection of political incentives rather than deep preferences. For example, the governmental leader might ultimately care about re-election, as is the norm in political economy models of career concerns (e.g. Besley, 2007). If the politician is headed towards an easy victory, then $\alpha < 0$ because added volatility will increase the possibility of losing power. If the politician is likely to lose office without some major success, then $\alpha > 0$.

The value of π_t is either $\underline{\pi}$ or $\overline{\pi}$. The state is always observed accurately by the public at the end of each period if consumption is allowed. When consumption is not allowed, we assume that the government receives a private signal of the state of the world. The probability transitions from $\pi_t = \underline{\pi}$ to $\pi_t = \overline{\pi}$ with probability $\delta_O \approx 0$ and from $\pi_t = \overline{\pi}$ to $\pi_t = \underline{\pi}$ with probability $\delta < .5$. We treat $\delta_O = 0$ to capture the highly unexpected nature of the emergence of the high-risk state.

2.2 Timing

We assume the following sequence of events. At time t=-2, the state of the world changed from $\pi_t = \underline{\pi}$ to $\pi_t = \overline{\pi}$, and this change was observed at the start of period t=-1. Consequently, demand fell from $1 - F(\underline{\pi}\ C)$ to $1 - F((\ \delta\underline{\pi} + (1-\delta)\overline{\pi})C)$ between periods -2 and -1. The government then imposed a ban on the action that caused the level of activity to fall to zero at time t=0.

Why did the government wait a time period, and why did a ban maximize the government's welfare? We assume that the time delay was technological, and that the government could not immediately institute a ban. We also assume either that the government solved a dynamic programming problem and determined that the ban was in its interests, or that external forces induced the government to undertake the ban. We are not interested in that initial decision, and so we will not microfound the government's optimization problem at the point of the initial ban.

Instead, we are interested in the government's decision to reopen, which will allow the action to be taken at t=1. We assume that after t=1, the game ends. Consumers have no direct information on the risks of the action, because the action has been shut down, but do know the government's objective function, transition probabilities, and that $\pi_{-1} = \overline{\pi}$. We further assume that the government has access to a private signal that informs it of the state of the world as of time t=0.

2.3 Equilibrium

We define a Bayesian equilibrium in this setting as one in which (1) if the government reopens, then consumers' believe that the probability of harm $\overbrace{\pi}_1$ satisfies Bayes' rule so that $\overbrace{\pi}_1 = \frac{\delta P_{Low\,Risk}^{Reopen} + (1-\delta)\delta P_{High\,Risk}^{Reopen}}{\delta P_{Low\,Risk}^{Reopen} + (1-\delta) P_{High\,Risk}^{Reopen}}$, where $P_{Low\,Risk}^{Reopen}$ and $P_{High\,Risk}^{Reopen}$ reflect the probability that the government will reopen conditional upon receiving a low risk or high risk signal (with probability δ, a high risk situation becomes low risk in the next period, and the situation was high risk at t=-1), (2) consumer demand equals $1 - F(\overbrace{\pi}_1\, C)$, (3) the government always opens if the expectation of its payoff upon reopening is higher than zero, never opens if it is lower than zero, and may randomize between opening and reopening if its expected payoff conditional upon reopening equals zero.

We also assume that B is uniformly distributed on the unit interval, so that average social welfare less social cost of harm equals $(1 - \overbrace{\pi}_1\, C)\left(\frac{1+\overbrace{\pi}_1\, C}{2} - \underline{\pi}(C + \Delta)\right)$ if risk is low and known to be low and $(1 - \overbrace{\pi}_1\, C)\left(\frac{1+\overbrace{\pi}_1\, C}{2} - \overline{\pi}(C + \Delta)\right)$ if the risk is high. We assume that $\frac{1}{\underline{\pi}} > 2\Delta + C > \frac{1}{(1-\delta)\overline{\pi} + \delta\overline{\pi}}$, which ensures that the net social welfare from allowing the action is negative if risks were high last period. This assumption helps justify why the prohibition was instituted in the first place.[i]

The following proposition refers to high demand as the demand at time -2 (i.e. $1 - \underline{\pi}\, C$) and low demand as the demand if there is no information content from reopening (i.e. $1 - C((1-\delta)^2\overline{\pi} + (2-\delta)\delta\underline{\pi})$), which equals the demand before closing if $\delta = 0$. We identify low demand after reopening with a symmetric response to opening and closing and high demand after reopening as an asymmetric response.

Proposition 1: There exist two values $\alpha_2 > \alpha_1 > 0$, so that if $\alpha_1 > \alpha$, then the public sector only reopens if the risk fell in period 0, and in that case, there is high demand after reopening. If $\alpha > \alpha_2$, then the government always reopens and there is low demand after reopening. If $\alpha_2 > \alpha >$

α_1, there are three possible Bayesian equilibrium: (1) the government always reopens and there is low demand after reopening, (2) the government reopens only if the risk has declined and there is high demand after reopening and (3) the government always reopens if the risk has declined and sometimes reopens if the risk has not declined, and in this semi-pooling equilibrium, the demand after reopening lies between high and low demand and is increasing with δ, and decreasing with α, Δ, C, $\overline{\pi}$, and $\underline{\pi}$.

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Proposition 1 predicts that symmetry occurs in a pooling equilibrium, where there is no new information from the act of reopening, or if the government had no information. If the government has information and opens only when it has learned that conditions have shifted to safety, then the proposition predicts asymmetry. The fundamental difference between opening and closing is that when the action is ongoing, the consumer knows as much as the government. During lockdown, the government continues to receive a signal while consumers do not. Thus, consumers rely on the reopening decision to learn the state of the world. However, when there is semi-pooling, that inference will be imperfect, though it still is positive and produces some demand response to reopening. Unless there is complete pooling, there will be an asymmetric response to reopening, relative to the initial closing, because reopening generates some information.

The model also predicts that if the disease was thought to be less harmful, because either C or $\overline{\pi}$ or $\underline{\pi}$ are lower, then the response to reopening will be larger in a semi-pooling equilibrium. This is not so much a reflection of signaling, but rather that these variables capture the expected harm from reopening as long as there is some chance that the risk level is high. A higher value of δ increases demand in a semi-pooling equilibrium, because it increases the chances that the high-risk situation has evolved into a low risk situation. Higher values of α imply more pooling, which generate weaker demand, and low values of α imply separating, which will generate strong demand.

While the model is framed around a single action, it could easily be modified to allow multiple actions, all of which generate risk that depends on the state of the world. The asymmetry should appear most stark in those activities that are most dependent upon the perceived state of the world. For example, visits to a park or an open beach would, independent of whether or not the world is high risk, experience a more symmetric effect of the lockdown and reopening. Restaurants visits, which present greater risks than more general activity, would be expected to exhibit a more asymmetric reopening effect, if reopening is taken to mean that the risk is lower.

Even within restaurant visits, one would expect variation in the asymmetry of the response. For example, if there were some restaurants that were known to provide safety through social distancing, or if some restaurants only provided take-out service, then those restaurants should experience a more symmetric effect of the lockdown and the reopening than restaurants that primarily offer indoors, dine-in service.

3. SAFEGRAPH AND YELP DATA BEFORE AND AFTER STAY-AT-HOME REGULATIONS

We construct a daily, state-level time series for six measures of mobility and restaurant activities using cell phone location data from the geospatial data company SafeGraph[ii] and the online review platform Yelp. Our data range from December 1, 2019 to June 14, 2020. Due to the proprietary nature of our data from Yelp, all of our measures of mobility and restaurant activities are calculated relative to a state's December 2019 daily average. For example, a value of 0.75 indicates a 25% reduction in activity relative to the December 2019 daily average.

First, to understand how states' lockdown and reopening orders affected general mobility, we construct an "All Visits" measure, which uses SafeGraph's geospatial data to count the number of daily visits to all places-of-interest within a state. We then construct two mobility measures related to restaurant activity. The first, "Restaurant Visits" measures the total number of visits to all restaurants and other food-service locations. The second, "Full-Service Restaurant Visits" further restricts this sample to only those businesses classified by NAICS as full-service restaurants; cafeterias, grill buffets and buffets; or drinking places (alcoholic beverages).[iii]

PLACE TABLE 1 HERE

We present summary statistics for these three measures of mobility in Table 1. Of primary relevance to our analysis is how these measures changed around the states' stay-at-home and reopening orders. To illustrate these changes, we calculate ΔStay Home and ΔReopen, as the difference between the average value of each measure in the week after and the week before an order was issued. We present the mean and standard deviation of these differences in the last four columns of Table 1. As expected, we see evidence of a decline in mobility after the issuance of a stay-at-home order, and an increase in mobility following reopening decisions. Notably, the change in mobility around reopening, relative to the decline in mobility around the stay-at-home order is increasing in the relative risk of the activity, as we move from general mobility ("All Visits") to visiting a full-service restaurant ("Full-Service Restaurant Visits").

We next construct three measures of restaurant activity using data from Yelp. First, we use data from the Yelp Reservations platform to understand how people's willingness to dine *at* a restaurant changes in response to the COVID-19 pandemic, and states' local orders. This measure represents the total number of reservations scheduled to occur on a given date.[iv] Second, we use data from the Yelp Transactions Platform, which allows users to place delivery and takeout orders with local restaurants, to construct our "Orders" sample. Our final measure of restaurant activity is our "Page Views" sample, which totals the number of restaurant page views online or via mobile apps. We take this as a measure of user intent—to visit or order from a restaurant.

Summary statistics for each of these three measures are presented in Table 1. Similar to the Full-Service Restaurant Visits measure from the SafeGraph data, we see initial evidence of a decline in reservations and page views at the time of shutdown, and a relatively large increase in activity at the time of the shutdown, consistent with the asymmetric response predicted by our model. The Orders sample, however, shows a different response. On average, we see an increase in the number of delivery and takeout orders in the week following the issuance of a stay-at-home order[v], although, as illustrated in Figure 1a, this reflects a general upward trend that predates stay-at-home orders.

In comparison, it is difficult to tell whether the response to lockdown and reopening is symmetric from macro data. According to the U.S. Census Monthly Retail Trade reports[vi], total retail and food services have dropped in March (-8.22% from previous month) and April (-14.71%) but increased in May (18.16%) and June (7.50%). The month-to-month changes on food services and drinking places are of greater magnitudes, namely -30.04% and -34.32% in March and April, and +31.55% and +20.04% in May and June. While the percentage changes seem comparable before and after April, the absolute terms tell a different story. According to the National Restaurant Association, although June 2020 represented the highest monthly sales volume since the beginning of the pandemic lockdowns in March, it still remained roughly $18 billion (27.6%) down from the pre-coronavirus sales levels posted in January and February.[vii] These macro statistics differ from our data, because they tend to reflect national changes and do not account for the timing of local lockdown and reopening orders.

In addition to the mobility and restaurant activity measures described above, we also use data on COVID-19 penetration, and the vote share for the Republican candidate in the 2016 presidential election. To measure COVID-19 penetration, we collect daily, cumulative COVID-19 case counts from New York Times[viii] and the vote share data from Townhall.com.[ix] Finally, we identified the date of each state's stay-at-home and reopening orders from a combination of local news reports and official government documentation.[x] Since states typically adopted a multi-phase approach to reopening, we classify a state as having reopened on the first date that restaurants were permitted to serve customers on premises (either indoors or outdoors).

4. THE ASYMMETRIC IMPACT OF STARTING AND ENDING LOCKDOWN POLICIES

In this section, we first document an asymmetric response to states' stay-at-home orders across all six of our measures of mobility and restaurant activity. We then explore two implications of our model: 1) that the asymmetry should appear larger in activities that are most dependent on the perceived amount of risk in going out, and 2) that there is a larger reopening response under a semi-pooling equilibrium when the population believes that the virus risk is lower. Following that, we consider the role of publicly available information about COVID-19 cases in the public's daily behavior. Finally, we provide suggestive evidence that some states are indeed in the semi-pooling equilibrium described in Proposition 1.

4.1 Empirical specification

To document the asymmetric response to states' shutdown and reopening orders, we estimate the following regression equation,

$$y_{s,t} = \alpha + \beta_1 1(Stay\ Home)_{s,t} + \beta_2 1(Reopen)_{s,t} + \gamma_1 1(No\ Covid)_{s,t} + \gamma_2 Rel.Covid_{s,t} + \delta_s + \delta_t + \epsilon_{s,t}.$$

Where $y_{s,t}$ is the outcome of interest for state s on day t, and $1(Stay\ Home)_{s,t}$ and $1(Reopen)_{s,t}$ are indicators for whether the state was under a stay-at-home or reopening order on day t. To account for COVID-19 penetration, we include a control equal to one if the state has not disclosed the number of COVID-19 cases by day t, $1(No\ Covid)_{s,t}$, and a relative measure of COVID-19 if the information on COVID-19 cases is available, $Rel.\ Covid_{s,t}$.

More specifically, to construct $Rel.\ Covid_{s,t}$, we first compute a state's total number of COVID-19 cases per million population as of day t, and then divide it by the nationwide number of COVID-19 cases per million population on the same day. [xi] We use this relative measure because the absolute number of COVID-19 cases follows a similar trend in many states and the relative measure emphasizes the heterogeneity of state experiences. Finally, we include state and day fixed effects, δ_s and δ_t.

We consider three mobility outcomes: all visits to places-of-interest, visits to all restaurants, and visits to full-service restaurants, using data from SafeGraph, a geospatial data company. Additionally, we consider three measures from Yelp, the online review platform, which are more closely linked to restaurant activity: reservations, takeout and delivery orders, and the number of page views for restaurants' Yelp listings. These outcomes are discussed in more detail in Section III.

4.2 Results

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Table 2 shows that all six of our outcomes of mobility and restaurant activity display an asymmetric response to regulation. There is little or no response to the states' initial shutdown orders. The largest response is a 2.6 percentage point reduction in our SafeGraph measure of visits to all places-of-interest. We find no evidence of a response to stay-at-home orders across all three

Yelp outcomes. These findings are consistent with emerging research on the economic impact of stay-at-home orders, including Chudik, Pesaran, and Rebucci (2020), Brzezinski, Deiana, Kecht, and Van Dijcke (2020), Brzezinski, Kecht, Van Dijcke (2020), Gupta et al. (2020), Goolsbee and Syverson (2021), Cronin and Evans (2020), and Farboodi, Jarosch, and Shimer (2020). We also find that reopening decisions are associated with a meaningful increase in activity across all outcomes, ranging from a 4.1 percentage point increase in the broadest SafeGraph outcome, to a nearly 25 percentage point increase in reservations placed via the Yelp Reservations platform. This disparity between a near-zero response to stay-at-home orders and a more sizable reopening response to reopening decisions, is the asymmetric policy response characterized in Section 2.

We now consider two implications of our model. First, we test whether the asymmetric response to policy is more stark in those activities which are most dependent on the perceived state of the world. Table 2 shows that the asymmetry is smallest in our broadest measure of SafeGraph mobility, which includes both potentially high-risk activities that involve close contact between people, such as restaurant visits, or a visit to the dentist, as well as relatively low-risk activities, such as fishing and reading in a public park.[xii] As we narrow our focus to outcomes where the perceived state of the world becomes increasingly important, such as restaurant visits and visits to full-services restaurants, the size of the asymmetry increases. In the Yelp data, the asymmetry is largest for reservations, where the crowded nature of restaurant dining rooms makes the perceived state of the world particularly salient. The differential response is smaller for takeout and delivery orders, which better accommodate social distancing, and even smaller for Yelp page views which, while a signal of an intention to visit or order from a restaurant, do not incur any immediate risk.

Our results differ from concurrent work by Goolsbee and Svyerson (2021), who also use SafeGraph data but conclude that states that repealed their shutdown orders saw symmetric, modest recoveries in activity, because they focus on all kinds of activities, while we focus on restaurants. The asymmetry is more pronounced when looking at riskier activities, which are captured by full-service restaurants and reservations.

A second implication of our model is that in a semi-pooling equilibrium, demand after a state reopens will be decreasing with perceived risk of contagion. In the U.S., optimism about COVID-19, at least during the lockdown phase, was strongly associated with political party. On April 16, 2020, seventy-three percent of Democrats told Gallup Pollsters that they were somewhat or very worried about catching COVID-19.[xiii] Only thirty-six percent of Republicans told Gallup that they were worried about the disease. This difference across parties persisted throughout the shutdown period, with eighty-five percent of Democrats and only forty-seven percent of Republicans telling Gallup Pollsters that they were somewhat or very worried about being exposed to coronavirus.[xiv]

Allcott et al. (2020) show that locations with a greater number of Republicans were less likely to engage in social distancing. They also present survey results that indicate more conservative individuals anticipated fewer new cases than more left-leaning individuals. In light of these differences in perceived risk by political party, we use the 2016 Republican presidential vote share as a proxy for lower values of $\bar{\pi}$.

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In columns 1-4 of Table 3, we interact a normalized measure of each state's 2016 GOP presidential vote share with the indicators for the stay-at-home and reopening orders. In all three measures of restaurant visits, SafeGraph's Restaurants and Full-Service Restaurants measures, and Yelp's Reservations measure, we find that the asymmetry is increasing in GOP vote share. Indeed, the size of the asymmetry nearly doubles in the Full-Service and Reservations measures when moving from a state with an average level of 2016 GOP support, say Arizona or North Carolina, to a state one standard deviation above the mean, such as Nebraska. The one exception, though, is with Yelp Orders, where there is a larger increase in orders when stay-at-home directives went into effect, but we find no evidence of a relationship between GOP vote share and the reopening response. Our finding is consistent with Engle, Stromme and Zhou (2020), who only examine the effect of stay-at-home order and find the order reduces mobility more in the counties that vote less for the Republican Party in the 2016 presidential election.

Our model does not predict how restaurant activities should vary by the severity of COVID-19 risk throughout the shutdown, because it assumes the public receives no information about the risk during shutdown. In reality, COVID-19 information is updated daily, although different government agencies give different interpretations. For this reason, we treat $1(No\ Covid)_{s,t}$ and $Rel.Covid_{s,t}$ as pure controls. The baseline results (Table 2) suggest that a state's relative severity of COVID-19 risk has little effect on five of the six outcome variables. The only outcome that declines with relative Covid cases with a marginal significance is SafeGraph mobility to all restaurants.

In columns 5-8 of Table 3, we present two additional interactions between our relative COVID-19 cases and the indicators for the stay-at-home and reopening orders. During the stay-at-home period, mobility to all restaurants and online orders from Yelp decline with the COVID-19 risk. The other two outcomes, mobility to full-service restaurants and Yelp reservations, are insensitive to $Rel.Covid_{s,t}$ during shutdown. They already declined to a very low level at the beginning of shutdown. All four outcomes respond positively to reopening, but these positive responses are attenuated significantly in regions with higher values of relative COVID-19 cases. This suggests that, while the public may interpret the government's reopening order as a signal of safety, they also incorporate the ongoing information of COVID-19 in their daily behavior.

We now measure the relationship between the timing of reopening, and post-reopening COVID-19 rates. Under the semi-pooling equilibrium described in Proposition 1, some states will reopen after receiving a positive private signal, while others will reopen despite having not received a good signal. In that equilibrium, consumers will be unable to distinguish between these two sets of states, and so may infer from reopening that activity in their state is less risky than it actually is. If states are in fact in a semi-pooling equilibrium, there should be a greater acceleration of COVID-19 cases in states that opened without a positive signal of safety.

To investigate this, we construct two measures to proxy for states' private signals, both of which are based on changes in the rate of COVID-19 cases over time. Our first measure compares the growth rate of COVID-19 cases one week before the lockdown ends with the growth rate one week after states make their reopening decision. The growth rate one week after the lockdown ends should not reflect the reopening itself, which typically would impact cases after at least one week, but it could reflect private information possessed by the government. We segment states into two groups based on this measure: "Good Signal" states are those for which the growth rate falls around the reopening date, and "Bad Signal" states are those for which the growth rate increases around the reopening date.

As a second measure of private information: the growth in the number of cases two weeks before reopening relative to one week before reopening. While those cases could be seen by the public, for many citizens, their meaning may be opaque, and they may believe that the government is better at interpreting the meaning of those cases for restaurant safety.

PLACE FIGURE 2 HERE

In Figure 2, we plot the average growth rate of COVID-19 cases and the average number of daily visits to full-service restaurants, both relative to the growth rate on the date of reopening. Figure 2a employs the private signal proxy that is centered around the reopening date, and Figure 2b uses the proxy centered around a week before reopening. In both cases, our measure of the governmental signal strongly predicts the ex-post COVID-19 cases. We interpret this to suggest that some state governments chose to reopen, despite not receiving a positive signal about safety. Notably, consumers still interpreted reopening as a positive signal, regardless of which type of state they reside in, as restaurant activity increased across the board, and showed no evidence of a divergence in trend.

Our results do not imply that it was wrong to reopen, but just that reopening can have a different impact on behavior than an initial restriction. Some consumers appear to interpret reopening as a signal that activity is safe.

5. CONCLUSION

Overall, our results help to shed light on the impact of imposing and rescinding stay-at-home orders throughout much of the United States in the COVID-19 pandemic. Our model highlights that government actions can have both a direct effect (preventing people who want to go out from doing so) and an indirect effect (signaling to people when it is safe to go out again).

We then turn to the data, exploring the evolution of restaurant demand before, during, and after the lockdowns. Consistent with other emerging results, we find that most of the demand had already dropped before the lockdowns were in place. This suggests that the direct effect of lockdowns was small in the beginning and has led some policymakers to question the importance of the lockdowns. However, we also show that demand increases sharply shortly after the end of lockdowns—suggesting that the lockdowns were in fact relevant by the end. The observed asymmetry and timing of demand increases are consistent with the signaling mechanism proposed in the model.

Our analysis also shows heterogeneity in demand responses after the end of lockdowns, associated with political preferences and with the number of COVID-19 cases in an area. Within the United States, there has been an important partisan component to beliefs about COVID-19, with Republican-leaning citizens less likely to express concern about the virus. Consistent with this, we find larger increases in demand in areas with a higher Republican vote share. We also find smaller jumps in demand in areas with more cumulative COVID-19 cases.

Our work has implications for policymakers as well. As Gans (2020) notes, one "key to making people safe is knowledge." While some of this knowledge comes in the form of official guidelines and other direct information, there is also information conveyed through policymakers' regulatory decisions. Our work explores the signal value and information being conveyed through a regulatory decision and highlights the importance for policymakers of accounting for the signal being sent by the imposition and lifting of bans, such as the COVID-19 stay-at-home orders.

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[i] While that condition ensures that banning the action is optimal during that one period, it does not imply that the ban was dynamically optimal since the ban creates the scope for distortionary public action because of its private information.

[ii] Many COVID-19 research cited in this paper use cell phone tracking data from Safegraph, Unacast or other companies. Couture, Dingel, Green, Hanbury, and Williams (2021) compare smartphone data with conventional survey data such as the Census's estimated 2019 residential population, the 2014-2018 American Community Survey, the 2017-2018 IRS Migration Data, and the 2017 National Household Transportation Survey (NHTS). They show that smartphone data cover a significant fraction of the US population and are broadly representative of the general population in terms of residential characteristics and movement patterns.

[iii] These corresponding NAICS codes are 722511, 722514, 722410. Restaurants not included in our definition of full-service restaurants are limited-service restaurants (NAICS=722513) and snack and nonalcoholic beverage bars (NAICS=722515).

[iv] We exclude nine states from the Reservations measure, because the platform's coverage is relatively low in those states. The states are: AL, AR, DE, MS, ND, OK, RI, SD, WY.

[v] This finding is consistent with Raj, Sundararajan, and You (2020), who document the increase of online orders on Uber Eats after stay-at-home orders.

[vi] https://www.census.gov/retail/marts/www/timeseries.html.

[vii] https://restaurant.org/articles/news/restaurant-sales-hit-a-pandemic-high-in-june, National Restaurant Association.

[viii] We use the New York Times data posted at https://github.com/midas-network/COVID-19/tree/master/data/cases/united%20states%20of%20america/nytimes_covid19_data/.

[ix] Townhall.com published the 2016 general election results at https://townhall.com/election/2016/president/. We access county-level data from Townhall.com at https://github.com/tonmcg/US_County_Level_Election_Results_08-16 and aggregate it into states.

[x] There was considerable variation in the timing of stay-at-home and reopening orders by state, with stay-at-home orders ranging from March 17 (CA) to April 7 (SC), and restaurant reopenings ranging from April 24 (AK) to June 15 (NJ).

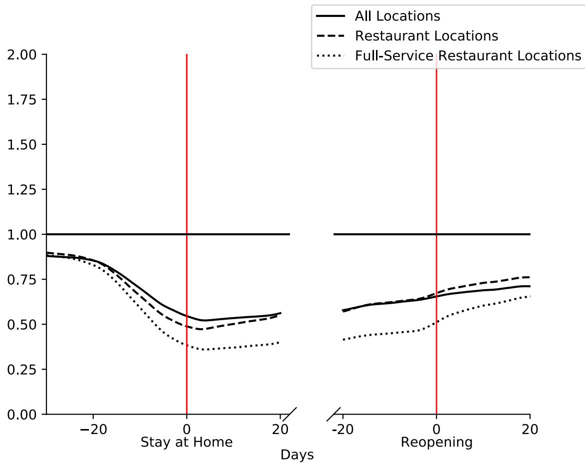
[xi] We use the 2019 state population as estimated by the US Census Bureau, accessed at https://www.census.gov/data/tables/time-series/demo/popest/2010s-state-total.html.

[xii] https://www.aei.org/op-eds/governments-incur-fury-by-banning-safe-activities-during-coronavirus-lockdown/

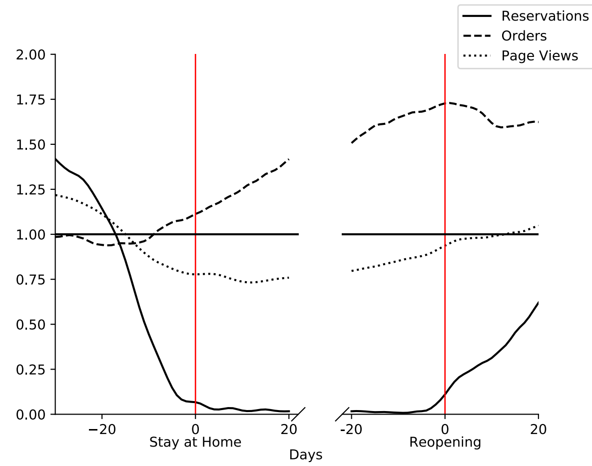
[xiii] https://news.gallup.com/poll/308504/fear-covid-illness-financial-harm.aspx

[xiv] https://news.gallup.com/poll/312680/americans-remain-worried-exposure-covid.aspx

Figure 1: Nationwide Average Responses

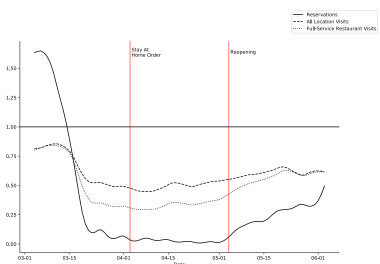


SafeGraph Outcomes

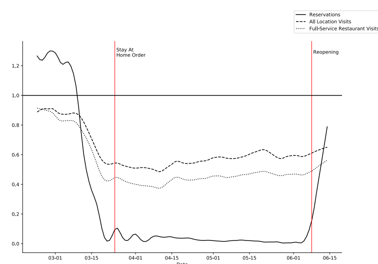


Yelp Outcomes

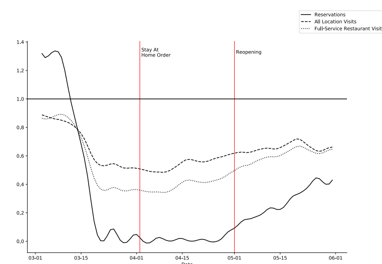
(a) SafeGraph Outcomes



Florida



Massachusetts



Texas

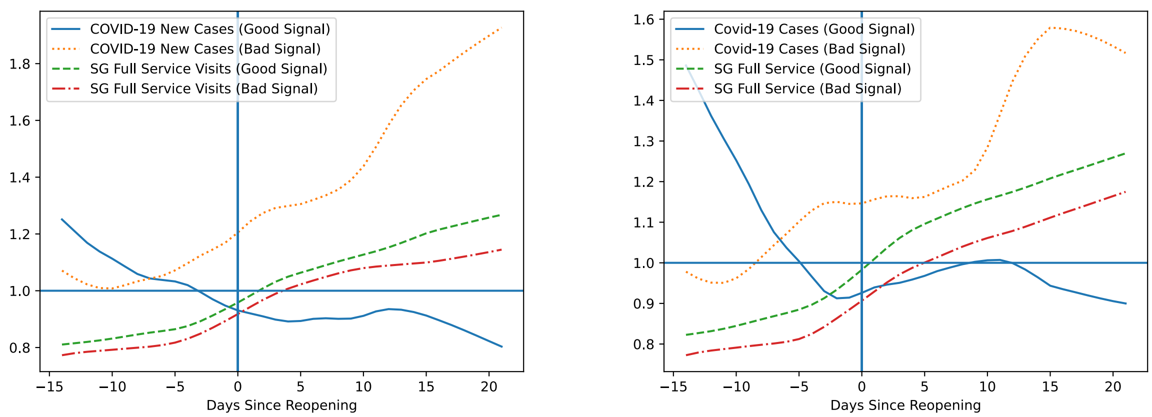
(b) Individual State Responses

Figure 1a presents the average of each measure of mobility and restaurant activity, centered around stay-at-home and reopening dates. Figure 1b presents Yelp Reservations, SafeGraph visits to all places-of-interest, and SafeGraph visits to full-service restaurants for three states. Due to a high degree of day-of-week variation, each line presents a trend line of the outcome of interest, calculated using a LOESS seasonal-trend decomposer.

Figure 2: COVID-19 Cases and Full Service Restaurant Visits After Reopening

(a) COVID-19 Growth Rate (Around Reopening)

(b) COVID-19 Growth Rate (Week Before Reopening)



In Figure 2a, states are grouped based on whether the number of new cases in the week after reopening is more ("Bad Signal") or less ("Good Signal") than the number of new cases in the week prior to reopening. Figure 2b takes the same approach, using the change in cases in the week prior to reopening relative to the change in cases two weeks prior to reopening. Data is relative to the value on each state's reopening. Each line presents a trend line of the outcome of interest, calculated using a LOESS seasonal-trend decomposer. The following states are excluded from these figures because of insufficient data in the post-reopening period: CO, DE, IL, MA, MI, MN, NH, NJ, NY, PA, RI, and WA.

Table 1: Summary Statistics

		All Observations			Δ Stay Home		Δ Reopen	
		N	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Safegraph Outcomes	All Visits	9,850	0.789	0.214	-0.0704	0.0751	0.0362	0.0320
	Restaurant Visits	9,850	0.791	0.235	-0.0633	0.105	0.0601	0.0423
	Full-Service Restaurant Visits	9,850	0.729	0.291	-0.0776	0.123	0.0924	0.0566
Yelp Outcomes	Reservations	8,077	0.721	0.911	-0.0748	0.121	0.1920	0.4060
	Orders	9,850	1.211	0.592	0.0866	0.13	0.0100	0.323
	Page Views	9,850	1.002	0.499	-0.0192	0.111	0.0945	0.0893
Explanatory Variables	1(No Covid Cases)	9,850	0.464	0.499	—	—	—	—
	Relative Covid Cases (per million)	9,850	0.606	2.403	-0.00576	0.625	0.0147	0.0703
	2016 GOP Presidential Vote Share	9,850	0.499	0.100	—	—	—	—
	$\mathbb{1}$ (Stay Home)	9,850	0.242	0.428	—	—	—	—
	$\mathbb{1}$ (Reopen)	9,850	0.161	0.368	—	—	—	—

Table 2: Baseline Regression Results

	SafeGraph Outcomes			Yelp Outcomes		
	(1)	(2)	(3)	(4)	(5)	(6)
	All	Restaurant	Full-Service	Reservations	Orders	Page Views
𝟙 (Stay Home)	-0.026***	-0.016***	-0.002	0.050	0.037	-0.005
	(0.006)	(0.006)	(0.008)	(0.031)	(0.054)	(0.024)
𝟙 (Reopen)	0.041***	0.089***	0.119***	0.249***	0.161**	0.077**
	(0.009)	(0.012)	(0.014)	(0.059)	(0.070)	(0.030)
𝟙 (No Covid)	-0.025***	-0.022**	-0.001	0.028	-0.036	-0.008
	(0.009)	(0.010)	(0.009)	(0.064)	(0.060)	(0.015)
Rel. Covid Cases (Z Score)	-0.003	-0.003*	0.000	-0.001	-0.030	-0.000
	(0.002)	(0.002)	(0.001)	(0.006)	(0.020)	(0.003)
Constant	0.800***	0.791***	0.711***	0.657***	1.193***	0.994***
	(0.004)	(0.005)	(0.005)	(0.033)	(0.032)	(0.012)
N	9,850	9,850	9,850	8,077	9,850	9,850

Observations are state-days. All outcomes are calculated relative to the state's December, 2019 daily average. Regressions include state and day fixed effects. Column (4) excludes nine states, due to relatively low coverage by the platform (AL, AR, DE, MS, ND, OK, RI, SD, WY). Standard errors are clustered at the state and date levels.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Interaction Regression Results

	Safegraph Outcomes		Yelp Outcomes		Safegraph Outcomes		Yelp Outcomes	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Restaurant	Full-Service	Reservations	Orders	Restaurant	Full-Service	Reservations	Orders
$\mathbb{1}$ (Stay Home)	-0.021***	-0.006	0.027	0.041	-0.012*	-0.003	0.040	0.075
	(0.007)	(0.008)	(0.029)	(0.048)	(0.007)	(0.008)	(0.030)	(0.056)
$\mathbb{1}$ (Reopen)	0.047***	0.088***	0.223***	0.142**	0.084***	0.121***	0.242***	0.101
	(0.014)	(0.015)	(0.064)	(0.070)	(0.014)	(0.015)	(0.062)	(0.083)
$\mathbb{1}$ (No Covid)	0.004	0.013	0.042	-0.007	-0.016*	-0.001	0.033	0.020
	(0.008)	(0.008)	(0.066)	(0.055)	(0.009)	(0.009)	(0.065)	(0.047)
Rel. Covid Cases (Z Score)	-0.001	0.001	-0.000	-0.027	-0.001	0.000	-0.001	-0.009***
	(0.001)	(0.001)	(0.006)	(0.018)	(0.001)	(0.001)	(0.005)	(0.003)
$\mathbb{1}$ (Stay Home) × GOP Share (Z Score)	0.033***	0.000	-0.037*	0.105*				
	(0.006)	(0.006)	(0.020)	(0.054)				
$\mathbb{1}$ (Reopen) × GOP Share (Z Score)	0.075***	0.073***	0.163**	-0.039				
	(0.008)	(0.010)	(0.062)	(0.059)				
$\mathbb{1}$ (Stay Home) × Rel. Covid Cases (Z Score)					-0.043***	0.010	0.018	-0.445***
					(0.012)	(0.010)	(0.044)	(0.079)
$\mathbb{1}$ (Reopen) × Rel. Covid Cases (Z Score)					-0.095**	-0.098**	-0.506***	-0.402**
					(0.040)	(0.042)	(0.163)	(0.171)
Constant	0.785***	0.707***	0.657***	1.188***	0.790***	0.710***	0.660***	1.185***
	(0.005)	(0.005)	(0.032)	(0.030)	(0.005)	(0.005)	(0.033)	(0.031)
N	9,850	9,850	8,077	9,850	9,850	9,850	8,077	9,850

Observations are state-days. All outcomes are calculated relative to the state's December, 2019 daily average. Regressions include state and day fixed effects. Columns (3) and (7) exclude nine states, due to low coverage by the platform (AL, AR, DE, MS, ND, OK, RI, SD, WY). Standard errors are clustered at the state and date levels.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Appendix 1: Proof of Proposition 1

The government either knows that $\pi_1 = \underline{\pi}$ or believes that $\pi_1 = \underline{\pi}$ with probability δ and $\pi_1 = \overline{\pi}$ with probability $1 - \delta$. We refer to the consumers endogenous beliefs about π_1 as $\tilde{\pi}_1$.

If the government knows that $\pi_1 = \underline{\pi}$, then surplus from reopening will be $(1 - \tilde{\pi}_1\ C)\left(\frac{1+\tilde{\pi}_1\,C}{2} - \underline{\pi}(C + \Delta)\right)$, and this is positive if and only if $\frac{1+\tilde{\pi}_1\,C}{2} > \underline{\pi}(C + \Delta)$, but as $\tilde{\pi}_1 \geq \underline{\pi}$ and $\frac{1}{\underline{\pi}} > 2\Delta + C$, then $\frac{1}{\underline{\pi}} > 2\Delta + \left(2 - \frac{\tilde{\pi}_1}{\underline{\pi}}\right)$, and so reopening is always optimal if the government has received a positive signal.

As the government with a positive signal will always reopen, Bayes' rule then implies that the maximal possible level of perceived risk in period 1 ($\tilde{\pi}_1$) is $(1 - \delta)^2\overline{\pi} + (2 - \delta)\delta\underline{\pi}$, which would be the risk if all governments reopened, and the lowest possible risk is $\underline{\pi}$. As $0 < \delta < 1$, simple algebra yields that $\underline{\pi} \leq \tilde{\pi}_1 < (1 - \delta)\overline{\pi} + \delta\underline{\pi}$, and hence $\frac{1}{\underline{\pi}} > 2\Delta + C > \frac{1}{(1-\delta)\overline{\pi}+\delta\overline{\pi}}$ implies that

$$\underline{\pi}(C + \Delta) < \frac{1+\underline{\pi}\,C}{2} \leq \frac{1+\tilde{\pi}_1\,C}{2} < \frac{1+\left((1-\delta)\overline{\pi}+\delta\overline{\pi}\right)C}{2} < \overline{\pi}(C + \Delta).$$

If the government receives a negative signal, then their payoff from reopening equals $\delta(1 + \alpha)(1 - \tilde{\pi}_1\ C)\left(\frac{1+\tilde{\pi}_1\,C}{2} - \underline{\pi}(C + \Delta)\right) + (1 - \delta)(1 - \alpha)(1 - \tilde{\pi}_1\ C)\left(\frac{1+\tilde{\pi}_1\,C}{2} - \overline{\pi}(C + \Delta)\right)$.

The $1 + \alpha$ term multiplies the first expression and the $1 - \alpha$ term multiplies the second expression because $\overline{\pi}(C + \Delta) > \frac{1+\tilde{\pi}_1\,C}{2} > \underline{\pi}$, As continuing the shutdown generate public welfare of zero, reopening will dominate continuing the shutdown for the government if and only if

$$\tilde{\alpha}(\tilde{\tilde{\pi}}_1) = \frac{\left((1-\delta)\overline{\pi}+\delta\underline{\pi}\right)(C+\Delta)-\frac{1+\tilde{\tilde{\pi}}_1 C}{2}}{\delta\left(\frac{1+\tilde{\tilde{\pi}}_1 C}{2}-\underline{\pi}(C+\Delta)\right)+(1-\delta)\left(\overline{\pi}(C+\Delta)-\frac{1+\tilde{\tilde{\pi}}_1 C}{2}\right)} < \alpha.$$

Both the numerator and denominator of $\tilde{\alpha}(\tilde{\tilde{\pi}}_1)$ must be positive, and the denominator is greater than the numerator so $\tilde{\alpha}(\tilde{\tilde{\pi}}_1)$ is bounded between zero and one. The function $\tilde{\alpha}(\tilde{\tilde{\pi}}_1)$ is monotonically decreasing in $\tilde{\tilde{\pi}}_1$, as reopening becomes more appealing if there are fewer people who respond to the opening. As $\tilde{\tilde{\pi}}_1$ is bounded between $\underline{\pi}$ and $(1-\delta)^2\overline{\pi}+(2-\delta)\delta\underline{\pi}$, the range of $\tilde{\alpha}(\tilde{\tilde{\pi}}_1)$ goes from $\frac{2\left((1-\delta)\overline{\pi}+\delta\underline{\pi}\right)\Delta+\left((1-\delta^2)\overline{\pi}+\delta^2\underline{\pi}\right)C-1}{2\left((1-\delta)\overline{\pi}-\delta\underline{\pi}\right)\Delta-(1-2\delta)+\left((1-3\delta+2\delta^2)(1-\delta)\overline{\pi}-\delta(4-3\delta+2\delta^2)\underline{\pi}\right)C} = \hat{\alpha}_1$, when $\tilde{\tilde{\pi}}_1 = (1-\delta)^2\overline{\pi}+(2-\delta)\delta\underline{\pi}$, to $\frac{\left((1-\delta)\overline{\pi}+\delta\underline{\pi}\right)(C+\Delta)-\frac{1+\underline{\pi}C}{2}}{\delta\left(\frac{1+\underline{\pi}\,C}{2}-\underline{\pi}(C+\Delta)\right)+(1-\delta)\left(\overline{\pi}(C+\Delta)-\frac{1+\underline{\pi}C}{2}\right)} = \hat{\alpha}_2$,when $\tilde{\tilde{\pi}}_1 = \underline{\pi}$,. If $\alpha < \hat{\alpha}_1$,then there is no feasible value of $\tilde{\tilde{\pi}}_1$ such that a government with a weak signal will reopen, and hence the only equilibrium is a separating one in which governments with positive signals open and governments with negative signals don't open. In that case, $\tilde{\tilde{\pi}}_1 = \underline{\pi}$ if there is reopening and demand is high.

If $\alpha > \hat{\alpha}_2$,then there is no feasible value of $\tilde{\tilde{\pi}}_1$ such that a government with a weak signal will not reopen, consequently there exists only a unique equilibrium with full pooling where $\tilde{\tilde{\pi}}_1 = (1-\delta)^2\overline{\pi}+(2-\delta)\delta\underline{\pi}$ and demand is low conditional upon reopening.

If $\hat{\alpha}_1 > \alpha$, then there is no feasible equilibrium value of $\hat{\bar{\pi}}_1$ such that a government with a weak signal will reopen, consequently there exists only a unique equilibrium with separating where $\hat{\bar{\pi}}_1 = \underline{\pi}$.

If $\hat{\alpha}_1 \leq \alpha \leq \hat{\alpha}_2$, then we are in a multiple equilibrium range where three equilibria exist: a pure pooling equilibrium, a pure separating equilibrium and a semi-pooling equilibrium

If $\hat{\bar{\pi}}_1 = (1-\delta)^2\overline{\pi} + (2-\delta)\delta\underline{\pi}$, then both high and low risk governments will reopen, those beliefs will be justified, and demand will be low. This is the pure pooling equilibrium.

If $\hat{\bar{\pi}}_1 = \underline{\pi}$, then only low risk governments will reopen, those beliefs will be justified, and demand will be high. This is the pure separating equilibrium.

Finally, there is a third semi-pooling equilibrium in which all low risk governments and some high risk governments reopen, and $\hat{\bar{\pi}}_1$ solves $\frac{\left((1-\delta)\overline{\pi}+\delta\underline{\pi}\right)(C+\Delta)-\frac{1+\hat{\bar{\pi}}_1 C}{2}}{\delta\left(\frac{1+\hat{\bar{\pi}}_1 C}{2}-\underline{\pi}(C+\Delta)\right)+(1-\delta)\left(\overline{\pi}(C+\Delta)-\frac{1+\hat{\bar{\pi}}_1 C}{2}\right)} = \alpha$, which implies that

$$\hat{\bar{\pi}}_1 C = \frac{2(C+\Delta)}{(2\delta\alpha+1-\alpha)}\left(\underline{\pi}\delta(1+\alpha) + \overline{\pi}(1-\delta)(1-\alpha)\right) - 1.$$

As the level of demand is captured by $\hat{\bar{\pi}}_1 C$, differentiating this expression gives us that demand is falling (i.e. $\pi_1 C$ is rising) with $\alpha, C, \Delta, \underline{\pi}$ and $\overline{\pi}$ and rising with δ.