Contractual Restrictions and Debt Traps*

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Abstract

Microcredit and other forms of small-scale finance have failed to catalyze entrepreneurship in developing countries. In these credit markets, borrowers and lenders often bargain over not only the interest rate but also implicit restrictions on types of investment. We build a dynamic model of informal lending and show this may lead to endogenous debt traps. Lenders constrain business growth for poor borrowers yet richer borrowers may grow their businesses faster than they could have without credit. The theory offers nuanced comparative statics and rationalizes the low average impact and low demand of microfinance despite its high impact on larger businesses.

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Capital constraints pose a substantial obstacle to small-scale entrepreneurship in the developing world. Experimental evidence from "cash drop" studies paints a remarkably consistent picture: across a broad range of contexts including Mexico, Sri Lanka, Ghana, and India, small-scale entrepreneurs enjoy a *monthly* return to capital in the range of 5%–10%.[1] Surprisingly, however, many experimental evaluations of microfinance find that it has only modest or even no impact on entrepreneurial income growth.[2] This may be especially puzzling in light of the fact that the interest rates charged for microloans are well below the estimates of marginal return to capital. If credit is available, and interest rates are below entrepreneurs' marginal return to capital, what prevents them from using it to pursue their profitable investment opportunities?

We address this puzzle through a theory that predicts that increasing access to credit can actually constrain entrepreneurship, in the sense that entrepreneurs with access to credit may experience *less* business growth than those without, and that these constraining effects may be strongest precisely when entrepreneurs have access to the most productive investment opportunities. We rely on two special features of informal credit markets for small-scale borrowers in the developing world.

First, we highlight that a borrower who successfully grows his business and builds a stock of pledgeable collateral may eventually gain access to a more active credit market and graduate from his informal lender. The borrower and his informal lender suffer from a hold-up problem; the borrower cannot commit to share any benefits he derives once he has stopped borrowing from his informal lender.

Second we assume that in the informal sector, the borrower and lender bargain not only over the interest rate, but also over a contractual restriction that determines whether the borrower can make investments in fixed capital or only in expanding his working capital (e.g. buying inventory). We assume that fixed capital investments help the borrower build his stock of pledgeable and productive assets while working capital investments generate a consumption good but are not useful for permanently growing the borrower's business.

While microfinance institutions rarely contract over specific investments that a borrower must make, it is common for microfinance institutions to impose rigid requirements that loan repayments begin immediately after disbursal and in frequent installments thereafter. A growing body of experimental research finds that by relaxing this requirement and allowing borrowers to match their repayments to the timing of their cashflows, borrowers exhibit higher demand for credit, make longer-term investments, and see substantial and persistent increases in their sales and profits (e.g. Field et al. (2013), Takahashi et al. (2017), Barboni and Agarwal (2018), and Battaglia et al. (2019)).[3]

[1]See e.g., De Mel et al. (2008), Fafchamps et al. (2014), Hussam et al. (2017), and McKenzie and Woodruff (2008).

[2]See Banerjee et al. (2015) and Meager (2017) for overviews of the experimental evaluations of microfinance.

[3]The insistence on early and frequent repayment is sometimes attributed to deterring default but there is little empirical support for this claim. Of the four studies cited above, only Field et al. (2013) finds an increase in default from

Following this empirical evidence, we assume that the fixed capital project takes longer to materialize output than does the working capital project, and that when a lender insists on initial payments early in the loan's tenure the borrower must choose his working capital project. We refer to contracts that insist on early repayment as *restrictive contracts* and those that allow for flexible repayment as *unrestrictive contracts.*

That the borrower may graduate from his informal lender generates an incentive for the informal lender to constrain the borrower's business growth. And that the lender can impose contractual restrictions on the borrower gives her the means to do so. We build a dynamic theory of informal lending on top of these two modeling ingredients to understand when a lender may hold her borrower captive.

We focus on the relationship between a single informal lender and her borrower. The borrower's outside option is to invest in either of his two projects without the help of the lender. The lender wishes to prolong the period over which she can extract rents from her borrower. By offering a restrictive contract, the lender can keep the borrower captive, but in doing so the lender must offer her borrower a relatively low interest rate to compensate him for forgoing the fixed capital project. In contrast, the borrower readily accepts a high interest rate for unrestrictive contracts, which help him to grow his business faster than he could have in the lender's absence.

Our first main result characterizes when the lender offers a restrictive contract in equilibrium and keeps her borrower captive. Whether restrictions arise in equilibrium depends on the confluence of two factors: the borrower's present bargaining position and the hold-up problem created by the borrower's growth, which in turn depends on borrower's future bargaining position. We show that the lender may offer restrictive contracts and stop the borrower from growing his business even when business growth is socially efficient and even when the borrower would have grown his business in autarky. This is the sense in which increasing access to credit may constrain entrepreneurship. Restrictive contracts are likely when the borrower can only grow his business very slowly on his own, so that his bargaining position is weak. Restrictive contracts are also likely when the borrower has access to very productive fixed capital investment opportunities that require the lender's capital, so that the hold-up problem limits how much rent the lender can extract through unrestrictive contracts. This may explain why microfinance has had so little impact on entrepreneurship despite, or perhaps because of, the high return to capital found amongst microentrepreneurs in the research cited above.

Our theory predicts that borrowers who are close to entering the formal sector will receive un-

allowing borrowers flexibility in the timing of repayment, and the additional default is quite small. That study reports that on average, borrowers who received a flexible repayment contract defaulted on an extra Rs. 150 per loan. However three years later, these same borrowers earned on average an additional Rs. 450 to Rs. 900 *every week.*

restrictive contracts. These are borrowers with strong bargaining positions, so lenders find it too costly to offer them restrictive loans they will accept. This helps reconcile a second pattern from the experimental literature on the impacts of microfinance, which finds that while on average microfinance has had little impact on entrepreneurship, relatively wealthier borrowers do enjoy business growth from microcredit.[4]

The model further yields nuanced comparative statics that shed light on the dynamic interlinkages of wealth accumulation. We show that improving the attractiveness of the formal sector unambiguously improves the welfare of relatively rich borrowers who are close to the formal sector: the bargaining position of these borrowers improves, so they are resultantly less likely to receive restrictive contracts, and even when they do the contracts are more generous. Yet, improving the formal sector may actually reduce the welfare of poorer borrowers: because of the hold-up problem, the very fact that borrowers have stronger bargaining positions if they grow larger reduces the rent the lender can extract from unrestrictive contracts to poorer borrowers, who may therefore become more likely to receive restrictive contracts and remain in captivity. Thus there is a "trickle-down" nature of the comparative statics in our model, highlighting that policies that seem to improve the welfare of some borrowers may backfire on the poorest borrowers they aimed to help. This result echoes the seminal work of Petersen and Rajan (1995); we discuss this connection in Section 3.

Interpreting borrower captivity as a poverty trap, our model offers a counterpoint to the standard intuition that poverty traps are driven by impatience. We show that increasing borrower patience relaxes the poverty trap for rich borrowers, yet higher patience may *amplify* the poverty trap for poorer borrowers, causing them to get trapped at even lower levels of wealth. This is again due to the "trickle down" effect whereby lenders react to richer borrowers becoming more demanding by tightening contractual restrictions on poorer borrowers and preventing their growth.

Our model also highlights a danger of pledgeable collateral. While on the one hand pledgeable collateral increases the informal lender's ability to extract rent from her borrower, it also enables the borrower to access formal credit; in effect the more pledgeable collateral the borrower controls, the weaker is the informal lender's comparative advantage in enforcement. We demonstrate that the informal lender restricts the borrower's business growth precisely so that she can preserve her monopoly power and prolong the period of rent extraction. If the borrower could grow his business and commit never to borrow from a formal lender, the informal lender would never restrict the borrower's business growth and both parties might be better off. This harm of pledgeable collateral is related to, but distinct from the analysis of Donaldson et al. (2020), which also highlights the potential harm of pledgeable collateral. We further discuss this connection in Section 2.4.

[4]See Angelucci et al. (2015), Augsburg et al. (2015), Banerjee et al. (2015), Crepon et al. (2015), and Banerjee et al. (2017). This is also consistent with abundant anecdotal evidence that MFIs relax contractual restrictions such as rigid repayment schedules for richer borrowers, although this fact may be explained by a number of other theories.

We note that while our model is motivated by the microfinance industry, in principle it could apply more broadly to settings in which a lender desires to keep her borrower captive. Importantly, our theory relies on the lender's desire to prolong the period over which she enjoys monopoly power over her borrower, the lender's ability to influence the borrower's circumstances through contractual terms other than the interest rate of the loan, and limits on the contractual environment that inhibit the borrower's ability to pledge his long-term payoff from escaping captivity. It seems plausible that debt traps from payday lending markets in the United States might satisfy these properties (e.g. Bertrand and Morse (2011), Skiba and Tobacman (2019)), but our theory is unlikely to apply in well-functioning credit markets that are competitive or where lenders have means to contract on future rents (e.g. venture capital).

Our paper contributes to the literature on debt traps resulting from limited pledgeability. The topic has a long tradition in the literature on development finance (e.g., Bhaduri (1973), Ray (1998)). The dynamic inefficiency in our model stands in contrast to other papers that study lending inefficiencies arising from information frictions (e.g., Stiglitz and Weiss (1981), Dell'Ariccia and Marquez (2006), Fishman and Parker (2015)), common agency problems (e.g., Bizer and DeMarzo (1992), Parlour and Rajan (2001), Brunnermeier and Oehmke (2013), Green and Liu (2017)), and strategic default (e.g., Breza (2012)). Our work also relates to He and Xiong (2013), who analyze restrictive investment mandates in the context of asset management. Donaldson et al. (2019) also examines an environment in which a borrower has access to two projects, one of which keeps him captive to his lender. Our analysis differs in two fundamental ways. First, we study a different mechanism by which the lender can keep her borrower captive; the lender can inhibit the borrower from accumulating collateral. Second, while in Donaldson et al. (2019) it is the borrower who chooses the project, in our analysis it is the lender who chooses the project subject to the borrower's outside option of accumulating collateral slowly.

Finally, our paper relates to Rigol and Roth (2021), which utilizes a field experiment to demonstrate that microfinance loan officers constrain the business growth of their borrowers due to limited pledgeability constraints parallel to those we study in this paper.

The rest of the paper proceeds as follows. In Section 1 we describe the baseline model in which the borrower has only two business sizes—one in which he interacts only with his informal lender in equilibrium and the other in which he is in the formal sector. Section 2 characterizes the equilibrium of this game and discusses connections to the empirical literature on microfinance and policy implications. Section 3 extends the model so that there are two business sizes in which the borrower only has access to the informal lender in equilibrium, and discusses how the bargaining position of relatively richer borrowers influences the welfare of poorer borrowers when subjected to a forward-looking lender. Section 4 summarizes the relationship of our results with existing empirical evidence and concludes. The appendix contains several model extensions, a discussion of institutional features of microfinance that are not captured by our model, and all of the proofs.

1 The Baseline Two-State Model

Preliminaries We study a dynamic game of complete information and perfectly observable actions. There are three players, a borrower (he), an informal lender (she), and a formal lender (she), all of whom are risk neutral. Time is discrete and players discount the future at rate δ. Each period is subdivided into an early and late portion of the period, though no discounting happens within a period. At baseline we study a game with two states $w \in \{1, 2\}$ representing the borrower's business size. Each business state is accompanied by a collateralizable, productive asset, A_w. We assume that $A_1 = \emptyset$ (i.e. the borrower in state 1 has no productive asset). Later in section 3 we extend the model to three states and study state-dependent dynamics.

Production Technologies Within each period the borrower has access to two production technologies: a working capital project and a fixed capital project. Due to limited attention he must choose only one project within each period.

The working and fixed capital projects transform resource inputs into consumption goods at the same rate, and both projects can operate at two scales. Without any outside capital (i.e. with only the borrower's labor), the borrower can produce y_w^{aut} consumption goods under autarky. With κ units of outside capital the borrower can produce y_w goods, with $y_w - \kappa > y_w^{aut}$.

There are two differences between working and fixed capital projects. First, the fixed capital project offers the entrepreneur the possibility to grow his business, whereas the working capital project does not. If the borrower invests in fixed capital without outside capital, his business grows from state 1 to state 2 at the end of the investment period with probability g^{aut}; the borrower grows faster at probability $g > g^{aut}$ if he receives κ additional units of outside capital. The borrower stays at $w = 1$ if his business does not grow. For expositional simplicity, we refer to the growth probabilities per period, g^{aut} and g, also as *growth rates.*

The second difference between the two project types is that the working capital one produces output in the early portion of the period and enables the borrower to repay his debt early; in contrast, the fixed capital project takes more time to complete and generates output only in the late portion of the period. This distinction implies that the borrower's project choice will be influenced by the timing of repayment that his lending contract specifies, as we detail now.

Contracts At the beginning of each period, each lender can make a take-it-or-leave-it offer of a loan contract $c = \langle R, a \rangle \in \mathcal{C} \equiv \mathbb{R}^+ \times \{0, 1\}$, which specifies an upfront transfer of κ to the borrower, a repayment R from the borrower to the lender, and a contractual restriction a.

If the borrower rejects both contracts, he chooses one of the two projects to perform without any outside capital. If the borrower accepts a contract, the chosen lender transfers κ to the borrower. If the contract specifies contractual restriction $a = 1$, the lender insists on being repaid in the early portion of the period, and the borrower must choose the working capital project. If instead the contract specifies $a = 0$, then the borrower is free to repay the lender in the late portion of the period, and therefore can choose either project. We refer to contracts which specify $a = 1$ as *restrictive* loans and those that specify $a = 0$ as *unrestrictive* loans.

If the borrower accepts the contract, the borrower chooses a project to implement and whether or not to repay his debt R to the lender. The formal lender can only enforce contracts by seizing collateral, A_w. Therefore, if the borrower defaults on his contract to the formal lender, the lender seizes his productive assets and he reverts to state $w = 1$. The informal lender has an enforcement advantage: in addition to being able to seize collateral, the informal lender can costlessly impose a non-pecuniary punishment on a defaulting borrower, which costs the borrower $P > 0$ disutility. This can be understood as community sanctioning or physical pressure.[5]

Regardless of whether the contract is accepted or rejected, and whether or not the borrower repays his debt, all players meet again in the next period, and can propose new contracts.

Timing and Summary of Setup

1. Each lender makes a take it or leave it offer $c = \langle R, a \rangle$ to the borrower.
2. The borrower decides whether to accept a contract.
   (a) If he rejects both contracts, he chooses one of his two projects.
      i. Each lenders' flow payoff is 0 and the borrower's flow payoff is y_w^{aut}.
      ii. If the state is $w = 1$ and the borrower invested in fixed capital then his business grows from state 1 to 2 with probability g^{aut}, and remains constant otherwise.

   (b) If he accepts the contract, the chosen lender transfers κ to the borrower. If the contractual restriction is $a = 1$ then he must choose the working capital project. Else he can choose either project.
      i. If the borrower abides by the contract then he repays R to the lender.
         A. The lender's flow payoff is $R - \kappa$. The borrower's flow payoff is the output of the consumption good, net of repayment, $y_w - R$. The rejected lender's flow

[5]In principle the informal lender could separately decide whether or not to impose the punishment and whether or not to seize the borrower's collateral, but in equilibrium the informal lender will always do both in the event that the borrower defaults.

payoff is 0.

B. If the state is $w = 1$ and the borrower invested in fixed capital then his business grows from state 1 to 2 with probability g, and remains constant otherwise.

ii. If the borrower defaults on the contract then he repays nothing to his lender

A. If he defaults on the formal lender, the lender seizes his collateral at the end of the period, reverting the borrower to state $w = 1$. The lender's flow payoff is 0 and the borrower's is y_w.

B. If the borrower defaults on a contract with the informal lender, the lender both seizes his collateral and imposes a punishment of size P. The lender's flow payoff is 0 and the borrower's is $y_w - P$.

3. The period concludes and after discounting the next one begins.[6]

Equilibrium Our solution concept is the standard notion of *Stationary Markov Perfect Equilibrium* (henceforth *equilibrium*)—the subset of the subgame perfect equilibria in which strategies are only conditioned on the payoff relevant state variables. An equilibrium is therefore characterized by the lenders' state-contingent contractual offers $(R_w, a_w) \in \mathcal{C}$, the borrower's state and contract-contingent accept/reject decision $d_w : \mathcal{C} \rightarrow \{\text{accept, reject}\}$, and the borrower's state, contract and acceptance-contingent investment decision $i_w : \mathcal{C} \times \{\text{accept, reject}\} \rightarrow \{\text{work, fixed}\}$ and repayment decision $r_w : \mathcal{C} \rightarrow \{\text{repay, default}\}$. At all states all strategies must be mutual best responses. We defer discussion of the value functions that pin down these equilibrium response functions to Section 2.

By studying Stationary Markov Perfect Equilibria, we impose that the lender uses an impersonal strategy: borrowers with the same business size must be offered the same (potentially mixed set of) contracts. This may be an especially plausible restriction in the context of large lenders, such as microfinance institutions and banks whose policymakers may be far removed from their loan recipients, thereby rendering it infeasible to offer personalized contracts that condition on the borrowers' investment histories.

Assumptions

We impose two parametric restrictions.

Assumption 1. *y_w^{aut} is increasing in the state, as is $y_w - y_w^{aut}$.*

Assumption 1 states that both the borrower's autarky output and the value of outside capital are

[6]Note that for tractability we have implicitly assumed that the borrower cannot save his consumption good between periods. The primary vehicle of savings in this model is investment in the fixed capital project.

increasing in the state. This implies it is socially efficient for the borrower to invest in fixed capital. The social planner would then have the borrower always invest in fixed capital, and make transfers to the two lenders as appropriate.

Assumption 2. $P \geq \kappa$ *and* $P < y_1$.

The first inequality in Assumption 2 guarantees that the informal lender can enforce repayments that exceed the principal, so that lending in state $w = 1$ by the informal lender is at least weakly profitable. The second inequality guarantees the sufficiency of short-term contracts in state $w = 1$, as the enforceable repayment P does not exceed the borrower's within-period cash flow. In general, if P exceeded y_1 then the lender could write a contract with the borrower that required repayment over multiple periods. We relax this assumption in Section 2.3.

Model Discussion

The model has several features that merit further discussion.

Fixed and Working Capital Projects

The borrower has access to two projects, a working capital project and a fixed capital project. The working capital project corresponds to buying a liquid asset such as inventory, which the borrower can sell quickly. The fixed capital project takes longer to realize a return, but also gives the borrower a chance to build up a stock of pledgeable assets and expand his business (e.g. the borrower could buy a durable asset, or expand his store front, and hence take longer to recoup his costs). We assume that the borrower can only choose one of these two projects within a given period, due to limited attention or energy. But the borrower can switch projects between periods. In Appendix A.2 we provide an extension in which the borrower can utilize both projects within the same period.

For simplicity, while the fixed capital project returns a consumption good only in the late portion of the period, we assume that it produces consumption goods at the same rate as the working capital project. Therefore, because there is no discounting within a period, when the borrower is in autarky the fixed capital project strictly dominates the working capital project. However the model can easily accommodate working capital and fixed capital projects that return consumption goods at different rates.

The Contractual Restriction $a = 1$

We assume that when the lender offers a contract that specifies $a = 1$ the borrower must repay his debt R in the early portion of the period, and therefore must choose the working capital project. In contrast, when the contract specifies $a = 0$ the borrower is free to repay his debt in the late portion of the period, and so can invest in either of his two projects. This formulation is consistent with a growing body of empirical evidence about microfinance. A standard microfinance contract requires that borrowers begin repaying their loans immediately after receiving them, and continue to do so in frequent installments throughout the loan cycle. Field et al. (2013), Takahashi et al. (2017), Barboni and Agarwal (2018), and Battaglia et al. (2019) find that by relaxing this feature of the loan contract and allowing borrowers to match the timing of their repayments to the cashflow of their businesses, borrowers exhibit higher demand for credit, make larger and longer-term investments, and their businesses grow faster as a result. Our theory will illuminate when and why the lender might design contracts to discourage her borrowers from making long-term investments even when such investments do not cause the lender any short-run loss.

We assume that if the borrower accepts a contract, he is free to default on repayment but he must choose the project dictated by the contractual restriction. This may represent that the borrower must incur some irreversible costs that commit him to his chosen project before the loan is made, to demonstrate to his lender that he has a viable repayment plan. In the appendix we extend the model to allow for the borrower to renege on the contractual restriction $a = 1$ and choose the fixed capital project, subject to some adjustment cost, and subject to the same punishments as in the present model. We demonstrate that our analysis is robust to this extension.

The Collateral

We assume that the borrower's productive asset in state 2, A_2, can be used as collateral; if the borrower fails to repay the amount specified by the contract, either lender can seize the asset so that the borrower is forced to return to $w = 1$. The collateral is valuable to the borrower because it increases his productivity; as we show below, a second, endogenous, source of value from this collateral arises because it may enable the borrower to access the formal lender.

For expositional simplicity, we assume that the collateral has no resale value, i.e., it is only useful to the lender as a disciplinary device to enforce repayment, and has no other value to the lender. The analysis would proceed similarly if instead we assumed that the collateral had positive resale value to the lender, so that she could recoup some or all of her principal in the event that the borrower defaulted.

The Informal and Formal Lenders

The only exogenous difference between the formal and informal lender is that in addition to being able to punish borrowers by seizing collateral, the informal lender can also costlessly sanction the borrower by imposing a non-pecuniary penalty P. This difference results in an endogenous asymmetry in the competitive environment in which these lenders operate. As we discuss in the analysis to follow, the informal lender acts as a monopolist when the endogenous value of the borrower's collateral is less than the principal loan amount κ. When the value of the borrower's collateral exceeds κ, both lenders can profitably lend, leading to a competitive credit market. This is the mechanism by which the informal lender loses her continuation surplus when the borrower gains access to the formal lender.

2 Analysis of the Two-State Model

First in Section 2.1 we study a baseline model in which there is no formal lender—only the borrower and his informal lender. In this baseline model the informal lender always helps the borrower grow his business faster than he could have in autarky, so as to take advantage of the borrowers increased productivity and additional collateral.

Next in Section 2.2 we examine the addition of a formal lender who can compete for borrowers who have sufficient collateral. In characterizing the equilibrium of this model we identify conditions under which the informal lender will keep her borrower captive in state 1, thereby inhibiting efficient business growth, not only relative to the baseline without a formal lender but also relative to autarky, when the borrower has access to no lender at all. In Sections 2.3 and 2.4, and in Section 2.5 we discuss policy implications.

In Section 3 we extend the model to have three states and explore the dynamic nature of comparative statics within the richer model.

2.1 Baseline With Only The Borrower and Informal Lender

First we show that, in the absence of a formal lender to compete for borrowers with collateral, the informal lender always helps the borrower grow his business faster than he could have in autarky.

State $w = 2$. We solve the model by backward induction, starting in state 2. As in state 2 the borrower's fixed and working capital projects are equivalent, the lender's contractual restriction is irrelevant. Thus we focus our attention on the repayment R that the lender demands. The lender's offer is constrained by two factors. First is the borrower's individual rationality constraint. If the borrower rejects the lender's loan and produces output on his own then his payoff is $y_2^{aut} + \delta B_2$ where B_2 is his continuation value in state 2. Therefore the borrower's *individual rationality* constraint for accepting the loan is:

$$\underbrace{y_2 - R + \delta B_2}_{\text{flow utility if borrow and repay}} \quad \geq \quad \underbrace{y_2^{aut} + \delta B_2}_{\text{flow utility in autarky}}$$

$$\iff R \leq y_2 - y_2^{aut}. \tag{IR 2}$$

The lender's second consideration is the borrower's incentive compatibility constraint. Namely if the borrower accepts the lender's contract and defaults, then the lender seizes his collateral, reverting him to state 1 and inflicts a punishment P. Therefore his payoff is $y_2 - P + \delta B_1$, where B_1 is the borrower's state 1 continuation value. The borrower's *incentive compatibility* constraint for repayment is

$$\underbrace{y_2 - R + \delta B_2}_{\text{flow utility if borrow and repay}} \geq \underbrace{y_2 - P + \delta B_1}_{\text{flow utility if borrow and default}}$$

$$\iff R \leq P + \delta \left(B_2 - B_1\right). \tag{IC 2}$$

In state 2 the lender offers the maximal repayment that satisfies both the IC and IR constraints, i.e., $R = \min\left\{y_2 - y_2^{aut}, P + \delta\left(B_2 - B_1\right)\right\}$.

State $w = 1$. In state 1 there is a meaningful distinction between the borrower's fixed and working capital projects. Therefore we separately analyze the lender's optimal restrictive and unrestrictive contracts, and then determine which one the lender offers in equilibrium. Once again each contract will be subjected to the borrower's individual rationality and incentive compatibility constraints.

If the lender offers a restrictive contract $\langle R, a = 1\rangle$ then the borrower's payoff from accepting and abiding by the contract is $y_1 - R + \delta B_1$, as he is forced to invest in the working capital project to meet the early repayment deadline. If the borrower rejects the contract then he always prefers to invest in fixed capital. Therefore his payoff upon rejecting the contract is $y_1^{aut} + \delta\left(B_1 + g^{aut}\left(B_2 - B_1\right)\right)$ as with probability g^{aut} he progresses to state 2. The individual rationality constraint is then

$$y_1 - R + \delta B_1 \geq y_1^{aut} + \delta\left(B_1 + g^{aut}\left(B_2 - B_1\right)\right)$$

$$\iff R \leq y_1 - y_1^{aut} - \delta g^{aut}\left(B_2 - B_1\right). \tag{IR 1-rest}$$

If the borrower accepts the contract and reneges, his payoff is $y_1 - P + \delta B_1$, as he consumes the full output of the working capital project but is then punished which costs P (recall the state 1 borrower has no collateral). The borrower's incentive compatibility constraint is then

$$R \leq P. \tag{IC 1-rest}$$

Together these constraints (IR 1-rest) and (IC 1-rest) pin down the lender's equilibrium value from offering a restrictive contract in state 1, as the lender requests the maximal repayment that satisfies the constraints.

Finally we must consider the lender's optimal unrestrictive contract. As the borrower is able to invest in the fixed capital project regardless of whether or not he accepts the loan, his individual rationality constraint is

$$R \leq y_1 - y_1^{aut} + \delta\left(g - g^{aut}\right)\left(B_2 - B_1\right) \tag{IR 1-unrest}$$

and his incentive compatibility constraint is

$$R \leq P. \tag{IC 1-unrest}$$

Comparing (IR 1-rest) to (IR 1-unrest) we see that the lender must offer a steeper concession in the demanded repayment amount to satisfy the borrower's individual rationality constraint when offering a restrictive contract, because to accept a restrictive contract is to forgo the possibility of business growth.

We are now ready to state our main result of this section.

Proposition 1. *In the unique equilibrium of the model with only the borrower and informal lender, the lender always offers unrestrictive contracts in state $w = 1$.*

That the lender always offers the borrower an unrestrictive loan in state $w = 1$ arises from the confluence of three factors. First, as discussed above, the lender can charge a higher repayment amount in state $w = 1$ with an unrestrictive contract, as she is not asking the borrower to forgo business growth. Second, the borrower is more productive in state $w = 2$, both in terms of the output that he can produce on his own and also in terms of the additional value the lender can provide. This increases the amount of value the lender can extract from the borrower, relative to state $w = 1$. Finally, the state 2 borrower has pledgeable collateral, which relaxes the borrower's incentive compatibility constraint and increases the amount of value the lender can extract from the borrower.

All of these forces work in the same direction, and induce the lender to help the borrower grow his business faster than he could have in autarky and (weakly, and sometimes strictly) increase his utility relative to autarky. As we will see in the next section, these conclusions may be reversed when a formal lender can compete with the informal lender for borrowers with sufficient collateral.

2.2 The Model With Both a Formal and Informal Lender

We now consider the introduction of a formal lender. As the borrower has no pledgeable collateral in state 1, he cannot commit to repay his formal lender and therefore in state 1 the informal lender is a monopolist. In state 2 the borrower's pledgeable collateral is worth $\delta\,(B_2 - B_1)$, which corresponds to the decline in the borrower's continuation utility if his collateral is seized. Therefore, the formal lender can make a weakly profitable loan to the borrower in state 2 if and only if $\delta\,(B_2 - B_1) \geq \kappa$, so that the lender can at least recoup the loan's principal. For now we assume that this inequality holds and we will confirm (or disconfirm) this endogenous condition after computing the borrower's continuation values B_1 and B_2.

Assuming that the formal lender can break even, the presence of two lenders implies that the state 2 borrower enjoys a perfectly competitive credit market. When this condition holds, we simply refer to state 2 as the *formal sector.* Both lenders offer the borrower a loan with repayment amount $R = \kappa$ (recall the state 2 contractual restriction is irrelevant). The borrower's equilibrium state 2 continuation utility is

$$B_2 = \frac{y_2 - \kappa}{1 - \delta},$$

and the formal and informal lenders enjoy state 2 continuation utilities of 0.

Moving backwards to state 1, the borrower's individual rationality and incentive compatibility constraints are exactly the same as in the previous section for both restrictive (IR 1-rest and IC 1-rest) and unrestrictive contracts (IR 1-unrest and IC 1-unrest). Now, however, Proposition 1 does not hold, and the informal lender may keep the borrower captive in state 1 by offering only restrictive contracts. To determine the lender's equilibrium behavior in state 1, in Appendix C we separately characterize the optimal contract that the lender offers if she is constrained to only offer restrictive contracts, and the optimal contract she offers if she is constrained to offer only unrestrictive contracts. By comparing the lender's payoffs in both circumstances we can determine her optimal equilibrium behavior. We use $\left(\tilde{B}_1, \tilde{L}_1, \tilde{R}_1\right)$ to denote, respectively, the borrower's and the informal lender's value functions and the requested repayment in state 1 in a hypothetical equilibrium if the informal lender were constrained to only offer restrictive contracts; likewise, we use $\left(\hat{B}_1, \hat{L}_1, \hat{R}_1\right)$ to denote the value functions and requested repayment in a hypothetical equilibrium if the informal lender were constrained to only offer unrestrictive contracts.

In effect, the decision of whether to offer an unrestrictive contract versus a restrictive one depends on the tradeoff between extracting a higher repayment in state 1 using an unrestrictive contract versus allowing the borrower to reach state 2 in finite time. The following lemma highlights an important force for understanding when the lender holds the borrower captive by offering restrictive contracts.

Lemma 1. *$\hat{B}_1 \geq \tilde{B}_1$ with strict inequality when the individual rationality constraint does not bind for unrestrictive contracts in state 1.*

The borrower's value is always weakly higher under an unrestrictive contract than under a restrictive one, and, so long as the borrower's individual rationality constraint does not bind for unrestrictive contracts, the inequality is strict. This asymmetry arises from a hold-up problem associated with the fixed capital project. When the borrower receives a restrictive contract and is captive in state 1, the lender can calibrate the repayment in perpetuity so that either the individual rationality or incentive compatibility constraint binds. In contrast, when the borrower receives an unrestrictive contract, much of his value accrues to him in state 2. In state 2, because the borrower has sufficient pledgeable capital to borrow from the formal lender, the borrower commands the full state 2 surplus. From the perspective of state 1, the hold-up problem arises because the borrower cannot commit not to exercise his improved bargaining position in state 2. This stands in contrast to the case where only the informal lender operated in state 2, so that the borrower's increased collateral in state 2 improved, rather than harmed, the lender's ability to extract rents.

We term this difference between the borrower's value under unrestrictive and restrictive contracts, $\hat{B}_1 - \tilde{B}_1$, the borrower's expansion rent, as it is the additional utility the borrower enjoys when he is allowed to expand his business. It will be critical in determining when the lender holds the borrower captive in state 1.

2.2.1 Characterizing the Equilibrium Contract

Let B_w^* be the borrower's equilibrium value in state w, and L_w^* be the informal lender's equilibrium value in state w. We are now ready to characterize the equilibrium. For now, we continue to conjecture that $\delta\left(B_2^* - B_1^*\right) \geq \kappa$ so that the formal lender can lend in state 2. This continues to pin down state 2 value functions. We will verify (or disverify) this conjecture below.

Proposition 2. *Conditional on the formal lender operating in period* 2, *i.e.* $\delta\left(B_2^* - B_1^*\right) \geq \kappa$, *equilibrium behavior is unique.*

Proposition 2 states that if there is an equilibrium in which $\delta\left(B_2^* - B_1^*\right) \geq \kappa$, then conditional on the formal lender operating in state 2, equilibrium behavior is unique. Nevertheless, even when such an equilibrium exists, there may also be an equilibrium in which the formal lender does not lend in state 2. In this latter equilibrium, because the borrower does not expect to be part of a competitive credit market in state 2, the borrower's state 2 continuation value would be lower, making his collateral less valuable, potentially falling below the level required for the formal lender to recoup her principal. That is, this is a model in which the value of the borrower's collateral is determined by his expectation about the value of operating in state 2. Because we are interested in the consequences of competition between the informal and formal lender, from here out we will focus attention on the equilibrium where the formal lender operates in state 2, when such an equilibrium exists.

Having characterized equilibrium behavior in state 2, we now characterize state 1. The informal lender may in general follow a mixed strategy between offering unrestrictive and restrictive contracts. Let p^* be the equilibrium probability that state 1 contracts are restrictive; p^* is a key object of our equilibrium and subsequent comparative static analysis.

Proposition 3. *Generically,*[7] *the lender offers restrictive contracts with probability*

$$p^* = \begin{cases} 1 & \text{if } \hat{L}_1 - \tilde{L}_1 \leq 0, \\ 0 & \text{if } \hat{L}_1 - \tilde{L}_1 \geq \frac{\delta}{1-\delta} g_1^{aut} \left(\hat{B}_1 - \tilde{B}_1\right), \\ \frac{\left(g_1^{aut}\left(\hat{B}_1 - \tilde{B}_1\right) - \left(\frac{1-\delta}{\delta}\right)\left(\hat{L}_1 - \tilde{L}_1\right)\right)(1-\delta+\delta g_1)}{(1-\delta+\delta g_1) g_1^{aut}\left(\hat{B}_1 - \tilde{B}_1\right) + \left(g_1^{aut} - g_1\right)(1-\delta)\left(\hat{L}_1 - \tilde{L}_1\right)} & \text{otherwise.} \end{cases}$$

The borrower's equilibrium value function is

$$B_1^* = s\tilde{B}_1 + (1-s)\,\hat{B}_1,$$

with $s \equiv \frac{p\left(1-\delta\left(1-g_1^{aut}\right)\right)}{1-p\delta\left(1-g_1^{aut}\right)-(1-p)\delta(1-g_1)} \in [0,1]$. *The lender's value function is* $L_1^* = \max\left\{\hat{L}_1, \tilde{L}_1\right\}$. *The unrestrictive contract offered is* $\left\langle \hat{R}_1, 0 \right\rangle$ *and the restrictive contract offered is*

$$\left\langle \min\left\{P, y_1 - \left(y_1^{aut} + \delta g_1^{aut}\left(B_2^* - B_1^*\right)\right)\right\}, 1 \right\rangle.$$

The proposition completely characterizes the equilibrium contracts and value functions. When $p^* = 1$, all contracts are restrictive, and the informal lender endogenously keeps the borrower captive, even though the borrower would eventually reach the formal sector in the absence of credit. Even when $p^* < 1$, the borrower grows at a rate $(1-p^*)\,g$, which may be slower than under autarky, g^{aut}.

To understand why p^* may be interior, note that the borrower's willingness to accept a restrictive contract at a given repayment amount depends on his value from staying in the current state. In turn, because of the borrower's expansion rent, his value from staying in the current state is increasing in the likelihood that he receives an unrestrictive contract in the current state. The more likely the borrower expects unrestrictive contracts to be, the lower is the repayment amount he requires to accept restrictive contracts. Therefore it is possible that when the borrower expects to receive restrictive contracts with probability 1 the lender prefers to give an unrestrictive contract, and when he expects to receive an unrestrictive contract with probability 1 the lender prefers to give a restrictive contract. This phenomenon can lead to an equilibrium probability of restrictive contracts p^* strictly between 0 and 1, but in such cases, conditional on the formal lender operating in equilibrium, p^* is still unique. Note that the informal lender's equilibrium value L_1^* is always equal to $\hat{L}_1$ whenever $p^* < 1$ and is equal to $\tilde{L}_1$ only when $p^* = 1$.

Proposition 3 admits the following corollary.

[7] The proposition holds except in the non-generic case where $\hat{B} = \tilde{B}$ and $\hat{L} = \tilde{L}$, under which both players are indifferent between restrictive an unrestrictive contracts, and there is a continuum of equilibria corresponding to all $p^* \in [0,1]$.

Corollary 1. *The informal lender offers a restrictive contract with probability $p^* = 1$ if and only if*

$$\underbrace{\beta\left(\frac{y_2 - y_1}{1-\delta}\right)}_{\substack{\text{"efficiency gain"} \\ \text{from investing in fixed capital}}} \leq \underbrace{\hat{B}_1 - \tilde{B}_1}_{\substack{\text{"expansion rent"} \\ \text{captured by the borrower}}}, \tag{1}$$

where $\beta \equiv \frac{\delta g}{1-\delta(1-g)}$ is the expected fraction of the borrower's discounted lifetime that he spends in state 2 when he invests in fixed capital and grows his business at the fast rate g enabled by the lender's capital.

The left-hand side of inequality (1) can be understood as the social efficiency gain of investing in fixed capital relative to working capital. The borrower and lender spend a discounted fraction β of their lives in the formal sector, where the joint payoff is $\frac{y_2-\kappa}{1-\delta}$, and they forgo their joint production in state 1, $\frac{y_1-\kappa}{1-\delta}$. The right-hand side is the borrower's expansion rent—the additional value he derives from unrestrictive contracts relative to restrictive contracts because of the hold-up problem. The lender's benefit from unrestrictive contracts is the entire social efficiency gain of helping the borrower grow his business minus the borrower's expansion rent. Therefore when Inequality (1) holds, the lender offers only restrictive contracts and keeps the borrower captive.

We have now fully characterized the equilibrium under the conjecture that $\delta\left(B_2^* - B_1^*\right) \geq \kappa$ and the equilibrium values B_1^* and L_1^* can be computed. Therefore, whether $\delta\left(B_2^* - B_1^*\right) > \kappa$ can now be verified. If this condition holds, then the equilibrium described above is valid. If this condition does not hold, then there is no equilibrium in which the formal lender can lend in state 2. Therefore, the informal lender is a monopolist in both states and the analysis in Section 2.1 characterizes the unique equilibrium. From now on, we restrict attention to the case where the borrower's state 2 collateral is sufficiently valuable such that the formal lender is active in state 2.

We close this section by noting that the equilibrium we study may not be renegotiation proof. While Proposition 2 demonstrates that conditional on the formal lender operating in state 2 the game has a unique Markov perfect equilibrium, the game does sometimes admit other subgame perfect equilibria that Pareto-dominate the Markov equilibrium we identify. However, because these equilibria rely on the stochastic nature of the fixed capital technology and require a high degree of coordination that is unlikely to be feasible in the microfinance context, we think the possibility that the borrower and informal lender would renegotiate to these Pareto-superior equilibria is unrealistic. We discuss these alternative equilibria in Appendix E.

Access to Credit can Inhibit Entrepreneurship Proposition 3 highlights that expanding access to credit can *inhibit* entrepreneurship. This statement can be interpreted in two ways. First, the informal lender can slow the borrower's business growth.

Proposition 4. *Relative to the case where only the formal lender is allowed to operate, introducing the informal lender (so that both lenders are allowed to operate) can slow the borrower's business growth.*

When there is no informal lender, the formal lender faces no competition. She can still never lend in state 1, as the borrower has no pledgeable collateral. In state 2, if the borrower's pledgeable collateral is sufficiently valuable, then the formal lender operates as a monopolist, else she cannot lend in state 2 either.

In either case, when the borrower does not have access to an informal lender, he always invests in fixed capital and reaches state 2 in finite time. Once we introduce the informal lender, the borrower would still like to grow his business but may not be able to; off the equilibrium path when the borrower rejects the informal lender's loan in state 1, he invests in fixed capital. And, under assumption 1, investing in fixed capital is socially efficient. Nevertheless, the informal lender may offer only restrictive contracts in equilibrium, thereby slowing the borrower's business growth relative to if the borrower had no access to informal credit.

Second, introducing the formal lender in state 2 can also slow the borrower's business growth in state 1.

Proposition 5. *Relative to the case where only the informal lender is allowed to operate, introducing the formal lender (so that both lenders are allowed to operate) can slow the borrower's business growth.*

This can be seen by comparing the equilibrium in Section 2.1 to that of Section 2.2. In the former, the informal lender is a monopolist in both states; hence, in state 1 she always lets the borrower grow his business, as doing so not only makes him more productive but also increases his stock of pledgeable assets, thereby increasing the repayment that the informal lender can demand. In contrast, when there is a formal lender in state 2, the fact that the borrower has an increased base of pledgeable assets becomes a deterrent for the informal lender. In this case, the advantage to the informal lender of increased pledgeability is offset by a reduction in the informal lender's comparative advantage in enforcement. While the informal lender is a monopolist in state 1 because of her informal enforcement power, in state 2 the borrower has sufficient pledgeable assets to attract the formal lender. This competition effect dominates the informal lender's benefit from increased pledgeability. Therefore, introducing the formal lender can induce the informal lender to offer restrictive contracts in state 1, and thereby diminish the rate of the borrower's business growth, not only relative to the case where the borrower only has access to informal credit, but, perhaps surprisingly, also relative to the case where the borrower has no access to credit at all.

We close this section by examining the impact of each lender on the borrower's welfare.

Proposition 6. *Introducing the formal lender may reduce the borrower's welfare relative to the model* *where the formal lender is absent. Introducing the informal lender always weakly increases the borrower's welfare relative to the model where the informal lender is absent.*

Introducing the formal lender may reduce the borrower's welfare, as it can induce the informal lender to keep the borrower captive in state 1. This echoes the hallmark result of Petersen and Rajan (1995), which we further discuss in Section 3. In contrast, introducing the informal lender always weakly improves the borrower's welfare relative to her absence because of the voluntary nature of the loans—the lender must always satisfy the borrower's individual rationality constraint.

Finally, we note that our finding, that expanding access to credit may inhibit entrepreneurship, may shed some light on the disappointing impacts of microfinance repeatedly found across studies around the world, cited in the introduction.

2.3 When Does The Informal Lender Keep The Borrower Captive?

In this section we study comparative statics of the equilibrium to understand the interplay between the the strength of the borrower's bargaining position and the hold-up problem. In doing so we shed light on the circumstances in which restrictive contracts and borrower captivity are especially likely and argue that this can explain a number of empirical facts about microfinance.

Captivity and the Growth Rate of the Fixed Capital Project

Our first comparative statics regard the growth rate of the fixed capital project.

Proposition 7. *The state 1 probability of a restrictive contract p^* is weakly decreasing in g^{aut}, and weakly increasing in g. In both cases the comparative static is strict for $p^* \in (0,1)$. Moreover, there exists a $\bar{g}$ such that for $g > g^{aut} > \bar{g}$, the borrower always receives unrestrictive contracts with positive probability.*

These comparative statics can be understood by examining Inequality (1). Increasing g^{aut}, which reflects improving the productivity of the fixed capital project or the borrower's access to capital without the lender's help, makes restrictive contracts less likely in equilibrium. Increasing g^{aut} improves the borrower's bargaining position as investing in fixed capital on his own is now relatively more attractive, and therefore this reduces the maximum repayment he is willing to accept along with a restrictive contract. This reduces the borrower's expansion rent. However, the efficiency gain from investing in fixed capital is unaffected. Hence unrestrictive contracts become relatively more attractive and therefore more likely in equilibrium.

In contrast, increasing g, which corresponds to increasing the productivity of the fixed capital project or the size of the loan, makes restrictive contracts more likely. Because of the hold-up problem, increasing g reduces the rent that the lender can extract from unrestrictive contracts. The lender can only extract rent from the borrower so long as he remains in state 1, and increasing g

reduces the amount of time after which the borrower leaves the lender when receiving unrestrictive contracts. In contrast, because the borrower can only take advantage of the improved fixed capital project with the lender's help, the borrower's bargaining position is left unchanged, so the lender's utility from restrictive contracts is unchanged.

Together the comparative statics on g^{aut} and g start to paint a picture about when introducing a lender is likely to inhibit entrepreneurship. In particular, borrowers with poor ability to grow without the lender's help (low g^{aut}), and strong prospects for business growth with the lender's help (high g) are likely to be offered restrictive contracts. This may be precisely the case for many microfinance borrowers around the world. In particular, while it may have at first seemed counterintuitive that microfinance has had low impact on entrepreneurship despite strong evidence that unconditional cash grants lead to substantial and persistent business growth, Proposition 7 suggests that it may be *because* of these attractive investment opportunities, and an inability for borrowers to grow their businesses in the absence of outside capital, that microfinance has had little impact.

Finally raising both g^{aut} and g to a sufficiently high level, which increases the rate of growth from the fixed capital project under any investment level and may be interpreted as reducing the borrower's distance from achieving the regime change and entering the formal sector, guarantees that the borrower receives an unrestrictive contract with positive probability. Effectively, for borrowers with sufficiently strong bargaining positions, the lender cannot extract substantial rents using restrictive contracts. This renders the hold-up problem irrelevant, as the lender can always extract positive rent through unrestrictive contracts.

That borrowers near the formal sector receive unrestrictive contracts is consistent with the large body of experimental evidence on the impact of microfinance. Many of these studies find that while the average impact of microfinance on entrepreneurship is low, borrowers with relatively larger businesses who are closer to graduating out of microfinance do see substantial business growth.[8]

Less Captivity In the Two-State Model if the Formal Sector is More Attractive. Our next comparative static concerns the attractiveness of the formal sector. The borrower's equilibrium utility in state 2 is $B_2^* = \frac{y_2-\kappa}{1-\delta}$. In what follows we discuss how the equilibrium probability of a restrictive contract in state 1, p^*, changes with respect to B_2^*. Interpreted literally, this exercise can be understood as examining how changing y_2 affects p^*. More generally, we would like to interpret this exercise as encapsulating any policy that might improve the borrower's welfare in the formal sector, such as making it more competitive or inducing lenders to offer contracts with better terms.

Proposition 8. *The state* 1 *probability of a restrictive contract* p^* *is weakly decreasing in* B_2^**, and strictly so for* $p^* \in (0,1)$.

[8] See Angelucci et al. (2015), Augsburg et al. (2015), Banerjee et al. (2015), Crepon et al. (2015), and Banerjee et al. (2017).

Increasing B_2^* increases the value that the borrower places on business growth, and hence strengthens his bargaining position. So as B_2^* increases, borrowers become more demanding when receiving restrictive contracts. In contrast, borrowers are weakly more willing to accept any unrestrictive contract as B_2^* increases, since unrestrictive contracts allow them to grow their business and reach the formal sector. Therefore increasing the attractiveness of the state in which the borrower gains access to the formal lender reduces the likelihood of borrower captivity. We will see in Section 3, when the borrower must grow his business more than once in order to reach the formal lender, that this conclusion may be reversed.

Less Captivity under Greater Enforcement. Our final comparative static concerns the informal lender's ability to enforce repayment P. Thus far we have assumed that $P \leq y_1$ so that the lender can fully extract the repayment R within a single period. What if P exceeds y_1? If P is sufficiently large, the lender always offers unrestrictive contracts.

Proposition 9. *There exists a $\bar{P}$ such that for $P \geq \bar{P}$ the informal lender offers unrestrictive contracts with probability 1, $i.e.$ $p^* = 0$.*

When P is sufficiently large, the borrower's individual rationality constraint binds for both unrestrictive and restrictive contracts in state 1. Therefore the borrower's expansion rent is 0 and by corollary 1 the lender offers unrestrictive contracts with probability 1. In effect, the borrower can commit to share the benefits of state 2 with his informal lender in exchange for unrestrictive contracts in state 1, thereby alleviating the hold-up problem.

Other Comparative Statics. For completeness we note that comparative statics can also be done with respect to the output of the consumption good, y_1^{aut} and y_1. Increasing y_1^{aut} improves the borrower's bargaining position, and just as in Proposition 7 this pushes towards unrestrictive contracts. Increasing y_1 makes state 1 more attractive. This increases the likelihood of restrictive contracts as it reduces the social efficiency gain of growing to the formal sector, and the lender is more willing to offer concessionary repayment amounts to keep the borrower captive.

2.4 The Danger of Pledgeable Collateral

Our model highlights a danger of pledgeable collateral in informal credit markets. When the borrower's state 2 collateral is worth less than κ, he does not have access to the formal lender, and, as we analyzed in Section 2.1, the informal lender always lets the borrower invest in fixed capital and grow his business. Marginally increasing the value of the borrower's state 2 collateral to exceed κ may induce the informal lender to keep her state 1 borrower captive, and may reduce the welfare of both the state 1 borrower and the informal lender. Intuitively, when the informal lender is a monopolist, she values the borrower's collateral as it increases the value she can extract from her loans. However, when a formal lender is present, increasing the borrower's collateral also reduces the informal lender's comparative advantage in enforcement, as the informal lender now has to compete with the formal lender for the borrower in $w = 2$. Precisely because the borrower in state 1 cannot commit not to exercise his improved bargaining position in state 2, the informal lender may find it optimal to keep the borrower captive in state 1, as we have shown. Therefore, increasing the pledgeable collateral of the state 2 borrower can lead to a Pareto disimprovement to both the informal lender and the borrower from the perspective of state 1.

The danger of pledgeable collateral that we highlight complements the analysis of Donaldson et al. (2020). In that paper the authors study a model in which a borrower seeks to finance two projects in sequence, the second of which may be socially inefficient. The authors highlight that by increasing the borrower's collateral, all else equal he is more likely to be able to finance the second project even when it is inefficient, and that this imposes a negative externality on his first lender who is now less likely to be repaid. Anticipating this, the first lender may demand so high a fraction of his collateral, so that the borrower has no chance of financing his second project, even if it is discovered to be socially efficient. Our mechanism is different, in that increasing the borrower's state 2 collateral can be harmful even though investing in fixed capital in state 1 is always socially efficient. In our case, the informal lender seeks to maximize rent extraction, and allowing the borrower to grow his business limits the amount of rent the informal lender can extract. This difference also manifests itself in the fact that in our setting, if the borrower were to make a take it or leave it offer rather than the lender, the borrower would never be captive in state 1 regardless of his pledgeable collateral, whereas in the model of Donaldson et al. (2020) the danger of pledgeable collateral materializes even though the borrower is making the take it or leave it offer.

2.5 Policy

Our analysis has demonstrated that introducing a lender can inhibit entrepreneurial growth relative to the borrower's autarkic benchmark. In this section we consider how this phenomenon is affected by two policy responses: introducing a cap on the interest rate and providing a subsidy to lenders. We show that even though these policies may appear to be borrower-friendly, they can both exacerbate borrower captivity, rather than relieving it, and reduce the borrower's welfare.

First, suppose a policy maker imposed a cap $\bar{R}$ on the allowable repayment amounts to be demanded by either lender. We restrict attention to $\bar{R} \geq \kappa$, as otherwise no lending can profitably occur. What is the effect of reducing $\bar{R}$, thereby limiting the amount that either lender can extract from the borrower? We have the following proposition.

Proposition 10. *Reducing $\bar{R}$ weakly increases the equilibrium probability p^* of a restrictive contract in state* 1*.*

Reducing the repayment cap $\bar{R}$ makes borrower captivity more likely. In state 2 the cap has no impact on payoffs, as both lenders offer a repayment of $R_2^* = \kappa$. In state 1, recall that the informal lender can demand a higher repayment amount with an unrestrictive contract than with a restrictive contract, as she does not need to compensate the borrower for forgoing business growth. Therefore, the repayment cap is more binding for unrestrictive contracts than for restrictive contracts. Put another way, the repayment cap reduces the amount that the lender can extract from both restrictive and unrestrictive contracts; if the lender cannot extract meaningfully more repayment from unrestrictive contracts then she may as well keep the borrower captive with the restrictive contract to prolong the period over which she can extract rents. On net, lowering the repayment cap can actually result in a Pareto disimprovement, harming the welfare of both the borrower and lender.

Next we consider the related policy of subsidizing lenders for each loan they make. Suppose that a policy maker offers any lender a transfer of s each time they make a loan to the borrower. What is the effect of increasing the subsidy? We show that the effect depends importantly on the state. We have the following proposition.

Proposition 11. *In state 2, increasing s improves the borrower's welfare. In state 1, increasing s weakly increases the equilibrium probability p^* of a restrictive contract in state 1, and weakly decreases the borrower's welfare.*

In state 2 the credit market is competitive. Therefore the subsidy is passed onto the borrower and increases his welfare. In state 1, however, the informal lender is a monopolist, and therefore keeps the entire subsidy. Increasing the subsidy has no effect on the lender's continuation value in state 2, as the state 2 subsidy is fully enjoyed by the borrower, but it increases the lender's value of remaining in state 1. This weakly increases the probability of borrower captivity and therefore weakly decreases the borrower's welfare.

The above two exercises demonstrate the importance of appreciating the interaction between the formal and informal sectors. If the analysis were restricted to the perfectly competitive credit market in state 2, then reducing the repayment cap and increasing the subsidy for lending both weakly improve the borrower's welfare. It is not until the informal sector is taken into account that we can appreciate how these policies might actually harm the borrower's welfare.

3 Contractual Restrictions and State Dependence

In this section we extend the model to have 3 states, so that in equilibrium there are 2 states where the informal lender is a monopolist before the borrower graduates to the formal lender. The goal is to develop new insights stemming from the forward-looking nature of the lending relationship. We demonstrate that comparative statics that improve the welfare of relatively rich borrowers may, but will not always, harm the welfare of poorer borrowers. In effect, because of the hold-up problem, improving the bargaining position of richer borrowers may reduce the rent the lender can extract from unrestrictive contracts to poorer borrowers and increase the likelihood that they are held captive. For instance, increasing the attractiveness of the formal sector may harm poorer borrowers by virtue of helping richer ones. This is similarly true when increasing the borrower's patience. We characterize when these forces can on net increase the likelihood of captivity for poorer borrowers.

Formally, we now extend the model to have 3 business sizes. We maintain Assumption 1, and strengthen Assumption 2 so that $P \leq y_1 - \kappa$. This strengthening ensures that the informal lender can extract her full repayment within a single period.[9] We also index g and g^{aut} by the state w, as the fixed capital project now helps the borrower grow in both of the first two states. We restrict attention to the case where there is an equilibrium in which state 3 is the first state w for which $\delta\left(B_w^* - B_1^*\right) \geq \kappa$ so that the formal lender can profitably lend, and we refer to state 3 as the formal sector.

To begin our analysis, note that we can directly apply the analysis from Section 2 to state $w = 2$. Moreover, we can extend Proposition 3 to state $w = 1$. Specifically, in equilibrium the borrower receives a restrictive contract in state $w = 1$ with probability 1 if and only if

$$\underbrace{\beta_1\left(\left(B_2^* + L_2^*\right) - \frac{y_1 - \kappa}{1-\delta}\right)}_{\substack{\text{"efficiency gain"} \\ \text{from investing in fixed capital}}} \leq \underbrace{\hat{B}_1 - \tilde{B}_1}_{\substack{\text{"expansion rent"} \\ \text{captured by the borrower}}} . \tag{2}$$

where $\beta_w \equiv \frac{g_w}{1-\delta(1-g_w)}$. The model can be solved via backward induction: the equilibrium can first be characterized in state $w = 2$ just as it was in Section 2; then, as a function of continuation values in state $w = 2$, the equilibrium can be analytically characterized in state $w = 1$.

The 3-state model features a richer manifestation of the hold-up problem. In the 2-state model the lender's state 1 value from unrestrictive contracts was limited by the speed g_1 at which the borrower grows his business when offered unrestrictive contracts. In the 3-state model the lender's value from unrestrictive contracts is also determined by her value L_2^* in state 2. In the final state the lender's value is still 0. However, in state 2 the lender's value is in part a reflection of the borrower's state 2 bargaining position. Thus because of the hold-up problem, increasing the borrower's state 2 bargaining position may reduce the lender's ability to extract rents through unrestrictive contracts in state 1. We will see in Proposition 12 that this is sometimes, but not always the case.

In general the borrower may receive restrictive contracts in either or both of the first two states. Propositions 3 and 7 continue to offer guidance as to when the borrower is likely to be captive in state $w = 1$ or $w = 2$ (or both, or neither).

[9] In state w the borrower's incentive compatibility constraint is $R \leq P + \delta\left(B_w - B_1\right)$. If $\delta\left(B_w - B_1\right) \geq \kappa$ then the formal lender can lend the borrower and lender can only demand $\kappa \leq y_1$. Otherwise our strengthening of Assumption 2 guarantees that $R \leq P + \delta\left(B_w - B_1\right) < P + \kappa \leq y_1$.

However, while Proposition 8 continues to characterize the comparative statics of the equilibrium in state 2 with respect to the borrower's formal sector value $B_3^* = \frac{y_3 - \kappa}{1-\delta}$, the comparative statics of the equilibrium in state 1 with respect to B_3^* are markedly different. Let p_w^* be the equilibrium probability of a restrictive contract in state w.

Proposition 12. *When the lender offers restrictive contracts in state 2, increasing B_3^* makes restrictive contracts more likely in state 1. Otherwise, increasing B_3^* makes unrestrictive contracts more likely in state 1. Formally,*

$$dp_1^*/dB_3^* \begin{cases} \geq 0 & \text{if } p_2^* = 1, \\ \leq 0 & \text{if } p_2^* < 1. \end{cases}$$

The inequalities are strict if $p_1^* \in (0, 1)$.

We know from Proposition 8 that increasing B_3^* always makes unrestrictive contracts more likely in state 2. However, Proposition 12 highlights that the comparative statics in state 1 now hinge on the equilibrium contract offered in state 2.

If $p_2^* = 1$, then increasing B_3^* always makes restrictive contracts more likely in state 1. This can be understood with reference to Inequality (2). The right hand side of the inequality—the borrower's expansion rent—is increasing in B_3^*. The left-hand side of the inequality—the social efficiency gain from unrestrictive contracts—is constant in B_3^*. To see this, note that because $p_2^* = 1$, the borrower is captive in state 2 and will never reach the formal sector on the equilibrium path. Therefore, in state 1 the social efficiency gain of unrestrictive contracts is unaffected by increasing B_3^*, as the state 2 joint surplus is fixed. So, while increasing B_3^* decreases the likelihood of borrower captivity in state 2, when $p_2^* = 1$ the very same force increases the likelihood of captivity in state 1.

Another way to understand this phenomenon is that increasing B_3^* increases the borrower's bargaining position in state 2. Because the lender offers the borrower a restrictive contract in state 2, strengthening the borrower's bargaining position results in a transfer from the lender to the borrower in the form of a lower repayment amount. From the perspective of state 1 this has two consequences. First, the borrower's improved state 2 bargaining position improves the borrower's state 1 bargaining position, by increasing the value of growing at the autarkic rate. This makes restrictive contracts less attractive to the lender. Second, because of the hold-up problem, the lender anticipates that unrestrictive contracts become less attractive, as the borrower cannot commit not to exercise his improved bargaining position in state 2. The preceding discussion implies that when $p_2^* = 1$ the latter force always dominates and the lender shifts towards restrictive contracts in state 1.

Conversely, when $p_2^* < 1$—the borrower receives unrestrictive contracts with positive probability in state 2—increasing B_3^* always reduces the probability of contractual restrictions *in both states.* As in the above case, increasing B_3^* increases B_2^* and therefore strengthens the borrower's bargaining position in state 1. This makes restrictive contracts less attractive to the lender in state 1. However the lender's payoff from unrestrictive contracts in state 1 is unaffected. To see this, note that the lender has a weak preference to offer unrestrictive contracts in state 2, and unrestrictive contracts exceed the borrower's outside option. Hence increasing the borrower's state 2 bargaining position does not reduce the lender's state 2 payoff. Therefore, when $p_2^* < 1$, increasing B_3^* causes the lender to shift toward unrestrictive contracts in state 1.[10]

The preceding discussion implies that when $p_2^* = 1$, increasing the attractiveness of the formal sector can cause a Pareto disimprovement as the welfare of both parties in state 1 diminishes due to the hold-up problem. Therefore, policies that improve the benefits of graduating from microfinance and formal lending may backfire.

Before proceeding to the next result it is worth contrasting Proposition 12 with the result of Petersen and Rajan (1995) (henceforth PR), who also study financing with limited commitment and show that improving borrower's future outside option—modeled through intensified competition from other lenders—can make the borrower worse off. The result of PR follows from the fact that early-stage financiers may not be able to recoup the initial costs of lending if they must operate in a competitive market once their borrowers' businesses have grown. In our model, the lender is motivated by prolonging the time over which she can extract rents from her borrower. This difference manifests itself in two key ways. First, in our model, borrowers always reach the formal sector under autarky; yet, in repeated lending relationships, lenders hold borrowers captive—through contractual restrictions that borrowers willingly accept—in order to prolong rent extraction. In contrast, borrowers in PR are only "trapped"—and their projects unfinanced—because the lender may refuse to lend. In other words, borrowers are trapped because of the *presence of lending* in our model and are trapped because of the *lack of lending* in PR.

A second conceptual distinction is that the severity of the poverty trap in our model depends crucially on borrower's bargaining position. In PR, intensifying future competition always worsens the initial financial friction. In contrast, in our model, once the formal sector becomes sufficiently attractive (so much so that the lender offers unrestrictive contracts in state 2, as is dictated by Proposition 8), then further increases in formal sector attractiveness always reduce the likelihood of captivity in state 1 and speed growth. Intuitively, once the borrower's state 2 bargaining position becomes sufficiently strong, the lender can no longer extract substantial rents by keeping the state 2 borrower in captivity. At this point, increasing the formal sector attractiveness improves the state 1 borrower's bargaining position without influencing the amount of rent the lender can extract from unrestrictive contracts, and so only serves to help the state 1 borrower.

[10] We note that Proposition 8 implies that the comparative statics of p_1^* with respect to B_3^* are non-monotone. When $p_2^* = 1$, increasing B_3^* increases p_1^*. However, once B_3^* becomes sufficiently large, p_2^* drops below 1, at which point p_1^* decreases in B_3^*.

Borrower Patience May Worsen Captivity. Our next result concerns the role of the borrower's patience in determining his captivity. When the equilibrium is such that the borrower receives a restrictive contract in state 1 it can be understood as an endogenous poverty trap. Borrowers that start in state 1 never grow out of it, despite finding it worthwhile to grow their business if they reach a larger state. The standard intuition about poverty traps is that they are driven by impatience, and that sufficiently patient agents never succumb to them (e.g. Azariadis (1996)). However, this is a model in which increasing the borrower's patience can make the poverty trap in state 1 more likely. Let δ^B denote the borrower's patience, and δ^L denote the lender's patience.

Proposition 13. *Increasing the borrower's patience δ^B can increase the likelihood of restrictive contracts in state* 1.

The intuition underlying Proposition 13 resembles that of Proposition 12. Increasing the borrower's patience increases his value for investing in fixed capital and growing his business. This in turn increases his state 2 bargaining position and decreases the repayment amount he is willing to accept for restrictive contracts in state 2. Just as in Proposition 12 because of the hold-up problem this can increase the likelihood of captivity (and hence a poverty trap) in state 1.

Captivity Arises Because of Repeated Interactions. Our final result of the section concerns the role of repetition in inducing borrower captivity. Borrower captivity arises not because of *lending* per se—having access to external funds always expands production sets—but because of the *repeated* nature of lending relationships. The lender has an incentive to trap the borrower only because of future rents, and the trap completely disappears if the relationship is short-lived. This result provides a counterpoint to the standard intuition that repeated interaction facilitates cooperation and efficient contracting (e.g. Tirole, 2010). Our model highlights that under motives to maintain dynamic rents, repeated interaction strictly lowers welfare.

Proposition 14. *If the borrower and lender only interact once, the lender always offers unrestrictive contracts.*

The borrower always accepts a higher repayment amount for unrestrictive contracts than for restrictive contracts. Therefore, a lender who anticipates no future relationship always myopically prefers to offer unrestrictive contracts and help her borrower grow. It is only because of the possibility of rent extraction in the future that the lender might incur a short-term loss in order to keep her borrower captive. This result resembles, but also stands in contrast with results from the relationship banking literature, which dictate that lenders with long-standing relationships with their borrowers may have informational advantages and can use this to extract rents from their borrowers. While that literature highlights that repetition may lead to information rents, it is the information rent and not the repetition per se that leads to rent extraction. In contrast our model highlights that *repetition* may be the key factor in exacerbating the lender's rent extraction motive.

4 Concluding Remarks

We have presented a model to formalize the intuition that, because informal lenders may not be able to offset the costs of supporting borrower growth by extracting the benefits in the future, they may impose contractual restrictions that inhibit long-term, profitable investments.

Our simple theory is able to organize many of the established facts about microfinance. First, the model reconciles the seemingly inconsistent facts that small-scale entrepreneurs enjoy very high return to capital yet are unable to leverage microcredit and other forms of informal finance to realize those high returns, despite moderate interest rates charged by the lenders. In our model, firms that borrow from the informal lender may see their growth stalled, and remain in the relationship indefinitely, even though they would have continued to grow in the absence of a lender. Put simply, in this model, having access to a lender can *reduce* business growth.

While the experimental studies cited above find, on average, low marginal returns to credit, a number of them find considerable heterogeneity in returns to credit across borrowers of different size. In particular, they consistently find that treatment effects are higher for businesses that are more established. Our model sheds light on this heterogeneity, as the borrowers nearest to the formal sector receive unrestrictive contracts and grow faster than they could have in autarky.

One further puzzling fact in the microfinance literature is that, despite the fact that loan products carry low interest rates relative to the returns to capital, demand for microcredit contracts is low in a wide range of settings.[11] Our model offers a novel explanation. Despite low interest rates, contractual restrictions that impose constraints on business growth push borrowers exactly to their individual rationality constraints; these borrowers do not benefit at all from having access to informal lending. In Appendix A.3, we formalize low take-up rate of informal loans through a model extension in which the lender is incompletely informed about the borrower's outside option. While our argument is intuitive, it stands in sharp contrast to standard intuitions based on borrower-side financial constraints, which predict that credit constrained borrowers should have high demand for additional credit at the market interest rate.

Our theory also offers nuanced predictions about the lending relationship as the economic environment changes, illuminates a new force by which increasing the borrower's access to pledgeable collateral can harm his welfare, and yields several policy implications.

In addition to the theories cited in the introduction, it is worth contrasting our theory with two other classes of theories prominent in development economics. The first might sensibly be labeled "blaming the borrower." These theories allude to the argument that many borrowers are not natu-

[11] See e.g. Banerjee et al. (2014), and Banerjee et al. (2015).

ral entrepreneurs and are primarily self-employed due to a scarcity of steady wage work (see e.g., Schoar (2010)). While these theories have some empirical support, they are at best a partial explanation of the problem as they are inconsistent with the large impacts of cash grants, as cited in our introduction.

Second are the theories that "blame the lender" for not having worked out the right lending contract. These theories implicitly guide each of the experiments that evaluate local modifications to standard contracts.[12] While many of these papers contribute substantially to our understanding of how microfinance operates, none have so far generated a lasting impact on the models that MFIs employ. Therefore it seems unlikely that a lack of contract innovation is the sole constraining factor on the impact of microfinance.

Our theory, in contrast, assumes that borrowers have the competence to grow their business and that lenders are well aware of the constraints imposed on borrowers by the lending paradigm. Instead we focus on the rents that lenders enjoy from retaining customers and the fact that sufficiently wealthy customers are less reliant on their informal financiers. Therefore our theory may offer a useful, and very different lens with which to understand the disappointing impact of microfinance. For instance our theory suggests that policymakers might increase the impact of microfinance by regulating the types of contracts that lenders can offer, rather than by offering business training to borrowers, or consulting to lenders. Part of the value of this theory, therefore, may stem from the distance between its core logic and that of the primary theories maintained by empirical researchers and policymakers.

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[12] See e.g. Gine and Karlan (2014), Attanasio et al. (2015), and Carpena et al. (2013) on joint liability, Field et al. (2013) on repayment flexibility, and Feigenberg et al. (2013) on meeting frequency.

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A Model Extensions

A.1 The Borrower Can Renege on the Contractual Restriction

In this section we consider an extension of the two state model in which the borrower can renege on the contractual restriction $a = 1$ by choosing the fixed capital project subject to an adjustment cost of $\gamma > 0$. This adjustment cost can represent the fact that for the borrower to be offered a restrictive contract, he must make an irreversible investment, which costs γ in beginning the working capital project. This may be understood as the borrower demonstrating the feasibility of his intended project to his lender. If he switches to the fixed capital project, this cost γ is sunk and his payoff is therefore reduced by γ relative to if he had not started the working capital project. At the end of this section we consider the case in which $\gamma = 0$.

State 2 behavior and payoffs are exactly the same as in our baseline model. Therefore we focus on state 1. The borrower's individual rationality constraints are unchanged relative to the baseline model, as is his incentive compatibility constraint from an unrestrictive contract.

If the borrower reneges on a restrictive contract his payoff is

$$\max \left\{y_1 - \gamma - P + \delta \left(B_1^* + (1-g)\left(B_2^* - B_1^*\right)\right), y_1 - P + \delta B_1^*\right\}$$

where the first argument corresponds to the case where the borrower reneges on both the repayment and the contractual restriction and the second argument corresponds to the case where the borrower reneges on only the repayment. Therefore the borrower's IC constraint upon receiving restrictive contracts is now

$$R \leq \max \left\{\gamma + P - \delta \left((1-g)\left(B_2^* - B_1^*\right)\right), P\right\}$$

Critically, Lemma 1 holds without alteration. That is, the borrower enjoys an expansion rent if and only if IR does not bind for unrestrictive contracts. Therefore our analysis will proceed with only minor adjustments to the expressions.

If $\gamma = 0$ then the when the borrower reneges on the restrictive contract he always invests in fixed capital. His payoff is therefore

$$y_1 - P + \delta \left(B_1^* + (1-g)\left(B_2^* - B_1^*\right)\right),$$

which is the same as his payoff from accepting the unrestrictive contract and reneging on repayment. Therefore the borrower's payoff is the same regardless of which type of contract he is offered; that is, Lemma 1 no longer holds and there is no expansion rent. Therefore, when $\gamma = 0$ the lender always offers the borrower unrestrictive contracts in equilibrium.

A.2 The Borrower Can Operate Both Projects Within a Period

In this section we consider an extension to the two state model in which the borrower can operate both projects within the same period. We focus on state 1. Suppose the borrower is endowed with 1 unit of labor in each period, and can choose how to allocate his labor between his two projects. Let l be the labor allocated to the working capital project and $(1-l)$ be the labor allocated to the fixed capital project. Suppose that when $k \in \{0, \kappa\}$ capital is allocated to the working capital project it returns ly_1^{aut} or ly_1 output in the early portion of the period, and similarly when $(1-l)$ labor and k capital are allocated to the fixed capital project it returns $(1-l)\, y_1^{aut}$ or $(1-l)\, y_1$ output in the late portion of the period and $(1-l)\, g_1^{aut}$ or $(1-l)\, g_1$ probability of growth.

Suppose further that when the lender offers a restrictive contract $c = \langle R, 1 \rangle$, the borrower must invest enough labor and capital to repay R in the early period but then is free to invest the remainder of both into the fixed capital project.

It is straightforward to see that Inequality (1) will still determine when the borrower receives a restrictive contract and remains captive so long as

$$\max_l \frac{ly_1 + (1-l)\, y_1^{aut} - R}{1-\delta} \left(1 - \frac{(1-l)\, \delta g_1^{aut}}{1 - \delta\left(1 - (1-l)\, g_1^{aut}\right)}\right) + \frac{(1-l)\, \delta g_1^{aut}}{1 - \delta\left(1 - (1-l)\, g_1^{aut}\right)} B_2^*$$

such that

$$ly_1 \geq R$$

is less than

$$\frac{y_1 - R}{1-\delta}$$

Even if the borrower is free to allocate some fraction of his labor to the fixed capital project when he receives a restrictive contract, the fixed capital project is socially efficient, and in autarky he would invest in fixed capital, the borrower may still find it worthwhile to invest all of his labor into the working capital when he receives a restrictive contract in equilibrium. This occurs because capital and labor are complements, so when the repayment demands sufficiently high labor in the working capital project, the borrower may find he prefers to invest the residual of his labor into the working capital project and enjoy the consumption output rather than growing his business slowly.

A.3 The Borrower is Privately Informed About His Outside Option

In this section we explore an extension in which the borrower maintains some private information about his outside option. We augment the model such that the borrower's growth rates g_1^{aut} and g_1 are privately known. If he invests in fixed capital we assume he grows at rate $g_{1,\nu}^{aut} = g_1^{aut} + \nu$ or $g_{1,\nu} = g_1 + \nu$. Let $\nu_t \overset{\text{iid}}{\sim} F$ be a random variable, privately known to the borrower and redrawn each period in an iid manner from some distribution F.

Now, if the lender offers the borrower a restrictive contract, she will face a standard screening problem. Because she would like to extract the maximum acceptable amount of income, borrowers with unusually good outside options will reject her offer. This is encoded in the following proposition.

Proposition 15. *The borrower may reject restrictive contracts with positive probability.*

This intuitive result offers an explanation for the low take-up of microcredit contracts referenced in the introduction. Lenders who offer restrictive contracts to borrowers aim to extract the additional income generated by the loan, but in doing so lenders are sometimes too demanding and therefore fail to attract the borrower. In contrast, because lenders who offer unrestrictive contracts leave the borrower with excess surplus, demand for these contracts is high.

B Additional Interpretation Issues

In this section we provide additional discussions on how our modeling assumptions map to the institutional features of microfinance.

Non-profit MFIs

The debt trap in our model arises when a profit-maximizing lender prolongs the period over which she can extract rents from her borrower. Yet the low returns from microfinance have been observed across a range of microfinance institutions spanning both for-profit and non-profit business models. We believe there are a number of ways in which the forces identified in our model might similarly apply to non-profit MFIs. First, because the two business models share many practices, features that are adaptive for profit-maximizing MFIs may have been adopted by non-profits. A second possibility operates through the incentives of loan officers who are in charge of originating and monitoring loans. Across for-profit and non-profit MFIs many loan officers are rewarded for the number of loans they manage, so losing clients through graduation may not be in their self-interest. Put another way, even in non-profit MFIs, the loan officers often have incentives that make them look like profit-maximizing lenders. This interpretation is supported by Rigol and Roth (2021), which uses an experiment with a large Chilean MFI to demonstrate that standard features of loan officer compensation cause them to inhibit borrower graduation out of microfinance.

A final possibility is that, while non-profits may not aim to maximize rent extraction, they may still need to respect a break-even constraint. If microfinance institutions incur losses to serve their poorest borrowers, then they may need to inhibit their wealthier borrowers' ability to graduate in order to maintain profitability.

Infinite stream of borrowers

One crucial feature of our debt trap is that when the borrower becomes wealthy enough to leave his lender, the lender loses money. In reality there are many unserved potential clients in the communities in which MFIs operate. Why, then, can't an MFI offer unrestrictive contracts and then replace borrowers who have graduated with entirely new clients? The proximate answer is that unserved clients are unserved primarily because they have no demand for loans (e.g. Banerjee et al. (2014)). This may be unsatisfactory, as demand for microfinance would presumably increase if the lender lifted contractual restrictions and allowed the borrowers to invest more productively. But even in this case, the pool of potential borrowers who would find this appealing is likely limited. For instance, Banerjee et al. (2017) and Schoar (2010) argue that only a small fraction of small-scale entrepreneurs are equipped to put capital to productive use. Thus, it is reasonable to assume that MFIs lose money when a borrower terminates the relationship.

C Characterizing Equilibrium Value Functions

In this section we characterize the borrower and lender's equilibrium value functions, first assuming that the lender is constrained to only offer restrictive contracts and then only unrestrictive contracts. We use $\left(\tilde{B}_1, \tilde{L}_1, \tilde{R}_1\right)$ to denote, respectively, the borrower's and the informal lender's value functions and the requested repayment in state 1 in a hypothetical equilibrium if the informal lender were constrained to only offer restrictive contracts; likewise, we use $\left(\hat{B}_1, \hat{L}_1, \hat{R}_1\right)$ to denote the value functions and requested repayment in a hypothetical equilibrium if the informal lender were constrained to only offer unrestrictive contracts. We subsequently use this analysis to derive the borrower and lender's equilibrium behavior without the additional constraint on the lender.

We begin by computing the borrower's continuation value if he were to reject the informal lender's offer at every period in state 1, as this is his utility from exercising his outside option. In this case the borrower's state 1 continuation utility would be

$$\begin{aligned} B_1^{aut} &= y_1^{aut} + \delta\left(B_1^{aut} + g^{aut}\left(B_2 - B_1^{aut}\right)\right) \\ &= (1-\alpha)\frac{y_1^{aut}}{1-\delta} + \alpha B_2 \end{aligned}$$

where $\alpha \equiv \frac{\delta g^{aut}}{1-\delta(1-g^{aut})}$ is the expected fraction of the borrower's discounted lifetime that he spends in state 2 when he invests in fixed capital and grows his business at the slow autarkic rate g^{aut}. The borrower's autarky value in state 1 is therefore a weighted average between $\frac{y_1^{aut}}{1-\delta}$ and B_2, as the borrower spends an expected discounted fraction $(1-\alpha)$ of his life in state 1 and an expected discounted fraction α of his life in state 2.

If the Lender Offers Only Restrictive Contracts in State 1. First, if the lender offers a restrictive contract in equilibrium she must satisfy (IR 1-rest) and (IC 1-rest). Therefore the repayment amount she demands is

$$\tilde{R}_1 = \min\left\{y_1 - y_1^{aut} - \delta g^{aut}\left(B_2 - B_1\right), P\right\}$$

The borrower's state 1 continuation utility is then

$$\begin{aligned}\tilde{B}_1 &= y_1 - \tilde{R}_1 + \delta \tilde{B}_1 \\ &= \max\left\{B_1^{aut}, \frac{y_1 - P}{1-\delta}\right\}\end{aligned}$$

The first argument corresponds to the case when the borrower's individual rationality constraint binds. If the borrower receives a restrictive contract in every period then he never grows his business, but the repayment amount is set so that the borrower is indifferent between accepting his restrictive contract versus rejecting it and growing his business at the slow autarkic rate. The second argument corresponds to the case when the borrower's incentive compatibility constraint binds. When the informal lender's ability to punish the borrower is large (i.e. P is high), then the borrower's IR constraint binds and the first argument is the relevant one. Otherwise, the lender's IC constraint will bind at the second argument is relevant.

In the scenario that the lender only offers restrictive contracts in equilibrium, the lender's continuation value is

$$\tilde{L}_1 = \frac{\tilde{R}_1 - \kappa}{1-\delta} = \frac{y_1 - \kappa}{1-\delta} - \tilde{B}_1$$

where the second equality follows from the fact that $\tilde{B}_1 = \frac{y_1 - \tilde{R}_1}{(1-\delta)}$.

If the Lender Offers Only Unrestrictive Contracts in State 1. Next consider the case where the lender offers an unrestrictive contract in every period. The lender's offer must satisfy (IR 1-unrest) and (IC 1-unrest) so the repayment amount must satisfy

$$\hat{R}_1 = \min\left\{y_1 - y_1^{aut} + \delta\left(g - g^{aut}\right)\left(B_2 - B_1\right), P\right\}$$

The borrower's continuation value is

$$\begin{aligned}\hat{B}_1 &= y_1 - \hat{R}_1 + \delta\left(\hat{B}_1 + g\left(B_2 - \hat{B}_1\right)\right) \\ &= \max\left\{B_1^{aut}, (1-\beta)\frac{y_1 - P}{1-\delta} + \beta B_2\right\}\end{aligned}$$

where $\beta \equiv \frac{\delta g}{1-\delta(1-g)}$ is the expected fraction of the borrower's discounted lifetime that he spends in state 2 when he invests in fixed capital and grows his business at the fast rate g enabled by the lender's capital. The first argument corresponds to the case where the borrower's individual rationality constraint binds, whereas the second argument corresponds to the case where the borrower's incentive compatibility constraint binds. In the second case, the borrower's state 1 continuation value is a weighted average of his state 1 payoff $\frac{y_1-P}{1-\delta}$ and his state 2 payoff B_2 where the weights are determined by the speed at which he can grow to state 2 with the lender's capital.

The lender's continuation value is

$$\begin{aligned}\hat{L}_1 &= (1-\beta)\frac{\hat{R}_1-\kappa}{1-\delta} \\ &= (1-\beta)\frac{y_1-\kappa}{1-\delta}+\beta\frac{y_2-\kappa}{1-\delta}-\hat{B}_1\end{aligned}$$

The first equality follows from the fact that the lender expects to stay in state 1 for a fraction $(1-\beta)$ of her discounted lifetime, and to be in state 2 for a fraction β of her expected lifetime, at which point her continuation value is 0 as the borrower has entered a competitive credit market. The second equality comes from the fact that the lender's state 1 value can be computed by subtracting the borrower's state 1 value from their joint state 1 surplus, $(1-\beta)\frac{y_1-\kappa}{1-\delta}+\beta\frac{y_2-\kappa}{1-\delta}$.

D Omitted Proofs

This section contains all omitted proofs. Recall that $\alpha \equiv \frac{\delta g^{aut}}{1-\delta(1-g^{aut})}$, $\beta \equiv \frac{\delta g}{1-\delta(1-g)}$.

Proof of Proposition 1

First consider equilibrium behavior in state 2. The lender's state 2 value is

$$L_2^* = \min\left\{\frac{y_2-y_2^{aut}-\kappa}{1-\delta}, \frac{P+\delta(B_2-B_1)-\kappa}{1-\delta}\right\}$$

where the first argument is relevant if IR binds and the second is relevant if IC binds. First, suppose that IR binds, so that $L_2^*=\frac{y_2-y_2^{aut}-\kappa}{1-\delta}$.

Now consider equilibrium behavior in state 1. Let B_w^* be the borrower's equilibrium state w value. If the borrower rejects a contract then his outside option utility is

$$\bar{u} = y_1^{aut}+\delta\left(g_1^{aut}B_2^*+\left(1-g_1^{aut}\right)B_1^*\right)$$

We now consider three cases.

- Suppose in equilibrium that the borrower's IR constraint binds for both the optimal restrictive and unrestrictive contract. Then if the lender were to offer a restrictive contract in every period her payoff would be

$$\frac{y_1-\kappa}{1-\delta}-\bar{u}$$

while unrestrictive contracts give the lender

$$(1-\beta)\,\frac{y_1-\kappa}{1-\delta}+\beta\frac{y_2-\kappa}{1-\delta}-\bar{u}$$

In this case the lender would offer only unrestrictive contracts in equilibrium, by Assumption 1.

- Next suppose that in equilibrium the borrower's IC constraint binds for both contracts. Then the restrictive contract gives the lender a payoff of

$$\frac{P-\kappa}{1-\delta}$$

and the unrestrictive contract gives the lender

$$\frac{P-\kappa}{1-\delta}\,(1-\beta)+\beta L_2^*$$

If $L_2^*=\frac{y_2-y_2^{aut}-\kappa}{1-\delta}$, then the lender prefers to offer unrestrictive contracts. To see this note $y_2-y_2^{aut}>y_1-y_1^{aut}>P$, where the final inequality follows because the borrower's IC constraint binds for restrictive contracts. If $L_2^*=\frac{P+\delta(B_2-B_1)-\kappa}{1-\delta}$, then the lender clearly prefers unrestrictive contracts.

- The final case is that in equilibrium IR binds for the optimal restrictive contract, and IC binds for the optimal unrestrictive contract. The restrictive contract gives lender

$$\frac{R^{rest}-\kappa}{1-\delta}$$

for some R^{rest} that pushes the borrower to his IR constraint. The optimal unrestrictive contract gives the lender

$$\frac{P-\kappa}{1-\delta}\,(1-\beta)+\beta L_2^*$$

First observe that $P>R^{rest}$ by the fact that IR binds for the restrictive contract. If $L_2^*=\frac{y_2-y_2^{aut}-\kappa}{1-\delta}$, then the lender prefers unrestrictive contracts because $y_2-y_2^{aut}-\kappa>y_1-y_1^{aut}-\kappa>R^{rest}$ where the last inequality follows by the fact that the IR constraint binds for restrictive contracts.

If $L_2^*=\frac{P+\delta(B_2-B_1)-\kappa}{1-\delta}$ then the lender clearly prefers unrestrictive contracts. □

Proof of Propositions 2 and 3

The existence of an equilibrium follows standard arguments (see Maskin and Tirole (2001)). In this section we prove that generically the equilibrium is unique.

First consider the case that in equilibrium the borrower's IR constraint binds for both the optimal restrictive and the optimal unrestrictive contracts. Then, letting $\bar{u}$ be the borrower's utility if he were to take his outside option, the lender's state 1 value from offering restrictive contracts is

$$\frac{y_1 - \kappa}{1-\delta} - \bar{u}$$

while unrestrictive contracts give the lender

$$(1-\beta)\frac{y_1 - \kappa}{1-\delta} + \beta\frac{y_2 - \kappa}{1-\delta} - \bar{u}$$

In this case the lender would offer only unrestrictive contracts in equilibrium, by Assumption 1.

Next consider the case that in equilibrium the borrower's IC constraint binds for both the optimal restrictive and the optimal unrestrictive contracts. Then the lender's state 1 value from offering restrictive contracts is

$$\frac{P-\kappa}{1-\delta}$$

and from unrestrictive contracts is

$$(1-\beta)\frac{P-\kappa}{1-\delta}.$$

In this case the lender offers only restrictive contracts in equilibrium.

The final case is that in equilibrium the borrower's IR constraint binds for the optimal restrictive contract and his IC constraint binds for the optimal unrestrictive contract. In this case, we solve for the unique $p^* \in [0,1]$ representing the probability the lender offers a restrictive contract.

The borrower's state 1 value in equilibrium satisfies

$$\begin{aligned}
B_1^* &= p\left(y_1^{aut} + \delta\left(B_1^* + g_1^{aut}\left(B_2 - B_1^*\right)\right)\right) + (1-p)\left(y_1 - P + \delta\left(B_1^* + g_1\left(B_2 - B_1^*\right)\right)\right) \\
&= p\left(\tilde{B}_1 + \delta\left(1-g_1^{aut}\right)\left(B_1^* - \tilde{B}_1\right)\right) + (1-p)\left(\hat{B}_1 + \delta\left(1-g_1\right)\left(B_1^* - \hat{B}_1\right)\right) \\
&= \frac{p\left(1-\delta\left(1-g_1^{aut}\right)\right)\tilde{B}_1 + (1-p)\left(1-\delta\left(1-g_1\right)\right)\hat{B}_1}{1-p\delta\left(1-g_1^{aut}\right) - (1-p)\,\delta\left(1-g_1\right)} \\
&= s\tilde{B}_1 + (1-s)\,\hat{B}_1
\end{aligned}$$

where

$$s \equiv \frac{p\left(1-\delta\left(1-g_1^{aut}\right)\right)}{1-p\delta\left(1-g_1^{aut}\right) - (1-p)\,\delta\left(1-g_1\right)}$$

The lender's payoff from offering unrestrictive contracts is

$$\hat{L}_1 = (1-\beta)\,\frac{\hat{R}_1-\kappa}{1-\delta} = (1-\beta)\,\frac{P-\kappa}{1-\delta}$$

To solve for the lender's payoff upon offering a restrictive contract, note that the corresponding repayment amount $\tilde{R}_1(p)$ satisfies

$$\begin{aligned} y_1 - \tilde{R}_1(p) + \delta B_1^*(p) &= y_1^{aut} + \delta\left(\left(1-g_1^{aut}\right)B_1^*(p) + g_1^{aut}B_2^*\right) \\ \tilde{R}_1(p) &= y_1 - y_1^{aut} - \delta g_1^{aut}\left(B_2^* - B_1^*(p)\right) \\ &= y_1 - (1-\delta)\,\tilde{B}_1 + \delta g_1^{aut}\left(B_1^* - \tilde{B}_1\right) \end{aligned}$$

Therefore

$$\begin{aligned} L_1^*(p) &= \frac{\tilde{R}_1(p)-\kappa}{1-\delta} \\ &= \tilde{L}_1 + \frac{\delta}{1-\delta}g_1^{aut}(1-s)\left(\hat{B}_1 - \tilde{B}_1\right) \end{aligned}$$

When $p \in (0,1)$ it solves

$$\hat{L}_1 = L_1^*(p)$$

so $s = 1 - \frac{(1-\delta)\left(\hat{L}_1-\tilde{L}_1\right)}{\delta g^{aut}\left(\hat{B}_1-\tilde{B}_1\right)}$. We can now solve for p^*.

$$\begin{aligned} p^* &= \frac{s\left(1-\delta\left(1-g_1\right)\right)}{(1-s)\left(1-\delta\left(1-g_1^{aut}\right)\right) + s\left(1-\delta\left(1-g_1\right)\right)} \\ &= \frac{\left(\delta g_1^{aut}\left(\hat{B}_1-\tilde{B}_1\right) - (1-\delta)\left(\hat{L}_1-\tilde{L}_1\right)\right)\left(1-\delta\left(1-g_1\right)\right)}{(1-\delta)\left(\hat{L}_1-\tilde{L}_1\right)\left(1-\delta\left(1-g_1^{aut}\right)\right) + \left(\delta g_1^{aut}\left(\hat{B}_1-\tilde{B}_1\right) - (1-\delta)\left(\hat{L}_1-\tilde{L}_1\right)\right)\left(1-\delta\left(1-g_1\right)\right)} \end{aligned}$$

When there is no p for which $\hat{L}_1 = L_1^*(p)$ then either $L_1^*(1) > \hat{L}_1$ in which case $p^* = 1$ or $L_1^*(0) < \hat{L}_1$ in which case $p^* = 0$.

The proof can trivially be extended to the three state model via backward induction. □

Proof of Proposition 4

When there is no informal lender, the borrower never receives a loan in state 1, as the formal lender cannot enforce repayment without collateral. Therefore in state 1 the borrower is in autarky and chooses flexibly between his working and fixed capital projects. Assumption 1 guarantees that he invests in fixed capital and reaches state 2 in finite time.

In contrast, when there is both a formal and informal lender, our analysis in Section 2.2 indicates that the borrower may receive a restrictive contract in state 1 and never reach 2. Specifically this is the case when Inequality 1 is satisfied. Therefore introducing the informal lender to the model may reduce the borrower's business growth. □

Proof of Proposition 5

We analyze the case where there is no formal lender in Section 2.1 and demonstrate that the borrower always receives unrestrictive contracts and reaches state 2 in finite time. In contrast, we analyze the case where both lenders are present in Section 2.2 and show that when Inequality 1 is satisfied the borrower always receives restrictive contracts and never grows his business out of state 1. Therefore introducing the formal lender to the model may reduce the borrower's business growth. □

Proof of Proposition 6

We construct an example to demonstrate that introducing the formal lender can reduce the borrower's welfare. Consider the case where $y_1^{aut} = g_1^{aut} = 0$, and $y_2^{aut} = \varepsilon$ for ε arbitrarily close to 0, and $P < y_1$. Then using the analysis from Section 2.1 it is straightforward to show that when there is only an informal lender the borrower's state 1 equilibrium payoff is $(1-\beta)\frac{y_1-P}{1-\delta} + \beta\frac{y_2-P}{1-\delta}$. In contrast, when there is both a formal and informal lender, and where the formal lender operates in state 2, the borrower will always receive restrictive contracts. His state 1 equilibrium payoff is then $\tilde{B}_1 = \frac{y_1-P}{1-\delta} < (1-\beta)\frac{y_1-P}{1-\delta} + \beta\frac{y_2-P}{1-\delta}$. Therefore the borrower's state 1 welfare is lower in the presence of formal lender than in her absence.

That introducing the informal lender can never harm the borrower's welfare relative to the informal lender's absence is an immediate consequence of the fact that the lender must satisfy the borrower's individual rationality constraint. □

Proof of Propositions 7 and 8

When p^* is interior it is defined by

$$p^* = \frac{\left(\delta g_1^{aut}\left(\hat{B}_1 - \tilde{B}_1\right) - (1-\delta)\left(\hat{L}_1 - \tilde{L}_1\right)\right)(1-\delta(1-g_1))}{(1-\delta)\left(\hat{L}_1 - \tilde{L}_1\right)(1-\delta(1-g_1^{aut})) + \left(\delta g_1^{aut}\left(\hat{B}_1 - \tilde{B}_1\right) - (1-\delta)\left(\hat{L}_1 - \tilde{L}_1\right)\right)(1-\delta(1-g_1))}$$

The comparative statics with respect to g_1^{aut}, g_1 and B_2^* can be verified directly. To see the result regading $\bar{g}$, recall that when the lender offers restrictive contracts with probability 1, her value is

$$\tilde{L}_1 = \frac{y_1 - \kappa}{1-\delta} - \tilde{B}_1.$$

For g_1^{aut} sufficiently large, $\tilde{L} < 0$. In contrast, $\hat{L} > 0$ for all values of g_1^{aut} and g_1. Hence when both are large the lender never offers restrictive contracts with probability 1. □

Proof of Propositions 9

When P is sufficiently large, the borrower's individual rationality constraint binds for both restrictive and unrestrictive contracts in state 1. Therefore, the lender's equilibrium payoff from offering an unrestrictive contract in state 1 is

$$\begin{aligned}\hat{L}_1 &= (1-\beta)\frac{y_1-\kappa}{1-\delta} + \beta\frac{y_2-\kappa}{1-\delta} - \hat{B}_1 \\ &= (1-\beta)\frac{y_1-\kappa}{1-\delta} + \beta\frac{y_2-\kappa}{1-\delta} - B_1^{aut}\end{aligned}$$

And, borrowing notation from the proof of Proposition 3, the lender's equilibrium payoff from offering a restrictive contract when the borrower expects one with probability p is

$$\begin{aligned}L_1^*(p) &= \frac{\tilde{R}_1(p)-\kappa}{1-\delta} = \tilde{L}_1 + \frac{\delta}{1-\delta}g_1^{aut}(1-s)\left(\hat{B}_1 - \tilde{B}_1\right) \\ &= \tilde{L} \\ &= \frac{y_1-\kappa}{1-\delta} - B_1^{aut}\end{aligned}$$

where the second equality follows from the fact that $\hat{B}_1 = \tilde{B}_1$ so long as the individual rationality constraint binds for both restrictive and unrestrictive contracts. By Assumption 1, $\hat{L}_1 > L_1^*(p)$ for all p, so the lender always offers unrestrictive contracts with probability 1. □

Proof of Proposition 10

In state 2 the repayment cap has no impact because the market is competitive and both lenders charge $R = \kappa$. In state 1, if the repayment cap $\bar{R}$ does not bind for either contract then it has no impact on the probability of a restrictive contract. If it binds for both the optimal restrictive and unrestrictive contracts, then the lender offers restrictive contracts with probability 1. Else the repayment cap only binds for unrestrictive contracts. In this case, reducing the repayment cap makes unrestrictive contracts less attractive without changing the lender's payoff to restrictive contracts, which makes restrictive contracts weakly more likely (and strictly so if the lender is following a mixed strategy).

Proof of Proposition 11

In state 2 the subsidy is passed through to the borrower and both lenders charge a repayment amount $R = \kappa - s$. Therefore the subsidy has no influence on the informal lender's state 2 value. In state 1, the informal lender is a monopolist so the subsidy has no impact on the repayment amount she charges. Therefore her payoff to unrestrictive contracts is

$$\hat{L}_1 = (1-\beta)\,\frac{\hat{R}_1 + s - \kappa}{1-\delta}$$

and her payoff to restrictive contracts is

$$\tilde{L}_1\left(p^*\right) = \frac{\tilde{R}_1\left(p^*\right) + s - \kappa}{1-\delta}$$

where $\tilde{L}_1(p)$ and $\tilde{R}_1(p)$ are defined as in the proof of Proposition 3. Clearly $\tilde{L}_1(p^*)$ increases in s faster than does $\hat{L}_1$, so increasing s makes restrictive contracts weakly more likely and strictly so for interior p^*.

Proof of Proposition 12

When p_1^* is interior, it is implicitly defined by the following equation, which sets the lender's equilibrium value from unrestrictive contracts, $\hat{L}_1$, equal to her value from offering restrictive contracts with probability p_1^* (recall when p_1^* is interior, the borrower's IR constraint binds for restrictive contracts, and his IC constraint binds for restrictive contracts).

$$(1-\beta_1)\,\frac{P-\kappa}{1-\delta} + \beta_1 L_2^* = \tilde{L}_1 + \frac{\delta}{1-\delta} g_1^{aut}\left(1-s_1\right)\left(\hat{B}_1 - \tilde{B}_1\right) \tag{3}$$

First suppose $p_2^* = 1$, so that

$$\begin{aligned} L_2^* &= \tilde{L}_2 \\ &= \frac{y_2-\kappa}{1-\delta} - \tilde{B}_2 \end{aligned}$$

Clearly $\frac{d}{dB_3^*}L_2^* \leq 0$ and $\frac{d}{dB_3^*}B_2^* = \frac{d}{dB_3^*}\tilde{B}_2 \geq 0$. Moreover, because the lender offers a restrictive contract with probability 1 in state 2, increasing B_3^* does not change the sum of $L_2^* + B_2^*$. Hence, $\frac{d}{dB_3^*}L_2^* = -\frac{d}{dB_3^*}B_2^*$. Rearranging Equation (3) we have

$$(1-\beta_1)\,\frac{P-\kappa}{1-\delta} - \frac{y_1-\kappa}{1-\delta} + (1-\alpha)\,\frac{y_1^{aut}}{1-\delta} = -\left(\alpha_1 B_2^* + \beta_1 L_2^*\right) + \frac{\delta}{1-\delta} g_1^{aut}\left(1-s_1\right)\left(\hat{B}_1 - \tilde{B}_1\right)$$

where we used the fact that IR binds for the case where the borrower expects restrictive contracts with probability 1 (or else the lender would offer restrictive contracts with probability 1). The left-hand side is invariant in B_3^* and the right-hand side is increasing in B_3^*. Hence $\frac{dp_1^*}{dB_3^*} \geq 0$.

Now consider the case where $p_2^* < 1$. Then $L_2^* = \hat{L}_2$ is weakly increasing in B_3^*. Revisiting the equation

$$\hat{L}_1 = \tilde{L}_1(p)$$

we see that p_1^* must adjust to lower $\tilde{R}_1(p)$, which implies that $\frac{dp_1^*}{dB_3^*} \leq 0$. □

Proof of Proposition 13

Consider an example in which $p_2^* = 1$ and the lender has a strict preference for restrictive contracts, $p_1^* = 1$ and in state 1 the lender is indifferent between the two contracts, and $\delta^L = \delta^B$.

It is straightforward to show that the lender's indifference condition in state 1 implies that

$$\beta_1^B B_2^* + \beta_1^L L_2^* - \beta_1^L \frac{P-\kappa}{1-\delta^L} = \beta_1^B B_2^* + \frac{y_1 - P - y_1^{aut}\frac{1-\delta^L}{1-\delta^B}}{1-\delta^L} - \alpha_1^B\left(B_2^* - \frac{y_1^{aut}}{1-\delta^B}\right) \tag{4}$$

where $\beta_w^L = \frac{\delta g_w}{1-\delta^L(1-g_w)}$, and β_w^B and α_w^B are defined similarly.

Consider first the thought experiment of raising the borrower's patience (increasing δ^B) in state 2 but holding it fixed in state 1. This makes the borrower more demanding of restrictive contracts in equilibrium as he values growth more highly, and therefore has the same consequence as increasing B_3^*. So just as in Proposition 12, this pushes the lender to offer more restrictive contracts in state 1. This can be verified with reference to the above equation by noting that the left hand side grows more slowly than the right hand side whe incresaing the borrower's future patience, which implies that p_1^* must increase to compensate.

Now consider the thought experiment of raising the borrower's patience in state 1 but holding it fixed in state 2. This has the reverse consequence of raising the borrower's value of business growth in state 1 and increasing the lender's preference for unrestrictive contracts.

Examination of Equation (4) demonstrates that this latter force is second order when δ^B and δ^L are small. Hence increasing δ^B can on net increase the probability of restrictive contracts in equilibrium. □

Proof of Proposition 14

This is a straightforward consequence of the fact that the lender's flow payoff from unrestrictive contracts is always weakly higher than her flow payoff from restrictive contracts. □

Proof of Proposition 15

Let the distribution F be such that $\nu = 0$ with probability $1-\varepsilon$ and $\nu = \frac{1}{\varepsilon}$ with probability ε. Fix all parameters such that when $\varepsilon = 0$ the lender offers restrictive contracts with probability 1. Then fixing all other parameters and for ε sufficiently low, if the lender continues to offer restrictive contracts with probability 1. Moreover, for ε sufficiently low, the lender cannot afford to satisfy the borrower's IR constraint when $\nu = \frac{1}{\varepsilon}$. Hence the borrower rejects the restrictive contract with probability ε on the equilibrium path. $\square$

E Construction of a Non-Markov Subgame Perfect Equilibrium

While our model admits a unique Markov perfect equilibrium (MPE) conditional on the formal lender operating in state 2, there are other non-Markov subgame perfect equilibria (SPE) as well. These equilibria rely on the stochastic nature of the fixed capital project. Specifically, in this section we study an example with a unique MPE in which the borrower is captive in state 1. However there is also an SPE in which the lender lets the borrower grow in state 1 in exchange for a higher repayment amount than can be sustained by the threat of the punishment P. The borrower is willing to make this repayment because he knows that there is a positive probability he will remain in state 1 in the next period, and will value the lender's continued cooperation. If the borrower succeeds in growing, and graduates to the formal sector then it is no longer incentive compatible for the borrower to share any surplus with the informal lender, as he has entered a competitive lending market. Therefore this equilibrium construction would not work if growth was deterministic.

We now formally construct our example. Consider the two state model of Section 1. We study an example where in the unique MPE the formal lender operates in state 2, and in state 1 for the informal lender holds the borrower captive (i.e. offers restrictive contracts with probability 1). To ensure this, we fix all state 1 parameters other than P and g_1^{aut} at arbitrary values satisfying our Assumptions 1 and 2, we choose state 2 parameters such that the borrower's equilibrium state 2 value is $B_2^* = \frac{1}{\varepsilon}$ (and we set y_2^{aut} sufficiently high to ensure that the collateral is valuable enough that the formal lender operates in state 2 in MPE) , we set the punishment $P = \kappa + \varepsilon$, and $g_1^{aut} = \varepsilon^2$, and we take ε to be small.

For ε sufficiently small this assures that the MPE involves borrower captivity in state 1 and that his incentive compatibility constraint binds. Every period the lender offers the borrower a restrictive contract with repayment $R = P$. The borrower's MPE value is

$$B_1^* = \frac{y_1 - P}{1 - \delta}$$

and the lender's MPE value in state 1 is

$$L_1^* = \frac{P - \kappa}{1 - \delta} = \frac{\varepsilon}{1 - \delta}$$

Now we construct an SPE in which the lender does not hold the borrower captive in state 1. In the "cooperative phase" the lender always offers the borrower an unrestrictive contract with repayment $R = y_1 > P$, and the borrower always accepts it and repays her loan. If either party deviates from this behavior then both parties revert to MPE behavior. In state 2 all parties follow MPE behavior.

To see that this is an SPE, note that the borrower's state 1 value in this SPE is

$$B_1^S = \beta B_2^* > B_1^*$$

where $\beta \equiv \frac{\delta g}{1-\delta(1-g)}$. The lender's state 1 value in this SPE is

$$L_1^S = \frac{y_1 - \kappa}{1 - \delta} (1 - \beta) > L_1^*$$

If the borrower reneges on repayment then his payoff is

$$y_1 - P + \delta \left((1 - g) B_1^* + g B_2^* \right)$$

as he enjoys the full output of his consumption good y_1, he suffers a punishment disutility P, and then next period if he does not succeed in growing he reverts to his MPE payoff. The borrower's incentive compatibility constraint is therefore

$$\begin{aligned} y_1 - P + \delta \left((1 - g) B_1^* + g B_2^* \right) &\le \delta \left((1 - g) B_1^S + g B_2^* \right) \\ y_1 - P &\le \delta (1 - g) \left(\beta B_2^* - \frac{y_1 - P}{1 - \delta} \right) \end{aligned}$$

which is satisfied for sufficiently small ε.

The lender's payoff from offering an unrestrictive contract (i.e. abiding by his dictated behavior in the cooperative phase) is L_1^S, and his payoff from offering a restrictive contract is L_1^* as the borrower would renege on any restrictive contract with repayment $R > P$.

Therefore, the lender's incentive compatibility condition is that

$$L_1^S > L_1^*$$

which is assured for sufficiently small ε.

This completes the description and verification of the equilibrium.