Corporate Refinancing, Covenants, and the Agency Cost of Debt

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Abstract

How valuable are restrictive debt covenants in reducing the agency costs of debt? I exploit the revealed preference decision to refinance fixed-coupon bonds, which weighs observable interest rate savings against the unobservable costs of a change in restrictive covenants. Variation in this trade-off reveals that firms require higher interest rate savings to refinance when it would add restrictive covenants. I structurally estimate a model of debt refinancing and find that a high-yield bond's restrictive covenant package increases the value of speculative-grade firms by 2.4 percent. This suggests that covenants are essential for allowing the tax benefits of debt to offset costs of financial distress.

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1 Introduction

A large theoretical literature demonstrates that the state-contingent allocation of control and cash flow rights is useful in preempting agency conflicts between creditors.[1] In debt contracts, this state-contingent allocation is achieved in large part through restrictive covenants (Smith and Warner, 1979). Empirically, studies of such restrictive covenants have largely focused on the ex-post impacts of covenant violations (Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009)). This leaves unanswered questions regarding the *ex-ante* efficiency gains achieved by using debt covenants to allocate control rights. In practice, how effective is this mechanism in ameliorating the agency costs of debt, or put differently, how much surplus is generated by the use of covenants? Given covenants are more likely to be used in the riskiest debt, how important are they in allowing value to be created from a high-leverage capital structure?

This paper provides quantitative answers to these questions. I develop a dynamic revealed preference framework that allows me to estimate the value of high-yield corporate bond covenants by relating the observed timing of bond refinancings to changes in interest rates and to whether refinancing would impose or remove covenants from the firm's debt. I find that high-yield bond covenants add significant value. My baseline estimates suggest that they increase the total enterprise value of speculative-grade firms by approximately 2.4 percent. I also document substantial variation in the magnitude of the value added by these covenants: large firms and growth firms with speculative-grade capital structures benefit from restrictive covenants by as much as seven percent of book value. Taken in concert with estimates in the literature of the net benefits of leverage, these estimates suggest restrictive debt covenants are essential in allowing high-leverage capital structures to add value to firms.

My methodology leverages a directly observable tradeoff in firms' decisions to refinance

[1] Jensen and Meckling (1976) provides a foundation for the notion of inter-creditor agency costs. Aghion and Bolton (1992) and Zender (1991) show the value of debt in aligning incentives comes in part from the allocation of control rights.

callable corporate bonds. This framework allows me to exploit time-series variation in risk-free interest rates and plausibly exogenous variation in how refinancing would change covenants to estimate the ex-ante surplus generated by the restrictive covenants in high-yield bond indentures. Like mortgage refinancing, the textbook consideration of fixed-coupon bond refinancing is to optimally exploit declines in borrowing costs. However, for firms that have gained or lost access to the investment grade bond market, refinancing also substantially alters the restrictive covenants imposed on the firm. I provide detailed evidence that only high-yield, and not investment-grade, bonds contain covenants that substantially limit the control rights of the firm management, for example by preventing the firm from making equity distributions, raising additional debt, or undertaking new acquisitions. For firms that have been upgraded to investment-grade, which are referred to in capital markets as "rising stars," refinancing is an opportunity not only to exploit interest rate savings, but also to shed restrictive high-yield covenants, which may be inefficiently restricting the firm. For firms that have lost their investment-grade rating ("fallen angels"), refinancing to exploit declines in interest rates comes at the cost of adopting new limitations on managerial discretion.

A recent report by Moody's, a credit rating agency, highlights exactly this tradeoff:

> "Investment-grade covenants typically do not restrict a company's ability to make dividends, buyback shares or incur unsecured debt–a fact that investors in investment-grade securities often overlook because of the issuers' strong credit profiles. But when investment-grade companies fall to speculative grade, the flexibility afforded by the covenants included in their bonds–which can have maturities of 20-40 years–can become a factor as these fallen angels seek to refinance."
> – Moody's (2016)

I begin my empirical analysis by documenting that, consistent with theory and anecdotal evidence, covenants are indeed an important factor in the decision to refinance callable corporate bonds. I do this by considering the refinancing decisions of firms that have experienced substantial credit rating downgrades or upgrades since issuance relative to decisions of firms that have not experienced such ratings changes but have the same potential interest rate savings from refinancing and the same current fundamentals. The only difference between these firms that is relevant for the decision to refinance an individual bond issue is how the debt covenants binding on the firm will change if the refinancing is undertaken.

For each bond in my sample I estimate the dynamically optimal refinancing strategy the issuing firm should follow in the absence of any covenant considerations. I find that this benchmark model of bond call policy is systematically biased when refinancing will significantly change the covenants binding on the firm. Firms that would face tighter covenants upon refinancing require larger declines in interest rates to initiate refinance than is implied by the model, relative to otherwise similar refinancing opportunities that do not involve changing covenants. Conversely, bonds which can *shed* covenants refinance too "early," that is, they are called even though there is remaining time value in their option to wait for further declines in interest rates.

These refinancing patterns suggest that covenants have the effect on debt value hypothesized in the seminal work of Smith and Warner (1979): they limit firm actions to protect the value of debt. Fallen angel firms delay refinancing relative to always-junk firms because loose covenants allow shareholders to usurp wealth from debtholders (for example via asset substitution) and thus increase the cost of calling the bond relative to the opportunity cost of continuing to service it. Rising star firms refinance before the time value of their refinancing option has expired because restrictive covenants prevent these firms from taking profitable investment opportunities; refinancing eliminates these constraints.

That firms are willing to sacrifice interest-rate savings to shed or avoid covenants reflects the fact that restrictive covenants materially limit control rights in a way that affects the distribution of value between debt and equity claims. It does not by itself identify the *total* value these covenants create or destroy, quantification of which is the central goal of my paper. I recover the surplus generated by restrictive covenants by considering both the impact of covenant changes on the sensitivity of refinancing to changes in interest rates and the extent to which covenants increase the value of debt.

Why does the tradeoff between interest-rate savings and covenant considerations in refinance reveal the surplus generated by restrictive covenants? I show that the "delay" of fallen-angel firms to refinance to adopt covenants is an overhang problem.[2] Refinancing transfers significant value to existing bondholders unless interest-rate savings are large, precisely because loose covenants allow the firm to take actions that erode the value of debt claims relative to the pre-specified call price of the bond. The firm will only refinance if the surplus generated by adopting new restrictive covenants is at least as large as the the net transfer to outstanding bondholders. Thus, combined with estimates of how covenants affect the value of corporate bonds, variation in interest rates and observed refinancing behavior identifies the surplus generated by fallen angel firms adopting restrictive debt covenants. A similar argument holds for firms that have attained investment-grade status: they could increase surplus by shedding high-yield bond covenants.

To exploit this insight, I structurally estimate a dynamic model of corporate bond refinancing that combines the intuition of the revealed preference tradeoff between interest rate savings and covenants with the dynamic considerations necessary to apply the model to the observed prices and refinancing decisions of real-world corporate bonds. In the model, covenants affect the stochastic process for the total enterprise value of the firm in a state-dependent fashion. This allows (but does not impose) restrictive covenants to affect the total value of the firm and the way firm value is distributed between equity and debt claims. For example, loose covenants could allow a highly levered firm to increase the variance of its profits at the expense of their mean, inefficiently transferring wealth from bondholders.

For a given parameterization, I can solve the model for each bond in my sample at each time the bond is callable to generate predictions of the bond's optimal refinancing policy and

[2]Because bonds are diffusely held, covenants are difficult to renegotiate, and the most effective way to modify them is to retire and replace debt issues. Indeed, this is one of the theories of why corporate debt is callable in the first place. If an investment opportunity were to arise for which excessive value would accrue to debt holders, debt overhang can be avoided by calling the original debt issue (Bodie and Taggart, 1978).

market price. The parameters of the model related to covenants are identified by intuitive and directly observable variation in the data: the difference in refinancing policies and bond prices of issuers that stand to gain or shed covenants in refinancing relative to those of issuers whose covenants would be unchanged by refinancing. I structurally estimate these parameters in a maximum likelihood framework to reconcile the predictions of my model for bond prices and refinancing dates with what I actually observe in the data.

I also explore heterogeneity in the value of high-yield restrictive covenants across my sample of firms. I find that there are substantial differences in the value of these covenants by firm size, industry, and growth opportunities. Large firms and firms with the highest growth opportunities gain as much as seven percent of asset value from restrictive covenants. The value of high-yield covenants for small speculative-grade firms is statistically indistinguishable from zero.

Finally, the effects that I find of restrictive debt covenants on risky firm asset value are quantitatively large. Most notably, they are on the same order of magnitude as estimates of the overall *net* value debt adds to the capital structure, accounting for the costs of financial distress. My contribution expands this in an important dimension: I show that the use of debt covenants in risky capital structures is essential to solving agency problems that allow debt to generate positive value for the firm.

Relation to Prior Literature

My paper builds off of the large theoretical literature studying the design of securities in an incomplete contracting setting. Aghion and Bolton (1992) and Zender (1991) show that the allocation of control rights can improve the alignment of incentives between claimants. In these papers debt contracts, which allocate control rights in the event of default, emerge as optimal securities. Covenants can be understood in this context to operate through two mechanisms: increasing contractual completeness by restricting management control rights in a state-contingent manner and by inducing ex-post renegotiation.

The salience of these two mechanisms, and thus the design of covenants, depends on the cost of renegotiation. Corporate loans are narrowly monitored and controlled by individual or syndicate lenders, while bonds are publicly issued securities with disbursed ownership; therefore renegotiation is relatively more costly for bonds.[3] It is thus not surprising that loan and bond covenants are substantially different. Violation of financial covenants in loans induces technical default, and thus these covenants serve as tripwires that induce renegotiation between firms and creditors.[4] Bond covenants are instead "incurrence" based, and violation only restricts firms from taking certain actions that could reduce their ability to service debt. My paper studies bond covenants and thus speaks generally to the value of state-contingent control rights in general than specifically through renegotiation.[5]

Garleanu and Zwiebel (2009) study the optimal allocation of control rights in the context of asymmetric information about the magnitude of potential asset substitution. They show that when renegotiation costs are low, this information asymmetry results in optimal covenants that are tight and thus frequently violated, but also frequently relaxed upon violation. Murfin (2012) documents that bank lenders increase the tightness of these tripwires after experiencing defaults in their loan portfolios.

Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009) empirically assess how firm activity responds to covenant violations. They show that covenant

[3]Further, The Trust Indenture Act of 1939 prevents issuers of public debt securities from modifying the terms of principal, interest, and maturity after issuance, which further limits the ability of bondholders and issuers to renegotiate debt structure.

[4]This idea dates back to Zinbarg (1975), which observes that the relatively restrictive covenants of loan agreements "provide a series of checkpoints that permit the lender to review proposed actions by the borrower with potential for substantially impairing the lender's position."

[5]Becker and Ivashina (2016) argue that the recent loosening of covenant strength in the leveraged loan market is due to the increasingly diffuse ownership structure of syndicated private debt. In fact, this market is becoming indistinguishable from the high yield bond market. The majority of the covenant loosening that has occurred recently in the leveraged loan market has indeed been the shift from maintenance covenants to "bond style" incurrence covenants, which is known in the industry as "covenant-lite." Investment banks are increasingly arranging high-yield bond and leveraged loan financing from the same desk, further blurring the distinction between these two markets. Interestingly, the continued spread of covenant-lite terms in the private debt market, beyond just widely held institutional loan tranches, suggests that the importance of inducing state-contingent renegotiation may be declining.

violations cause lenders demand concessions that materially restrict the activity of the firm. For example, Chava and Roberts (2008) uses a regression discontinuity design to show firm investment declines sharply around financial covenant violations. While this reveals that the state contingent allocation of control rights affects the operations of firms, it cannot quantify the value created by this mechanism.

Several recent papers have made progress in attempting such quantification. Bradley and Roberts (2015) use a two-equation regression methodology to estimate how much restrictive covenants lower the interest rate spreads of loan securities, accounting for the selection of riskier firms into more restrictive covenants. Reisel (2014) performs a similar analysis for bond covenants. They show that all else equal, the inclusion of covenants is in fact associated with lower interest rates. Thus, this work demonstrates that covenants address agency problems because they lower the cost of debt capital.

Matvos (2013) is the closest paper to mine because it goes beyond the estimation of the price effect of covenants to provide an assessment of the total value created by loan covenants. It identifies covenant surplus creation through the indifference condition that the firm's perceived cost of marginal covenant inclusion is exactly offset by the reduced price of debt associated with this marginal covenant inclusion. Gamba and J. Triantis (2014) and Xiang (2018) develop capital structure models that explicitly consider the relationship between financial covenants in loans and financing and investment decisions of firms.

My paper innovates on these papers in several dimensions. I approach the issue of identification from a new perspective: instead of trying to explicitly account for the selection into tight or loose covenants at issuance, I consider the information revealed by firms when they face an opportunity to change the covenants binding on their firm. I exploit the large discontinuity in covenant strength across investment-grade and high-yield bonds to generate extensive margin variation in firms' ability to shed or avoid the restrictive incurrence covenants found in high-yield bonds but not in investment-grade bonds. This allows me to obtain estimates of the total value created by high-yield bond covenant packages.

Also related to this paper is a literature that studies the endogenous choice of covenants and the innovation of new capital market instruments, including Nash, Netter, and Poulsen (2003), Billett, King, and Mauer (2007), Kahan and Yermack (1998), Asquith and Wizman (1990), and Beatty, Ramesh, and Weber (2002). King and Mauer (2000) also studies the call policy of nonconvertible corporate bonds and finds evidence that one of the determinants of calling a corporate bond is to remove restrictive covenants.

The rest of this paper is organized as follows. In Section 2, I introduce a static model to elucidate how refinancing decisions are influenced by inter-creditor conflicts and how the observed refinancing of firms reveals the magnitude of these conflicts. Section 3 describes the data I will use to explore this empirically. Section 4 presents evidence that firms' decisions to refinance debt are inconsistent with a benchmark model of refinancing in which covenants play no role. Instead, I show refinancing patterns support the hypothesis that firms are willing to pay to avoid new restrictive covenants and to shed old ones. In Section 5, I outline a dynamic structural model of refinancing that explicitly incorporates firms' valuation of covenants in the refinancing decision, as motivated by the model in Section 2. Section 6 reports on the estimation of this model and what it reveals about the value of covenants in addressing inter-creditor agency conflicts. Section 7 concludes.

2 A Static Model of Covenants and Refinancing

I now introduce a stylized model that endogenizes the use of restrictive covenants. This model explains why they are only included in risky debt and illustrates the tradeoff that arises between the agency costs of debt and interest rate savings in firms' decisions to refinance callable bonds.

State	Action	Value	Equity Payout	Debt Payout
H	A	$X + \Delta$	$X + \Delta - D$	D
H	B	X	$X - D$	D
L	A	$(X + \Delta)$w/ prob $1 - q$	$(1 - q) \times (X + \Delta - D)$	$(1 - q) \times D$
L	B	X	$(X - D)$	D

Table 1: **States, Actions, and Payoffs in the Static Model**

2.1 Model Setup

There are three periods $t \in \{1, 2, 3\}$. A firm has an investment opportunity and needs to issue debt K to finance the investment. The project is financed in period 1 by issuing a two period callable coupon bond, with coupon payment c_0 per dollar of face value due each period. In period 2 there is a shock to the risk-free rate and a shock to the fundamentals of the project. There is also at this time an opportunity to refinance the bond by calling it at par plus a call premium, CP per dollar of par, and issuing a new one period bond. The state of the project $s \in \{H, L\}$ is realized in period 3 and the firm takes action $a \in \{A, B\}$ after the state is realized. The capital market is competitive, so bonds are issued at a yield that ensures zero profit in expectation for bond investors.

The expected probability, as of period t, of state L being realized is p_t. The payoff of the project depends jointly on the realized state and chosen action, as shown in Table 1. The quantity q denotes the probability that action A is unsuccessful in state L and returns zero.

The payoff structure in Table 1 captures the idea that action A is uniformly good in state H but in state L the interests of equity and debt may diverge. This requires the following parameter assumption.

Assumption 2.1. *(Inter-Creditor Agency Conflict)*

$$qX > (1 - q)\,\Delta > q\,(X - D)$$

Under Assumption 2.1, in state L shareholders prefer action A, but action B is value-maximizing. Again note that in state H it is optimal to take action A and there is no conflict between equity and debt holders.

2.2 Risk Dependent Optimality of Covenants

Covenants are imperfect and do not replicate complete contracting. I model this by assuming it is not possible to restrict the firm's action in a state-dependent manner. However, a covenant can be written that forces the issuer to always take action B. This gives rise to a tradeoff. By imposing this covenant, the firm solves its agency problem that would arise in state L, but at a cost of limiting its flexibility to make the value-maximizing decision in state H. This is intuitively realistic for many covenants in high-yield corporate bonds, for example those that restrict asset sales, mergers and acquisitions, and investment in non-core business lines. I define the total surplus generated by the covenant to be the difference in the expected project payout with and without the restrictive covenant in place. The total surplus generated by the covenant is given by:

$$S(p) = Xpq - \Delta(1 - pq). \tag{1}$$

Conditional on debt being issued, either to fund the project initially or to refinance previous debt, the covenant will be included in the new issue if and only if the covenant provides positive total surplus. This follows directly from the assumption that new debt is issued at a fair market price, which aligns equity holder's incentives with the maximization of total asset value. The above equation implies there is a threshold value of p above which any debt issuance will include the covenant and below which no debt will include the covenant.

Proposition 2.1. *Whenever debt is issued, it will contain the restrictive covenant if and only if* $p > \bar{p}$*, where* $\bar{p} \equiv \Delta / [q(X + \Delta)]$.

This prediction is consistent with the fact highlighted in the Section 1 that riskier firms' debt issues have strict covenants but safer debt does not. Section 4.1 examines this empirically.

2.3 Covenants and the Refinancing Decision

Now consider the model at period 2 and what I denote as an *always junk* firm. An *always junk* firm in the model is one where the level of risk was initially high enough to warrant the inclusion of the covenant at time 1, and whose risk has remained high enough so that the covenant is still optimal, ie $p_1 > \bar{p}$ and $p_2 > \bar{p}$. Thus in a refinancing this firm does not change its covenant status and needs only to consider the interest rate savings associated with such a decision. The *always junk* firm refinances when:

$$\left[\frac{(1+c_0)}{(1+rf)} - (1+CP)\right] \times K > 0. \quad (2)$$

Simply put, the firm refinances when the present value of its future debt payments exceeds the present value of prepaying those obligations today at the pre-specified call premium. Note the implicit assumption that firms can issue replacement debt securities at fair value eliminates the need to explicitly consider the cost of the replacement debt issue.

I define a firm to be a *fallen angel* if its level of risk, parameterized by p, has increased beyond the threshold level $\bar{p}$, i.e. a fallen angel satisfies $p_2 > \bar{p} > p_1$. If a fallen angel firm decides to refinance, then by Proposition 2.1 it will replace its old debt that lacks the covenant with new debt that contains the covenant. What determines the decision of these firms to refinance their debt? I assume management decision making is aligned with the interests of shareholders, so firms will refinance only if doing so increases the value of the firm's equity. A fallen angel firm's equity value after refinancing or choosing not to refinance are given by, respectively:

$$E^{FA,Refi} = (1+rf)^{-1} X - (1+CP) \times K,$$
$$E^{FA,NoRefi} = (1+rf)^{-1} (1-pq) (X + \Delta - (1+c_0) \times K).$$

Rearranging, the fallen angel firm thus refinances if:

$$\frac{S(p)}{(1+rf)} + \left[\frac{(1+c_0)}{(1+rf)} - (1+CP)\right] \times K > \frac{pq(1+c_0)K}{(1+rf)} \tag{3}$$

The decomposition in Equation 3 revealing. The fallen angel refinances only if the sum of the increase in enterprise value from adopting the covenant, $S(p)$, together with the interest rate savings achieved by refinancing (left hand side), exceeds the value of the foregone opportunity to expropriate value from debt holders that was only possible in the absence of the covenant (right hand side). Intuitively, while the efficient action is for the firm to adopt the covenant whenever $S(p) > 0$, because changing covenants is only possible with refinancing, there are two different sources of debt overhang (Myers, 1977), that cause equity-aligned managers to deviate from this policy. First, all else equal, higher interest rate savings increase the attractiveness of refinancing. Second, given any level of interest rate savings, fallen angel firms are less intrested in refinancing the larger the fraction of the total value of the covenant that is effectively given up to current debt holders upon refinancing.

This framework provides empirical predictions for the refinancing patterns of firms which face changes in covenants upon refinance relative to those that do not. The simplest way to benchmark refinancing activity is with respect to movements in risk free rates. All else equal, the lower the current risk free rate, the more likely any firm with fixed coupon debt is to refinance. Thus, the model implies a *refinancing boundary*, the risk free rate at which the firm is indifferent to refinancing, refinancing is always optimal below this boundary, and is never optimal above the boundary. A simple rearrangement of the above expressions gives the refinancing boundarys for *always junk* and *fallen angel* firms:

$$\begin{aligned}
\left(1+\bar{r}^{AJ}\right) &= (1+CP)^{-1}(1+c_0)K, \\
\left(1+\bar{r}^{FA}\right) &= (1+CP)^{-1}(1+c_0)K - (1+CP)^{-1}\left[pq(1+c_0)K - S(p)\right].
\end{aligned}$$

Relative to the always junk firm, in order to refinance the fallen angel firm requires additional interest rate savings compensation to offset the loss of appropriation rights given up by adoptiong the covenant, net the increased firm value created by adopting the covenant. Similar analysis delivers the comparison of *always investment grade* firms ($p_1 < \bar{p}$ and $p_2 < \bar{p}$) and *rising star* firms ($p_1 > \bar{p} > p_2$):

$$\left(1+\bar{r}^{AIG}\right) = (1+CP)^{-1}(1+c_0)\,K\,(1-pq)\,,$$
$$\left(1+\bar{r}^{RS}\right) = (1+CP)^{-1}(1+c_0)\,K\,(1-pq) + (1+CP)^{-1}\left[pq\,(1+c_0)\,K - S\,(p)\right].$$

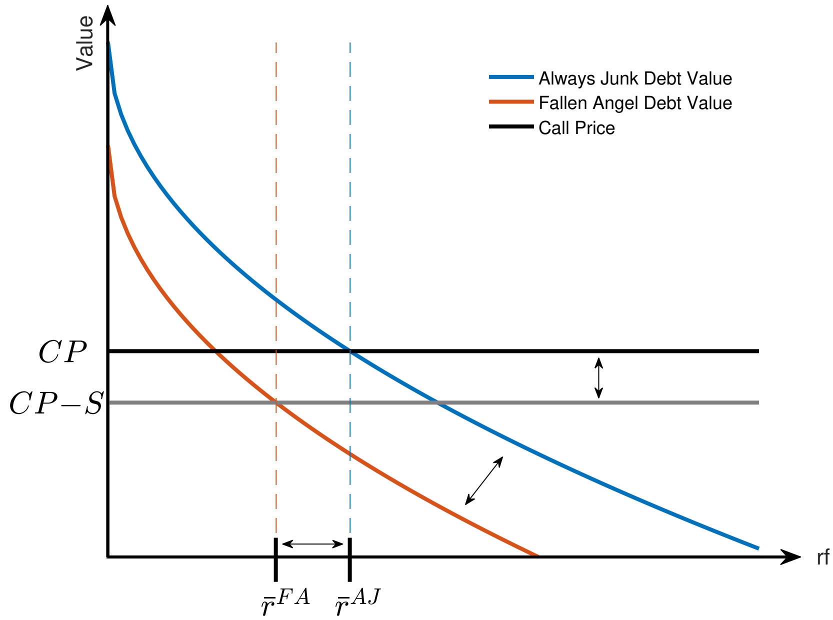
The following proposition characterizes an empirically testable systematic relationship between the refinancing behavior of these types of firms.

Proposition 2.2. *All else held equal, the refinance boundary of a fallen angel firm is lower than an always junk firm. All else held equal, the refinance boundary of a rising star firm is higher than that of an always investment grade firm.*

The proof of this proposition amounts to showing that the expression $(1+CP)^{-1}\left[pq\,(1+c_0)\,K - S\,(p)\right]$ is positive. This is demonstrated formally in Appendix C. Informally, however, the idea is that by definition of an agency conflict, the value equity holders can appropriate from debtholders by taking an inefficient action exceeds the total value destruction from this activity. Thus, fallen agnel firms delay refinancing until risk free rates fall by a sufficient amount so that interest rate savings compensate for this difference. *Rising star* firms, in contrast, are willing to refinance for a smaller amount of interest rate savings than otherwise identical *always investment grade* firms because they are willing to pay (in terms of foregone interest rate savings) to have the opportunity to remove the covenant and gain from the ability to take action A.

Figure 1 provides a graphical exposition of how the surplus generated by the restrictive covenant is identified in this framework. Imagine one could use refinancing decisions to

Figure 1: **Identification of Surplus Generated by the Restrictive Covenant**



observe the values of the refinancing boundaries $\bar{r}^{FA}$ and $\bar{r}^{AJ}$, and use secondary market pricing to observe as the difference in the value of the (risky) debt cash flows of fallen angel and always-junk firms with the same current value of p. The fact that the lack of the covenant reduces the value of debt implies that the fallen angel firms will only refinance for larger declines in the risk-free rate. However, because conditional on refinance the firm would be able to capture the surplus S generated by the newly imposed covenant, the firm will be willing to sacrifice some interest rate savings to refinance. This can be seen in Figure 1 by noting that when $rf = \bar{r}^{FA}$, the value of the firm's outstanding debt claims is still below the call price CP. If the firm is indifferent to refinancing at this interest rate, it implies the surplus the firm can capture by accepting the restrictive covenant is exactly equal to this difference. Again, a risky firm without the protective covenant places a lower value on the debt claims it owes precisely because the lack of covenant allows the firm to undertake the risky activity and thus raise its probability of default. Only when the interest rate decline is large enough are equity holders incentivized to refinance. Fixing the value of the expropriation option, the larger the total value created by the covenant, the smaller the level of interest rate savings are required to induce refinance. The remainder of the paper builds on this insight to construct a framework from which data on real-world bond refinancings can be used to estimate the surplus generated by restrictive covenants. Section 4 documents in reduced form the variation in the data, on refinancing behavior differences across firms and on differences in the value of their debt, that is conceptually necessary to back out the value of covenants. Section 5 develops the dynamic structural model required interprets this variation in the context of real world corporate bonds, and Section 6.5 estimates this model to uncover the value of the covenants included in high yield corporate bonds.

3 Data

This paper empirically analyzes inter-creditor agency problems using firms' revealed preferences about when they should refinance debt. To do this I assemble data from several sources. First, I use FISD's Mergent Corporate Bond Securities Database to construct a sample of corporate bonds. This dataset, subsequently referred to as Mergent, contains detailed data on the issuance, outstanding amount, and ratings of a large sample of corporate bond issues. Mergent provides no formal documentation of their data collection process, but conversations with the company and analysis of the data reveal that the coverage of corporate bonds is near universal after 1993 and very good for older bonds, but even better for bonds still outstanding as of 1993. Where available, the dataset contains bond ratings issued by the three major credit agencies, including ratings assigned at issue and subsequent ratings revisions throughout the life of the bond. I augment these ratings data with additional historical firm and bond ratings from Standard and Poor's S&P RatingsXpress data package.

Using the Mergent data I construct a sample of bonds that meet the following criteria: the bonds are dollar denominated, issued by a corporation, are not convertible to equity, exchangeable, or puttable, pay a fixed coupon, are publicly placed, are non-callable or are continuously callable and have a call period without "make-whole" protections. I exclude bonds issued by utility companies from the analysis because they have different covenant patterns and utility rate base regulation complicates refunding decisions.[6] I also require that I be able to match each bond to an initial credit rating, that the bond have a face value of at least $5 million, and for callable bonds that they have data on the call price schedule for the bond. This results in a final sample of 8,197 callable and 4,801 non-callable corporate bonds. Table A3 tracks the application of these screens sequentially to the sizes of the two samples.

Summary statistics for the sample of callable corporate bonds are displayed in Table 2. These bonds are issued by 3,437 distinct firms. The average size of a bond issue in the sample is $319 million and the largest bond has a face value of $11 billion. The median bond maturity is 8 years. Over 60 percent of the bonds in the sample are speculative grade at origination, with the median S&P equivalent credit rating at issuance being BB-, two notches below the highest speculative-grade rating, BB+.

For this sample of callable bonds I collect data on the bonds issuance characteristics, the terms at which the bond can be called, the ratings history of the bond, and any subsequent action of the firm that affects the amount of outstanding principal remaining of an issue. Such actions include early repayments due to exchanges, calls, and puts, as well as repayments at scheduled maturity, defaults, and restructurings. Highlighting the substantial decline in interest rates over my sample period, only 10 percent of bonds in my sample were outstanding through their original maturity date. Instead, over 70 percent of the bonds that are no longer outstanding were called, 11 percent ended due to restructuring, and the remainder ended for

[6]Make-whole protections are a relatively recent innovation in the optionality of corporate bonds. Bonds with make whole provisions are called at the maximum of the stated call price and a price computed by discounting all future coupon and principal payments using a discount rate specified as a fixed nominal spread above a benchmark Treasury rate. Thus, make-whole protected bonds have effectively variable call prices that increase as when interest rates fall. This has the effect of substantially reducing a bond's optionality on risk free rates. Since my identification comes in part from exploiting variation in option value arising from changes in interest rates, I ignore these bonds.

other reasons.

I also use Mergent's Bond Covenant file to study the prevalence of various covenants in corporate bond issues. This file tracks the presence of the most common restrictive covenants in a large subset of the bond's in the Mergent database. Covenant coverage is unfortunately fairly crude and for the majority of the sample is simply an indicator of if a given bond does or does not contain certain common debt covenants. There is a substantially more limited set of detailed notes on specific covenant provisions, but I only use these data to develop further my qualitative understanding of the typical nature of these covenants.

I also use secondary market pricing data from TRACE, Moody's Default and Recovery Database, and from Bank of America Merrill Lynch's Corporate Bond Index constituents. I use these data sources to construct estimates of the replacement cost of debt of each callable bond in my sample, and to provide auxiliary evidence on the relationship between restrictive covenants and loss given default.

4 Supporting Evidence

In this section I provide suggestive evidence that firm's decision to refinance debt is not only a function of interest rate savings, but also of changes in restrictive covenants such a refinancing would entail. The analysis is comprised of two distinct parts. First, I describe the range of covenants that appear in corporate bond indentures, and document that only the high-yield bond market includes restrictive covenants. Second, I show that the refinancing behavior of firms reveals the patterns we should expect if bond covenants ameliorate agency costs of debt.

4.1 Covenant Strength in Investment Grade and High Yield Bonds

In this section I describe the restrictive covenants most commonly found in corporate debt. I limit this analysis to my sample of callable and non-callable bonds described in

Section 3 for which I have data on bond covenants. This results in sample of 8,400 bonds. Figure 3 plots, by initial credit rating, the fraction of bonds that contain a given type of covenant. First, we can see that the presence of negative pledge or stronger covenants protecting the seniority of the bond issue is uniform across bond ratings. However, limiting the analysis to covenants restricting debt that are *stronger* than a negative pledge covenant, we see that this fraction jumps precisely at the high yield boundary (BB+), and nearly all high yield bonds contain such a covenant. Other covenants, including restrictions on non debt payments, restrictions on asset sales, and restrictions on transactions with affiliates, show significant discontinuity around the investment grade boundary. One exception is the takeover protective covenant, which typically gives bondholders a right to demand immediate repayment upon a change of control event. These covenants are designed to protect against debt dilution, for example in the event of a leveraged buyout. Asquith and Wizman (1990) study the ex-post value of this particular covenant in the event of leveraged buyouts. It is not surprising that takeover protection covenants are included in investment grade bond issues, as leveraged buyouts are conducted precisely to significantly modify the capital structure of a firm and often result in a substantial credit downgrade. In general, a defining feature of a high-yield covenant package is the inclusion of restrictive covenants that impose limits on future debt issuance and investment and payout policy.

These patterns may seem striking in the graphical form I have presented them. The best explanation is that the investment-grade and high-yield bond markets are quite segregated and different standards emerged in the two markets that reflect average differences in credit risk across these markets. Market participants I spoke with emphasized that the high-yield and investment-grade markets are indeed very separate, stressing that underwriting in these markets is deals with different sets of investors and firms. Despite these markets being distinct, there is ample evidence, anecdotally and empirically, that firms do gain and lose access to different subsets of capital markets as their credit ratings change. Graham and Harvey (2001) report that firms are conscious of their debt rating and its impact on their cost of capital.

These results show that there is a stark difference between the strength of covenants offered across high-yield and investment-grade corporate bonds in the cross section. However, do the covenants available for refinancings to specific firms whose debt is upgraded or downgrade also change this way? I further confirm that these differences remain within the bond issues of a given firm that crosses the investment-grade/high-yield boundary.

To do this, I start with my sample of called corporate bonds and, when possible, match them to the refunding bonds issued to replace them. My criterion for this match is that the firm issues the replacement bond within one year it repays the old bond issue. Figure A1 shows the distribution of timing gaps between my matches of called and refunding bond issues. In a textbook bond refunding a firm will simultaneously issue its new bond and announce it will call its old bond in thirty to sixty days. Thus, a majority of the timing gaps are in the one month range. However, if the firm has the financial flexibility to temporarily increase its debt, it is also possible to prepay earlier than the new bond is financed; for example by temporarily drawing down a credit facility, or issuing new debt first and calling the bond months later.

In each case, I indicate if the called bond was in a different rating class at the time of refinancing than it was at origination, either upgraded to investment grade (a rising star bond) or downgraded to high yield (a fallen angel bond). In this matched sample of bonds I collect information from Mergent, when it exists, on the covenants contained in each of the original and replacement bonds.

Table 4 reports average differences between covenants in the called and refunding bonds by the rating change status of the called bond. In the first column we can see that when fallen angel bonds are refunded, their replacement bonds contain around six more restrictive covenants. Similarly, when rising star bonds are refunded, the new issue sheds three restrictive covenants on average. Which covenants are the most likely to be added or dropped? The remaining columns of Table 4 are averages of differences in indicators for the presence of specific covenants between the called and refunding issue. For example, we see that nearly three quarters of called rising star bonds shed covenants that limit debt issuance, specify restricted payments, or limit transactions with affiliates. The most commonly adopted covenant in fallen angel refinancings is a limitations on asset sales, but the use of all other common covenants increases markedly as well.

Overall, these results support the notion that capital markets supply loose covenant restrictions to highly rated firms and strong restrictive covenants to poorly rated firms, and that these patterns also hold *within firm* as firms cross the investment-grade/high-yield boundary, indicating that individual firms do face large differences in covenants as a function of bond refunding decisions.

4.2 Covenants and Debt Refinancing

In this section I test the second basic prediction of the model in Section 2–if covenants help to solve agency problems but are imperfect in doing so, firms' decisions to refinance callable debt should reflect not only interest rate savings but also their valuation of shedding or avoiding restrictive covenants.

Table 3 shows some basic summary statistics about called bonds as a function of major ratings changes experienced from issuance to the time the bond ends (to to maturity, call, or restructuring). The sample is limited to the 5,140 callable bonds that were no longer outstanding as of December 31, 2016. Rising star bonds are the most likely to be called, and are called at the highest call price and lowest decline since the bond's issuance in the 10 year Treasury yield. In contrast, fallen angel bonds are on average called at lower call prices and when Treasury yields have declined by 114 basis points more than it takes for rising star bonds to be called. Comparing rising star bonds to always investment grade bonds, rising star bonds are called more aggressively (at higher call prices and lower interest rate savings). Comparing fallen angel bonds to always junk bonds, fallen angel bonds require larger interest rate savings and lower call premiums to be refinanced.

These facts are *consistent* with the idea that covenants affect the refinancing decision, but testing this prediction is complicated by the dynamic nature of the firm's decision to refinance long-term debt in the presence of stochastic borrowing costs. In the single period model, firms decide to refinance by weighing the one-period interest cost savings of refinancing against the cost or benefit in changes to restrictive debt covenants. In reality, the interest rate savings is represented by the value of the option embedded in callable debt to repay the bond ahead of maturity. I now introduce a simple statistical model of the refunding decision of long-maturity debt under stochastic interest rates that ignores covenant considerations. I then estimate this model and show that there are observed systematic deviations from the model-implied optimal refinancing policy that exactly coincide with a role for changing covenants in the refinancing decision.

A Dynamic Refunding Model Ignoring Covenants

Consider a callable bond with maturity τ_m, continuous coupon process $c(t)$, principal normalized to 1, and call price schedule $CP(t)$. To reduce notation define $CP(\tau_m) = 1$ to capture the notion that at maturity the bond pays back its principal. Assume the bond is subject to exogenous default intensity $h(t)$. In the event of default at t, debt holders recover a fraction of market value of the bond $(1 - L(t))$. The price (per unit of principal) of this callable risky bond can be expressed as:

$$P(t) = \min_{\tau < \tau_m} \mathbb{E}_t^Q \left[\int_t^{\tau} e^{-\int_t^s R_m dm} c(s)\, ds + e^{-\int_t^{\tau} R_m dm} CP(\tau) \right]$$
$$R(t) = r^f(t) + h(t)\, L(t)$$

where $R(t)$ denotes the "default adjusted" short rate process, which can be used to price default-able claims as if they were risk free, as shown by Duffie and Singleton (1999). I assume the default adjusted short rate process follows a single-factor diffusion process under the risk neutral measure.

$$dR_t = \mu(R_t)\,dt + \sigma(R_t)\,dZ_t^Q$$

The single-factor specification of the default-adjusted short rate process implies that the optimal call policy can be expressed as an *exercise boundary* $b(t)$ in the state variable $R(t)$:

$$\tau_c = \min\ t \ \text{ s.t. } \ b(t) > R(t)$$

where the exercise boundary is a function of time to maturity, the coupon and call schedule of the bond, and the parameterization of the stochastic process for the default adjusted short rate. Assuming no role of covenants in the refunding decision, the process for R embeds the aspects of the firm's cost of capital that are important for the refunding decision. When the short rate falls, the firm assigns a higher valuation to the stream of interest and principal payments, relative to the fixed call price schedule at which the firm can retire the bond.

Hypothesis Specification

I now use this model to test if covenants play a role in the refinancing decision as described in Section 2. Consider two firms that are identical in every way except one has debt with loose covenants and the other has debt with strict covenants. Assume that if either firm were to refinance its debt, the new debt issue would contain the strict covenants. If firms are averse to adopting restrictive covenants, the firm that currently has loose covenants will not be as willing to refinance as the firm that already has covenants. However, the refunding model presented above has no way to account for the difference between these two firms and would deliver identical optimal refunding decisions for each firm. Thus, a simple and direct test of the hypothesis that covenants play a role in the refunding decision is to see if this model's prediction of refunding decisions of fallen angel and rising star bonds are systematically biased relative to the model's refinancing predictions of other bonds.

Specifically, I will consider the following statistical model specification to evaluation the naive bond refunding model:

$$
\begin{gathered}
Call_{it} = \mathbf{1}\left\{b^*_{it} > R_{it}\right\}, \\
b^*_{it} = \hat{b}_{it} + \delta^{FA} FA_{it} + \delta^{RS} RS_{it} + \beta X_{it} + \epsilon_{it} \\
\epsilon_{it} \sim N\left(0, \sigma^2\right)
\end{gathered} \tag{4}
$$

Where $Call_{it}$ is an indicator of if bond i is called at time t, $\hat{b}_{it}$ is the refinancing boundary for bond i at time t implied by the dynamic refinancing model, b^*_{it} is the true policy followed by the firm, and R_{it} is an estimate of the default-adjusted short rate of bond i at time t. The difference between b^*_{it} and $\hat{b}_{it}$ models the deviations of firm's *actual* refinancing decisions from this model. It accounts for linearly additive miss-specification in both the model implied boundary and the estimated default adjusted short rate. If firms are willing to forgo interest rate savings to avoid restrictive covenants then we should expect $\delta^{FA} < 0$, or that the model is over-estimating the refinancing boundary of fallen angel firms. Similarly, if firms are willing to give up option value of future interest rate savings to *shed* restrictive covenants, we should expect $\delta^{RS} > 0$.

Estimation Strategy

To implement this test, I need to solve the optimal refinancing model for each bond and assemble an empirical estimate of each bond's default adjusted short rate at each time the bond is callable. This requires estimation of the risk-neutral dynamics of firms' default adjusted short rates and a procedure for mapping firm and bond observables into default adjusted short rates. I provide a detailed explanation of this procedure in Appendix C, but summarize it briefly here.

I first assume the default adjusted short rate follows a single factor affine diffusion process under the risk-neutral measure, parameterized by θ. This allows the model to be solved for optimal exercise boundaries, now expressed as $b_{it}\left(\theta\right)$. I assume the same process for the default adjusted short rate R also prices non-callable corporate bonds, and that the price of risk is also linear in this process. This allows me to exploit the tools of affine term structure modeling to estimate θ by maximum likelihood, as outlined in Singleton (2001) and implemented in Duffee (2002).

I next construct estimates of default adjusted cost of capital R_{it}. I first obtain estimates of default adjusted costs of capital for my panel of non-callable bonds at each point in time I observe a price for the bond. I then estimate a flexible parametric relationship between these observable default adjusted short rates and observable characteristics at the firm-bond-time level, and project this relationship onto the callable bond sample to obtain short rate estimates for each callable bond at each time the bond is callable.

Results

Tables 5 reports the results of the estimation of Equation 4 under various specifications of the vector of control variables X_{it}. The sample is comprised of all bond-months for which the bond is outstanding and currently callable. Consistent with a role of covenants in the refinancing decision, fallen angel firms delay their refinancing and rising star firms accelerate refinancing relative to firms for which ratings have not changed materially. Column 1 reports baseline estimates in which X_{it} contains controls for the origination ratings class of the bond and the number of months since the bond was issued. By controlling for current rating fixed effects, the coefficients on the fallen-angel and rising-star dummies can be interpreted as systematic deviations in observed refinancing behavior from the model-implied refinancing boundary relative to any systematic deviations of other bonds in the same current rating category that have not been upgraded or downgraded significantly since origination.

Column 2 adds current rating fixed effects, and column 3 further adds industry fixed effects. The fixed effects specifications imply that fallen angel firms, relative to always junk firms, on average wait until their default adjusted short rate is on average roughly 48 basis points below the model implied boundary. In contrast, rising star firms exercise when the default adjusted short rate is around 58 basis points above investment grade firms that would not be shedding covenants in a refinancing. The final column adds financial ratios from Compustat which may contain omitted variation about credit risk that is related to the firm's decision to refinance. The inclusion of these controls does not reduce the magnitude or statistical significance of the rising star and fallen angel coefficients.

I also explore how the individual restrictive covenants at stake in a refinancing decision affects the call exercise boundary. Table 6 repeats the specifications of Table 5 limited to the subset of bonds for which I am able to match data on individual covenants included in the bonds. The first column of Table 6 repeats the baseline fixed effects specification (column 3 of Table 5) on the subset of bonds for which covenant data is available. The second column adds the number of restrictive covenants and its interaction with rising star and fallen angel indicators. The more covenants included in a rising star bond, the more eager the now-investment grade firm is to refinance it, holding fixed the interest rate incentive to refinance. In fact, variation in the number of covenants of in the current rising star indenture explains the entire early propensity of rising star firms to refinance earlier than comparable firms that have bonds issued as investment grade. This further supports the notion that it is exactly changes in covenants that are driving the differences in refinancing patterns observed here. The interaction of the fallen angel indicator and number of covenants is not precisely estimated. This is due to the fact that there is very little variation in the covenants of bonds issued as investment grade.

The third column of Table 6 consider the effects of the presence of specific covenants. Because there is so little variation in the covenants of fallen-angel bonds I report the interactions only for rising star firms' incentives to refinance. The possibility of shedding restricted payments, cross acceleration, and minimum net worth covenants seem to be particularly valuable to firms, though only the effect of the cross acceleration covenant is statistically significant. These types of covenants directly prevent shareholders from being able to extract value from the firm in bad states. It is thus not surprising that shareholder's revealed valuation of this restriction is higher than that of other covenants, which impose less direct limits on equity value maximization.

Robustness

The previous analysis ignored the fact that firms can have multiple bond issues outstanding and thus refinancing an individual debt issue may not change the set of restrictive covenants binding on the firm as a whole. Specifically, a fallen-angel firm may have little incentive to delay refinancing one bond if it has previously issued other speculative-grade debt that imposed covenants on the firm. On the other hand, some restrictive covenants act to protect the individual, and their absence in a fallen angle issue may provide the firm financial flexibility it would lose in a refinancing. To explore this channel I augment the previous analysis with interactions of the fallen angel indicator and variables that measure if the firm has other debt that was issued as speculative grade. Table 7 presents this analysis. The first column replicates column 3 of Table 5 and each additional column estimates a different parameterization of the presence on the balance sheet of debt likely to contain covenants. Column 2 shows that a fallen angel bond issued by a firm that has subsequently issued high yield debt significantly reduces the delay in refinancing relative to firms with no subsequent high-yield debt issues. However, firms with outstanding high yield debt still delay refinancing. Column 3 considers a continuous measure, the fraction of the firm's outstanding debt that was originated as high-yield. The interpretation is that if essentially all of a firm's outstanding debt has restrictive covenants, there is very little incentive for the firm to delay refinancing its fallen-angel bonds. Finally, column 4 considers a "slope-intercept" parameterization by including interactions with both the existence of any high-yield issued debt and the fraction of this debt in the firm's capital structure. Tables A1 and A2 add the interaction of the fallen angel indicator and the fraction of the firm's debt issued as high-yield to specifications of Tables 5 and 6 and confirm the qualitative findings are unchanged. I interpret these findings as a powerful verification that my main results are entirely driven by covenant considerations.

I also conduct a placebo test to consider the possibility that these results are spurious or driven by an omitted mechanism. The model specifications in Tables 5 and 6 all (correctly) assume that crossing the investment-grade/high-yield boundary induces differences in the covenants of the outstanding and potential refunding bond issue. As a placebo check, I instead estimate the model assuming this difference in covenants occurs at various different credit ratings. Figure A2 reports the log-likelihood of these estimated models as a function of the hypothetical investment-grade/high-yield boundary. The fact that the log likelihood peaks exactly at the true boundary between the investment grade and high-yield market, beyond which covenants first are included in bonds, provides further evidence that this model is capturing a real relationship between covenants and the observed refinancing behavior of bond issuers. A related interpretation of this exercise is that it is using the model and observed refinancing data to estimate where in the distribution of credit ratings covenants on new bond issues change substantially, under the assumption that covenants are an important aspect of the refinancing decision. If potential covenant changes did not affect the refinancing decision we would not expect an estimation procedure using data on bond refinancings to reveal the true difference credit rating boundary between loose and restrictive covenants.

Unfortunately, the results presented in this section do not *quantify* how much covenants help reduce agency costs of debt or the magnitude of the costs covenants would impose on firms that do not need them. In order to do this, I now turn to estimating a dynamic model that explicitly considers the role of restrictive covenants.

5 Dynamic Model of Refinancing and Covenants

In the previous section, I document that refinance interest rate boundaries are *lower* for fallen-angel firms, which face new restrictive covenants when refinancing, and boundaries are *higher* for rising-star firms, which can shed covenants by refunding debt, relative to a model that ignores covenants. This implies that loose covenants transfer value from debt to equity claimants. However, this analysis does not quantify the extent of this transfer of value, nor the value created or destroyed by the inclusion of restrictive covenants. As shown in the static model of Section 2, to answer these questions we need to understand both how restrictive covenants shift these exercise boundaries and affect the value of debt. This requires a model of refinancing that explicitly considers how covenants resolve agency conflicts in a dynamic setting. I now introduce such a model.

5.1 Model Primitives

The model is in discrete time and continues indefinitely until the firm defaults at random default time τ^D. The firm's only debt is a callable coupon bond with face value K which pays a per-period coupon c. The firm has assets in place that generate per-period revenue $A_t = A(X, \gamma)$ for $t < \tau_D$. X is a state variable capturing the distribution of per-period cash-flows and their dynamics and $\gamma \in \{0, 1\}$ is an indicator of if the firm is subject to restrictive debt covenants. There are N non-default states and one default state, and the firm can transition between non-default states and into the default state. I model the flow value of cash-flows to the non-defaulted firm as:

$$A(X_t, \gamma_t) = a(X_t) + f(X_t) \times \mathbf{1}\{\gamma_t = 0\}.$$

Here $a()$ and $f()$ are general functions of the state variable X. The firm only receives $f(X)$ if it is not subject to restrictive covenants. This term captures the state-dependent increase in asset returns allowed by not having restrictive covenants, conditional on not defaulting.

The covenant status of the firm also affects the firm's probability of default, because loose covenants allow the firm to engage in projects with higher risk. The risk-neutral probability of defaulting in period t (conditional prior survival), $p_t^D = p^D(X, \gamma)$, is given by:

$$p^D(X_t, \gamma_t) = p(X_t) + s(X_t) \times \mathbf{1}\{\gamma = 0\}.$$

In the event of default, the firm permanently ceases debt payments and receives no future returns from assets. If the firm is not in default it pays per-period claims to debt-holders $d_t = d(\tau^m, c, \phi)$, where τ^m is the time to maturity of the firm's single debt issue, c is the per-period coupon, and $\phi \in \{0, 1\}$ is the firms decision to call its outstanding debt. The required payment is either the coupon on its outstanding debt, the principal and final coupon at maturity, or the coupon and early prepayment of the debt if the firm exercises its option to call the bond. The debt service in each period is thus given by

$$d(\tau, c, a) = \begin{cases} cK & \phi = 0 \\ (1 + CP(\tau) + c) K & \phi = 1, \end{cases}$$

where the call premium due at maturity is always zero. In the case the outstanding bond is retired (either at maturity or due to the firm exercising its call option on the bond), the firm issues new debt with coupon $c'(r, x)$ set so that the new bond is offered at par and and principal amount to finance the replacement cost of the previous bond.

The state variable capturing the firm's inherent risk X_t evolves as a Markov process with transition matrix Π specified under the risk neutral measure. Following the results of Proposition 2.1 and the empirical evidence in Section 4.1, the new debt contains covenants if and only if the firm's credit rating is below investment grade. The replacement bond has a maturity $\bar{\tau}^m(X_t)$ and call premium schedule $CP(\tau)$ that are set exogenously.

The risk-free short rate given by r_t and is assumed to evolve as an affine diffusion process under the risk-neutral measure. The firm is assumed to be incentivized to maximize the present value of equity claims. The only control variable of the firm in this model is the decision to refinance its debt each period. There are two reasons a firm would want to refinance: to lower its debt servicing costs or to change the covenants imposed on the firm, which changes the distribution of asset returns, as well as the decomposition of expected asset returns between equity and debt. Given the primitives of the model the objective of the firm is to choose its refinancing to maximize the present value of equity. The equity value maximization problem can be written as:

$$V_t^E\left(r_t, X_t, \gamma, \tau^m\right) = \max_{\{\phi_s\}_{s>t}} \mathbb{E}_t^Q\left[\sum_{s=t}^{\tau^D} e^{-\int_t^s r_u du}\left[A\left(X_s, \gamma_s\right) - d\left(\tau_s^m, c_s, a_s\right)\right]\right] \tag{5}$$

$$s.t.$$

$$\tau_{s+1}^m = \begin{cases} \left(\tau_s^m - 1\right) & \phi_s = 0 \\ \bar{\tau}^m\left(p\right) & \phi_s = 1 \end{cases} \tag{6}$$

$$\gamma_{s+1} = \begin{cases} \gamma_s & \phi_s = 0 \\ \mathbf{1}\left(X_s \leq \bar{X}^{HY}\right) & \phi_s = 1 \end{cases} \tag{7}$$

$$c_{s+1} = \begin{cases} c_s & \phi_s = 0 \\ c'\left(r, p\right) & \phi_s = 1 \end{cases} \tag{8}$$

$$\tau_s = 0 \implies \phi_s = 1 \tag{9}$$

$$\Pr\left(\tau^D = s | \tau^D > s - 1\right) = p^D\left(X_t, \gamma_t\right). \tag{10}$$

Equation 5 specifies the state-dependent value of equity, and Equations 6-10 specify the constraints on the evolution of the endogenous state variables. Specifically, remaining maturity decreases until the bond is replaced, at which point it resets to the exogenously specified level. Covenants on debt do not change until debt is refinanced, in which case they are present only if the firm has a below investment-grade rating. To reduce notation, I specify that the bond is semantically refinanced ($\phi = 1$) upon maturity if it has not been refinanced already. Given an optimal refinancing strategy $\{\phi_t^*\}$, the value of the callable bond can be expressed as

$$P_t^D\left(r_t, X_t, \gamma, \tau^m\right) = \mathbb{E}_t^Q\left[\sum_{s=t}^{\tilde{\tau}^m} e^{-\int_t^s r_u du} d\left(\tau_s^m, c, \phi_s^*\right)\right],$$
$$\tilde{\tau}^m = \min\left(\tau^m, \tau^D\right).$$

It is also useful to develop an expression for the agency costs of debt solved by covenants in this model. A natural expression for this is simply the difference between actual firm asset value and the counterfactual asset value assuming it is never possible to use debt covenants. Denote this quantity as the asset value of covenants (AVC):

$$AVC\left(r_t, X_t\right) \equiv \mathbb{E}_t^Q\left[\sum_{s=t}^{\tau^D} e^{-\int_t^s r_u du} A\left(X_s, \gamma_t\right)\right] - \mathbb{E}_t^Q\left[\sum_{s=t}^{\tau_{\gamma=0}^D} e^{-\int_t^s r_u du} A\left(X_s, \gamma = 0\right)\right], \tag{11}$$

where $\tau_{\gamma=0}^D$ signifies that the random default time is given the firm never has restrictive covenants.

5.2 Model Discussion

The goal of this model is to capture the relationship between covenants, the distribution of asset returns, and optimal debt refinancing in a dynamic infinite horizon setting with as minimal complexity as possible. The model assumes a very simple parameterization of the distribution of asset returns: each period the firm either survives and generates revenue or it defaults permanently. The flow revenue and default probability are functions of two state variables: one capturing the inherent fundamentals of the firm and the other indicating if the firm is subject to restrictive covenants.

This model can be thought of as a generalization and multi-period extension to the static model presented in Section 2. In both models the distribution of firm cash-flows depend on the level of inherent risk in the firm and if the firm has restrictive debt covenants. The difference between these models is that here the periods are dynamically linked: by the fact that default affects all future cash-flows and because the firm's debt has a maturity longer than one period. These dynamics are essential to exploiting the variation in the data that credibly identifies the effect of covenants on the agency cost of debt.

It is important to note how the model is scaled: it is expressed in terms of $1 of par value of debt. For every dollar of debt a firm has, the firm generates $A(X_t, \gamma)$ in cash-flow. Thus, the model objects' relationships with the state variable X_t capture the reduced-form relationship between earnings, leverage, maturity, and the probability of default. These features of the firm are not modeled explicitly, but rather taken as given to model the refinancing decision of a firm.[7] Further, note that the model does not explicitly distinguish between probability of default and loss given default. Instead, the state transition process and probability of default parameterization $p^D(\cdot)$ capture the stochastic process for the continuation value of servicing a bond. Further decomposition between probability of default and loss given default is neither identified nor relevant in this framework. Thus the estimated default probabilities should be interpreted as risk-neutral probabilities of default *as if* there is no recovery value of the bond in default.

My modeling choices necessitate a comparison to those more common in the dynamic capital structure literature, for example Strebulaev (2007), Hennessy and Whited (2005), DeAngelo, DeAngelo, and Whited (2011), and Morellec, Nikolov, and Schurhoff (2012).[8] These models typically introduce various trade-offs of capital structure considerations in a dynamic setting and are calibrated or estimated to match empirical moments of firm capital structure, such as leverage ratios and their dynamics. The ability or inability of these models to fit aggregate moments is then used as support for or against their underlying economic mechanisms. In contrast, many (but not all) elements of my model are statistical and not

[7] See Billett, King, and Mauer (2007) for a comprehensive overview of the empirical relationship between corporate leverage, maturity, and covenant usage.

[8] See Strebulaev and Whited (2012) for an excellent review of dynamic models and structure estimation in corporate finance.

structural in nature, and I use the model to estimate parameters that are reduced-form representations of objects of real world importance. Regarding model structure, I do not endogenize capital structure elements such as leverage, debt maturity, and their dynamics, or even consider a framework in which a certain capital structure is optimal. In theory the decision to perform a simple debt refunding is isolated from other aspects of firm capital structure. Thus, for the purpose of identifying how covenants shift the distribution of asset value between claimants, it is sufficient to treat the other decisions of the firm as exogenous and to let the exogenous state variables capture the implications of these decisions in reduced form. Further, I do not attempt to endogenize other aspects of capital structure because the variation I will exploit to identify my model only intuitively identifies how covenants change the distribution of asset returns.

5.3 Simplifying Assumptions

The above model can be solved numerically by value function iteration to a fixed point on the state variables (r, X, γ, τ). Since the ultimate goal is to estimate key parameters of this model from data by applying it to each bond in my sample at each time the bond is callable, solving the model by fixed-point iteration on four state variables is computationally infeasible. Instead, I now introduce two assumptions that greatly reduce the complexity of solving the model but impose minimal loss of flexibility or realism.

Assumption 5.1. Any subsequent replacement bond issues after the retirement of the first bond are non-callable.

Assumption 5.2. The replacement bond issues are fairly priced. The firm and capital markets agree on the valuation of promised coupon and principal payments.

Assumption 5.3. The risk-free short rate evolves as a single-factor affine diffusion process.

Assumptions 5.1 and 5.2 together allow for the model to be solved by a combination of value function iteration and backward induction of smaller two-state variable problems. If replacement debt issues are priced fairly, then the cash-flow the firm receives from issuing the new bond is exactly equal to the firm's valuation of the principal and interest payments, and we can thus these terms cancel out beyond the initial refinancing. Also note that *some* assumption on the pricing of future debt needs to be made in order to estimate this forward looking dynamic model, and this particular assumption is both natural and convenient.

However, because covenants affect the equity value of the firm, Assumption 5.1 alone does not ensure the model can be solved by backward induction from maturity: we still need to know the continuation value of equity associated with the flexibility of covenants imposed in the future by all subsequent bond issues. If the replacement debt is callable, this involves pricing the option value of covenant flexibility afforded by the refinancing option embedded in all *future* bond issues. Specifying replacement debt as non-callable allows one to solve for the continuation value due to covenants in a straightforward fashion without explicitly modeling these dynamics. I do however, incorporate the realistic feature that speculative grade firms extend debt maturity in refinance while speculative grade firms do not alter maturity in refinance, as documented by Xu (2016).

Fortunately, the implications of this simplifying assumption are minor. Related to the intuition behind the results in Dunn and Spatt (2005), when debt can be refinanced more than once, there is a limited sensitivity of the contemporaneous refinancing decision to properties of subsequent refinancing options when the subsequent refinancing options are being priced into the replacement debt issue.

Assumption 5.3 is not strictly necessary, but drastically simplifies the model. By assuming the term structure of risk free rates is captured by a single factor, the firm's refinance decision can be expressed as an *exercise boundary* in the current level of the short rate for a given value of the other state variables (credit rating, remaining maturity, and covenant status). A more realistic model would allow a multi-factor term structure to match the term structure of interest rates, but in such a model optimal call policies would instead be an exercise *surface* in the interest rate factors and introduce additional state variables to the model.

As mentioned previously, my model is already innovative in extending dynamic models of firm decisions to account for stochastic interest rates. Expanding dynamic corporate finance models to further capture the term structure of rates is an important direction for continued work in this area. For this paper, however, I assume the short rate itself is an affine diffusion process and evolves under the risk-neutral measure as a single factor Ornstein-Uhlenhbeck process:

$$dr_t = \kappa \left(\bar{r} - r_t \right) dt + \sigma dZ_t^Q.$$

5.4 Solution

I now describe how the model is solved for each bond to determine the bond's optimal refinancing policy $\phi \left(\tau, r, X, \gamma \right)$ and price $P \left(\tau, r, X, \gamma \right)$ given the parameters of the model.

Given a parameterization, the first step in solving the model is to solve for the continuation value of assets after the initial refinancing. Given a solution value function, the optimal exercise policy is solved by backward induction from all states in which the bond matures, defaults, or is refinanced. This begins at the month of bond maturity, and proceeds backward each month to the first month the bond is callable. At each time step, the model produces functions for the value of the equity and debt, as well as the optimal refinancing policy as a function of the state variables. For a given remaining maturity, covenant status, and credit rating, the optimal refinancing policy takes the form of an *exercise boundary* in the risk-free short rate:

$$b \left(\tau, X, \gamma \right) \equiv \max \, r \text{ st } \left\{ a \left(\tau, r, X, \gamma \right) = 1 \right\},$$

$$\phi \left(\tau, r, X, \gamma \right) = 1 \iff r \leq b \left(\tau, X, \gamma \right)$$

6 Structural Estimation of Covenant Value

6.1 Model Parameterization

I now specify the exact parameterization of the model of Section 5 that I will bring to the data. In the general model the state space and number of parameters are both potentially large. To simply estimation and to ensure the model is identified, I introduce several parametric assumptions.

Empirically, I proxy for the state variable X with bond credit ratings.First, I assume the state variable X takes one of eight discrete values $X \in \{1, 2, .., 8\}$ where 1 is the highest rating. The transition matrix Π describes the evolution between the eight non-default states of X. I further assume that the firm can only transition into default from the worst state. Thus, the probability of default specification of the model is

$$p^D(X, \gamma) = \begin{cases} 0 & X \leq 7 \\ p + s \times \mathbf{1}\{\gamma = 1\} & X = 8 \end{cases}$$

Let$\bar{X}^{HY}$ denote the cutoff credit rating at and below which firms receive covenants on new bond issues. I parameterize the relationship between covenants and non-default asset returns by assuming this value is only a function of if the firm's current credit rating is above or below this credit rating cutoff:

$$f(X) = \begin{cases} f^B & X \leq \bar{X}^{HY} \\ f^G & X > \bar{X}^{HY} \end{cases}$$

Thus, firms generate additional asset return f^G when they are investment-grade and have no covenants, and f^B when they are speculative grade but have no covenants. I further assume that conditional on survival, the level of asset returns each period *without* debt covenants is a fixed constant:

$$a\left(X\right)=\bar{a}.$$

As previously specified, the risk free short rate is assumed to follow a single-factor Ornstein-Uhlenhbeck process with parameters $(\bar{r},\kappa,\sigma)$.

I have now described all unknown parameters in the model. Given properties of the callable coupon bond (its coupon, maturity, and call price schedule), the parameter vector θ completely describes the model:

$$\theta=\left(\bar{r},\kappa,\sigma,\Pi,f^{G},f^{B},\bar{a},p,s\right)$$

6.2 Calibrated Parameters

I calibrate the parameters of the model that are not intuitively identified by the model: those governing the dynamics of the exogenous state variables of the model, and the average asset returns per dollar of debt. For the benchmark specification the parameter $\bar{a}$ is calibrated the sample median ratio of monthly $EBITDA$ to long term debt, which is 0.025.

The risk free rate variable takes the form of a discretized Vasicek (1977) short rate process. I use 75 grid points spaced between 0 and 25 percentage points and fit the parameters to the monthly innovations of the three-month Treasury rate over the period 1970-2016 The estimated parameters (expressed in annualized units) are

$$\bar{r}=0.035,\kappa=0.12,\sigma^{2}=0.00028.$$

I calibrate the credit rating transition density using external data. The primary purpose of this is to keep the number of estimated parameters reasonable and ensure the estimated parameters are identified in the model. I map the eight levels of X in the model to observed S&P equivalent long term bond ratings according to the concordance in Table A4. There are four investment grade ratings and four speculative grade ratings. I use annual ratings transitions probabilities reported by S&P (1995) and estimate a generator matrix using the method described by Israel, Rosenthal, and Wei (2001) to create a monthly transition matrix. Table A5 reports the original data, estimated generator, and monthly transition probabilities.

6.3 Model Estimation

Given a calibration of $(\Pi, \bar{r}, \sigma, \kappa)$ we can now proceed to estimate $\left(f^G, f^B, p, s\right)$ by matching the model implied bond prices and refinancing decisions to their observable counterparts. I estimate the above model using a maximum likelihood estimation strategy.

Recall that for given values of known and unknown parameters, and for a given realization of the state variables describing the firm's credit risk, the model solution is an exercise boundary for the risk free short rate. The firm should initiate the repayment of its bond only if the short rate is below this state contingent boundary. To formulate a likelihood function that identifies the parameters, I assume there is a source of unmeasured error in this exercise boundary that is not captured by the model. The true boundary is:

$$b^* \left(X_{it}, \tau, \theta\right) = b \left(X_{it}, \tau, \theta\right) + \epsilon_{it}^b.$$

Similarly, the true value of the callable bond i at time t is

$$P^* \left(r_t, X_{it}, \gamma, \tau^m\right) = P \left(r_t, X_{it}, \gamma, \tau^m\right) + \epsilon_{it}^p$$

Where the measurement error vector $\left(\epsilon_{it}^b, \epsilon_{it}^p\right)$ is assumed to be jointly normally distributed with mean zero and covariance matrix Σ_ϵ. Recall ϕ_{it} is an indicator of if bond i is actually refinanced at time t. Then the model implies

$$\Pr\left(\phi^*_{it}=1\right)=\Pr\left(r_t-\epsilon^b_{it}<b_{it}\left(\theta\right)\right)$$
$$\Pr\left(\phi^*_{it}=0\right)=\Pr\left(r_t-\epsilon^b_{it}>b_{it}\left(\theta\right)\right)$$

Now consider the likelihood of the model implied bond price matching the data

$$\Pr\left(P^*_{it}\right)=\Pr\left(\epsilon^p_{it}=P^*_{it}-P_{it}\left(\theta\right)\right)$$

Let ϕ^*_{it} be an indicator for if bond i is actually called at time t. And for brevity let $b_{it}\left(\theta\right)$ denote the model implied call boundary with state dependency implicit in the subscripts. The likelihood of an individual observation is:

$$\mathcal{L}_{it}=\Pr\left(\epsilon^b_{it}>r_t-b_{it}\left(\theta\right)\ \&\ \epsilon^p_{it}=P^*_{it}-P_{it}\left(\theta\right)\right)^{\phi^*_{it}}\Pr\left(\epsilon^b_{it}<r_t-b_{it}\left(\theta\right)\ \&\ \epsilon^p_{it}=P^*_{it}-P_{it}\left(\theta\right)\right)^{1-\phi^*_{it}}$$

Because the errors are assumed independent across observations we can write the log likelihood function of all the data as

$$\ell\left(\theta,\Sigma_\epsilon\right)=\sum_{i,t}\omega_{it}\log\left(\mathcal{L}_{it}\right) \tag{12}$$

The weights ω_i are constructed to ensure each bond receives the same influence in the likelihood function no matter how long the particular bond was callable in the data.

Likelihood evaluation is performed in a two step procedure. In the inner step, given a candidate value of θ the model produces policy functions of $b_{it}\left(\theta\right)$ and prices $P_{it}\left(\theta\right)$ that are not a function of Σ_ϵ.[9] The likelihood function can thus be quickly maximized over Σ_ϵ in the inner step without recomputing the optimal bond policies, giving $\hat{\Sigma}_\epsilon\left(\theta\right)$. In the outer step,

[9]Embedding structural errors into the nonlinear bond pricing and refinancing model would improve the model's ability to deal with selection, but comes at the cost of greatly increased computational complexity.

the likelihood function is simply maximized over parameters to be estimated in θ. Standard errors of the maximum likelihood estimates are calculated by evaluating the Hessian matrix of the likelihood function at the maximum likelihood estimates.

6.4 Identification of Estimated Parameters

The estimated parameters f^B, f^G, p and s are identified by reconciling observed values of debt and observed refunding decisions with their model-implied counterparts. The most intuitive source of variation comes from comparing the data to the model across the four types of refinancing classified by the covenant transition that a refinancing would induce: fallen-angel refinancings *add* restrictive covenants, always-junk refinancings *maintain* restrictive covenants, rising star refinancings *remove* covenants, and always investment-grade refinancings *avoid* covenants.

Figure 2 plots comparative statics of the model for a representative bond issue at a given point in time. The top left panel shows the optimal exercise boundary for refinancing in each of the four categories as a function of the parameter f^B. The refinancing boundary gap between always-junk and fallen-angel refinancings is increasing in f^B, consistent with the reduced-form evidence in Section 4 that firms which will adopt covenants in a refinancing delay refinancing relative to firms that will maintain restrictive covenants in a refinancing.

Similarly, the top right figure shows that higher values of f^G increase the gap between the refinancing decision of rising star and always investment grade bonds: the restrictive covenants limit flexibility the more eager rising-star firms are to refinance and shed covenants for a given level of interest rate savings.

Increasing f^G also causes the optimal refinance boundaries of junk-rated bonds to decline, though in parallel to each other, and reflects a precautionary motive to locking in restrictive covenants when they would be harmful if the firm were to be upgraded to investment-grade. This sensitivity is a result of the simplifying assumption that replacement debt issues are non-callable and of a long maturity.

The middle panels of Figure 2 plot refinancing boundaries as a function of the parameters p and s. The bottom panels show the corresponding comparative statics for the price of the bond. Bond prices are averaged over all grid points of the state variable capturing the risk-free short rate. Higher values of p decrease call exercise boundaries for poorly-rated firms because a higher probability of default reduces the actual value of debt liabilities relative to the pre-specified call price. But unlike the effect of f^B on refinancing boundaries, the effect of p is the same for both junk-rated debt with and without covenants. We can also see that the price of junk debt is decreasing in the probability of default parameter p. The parameter s captures the differential probability of default for firms without covenants. The fact that the refinancing boundary for fallen angel debt is increasing in s reflects the fact that as s increases and holding f^B fixed the value generated by covenants rises, and for a fixed price to refinance debt the firm is able to capture more of the value associated with covenants.

Again note that these figures describe the intuitive moments being matched by the model. These comparative statics are presented for specific bond coupon, remaining maturity, and call price schedule. Each observation used to estimate the model will generate a slightly different version of these figures. For example, for fixed unknown parameters, the gap between refinancing boundaries of fallen angel and always-junk bonds will be larger the longer the remaining maturity of the bond. These features vary in a non-linear way that reflects the optionality of the refinance decision. An alternative way to estimate the model would be to collapse this non-linearity and match aggregate moments of the model to the data, for example average refinancing boundary gaps or differential sensitivities of bond prices to interest rates between fallen angel and always-junk or between rising star and always investment-grade bonds. Instead, my maximum likelihood estimation strategy explicitly accounts for variation in these observed moments at the observation level.

6.5 Estimation Results

I begin by estimating the baseline model for my full sample of callable corporate bonds. As described in Section 3, this sample contains 4,185 callable corporate bonds for which I observe information on the key state variables of the model—the risk free rate and the bond's credit rating–during the period in which the bond can be called. This sample comprises 289,794 bond-month observations, 23,233 of which I have data on the bond's secondary market price that month.

The results of the estimation are displayed in Table 8. The parameter estimates are multiplied by a scaling factor of 100 to aide readability. Estimates of f^B and s loosely capture how covenants change in the distribution of asset returns for speculative-grade issuers. Firms in this category without covenants have a higher per-period cash-flow return, but also a higher probability of default. The non-scaled estimate of f^B of 0.00105 is approximately four percent of the calibrated value of $\bar{a}$. The trade-off is that loose covenants increase the probability of default (conditional on being in the worst rating category) by s, which is estimated to be 64 basis points. The estimated value of f^G implies that investment-grade firms subject to debt covenants earn about eight percent lower asset returns than similar firms not burdened by covenants. The estimate of p of 356 basis points is the monthly risk-neutral implied probability of default (assuming zero recovery) of a firm in the worst credit rating *with* restrictive covenants. Again, the model estimation implies that this probability of default rises by 64 basis points for firm with comparable intrinsic risk but without debt covenants. This parameter is identified by the difference in prices of callable bonds with and without restrictive covenants, accounting for differences in optimal exercise policy of the bonds' embedded call options and how this exercise policy is affected by covenants.

These parameters should not be interpreted literally, but rather as what bond prices and firm decisions about refinancing callable bonds would imply if this were the true model of asset returns. The cash-flow process in the model is overly simplistic but provides a low-dimensional parameterization of how covenants change total asset value and the allocation of value between debt and equity for a given capital structure.

A better way to interpret the model estimates is to recast them as estimates of the agency costs of debt solved by covenants. To do this, I use the model to calculate the book value of firms with the same state variable X_t but with and without restrictive covenants, as specified in Equation 11. I compare percent differences in value of investment grade and speculative grade firms with and without covenants, averaged across the remaining state variables of the model. These results are presented in Table 9. The main sample estimate reveals that the average speculative grade firm would be worth 2.4% less if it did not have debt covenants. This quantity reflects my model's estimate of the agency costs of debt solved by a typical high-yield covenants package. The 95% confidence interval of this estimate is [1.65, 3.14], implying the model quite widely rejects the neutral mutations hypothesis that covenants add positive but only quantitatively small value.

The model also reveals that imposing restrictive covenants on investment grade firms (which as an empirical regularity do not receive restrictive covenants in new debt issuance) would *decrease* firm value by 1.3%. I also interpret this number as quantitatively large. My estimates of the value of covenants for high-yield and investment grade bonds thus together reconcile why strict covenants appear in only riskier debt issues. These covenants reduce agency costs for highly levered firms but inefficiently limit flexibility of firms not prone to agency conflicts. For low-risk firms, the benefits of typical high-yield covenants in reducing the firm's cost of capital are outweighed by the restrictions on investment activity they impose. For higher risk firms, the commitment induced by covenants *ex-ante* allows investors to demand a lower cost of capital that leverages the

It is useful to compare the estimate of the value of covenants to other estimates in the literature of the costs of financial distress and the value of debt. While my paper does not explicitly model the determinants of firm capital structure, its estimates are still useful for thinking about the cost benefit analysis in a standard trade-off theory. Korteweg (2010) and Van Binsbergen, Graham, and Yang (2010) estimate the net benefit of debt in firm capital structure to be 5.5% and 3.5% of book value, respectively. Considered jointly with my estimate of the agency cost of covenants, this implies that in the absence of covenants the net value of debt would be significantly reduced. Assuming these figures are representative of the typical speculative-grade firm, my baseline estimates imply that 40 to 70 percent of the net benefits of debt are attributable to covenants. Thus, restrictive debt covenants are not only quantitatively important for firm asset value, but essential in allowing the typical firm from even being able to benefit from debt in its capital structure. Stated differently, these estimates together imply that the *existence* of risky high-leverage capital structures would not be possible without debt covenants.

6.6 Heterogeneity

The model estimated in Section 6.5 is sparsely parameterized. This raises the possibility that results may be biased due to selection and heterogeneity of the value of covenants in the sample. To partially address this, I estimate the model on various subsets of the sample of bond issues. The results are reported in Table 10. To streamline exposition I directly report in this table the implication of the model estimates for the value of strict covenants (as a function of firm book value). First I report results for the subset of bonds for which I was able to match to issuer data in Compustat, which results in a slightly higher value of covenants for speculative firms. Because Compustat matches are likely to be larger, thus suggests that restrictive covenants are more valuable for larger firms. Indeed, I confirm this is true within the Compustat sample by splitting firms based on the median book value of the issuer at bond issuance. The value of covenants for risky firms estimated in the large firm sample is 4.3% while the estimate from the smaller firm sample is estimated to be near zero and not statistically significant.

I also explore splitting the sample by the issuer's book-to-market ratio at origination. Interestingly, growth firms seem to benefit more from restrictive covenants than do value firms. This is consistent with Billett, King, and Mauer (2007), which finds that firms with growth opportunities take more bond covenants and that covenants help growth firms use leverage profitably. Again, my estimation contributes to this finding by quantifying the importance of debt covenants. My results suggest restrictive debt covenants create significant value relative to a similarly leveraged firm without them.

Finally, I split my sample into broad industry categories, based on the designations in the Mergent issuer data. Manufacturing firms, as well as firms in Retail, Services, and Leisure seem to benefit significantly from restrictive covenants relative to other firms. In contrast, media and communications issuers refinancing behavior and bond prices suggest there is little if any gain from restrictive covenants. Even the value of covenants to investment-grade issuers is estimated to be significantly more negative for media and communications issuers than firms in other industries.

6.7 Robustness

I also consider two alternative parameterizations of the structural model to investigate whether the chosen calibration is significantly influencing the results. First, instead of using the physical probability ratings transition density matrix from S&P, I consider a risk-neutral version derived in from Lando (2004, pg. 154). I chose to use the physical matrix instead because the methodologies for computing risk-neutral transition density matrices are relatively sensitive to assumptions and input data. Column 2 of Table A6 reports the estimates of the model with this alternative transition matrix. The parameters are relatively unchanged, most notably the estimated baseline probability of default and increase in probability of default due to lack of covenants decrease. The estimated value of f^G also increases by approximately twenty percent. These changes result in somewhat larger magnitudes of the estimates of the value of covenants to investment grade and speculative grade firms. Next, I consider allowing more flexibility in the parameterization of a, the baseline ratio of firm cash flows to debt. Instead of calibrating this as a fixed parameter, I allow it to take a different value for each value of X. I calibrate these as the median ratio of monthly EBITDA to long term debt in each of the eight model-mapped credit ratings. This has an even smaller effect on the resulting estimation than the choice of transition matrix.

7 Conclusion

This paper is the first in the literature to structurally estimate a dynamic model to quantify the value of restrictive bond covenants. To do this, I abstract from other capital structure considerations of the firm and exploit an intuitive and clean trade-off between what I have shown are the two first order determinants of their decision to refinance debt: resulting changes in restrictive covenants imposed on the firm and the interest rate savings obtained through refinancing. When firms will face substantially tighter covenants, as proxied for by a potential refinancing of fallen-angel bonds, firms are willing to forgo substantial interest rate savings relative to firms refinancing debt originated as speculative grade into new speculative grade issues.

In my dynamic model this differential sensitivity to interest rate savings identifies the value of covenants in ameliorating agency costs of debt. The intuition behind this identification is that debt covenants are explicitly designed to protect the ability of firms to service debt when they are relatively close to default, at the expense of potentially preventing the firm from taking good investment opportunities. The absence of covenants close to default thus lowers recovery values and decreasing the firm's perceived value of its debt liabilities, which makes potential interest rate savings associated with refinancing less attractive. The more successful covenants are in reducing agency costs of debt, the lower the value of the debt of a firm without covenants in the same position as a firm with covenants, and the higher the difference is between the actual value of debt and the cost of refinancing the bond. To the extent that covenants would increase value, the higher the gap between the value of debt and the cost of repaying the debt, the less of this increase in value would accrue to the equity claimants of the firm, and the less likely they are to refinance. This explains the observed gap in refinancing behavior between fallen angel and always junk bonds documented in Section 4. By instead holding the refinancing cost gap across firms with and without covenants fixed the correlation between the size of this gap and the differential propensity of firms with and without covenants to refinance into debt with covenants identifies the value that would be generated by covenants. A similar argument holds for the refinancing decision of firms able to shed covenants relative to that of firms that do not and will not have covenants after a refinancing.

I use a dynamic model that embeds the pricing of fixed income derivatives in a stochastic interest rate environment to collapse the nonlinear identifying variation described above into intuitive and interpretable quantities that are directly observable in the data–the decision to refinance debt and the market price of this debt. By explicitly considering how covenants affect the distribution of firm cash-flows in this model of optimal refinancing I am able to recover quantitative estimates of the costs and benefits of covenants.

My findings suggest that typical high-yield restrictive debt covenants add significant value to risky firms. In fact, because the value of covenants implied with my model is comparable to estimates in the literature of the net benefits of debt itself, I conclude that restrictive covenants are absolutely essential for allowing the high leverage capital structure to generate value. This finding is consistent with the rapid rise and collapse of the original junk bond market in the 1980s, in which many of these bonds lacked restrictive covenants but were part of highly leveraged capital structures.

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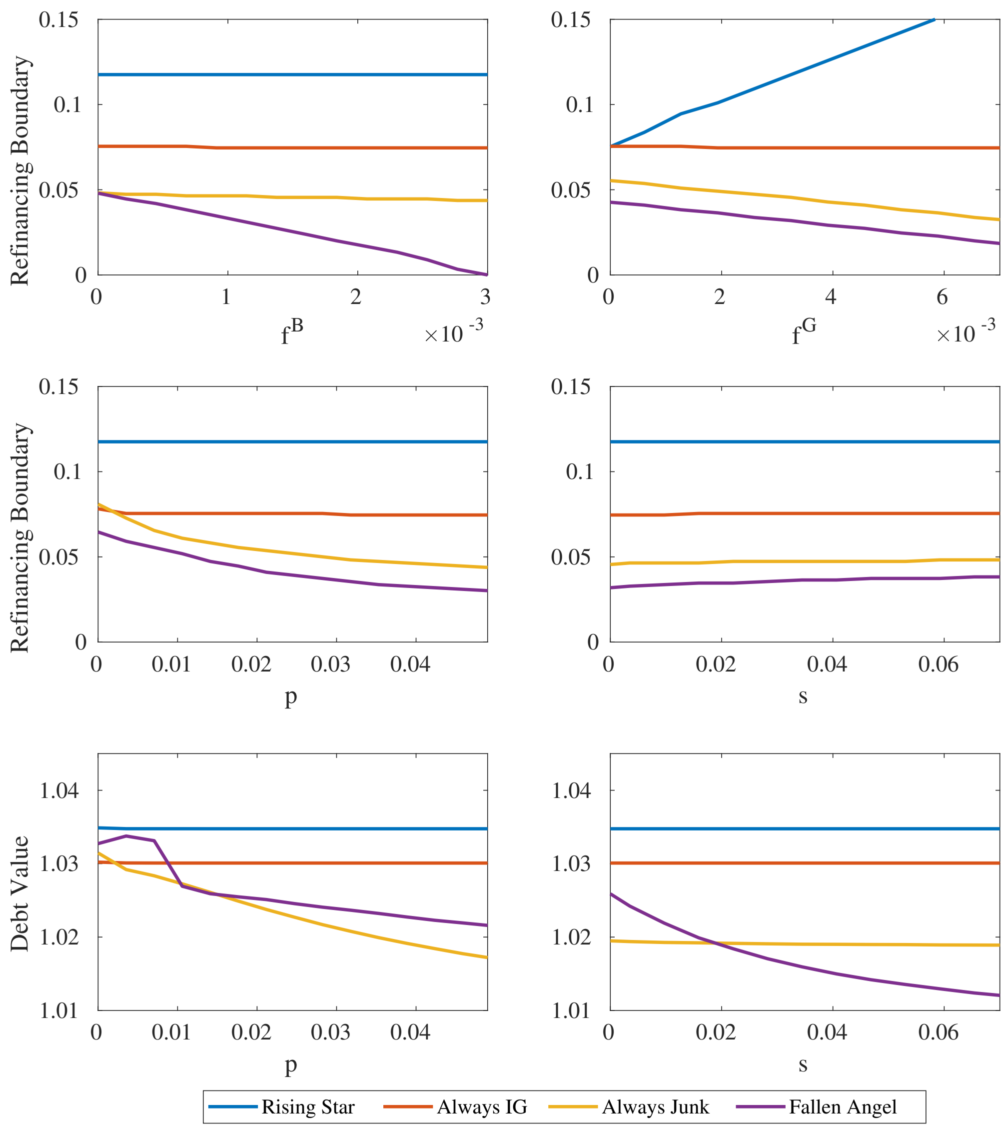
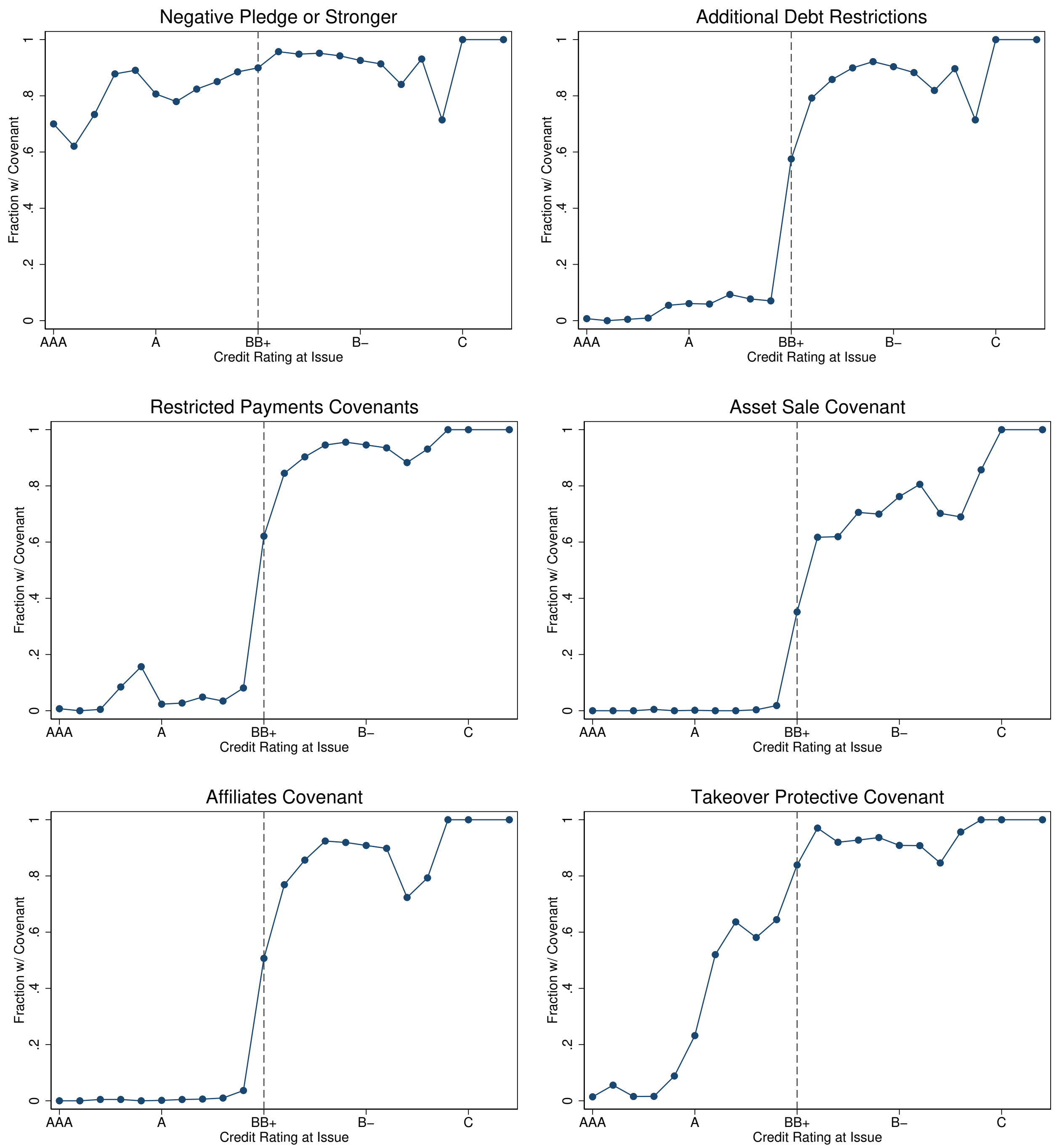
Figures and Tables

Figure 2: **Identification of parameters f^B, f^G, p, and s**



Notes: These figures show representative comparative statics of optimal exercise boundary debt value as a function of the parameters to be estimated. These comparative statics are for a hypothetical 9% coupon bond with 5 years until maturity and call price schedule: currently callable at 103.75%, after 1 year at 102.5%, after two years at 101.25%, and after three years at par. Rising Star and Always Investment-Grade bonds are AA rated and Always-Junk and Fallen Angel bonds are BB rated. Comparative statics of bond price are computed by averaging over all the bond price at all grid points of the risk-free short rate.

Figure 3: **Covenant Quality at the Investment-Grade Boundary**



Notes: These figures shows the fraction of bonds in the covenant matched sample that contain a given type of covenant, sorted by S&P credit rating at bond issuance. BB+ is the highest S&P rating not considered to be investment grade.

Table 2: **Callable Bond Sample Summary Statistics**

	N/Mean	Median	Min	p25	p75	Max
Number of Bonds	8,197					
Number of Issuers	3,437					
Size (millions)	$319	200	5	100	400	11,000
Tenor (years)	14.0	10	1	8	18	100
Investment Grade	38.1%	BB	C	B	A-	AAA

Table 3: **Corporate Bond Calls by Bond Status**

	Corporate Bond Calls by Bond Status at Termination			
	N	Fraction Called	Call Price	10yr Treas. Decline (bps)
Always Investment Grade	1,648	0.87	102.31	166
Always Junk	3,080	0.71	103.18	121
Fallen Angel	191	0.80	101.27	226
Rising Star	221	0.94	103.63	102
Total	5,140	0.77	102.85	138

Notes: This table reports details of the bonds in sample that were called by the issuer by the ratings change status of the bond from origination to end. The first column reports, for each category, the number of bonds in the sample that are no longer outstanding. The second column shows the percentage of these bonds that were called by the issuer. The third column reports the price the issuer paid (per $100 of par value) to call the bond. The fourth column tracks the average difference, in basis points, between the 10 year treasury yield at bond issuance and at the time the bond was called.

Table 4: **Covenant Differences Between Called and Refunding Bonds**

	Average Difference Between Replacement and Called Bond Issue					
	Number of Covenants	Debt Restrictions	Restricted Payments	Asset Sales	Affilates	Change of Control
Always HY	-0.07	-0.04	-0.05	-0.01	-0.02	0.01
Always IG	0.75	-0.12	-0.02	0.08	-0.01	-0.06
Fallen Angel	6.13	0.63	0.25	0.75	0.25	0.25
Rising Star	-2.56	-0.56	-0.71	-0.16	-0.80	-0.11
Total	0.03	-0.07	-0.07	0.01	-0.05	-0.01

Notes: This table explores how covenants change between refunded and replacement bond issues. The main sample of called bonds is matched, where possible, to new bonds by the same issuer. The first column reports the number of covenants in the new bond issue relative to the refunded bond issue. The remaining columns report the average change in an indicator for the presence of specific categories of covenants.

Table 5: **Ratings Changes and Implied Shifts in Call Exercise Boundary**

	Dependent Variable: Bond Called			
	(1)	(2)	(3)	(4)
Boundary Shift: Fallen Angel	−0.177***	−0.482***	−0.471***	−0.475***
	(0.050)	(0.0643)	(0.0647)	(0.080)
Boundary Shift: Rising Star	0.604***	0.575***	0.564***	0.195
	(0.076)	(0.083)	(0.082)	(0.104)
Debt/EBITDA				−0.005
				(0.004)
Debt/Assets				0.212
				(0.135)
Interest Coverage				0.029***
				(0.007)
Price/Earnings				0.001**
				(0.0003)
Return on Assets				−0.066
				(0.357)
Profit/Assets				−0.020
				(0.152)
Observations	139,061	139,056	139,048	72,410
Bonds	3,615	3,615	3,614	2,083

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. The coefficients are estimates from the model of Equation 4. Each specification controls for the time elapsed from bond origination to the current month. Column (2) adds current credit rating fixed effects. Column (3) further adds industry fixed effects. Column (4) further adds controls for balance sheet characteristics of the issuing firm. Balance sheet variables are windsorised at the 1% level.

Table 6: **Specific Covenants and Implied Shifts in Call Exercise Boundary**

	Dependent Variable: Bond Called		
	(1)	(2)	(3)
Boundary Shift: Fallen Angel	-0.447^{***}	-0.547^{**}	-0.549^{**}
	(0.096)	(0.182)	(0.182)
# Num. Covenants		0.020	0.020
		(0.038)	(0.038)
Boundary Shift: Rising Star	0.649^{***}	-0.0151	-0.134
	(0.140)	(0.233)	(0.298)
# Num. Covenants		0.071^{***}	0.059
		(0.021)	(0.079)
# Restricted Payments			0.070
			(0.330)
# Restricted Subsidiaries			0.289
			(0.533)
# Restricted Debt Issuance			-0.825
			(0.560)
# Change in Control Put			-0.229
			(0.335)
# Cross Acceleration			0.977^{*}
			(0.426)
# Minimum Net Worth			0.274
			(0.329)
# Asset Sale Restrictions			-0.035
			(0.244)
Observations	43,521	43,521	43,521
Bonds	2,051	2,051	2,051

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. The coefficients are estimates from the model of Equation 4. Each specification controls for the time elapsed from bond origination to the current month, and current credit rating and industry fixed effects.

Table 7: **Call Exercise Boundary: Fallen Angels with Existing HY Debt**

	Dependent Variable: Bond Called			
	(1)	(2)	(3)	(4)
Boundary Shift: Fallen Angel	-0.471^{***}	-0.498^{***}	-0.494^{***}	-0.498^{***}
	(0.065)	(0.068)	(0.068)	(0.068)
# Any HY Debt		0.166		0.0620
		(0.138)		(0.228)
# Frac Debt HY			0.374	0.274
			(0.291)	(0.474)
Boundary Shift: Rising Star	0.564^{***}	0.564^{***}	0.564^{***}	0.564^{***}
	(0.082)	(0.082)	(0.082)	(0.082)
Observations	139,048	139,048	139,048	139,048
Bonds	3,614	3,614	3,614	3,614

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. This specification uses data at the bond-month level. The coefficients are estimates from the model of Equation 4. Each specification controls for the time elapsed from bond origination to the current month, and current credit rating and industry fixed effects.

Table 8: **Baseline Structural Estimates**

	Full Sample
f^B	0.105
	(0.011)
f^G	0.200
	(0.012)
p	3.575
	(0.083)
s	0.637
	(0.034)
N_{bonds}	$4,185$
N_{prices}	$23,233$
N_{bounds}	$289,794$

Notes: This table reports the baseline structural model estimates of Section 6. The coefficients and standard errors are reported after being multiplied by 100.

Table 9: **Percent Change in Asset Value from Adding Restrictive Covenants**

	Full Sample
Speculative Grade Firms	2.39%
95% CI	(1.65, 3.15)
Investment Grade Firms	-1.33
95% CI	(-1.49, -1.18)
N_{bonds}	4,185
N_{prices}	23,233
N_{bounds}	289,794

Notes: This table reports the model-implied counterfactual estimates of how much covenants affect the value of assets by credit risk class. The exercise used the estimated model to calculate asset value under the status quo relative to two counterfactual scenarios: that either all firms or no firms are subject to typical high-yield covenant restrictions. 95% confidence intervals are are calculated using the the covariance matrix of the estimates reported in Table 8 and applying the delta method to the model-implied mapping between parameter estimates and the effect of covenants on asset values. See Section 6.5 for more details.

Table 10: **Heterogeneity in the Value of Restrictive Covenants**

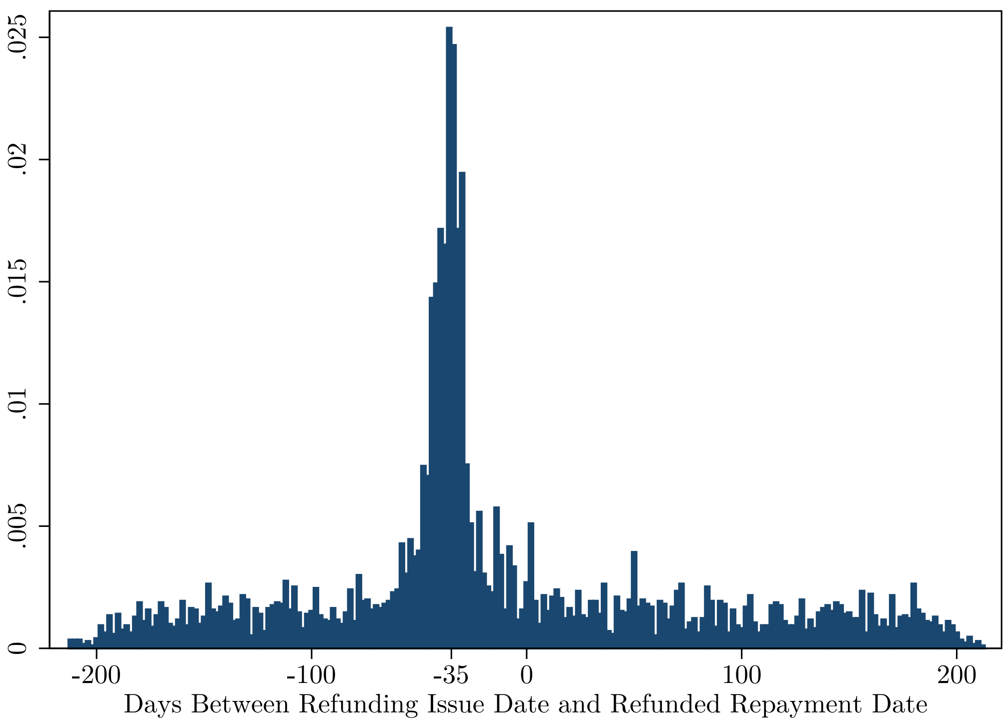
	Speculative Grade	*Investment Grade*
Baseline Estimates	2.39%	-1.33
	(1.65, 3.14)	(-1.49, -1.18)
Compustat Sample	3.89	-1.13
	(2.56, 5.21)	(-1.37, -0.88)
Large Firms ($>$ \$2.5bn)	4.33	-1.17
	(2.92, 5.73)	(-1.43, -0.90)
Small Firms ($<$ \$2.5bn)	0.47	-0.78
	(-1.00, 1.94)	(-1.05, -0.51)
Growth Firms ($BM < 0.65$)	6.44	-0.95
	(4.25, 8.63)	(-1.13, -0.79)
Value Firms ($BM > 0.65$)	3.12	-1.38
	(1.95, 4.30)	(-1.65, -1.10)
Manufacturing	7.17	-0.73
	(5.91, 8.44)	(-0.86, -0.59)
Retail, Services, Leisure	6.87	-0.38
	(5.41, 8.34)	(-0.68, -0.08)
Media	0.49	-2.07
	(-1.36, 2.33)	(-2.36, -1.77)
Energy, Mining, Transit	3.55	-1.15
	(2.14, 4.96)	(-1.29, 1.00)

Notes: This table reports the model-implied counterfactual estimates of how much covenants affect the value of assets by credit risk class. Each row represents a different subset of the data on which the model is separately estimated. 95% confidence intervals are are calculated using the the covariance matrix of the underlying parameter estimates and applying the delta method to the model-implied mapping between parameter estimates and the effect of covenants on asset values. See Section 6.5 for more details.

Internet Appendix

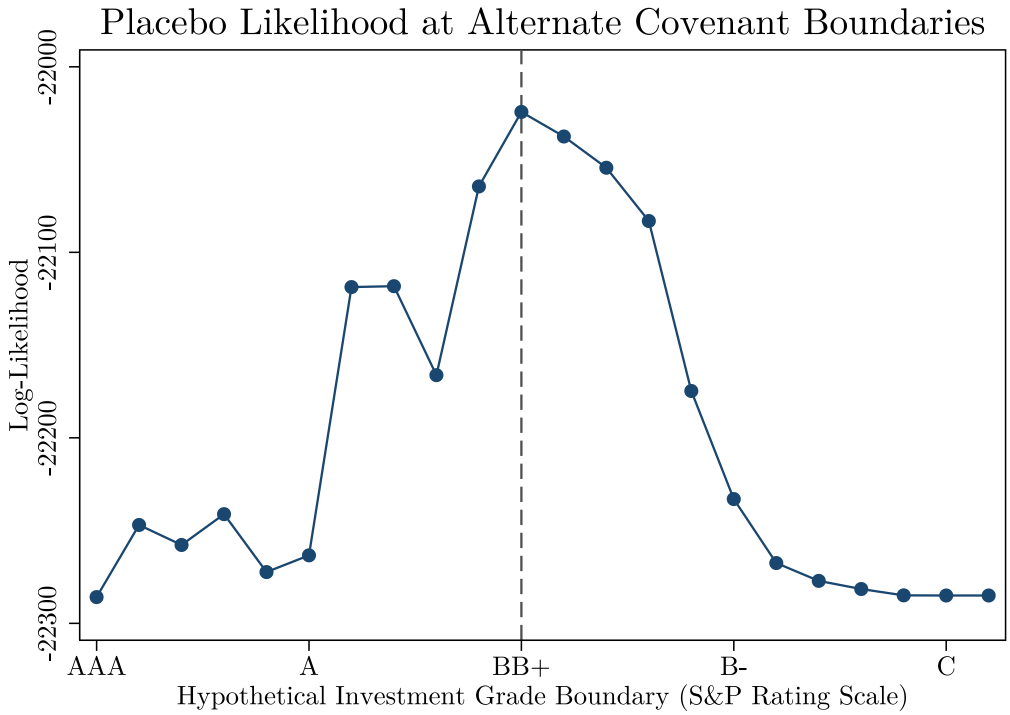
A Additional Tables and Figures

Figure A1: **Histogram of Timing Delay Between Called and Refunding Bond Issues**



Notes: This figure shows the distribution of the days between the issuance of a new bond and the repayment of a called bond issue. The standard refunding convention involves issuing a new bond simultaneously with the notification of the firm's intent to call the bond in 30 days, which is the typical minimum call notice period mandated in corporate bond indentures. I assemble matches by looking for all new bond issuances of a firm that fall within 200 days of the firm prepaying a callable bond. The significant mass around issuance of a new bond exactly 1 month before the early repayment of an outstanding bond suggests these transactions are textbook corporate bond refundings.

Figure A2: **Placebo Test**



Notes: This figure plots the estimated log-likelihoods of versions of the model described in Equation 4 at different hypothetical cutoff values of the rating boundary between investment grade and high yield rating categories. The exact specification used includes credit rating and industry fixed effects. The fact that the likelihood peaks at exactly the true investment-grade / speculative-grade boundary is suggestive that the there is not an omitted mechanism or interpretation behind the result that covenants affect corporate bond refinancing behavior.

Table A1: **Ratings Changes and Implied Shifts in Call Exercise Boundary**

	Dependent Variable: Bond Called			
	(1)	(2)	(3)	(4)
Boundary Shift: Fallen Angel	-0.300***	-1.205***	-1.273***	-1.243***
	(0.0511)	(0.150)	(0.141)	(0.224)
# Frac Debt HY	1.069***	1.077***	1.048***	1.179***
	(0.277)	(0.279)	(0.285)	(0.266)
Boundary Shift: Rising Star	0.438***	1.210***	1.209***	0.947***
	(0.0832)	(0.139)	(0.138)	(0.229)
Debt/EBITDA				0.00439
				(0.0104)
Debt/Assets				0.199
				(0.174)
Interest Coverage				0.0342***
				(0.00661)
Price/Earnings				0.000951**
				(0.000315)
RoA				-0.971*
				(0.409)
Profit/Assets				0.00709
				(0.163)
Observations	307,360	307,353	307,345	189,996
Bonds	6,459	6,459	6,458	4,104

Notes: The coefficients are estimates from the model of Equation 4. Each specification controls for the time elapsed from bond origination to the current month. Column (2) adds current credit rating fixed effects. Column (3) further adds industry fixed effects. Column (4) further adds controls for balance sheet characteristics of the issuing firm. Balance sheet variables are windsorised at the 1% level.

Table A2: **Specific Covenants and Implied Shifts in Call Exercise Boundary**

	Dependent Variable: Bond Called		
	(1)	(2)	(3)
Boundary Shift: Fallen Angel	-1.630***	-1.358***	-1.321***
	(0.167)	(0.239)	(0.241)
# Frac. Debt HY	0.847**	0.883**	0.936**
	(0.307)	(0.310)	(0.313)
# Num. Covenants		-0.0616	-0.0808
		(0.0430)	(0.0452)
Boundary Shift: Rising Star	1.603***	1.195***	0.906**
	(0.192)	(0.278)	(0.333)
# Num. Covenants		0.0501*	0.0239
		(0.0216)	(0.0771)
# Restricted Payments			0.376
			(0.345)
# Restricted Subsidiaries			0.0843
			(0.524)
# Restricted Debt Issuance			-0.737
			(0.525)
# Change in Control Put			-0.0856
			(0.348)
# Cross Acceleration			1.001*
			(0.435)
# Minimum Net Worth			0.341
			(0.363)
# Asset Sale Restrictions			-0.0511
			(0.246)
Observations	63,244	63,244	63,244
Bonds	2,682	2,682	2,682

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. This specification uses data at the bond-month level. The coefficients are estimates from the model of Equation 4. Each specification controls for the time elapsed from bond origination to the current month, and current credit rating and industry fixed effects.

Sequential Screens	Callable	Non-Callable
Dollar Denominated US Corporate	23,289	10,899
Not Convertible, Exchangeable, or Puttable	20,610	9,301
Fixed Coupon	19,355	7,983
Public Placement	13,508	6,661
Face value at least $5m	13,352	5,567
Matched initial credit ratings data	12,578	**4,801**
Continuously Callable	12,451	n/a
Has Non Make-Whole period	8,220	n/a
No Make-Whole protections	**8,197**	n/a

Table A3: Number of bond issues meeting sequential screening requirements. Source data from Mergent database.

S&P Long Term Rating	Model Rating	Ratings Class
AAA	1	IG
AA+	2	IG
AA	2	IG
AA-	2	IG
A+	3	IG
A	3	IG
A-	3	IG
BBB+	4	IG
BBB	4	IG
BBB-	4	IG
BB+	5	HY
BB	5	HY
BB-	5	HY
B+	6	HY
B	6	HY
B-	6	HY
CCC+	7	HY
CCC	7	HY
CCC-	7	HY
CC	7	HY
C	7	HY
D	8	HY

Table A4: Concordance Table of Model Ratings and S&P Long Term Bond Ratings

Table A5: **Ratings Transition Matrices**

Panel A: Standard and Poor's One Year Transition Matrix

	1	2	3	4	5	6	7	8
1	90.79%	8.29%	0.72%	0.10%	0.10%	0.00%	0.00%	0.00%
2	0.10%	91.22%	7.85%	0.62%	0.10%	0.10%	0.00%	0.00%
3	0.92%	2.36%	90.04%	5.44%	0.72%	0.31%	0.10%	0.10%
4	0.00%	0.32%	5.94%	86.95%	5.30%	1.17%	0.12%	0.21%
5	0.00%	0.11%	0.66%	7.69%	80.55%	8.79%	0.99%	1.21%
6	0.00%	0.11%	0.23%	0.45%	6.47%	82.75%	4.09%	5.90%
7	0.23%	0.00%	0.23%	1.25%	2.28%	12.86%	60.64%	22.53%
8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Panel B: Generator Matrix

	1	2	3	4	5	6	7	8
1	-0.085	0.081	0.005	0.000	0.000	0.000	0.000	0.000
2	0.012	-0.093	0.078	0.001	0.002	0.000	0.000	0.000
3	0.001	0.028	-0.095	0.059	0.005	0.002	0.000	0.000
4	0.000	0.001	0.060	-0.132	0.063	0.007	0.001	0.000
5	0.000	0.001	0.003	0.058	-0.166	0.088	0.004	0.013
6	0.000	0.001	0.001	0.004	0.071	-0.200	0.029	0.094
7	0.000	0.005	0.005	0.010	0.030	0.078	-0.390	0.262
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Panel C: Monthly Transition Probabilities

	1	2	3	4	5	6	7	8
1	0.993	0.007	0.000	0.000	0.000	0.000	0.000	0.000
2	0.001	0.992	0.006	0.000	0.000	0.000	0.000	0.000
3	0.000	0.002	0.992	0.005	0.000	0.000	0.000	0.000
4	0.000	0.000	0.005	0.989	0.005	0.001	0.000	0.000
5	0.000	0.000	0.000	0.005	0.986	0.007	0.000	0.001
6	0.000	0.000	0.000	0.000	0.006	0.983	0.002	0.008
7	0.000	0.000	0.000	0.001	0.002	0.006	0.968	0.022
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

Table A6: **Structural Estimates: Robustness**

	Baseline	*Alt. Trans. Matrix*	*Flexible* $a\,(X)$
		Parameter Estimates	
f^B	0.105	0.103	0.010
	(0.011)	(0.011)	(0.022)
f^G	0.200	0.238	0.199
	(0.012)	(0.011)	(0.016)
p	3.575	2.973	3.584
	(0.083)	(0.061)	(0.087)
s	0.637	0.580	0.657
	(0.034)	(0.101)	(0.038)
		Value of Covenants	
HY	2.39%	4.20	2.20%
	(1.65, 3.14)	(1.45, 6.94)	(-0.95, 5.35)
IG	-1.33	-1.91	-0.88
	(-1.49,-1.18)	(-2.06, -1.77)	(-1.00, -0.76)
N_{bonds}	4,185	4,185	4,185
N_{prices}	23,233	23,233	23,233
N_{bounds}	289,794	289,794	289,794

Notes: This table reports the estimates of the structural model estimates of Section 6 to alternative assumptions. The first column replicates the baseline estimates of Table 8. The second column uses the calibrated risk-neutral transition matrix derived from the one shown in Table 6.3 of Lando (2004, pg. 154). Estimates in the third column allow the parameter a to vary with the risk state X, and these values are calibrated to match the sample average EBITDA to long term debt ratio of each rating category. The model coefficients and standard errors are multiplied by 100.

B Proofs

Proof of Proposition 2.1

Given any new debt is fairly priced by capital markets, shareholders have the incentive to maximize total asset value in choosing covenants. Thus, following Equation 1, new debt will contain the covenant if and only if $S\left(p^L\right) > 0$. Solving $S(\bar{p}) = 0$ gives the covenant boundary:

$$\bar{p} \equiv \Delta / \left[(1-q)(X+\Delta)\right].$$

Proof of Proposition 2.2

The expressions for the refinancing boundaries are direct calculations of the difference between equity value if the firm does or does not refinance and solving for the equalizing interest rate. To show that rising star firms call at a higher boundary and fallen angel firms call at a lower boundary, it is sufficient to prove $\delta > 0$.

$$\delta = (1+CP)^{-1}\left[\Delta\left(1-p_1^L(1-q)\right) - p_1^L(1-q)(X-D)\right]$$

From Assumption 2.1, $q\Delta > (1-q)(X-D)$. This and $\Delta > 0$ means we can write

$$\begin{aligned}\Delta\left(1-p_1^L(1-q)\right) &= \Delta\left(1-p_1^L\right) + p_1^L\Delta q \\ &> \Delta\left(1-p_1^L\right) + p_1^L(1-q)(X-D) \\ &> p_1^L(1-q)(X-D)\end{aligned}$$

which confirms $\delta > 0$.

C Detailed Appendix for Section 4.2

Discretization and Solution to the Bond Pricing Model

To solve for optimal exercise boundaries in the "reduced form" section I discretize the bond pricing model to the solve by backward induction using dynamic programming methods from bond maturity. The discritization is to the monthly frequency. The interest rate process follows a Vasicek (1977) process discretized using the standard Euler method:

$$R_{t+\Delta} - R_t = \kappa^Q \left(\bar{R}^Q - R_t\right) \Delta + \sigma^Q \sqrt{\Delta} \times N^Q (0, 1)$$

Where $\Delta = \frac{1}{12}$ and thus the parameters are expressed in annualized terms. Denote the value of a callable riskless coupon bond with τ periods remaining to maturity, at which it pays one dollar of pricipal, as $V_\tau (R_t)$. For expositional simplicity, if the bond pays bi-annual coupon c^{ba} define the effective monthly coupon as $c = c^{ba}/6$. Assume the price per dollar of principal to call the bond The optimal call policy solves the following recursive equations:

$$\begin{aligned} V_{\tau=0} (R_t) &= 1 + c \\ V_{\tau>0} (R_t) &= c + e^{-R_t \Delta} \times \min \left\{ CP_{\tau-1}, \mathbb{E}_t^Q \left[V_{\tau-1} (R_{t+\Delta}) \right] \right\} \end{aligned}$$

I solve this with dynamic programming methods by discritizing the default-adjusted short rate process R_t into an evenly spaced grid of 150 points between 0 and 0.25.

Calibration of Default Adjusted Short Rate Process

I calibrate the risk-neutral default adjusted short rate process by to match the distribution of price innovations in a large sample of non-callable corporate bonds. First, I assume that the same default adjusted short rate process prices callable and non-callable bonds. Consider a defaultable τ-period zero coupon bond that pays one dollar upon maturity in the event of no default. Assuming the default adjusted short rate process follows a single factor affine diffusion process, its price can be expressed as

$$P_\tau(R_t, \theta) = \exp\left(A(\tau, \theta) + B(\tau, \theta) R_t\right)$$

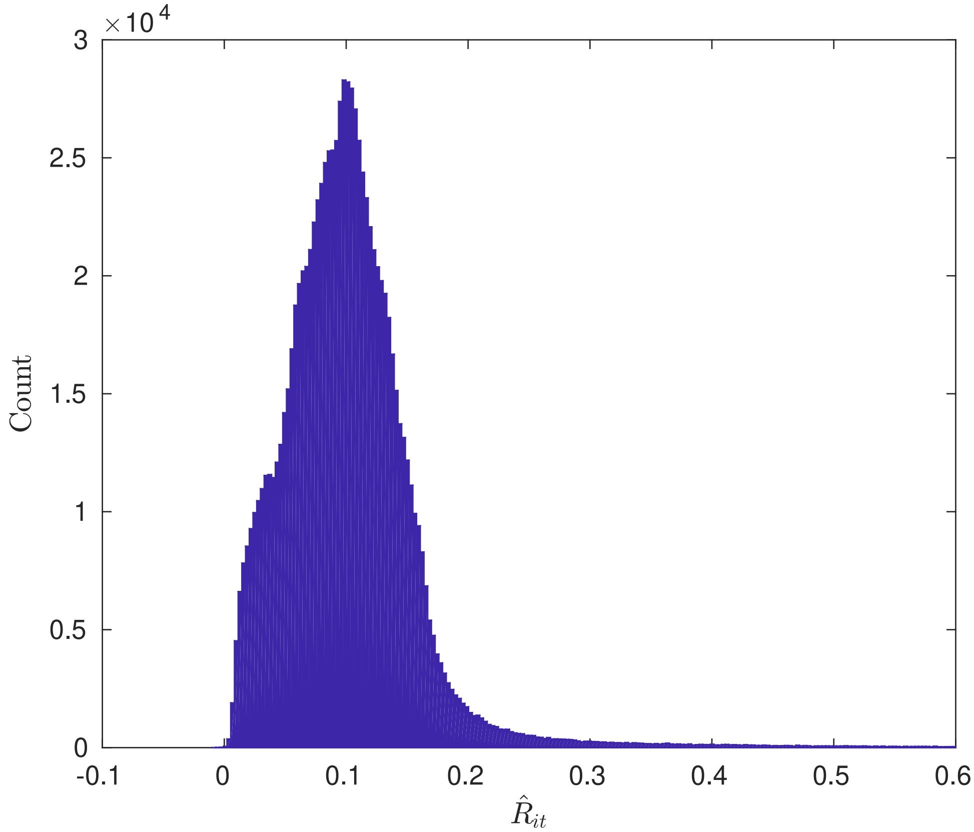
where θ parameterizes the default adjusted short rate process, and the coefficients A and B are well known as in Duffie and Kan (1996). A bond paying a coupon c every month thus has price given by

$$P_\tau^c(R_t, \theta) = \sum_{s=1}^{\tau} c \times P_s(R_t, \theta) + 1 \times P_\tau(R_t, \theta)$$

Since P_τ is monotone in R_t this equation can be solved uniquely for R_t given the current price of a coupon bond P_τ^c:

$$\hat{R}_t(\theta) = P^{-1}\left(P_\tau^c, \theta\right).$$

Figure A3: Histogram of $\hat{R}_{it}$ in Non-Callable Pricing Sample



Thus, given a panel of corporate bond prices, assuming a single factor default adjusted short rate allows us to invert the pricing equation to obtain estimates of the current level of the short rate process. Conceptually this state variable is a sufficient statistic about the term structure of interest rates to price the risky coupon bond. Of course, multi-factor models will do a better job capturing the true term structure of interest rates, but at the expense of significantly increasing the complexity of these methods. Duffee (2002) applies a multi-factor version of these methods to *indices* of corporate bond yields.

Given levels of the short rate implied by bond prices and a candidate θ we can express the likelihood of the bond price data as

$$\Pr\left(P_{i,t+1}|P_{i,t}\right) = \Pr\left(\hat{R}_{i,t+1}|\hat{R}_{i,t}\right)\left|\frac{\partial R_{i,t+1}}{\partial P_{i,t+1}}\right|.$$

Of course, we only observe prices under the physical measure, but have only specified the stochastic process for the default-adjusted short rate under the risk-neutral measure. Thus, I assume the price of risk in this economy satisfies $\eta_t = \lambda R_t$. This implies that the physical dynamics of the default adjusted short rate also follow a Vasicek process, where the mapping between risk neutral and physical parameters involves the price of risk coefficient λ :

$$\begin{aligned} \kappa &= \kappa^Q - \lambda\sigma^Q \\ \bar{R} &= \bar{R}^Q \frac{\kappa^Q}{\kappa} \\ \sigma &= \sigma^Q \end{aligned}$$

I then apply this maximum likelihood procedure to a large sample of monthly bond prices obtained from TRACE and Bank of America Merrill Lynch's bond index constituents data. I use the simplex method to find maximum likelihood estimates of the four parameters $\theta = \left(\kappa, \bar{R}, \sigma, \lambda\right)$. Figure A3 shows a histogram of the recovered values of the short-rate process given the maximum likelihood estimates $\hat{\theta}$. The maximum likelihood estimates are:

$$\hat{\kappa} = 0.0174, \quad \hat{\bar{R}} = 0.0774, \quad \hat{\sigma} = 0.0345, \quad \hat{\lambda} = -0.795$$

Hedonic Regression Model of Default Adjusted Cost of Capital

I now have maximum likelihood estimates of the stochastic process for the default-adjusted cost of capital and a way to solve the debt pricing model for optimal call policy. Optimal call policy is a boundary in the default adjusted cost of capital: bond i at time t should call the bond whenever its default adjusted replacement cost of capital is below the boundary $b_{it}(\theta)$. Thus to evaluate the refinancing decisions of callable bonds I need to obtain an estimate of the default adjusted cost of capital for every bond at every time the bond is callable.

I do this by inverting non-callable bond prices to obtain estimates of the default adjusted short rate and mapping these estimates onto the sample of callable bond months. To do this, I develop a hedonic regression model that estimates default adjusted cost of capital as a flexible function of a wide range of bond, firm, and time specific characteristics. I estimate this model on the non-callable bond $\hat{R}_{it}$ obtained as described in the previous subsection using the maximum likelihood estimates of the parameters obtained there.

The model includes data on bonds' current credit rating, credit rating at issuance, industry, parent company credit rating, maturity-matched treasury yields, as well as time, rating by time, and industry by time fixed effects. Importantly, the model's inclusion of origination and current ratings captures, to the extent it exists in the data, any differences in the cost of capital of firms with different origination ratings but identical current ratings. This controls for the main source of potential bias in my methodology, that I am not capturing differences in the replacement cost of capital of firms between firms with dramatic ratings changes (that thus face difference covenants upon refinance) and firms that have not experienced such ratings changes.

The model is estimated on a sample of nearly 2 million bond-month observations, and contains thousands of parameters, most of which are the interacted fixed effects described above. The R^2 of the model is over 86%, suggesting that my calibrated single factor model of default adjusted short rates is able to capture a large majority of the variation in bond prices I then project these estimates onto the bond-month observations in my callable bond sample. To form estimates of the default adjusted cost of capital for each callable bond at each point in time.