The Allocation of Socially Responsible Capital

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Abstract

Portfolio allocation decisions increasingly incorporate social values. We develop a tractable framework to study how competition between investors to own socially valuable assets affects social welfare. Relative to the most common social-investing strategies, we identify alternative strategies that result in higher impact and higher financial returns. From the firm perspective, increasing profitability can have a greater impact than directly increasing social value. Whether investors and firms exhibit positive or negative assortative matching depends on the nature of social preferences. We present empirical evidence that socially-guided mutual funds allocate their capital inefficiently from the perspective of generating impact and financial returns.

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1 Introduction

The last decade has seen an invigoration of investing in companies that rank favorably on metrics of social value, such as environmental stewardship, social responsibility, and good governance practices (collectively referred to as ESG). This shift in investment strategies has the potential alter the allocation of capital in the economy. In fact, many argue the entire purpose of this movement is to help reallocate resources to socially beneficial uses and away from socially harmful ones. It is thus centrally important to understand whether and how this style of investing generates its intended impact. This paper develops a theoretical framework to explore how investing with social convictions results in the creation of social value, and illustrates how competition between investors plays an important role.

We focus on strategies based on portfolio composition rather than engagement. The most common of such strategies in practice are constructed with attention to the financial returns and the social value of the companies included in an investor's portfolio. For example, ESG index funds attempt to track the returns of a benchmark index while maximizing some composite measure of the social good of the companies in the portfolio. Proponents of such "values-aligned" investing claim that they increase the valuation of (or equivalently decreases the cost of capital for) economic endeavors that contribute the most positively to society. This in turn shifts the set of projects that markets will finance towards those that create social value and away from those that destroy it.

We argue that the folk wisdom justifying values-aligned investing is misguided, and such investment strategies are an inefficient way to use asset allocation decisions to create social value. As the prevalence of socially-motivated investing grows, investors will invariably compete for the right to invest in, or own securities issued by, more socially valuable firms. We show this competition can limit investor impact and make it more expensive.

The basic logic is captured by the following example. Suppose there is a commercial capital market willing to supply financing for a return of 5%. There are also two *social*

investors, who care about both financial returns and social value creation. Suppose further that there are three firms, each of which needs one unit of capital to operate:

- Firm A generates a 6% profit and 10 units of social value.
- Firm B generates a 6% profit and 12 units of social value.
- Firm C generates a 4% profit and 9 units of social value.

Investors finance companies in exchange for a share of their profits. Imagine the social investors employ *values-aligned* strategies, meaning they make investment decisions based on the social value created by the company in which they are investing and the financial return they will receive. Specifically, investors allocate their capital to maximize the sum of their financial return and the social value of the firm they finance. The social investors will thus compete to finance Firms A and B. Firm B will receive capital at a lower cost than Firm A because it generates more social value. Firm A is financed by social investors at a cost of capital of 5%, the cost of the firm's outside option in the commercial capital market. Firm B is financed by social investors at a cost of capital of 3%, so that the benefits to social investors of investing in A and B are equalized. Firm C is unfinanced and thus does not generate social value.

In this example the social investors have failed to generate social impact. In their absence, the equilibrium allocation of capital would be unchanged, as Firms A and B could be commercially financed. Competition between commercial and social investors *displaced* commercial investors and thus limited social investor impact. Further, competition *between* the social investors lowered the returns to the social investor in Firm B even though this investor had no social impact.

Now imagine the investor financing Firm B were to deviate and instead finance Firm C. Facing no competition from other social or commercial investors, this investor could earn a return of 4%, an improvement over the return they received financing Firm B. And, since

investing in Firm C does not displace a commercial investor, all three firms would be funded and total social value would increase. That is, there exists a deviation from a values-aligned investing strategy that increases not only impact but also financial returns.

While this example is stark and highly stylized, we show these forces persist in a more general model, in which many commercial investors and social investors compete and in which firms' costs of capital are determined in equilibrium. To highlight the limits of typical socially motivated investment strategies in the presence of competing investors, our framework encompasses two types of social investors. *Values-aligned* social investors, as in the example, and consistent with observed real-world social investing strategies, form portfolios as if they only care about their financial returns and the social value generated by the firms in which they invest. *Impact-aligned* social investors, in contrast, care about their financial returns and the consequences of their investment decisions for total social welfare.

Beyond admitting a tractable analysis of equilibrium behavior, our model yields several positive and normative implications for social investors and entrepreneurs. First, we show that in equilibrium, values-aligned social investors and entrepreneurs exhibit positive assortative matching – investors who care more about social welfare match to entrepreneurs that create more social value. More interestingly, when social investors are impact-aligned, a form of negative assortative matching emerges whereby the investors who care more about social welfare match to entrepreneurs who create less social value. This can be viewed as an extension of the core logic that impact-aligned investors do not want to displace investors with less concern for social value, whether that be commercial investors, or other social investors with a lower degree of altruism. More generally, this result arises from the fact that impact-aligned social investors have interdependent utility, in that they internalize the social value created by all firms regardless of who financed them.

Second, as highlighted in the example, we identify improvements to the equilibrium investment strategies of values-aligned social investors that create more social impact *and* deliver higher financial returns. When investment opportunities with high social value are

also profitable enough to attract commercial investment, values-aligned investors compete with and displace commercial investors who would have otherwise financed these activities. Further, competition *between* social investors generates a financial concession that is wasteful from the perspective of generating impact. We show that a portion of the price impact generated by values-aligned investment results in a transfer from investors to firms' existing owners, without expanding the pool of socially valuable projects. This generates scope not only to increase the impact of values-aligned investment strategies, but also the financial return.

Third, we show that when firms have an intensive margin of scale, there is scope for creating impact by investing in commercially viable firms. We illuminate the role of "blended finance"–issuing claims at different prices for different investors—in maximizing a social investor's impact. Securities resembling green bonds emerge in equilibrium to allow impact-aligned social investors to provide low-cost financing for socially valuable projects without displacing a firm's existing commercial investors. In contrast, there is no role for blended finance when social investors are values-aligned. Further, our analysis highlights that green bonds are only impactful if they are used to finance investments that are not profitable at the commercial cost of capital–an aspect not considered by green bond certification agencies and standards.

Finally, our framework has implications for evaluating the social impact of a firm, sometimes called its *enterprise impact* (Brest et al., 2016). Enterprise impact depends not only on the amount of capital used by the enterprise, but also on the *type* of capital used by the enterprise. All else equal, enterprises that attract the capital of socially minded investors have a lower contribution to social welfare than those that attract the capital of purely commercial investors. Holding fixed the social value created by a firm, it can raise its enterprise impact by reducing its dependence on social capital, freeing social capital to fund another enterprise that is unable to obtain commercial financing. The more profitable a firm, the less likely it is to rely on scarce, socially valuable capital. Our framework thus provides a new

connection between the profitability of an enterprise and its contribution to social welfare.

How important are our findings in practice? This depends on the extent to which socially-minded adopt values-aligned strategies. We present empirical evidence that the investment strategies adopted by "sustainable" equity mutual funds strongly mirror those of the values-aligned investors in our model. In particular, we compare holdings of funds incorporating sustainability objectives to the broader universe of mutual funds, and find no evidence that these sustainable funds have reduced holdings in highly profitable high social-value companies that could have obtained commercial capital. Our results therefore suggest investors in these funds could improve both financial returns and contribution to social value.

This paper contributes to the growing literature on investing with social preferences, and in particular whether these preferences can induce more socially desirable economic activity. The majority of the literature studies investors who have a taste for investing in socially responsible activities (values alignment), rather than an explicit preference for generating impact through their investment strategies. A key theme of this literature is that the high degree of substitutability of capital limits the ability of socially motivated investors to have impact. In standard equilibrium asset pricing models, the extent to which a tilt to more socially desirable investments can affect their relative cost of capital depends on the degree to which this limits aggregate risk sharing in the economy (e.g. Heinkel et al., 2001, Pastor et al., 2020, Pedersen et al., 2021, Zerbib, 2020). Berk and van Binsbergen (2021) quantifies this mechanism and argues the effects are diminutive unless a large majority of investors adopt social preferences.

Building on this work, we highlight a new mechanism through which impact of values-aligned investment strategies is limited that is distinct from risk-sharing arguments. Consistent with the above literature, we find values-aligned investment strategies can in principal shift capital to more socially valuable activities. However, we show that these strategies are inefficient at generating impact in terms of the return concession required. Competition between social investors to hold socially responsible investments bids up prices more than

necessary for an investor to achieve a given social impact. Alternative investment strategies, that do not involve competing for the right to make a socially valuable investment, can achieve more impact for the same return concession. By characterizing the optimal investment strategy of investors who want to achieve social impact, we highlight this mechanism that limits the impact achieved by more pervasive values-aligned investment strategies.

This paper connects more closely to a smaller theoretical literature asking how social investors should behave to maximize their impact in other settings. Broccardo et al. (2020) argues that engagement is more effective than divestment in generating impact. Oehmke and Opp (2020) and Landier and Lovo (2020) study activist social investors who aim to induce firms to adopt green investment choices in the presence of managerial moral hazard. In contrast to these papers, we study passive investors in a complete information environment, whose goal is to enable new projects by offering cheaper capital to firms with socially valuable projects. In their setting, Oehmke and Opp (2020) show that values-aligned social investors cannot generate social impact because they would rather not invest at all than invest in an improved but still polluting company. Our results, in contrast, stem from the role of competition between investors for socially valuable investments, which is not modeled in their paper. Gupta et al. (2021) also study impact-aligned social investors, focusing on how their presence in capital markets influences the timing of firms' decisions to reduce negative externalities. In particular, they highlight that the desire of investors to be impactful can be counter-productive because it can incentivize firms to delay externality reduction. Moisson (2020) contrasts the behavior of impact-aligned social investors with that of investors who hold other moral criteria.

Several additional papers analyze the behavior of individual firms and their prosocial investors. Focusing on the single investor case, Chowdhry et al. (2019) and Roth (2020) analyze when a socially minded investor can have more impact through an investment in a social enterprise than they can through a grant. Hart and Zingales (2017) fleshes out several cases for a stakeholder view of the firm, and Morgan and Tumlinson (2019) and Friedman

and Heinle (2021) highlight collective action concerns among firms' prosocial shareholders. Dewatripont and Tirole (2020) studies how competition affects the degree to which firms' behaviors reflect the ethical concerns of their stakeholders. Edmans et al. (2022) characterize the tradeoff between blanket divestment strategies and alternative investment strategies that reward socially harmful companies for taking positive actions, arguing such "tilting" strategies can be more impactful.

Our study also relates to the economic literature on altruistic motives. Andreoni (1990) highlight the distinction between "pure altruists," who derive utility from social welfare, and "impure altruists," who derive utility, or "warm glow" from having directly improved social welfare. In this light, our impact-aligned social investors can be understood as pure altruists, and our values-aligned social investors can be understood as impure altruists. We contribute to this literature by embedding these various altruistic motives into a model of capital allocation.

Our analysis bears a technical resemblance to assignment matching models, commonly employed in trade and labor economics (e.g. Roy, 1951, Becker, 1973, Sattinger, 1979, Costinot and Vogel, 2010). We contribute to this literature by providing a model in which agents sort along two dimensions of heterogeneity, as in Gola (2020), and by studying an environment where one side of the market has interdependent utility in the sense that they care not only about their own match, but also the matches of others. We show that this latter feature can partially reverse the classic result of positive assortative matching.

The rest of the paper proceeds as follows. Section 2 outlines the model for the case where entrepreneurs have binary projects and Section 3 presents the results in this setting. In Section 4 we consider the case where entrepreneurs have an intensive margin of scale. Section 5 presents empirical evidence. Section 6 concludes.

2 Baseline Model

Players, Technology, and Contracts

There is a finite set E of entrepreneurs. Each one is endowed with a project that requires one unit of capital. If entrepreneur i receives the requisite capital, their project returns $\pi_i \in \mathbb{R}^+$ profit and $w_i \in \mathbb{R}$ "social value," where π_i and w_i represent the private and social return of the project respectively.[1] We assume that the features of each project are perfectly observable to all players.

There is a finite set S of social investors, each of whom allocates one unit of capital.[2] There is also a market for commercial capital that elastically supplies financing to all firms at required rate of return of r^C.

A contract between some investor and an entrepreneur i specifies the transfer of one unit of capital from the investor to the entrepreneur in exchange for financial return r_i on their invested capital. The entrepreneur receives a share of profits $\pi_i - r_i$, and w_i social value is created. We will sometimes refer to r_i as the price or cost of capital offered to an entrepreneur. Because we are studying a complete information environment without contracting frictions, this contract can be understood as either debt or equity. In addition to being able to finance the entrepreneurs in E, social investors can also allocate their capital to a "social value-neutral" asset with financial return r^C and 0 social value.

Preferences

We index investors and entrepreneurs such that investor i matches with entrepreneur i. Each entrepreneur's utility is their share of the profit, $(\pi_i - r_i)$. We will separately examine

[1] We assume that w_i encompasses the full social return of the project, including the private return π_i, as well as any consumer and employee surplus and externalities arising from the project. However the theory would proceed unchanged were we to interpret w_i more narrowly, as representing say, climate externalities.

[2] We discuss how the analysis can be extended to a model with a continuum of investors and projects in Appendix A.4.

two classes of social investors.

Values-aligned social investors make investment decisions based on the financial returns they receive and the social value created by the entrepreneur they have financed. That is, they choose their investment strategy to maximize:

$$r_i + \theta_i w_i, \tag{1}$$

where $\theta_i \in \mathbb{R}^+$ represents the strength of investor i's social preference.

Impact-aligned social investors, in contrast, make investment decisions based on the financial returns they receive and the consequence of their investment for aggregate social value. In particular, they maximize:

$$r_i + \theta_i \sum_{j \in \bar{E}} w_j = (r_i + \theta_i w_i) + \theta_i \sum_{j \in \bar{E} \setminus i} w_j, \tag{2}$$

where $\bar{E}$ is the set of entrepreneurs who receive financing.[3] We can observe that the difference between the objective functions of values-aligned and impact-aligned social investors is that impact-aligned social investors derive utility equally from all social output regardless of who financed it. The implication of this difference is that impact-aligned investors internalize the consequences of their actions on total social welfare, while values-aligned investors only consider the social value of the firm they finance.

There are two interpretations of the assumption that values-aligned investors do not fully internalize the implications of their investment decision on social welfare. Values-aligned investors may derive intrinsic utility from owning firms that create social value, similar to the conception of warm-glow altruists in Andreoni (1990). In such a case, the analysis to

[3]We note that as w_i includes r_i, both preferences above "double count" r_i. The values-aligned preferences can be rewritten as $r_i + \tilde{\theta}_i (w_i - r_i)$ and the impact-aligned preferences can be rewritten as $r_i + \tilde{\theta}_i \left(w_i - r_i + \sum_{j \in \bar{E} \setminus i} w_j \right)$, where $\tilde{\theta}_i = \frac{\theta_i}{1+\theta_i}$. Hence the two preferences can be equivalently stated, up to a re-normalization, without double counting r_i.

follow should be understood as exploring the positive implications of these two modes of investment behavior. Alternatively, values-aligned preferences may represent the *behavior* of socially conscious investors, while impact-aligned social preferences may more faithfully represent the *intentions* of socially conscious investors to affect social change. Under this interpretation, our analysis of the behavior of impact-aligned social investors offers normative guidance to real-world investors with social preferences. In Section 5 we discuss the empirical evidence, which suggests that the normative interpretation is more likely to reflect reality.

Timing of Actions

First, in the *offer stage*, social investors offer contracts to entrepreneurs.[4] Simultaneously, all firms receive an offer for commercial financing at rate r^C. Then, in the *acceptance stage,* entrepreneurs choose at most one contract to accept, and payoffs are realized.

Equilibrium

The solution concept is pure-strategy Subgame Perfect Equilibrium. In the acceptance stage, all entrepreneurs choose the contract offered to them that maximizes their share of the profits. In the offer stage, all investors choose a contract that maximizes their utility among all contracts that will be accepted, given the contracts offered by other investors.

Social Welfare

Our measure of social welfare is $W = \sum_{i \in \bar{E}} w_i$, where $\bar{E}$ is the set of entrepreneurs that receive financing. Our interpretation is that w_i is the total social value created by firm i if it receives financing, including the value to the firm's owners.[5] Impact-aligned social investors can therefore be understood to be maximizing a modified variant of social welfare

[4]For technical convenience, we allow social investors to offer more than one contract. In the event that multiple contracts are accepted, one is chosen uniformly at random to be implemented.

[5]Under this interpretation, the value accruing to the firm's owners is determined independently of how ownership is divided, i.e. the welfare weights placed on entrepreneurs and investors are the same.

that increases the weight placed on their own consumption. Also note that, consistent with Hart and Zingales (2017) and Broccardo et al. (2020), our measure of welfare does not include the "altruistic" utility that social investors derive from the creation of social value w_i. The exclusion of investors' altruistic utility from the social welfare function can be motivated by assuming that the utility that investors derive from supporting socially valuable firms is "small" relative to the other ways in which those firms contribute to social value.

Discussion of Modeling Choices

The goal of our analysis is to illuminate the equilibrium implications of values-aligned investment strategies and the limits of how they can create impact. To present our results in a parsimonious manner, our model omits several important features of financial markets that are relevant for social investing. First, our model only accommodates a limited form of endogenous firm entry. In our model the universe of potential firms is fixed. The allocation of commercial and social capital determines which of these firms ultimately enter the market, but entrepreneurs cannot respond by altering the distribution of potential firms. Second, entrepreneurs cannot endogenously change the properties of existing firms in response to market prices, though this constraint is relaxed in Section 4. Finally, because we have assumed that the commercial market elastically supplies capital at rate r^C, rather than assuming a finite supply of commercial capital, there is no channel for social investors to raise the cost of capital of firms with negative social value through divestment. There is however a channel for investors to reduce the cost of capital for firms with high social value.

The inclusion of each of these modeling features would allow for new channels by which values-aligned social investors could create social value. However, we emphasize that the goal of our analysis is to illustrate the role of competition between social investors in generating social value, and conjecture that each of these excluded modeling features could be incorporated without qualitatively altering our main results.

3 Analysis of Baseline Model

To understand the behavior of values-aligned and impact-aligned social investors we first characterize the equilibrium of the model in which all investors are either values-aligned or impact-aligned. Beginning in Section 3.3 we present results in the model in which both types of social investors coexist.

3.1 Values-Aligned Social Investors

We begin by characterizing the equilibrium of the model where all social investors are values-aligned. For any two entrepreneurs i and j who are both supported by social investors with the same subscripts, the investors' equilibrium incentive compatibility condition dictates that

$$r_i + \theta_i w_i \geq r_j + \theta_i w_j. \tag{3}$$

This incentive compatibility condition dictate that at the equilibrium prices r_i and r_j, no social investor prefers to undercut another one.[6] This inequality implies that investors and entrepreneurs follow positive assortative matching in equilibrium.

Lemma 1. *For any two values-aligned social investors i and j (matched in equilibrium to entrepreneurs i and j), if $\theta_i \geq \theta_j$, then $w_i \geq w_j$.*

Lemma 1 is an implication of the fact that investors' preferences are supermodular in θ_i and w_i. Social investors with relatively higher altruism θ_i have relatively higher willingness to pay per unit of social value of a given enterprise. Therefore, in equilibrium the investors with highest altruism finance the enterprises with the highest social value. This will present a point of contrast to the case where investors are impact-aligned.

[6] We adopt the convention that an entrepreneur i who accepts no offers for financing has a cost of capital $r_i = \pi_i$.

Our next observation, once again following from the above incentive compatibility condition, is that cost of capital is decreasing in an entrepreneur's social value w_i.

Lemma 2. *For any two entrepreneurs financed in equilibrium by values-aligned social investors, if $w_i > w_j$ then $r_i \leq r_j$. Furthermore this inequality is strict for any two entrepreneurs with $r_i > 0$ and $r_j > 0$.*

To see the logic of this lemma, note that for any two entrepreneurs i and j financed by social investors with the same altruism parameter $\theta_i = \theta_j = \theta$, social value is priced at rate θ. That is, the investors' incentive compatibility conditions demand that $r_i = r_j + \theta\,(w_j - w_i)$, so that an entrepreneur with one unit higher social value receives capital at a cost reduced by θ units. Moreover, Proposition 1 dictates that as entrepreneurs become more socially valuable they match with more altruistic investors, which serves to further steepen the rate at which their cost of capital declines with their social value. We note that this inequality does not hold strictly for entrepreneurs supported by the commercial market in equilibrium, as all of these entrepreneurs receive a cost of capital of r^C.[7]

Lemma 2 will be play an important part of the analysis to follow, and an important point of contrast to the case where social investors are impact-aligned. When investors are values-aligned, the fact that they compete with one another to be the owners of socially valuable companies causes them to bid up the prices of companies with higher social value. As we will discuss further, in some cases this expands the financing frontier for socially valuable companies, while in other cases it only serves to increase the share of profits retained by entrepreneurs with socially valuable companies.

Figure 1 depicts the equilibrium matching of investors to entrepreneurs. The graph shows the space of available investment opportunities, parameterized by π and w, and the shaded regions show the sets of entrepreneurs financed by different types of investors. Let

[7]This inequality also does not hold strictly for entrepreneurs whose social value w_i is high enough that their equilibrium cost of capital is $r_i = 0$. By assumption there is no opportunity for investors to undercut one another when the equilibrium cost of financing is 0 – we interpret these instances as akin to philanthropy.

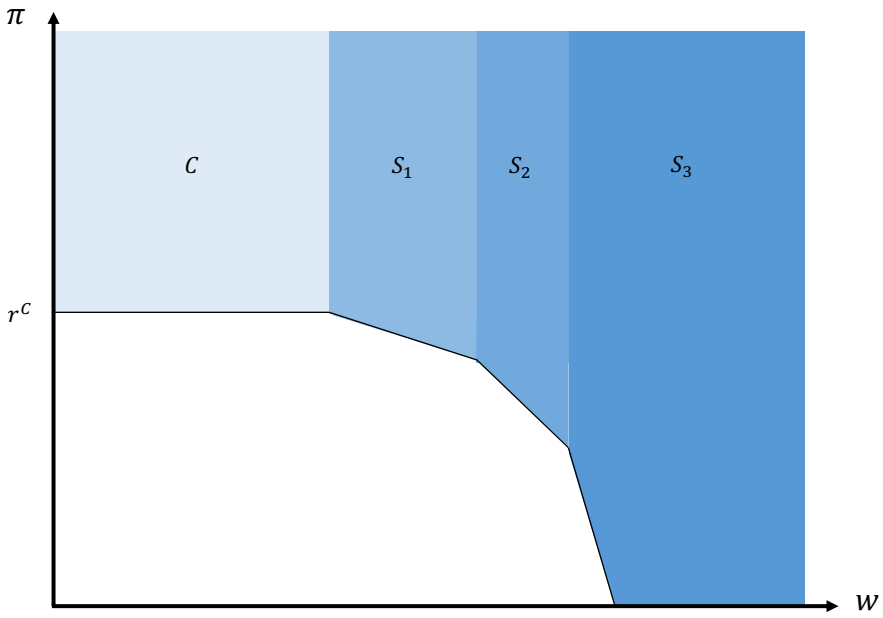

Figure 1: Equilibrium investment with values-aligned social investors

$\theta^1 \equiv \min_{i \in S} \theta_i$ be the lowest level of altruism among all social investors, noting that several investors may have the same level of altruism. Let $S_1 \equiv \{i \in S : \theta_i = \theta^1\}$ be the set of investors with the lowest level of altruism. Similarly let S_k be the set of investors with the k'th lowest level of altruism.

Entrepreneurs in the lightest blue region, with profits higher than the commercial hurdle rate r^C but relatively low social value, are financed by the commercial market. Entrepreneurs with progressively higher social value w_i are financed by social investors with progressively higher levels of altruism, and entrepreneurs in the unshaded region, with profits lower than their equilibrium cost of capital, are unfinanced in equilibrium. The financing frontier becomes progressively steeper as the investors become more altruistic (with slope $\frac{-1}{\theta^i}$), reflecting their increased willingness to trade off financial return for social value.

In Appendix Section A.1 we describe the precise strategies that support this equilibrium.

3.2 Impact-Aligned Social Investors

Next we analyze the equilibrium allocation of capital when social investors are all impact-aligned. One of the principal differences to the case where social investors are values-aligned is that, so long as social capital is sufficiently scarce, impact-aligned investors do not finance commercially viable entrepreneurs regardless of their social value. We maintain the following assumption for the remainder of Section 3.2.

Assumption 1. *Social capital is scarce:* $S \leq \left|\left\{i \in E : \pi_i + \theta_1 w_i > r^C, \pi_i < r^C\right\}\right|$

Assumption 1 regards the number of entrepreneurs that are not commercially viable (i.e. $\pi_i < r^C$), with projects whose full profit plus the social value as judged by the social investor with lowest altruism exceeds the commercial rate of return (i.e. $\pi_i + \theta_1 w_i > r^C$). If the number of such entrepreneurs is greater than the number of social investors, (i.e. when Assumption 1 holds), then impact-aligned investors never finance entrepreneurs that could have attracted commercial capital.

Lemma 3. *In equilibrium no impact-aligned investor finances an entrepreneur with* $\pi_i \geq r^C$.

To see the logic underlying this observation, suppose it were to be violated, and some impact-aligned social investor j were in equilibrium to support a firm j with profits $\pi_j \geq r^C$. In this conjectured equilibrium, investor j receives a financial return of at most r^C. By Assumption 1 there must be at least one firm $\tilde{j}$ that does not receive financing in equilibrium for whom $\pi_{\tilde{j}} + \theta_1 w_{\tilde{j}} > r^C$. Firm $\tilde{j}$ would accept any offer of financing with $r_{\tilde{j}} \leq \pi_{\tilde{j}}$. Investor j then has a profitable deviation from their equilibrium offer to instead offer capital to firm $\tilde{j}$ at cost $r_{\tilde{j}} = \pi_{\tilde{j}}$. Investor j's financial return would decrease by at most $r^C - \pi_{\tilde{j}}$,while total social value would increase by $w_{\tilde{j}}$, as firm j would still receive capital from the commercial market. This is a profitable deviation by assumption, as $r^C - \pi_{\tilde{j}} < \theta_1 w_{\tilde{j}}$.

While Lemma 3 holds across all equilibria, the precise set of firms who receive financing from impact-aligned social investors is not fixed across all equilibria. Nevertheless, we have

the following characterization of the capital allocation in the investor-optimal and welfare-optimal equilibrium, which reverses the positive assortative matching result for values-aligned investors.

Proposition 1. *Consider the investor- and welfare-optimal equilibrium. Take any two enterprises i and j that receive capital from impact-aligned social investors with $\pi_i = \pi_j$: if $w_i > w_j$ then $\theta_i \leq \theta_j$.*[8]

Proposition 1 implies that when social investors are impact-aligned, they exhibit a form of negative assortative matching in equilibrium. Among firms with fixed a level of profits π, the higher is the social investor's altruism parameter θ_i, the lower is the social value w_i of the firm they support. This negative assortative matching holds despite the fact that the utility of impact-aligned social investors is supermodular in their altruism parameter θ_i and the social value w_i of the firm they support (as was the case with values-aligned social investors).

We defer a formal proof of Proposition 1 to the appendix but provide a discussion here. In equilibrium, in order for an impact-aligned social investor i not to deviate and support a firm that could have attracted commercial investment, it must be that

$$\pi_i + \theta_i w_i \geq r^C. \tag{4}$$

This incentive compatibility condition is easier to satisfy for social investors with higher altruism parameters. Therefore in the welfare-optimal equilibrium, it is the impact-aligned social investors who care the least about social welfare that match to the most impactful entrepreneurs for a given level of profitability, as these are the entrepreneurs who are most able to entice social investors away from commercial markets. In contrast, impact-aligned

[8] All equilibria with the same investment frontier depicted in Figure 2 are investor- and welfare-optimal. Therefore, formally, there exists an investor- and welfare-optimal equilibrium such that holding the level of entrepreneur profit fixed, if $w_i > w_j$ then $\theta_i \leq \theta_j$. But there may be other equilibria with equivalent allocations that do not feature negative assortative matching.

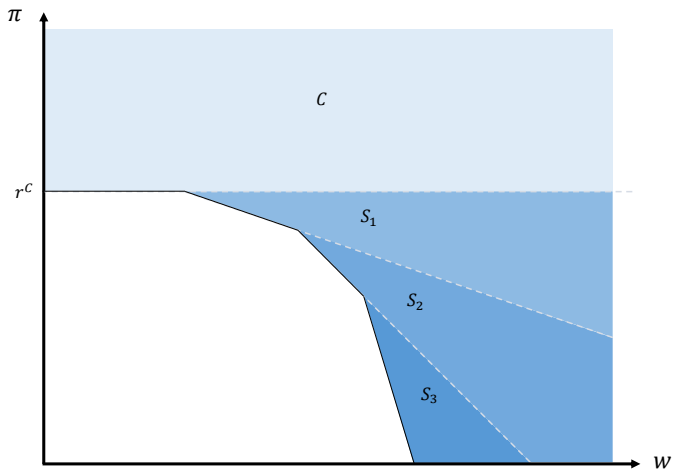

Figure 2: Equilibrium investment with impact-aligned social investors

social investors with higher altruism parameters are willing to forgo commercial returns to support entrepreneurs with lower contribution to social welfare for a given profit level. And because impact-aligned social investors derive utility from the social value created by all firms supported in equilibrium, social investors with high altruism parameters do not undercut social investors with low altruism parameters, as they recognize that doing so would not expand social value.

Therefore, that positive assortative matching breaks down when social investors are impact-aligned arises from the fact that impact-aligned social investors have interdependent utilities in the sense that their utility depends not only on the firm they match to and their financial return but also on the matching of other investors and entrepreneurs. This result can be understood as an extension of the core logic that impact-aligned social investors do not want to displace investors with less concern for social welfare, whether they be commercial investors or other impact-aligned social investors with a lower degree of altruism.

The investor- and welfare-optimal equilibrium allocation is depicted in Figure 2. In Appendix A.2 we formally characterize the allocation of impact-aligned social capital.

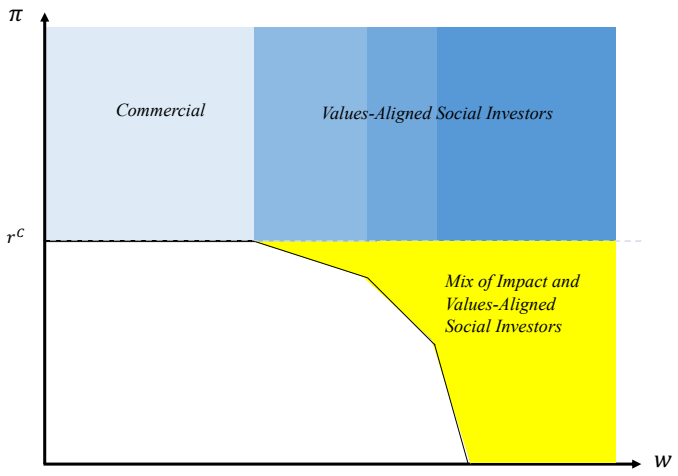

Figure 3: Equilibrium investment with both types of social investors

3.3 Equilibrium Structure with Both Types of Social Investors

In the following sections we discuss a number of normative results about social investing, in a market in which both impact-aligned and values-aligned social investors coexist. Much of the above analysis continues to hold when both types of social investors are present within the same market. Lemmas 1 and 2 carry over to the case with both types of social investors. Values-aligned social investors and entrepreneurs continue to engage in positive assortative matching, and the cost of capital is decreasing in social value w for any two entrepreneurs financed by values-aligned social investors. And Lemma 3 also carries over to the case with both types of social investors. So long as social capital is sufficiently scarce, impact-aligned social investors never finance an entrepreneur that could have attracted commercial capital.

A generic equilibrium allocation is depicted in Figure 3.

3.4 Reallocating Values-Aligned Social Capital to Improve Social Welfare and Financial Returns

In this section we consider two thought exercises. First, holding fixed the equilibrium behavior of all other investors, we consider the possibility of reallocating the investment of a single values-aligned social investor. We demonstrate that any values-aligned social investor who supports a firm with $\pi_i \geq r^C$ and who earns a financial return of $r_i < r^C$ could reallocate their capital to increase total social welfare *and* increase their financial return. In this sense values-aligned investors leave both money and impact on the table.

Proposition 2. *Consider any values-aligned social investor i that supports a firm with $\pi_i > r^C$ and earns a return $r_i < r^C$ in equilibrium. If the distribution of firms is sufficiently dense, there exists an unfinanced firm j with profits $\pi_j > r_i$, such that if the values-aligned social investor i were to deviate and offer firm j financing at cost π_j, total social welfare would increase as would investor i's financial return.*

In the proof of Proposition 2 we formalize the notion that the distribution of entrepreneurs is *sufficiently dense*. Intuitively, it guarantees that for any combination of π and w, there is an entrepreneur with profits π_i near π and social value w_i near w. Proposition 2 can be understood with reference to Figure 4. Fix any values-aligned social investor i that supports a firm i with profits $\pi_i \geq r^C$ and who earns financial return $r_i < r^C$ (generically this holds for all values-aligned investors who support firms with $\pi_i \geq r^C$). These investors support the firms highlighted in the three darker shades of blue. And consider among the set of unfinanced firms some firm j with profits $\pi_j > r_i$ and with social value $w_j > 0$.[9] This firm is guaranteed to exist by the assumption that the distribution of firms is dense. One such firm is highlighted in green.

[9]Or, more generally, we identify a firm j with profits $\pi_j > r_i$ and with social value w_j greater than the social value of the marginal firm receiving commercial support.

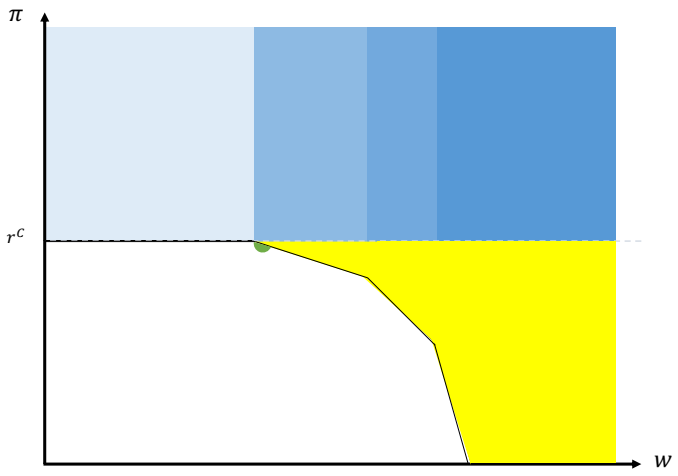

Figure 4: Reallocating Values-aligned Social Capital Out of Equilibrium

The contribution to social welfare of the equilibrium investment for investor i is 0, regardless of the social value w_i of firm i, as investor i is merely displacing commercial investment. Firm j creates less social value than any firm in the blue region of the diagram, but by reallocating investor i's capital to firm j social welfare increases, as firm j was previously unfinanced.

Further, by offering firm j a cost of capital $r_j = \pi_j$, investor i can earn higher financial return as well. As with social value, firm j earns lower profits than any firm in the blue region of the diagram. But because $\pi_j > r_i$, the subsidy required to make firm j profitable is smaller than the financial concession (relative to the commercial rate of return) that investor i made to own firm i. Critically, values-aligned social investors compete down the price of capital of firms with large contributions to social value even when these firms could have attracted commercial financing. The financial compromise made by values-aligned investors to support such firms results in a transfer of wealth to the entrepreneur rather than expanding the pool of socially valuable firms. In contrast, the financial compromise made to support a firm that could not attract commercial financing goes entirely toward expanding the pool of socially

valuable firms rather than transferring rents to entrepreneurs whose projects would anyway have been feasible.

We note that the proof of Proposition 2 relies on allowing the deviating values-aligned investor to extract the full profit of the previously unfinanced firm highlighted in green in Figure 4. However this feature of the model is not necessary for the result. The critical ingredient is that values-aligned investors bid up the price of some firms in a way that does not contribute to aggregate social value (i.e. by competing up the price of socially valuable projects that would have occurred even at the commercial cost of capital). So long as there is a way to reallocate a portion of that financial concession in a way that *does* contribute to social value (e.g. by donating it to charity), then the deviating values-aligned investor can reduce the total financial concession and allocate the remainder in a way that creates social value – this increases both impact and financial return.

Finally, we note that while there exist opportunities for values-aligned investors to increase their financial return and social impact, these opportunities do not increase the utility of values-aligned social investors (who place intrinsic value on the social value of the company they finance). Nevertheless, Proposition 2 offers encouraging news about the prospect of converting values-aligned social investors to impact-aligned social investors in practice. A substantial amount of effort has gone into investigating the hypothesis that ESG investing can increase impact and financial returns (e.g. Eccles et al., 2014), suggesting investors are sensitive to the financial and social implications of values investing. Our model demonstrates that relative to conventional ESG strategies there is room for improvement in both dimensions.

3.5 Enterprise Impact

How should one judge the contribution to social welfare of a particular entrepreneur, sometimes referred to as *enterprise impact* (e.g. Brest et al., 2016)? On first pass it might

seem natural for w_i to be the measure of enterprise impact. However, we argue that a firm's enterprise impact should also account for the social value of the capital it employs.

Let $W(S_V, S_I)$ be the total social value created in equilibrium given masses of values-aligned investors S_V, and impact-aligned investors S_I. Define ν_{S_V} to be the increase in social value corresponding to adding one additional values-aligned social investor, and ν_{S_I} to be the increase in social value corresponding to adding one additional impact-aligned social investor. The social value of commercial capital, ν_C, is normalized to 0. It is straightforward to show that $\nu_C < \nu_{S_V} < \nu_{S_I}$.

We define the enterprise impact of firm i to be $e_i \equiv w_i - \nu_i$ where ν_i is the social value of capital attributable to the investor who supports entrepreneur i in equilibrium. We define the enterprise impact to be 0 for firms that do not receive financing. Enterprise impact e_i corresponds to the change in total social value created in equilibrium from adding firm i to the market.

This definition of enterprise impact might have practical value for socially motivated investors aiming to quantify the social value of a particular enterprise. Frontier efforts in the impact investing industry often attempt to account for the social value created by the enterprise and the amount of capital employed by the enterprise, such as in the *impact multiple of money* method (Addy et al., 2019). Our analysis highlights that it is also critical to account for the composition of social capital versus commercial capital raised by an enterprise in judging its impact.

This definition of enterprise impact also highlights an alignment between the enterprise impact and profitability of a firm. Firms can increase their enterprise impact by increasing their profitability *even holding fixed their social value* w_i. Increasing the profitability of the firm makes it more likely to attract commercial capital, freeing up capital that is willing to accept lower returns to fund higher social value endeavors. In particular, we have the following result.

Proposition 3. *Suppose firm i attracts financing from an impact-aligned social investor in equilibrium. Increasing its profits π_i while holding fixed its social value w_i increases its enterprise impact e_i and total social value created in equilibrium.*

Proposition 3 states that making a firm more profitable increases its enterprise impact *even holding its social value w_i fixed.* Importantly, this result is not driven by an assumption that a firm's profitability and its social value are correlated. Instead, this result is driven by the observation that once a firm becomes profitable enough to attract commercial financing, impact-aligned social investors will step aside, freeing up their capital to support another socially valuable firm. Therefore, more profitable firms use less socially valuable capital, and have higher enterprise impact. Note that this phenomenon does not hold for firms supported by values-aligned social investors, as values-aligned social investors pay no regard to whether a firm could attract commercial capital in their absence.

4 Impact on the Intensive Margin

So far we have assumed that every firm has a single project, which is completed if and only if it raises a unit of capital. Within this setting, we demonstrated that there are deviations from equilibrium values-aligned investment strategies that improve both financial return and social impact. However, our proposed alternative investment strategy required that investors allocate their capital to firms that are not commercially viable. In reality, this may only be possible in private capital markets, which is likely infeasible for small investors.[10] In this section we consider a variant of the model in which firms have continuous, concave production functions and demonstrate that social investors *can* have impact by inducing commercially viable firms to change their scale of operation. Therefore, there may be room for social investors to induce change in public markets.

[10] Jeffers et al. (2021) analyze the financial returns of private market investment funds with a mandate to generate social impact.

The economic logic in the continuous-project case is largely the same as in the binary-project case. Values-aligned investors bid up the prices of firms with high average social value while impact-aligned investors target their subsidies to firms with socially valuable but marginally unprofitable projects. Relative to the binary-project case, two new insights emerge. First, impact-aligned investors maximize their impact by leveraging commercial capital. In equilibrium a capital structure emerges in which commercial and impact-aligned investors co-invest in the same firms at different terms, referred to as "blended finance" in the impact investment community.[11] In contrast, values-aligned investors never support the same firms as commercial investors. Second, impact-aligned investors utilize a security that resembles green bonds as a mechanism to subsidize firms without displacing commercial investors.

4.1 Model

Agents, Technology, and Contracts

The model in Section 2 is now modified in the following ways. First, rather than assuming that firms have projects of binary scale, we now assume that each firm i can absorb any positive mass of capital k. Firm i then produces $\pi_i(k)$ profit and $w_i(k)$ social value. Both functions are increasing, concave, and continuously differentiable, although the case where $w(\cdot)$ is at some points decreasing could be easily accommodated. We maintain the assumption that there is a finite set of social investors denoted by S, each of whom owns one unit of capital. However, now capital is divisible so that in principle one investor could support several firms. There is a market for commercial capital that elastically supplies financing to all firms at rate r^C. Social investors can also allocate their capital to a "social value-neutral"

[11] The term "blended finance" is increasingly prevalent amongst impact investors in practice. The International Finance Corporation of the World Bank defines blended finance as "the use of relatively small amounts of concessional donor funds to mitigate specific investment risks and help rebalance risk-reward profiles of pioneering investments that are unable to proceed on strictly commercial terms. Concessional funds are structured as co-investments, with an expectation of reflows for future investments or other uses."

asset with financial return r^C and 0 social value.[12]

A contract between an investor and entrepreneur now specifies not only the transfer of capital from the investor to the entrepreneur at a cost of $r \geq 0$, but also a minimum-scale contingency k, discussed below. A contract is therefore represented by $\langle r, k \rangle$, where r represents the cost of 1 unit of capital. The minimum-scale contingency can also be left unspecified, represented by $\emptyset$.

Preferences

For simplicity we focus on the case where investors have a homogenous altruism parameter $\theta \in \mathbb{R}$. Let x_i^j be the amount of capital that investor i allocates to firm j in equilibrium, r_i^j be the cost of capital that investor i charges firm j and $\bar{k}_j$ be the mass of capital raised by firm j. A values-aligned social investor i's utility is represented by

$$\sum_j x_i^j \left(r_i^j + \theta \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j} \right). \tag{5}$$

That is, values-aligned social investors care about their total financial return and social value of the firm they support, weighted by their ownership share.

In contrast, an impact-aligned social investor i's utility is represented by

$$\sum_j x_i^j r_i^j + \theta \sum_j w_j\left(\bar{k}_j\right) \tag{6}$$

That is, impact-aligned social investors care about their financial return and the total social value created by all firms, regardless of who supports them.

Entrepreneur j has preferences represented by $\pi_j\left(\bar{k}_j\right) - r^j$, where $r^j \equiv \sum_i x_i^j r_i^j$, and the sum is taken over all contracts the entrepreneur accepts from investors i. That is,

[12]The results would be nearly unchanged if social investors did not have access to a social value-neutral asset.

entrepreneurs maximize firm profit net of the cost of external capital.

Timing

First, all social investors offer a contract to an entrepreneur. Simultaneously, entrepreneurs receive offers from the commercial market for an arbitrary amount of capital at rate r^C. Second, entrepreneurs accept any number of such contracts and operate at scale $\bar{k}_i$, where $\bar{k}_i$ represents the mass of capital they have accepted. Entrepreneurs may only accept a contract that specifies a minimum-scale contingency k if they operate at a scale $\bar{k}_i \geq k$.[13]

Finally, we maintain the solution concept is pure-strategy Subgame Perfect Equilibrium.

Discussion of Minimum-Scale Contingency

The minimum-scale contingency gives investors the ability to influence a firm's scale on the margin. Without it, if an investor offered a firm cheaper capital than it could attract on the commercial market, the firm may merely accept that capital as a substitute for more expensive capital without changing its scale. A minimum-scale contingency was implicit in the binary-project model because by definition, if an investor offered subsidized capital to a firm that could not attract commercial financing, it would necessarily operate at a larger scale. Unlike in the binary-project model, in the continuous case social investors may desire to change the scale of a firm that could attract a non-zero amount of commercial capital, and the minimum-scale contingency offers them a route to do so. Minimum-scale contingencies resemble green bonds in that they require a firm undertake a specific project (or reach a specific scale) in exchange for financing.[14]

[13] As in the model of Section 2, we allow social investors to offer more than one contract.

[14] In principle one could study a broader contracting space. For instance, a firm's cost of capital could be made contingent on its social impact. We leave study of the optimal contract design to future work.

4.2 Equilibrium Structure

For simplicity we focus separately on the cases where all social investors are values-aligned and where all social investors are impact-aligned.

Values-Aligned Social Investors

The equilibrium when social investors are values-aligned works in much the same way as the binary-project case. Among all firms that receive financing, equilibrium sorting is such that there is a cutoff $\bar{w}$ for which social investors finance all firms with average social value $\frac{w_i(\bar{k}_i)}{\bar{k}_i} > \bar{w}$, and commercial investors finance the rest. Firms financed by the commercial market have a cost of capital r^C, and firms i and j financed by social investors have a cost of capital pinned down by the following indifference condition among social investors

$$r_i + \theta \frac{w_i\left(\bar{k}_i\right)}{\bar{k}_i} = r_j + \theta \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j}. \tag{7}$$

which implies that firms with higher average social value have lower costs of capital.

Firms choose their scale in one of two ways. Firms that are unconstrained by a minimum-scale contingency choose their scale $\bar{k}_i$ to maximize $\pi_i(k) - r_i k$, so that $\bar{k}_i$ solves

$$\pi_i'\left(\bar{k}_i\right) = r_i. \tag{8}$$

Firms that are constrained by a minimum-scale contingency need not set their marginal profit equal to their cost of capital. Specifically, let a firm's commercial scale k_i^C solves $\pi_i'\left(k_i^C\right) = r^C$, and define its commercial share of profits to be $q_i^C \equiv \pi_i\left(k_i^C\right) - r^C k_i^C$. A firm constrained by a minimum-scale contingency $\bar{k}_i$ need only satisfy

$$\pi_i\left(\bar{k}_i\right) - r_i \bar{k}_i \geq q_i^C \tag{9}$$

Equilibrium is determined by a set of costs of financing $\{r_i\}$ that satisfies the above equation 7, and a set of firm scales $\left\{\bar{k}_i\right\}$ each of which either satisfies Equation 8 or 9. We note that there exists an equilibrium where no investor utilizes scale-contingencies, and Equation 8 determines the scale of all firms.

Finally, we note that across all equilibria, there is no co-investment between commercial investors and social investors within any firm that receives a subsidy relative to the commercial cost of capital.

Lemma 4. *Firms that receive a cost of capital $r_i < r^C$ from any values-aligned social investor are financed wholly by values-aligned social investors.*

Social and commercial investors disagree on the relative value of companies with the same profits but different contributions to social value, so there is no equilibrium price at which both sets of investors would be happy to finance the same investment.[15] While this extreme separation would not arise in a model with, for example, diversification motives, it illustrates an important point. Disagreement about the value of a company among investors implies that to change the scale of the company requires displacing some of its existing investors. This idea is closely related to the observations of Heinkel et al. (2001) and Broccardo et al. (2020), that commercial investors will partially "undo" the actions of social investors, insofar as social investors may partially crowd out commercial investors in the firms they support. We will see in the following section that impact-aligned social investors do not displace commercial investors, and co-investment does occur in equilibrium.

Impact-Aligned Social Investors

Next we analyze how the allocation of capital differs when social investors are impact-aligned. As there are multiple equilibria, in this section we focus on the investor-optimal

[15]The one exception is the firm for which $\frac{w_i(\bar{k}_i)}{\bar{k}_i} = \bar{w}$. This firm may be financed by both commercial and social investors at cost of capital r^C.

equilibrium. Our intention in analyzing this equilibrium in particular is not to make sharp predictions about investor behavior, but rather to highlight some of the key differences in investor behavior that may emerge when investors are impact-aligned. The propositions in Section 4.3 hold across all equilibria.

We first observe that unlike in the case of values-aligned social investors, impact-aligned social investors do co-invest with commercial investors.

Lemma 5. *In the investor-optimal equilibrium when social investors are impact-aligned, all firms raise at least k_i^C capital from commercial investors.*

If an impact-aligned social investor were to marginally undercut a commercial investor, they would earn a return of r^C, and create 0 additional social value. Therefore, the most efficient way to create impact is to leverage commercial capital, rather than displacing it. Firms that receive social investment raise at least k_i^C capital from commercial markets, and the remaining capital from social investors. In fact, as we demonstrate in the proof of Lemma 5, in equilibrium firms supported by social investors may raise more than k_i^C commercial capital. In such settings impact-aligned social investors subsidize the entry of commercial investors to increase the scale of firms with high marginal social value.

As in the case with binary projects, because impact-aligned social investors care about total social value creation rather than the social value of the firm they support, they do not compete with one another or with commercial investors. Rather than being determined by competitive forces, equilibrium prices of capital are determined by a no-rents condition.

Lemma 6. *Across all equilibria when social investors are impact-aligned, all entrepreneurs earn a payoff of q_i^C.*

Impact-aligned social investors demand a return r_i^S that solves $\pi_i\left(\bar{k}_i\right) - r^C \bar{k}_i^C - r_i\left(\bar{k}_i - \bar{k}_i^C\right) = q_i^C$, where $\bar{k}_i^C$ is the amount of commercial capital raised by firm i in equilibrium. If social investors demanded a higher return, firm owners would prefer to invest at their commercial

scale k_i^C and to rely exclusively on commercial capital. And because impact-aligned social investors recognize that by undercutting one another they are not contributing to total social value creation, required returns are set so as to make entrepreneurs indifferent between accepting social capital versus relying exclusively on commercial financing.

We now turn to characterizing the use of minimum-scale contingencies.

Lemma 7. *Impact-aligned social investors utilize minimum-scale contingencies in equilibrium.*

Because impact-aligned social investors set prices so as to leave entrepreneurs with their commercial payoff, the firms they support are always faced with a marginal cost of capital that is above their marginal return on investment at $\bar{k}_i$. If they were free to choose their own scale, they would accept the subsidized social capital in lieu of commercial capital, and still choose a smaller scale than social investors desired. Therefore, unlike in the case with values-aligned investors, impact-aligned social investors always utilize scale-contingent contracts.

Finally, we turn to equilibrium capital allocation.

Lemma 8. *In the investor optimal equilibrium when social investors are impact-aligned, for any two firms i and j that receive capital from social investors, we have*

$$\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = \pi_j'\left(\bar{k}_j\right) + \theta w_j'\left(\bar{k}_j\right). \tag{10}$$

Impact-aligned social investors allocate their capital so as to equalize the marginal profits plus the marginal social value of all firms that receive a subsidy.

Green Bonds

We close this section with a discussion of security design. In equilibrium, impact-aligned social investors co-invest with commercial investors but utilize different terms. Impact-

aligned social investors offer cheaper financing, and include a scale contingency requiring the firm to operate at a certain scale if it is to accept their capital. This contingent financing scheme closely resembles the real world institution of green bonds. Green bonds are debt securities issued by companies to finance investments with high social or environmental value, and are potentially issued at a premium to non green securities. In the model, such financing generates impact because it allows the financing of projects that are socially, but not commercially viable, without displacing commercial investors. Green bonds also emerge as optimal securities in Oehmke and Opp (2020).

Our analysis highlights an important condition for green bonds to generate impact–that they only be used to fund socially valuable investments that would not be attractive to commercial investors. In practice, however, existing certifications and standards in the green bond industry focus entirely on whether the project being financed by the green bond is socially desirable. In the absence of a certification verifying the use of proceeds could not have been financed commercially, investors in the green bond are simply displacing commercial investors and paying a premium for the privilege, consistent with the strategies adopted values-aligned rather than impact-aligned investors in our model.

4.3 Results

In this section we demonstrate that natural analogues of Propositions 2 and 3 extend to this setting. Namely, we demonstrate that in equilibrium values-aligned investors leave both money and impact on the table, and that increasing a firm's profitability may also increase its enterprise impact.

Proposition 4. *In equilibrium there may exist a deviation for a values-aligned social investor that would result in higher financial return and increase total welfare.*

When there is an intensive margin of scale, values-aligned investors still crowd out commercial capital to finance firms with high social value. Even if doing so increases the scale

at which the firm operates and thus increases the social value it creates, competition among social investors means that this involves a financial concession. Again, following the same logic as Proposition 2, investors can increase their return and impact by instead supplying their capital to the firm that can most efficiently generate impact on the margin for a given financial concession.

Proposition 4 is stated as a possibility result, rather than generically as in Section 3, because when firms have an intensive margin of scale there is no natural analogue of the sufficient density assumption about the distribution of firms that was employed in Proposition 2. Therefore, even though values-aligned investors make a financial concession that does not contribute to social value, we cannot guarantee that there exists a firm to which they can unilaterally deviate to both increase their financial return and total social welfare.

Next we consider an analogue to Proposition 3. We extend the definition of enterprise impact to account for the possibility that firm i attract capital from more than one type of investor. We define the enterprise impact of firm i to be $e_i(k) \equiv w_i(k) - \nu_i k$, where ν_i is now the *average* social value of capital utilized by entrepreneur i. We have the following result.

Proposition 5. *Suppose firm i attracts financing from an impact-aligned social investor in equilibrium. Increasing its profitability while holding fixed its social value $w_i(\cdot)$ increases its enterprise impact e_i and total social value created in equilibrium.*

The logic of this proposition is exactly parallel to that of Proposition 3. Take any entrepreneur supported by impact-aligned social investors in equilibrium. As it becomes more profitable, commercial investors will finance a larger fraction of its output, which frees impact-aligned social capital to invest elsewhere and increase total social value.

5 Empirical Evidence and Practical Implications

One fundamental message of this paper is that investors who care about impact should not pay a premium to make investments that could have been financed by any investor less interested in social impact. In our model, values-aligned investment strategies lead to exactly such a premium. This creates a deviation by which a social investor can increase both financial return and aggregate social value. The extent to which these insights matter and are applicable to the real world depend on three factors. First, do social investors adopt values-aligned investment strategies? Second, do social investors actually have preferences for social value *creation* rather than social value *association*? And third, are these investors indeed paying a "greenium" for their investments in high social value companies that could have been financed with commercial capital? Further, what can social investors actually do in practice to increase their impact? In this section we review the literature and present new empirical analysis that sheds light on these questions.

5.1 Strategies of Social Investors: Evidence from US Equity Mutual Funds

Are socially minded investors adopting values-aligned or impact-aligned investment strategies? Figure 3 provides guidance for answering this question. Values-aligned investors should exhibit a preference for investments associated with higher social value, as proxied by companies having higher ESG score, for example. In contrast, impact-aligned investors may also favor these companies, but will place special emphasis on the socially valuable firms that are not sufficiently profitable to easily attract commercial capital.

To operationalize this insight we study the portfolios of U.S. equity mutual funds, comparing holdings of funds that describe themselves as having sustainability goals or orientations to the broader universe of mutual funds.[16] We begin with all US mutual funds with

[16]Mutual funds have a fiduciary duty to invest only for the *financial* benefit of their clients. It is thus

Morningstar fund ratings as of 2019. To maximize data concordance, we further limit the sample to funds that Morningstar classifies as primarily investing in U.S. Equities. To study the portfolio allocation of funds, we also require funds in our sample to match to 2019Q4 security-level holdings data in the CRSP Mutual Fund Holdings Database. We combine this fund-level data with security-level data on sustainability scores from MSCI ESG Research, financials from CapitalIQ, and consensus analyst earnings forecasts from I/B/E/S. As a further restriction on the sample of mutual funds we limit to funds for which we can match at least 75 percent of their holdings by volume to our company-level financial and sustainability data.

Finally, we designate a fund as "sustainable" if it is one of the 303 mutual funds identified by Hale (2020) as an explicitly sustainable mutual fund active in 2019. This classification is designed to focus on mutual funds that go a step further than merely mentioning social considerations in their prospectuses (of which Hale (2020) counts 564 in 2019, up from only 81 in 2018). Funds designated as sustainable must make sustainability an "intentional focus" and central part of the investment strategy. Our final sample consists of 1,870 mutual funds, of which 82 are classified as sustainable.

Table 1 provides summary statistics of the mutual fund data used in the analysis. The (value-weighted) portfolio holdings of sustainable mutual funds are distinct in several dimensions. Unsurprisingly, sustainable funds are younger, smaller, and hold stocks with higher ESG ratings. More interestingly, sustainable mutual funds hold companies that have a higher return on assets – our proxy for profitability – and lower analyst growth expectations.

Figure 5 plots the portfolio weighted average ESG score and return on assets of mutual funds in our sample. This is a close empirical analogue to our equilibrium characterization in Figure 3; profits are on the y-axis and social value is on the x-axis. For the most part,

unlikely a mutual fund will explicitly state it has an objective of social value creation. Instead, these funds often argue that investment strategies that generate social impact will also generate attractive financial returns.

Table 1: Summary Statistics of Mutual Fund Sample

	(1)	(2)	(3)
		Means	
Variable	Full Sample	Sustainable Funds	Difference
Fund Size (millions)	3,682	1,731	-2,041
	(16,256)	(4,092)	(1,883)
Number of Securities Held	197	193	-4
	(269)	(192)	(31)
Fund Age	16.5	10.4	-6.4***
	(13.6)	(10.6)	(1.6)
Morningstar Fund Rating	3.17	3.38	0.21
	(1.03)	(1.00)	(0.14)
Average Holding ESG Score	4.86	5.20	0.36***
	(0.22)	(0.25)	(0.02)
Environmental Score	5.49	6.11	0.65***
	(0.87)	(0.72)	(0.10)
Social Score	4.52	4.90	0.39***
	(0.28)	(0.30)	(0.03)
Governance Score	5.89	5.96	0.07***
	(0.19)	(0.16)	(0.02)
Return on Assets	5.79	6.85	1.11***
	(2.06)	(1.50)	(0.24)
Earnings Growth Forecast	11.40	10.51	-0.93**
	(3.68)	(2.46)	(0.43)
Observations	1,758	78	1,758

Notes: This table reports summary statistics for the sample of mutual funds used in the empirical analysis in Section 5. The sample and data are constructed based on end of year 2019 data. A mutual fund is in the sample if it has a Morningstar rating, is classified by Morningstar as investing primarily in U.S. Equities, can be matched to the CRSP Mutual Fund Holdings Database, and for which we can match at least 75 percent of holdings by volume to sustainability and financial data from MSCI ESG and Capital IQ, respectively. A fund is classified as "Sustainable" if it is on the list of sustainable mutual funds in Hale (2020). The table reports group means and differences, as well as standard deviations in parenthesis.

Table 2: Determinants of Sustainable Fund Ownership of Securities

Dependent variable =	Sustainable Fund Relative Portfolio Weight				
	(1)	(2)	(3)	(4)	(5)
ESG Score	0.708***			0.710***	0.739***
	(0.089)			(0.088)	(0.097)
Environmental Score		0.158***			
		(0.023)			
Social Score		0.195***			
		(0.037)			
Governance Score		0.090*			
		(0.053)			
Profitability			0.011	-0.000	0.005
			(0.010)	(0.010)	(0.009)
Growth Expectations			0.005	0.005	0.008
			(0.005)	(0.005)	(0.005)
Industry FE	No	No	No	No	Yes
Number of Companies	1,342	1,342	1,342	1,342	1,342
R^2	0.072	0.046	0.003	0.074	0.132

Notes: This table reports cross-sectional regressions describing the relationship between attributes of US equity issuers and the extent to which their stock is held by a sample of "Sustainable" mutual funds, as identified by Hale (2020). The dependent variable is the ratio of a stock's weight across the aggregate portfolio of "Sustainable" mutual funds relative to the stock's weight in the overall mutual fund sample. *ESG Score*, and the three component subscores, are company-level metrics compiled by MSCI ESG Research as of 2019. *Profitability* is the company's return on assets in 2019 and *Growth Expectations* is the I/B/E/S median consensus forecast for long term earnings growth. Both variables are measured as percentages. Column 5 controls for industry fixed effects. Robust standard errors are reported in parenthesis.

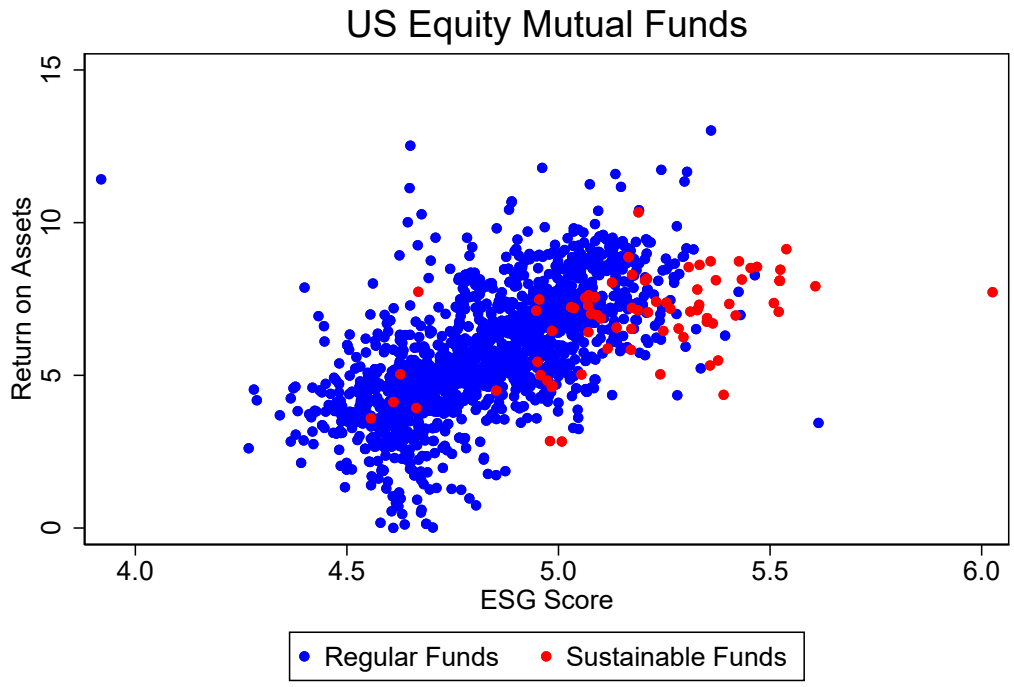

Figure 5: Portfolio Characteristics of Equity Mutual Funds

Notes: This figure shows a scatter plot of, for a sample of mutual funds, the portfolio weighted ESG score of the mutual fund and the portfolio weighted return on assets of companies in the fund. The data is cross-sectional as of end of year 2019. Mutual funds identified by stock is held by Hale (2020) are plotted in red, and all other funds identified by Morningstar as primarily investing in US Equities are plotted in blue. ESG scores are from MSCI ESG Research.

sustainable mutual funds appear to act as if they are values-aligned. At least on average, sustainable mutual funds do not invest in companies that are less profitable and thus less able to raise capital from commercial investors. This is in part because of a positive correlation between ESG scores and return on assets. But as we show in regression analysis below, even controlling for ESG scores, we do not find that sustainable mutual funds invest in companies with lower profitability.

Table 2 reports regression results exploring which company characteristics are associated with overweighting in sustainable fund portfolios. These cross-sectional regressions are estimated on the sample of publicly traded companies for which we have sufficient financial and ESG score data and are able to match to our data on mutual fund holdings. The dependent variable is the weight of the stock in the aggregate portfolio of sustainable mutual funds relative to the stock's portfolio weight in the broad sample of funds. A value of one means sustainable funds hold a stock in the same proportion as the broader sample of mutual funds, and a value of two indicates sustainable funds hold a stock in double the weight of the average mutual fund. Table 2 shows that ESG scores alone, and not measures of profitability and growth, are the only economically and statistically significant predictors of sustainable mutual fund holdings. Thus, there is no detectable evidence that sustainable mutual funds take into consideration the ability of the companies they invest in to attract commercial capital. As sustainable mutual funds behave as if they are values-aligned, our theoretical results may offer guidance as to how they can increase both their financial returns and impact.

5.2 The Preferences of Social Investors

The previous analysis showed US Equity mutual funds describing themselves as investing with sustainability considerations appear to adopt values-aligned investment strategies. Does this reflect the underlying preferences of the investors in these funds? Or, does it highlight an opportunity to better align investment strategies with the underlying objectives of investors

who seek to create an impact in the world, rather than merely being associated with it? In this section we provide new narrative evidence suggesting that investors do value impact, and we survey the academic literature investigating the social preferences of investors.

A first indication that social investors value impact comes from the marketing statements of ESG and sustainable investment funds, which routinely allude to the positive impact of their investment strategies. For a few examples of many, Nuveen, an investment management firm with one trillion dollars in assets under management, asserts that ESG investing is the approach "that is most likely to produce optimal financial and societal outcomes."[17] Candriam, a socially responsible investment fund with €130 billion under management, asserts on its webpage that they "invest in the future, channeling capital for the common good."[18] Calvert, another provider of ESG mutual funds with $23 billion in assets under management, goes one step further; their website allows investors to calculate the impact of their investment in a Calvert mutual fund across a variety of outcomes.[19] These examples indicate that ESG investment funds aim to attract investors with a direct preference for the creation of social value.

Approaching the question from a different angle, several academic papers examine the motivation of investors using laboratory experiments. Drawing on a population of real-world investors, Heeb et al. (2021) report on an experiment in which subjects were asked their willingness to pay for an asset, and the authors varied both the asset's financial return and environmental impact. The authors find that willingness to pay is higher for assets with a positive impact, but that willingness to pay is insensitive to the level of impact. While

[17]See: https://www.tiaa.org/public/pdf/how_nuveen_uses_responsible_investing_across_asset_classes.pdf Last Accessed: November 27, 2020

[18]See: https://www.candriam.com/en/private/about-us/ Last Accessed: November 27, 2020

[19]In fact, Calvert's calculation of the impact of an investment conflates measures of social value of portfolio companies with the impact of an investor in these companies. For example, it reports that a $10,000 investment in Calvert US Large-Cap Core Responsible Index Fund results in an annual reduction in emissions equivalent to burning 147 gallons of gasoline. This figure is based on the difference between the value-weighted emissions of constituents in the Calvert fund and the Russell 1000 Index. See: https://www.calvert.com/what-is-your-impact.php Last Accessed: November 27, 2020

the authors did not vary whether investment in the asset creates an impact or is merely an association with impact, this evidence still suggests that investors do not have fully impact-aligned preferences. Bonnefon et al. (2019) report on a similar exercise with a sample of respondents drawn from Amazon's Mechanical Turk. The authors elicit respondents' willingness to pay for a synthetic asset that offers both a financial return and a donation to charity. Counter to the findings of Heeb et al. (2021), the authors find that willingness to pay does scale with the impact of the asset. However they also find that varying whether respondents are pivotal in the impact, or whether they are merely paying for association with the impact does not influence their willingness to pay, suggesting that their sample was primarily motivated by values-association.

Outside of the investment context, a large literature investigates whether donors are motivated by warm glow (what we refer to as values-alignment) or pure altruism (what we refer to as impact-alignment). One of the most popular tests of whether donors are pure altruists relies on the extent to which a subject's donations are crowded out by the donations of others to the same cause. If a subject is only motivated by her own consumption and total social impact, then if a social planner (or laboratory experimenter) were to tax the subject by \$1 and donate it to charity, that should crowd out her subsequent donation to the same charity by exactly \$1. In contrast, if she values being associated with the output of the charity, then taxing her \$1 and reallocating it to the charity should reduce her donation by less than \$1. A number of papers find experimental evidence of imperfect crowd out (see e.g. Andreoni (1993), Bolton and Katok (1998), Chan et al. (2002), and Gronberg et al. (2012)) indicating that warm glow is a motive in charitable giving. Ottoni-Wilhelm et al. (2017) conduct an experiment that utilizes this same crowding out test, but does so at varying baseline levels of donations to a particular charity. The authors demonstrate that this allows them to quantify the relative importance of warm glow and pure altruism as motives for giving, and they conclude that pure altruism is the more important factor among their sample.

While the evidence above paints a mixed picture about the nature of altruism, our assessment is that it suggests that impact-alignment (or pure-altruism) is an important factor in the motivations of social investors.

5.3 Is There a Greenium for High Social Value Companies?

The final ingredient necessary for our main result is that investors bid up the prices of socially valuable companies, or equivalently lower their cost of capital, relative to the valuations and costs of capital that these companies would have achieved in the absence of social investors. Detecting such effects in the data is complicated by the fact that, regardless of social motivations, there is substantial debate about the degree to which the social value of companies is associated with fundamental risks, and if these risks are correctly priced (e.g., Andersson et al. (2016).)

Unsurprisingly, the evidence is mixed. Studies measuring ex-post performance of companies ranking highly in sustainability or social responsibility measures often find they outperform other stocks on a risk-adjusted basis, i,e. Eccles et al. (2014). In general, such out-performance could come from a higher cost of capital, changes in fundamentals, or a decrease in the required return of investors going forward. Other studies find it is *low* social value companies that outperform (Hong and Kacperczyk (2009)). Using ex-ante cost of capital measures, El Ghoul et al. (2011) find socially responsible companies face a lower cost of equity capital than other companies by 56 basis points. Studying green bonds, Baker et al. (2018) finds a small "greenium" in municipal bonds, while Larcker and Watts (2020) and Flammer (2021) find no premium for green bonds in municipal and corporate bond markets, respectively. Flammer (2021) does find that the issuance of green bonds is associated with a positive equity price reaction and an increase in equity ownership by "green" investors. Hartzmark and Sussman (2019) show that the introduction of Morningstar mutual fund sustainability ratings lead to significant flows into funds with high sustainability ratings and out

of funds with low sustainability ratings, consistent with investor preference for this attribute, but did not find significant pricing effects.

While there is no clear consensus that socially valuable companies face a lower cost of capital, measurement issues are a significant obstacle to resolving this debate. Existing studies have not isolated potential pricing effects arising from investors' non-fundamental taste for these assets from other confounding determinants of valuation and cost of capital.

5.4 Practical Considerations

Our theoretical results suggest that social investors could have more impact and higher financial returns if they were to reallocate their investments away from projects that are profitable at commercial rates and towards projects that require subsidy to be viable. How would this work in practice? In private markets this would principally require that social investors not undercut socially-neutral investors when searching for companies to finance.[20] Our results do not imply that impact-aligned investors must deploy all of their capital in deals that have not attracted the interest of socially-neutral investors, but to the extent that an impact-aligned investor is not able to deploy all of their capital in this way, they have effectively exhausted their opportunities for impact (or at least those that justify the associated financial concessions).

In public markets there are no opportunities to finance firms that have not attracted socially-neutral investment. Nevertheless, our analysis in Section 4 demonstrates that investors can have an impact by subsidizing impactful projects that would not be profitable within firms that have attracted some socially-neutral capital. As discussed in Section 4 this could be achieved through a variant of green bonds that offer project-specific financing and not only verify that the project is socially or environmentally valuable but also verify that

[20] In our model, impact-aligned investors also do not undercut one another, however we conjecture that in a dynamic variant of the model, displacing an impact-aligned investor would still contribute to higher social value creation than displacing a commercial investor.

it would not have been profitable without subsidy.

6 Conclusion

This paper provides a new framework to understand how values-based investing generates social impact in an environment where social and commercial investors compete. We analyze a model in which investors influence social outcomes through their asset allocation, and show that equilibrium asset allocation differs in important ways depending on precisely how social investors think about social value creation.

Investors following values-aligned investment strategies, which closely resemble the construction of conventional ESG and emissions reduction portfolios, have limited impact because they displace commercial investors who do not care about social value creation but would have supported some socially valuable companies anyway. Further, because values-aligned investors place intrinsic value on owning socially valuable firms, they compete with one another and push up the price of firms that could have been financed by commercial investors. From the perspective of generating impact, we show that this financial concession is inefficient. We identify an alternative investment approach, which only makes financial concessions to subsidize firms that would not be viable at the commercial cost of capital, and which generates more impact and also higher financial returns.

We demonstrate two further implications of the idea that impact can arise from not displacing investors who care less about social value. First, we show that when there is heterogeneity in the altruism of impact-aligned investors, the equilibrium involves negative assortative matching–high altruism investors finance lower social value projects so as to avoid displacing lower altruism investors from investing in high social value projects. Second, from a firm's perspective, we show that making a firm more profitable can also make it more impactful. Social investors who care about their financial return and their contribution to social value seek not to invest in firms that could have attracted commercial capital. Taking

a firm that would have been financed by an impact-aligned investor, and making it more profitable, allows it to utilize less socially valuable capital and frees up the impact-aligned capital to support a new firm.

When firms have an intensive margin of scale, impact-aligned investors' desire not to displace commercial investors also gives rise to "blended finance." Social investors can maximize their impact by co-investing with commercial investors at different terms, so as to leverage commercial investment rather than displacing it. This resembles existing green bonds, but with an important difference. In practice green bonds must only show that the use of proceeds goes towards funding investment with desirable social attributes. Our framework highlights that these securities should be further restricted to only fund investments that are also not viable at the commercial cost of capital.

The practical importance of our findings hinges on three factors–the degree to which social investors have preference for impact creation, whether social investors follow values-aligned strategies, and whether social investors contribute to an overvaluation of companies with high social value whose projects would otherwise be profitable at the commercial cost of capital. We show empirically that a large class of social investors–ESG equity mutual funds–seem to exhibit values-aligned investment strategies in that they favor companies associated with high social value, but do not avoid investment in the most commercially viable of these firms. Surveying the literature, we find inconclusive and mixed evidence, however, on whether this reflects the true preferences of investors, and whether this behavior inflates the valuation of high social value companies.

Our analysis raises several unanswered questions. First, it highlights the importance of better measurement of the behavior and valuation effects of socially minded investment strategies. Further, it raises the question of how investors should measure their own impact, given that the impact of an investor depends not only on the social value of the firms they support, but also on whether those firms could have received financing from value-neutral investors. These questions are fertile ground for future work.

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A Equilibrium Characterizations and Proofs

A.1 Strategies that Support the Equilibrium with Values-Aligned Investors in Section 3.1

In this section we describe the strategies that support the equilibrium allocation characterized in Section 3.1. Each entrepreneur i who receives financing from a social investor at cost r_i must receive at least two such offers. If not, the social investor who supports entrepreneur i would be incentivized to deviate and offer financing at cost r^C instead.

Specifically, let the entrepreneur with the lowest social value among all those who receive financing from a social investor be denoted $\bar{i}$. The investor assigned to finance entrepreneur $\bar{i}$ makes an offer to all entrepreneurs at their assigned cost of capital r_i. All other investors make just one offer to their assigned entrepreneur. To see that a social investor has no incentive to deviate from their offer of r_i, first consider the social investors who each make one offer. For each of them, note that by construction,

$$r^C + \theta_i \bar{w} \leq r_i + \theta_i w_i$$

A direct implication is that

$$\frac{1}{2}\left(r^C + \theta_i w_i\right) < r_i + \theta_i w_i$$

which is the incentive compatibility constraint for the social investor of entrepreneur i when i receives two offers.

Now consider the investor who finances entrepreneur $\bar{i}$. This investor has no incentive to deviate as any of her contracts, if accepted, would offer her weakly less utility. As in equilibrium her only offer to be accepted is that of entrepreneur $\bar{i}$, she has no incentive to deviate by offering a cheaper cost of capital to another entrepreneur.

A.2 Characterization of the Welfare and Investor-Optimal Equilibrium with Impact-Aligned Investors in Section 3.2, and Proof of Proposition 1

In this section we characterize the welfare- and investor-optimal equilibrium; the equilibrium that maximizes the sum of social value and maximizes the sum of investor utilities.

Define $\sigma(i)$ to be any ordering over all impact-aligned social investors $i \in S$.

At step 1, social investor $\sigma^{-1}(1)$ is called to support an entrepreneur. If she chooses an entrepreneur i with profits $\pi_i \geq r^C$, or the outside option asset, assign her a price of $r_i = r^C$, and a social value of $w_i' = 0$. Else assign her a price of $r_i = \pi_i$ and a social value of $w_i' = w_i$. Social investor $\sigma^{-1}(1)$ chooses the entrepreneur i that maximizes $r_i + \theta_{\sigma^{-1}(1)} w_i'$.

At step k, social investor $\sigma^{-1}(k)$ is called on to support an entrepreneur. She chooses an entrepreneur to support according to the same process, excluding any entrepreneur that has been chosen in a previous step.

This allocation of capital can be supported in equilibrium by the following strategies: every investor offers a single contract to their assigned entrepreneur, at cost $r_i = \begin{cases} r^c & \text{if } \pi_i \geq r^C \\ \pi_i & \text{else} \end{cases}$.

Now, assign every social investor an index that is increasing in their altruism parameter θ_i, so that the least altruistic social investors have the lowest indices. Then the welfare- and investor-optimal equilibrium depicted in Figure 2 corresponds to the ordering $\sigma(i) = i$. First we will demonstrate that the equilibrium arising from any other ordering σ' that results in a different allocation produces lower social welfare than σ.

Take some σ'. Identify two social investors, j and k such that

- $\theta_j > \theta_k$
- $\sigma'(k) = \sigma'(j) + 1$

That is, j is more altruistic than k, but j chooses an investment one step before k in the

ordering σ'. If no such pair can be found then $\sigma' = \sigma$. Now consider an alternative ordering σ'', which is the same as σ' except that $\sigma''(j) = \sigma'(k)$ and $\sigma''(k) = \sigma'(j)$ (i.e. j and k are reordered but everything else is preserved). Let a denote the entrepreneur chosen by k under σ' and b denote the entrepreneur chosen by j under σ'. If in the ordering σ'', k chooses a at step $\sigma''(k)$, then the two orderings result in the exact same allocation. Else, under the ordering σ'', at step $\sigma''(k)$, k chooses b. At step $\sigma''(j)$ under the ordering σ'', j chooses an entrepreneur c such that $w_c \geq w_a$, as j is more altruistic than k. In this case, it is straight forward to show that σ'' results in an allocation with weakly higher welfare than does σ'.

Now take σ'' and repeat the above process (i.e. identify mis-ordered pairs of investors and re-order them). Continue to do so until σ'' results in the same allocation as σ. So long as the allocation arising from σ' and from σ are different, it is straightforward to show that at least one transformation resulted in a strict welfare improvement. Therefore σ induces the welfare-optimal equilibrium.

To see that σ also induces the investor-optimal equilibrium, we need only demonstrate that the shift from σ' to σ'' described above also weakly improves aggregate investor welfare. If in σ'' k chooses a then the allocation and aggregate investor welfare are the same in σ' and σ'' . Otherwise, k chooses b. This improves k's welfare by revealed preference. If in σ'' j then chooses a, then aggregate investor welfare is unchanged from σ' to σ''.

The remaining case is that under σ'', k chooses b, and j chooses $c \neq a$ with $w_c \geq w_a$ and $\pi_c \leq \pi_a$. We complete the proof by demonstrating that this results in higher aggregate investor welfare than the case where j chose a under σ''. Relative to if j had chosen a under σ'', j's welfare is higher by revealed preference. And all investors who chose before j also have weakly higher welfare, because $w_c \geq w_a$. It remains to show that this also weakly improves the welfare of investors who choose after j under σ''. Relative to the case where j had chosen a, the set of entrepreneurs that these investors can choose from is fixed, except that now c is guaranteed financing, and a remains eligible for financing. Because c has higher social value and lower profits, this can only improve the aggregate welfare of the remaining investors.

Finally, to see that prices are not uniquely pinned down across equilibria, we construct an alternative equilibrium. Consider the following example with two investors and two entrepreneurs. $r^C = 2$, $\theta_1 = \theta_2 = 1$, $w_1 = 10$, $\pi_1 = 1$, $w_2 = 10$, $\pi_2 = 0.5$. Social investor 1 makes one offer to entrepreneur 1 at price $r_1 = 0.5$. Social investor 2 makes an offer to each entrepreneur at $r_1 = r_2 = 0.5$. Entrepreneur 1 accepts investor 1's offer, and entrepreneur 2 accepts investor 2's offer. It is straightforward to verify that no party has a payoff-increasing deviation.

A.3 Omitted Proofs From Sections 3.4, 3.5 and 4

Proof of Proposition 2

First, we provide a formal definition of "sufficient density" referenced in Proposition 2. We say that the distribution of firms is *ε-dense* if for every $\pi \in [0, \hat{\pi}]$ and for every $w \in [-\hat{w}, \hat{w}]$, there is a firm i with $|\pi_i - \pi| + |w_i - w| < \varepsilon$. Proposition 2 is formally stated as follows: *Consider any values-aligned social investor i that supports a firm with $\pi_i > r^C$ and earns a return $r_i < r^C$ in equilibrium. There exists an $\varepsilon > 0$ such that if the distribution of firms is ε-dense, there exists an unfinanced firm j with profits $\pi_j > r_i$, such that if the values-aligned social investor i were to deviate and offer firm j financing at cost π_j, total social welfare would increase as would investor i's financial return.*

We sketch the proof of Proposition 2 with reference to Figure 4, where a values-aligned investor in the blue region is moved to instead support an entrepreneur in the green region. For any level of financial compromise that a values-aligned social investor makes to support a firm that could have attracted commercial financing, ε-density guarantees that there exists a firm that is not supported by any investor but that could offer a higher return than the values-aligned social investor is earning, and such that if the values-aligned social investor were to reallocate her capital to the new firm total social welfare would increase. This new firm has lower w_i than the one that the values-aligned social investor supported, but the

values-aligned social investor's contribution to social welfare is higher when supporting the new firm because it could not attract commercial financing.

Proof of Proposition 3

This is a straightforward implication of the equilibrium depicted in Figure 3. Fix any entrepreneur i who is being supported by an impact-aligned social investor. Raising his profit π_i to $\pi_j < r^C$ does not change the type of capital he attracts. But raising his profit to $\pi_j > r^C$ causes him to instead be supported by a commercial investor and his enterprise impact increases. Social welfare increases because the impact-aligned social investor can now support another entrepreneur.

Proof of Lemma 4

In any equilibrium all values-aligned social investors are indifferent between supporting any two firms with average social value exceeding $\bar{w}$. Suppose that for one such firm i, where a social investor j offers a cost of capital $r_i < r^C$ there was co-investment. That is, the firm receives some capital at $r_i < r^C$ and some capital at r^C. Then a social investor supporting another firm could deviate and offer firm i a unit of capital at $r_i' = r_i + \varepsilon < r^C$, thereby displacing one unit of commercial capital that firm i previously accepted. This social investor would have strictly higher utility than social investor j, violating the equilibrium indifference condition.

Proof of Lemmas 5, 6, 7, and 8

We directly construct the investor-optimal equilibrium. Let $\bar{k}_i$ be the equilibrium scale of firm i, representing the total social capital it is offered in equilibrium plus $\bar{k}_i^C$, which represents the amount of commercial capital firm i raises in equilibrium. Each social investor

who offers firm i a contract specifies the minimum-scale contingency $\bar{k}_i$, and the required return r_i^S solving $\pi_i\left(\bar{k}_i\right) - r^C\bar{k}_i^C - r_i^S\left(\bar{k}_i - \bar{k}_i^C\right) = q_i^C$.

There is a unique allocation of capital such that

1. $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = \pi_j'\left(\bar{k}_j\right) + \theta w_j'\left(\bar{k}_j\right)$ for all firms i and j that receive social capital
2. $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) \geq \pi_k'\left(\bar{k}_j\right) + \theta w_k'\left(\bar{k}_j\right)$ for firm i that receives social capital and firm k that does not
3. Either,
   (a) For all firms that receive social capital, $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = r^C$, or
   (b) $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) > r^C$ and $r_i^S = 0$.

Social investors offer $\bar{k}_i - k_i^C$ capital to firm i and charge r_i^S satisfying 1-3 above.

We first verify that this is an equilibrium and second verify that it is the investor-optimal equilibrium.

To see that it is an equilibrium, note that it is incentive compatible for firms to accept all contracts that they are offered, as they earn q_i^C regardless of whether they accept all of their social investment contracts and operate at $\bar{k}_i$ or whether they accept only the commercial capital and operate at k_i^C. To see that this is incentive compatible from the investor's perspective, note that the marginal social return plus the marginal profits are equalized across all firms that receive social investor financing, and higher than all of those that do not. If a social investor were to deviate, she would gain at most the marginal social return plus marginal profits of the firm she deviates to, and lose that corresponding value from the firm she supports in equilibrium. By concavity of the firms' production functions this would lower her payoff. Finally, note that social investors cannot raise their required return r_i^S at the firm they are assigned to as they would violate the firm's incentive compatibility constraint.

Now, to see that this is the investor optimal equilibrium, we note that if, holding the allocation of commercial capital fixed, the capital allocation of social investors across firms was changed, this would by definition reduce the sum of total social investor returns plus total social welfare. Reducing the amount of commercial capital that any firm i raised would weakly reduce social investor welfare, and strictly so if social investors in this equilibrium all strictly prefer their allocation to the outside option asset, as either it would result in firm contraction, or it would need to be replaced by social capital previously allocated to another firm, both of which reduce the sum of investors' welfare by more than r^C by construction. Finally, if $r_i^S = 0$ then increasing the amount of commercial capital held by any firm i is not feasible, as it would result in a social investor earning a negative return. And if $r_i^S > 0$ then by construction we have $\pi_i'\left(\bar{k}_i\right) + \theta w_i'\left(\bar{k}_i\right) = r^C$ for all firms that receive social capital, and therefore raising the level of commercial capital would not increase social investors' welfare.

Proof of Proposition 4

Consider the unique equilibrium of the model with values-aligned investors in which no investor utilizes a scale-contingency. In this setting, each firm supported by a social investor operates at the scale $\bar{k}_i$ such that $\pi_i'\left(\bar{k}_i\right) = r_i$. Now consider the case where there are two firms i and j in equilibrium that are both supported by social investors and for whom $\frac{w_i\left(\bar{k}_i\right)}{\bar{k}_i} > \frac{w_j\left(\bar{k}_j\right)}{\bar{k}_j}$ and $w_i'\left(\bar{k}_i\right) < w_j'\left(\bar{k}_j\right)$. The former inequality implies that the equilibrium cost of capital will be lower for firm i, i.e. $r_i < r_j$, and the latter implies that the marginal social value of expanding firm j is higher than for expanding firm i. Now, consider the an arbitrary social investor who supports firm i, and suppose that this investor were to deviate in the following way. Firm i is offered ε less capital, a scale-contingent contract of $\bar{k}_i - \varepsilon$, and charged $\pi_i\left(\bar{k}\right) - \pi_i\left(\bar{k} - \varepsilon\right)$ less. Firm j is offered ε more capital, a scale contingent contract of $\bar{k}_j + \varepsilon$, and charged $\pi_j\left(\bar{k} + \varepsilon\right) - \pi_j\left(\bar{k}\right)$ more. Both firms would accept these contracts, as

they do not change the entrepreneurs' share of the profits. And for sufficiently small ε, the deviation would result in higher social welfare as $w_i'\left(\bar{k}_i\right) < w_j'\left(\bar{k}_j\right)$, and increased profit for the investor as $r_i < r_j$.

Proof of Proposition 5

Consider the case where social investors are impact-aligned, and in equilibrium each investor has a strict preference to support some firm i rather than investing in the outside option asset. And now take one such firm i that receives social capital. Let $\bar{k}_i$ be its equilibrium scale. Modify firm i's profitability to a new function $\tilde{\pi}$ in the following way.

$$\frac{d}{dk}\tilde{\pi}\left(k\right) = \begin{cases} \max\left\{\frac{d}{dk}\pi\left(k\right), r^C\right\} & \text{if } k \leq \bar{k}_i \\ \frac{d}{dk}\pi\left(k\right) & \text{else} \end{cases}$$

This modification increases the profitability of firm i so that it can attract commercial capital up to its former equilibrium scale $\bar{k}_i$, and holds fixed the marginal profitability of firm i at all higher scales. Impact-aligned social investors now recognize that their investment up to $\bar{k}_i$ in equilibrium has no impact. Thus the full mass of impact-aligned social investors who used to support firm i up to $\bar{k}_i$ are now free to allocate their capital elsewhere (potentially in part by increasing firm i's scale beyond $\bar{k}_i$), increasing social welfare.

A.4 Impact-Aligned Preferences in a Model with A Continuum of Investors and Projects

In this section we discuss the behavior of impact-aligned social investors in a setting with a continuum of investors and projects. In a model with a continuum of projects, social investors cannot influence aggregate social welfare $\int_{\bar{E}} w_j dj$. An intuitive formulation of the utility of impact-aligned social preference – which would not conform with the impact-aligned

behavior in our analysis – would be

$$r_i + \theta \int_{\bar{E}} w_j dj \tag{11}$$

Because a single investor cannot influence aggregate social welfare, social investors with the above preferences would single-mindedly optimize their financial return.[21] Nevertheless, the behavior of impact-aligned investors that we analyze can also persist in a continuum model.

Formally, we model the preferences of impact-aligned social investors as arising from the limit of a sequence of discrete models, each of which has a finite but increasing number of projects n that can be financed. In each of these models we assume that impact-aligned social investors have utility

$$\frac{1}{n}\left(r_i + \theta \sum_{j \in \bar{E}} w_j\right). \tag{12}$$

As the number of projects financed increases, the contribution of investor i to social welfare *as a fraction of total social welfare* vanishes, yet so does the amount that she values her own financial return. These preferences might be understood to represent the fact that an impact-aligned social investor places the same relative value on her own financial return and the welfare of a fixed set of others no matter how large is the set of total financed projects.

In contrast, the social preferences represented by Equation 11 can be understood as arising from the continuous limit of the same set of discrete models, in which social investor preferences are

$$r_i + \frac{1}{n}\theta \sum_{j \in \bar{E}} w_j.$$

This would correspond to the idea that relative to her own financial return, a social investor places less value on the welfare of a fixed set of others, as the set of all financed projects grows.

[21] Indeed, this aligns with Pastor et al. (2020), which considers social investors who have preferences for aggregate social value and concludes that this preference does not influence investor behavior.