Competitive Two-Part Tariffs

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Abstract

We study competitive two-part tariffs in a model of asymmetric duopoly firms that offer (vertically and horizontally) differentiated products. We show that the sign of the markup for each product depends on the average expected demand among all customers as well as the marginal rate of substitution of the demand for access between the marginal price and fixed fee. We also provide necessary and sufficient conditions for marginal-cost pricing to be an equilibrium. Under the logit demand system with an outside option, we show that competitive two-part tariffs, even in the symmetric setting, are not efficient. When firms are asymmetric, our results indicate that the equilibrium strategy in two-part tariffs involves "cross-subsidization" between the marginal price and fixed fee for the less efficient firm, with the efficient firm offering a marginal price above its marginal cost.

Keywords: Product differentiation, competition, two-part tariffs, marginal-cost pricing, cross-subsidization

JEL Codes: L11, L13, L15

1 Introduction

Two-part tariffs (2PTs) are prevalent in many industries. Examples include credit cards, membership retail stores (e.g., *Costco* and *Sam's Club*), and TV and wireless carrier subscriptions, among others.[1] The liberalization of the *British electricity market* at the end of the 1990s is perhaps one of the best examples of competition in 2PTs between asymmetric firms. The retail sector was divided into 14 monopolized regional markets before it was opened to competition. In the years before and after liberalization, firms predominantly offered single 2PTs. However, after liberalization, these markets were characterized by the considerable variability of the tariffs offered in each region at each point in time.

How do firms compete in 2PTs in the presence of consumer heterogeneity? Are competitive 2PTs efficient in the sense that the price is equal to marginal cost? When do the firms engage in cross-subsidization strategy by pricing below marginal cost and extracting consumer surplus using a high fixed fee? In this paper, we answer these questions by studying competitive 2PTs in a general model of multidimensional consumer heterogeneity, using both the Hotelling and a general discrete choice approach to horizontal differentiation.

To study these issues in more detail, we construct a model with two asymmetric firms (with asymmetric marginal costs and differentiated products), both offering 2PTs and competing for horizontally differentiated consumers with heterogeneous tastes for product quality and variable demands.[2] Initially, we study a model in which consumers have uniformly distributed horizontal brand preferences, à la Hotelling.[3] We show that, under this assumption, the sign of the markup of the marginal price for each firm is determined by the average expected demand among all customers who choose the firm's product as well as the marginal rate of substitution of the demand for access (MRSA)—or the probability of participation—between the marginal price and the fixed fee. The MRSA describes the demand of the set of *marginal consumers* who are indifferent between accepting the 2PTs from the firm and from its rivals. We further provide necessary and sufficient conditions for marginal-cost-based 2PTs to be an equilibrium. The conditions require that, for each firm's

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[1] The largest membership (or subscription) business model, *Amazon Prime*, *Amazon's* loyalty program, has driven *Amazon's* share of U.S. internet retailing to up to 52% of the market in 2018 (Wells et al., 2019). In January 2020, CEO Jeff Bezos announced that "the company's Amazon Prime membership program now boasts more than 150 million paying customers across the globe" (Del Rey, 2020). Amazon has expanded *Amazon Prime*, offering various benefits: access to Amazon Instant Video, free cloud storage through Amazon Web Services, access to special deals (*Lightning Deals*) on *Prime Days*, shipping on everyday essentials (*Prime Pantry*) and groceries (*Amazon Fresh*). More recently, internet-enabled subscription services have expanded exponentially: business-to-consumer (B2C) subscription services have been growing at 200% annually since 2011 (McCarthy and Fader, 2017). B2C subscription businesses sell a wide variety of products, including meal kits (*Hello Fresh* and *Blue Apron*), grooming products (*Dollar Shave Club*), and clothes (*Stitch Fix and Trunk Club*) (for details, see McCarthy and Fader (2017)). In all of the above examples, consumers pay a positive fixed fee that allows them to buy products and services at a given (positive or zero) unit price, a characteristic of 2PTs.

[2] We assume that consumer heterogeneity is described by a horizontal brand preference parameter and multidimensional taste parameters for product quality.

[3] For most of this paper, we assume that consumers are single-homing and that the market is fully covered; that is, all consumers buy from one firm and both firms sell strictly positive quantities. In Section 5, we consider a general discrete choice model of consumer demand allowing for outside options, which includes an example of the logit demand with an outside option.

product, when the price is equal to the marginal cost, the average expected demand among all customers who choose the product equals the MRSA between the marginal price and the fixed fee. Our analysis further indicates that if the marginal costs are asymmetric and the products of the two firms are symmetric, then the above conditions are easily violated.

We present two special settings of our model in which marginal-cost pricing is not a Nash equilibrium. In the first setting, two firms have symmetric demands but asymmetric marginal costs. We show that the optimal strategy for the less efficient firm (the firm with a higher marginal cost) is to set its marginal price below its own marginal cost and compensate for this loss with a fixed fee. On the other hand, the optimal strategy for the efficient firm is to set its marginal price above its own marginal cost but below that of its rival.[4] In the second setting, two firms have symmetric marginal costs but asymmetric demands. Here, the equilibrium involves the inferior firm pricing below its marginal cost, whereas the optimal strategy for the firm with superior vertical goods is to set its marginal price above its rival's price (and above the common marginal cost). Hence, in both settings, the disadvantaged firm cross-subsidizes between the tariffs (i.e., sets its marginal price below its marginal cost, and charges a positive fixed fee), and the advantaged firm sets its marginal price above its marginal cost.

We extend our analysis by considering a discrete choice model of consumer demand. The qualitative results described above still hold. For instance, the sign of the markup for each firm is determined by the average expected demand among all customers who choose the firm's product as well as the MRSA. Surprisingly, our results indicate that when consumers' horizontal preferences are represented by logit with an outside option, marginal-cost pricing, even in the symmetric setting, is not an equilibrium. The equilibrium, in this case, involves both firms pricing above the marginal cost, resulting in inefficiency.

2PTs were traditionally viewed as "price discrimination devices, employed exclusively by firms with market power" (see Hayes, 1987).[5] When consumers are homogeneous, a monopolistic firm could price efficiently at the marginal cost and use a fixed fee to fully extract the consumer's surplus. However, marginal-cost pricing under 2PTs may not be optimal for the monopolist facing heterogeneous consumers. In the presence of one-dimensional heterogeneity of consumer tastes, Schmalensee (1981) shows that the price-cost markup is determined by the difference between the average demand of the participating consumers and the demand of the marginal consumer (see Varian, 1989, for a summary). Thus, the optimal monopolistic 2PTs involve cost-based pricing only when the average consumer has the same efficient demand as the marginal consumer. If the marginal consumer demands more than the average consumer, the optimal price in the 2PTs would be less than the marginal cost. We provide a similar intuition for cross-subsidization in our model with competition and full market coverage in which consumers are horizontal and vertically differentiated. We find that in the context of multidimensional consumer heterogeneity, the MRSA

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[4]This result contrasts with the one in a model in which both firms use linear pricing (LP): as the number of tools available to firms increases (from one to two), they have incentives to establish "cross-subsidies" across the tariff instruments (fixed fee and marginal price), which is not possible in the LP model.

[5]A seminal contribution on monopolistic 2PTs is the study by Oi (1971).

of a firm provides a more general description of the consumers' participation incentives than the demand of the marginal consumer regarding the monopoly 2PTs in the context of one-dimensional consumer heterogeneity.[6]

The early literature on competitive price discrimination typically assumes horizontally differentiated consumers with homogeneous tastes for quality (or homogeneous demand) and symmetrically competing firms; that is, firms with symmetric marginal costs and product demands. However, these assumptions are restrictive for many applications of interest and do not match the main characteristics of the industries that use 2PTs. In many of the examples mentioned above, firms compete by offering 2PTs, and they share some common features. First, consumers are heterogeneous in multiple dimensions (e.g., their tastes for product quality as well as their horizontal brand preferences). Second and more importantly, firms are often asymmetric; that is, they usually have asymmetric marginal costs and offer asymmetrically differentiated products. The liberalization of the British electricity market at the end of the 1990s is perhaps one of the best examples of competition in 2PTs between asymmetric firms. According to Davies et al. (2014), in two-thirds of the cases, the entrant offered a lower unit price with a higher fixed fee than the incumbent.[7] Moreover, they show that this tariff asymmetry was persistent. Indeed, some of the industries in which 2PTs are widely practiced have evolved from being natural monopolies before the recent worldwide liberalization of their sectors (e.g., energy and communication) and have experienced competition from more efficient firms (i.e., lower marginal costs) with new products and differentiated demands.

The recent literature on competitive price discrimination shows that when the market is fully covered and when symmetric firms offer nonlinear pricing schedules, there exists an equilibrium in which each firm offers a simple 2PT contract with a marginal price equal to the marginal cost. A seminal contribution to this literature is Armstrong and Vickers (2001), who study competitive nonlinear pricing when consumers are differentiated à la Hotelling, have private information about their tastes for quality, and purchase all products from a single firm (one-stop shopping). Rochet and Stole (2002) interpret the quantity in Armstrong and Vickers (2001) as quality (so consumers choose a price-quality pair) and show that if firms are symmetric and transportation cost is low enough to guarantee full coverage, in equilibrium, firms offer a cost-plus-fee pricing schedule.[8] However, this surprisingly simple yet elegant result depends on the assumption of symmetry of the firms (and full market coverage), thus excluding cases in which firms may have different marginal costs or may offer asymmetrically differentiated products.[9]

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[6]Under monopoly, with one dimensional consumer heterogeneity the MRSA between the marginal price and fixed fee is the demand of the set of marginal consumers in the participation set.

[7]In this market, there are other types of asymmetries not considered in our paper. In particular, most of the electricity suppliers were also active in the gas market. Some of the firms were vertically integrated into electric generation; National Grid provides transmission, and there is a monopoly distributor in each of these regions. For a complete description of the British electricity market, see Davies et al. (2014).

[8]Note that if firms are symmetric and the market is competitive (all consumers buy from at least one firm), the results of Armstrong and Vickers (2001) and Rochet and Stole (2002) imply that there would be an efficient quantity (or quality) provision supported by the marginal-cost-based 2PTs.

[9]Armstrong and Vickers (2010) generalize the model in Armstrong and Vickers (2001) by assuming that consumers are allowed to multi-shop (buy from both firms or from just one) and find that in equilibrium, firms offer marginal-cost-based 2PTs. Hoernig and Valletti (2011) consider a version of the model in Armstrong and Vickers (2010) in

Our analysis extends the findings of Armstrong and Vickers (2001) and Rochet and Stole (2002) in two ways. First, we provide necessary and sufficient conditions under which marginal-cost-based 2PTs are an equilibrium, given horizontally differentiated consumers with heterogeneous quality preferences and asymmetric firms. These necessary and sufficient conditions allow us to identify environments in which marginal-cost-based 2PTs are not an equilibrium. Given that the strategy space of non-linear tariffs used by Armstrong and Vickers (2001) is larger than the set of 2PTs that we have focused on, our results therefore also lay out the conditions under which a pair of 2PTs cannot be a Nash equilibrium in a larger strategy space, such as nonlinear tariff space. Similarly, we show that even if firms are symmetric, marginal-cost-based 2PTs may not be an equilibrium, such as in the logit model with an outside option. Second, we characterize the equilibrium outcome of the model when a pair of marginal-cost-based 2PTs is not an equilibrium and show that in two special settings—asymmetric marginal costs or asymmetric demands—the equilibrium involves cross-subsidization between the marginal price and the fixed fee for the less efficient firm.

Yang and Ye (2008) consider a model similar to Armstrong and Vickers (2001) and Rochet and Stole (2002), and study the case in which consumer types on the vertical dimension are not fully covered; that is, they consider a model in which the lowest consumer type covered (in the market) is endogenously determined. They show that when the market structure moves from monopoly to duopoly, more types of consumers are served and quality distortions decrease. Based on a model similar to Yang and Ye (2008), Shen et al. (2016) provide conditions under which entry prompts an incumbent to expand or contract its low end of the product line. Note that in our model, we assume that all consumer types on the vertical dimension are covered; that is, firms do not exclude the low/mid end of the market.

Yin (2004) analyzes a model of 2PT competition with horizontal consumer heterogeneity in which the quantity interacts with the transportation cost (for example, a "shipping" cost) in consumers' utility and consumers have homogeneous tastes for quality. He shows that marginal prices are equal to marginal costs if and only if the demand of the marginal consumer is equal to the average demand. For instance, if the horizontal taste parameter is additively separable from the price (transportation cost is a "shopping" cost), marginal price is equal to the marginal cost in equilibrium. We show that this result does not hold if consumers have heterogeneous taste preferences and firms have asymmetric marginal costs or asymmetric demands. In this case, the less efficient firm (the one with the higher marginal cost) sets its prices below its own marginal cost.

Hoernig and Valletti (2007) consider a model where consumers are horizontally differentiated, à la Hotelling, and mix goods offered by two firms. They show that when both firms use 2PTs, marginal prices are equal to the marginal costs if and only if both firms are located at the same spot. Griva and Vettas (2015) study a duopoly model in which firms use 2PTs and offer homogeneous goods to a population of vertically differentiated consumers (heterogeneous usage rate). They show that when one price of the components is fixed for both firms, the market is segmented; that is,

which vertical and horizontal taste parameters are correlated. They show that neither 2PTs, nor full exclusivity, can arise in equilibrium. For a review of this literature, see Armstrong (2016).

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low-usage consumers choose the low-fee firm and high-usage consumers choose the low-rate firm. Our analysis does not consider any of these factors (e.g., interaction of the transportation cost with the quantity or location decisions), and thus the reasons for marginal-cost-based 2PT being an equilibrium in our study are different from those in their models.

Also related to our study is the literature on cross-subsidization under linear pricing by multiproduct firms that often price some products below marginal costs, and subsidize the resulting loss of the profits from other products. This literature provides different explanations for competitive cross-subsidization. DeGraba (2006) shows that pricing below cost could serve as a strategy to screen the most profitable consumers in a setting in which firms face heterogeneous consumers. Chen and Rey (2012) show that pricing below marginal cost for products on which a large firm competes with a smaller rival, and increasing the price on other products, allows the large firm to discriminate between multi-shoppers and one-stop shoppers. Note that in this context, loss-leading serves as an exploitative device rather than as an exclusionary instrument. Chen and Rey (2019) study multiproduct firms with different comparative advantages competing for customers with heterogeneous transaction costs. They show that firms price strong products (on which they have a comparative advantage) above cost, and price weak products below cost. Ellison (2005) examines an "add-on pricing" game in which add-on prices are unobserved and firms advertise a base good in the hope of selling add-ons at high unadvertised prices. In equilibrium, firms may price the base product below cost to subsidize the loss with the profit from add-on prices. A driving force behind this result is the correlation between the vertical taste and the horizontal preferences. Assuming independent vertical and horizontal consumer heterogeneity, Verboven (1999) obtains a similar result of high prices for add-ons. Complementary to the above studies in this literature, our paper provides a different rationale for cross-subsidization: the less efficient firm is the one that has incentives to cross-subsidize between the tariffs (fixed fee and marginal price) as an optimal strategy to extract consumer surplus.

The paper proceeds as follows. Section 2 sets up the model. In Section 3, we study some general properties of our model. Section 4 provides two settings in which marginal-cost-based 2PTs are not a Nash equilibrium and the equilibrium involves cross-subsidization by the less efficient firm. Section 5 extends our analysis based on the Hotelling specification to allow for general market share functions. Section 6 concludes.

2 Model

Two firms, A and B, offer differentiated products to a population of heterogeneous consumers. We assume that both firms can produce their products at constant marginal costs, denoted by c_A and c_B, respectively. We start with a single-homing (one-stop shopping) Hotelling model with consumers buying all products from one or the other firm or else consuming their outside option.[10]

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[10]In Section 5, we analyze a general discrete choice model of random utility maximization.

Each consumer is endowed with a type $(x, \boldsymbol{\theta})$, where x is uniformly distributed on the unit interval independently of the distribution of $\boldsymbol{\theta} \equiv (\theta_1, ..., \theta_n) \in \boldsymbol{\Theta} \equiv [\underline{\theta}, \bar{\theta}]^n$, which is continuously distributed with cumulative distribution $\mathbf{G}(\cdot)$. The consumer's preferences for the two differentiated products can be represented by the utility function $u_A(q_A, \boldsymbol{\theta}) - tx$ if she buys from A and $u_B(q_B, \boldsymbol{\theta}) - (1-x)t$ if she buys from B, where x is the distance to firm A (and $1-x$ the distance to firm B), $t > 0$ is the consumer transportation cost per unit of distance, measuring the degree of horizontal product differentiation, and $\boldsymbol{\theta}$ represents the "vertical" taste parameter for quality.

The next assumption describes the set of utility functions.

Assumption 1. *The utility function $u_i(q_i, \boldsymbol{\theta}) : \mathbb{R}_+ \times \boldsymbol{\Theta} \to \mathbb{R}_+$ is twice continuously differentiable and satisfies $\frac{\partial u_i(q_i, \boldsymbol{\theta})}{\partial q_i}\Big|_{q_i=0} > c_i$, $\frac{\partial^2 u_i(q_i, \boldsymbol{\theta})}{\partial q_i^2} < 0$ for $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and $\frac{\partial^2 u_i(q_i, \boldsymbol{\theta})}{\partial q_i \partial \theta_k} > 0$, for $k \in \{1, 2, ..., n\}$.*

The firms use 2PTs, which include a marginal (unit) price, p_i, and a lump-sum fee, F_i, for $i \in \{A, B\}$. To avoid expositional complications, we define the set of feasible unit prices of both firms as $\mathcal{P}$. Given (p_i, F_i), a consumer with taste parameter $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ decides to buy $q_i : \mathcal{P} \times \boldsymbol{\Theta} \to \mathbb{R}_+$ units from firm $i \in \{A, B\}$, where

$$q_i(p_i, \boldsymbol{\theta}) = \arg\max_{q_i \in \mathbb{R}_+} \{u_i(q_i, \boldsymbol{\theta}) - p_i q_i\}.$$

The net utility $U_i(p_i, F_i, \boldsymbol{\theta})$ is
$$U_i(p_i, F_i, \boldsymbol{\theta}) \equiv v_i(p_i, \boldsymbol{\theta}) - F_i,$$

where $v_i(p_i, \boldsymbol{\theta})$ is the indirect utility "offered" by firm i, defined by

$$v_i(p_i, \boldsymbol{\theta}) \equiv \max_{q_i \in \mathbb{R}_+} \{u_i(q_i, \boldsymbol{\theta}) - p_i q_i\}.$$

We will focus on the case with $E[v_i(c_i, \boldsymbol{\theta})] > 0$, where $v_i(c_i, \boldsymbol{\theta})$ is the maximum surplus offering a good at the marginal cost, c_i, by firm $i \in \{A, B\}$ for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$.

Note that (A1) implies that the buyer's demand function and the monopoly profit function—$q_i(p_i, \boldsymbol{\theta})$ and $\pi_i(p_i, \boldsymbol{\theta}) = (p_i - c_i) q_i(p_i, \boldsymbol{\theta})$, respectively—are continuously differentiable and that $q_i(p_i, \boldsymbol{\theta})$ is strictly decreasing in p_i and increasing in $\boldsymbol{\theta}$, for $i \in \{A, B\}$. Moreover, the indirect utility function, $v_i(p_i, \boldsymbol{\theta})$, satisfies $q_i(p_i, \boldsymbol{\theta}) = -\partial v_i(p_i, \boldsymbol{\theta})/\partial p_i$ by Roy's identity $\frac{\partial^2 v_i(\cdot)}{\partial p_i \partial \theta_k} < 0$ for all $i \in \{A, B\}$ and for $k \in \{1, 2, ..., n\}$ (due to A1), which implies that $-v_i(p_i, \boldsymbol{\theta})$ satisfies the increasing differences property. That is, $v_i(p_i, \boldsymbol{\theta}) - v_i(p_i', \boldsymbol{\theta})$ must be monotonically nondecreasing in $\boldsymbol{\theta}$ for all $p_i, p_i' \in \mathcal{P}$ and $p_i \leq p_i'$ for $i \in \{A, B\}$.

In order to simplify our analysis, we assume full market coverage in which all consumers buy from at least one firm $i \in \{A, B\}$, and both firms sell strictly positive quantities. This assumption is equivalent to assuming a lower and an upper bound for t, which will depend on the model considered in each section.

Let $\mu_i(p_i) \equiv -\frac{E[q_i(p_i, \boldsymbol{\theta})]}{E[q_i'(p_i, \boldsymbol{\theta})]}$ to be the reciprocal of the quasi-elasticity of expected demand, where $q_i'(p_i, \boldsymbol{\theta}) \equiv \frac{\partial q_i(p_i, \boldsymbol{\theta})}{\partial p_i}$.

Assumption 2. $\frac{d\mu_i(p_i)}{dp_i} < 1$, *for* $i \in \{A, B\}$.[11]

Under (A2), the expected value of the conditional monopoly profit function,

$$E\left[\pi_i\left(p_i, \boldsymbol{\theta}\right)\right] = E\left[q_i\left(p_i, \boldsymbol{\theta}\right)\right]\left(p_i - c_i\right),$$

is single-peaked in p_i, and hence there is a unique optimal monopoly price $p_i^m \in \mathcal{P}$.

Due to the full market coverage assumption, the share of $\boldsymbol{\theta}$-consumers who decide to buy from firm $i \in \{A, B\}$ is

$$s_i\left(p_i, F_i, p_j, F_j; \boldsymbol{\theta}\right) \equiv \frac{1}{2} + \frac{v_i\left(p_i, \boldsymbol{\theta}\right) - v_j\left(p_j, \boldsymbol{\theta}\right) - F_i + F_j}{2t}, \tag{1}$$

and the share of firm $j \neq i$ is $s_j\left(p_j, F_j, p_i, F_i; \boldsymbol{\theta}\right) = 1 - s_i\left(p_i, F_i, p_j, F_j; \boldsymbol{\theta}\right)$. The problem of each firm $i \in \{A, B\}$ is

$$\max_{p_i, F_i} E\left\{s_i\left(p_i, F_i, p_j, F_j; \boldsymbol{\theta}\right)\left[\pi_i\left(p_i, \boldsymbol{\theta}\right) + F_i\right]\right\} \tag{2}$$

for $j \neq i$.

In our single-homing model with 2PTs, we can interpret the permission to allow consumers to enter the shop as the first product (product 1) and its price to be equal to the fixed fee F_i, and treat the real product offered by firm i as product 2, with a price equal to p_i. In that sense, the expected demand for product 1 of firm A is $E[s_A(p_A, F_A, p_B, F_B; \boldsymbol{\theta})]$. Note that $\partial E[s_A]/\partial p_A = -E[q_A(p_A, \boldsymbol{\theta})]/2t$ and $\partial E[s_A]/\partial F_A = -1/2t$, so that the marginal rate of substitution of the demand for access (MRSA) between p_A and F_A is

$$\text{MRSA} \equiv \frac{\partial E[s_A]}{\partial p_A} \bigg/ \frac{\partial E[s_A]}{\partial F_A} = E[q_A(p_A, \boldsymbol{\theta})], \tag{3}$$

which is the expected demand for the product (product 2) conditional on access.

In our analysis, we consider both homogeneous and heterogeneous taste preferences.[12] We explore also the implications of two special asymmetric settings: In the first setting, we assume that the indirect utilities provided by both firms are equal—that is, $v_i\left(p, \boldsymbol{\theta}\right) = v_j\left(p, \boldsymbol{\theta}\right) = v(p, \boldsymbol{\theta})$ for all $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ where $v\left(p, \boldsymbol{\theta}\right)$ satisfies (A1)—but that the marginal cost for firm A, the *efficient* firm, is lower than the marginal cost for firm B, the *less efficient* firm, that is, $c_A < c_B$. The second special setting assumes that both firms have symmetric marginal costs but offer differentiated goods in the sense that the product offered by firm A is *vertically* superior to the product offered by firm B; that is, $v_A\left(p, \boldsymbol{\theta}\right) > v_B\left(p, \boldsymbol{\theta}\right)$ for all $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$.

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[11]Armstrong and Vickers (2001) have a similar assumption in a model with consumers with homogeneous tastes for quality and symmetric firms with a common marginal cost, c. They assume $\varsigma'(u) \leq 0$, where $\varsigma(p) = -\frac{q'(p)}{q(p)}(p - c)$ for $u = v(p)$. The function $\varsigma(p)$ represents the elasticity of demand expressed in terms of the markup $(p - c)$ rather than in terms of the price p. It is straightforward to show that $\mu'(p) < 1$ implies that $\varsigma'(u) \leq 0$. Likewise, Carrillo and Tan (2015) use such an assumption in a model of platform competition.

[12]Note that the terms "homogeneous" and "heterogeneous" refer to the taste parameter $\boldsymbol{\theta}$. We will denote "homogeneous preferences" when $\boldsymbol{\theta}$ is constant and "heterogeneous preferences" when $\boldsymbol{\theta}$ follows a distribution $\mathbf{G}(\cdot)$ independent of x. In both cases, consumers' preferences are horizontally differentiated.

3　Marginal-cost Pricing in Two-part Tariffs

In this section, we study first some general properties of our model. The set of feasible unit prices is $\mathcal{P} = [0, \bar{p}]$, where $\bar{p} = \max\{p_A^m, p_B^m\}$. Due to the full market coverage assumption, the market share and the problem of firm $i \in \{A, B\}$ are defined by (1) and (2), respectively.

The first-order condition of firm i with respect to p_i yields

$$(p_i - c_i) E \left[2t \cdot q_i'(p_i, \boldsymbol{\theta}) \, s_i(p_i, F_i, p_j, F_j; \boldsymbol{\theta}) - q_i(p_i, \boldsymbol{\theta})^2 \right] \tag{4}$$
$$+ E \left[q_i(p_i, \boldsymbol{\theta}) (t + v_i(p_i, \boldsymbol{\theta}) - v_j(p_j, \boldsymbol{\theta})) \right] + E \left[q_i(p_i, \boldsymbol{\theta}) \right] (F_j - 2F_i) = 0.$$

There are two main differences between using 2PT and LP. When both firms use LP (i.e., $F_i = F_j = 0$), the first and the second term on the left-hand side of (4) characterize the best response function of each firm i.[13] Note that if F_i is a positive number, there is a direct effect of using 2PT that shifts the *quasi* best-response curve, defined by (4) for firm i, to the left, in the (p_i, p_j) plane for $j \neq i$.[14] This implies that firm i reacts more aggressively with its marginal price for each value of p_j. Now, if F_j is also positive, there is an indirect effect that shifts the *quasi* best-response curve for firm i in the opposite direction: as F_i increases, firm j reacts more aggressively by decreasing p_j for each value of p_i, and by increasing F_j. In other words, when firms are allowed to use 2PTs, firms react aggressively by setting low marginal prices, allowing them to attract consumers and extract surplus more efficiently with the fixed fee, which does not depend directly on the curvature of the demand. The fixed fee is determined by the following first-order condition:

$$F_i + E \left[\pi_i(p_i, \boldsymbol{\theta}) \right] - t - \frac{1}{3} E \left[v_i(p_i, \boldsymbol{\theta}) + \pi_i(p_i, \boldsymbol{\theta}) - v_j(p_j, \boldsymbol{\theta}) - \pi_j(p_j, \boldsymbol{\theta}) \right] = 0. \tag{5}$$

Due to the Hotelling specification, the left-hand sides of (4) and (5) are linear in F_i and F_j. As such, we can easily eliminate F_i and F_j to get the following conditions which determine the *quasi* best-response functions,

$$p_i - c_i = \omega_i(p_i) E[s_i] \left(\frac{E[s_i \cdot q_i(p_i, \boldsymbol{\theta})]}{E[s_i]} - E[q_i(p_i, \boldsymbol{\theta})] \right), \tag{6}$$

where $1/\omega_i(p_i) \equiv \mathrm{Var}\left[q_i(p_i, \boldsymbol{\theta})\right] - 2t \cdot E\left[q_i'(p_i, \boldsymbol{\theta}) s_i\right] > 0$, and

$$s_i \equiv \frac{1}{2} + \frac{1}{2t} \left(v_i(p_i, \boldsymbol{\theta}) - v_j(p_j, \boldsymbol{\theta}) \right) + \frac{1}{6t} E \left[\pi_i(p_i, \boldsymbol{\theta}) - \pi_j(p_j, \boldsymbol{\theta}) - 2v_i(p_i, \boldsymbol{\theta}) + 2v_j(p_j, \boldsymbol{\theta}) \right] \tag{7}$$

is the market share of firm i, which depends on p_i, p_j and $\boldsymbol{\theta}$, after taking into account the optimal choices of fixed fees by both firms.

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[13]If both firms use LP, the problem of firm i is: $\max_{p_i} \ E\left\{ \left(\frac{1}{2} + \frac{v(p_i, \boldsymbol{\theta}) - v(p_j, \boldsymbol{\theta})}{2t} \right) \pi_i(p_i, \boldsymbol{\theta}) \right\}$.

[14]By *quasi* best-response functions, we refer to the best-response functions only in terms of p_A and p_B after taking into account the optimal choices of the fixed fees.

Proposition 1. *Suppose that (A1) and (A2) hold. In any pure strategy Nash equilibrium in two-part tariffs, for each $i \in \{A, B\}$, the markup $p_i - c_i$ has the same sign as $\frac{E[s_i \cdot q_i(p_i, \boldsymbol{\theta})]}{E[s_i]} - E[q_i(p_i, \boldsymbol{\theta})]$.*

Proposition 1 indicates that the sign of the markup for each firm is determined by two factors: the first is the *average expected demand* among all customers who choose the firm's product, and the second is the marginal rate of substitution of the demand for access (or the probability of participation) (MRSA) between the marginal price and fixed fee as defined in (3).[15] Under the full market coverage, the proportion of customers who choose the firm's product is the expected market share of the firm. Then, the average expected demand is the expected unconditional demand divided by the expected market share. Also, under full market coverage, the MRSA of a firm describes the demand of the set of marginal consumers who are indifferent between accepting the 2PTs from the firm and from its rivals. In our setting, consumer heterogeneity is described by $n+1$ dimensional types: consumers' preferences for the firms' products are horizontally differentiated and they also have heterogeneous vertical tastes (n-dimensional). Thus, the set of marginal consumers, is described by an n-dimensional manifold. Note that if the average demand is higher than the MRSA, the firm sets its prices above the marginal cost and extract surplus with both the marginal price and the fixed fee. On the other hand, if the average demand is lower than the MRSA, the only way the firm can increase its market share and profits is by decreasing its price *even below* its marginal cost and compensate the loss with the fixed fee.

Schmalensee (1981) and Varian (1989) provide a similar condition under monopoly. In their setting with one-dimensional type of heterogeneous consumer tastes, the MRSA between the marginal price and fixed fee is simply the demand of the marginal consumer. We find that it is better to use the MRSA between the marginal price and fixed fee to describe the consumers' participation incentives and compare it to the average expected demand. This new terminology of MRSA also works better in more general settings like the one described in Section 5. Moreover, under the full market coverage for all vertical types, the MRSA between p_i and F_i is completely determined by consumers' preferences for firm i's product (which is the unconditional expected demand) but is independent of firm j's offer. However, the average expected demand for i's product depends on its market share, which depends on consumers' preferences for both product i and product j. In other words, the average expected demand for firm i's product relies also on firm j's price p_j. If the market is not fully covered (or under monopoly), the MRSA between the instruments p_i and F_i describes the demand of the set of marginal consumers in the participation set.

As we mentioned earlier, in the Hotelling model with linear transportation cost and uniform distribution of consumer locations, the market share for each firm (defined in (1)) is linear in the difference between the two indirect utilities, $v_i(p_i, \boldsymbol{\theta}) - v_j(p_j, \boldsymbol{\theta})$, and linear in the difference between the two fixed fees, F_i and F_j. Therefore, a simpler condition for the sign of the markup for each

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[15]Note that the market share s_i, defined in (1), is linear in both the indirect utility and fixed fee of firm i, $v_i(p_i, \boldsymbol{\theta})$ and F_i, respectively. Then, marginal changes in s_i, after changes in $v_i(p_i, \boldsymbol{\theta})$ or F_i, are constant and independent of $\boldsymbol{\theta}$, explaining why the MRSA between p_i and F_i is $E[q_i(p_i, \boldsymbol{\theta})]$, which is the conditional expected demand. If the market is not fully covered for all $\boldsymbol{\theta}$, both terms will need to be slightly modified.

firm is presented in the following corollary.

Corollary 1. *Suppose that (A1) and (A2) hold. In any pure strategy Nash equilibrium in two-part tariffs, the markup $p_i - c_i$ of each firm has the same sign as* $\text{Cov}\left(v_i\left(p_i, \boldsymbol{\theta}\right) - v_j\left(p_j, \boldsymbol{\theta}\right), q_i\left(p_i, \boldsymbol{\theta}\right)\right).$

3.1 Marginal Cost Pricing

Proposition 1 and Corollary 1 imply a necessary condition (without second-order conditions) for any pure-strategy Nash equilibrium in 2PTs involving marginal cost pricing. If marginal-cost pricing arises in any pure-strategy Nash equilibrium in 2PTs, then the following condition holds:

$$\frac{E\left[s_i \cdot q_i\left(p_i, \boldsymbol{\theta}\right)\right]}{E\left[s_i\right]} - E\left[q_i\left(p_i, \boldsymbol{\theta}\right)\right] = 0$$

for $p_i = c_i$ for $i \in \{A, B\}$, or equivalently

$$\text{Cov}\left(s_i, q_i\left(p_i, \boldsymbol{\theta}\right)\right) = 0, \tag{8}$$

for $p_i = c_i$ for $i \in \{A, B\}$, where s_i is defined in (7).[16] Thus, in any Nash equilibrium in 2PTs that involves marginal-cost pricing, the average expected demand for the firm i is equal to the MRSA between p_i and F_i.

In the next proposition, we provide sufficient conditions for marginal-cost-based 2PT to be a unique equilibrium.

Proposition 2. *Suppose that (A1) and (A2) hold. If for any $p_i, p_j \in \mathcal{P}$, $i, j \in \{A, B\}$ and $i \neq j$* $\text{Cov}\left(v_i\left(p_i, \boldsymbol{\theta}\right) - v_j\left(p_j, \boldsymbol{\theta}\right), q_i\left(p_i, \boldsymbol{\theta}\right)\right) = 0$, *marginal-cost-based 2PT is a unique equilibrium.*

Proposition 2 states that if for firm $i \in \{A, B\}$ the demand is independent of the market share for all feasible prices then marginal-cost-based 2PT is a unique equilibrium.[17] In this case, there are no gains of reducing or increasing the marginal price below or above the marginal cost to increase the number of participating consumers.

Proposition 2 is related to the result in Mathewson and Winter (1997) for goods that are strongly complementary in demand. To see the connection we can interpret the permission to allow consumers to enter the shop by firm i as the first product with price F_i, and treat the real product offered as product 2 with price p_i. The demand for product 1 is the *expected* market share for firm i's product, $E\left[s_i\left(p_i, F_i, p_j, F_j, \boldsymbol{\theta}\right)\right]$, and the demand for product 2 is the *expected* value of the market share multiplied by the individual demand for that product, $E\left[s_i\left(p_i, F_i, p_j, F_j, \boldsymbol{\theta}\right) q_i\left(p_i, \boldsymbol{\theta}\right)\right]$ (i.e.,

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[16]Note that condition (8) is a sufficient statistic for general demand patterns in our duopoly setting. Since both market shares and sales quantities can be observed, condition (8), and the general condition in Proposition 1 can be potentially tested empirically. We show in Section 5 that condition (8) needs to be modified under a general specification of market share functions.

[17]Note that our game is neither a game with strategic complementarities as in Vives (1990) nor a supermodular game as in Milgrom and Roberts (1994). The reason is that the product under consideration and access by each firm are complements to consumers, not substitutes. These two "products" are therefore substitutes across the firms but complements within each firm.

the expected unconditional demand). Note that due to the heterogeneity of the consumer's vertical preferences, the ratio of the demands for the two products may or may not be independent of F_i. If condition (8) holds, then the ratio of the two demands is equal to $E\left[q_i\left(p_i, \boldsymbol{\theta}\right)\right]$, which is independent of the fixed fee, F_i. Hence the two "products" are strong complements in the sense of Mathewson and Winter (1997). Using their Proposition 5, we could conclude that the profits for firm i are maximized at $p_i = c_i$. However, as we will illustrate later, in the presence of heterogeneity in consumers vertical preferences, condition (8) can easily be violated.

To illustrate Proposition 2, we consider two important cases. In the first case, consumers are homogeneous in their tastes for quality, whereas their horizontal brand preferences remain unknown to the firms. In the second case, we assume that firms are symmetric.

Corollary 2. *Suppose the analogues of (A1) and (A2) for $\boldsymbol{\theta}$ identical for all consumers hold. Then, marginal-cost-based 2PT is a unique equilibrium.*[18]

Corollary 2 shows that if consumers are homogeneous in their tastes for quality, under the assumption of full market coverage, the equilibrium strategy for each firm is to set its prices equal to its marginal costs and extract surplus through the fixed fee. Following the above interpretation, the demand for product 1 is the market share of firm i's product, $s_i\left(p_i, F_i, p_j, F_j\right)$, and the demand for product 2 is the market share multiplied by the individual demand for that product, $s_i\left(p_i, F_i, p_j, F_j\right) q_i\left(p_i\right)$. Note that the ratio of the two demands is $q_i\left(p_i\right)$, which is independent of the fixed fee, F_i, and then Mathewson and Winter (1997) result applies. Hence, each firm always charges the price of its second product at marginal cost, independently of its rival's choices. Note that in this case, the marginal costs of the two firms may be different, which implies that the marginal prices (and fixed fees) may also be different.[19]

In the second case, we show in the next corollary that if firms are symmetric the ratio of the demands for the two products at $p_i = c_i$ remains independent of F_i. Hence, from Mathewson and Winter's result we know that marginal-cost pricing is part of the equilibrium.[20]

Corollary 3. *Suppose that (A1) and (A2) hold and that $c_i = c_j = c$ and $v_i\left(p, \boldsymbol{\theta}\right) = v_j\left(p, \boldsymbol{\theta}\right)$ for all $p \in \mathcal{P}$, $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and $j \neq i$. Then, any pure-strategy Nash equilibrium in 2PT involves marginal-cost pricing.*

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[18]In equilibrium, $F_i^* = t + \frac{v_i(c_i) - v_j(c_j)}{3}$ for $i \in \{A, B\}$ and $j \neq i$. Note that if $t < \frac{v_A(c_A) - v_B(c_B)}{3}$, then there exists a corner equilibrium in which firm B sets $p_B = c_B$ and $F_B = 0$ while firm A sets $p_A = c_A$ and $F_A = \frac{t}{2} + \frac{v_A(c_A) - v_B(c_B)}{2}$. For the rest of the paper we consider only interior equilibria.

[19]Corollary 2 is also related to Yin (2004), who considers a model where consumers are horizontally differentiated and have homogeneous taste preferences (i.e., one-dimensional horizontal consumer heterogeneity). He shows that if there is an interaction between the horizontal taste (or location) parameter and the consumer's utility with variable quantity (i.e., the cross-partial of the net utility with the quantity and location parameter is different from zero), marginal-cost pricing is an equilibrium if the demand of the marginal consumer is equal to the average demand. Also, if if there is no interaction between the horizontal taste parameter and the consumer's utility with variable quantity (e.g., the effect of the horizontal preference parameter on utility is additively separate from the effect of the quantity)—the transportation cost is a shopping cost—firms set marginal prices equal to marginal cost.

[20]Marginal-cost-based 2PT is also an equilibrium if, for example, the indirect utilities offered by the two firms are such that $v_i\left(p, \boldsymbol{\theta}\right) - v_j\left(p, \boldsymbol{\theta}\right) = K$, where K is a constant for all $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$.

Corollary 3 is consistent with the result by Armstrong and Vickers (2001) and Rochet and Stole (2002) that a pair of 2PTs with marginal-cost pricing is an equilibrium in a symmetric model in which firms choose nonlinear tariffs. For instance, Rochet and Stole (2002) show that the upward and downward incentive constraints do not bind for the firms and hence there is an equilibrium in which each firm sells efficient quality levels (e.g., cost-plus-fixed-fee pricing). This general result by Armstrong and Vickers (2001) and Rochet and Stole (2002) depends on the assumption of the symmetry of the firms. Although we consider a smaller strategy space (2PT instead of nonlinear pricing), our result does not rely on symmetry. Moreover, if marginal-cost pricing is not a Nash equilibrium in the space of 2PTs, it will not be a Nash equilibrium in a larger space of nonlinear pricing. Finally, as pointed out by Rochet and Stole (2002), departures from the assumption of independence between the vertical taste parameter and the horizontal preferences, may alter the conclusion that a pair of 2PTs is a Nash equilibrium.

3.2 Deviation from Marginal-cost Pricing in Two-part Tariffs

As discussed earlier, (8) is a necessary condition for marginal-cost-based 2PT as a Nash equilibrium. This condition can easily be violated.[21] We would normally expect that the MRSA between p_i and F_i would be lower than the average expected demand; in this case, the price charge would be greater than the marginal cost. That is, if the MRSA between the instruments p_i and F_i is low relative to the average expected demand, firms do not need to set prices at or below the marginal cost to attract more customers. Instead, firms can extract surplus with both marginal price and fixed fee. However, the MRSA between p_i and F_i may be higher than the average expected demand. In this case, the equilibrium marginal price would be less than the marginal cost. The greater the demand response to increases in the marginal price, the more likely it would be the firm decreases its marginal price below its marginal cost to attract more customer, subsidizing the losses with the fixed fee.

Beyond the above two special cases, the ratio of the two demands by heterogeneous consumers in general depends on fixed fees. To evaluate possible deviations from marginal-cost pricing in 2PTs, we use the following assumption on consumers' vertical tastes preferences.

Assumption 3. $\boldsymbol{\theta}$ *is strictly associated.*

A vector $\boldsymbol{\theta}$ of random variables is associated if $\text{Cov}[f(\boldsymbol{\theta}), g(\boldsymbol{\theta})] \geq 0$ for all nondecreasing functions f and g for which $E[f(\boldsymbol{\theta})]$, $E[g(\boldsymbol{\theta})]$, and $E[f(\boldsymbol{\theta})g(\boldsymbol{\theta})]$ exist. We use a strict version of association for the rest of the paper.[22]

If $\boldsymbol{\theta}$ is strictly associated and if $v_i(c_i, \boldsymbol{\theta}) - v_j(c_j, \boldsymbol{\theta})$ is monotonic increasing or decreasing (depending on marginal costs and the functional form of $v_i(\cdot)$ for $i \in \{A, B\}$) with respect to $\boldsymbol{\theta}$, since

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[21]Note that in our previous example, the two goods (product 1 and 2) are complements, then p_i and F_i should go in opposite directions and their directions depend on the average expected demand and the MRSA between them.

[22]For a complete reference on association of random variables and its properties, see Esary et al. (1967). See also Holmstrom and Milgrom (1994) and Milgrom and Weber (1982) for economic applications.

$q_i(c_i, \boldsymbol{\theta})$ is monotonic increasing in $\boldsymbol{\theta}$, then (8) can easily be violated. In these cases, marginal-cost pricing is not an equilibrium. Two special cases are provided in the following corollary.

Corollary 4. *Suppose that (A1)-(A3) hold. Marginal-cost-based 2PT is not a Nash equilibrium if either (i) $c_i \neq c_j$ and $v_i(p, \boldsymbol{\theta}) = v_j(p, \boldsymbol{\theta}) = v(p, \boldsymbol{\theta})$ for all $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$; or (ii) $c_i = c_j$ and $v_i(p, \boldsymbol{\theta}) - v_j(p, \boldsymbol{\theta})$ is strictly monotonic with respect to $\boldsymbol{\theta}$ for all $p \in \mathcal{P}$.*[23]

Corollary 4 shows that if marginal costs are asymmetric and the products of the two firms are symmetric, then (8) evaluated at the marginal costs does not hold and hence marginal cost-based 2PT is not an equilibrium. Likewise, if marginal costs are symmetric but the difference in the indirect utilities, $v_i(p, \boldsymbol{\theta}) - v_j(p, \boldsymbol{\theta})$, is monotonic with respect to $\boldsymbol{\theta}$, then (8) does not hold either and hence marginal-cost pricing is not an equilibrium.

To illustrate Corollary 4, consider the following class of examples (considered in Section 4) in which firms have symmetric marginal costs and for any $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, the indirect utility offered by firm i is $v(p, \boldsymbol{\theta})$ (which satisfies A1) and the indirect utility offer by firm j, $j \neq i$, is $\alpha \cdot v(p, \boldsymbol{\theta})$ for any $\alpha \in (0, 1)$, which means that product A is "vertically" superior to product B. Then, marginal-cost pricing is not an equilibrium.[24]

The reason for the contrast between Corollaries 3 and 4 is related to the dependence of the fixed fees and marginal prices on the distribution of $\boldsymbol{\theta}$. Note that from Corollary 2 we know that marginal-cost-based 2PT is a unique equilibrium if $\boldsymbol{\theta}$ is complete information for firms. If both marginal costs and indirect utilities (demand for the two goods) are symmetric, both the equilibrium marginal price and the fixed fee do not depend on the distribution of $\boldsymbol{\theta}$. Thus, the marginal price and the fixed fee remain an equilibrium even when $\boldsymbol{\theta}$ is unknown for both firms. However, if marginal costs or the products offered by the two firms are asymmetric, the fixed fee or the marginal price would depend on the distribution of $\boldsymbol{\theta}$.

4 Cross-subsidization by the Less Efficient Firm

In this section we provide two classes of examples in which marginal-cost-based 2PT is not a Nash equilibrium and the equilibrium involves the less efficient firm prices below its marginal cost. In the first class, the indirect utilities offered by the two firms are symmetric but the firms have different marginal costs. In the second class, both firms have the same marginal cost, c, but the demands

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[23]The monotonicity of the utility difference between the two products with respect to $\boldsymbol{\theta}$ can be interpreted as follows: $v_i(p, \boldsymbol{\theta}) - v_j(p, \boldsymbol{\theta})$ is monotonic with $\boldsymbol{\theta}$ means that $v_A(p, \boldsymbol{\theta}) + v_B(p, \boldsymbol{\theta}') > v_A(p, \boldsymbol{\theta}') + v_B(p, \boldsymbol{\theta})$ for $\boldsymbol{\theta}, \boldsymbol{\theta}' \in \boldsymbol{\Theta}$ such that $\boldsymbol{\theta} > \boldsymbol{\theta}'$ ("high" and "low" type).

[24]In previous versions of this paper, we also studied under what conditions marginal-cost-based 2PT is an equilibrium if both firms use nonlinear tariffs. We showed that any equilibrium involves marginal-cost-based 2PTs if and only if for $i, j \in \{A, B\}$ and $j \neq i$, $v_i(c_i, \boldsymbol{\theta}) - v_j(c_j, \boldsymbol{\theta})$ is constant over $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ (i.e., the difference between the utilities offered by the two firms at their marginal costs, or the efficient utilities, is independent of consumer taste $\boldsymbol{\theta}$)—this result extends Proposition 5 in Armstrong and Vickers (2001) and Proposition 6 in Rochet and Stole (2002) to allow for general asymmetric demands and costs). The following two examples satisfy this condition. In the first, firms have symmetric costs, $v_A(p, \boldsymbol{\theta}) = v(p, \boldsymbol{\theta})$ and $v_B(p, \boldsymbol{\theta}) = v(p, \boldsymbol{\theta}) + k$, where $k \in \mathbb{R}$ and $v(p, \boldsymbol{\theta})$ is derived from a utility function that satisfies (A1). In the second example, firms have asymmetric marginal costs, $v_A(p, \boldsymbol{\theta}) = v(p, \boldsymbol{\theta})$ and $v_B(p, \boldsymbol{\theta}) = \alpha v(p, \boldsymbol{\theta})$, where $\alpha \in (0, 1)$, $v(p, \boldsymbol{\theta}) = h(\boldsymbol{\theta}) v(p)$, $v(\cdot)$ is strictly decreasing and $h(\cdot)$ is strictly increasing.

for their products are not symmetric. We show that when firms have asymmetric marginal costs or asymmetric demands, information about vertical taste preferences has a substantial effect on the equilibrium pricing strategy, and in particular, in equilibrium the less efficient firm "cross-subsidizes" between the fixed fee and the marginal price.

4.1 Asymmetric Costs and Symmetric Demands

Without loss of generality, we assume that the marginal cost of firm A, the efficient firm, is lower than that of firm B, the less efficient firm; that is, $c_B > c_A \geq 0$. The set of feasible unit prices for both firms is $\mathcal{P} = [0, p_B^m]$, where p_B^m corresponds to the monopoly price of firm B.[25] Given the symmetric demands, the equilibrium conditions (5) and (6) can be simplified as follows

$$F_i = t + \frac{1}{3} E\left[v\left(p_i, \boldsymbol{\theta}\right) + \pi_i\left(p_i, \boldsymbol{\theta}\right) - v\left(p_j, \boldsymbol{\theta}\right) + \pi_j\left(p_j, \boldsymbol{\theta}\right)\right] - E\left[\pi_i\left(p_i, \boldsymbol{\theta}\right)\right], \tag{9}$$

and

$$p_i - c_i = \omega_i\left(p_i\right) \text{Cov}\left(v\left(p_i, \boldsymbol{\theta}\right) - v\left(p_j, \boldsymbol{\theta}\right), q\left(p_i, \boldsymbol{\theta}\right)\right), \tag{10}$$

where $1/\omega_i(p_i) \equiv \text{Var}\left[q\left(p_i, \boldsymbol{\theta}\right)\right] - 2t \cdot E\left[q'\left(p_i, \boldsymbol{\theta}\right) s_i\right]$ and s_i is defined as in (7).

The equilibrium condition (10) has several implications. First, in equilibrium the two firms cannot charge the same marginal prices. Second, the efficient firm A should charge a lower marginal price than the less efficient firm B. Otherwise, $p_A > p_B$ would imply that the covariance on the right-hand-side of (10) is negative for $i = A$ and positive for $i = B$, since $-v(p, \boldsymbol{\theta})$ has increasing difference property and $\boldsymbol{\theta}$ is associated, which in turn implies negative markup for firm A and positive markup for firm B. This contradicts the assumption that firm A has lower marginal cost than firm B has. Moreover, the same logic implies that in equilibrium, the efficient firm indeed has positive markup while the less efficient one has negative markup. We summarize these findings in the following proposition and the formal proof is presented in the Appendix.

Proposition 3. *Suppose that (A1)-(A3) are satisfied. In any pure-strategy Nash equilibrium in 2PTs, the following hold:*

(i) $c_A < p_A^* < p_B^* < c_B$ *and*

(ii) the expected market share, per-customer profits and total revenue are greater for firm A than those for firm B.[26]

Why the more-efficient firm, A, sets its marginal price above its marginal cost and the less efficient firm, B, below its marginal cost? The intuition follows from Proposition 1. Suppose that

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[25]Since B is the less efficient firm, $\bar{p} = \max\{p_A^m, p_B^m\} = p_B^m$.

[26]We show in the proof of Proposition 3 that if the difference between c_B and c_A is *small*, then $F_A > F_B$. A formal definition of the upper bound of c_B, as a function of c_A, is provided in the proof of Proposition 3. From the Implicit Function Theorem, it follows that given c_A, as c_B increases, F_A increases and F_B decreases. In summary, in any equilibrium, the efficient firm charges a lower marginal price and a higher fixed fee than those of the less efficient firm.

initially both firms offer their products at the marginal cost and charge a positive fixed fee. Then, it follows that the average expected demand is greater (lower) than the MRSA between p_A and F_A for firm A (B).[27] Thus, in order for firm B to attract more customers it needs to decrease the marginal price even below its own marginal cost and compensate the losses with the fixed fee. On the other hand, firm A's average expected demand is higher than the MRSA between p_A and F_A. Then, firm A does not need to decrease its marginal price below its own marginal cost to increase its market share and the share of revenues extracted with the fixed fee.

Part (ii) follows from (i), (9), and the fact that the expected market share is a linear function of the difference of the expected surplus from both firms. In any equilibrium $p_A^* < p_B^*$, then, the expected total surplus and the market share are greater for firm A that for firm B. Moreover, since $c_A < p_A^* < p_B^* < c_B$, the expected revenue per consumer is greater for firm A than for firm B.

In sum, in any equilibrium, the expected market share, profits, total revenue per consumer, and total revenue are greater for firm A than for firm B. In particular, note that the marginal price is lower and the fixed fee is higher for the efficient firm than for the less efficient one (see Footnote 26). These results may explain the empirical regularities observed in the British electricity market and highlighted by Davies et al. (2014); if the entrant firms are more efficient than the incumbent, we should expect lower marginal prices and higher fixed fees for the entrant.[28]

Proposition 3 contrasts with Chen and Rey (2012) who consider a model in which a large retailer, supplying a broad range of products, competes with a smaller retailer that focuses on a narrower product line. They consider two products (or product lines) and assume that the first product is monopolized by the large firm, whereas both firms can supply the second product. They assume that consumers incur a shopping cost for visiting a store.[29] They show that whenever the large retailer enjoys a comparative advantage over the small retailer, the pricing strategy involves loss leading: the large retailer sells the competitive product below cost in order to discriminate between single-homing shoppers and multi-homing shoppers. Similarly, Proposition 3 contrasts with Chen and Rey (2019) who consider a model in which two firms with different comparative advantages (each firm has a strong and a weak product) compete for consumers with heterogeneous shopping patterns. In equilibrium, firms sell their weak products below cost, earn zero profit from one-stop shoppers, and extract profits from multi-homing shoppers, who pay a higher price for their strong product. Note that in our case, in equilibrium, the less efficient firm sets its price below its marginal cost and compensate the loss with the fixed fee, whereas the firm with the advantage sets its price above its marginal cost.

To determine the sufficient condition for the uniqueness of equilibrium, we need to introduce

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[27]Remember that in the symmetric case the average expected demand is equal to the MRSA between the instruments p_i and F_i for both firms.

[28]As we mentioned before, there are other types of asymmetry that are important in the British electricity market. Some of the firms were integrated upstream into generation and some were active in the gas market. Although Davies et al. (2014) suggest small costs asymmetries between firms, we need to assume that these other type of asymmetry can be projected into the firms' marginal costs. This may result in firms with asymmetric marginal costs.

[29]Intuitively, consumers with a low shopping cost take advantage of multi-homing, whereas those with higher shopping cost favor one-stop shopping.

a new assumption that helps us to analyze the slope of the *quasi* best-response functions.[30] We introduce the following definition:

Definition 1. $v(p, \boldsymbol{\theta}) : \mathcal{P} \times \boldsymbol{\Theta} \to \mathbb{R}_+$ is *separable* if there exist functions $v : \mathcal{P} \to \mathbb{R}_+$, $h : \boldsymbol{\Theta} \to \mathbb{R}$ and $l : \boldsymbol{\Theta} \to \mathbb{R}$, where $v(\cdot)$ is strictly decreasing such that for all $(p, \boldsymbol{\theta}) \in \mathcal{P} \times \boldsymbol{\Theta}$,

$$v(p, \boldsymbol{\theta}) = v(p) h(\boldsymbol{\theta}) + l(\boldsymbol{\theta}).$$

Assumption 4. $v(p, \boldsymbol{\theta}) : \mathcal{P} \times \boldsymbol{\Theta} \to \mathbb{R}_+$ *is separable.*

Examples of the classes of indirect utilities that satisfy (A4) are: (i) the power functions (or constant elasticity demand) e.g., suppose that $u(q, \theta) = \theta \sqrt{q}$ then $v(p, \theta) = \frac{\theta^2}{4p}$; ($ii$) the log function, e.g., $u(q, \theta) = \theta \log q$, then $v(p, \theta) = \theta(\log \theta - 1) - \theta \log p$; and ($iii$) the linear demand-type function, e.g., $u(q, \theta) = \alpha q - \frac{\theta q^2}{2}$, then $v(p, \theta) = \frac{(\alpha - p)^2}{2\theta}$. Also, (A4) allows us to simplify (10) and show in Lemma 1 (see the Appendix) that the slope of the implicit functions defined by (10) for each firm i, $R^i(p_A) : \mathcal{P} \to \mathcal{P}$, is positive, where $R^i(\tilde{p}_A^i) = \tilde{p}_B^i$ is such that $\tilde{p}_A^i$ and $\tilde{p}_B^i$ satisfy (10) for each $i \in \{A, B\}$.

Although both firms are using fixed fees to extract surplus, both *quasi* best-response functions in terms of the unit prices are increasing, as in the standard LP game. Note that if consumers have homogeneous taste parameters for quality, the *quasi*-best response function is vertical for firm A and horizontal for firm B, which contrasts with Lemma 1.

Note that (A4) allows us to express (10) as a function of $\bar{\theta} \equiv E[h(\boldsymbol{\theta})]$ and $\sigma \equiv \text{Var}[h(\boldsymbol{\theta})]$. Using Lemma 1, we show the uniqueness of the pure strategy Nash equilibrium in 2PTs in the following proposition.

Proposition 4. *Suppose that (A1)-(A4) are satisfied and that $3\sigma > \bar{\theta}^2$ and $c_B - c_A < -q(c_B)/3q'(c_A)$.[31] Then there exists a unique equilibrium in 2PTs in which $p_i^* \in \mathcal{P}$ is determined by (10) and F_i^* satisfies (9) for $i \in \{A, B\}$.*

From Proposition 3 we know that the two implicit functions, $R^A(p_A)$ and $R^B(p_A)$ derived from (10) for $i \in \{A, B\}$ cross at least once in the set $(c_A, c_B)^2$ (see Figure 1). Next, from Lemma 1 we know that the slopes of the implicit functions $R^A(p_A)$ and $R^B(p_A)$ are positive. For uniqueness, we show that in the set $(c_A, c_B)^2$ the slope of $R^A(p_A)$ is greater than the slope of $R^B(p_A)$.

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[30]Note that (10) implicitly defines a *quasi* best-response function for each $i \in \{A, B\}$ as a function of (p_A, p_B).

[31]The condition, $c_B - c_A < -q(c_B)/3q'(c_A)$, establishes an upper bound for $c_B - c_A$. The latter condition combined with $3\sigma > \bar{\theta}^2$ allows us to simplify our proof of uniqueness. A more tedious proof, in which these two assumptions are not used is available upon request.

Figure 1: Equilibrium with Asymmetric Cost
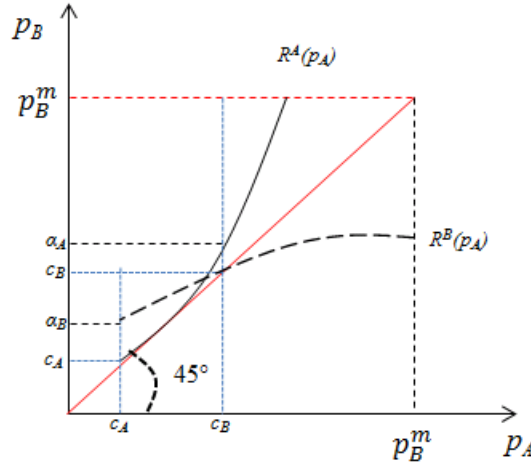
Note: Figure 1 shows the inverse of the *quasi* best-response function of firm A as a function of p_A and the *quasi* best-response function of firm B as a function of p_A.

Given the uniqueness of the pure-strategy Nash equilibrium in 2PTs, we next provide a set of comparative static properties of the equilibrium with respect to asymmetry of the firms and the heterogeneity or consumers' vertical tastes.

Corollary 5. *In equilibrium, as both c_B and c_A go to c, p_i^* converges to c and F_i^* to t for $i \in \{A, B\}$.*

Corollary 5 follows from Corollary 3 and Proposition 3. As the marginal costs for both firms converge to a common value c (i.e., as the asymmetry between the two firms vanishes) both marginal prices tend to the marginal cost and both fixed fees to the transportation cost, t (see, e.g., Armstrong and Vickers, 2001).

Corollary 6. *In equilibrium,*
  (i) as $\sigma \to 0$, $p_A \to c_A$ and $p_B \to c_B$;
  (ii) as $\sigma \to \infty$, $p_A \to \bar{p}_A$ and $p_B \to \bar{p}_B$ where $c_A < \bar{p}_A < \bar{p}_B < c_B$.

Corollary 6(i) follows from Corollary 4 and the monotonicity of the *quasi* best-response functions with respect to the marginal prices for both firms. Note that when $\sigma = 0$, the *quasi* best-response function for firm A is a vertical line at $p_A = c_A$ in the (p_A, p_B) plane, and for firm B it is a horizontal line at $p_B = c_B$. From numerical simulations, we find that as σ increases, p_A increases and p_B decreases. That is, as σ increases, the *quasi* best-response function rotates to the right around (c_A, c_A). Similarly, for firm B, as σ increases, the *quasi* best-response function rotates to the left (counterclockwise) around (c_B, c_B). Thus, for $p_i > c_i$, as σ increases, firm i reacts less aggressively (sets a higher price) for each p_j, for $j \neq i$. However, for $p_i < c_i$, as σ increases, firm i reacts more aggressively (sets a lower price) for each p_j, for $j \neq i$. This explains why marginal-cost-based 2PT is not a Nash equilibrium, when consumers are heterogeneous in their tastes. In particular, it explains why the optimal strategy for the less efficient firm is to set its marginal price

below its own marginal cost and to compensate for this loss with the fixed fee. On the other hand, the optimal strategy for the efficient firm is to set its marginal price above its own marginal cost but below that of its rival. Finally, from Corollary 6(ii), note that as σ increases, the marginal change of p_A and p_B decreases.

4.2 Asymmetric Demands and Symmetric Costs

This subsection introduces the second type of asymmetry related to the goods offered (or equivalently to the demand) by the two firms, but assumes that both firms have the same marginal cost, c. Without loss of generality, we assume that the indirect utility offered by firm A is higher than the one offered by firm B: $v_A(p, \boldsymbol{\theta}) - v_B(p, \boldsymbol{\theta}) > 0$ for all $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. To simplify the analysis, we introduce the following assumption:

Assumption 5. *Let $v_A(p, \boldsymbol{\theta}) = v(p, \boldsymbol{\theta})$ and $v_B(p, \boldsymbol{\theta}) = \alpha v(p, \boldsymbol{\theta})$ for $\alpha \in (0, 1)$, where $v(p, \boldsymbol{\theta})$ is separable.*

Intuitively, (A5) implies that for any $p \in \mathcal{P}$ and for $\boldsymbol{\theta}, \boldsymbol{\theta}' \in \boldsymbol{\Theta}$ such that $\boldsymbol{\theta} > \boldsymbol{\theta}'$ (e.g., high and low type, respectively), the sum of the indirect utilities offered by firms A and B to the high and low type, respectively, is higher than the sum of the indirect utilities offered by firm A and B to the low and high type, respectively, so that, $v_A(p, \boldsymbol{\theta}) + v_B(p, \boldsymbol{\theta}') > v_A(p, \boldsymbol{\theta}') + v_B(p, \boldsymbol{\theta})$. Thus, product A is "vertically" superior to product B.[32] Let α_A and γ_B be such that

$$v(\alpha_A) = \alpha v(c) \quad \text{and} \quad v(c) = \alpha v(\gamma_B). \tag{11}$$

From (A5) and (11), it follows that $\alpha_A > c$ and $\gamma_B < c$. We restrict our analysis to the set of indirect utilities that satisfy (A1) such that γ_B is strictly positive. This condition implies that the difference between the two indirect utilities is bounded.[33] We proceed to characterize the equilibrium of the game following a strategy similar to that in the previous subsection. Given the symmetric cost c and (A5), the equilibrium conditions (5) and (6) can be simplified as follows

$$F_i = t + \frac{1}{3} E\left[v_i(p_i, \boldsymbol{\theta}) + \pi_i(p_i, \boldsymbol{\theta}) - v_j(p_j, \boldsymbol{\theta}) + \pi_j(p_j, \boldsymbol{\theta})\right] - E\left[\pi_i(p_i, \boldsymbol{\theta})\right], \tag{12}$$

and

$$p_i - c = \omega_i(p_i) \operatorname{Cov}\left(v_i(p_i, \boldsymbol{\theta}) - v_j(p_j, \boldsymbol{\theta}), q_i(p_i, \boldsymbol{\theta})\right), \tag{13}$$

where $1/\omega_i(p_i) \equiv \operatorname{Var}\left[q(p_i, \boldsymbol{\theta})\right] - 2t \cdot E\left[q'(p_i, \boldsymbol{\theta}) s_i\right]$ and s_i is defined as in (7).

The equilibrium condition (13) has several implications. First, in equilibrium the two firms cannot use marginal cost pricing. Second, firm B should charge a marginal price below the marginal

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[32]For example, if $u_A(q, \theta) = \theta \sqrt{q}$ and $u_B(q, \theta) = \theta \sqrt{\alpha q}$, then $v_A(p, \theta) = \frac{\theta^2}{4p}$ and $v_B(p, \theta) = \alpha \frac{\theta^2}{4p}$, which satisfies (A5). Note that (A5) excludes functions such that the two indirect utilities offered by both firms differ by an additive constant.

[33]Equivalently, we can say that the difference between the demands of the two products offered by the firms is bounded.

cost, while firm A should charge a marginal price above the marginal cost; this is implication of assumption (A5). We summarize these findings in the following proposition and the formal proof is presented in an Appendix.

Proposition 5. *Suppose that (A1)-(A3) and (A5) are satisfied and that $3\sigma > \bar{\theta}^2$ and $\alpha_A - \gamma_B < -q(\alpha_A)/3q'(\gamma_B)$. Then there exists a unique equilibrium in 2PTs in which $p_i^* \in \mathcal{P}$ is determined by (13) and F_i^* satisfies (12) for $i \in \{A, B\}$. Moreover, $p_B^* < c < p_A^*$.*

For the proof of Proposition 5, we first show that no solution exists for (p_A, p_B) outside the set Ω, where $\Omega \equiv \{p_A, p_B \in \mathcal{P} \,|(p_A, p_B) \in [c, \alpha_A] \times [\gamma_B, c]\}$, which implies that if any equilibrium of the game exists, it must be in the set Ω. Next we show that for $(p_A, p_B) \in \Omega$, the slope of the implicit functions defined by (13) for $i \in \{A, B\}$, $\tilde{R}^i(p) : \mathcal{P} \to \mathcal{P}$, where $\tilde{R}^i(\tilde{p}_A^i) = \tilde{p}_B^i$, are such that $\tilde{p}_A^i$ and $\tilde{p}_B^i$ satisfy (13) for $i \in \{A, B\}$, is positive.

Figure 2: Equilibrium with Asymmetric Demands
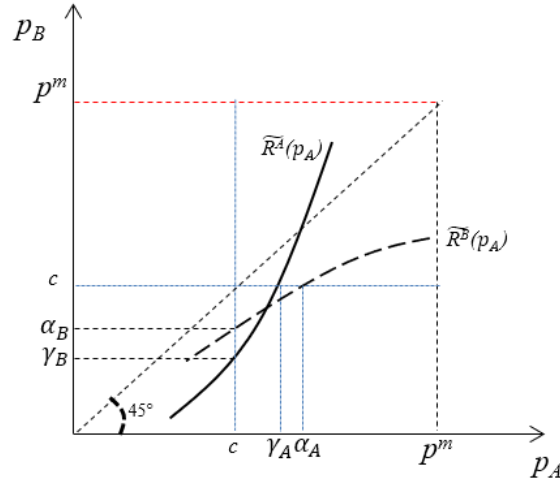
Note: Figure 2 shows the inverse of the *quasi* best-response function of firm A as a function of p_A and the *quasi* best-response function of firm B as a function of p_A.

We show that there exists at least one Nash equilibrium, that is, the two implicit curves defined by (13) for $i \in \{A, B\}$ always cross each other in the region Ω.[34] For uniqueness, we show that in the set Ω the slope of the implicit function $\tilde{R}^A(p_A)$ is greater than the slope of $\tilde{R}^B(p_A)$.

Proposition 5 says the disadvantaged firm (vertically inferior product demand) sets its marginal price below its rival's price and below its marginal cost, while the advantaged firm (vertically superior product demand) offers a unit price above its marginal cost and its rival's marginal price. The

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[34]Particularly, note that as $p_A \to c$ in (13) for $i = A$, we have that $p_B \to \gamma_B > c$, and as $p_A \to \gamma_A$, $p_B \to c$ where γ_A is such that (γ_A, c) satisfy (13) for $i = A$. Similarly, from (13) for $i = B$, as $p_A \to c$, we have that $p_A \to \alpha_B$, while as $p_A \to \alpha_A$, $p_B \to c$, where α_B is such that (c, α_B) satisfy (13) for $i = B$ and $\alpha_B > \gamma_B$. Similarly, we show that as $p_A \to \gamma_A$ in (13) for $i = B$, $p_B \to \delta_B < c$. Thus, both curves cross each other at least once in the set Ω (see Figure 2).

disadvantaged firm compensates for the loss of subsidizing the marginal price below the marginal cost with a positive fixed fee that is below that of its rival's fixed fee, which translates into an average expected demand strictly less than the MRSA between the instruments p_B and F_B. This result contrasts with Proposition 3 in which the disadvantaged firm sets a higher unit price (but below its own marginal cost) and a lower fixed fee than those of its rival. In the *Asymmetric Costs Model*, the efficient firm sets a marginal price below its rival's price but above its own marginal cost, while in the *Asymmetric Demands Model*, if the advantaged firm offers a price below the common marginal cost, it would have to compensate for this loss by increasing the fixed fee and decreasing its market share. Hence, firm A has incentives to deviate and offer a higher price than firm B, due to its advantage in product demand.

Note that as the difference between the indirect utilities offered by the two firms tends to 0 (i.e., α tends to 1) for any $p_i \in \mathcal{P}$, $i \in \{A, B\}$, and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, both marginal prices tend to the marginal cost, c, and the fixed fees become independent of $\boldsymbol{\theta}$, equal to t (the standard result of 2PTs).[35]

We may reconcile Proposition 5 with the empirical regularities observed in the British electricity market (described by Davies et al., 2014, i.e., the entrant firm offers a lower marginal price and a higher fixed fee than the marginal price and fixed fee, respectively, offered by the incumbent) in the following way: Suppose that the marginal cost of firm A is lower than that of firm B (i.e., $c_A < c_B$). Likewise, suppose that $v_A(p, \boldsymbol{\theta}) = v(p, \boldsymbol{\theta})$ and $v_B(p, \boldsymbol{\theta}) = \alpha v(p, \boldsymbol{\theta})$ for $\alpha \in (0, \bar{\alpha}]$, where $\bar{\alpha} > 1$ and is such that an interior equilibrium exists, $v(p, \boldsymbol{\theta}) = h(\boldsymbol{\theta}) v(p)$, $v(\cdot)$ is strictly decreasing, and $h(\cdot)$ is strictly increasing. Then, from Propositions 3 and 5 and the Implicit Function Theorem, it follows that there exist $\alpha^1 < 1$ and $\alpha^2 \in (1, \bar{\alpha})$ such that for $\alpha \in (\alpha^1, \alpha^2)$, $c_A < p_A^* < p_B^* < c_B$, and $F_A^* > F_B^*$. That is, if $\alpha \in (\alpha^1, \alpha^2)$, firm A offers a lower marginal price and a higher fixed fee than firm B, as in the British electricity market for the entrants and the incumbent firm, respectively.

5 General Market Share Functions

In this section, we extend our previous analysis based on the Hotelling specification to allow for general market share functions. Following Armstrong and Vickers (2001), we consider a discrete-choice model with a mass of consumers with types $(\boldsymbol{\xi}, \boldsymbol{\theta})$, where $\boldsymbol{\xi} \equiv (\xi_A, \xi_B, \xi_0)$ is distributed independently of the distribution of $\boldsymbol{\theta}$. Consumer's preference for the two differentiated products can be represented by the utility function $u_A(q_A, \boldsymbol{\theta}) + \xi_A$ if she buys from A, $u_B(q_B, \boldsymbol{\theta}) + \xi_B$ if she buys from B, and $u_0 + \xi_0$ if no purchase is made. Consumers buy all products from one or the other firm, or else take their outside option. Given two-part tariffs (p_i, F_i), the share of $\boldsymbol{\theta}$-consumers who choose to buy from firm A is $s(v_A(p_A, \boldsymbol{\theta}) - F_A, v_B(p_B, \boldsymbol{\theta}) - F_B)$ and the share of consumers who choose firm B is $s(v_B(p_B, \boldsymbol{\theta}) - F_B, v_A(p_A, \boldsymbol{\theta}) - F_A)$, where $v_i(p_i, \boldsymbol{\theta})$ is the indirect utility offered by firm i, defined as before, for $i \in \{A, B\}$. Moreover, both firms can produce their products at constant marginal costs, c_A and c_B, respectively.

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[35]Moreover, from the Implicit Function Theorem, it follows that p_A^* and F_A^* decrease and p_B^* and F_B^* increases as α increases, for α close to 1, which implies that $F_A^* > F_B^*$.

We impose the following regularity assumptions:[36] First, $s(u_A, u_B)$ is increasing with respect to u_A and decreasing with respect to u_B. Second, $s(u_A,u_B)/s_1(u_A,u_B)$ is weakly increasing with respect to u_A and weakly decreasing with respect to u_B.[37]

The problem of each firm is

$$\max_{p_i, F_i} \quad E\left[s\left(v_i\left(p_i, \boldsymbol{\theta}\right) - F_i, v_j\left(p_j, \boldsymbol{\theta}\right) - F_j\right)\left(\pi_i\left(p_i, \boldsymbol{\theta}\right) + F_i\right)\right] \tag{14}$$

for $i, j \in \{A, B\}$ and $j \neq i$.

We first provide a similar condition to the one presented in Proposition 1 to determine the sign of the markup $p_i - c_i$ in any pure strategy Nash equilibrium in 2PTs. Second, when consumers have homogeneous vertical taste preferences, we show that marginal-cost-based 2PT is a Nash equilibrium, similarly to Corollary 2. Third, we show that when the market shares are determined by logit with outside option, marginal-cost-based 2PT is not a Nash equilibrium even when firms are symmetric (i.e., identical marginal cost and symmetric product demand), contrary to Corollary 3. And finally, we consider a setting with general market share, asymmetric costs, and with discrete types and show that the qualitative results of Section 4 hold.

Note that in this case the MRSA between p_i and F_i is

$$\text{MRSA} \equiv \frac{\partial E[s_i]}{\partial p_i} \Big/ \frac{\partial E[s_i]}{\partial F_i} = \frac{E\left[s_1 \cdot q_i\left(p_i, \boldsymbol{\theta}\right)\right]}{E\left[s_1\right]}.$$

Proposition 6. *In any pure strategy Nash equilibrium in two-part tariffs, for each $i \in \{A, B\}$, the markup $p_i - c_i$ has the same sign as $\frac{E[s \cdot q_i(p_i, \boldsymbol{\theta})]}{E[s]} - \frac{E[s_1 \cdot q_i(p_i, \boldsymbol{\theta})]}{E[s_1]}$.*

Note that the sign of the markup for each firm is determined by two factors: the average expected demand among all customers who choose the firm's product, and the MRSA between the two instruments p_i and F_i, as in Proposition 1.

5.1 Marginal Cost Pricing

To study necessary conditions under which marginal-cost-based 2PT is an equilibrium, let

$$\phi_i\left(\boldsymbol{\theta}\right) \equiv \frac{s\left(v_i^*\left(\boldsymbol{\theta}\right), v_j^*\left(\boldsymbol{\theta}\right)\right)}{E\left[s\left(v_i^*\left(\boldsymbol{\theta}\right), v_j^*\left(\boldsymbol{\theta}\right)\right)\right]} - \frac{s_1\left(v_i^*\left(\boldsymbol{\theta}\right), v_j^*\left(\boldsymbol{\theta}\right)\right)}{E\left[s_1\left(v_i^*\left(\boldsymbol{\theta}\right), v_j^*\left(\boldsymbol{\theta}\right)\right)\right]}$$

where $v_i^*\left(\boldsymbol{\theta}\right) \equiv v_i\left(c_i, \boldsymbol{\theta}\right) - F_i^*$ and F_i^* is implicitly defined by

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[36]For the $\boldsymbol{\theta}$-consumer, the aggregate consumer utility is

$$V\left(u_A\left(\boldsymbol{\theta}\right), u_B\left(\boldsymbol{\theta}\right)\right) = E_\xi\left[\max\left\{u_A\left(\boldsymbol{\theta}\right) + \xi_A, u_B\left(\boldsymbol{\theta}\right) + \xi_B, u_0 + \xi_0\right\}\right].$$

By the envelope theorem, $V_1\left(u_A\left(\boldsymbol{\theta}\right), u_B\left(\boldsymbol{\theta}\right)\right) \equiv s\left(v_A\left(p_A, \boldsymbol{\theta}\right) - F_A, \ v_B\left(p_B, \boldsymbol{\theta}\right) - F_B\right)$ is the share of $\boldsymbol{\theta}$-consumers who choose to buy from firm A. We assume that consumers' tastes for the two firms' products are symmetrically distributed, i.e., $V\left(u_A\left(\boldsymbol{\theta}\right), u_B\left(\boldsymbol{\theta}\right)\right) = V\left(u_B\left(\boldsymbol{\theta}\right), u_A\left(\boldsymbol{\theta}\right)\right)$.

[37]Armstrong and Vickers (2001) make a similar assumption.

$$F_i^* = \frac{E\left[s\left(v_i^*\left(\boldsymbol{\theta}\right), v_j^*\left(\boldsymbol{\theta}\right)\right)\right]}{E\left[s_1\left(v_i^*\left(\boldsymbol{\theta}\right), v_j^*\left(\boldsymbol{\theta}\right)\right)\right]} \tag{15}$$

for $j \neq i$ and $i, j \in \{A, B\}$. Note that $\phi_i\left(\boldsymbol{\theta}\right)$ is equal to the average expected demand for the firm i minus the MRSA between the instruments p_i and F_i, at $p_i = c_i$. If a pure-strategy Nash equilibrium in 2PTs involves marginal-cost pricing then the following condition holds:

$$\text{Cov}\left(\phi_i\left(\boldsymbol{\theta}\right), q_i\left(c_i, \boldsymbol{\theta}\right)\right) = 0 \tag{16}$$

for $j \neq i$ and $i, j \in \{A, B\}$, which we formally summarize in the next corollary.

Corollary 7. *For a given $c_i, c_j \in \mathcal{P}$, if a pure-strategy Nash equilibrium involves marginal-cost-based 2PT then (16) holds for $i, j \in \{A, B\}$ and $j \neq i$.*

Corollary 7 follows from Proposition 6. That is, if the equilibrium in 2PTs involves setting prices equal to the marginal costs then the average expected demand for firm i equals the MRSA between the instruments p_i and F_i. Intuitively, since $q_i\left(\cdot\right)$ is increasing with respect to $\boldsymbol{\theta}$, condition (16) means that the equilibrium profit margins are constant over taste preferences, $\boldsymbol{\theta}$, at $p_i = c_i$.[38] For the specific case in which ξ_i is distributed uniformly (*à la* Hotelling), condition (16) reduces to the independence between each firm's efficient quantity $q_i\left(c_i, \boldsymbol{\theta}\right)$, and the difference between the efficient consumer surpluses offered by the two firms, $v_i\left(c_i, \boldsymbol{\theta}\right) - v_j\left(c_j, \boldsymbol{\theta}\right)$, as we showed in Corollary 1. For the general model presented here, the market share may not be linear with respect to the difference between the efficient consumer surplus offered by the two firms. That is, condition (16) depends on the behaviors of market share functions and not on the difference between the efficient consumer surpluses offered by the two firms as in Corollary 1.

If consumers have homogeneous tastes for quality, (16) is trivially satisfied for each firm $i \in \{A, B\}$. We show in the next corollary that in this case, marginal-cost-based 2PT is an equilibrium.

Corollary 8. *Suppose consumers have homogeneous vertical tastes and $v_i\left(c_i\right) > \frac{s(0,0)}{s_1(0,0)}$ for $i \in \{A, B\}$. Then, marginal-cost-based 2PT is an equilibrium, where the fixed fees are given by $F_i^* = v_i\left(c_i\right) - v_i^*$ and v_i^* are implicitly defined by $v_i\left(c_i\right) = v_i^* + \frac{s\left(v_i^*, v_j^*\right)}{s_1\left(v_i^*, v_j^*\right)}$ for $i, j \in \{A, B\}$ and $j \neq i$.[39]*

Corollary 8 extends Armstrong and Vickers' Proposition 1 to allow for asymmetric firms. Here, firms set their marginal prices equal to the corresponding marginal costs. Corollary 8 shows that,

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[38]Condition (16) provides a similar intuition to the one provided by Rochet and Stole (2002) for marginal-cost pricing to be an equilibrium. They consider a model in which under general market share functions, firms offer nonlinear pricing schedules. Their condition (equation 12 in Rochet and Stole, 2002) follows from two observations: First, in the unrestricted model where firms use nonlinear pricing, a necessary and sufficient condition for efficient quantities is that $S_i^*\left(\theta\right) - u_i\left(\theta\right)$ is constant, where $S_i^*\left(\theta\right)$ is the surplus generated from efficient consumption by consumer θ from trade with firm i, $u_i\left(\theta\right) \equiv \max_q \theta q - P_i\left(q\right)$, and $P_i\left(q\right)$ is the price schedule chosen by firm i. Second, an efficient equilibrium arises if and only if at the equilibrium utilities of the restricted game (where firms are restricted to cost-plus-fixed-fee pricing), the equilibrium profit margins are constant over taste preferences, θ.

[39]Additional conditions can be imposed to guarantee uniqueness of F_i^* and, in consequence, uniqueness of the Nash equilibrium. These conditions are satisfied by the logit model with and without outside option.

if consumers have homogeneous tastes for quality, the optimal strategy for each firm is to set the unit price equal to its marginal cost and extract surplus with the fixed fee. From Corollary 8, it follows that if, for example, ξ_i follows a type-I extremum distribution (i.e., logit market shares), marginal-cost-based 2PT is an equilibrium.[40]

Note that $v_i(c_i)$ is the efficient (maximum) surplus offered by firm i to the consumers and v_i^* is the net surplus when competing firms engage in efficient surplus extraction. In the case of a monopoly, the firm would set $F_i^* = v_i(c_i)$ and hence the net consumer surplus would be zero (full extraction). In the presence of competition, full extraction is not possible and hence consumers earn positive surplus, $v_i^* > 0$. The equilibrium net surpluses $\left(v_i^*, v_j^*\right)$ are determined by the above equations, which imply that $0 < v_i^* < v_i(c_i)$ for each $i \in \{A, B\}$. The ratio $\frac{s\left(v_i^*, v_j^*\right)}{s_1\left(v_i^*, v_j^*\right)}$ represents the competitive effect that prevents firms from full extraction. Moreover, the firm that provides the higher surplus (at its own marginal cost) has the higher fixed fee, market share and total profits, similar to the model in Section 3; that is, if $v_i(c_i) > v_j(c_j)$, then $F_i^* > F_j^*$, $s_i^* > s_j^*$ and $\Pi^i > \Pi^j$ for $i \neq j$ and $i, j \in \{A, B\}$.

In summary, if consumers have heterogeneous tastes for quality, and if firms are symmetric (i.e., $c_i = c_j$ and $v_i(p, \boldsymbol{\theta}) = v_j(p, \boldsymbol{\theta})$ for all $p \in \mathcal{P}$ and $\boldsymbol{\theta} \in \Theta$ and for $i \neq j$) then marginal-cost-based 2PT may not be an equilibrium. For the model presented here, even if the market share is constant with respect to $\boldsymbol{\theta}$ in equilibrium (e.g., symmetric firms and no outside option), the left-hand side of (16) may be different from zero (e.g., if $\frac{s_1(\cdot)}{E[s_1(\cdot)]}$ is monotonic with respect to $\boldsymbol{\theta}$). Thus, necessary and sufficient conditions such that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT depend on both $s(\cdot)$ and $s_1(\cdot)$.

5.2 Logit Market Shares with an Outside Option

To illustrate Proposition 6 and Corollary 7 in the case of symmetric firms, we allow for an outside option and assume that ξ_i follows a type-I extremum distribution. The market share of firm i is

$$s\left(u_i, u_j\right) \equiv \frac{e^{u_i}}{e^{u_i} + e^{u_j} + e^{u_0}},$$

where $u_i \equiv v\left(p_i, \boldsymbol{\theta}\right) - F_i$ for $i, j \in \{A, B\}$, $j \neq i$, and u_0 is the value of the outside option. Note that in a symmetric equilibrium, the covariance between $\left(\frac{s}{E[s]} - \frac{s_1}{E[s_1]}\right)$ and the firm's efficient quantity would be positive, since $s\left(v^*(\boldsymbol{\theta}), v^*(\boldsymbol{\theta})\right)$, and $s_1\left(v^*(\boldsymbol{\theta}), v^*(\boldsymbol{\theta})\right)$ are increasing with respect to $\boldsymbol{\theta}$, but the rate of increase is higher for $s\left(v^*(\boldsymbol{\theta}), v^*(\boldsymbol{\theta})\right)$ than for $s_1\left(v^*(\boldsymbol{\theta}), v^*(\boldsymbol{\theta})\right)$. Thus if $\boldsymbol{\theta}$ is associated, (16) is not satisfied.

Proposition 7. *In a symmetric model with logit market shares and an outside option, any symmetric pure-strategy Nash equilibrium in 2PTs involves pricing above marginal cost.*

Proposition 7 contrasts with Corollary 8, which shows that marginal-cost pricing is an equilibrium for a general market share setting (including logit) when consumers have homogeneous

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[40]See Yin (2004) for a similar result in the case of logit market shares without outside option.

taste preferences.[41] Likewise, Proposition 7 contrasts with Corollary 3, which shows that any pure strategy Nash equilibrium involves marginal-cost-based 2PT when consumers are differentiated *á la* Hotelling and have heterogeneous tastes for quality and firms have identical marginal costs and product demand (i.e., firms are symmetric, as in Proposition 7). Proposition 7 also contrasts with Armstrong and Vickers (2001) and Rochet and Stole (2002) who consider a setting in which firms offer nonlinear pricing schedules. The authors show that in the symmetric equilibrium each firm offers a cost-plus-fee pricing schedule and each customer consumes the efficient allocation from one of the two firms. Their result relies upon the inverse hazard rate of the market share being constant over $\boldsymbol{\theta}$ in equilibrium for each firm,[42] which in turn relies upon full market coverage and symmetry of firms.[43] Remember that in the Hotelling model, the market share is linear with respect to the difference in the consumer surplus offered by the two firms, which simplifies the necessary and sufficient conditions for marginal-cost pricing (Proposition 2).

5.3 General Market Shares and Asymmetric Marginal Costs

We explore whether the equilibrium strategy for the less efficient firm involves *cross-subsidization* between the unit price and the fixed fee in a general market share setting with asymmetric marginal costs and no outside option. We suppose that firms have identical product demand but asymmetric marginal costs. To illustrate, we suppose θ is drawn from the distribution on $\Theta = \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$ (low and high type), with probabilities λ and $1 - \lambda$, respectively. The next proposition generalizes Proposition 3, allowing for a general market share without outside option.

Proposition 8. *In any pure-strategy Nash equilibrium in two-part tariffs* $c_A < p_A^* < p_B^* < c_B$.

The result in Proposition 8 (and the strategy used for its proof) is similar to that of Proposition 3. That is, when consumers are heterogeneous in their tastes for quality, firms have incentives to deviate from marginal-cost pricing. The efficient firm increases its marginal price—keeping it below its rival's price—and slightly decreases its fixed fee. On the other hand, the less efficient firm has incentives to decrease the marginal price below its own marginal cost, so that the revenue losses arising from this strategy are more than offset by the revenue gains obtained from an inverse in fixed fee.

5.4 An Example

In the two previous subsections, we studied a symmetric logit market share model with outside option and an asymmetric general market share setting without outside option. The analysis with asymmetric firms and general market shares with outside option is complicated, so here, we present a numerical example for the logit market share model with asymmetric firms. We consider constant

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[41]When consumers are homogeneous, a 2PT game is formally equivalent to a linear pricing game with logit market shares—as was pointed out by Nocke and Schutz (2018),—which has a unique equilibrium.

[42]The inverse hazard rate for firm A is equal to $s\left(u_A\left(\boldsymbol{\theta}\right), u_B\left(\boldsymbol{\theta}\right)\right) / \frac{\partial s(u_A(\boldsymbol{\theta}), u_B(\boldsymbol{\theta}))}{\partial u_A}$.

[43]Rochet and Stole (2002) observed that their condition for *cost-plus-fixed-fee* pricing equilibrium (equation 12 in Rochet and Stole, 2002) does not apply if there is an outside option of fixed value in the logit market share case.

elasticity demand and two types of vertical tastes. That is, suppose θ is drawn from a distribution on $\Theta = \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$ (low and high type), with probabilities λ and $1 - \lambda$, respectively. We assume the following functional form: $u(q, \theta) = \theta \frac{\eta^{\frac{1}{\epsilon}} q^{1-\frac{1}{\epsilon}}}{1-\frac{1}{\epsilon}}$, and hence $q(p) = \eta \left(\frac{\theta}{p}\right)^{\epsilon}$, $v(p) = \frac{1}{(\epsilon-1)} \frac{\eta \theta^{\epsilon}}{p^{\epsilon-1}}$, and $q'(p) = -\eta \epsilon \frac{\theta^{\epsilon}}{p^{\epsilon+1}}$. We use the following parameters: $c_A = 0.2$, $c_B = 0.25$, $\eta = 0.2$, $\epsilon = 2$, $\theta_L = 0.3$, and $\theta_H = 0.5$. We normalized the value of the outside option equal to zero (i.e., $\exp(u_0) = 1$). Figures 3(i) and 3(ii) show the changes in marginal prices and fixed fees, respectively, when λ (x axis) varies from 0 to 1. Note that marginal prices are always above marginal costs for both firms, which is consistent with Proposition 7, and lower for firm A than for firm B, while fixed fees are always higher for firm A than for firm B.
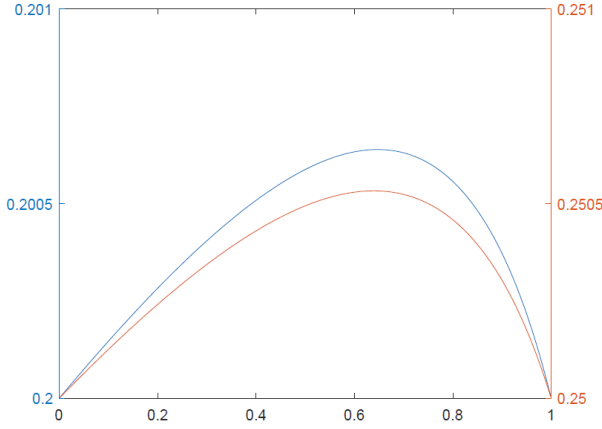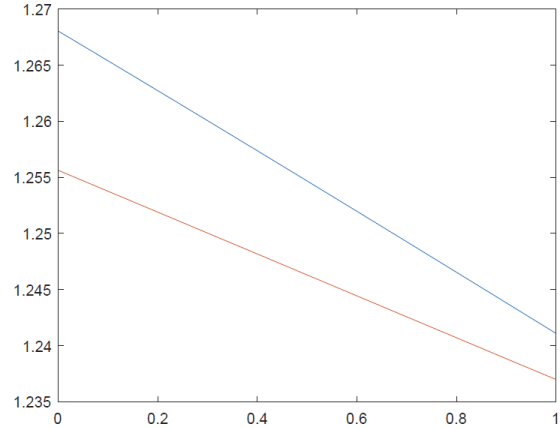
Figure 3(i): Marginal Prices

Figure 3(ii): Fixed Fees

Note: Figure 3(i) shows the changes in marginal prices for firm A (left y-axis, blue line) and firm B (right y-axis, red line) when λ (x axis) varies from 0 to 1. Similarly, Figure 3(ii) shows the changes in fixed fees for firm A (blue line) and firm B (red line), when λ (x axis) varies from 0 to 1

In the logit model with an outside option, firms also compete with the outside option, but the outside option does not react to the changes in marginal prices (or fixed fees) of any of the firms. Hence, firms react less aggressively to their competitor's pricing strategy than they do in the logit model without an outside option, allowing them to set prices above the marginal costs.

From previous results, we may expect that as the value of the outside option u_0 varies from $-\infty$ to 0, the less efficient firm again sets prices below the marginal cost. Figures 4(i) and 4(ii) demonstrate equilibrium values of p_A and p_B, respectively, when the outside option u_0 varies from $-\infty$ to 0 (x axis is equal to $\exp(u_0)$). Notice that there is a non-monotonic, inverse U-shaped between the outside option value and p_A and p_B, respectively. Moreover, as $u_0 \to -\infty$, $p_A \to \hat{p}_A > c_A$ and $p_B \to \hat{p}_B < c_B$. When the outside option is not attractive (i.e., u_o is sufficiently low), the less efficient firm sets its marginal price below the marginal cost, consistent with Proposition 8.
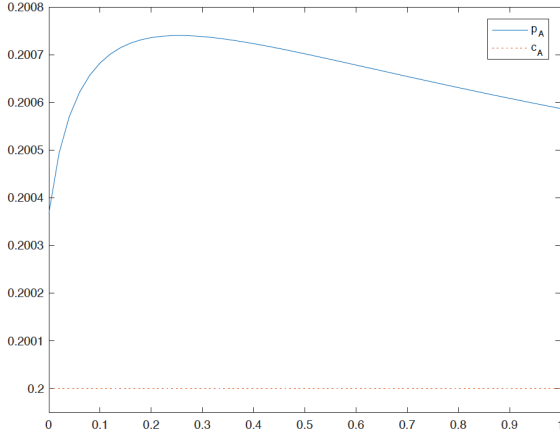
Figure 4(i): p_A Figure 4(ii): p_B
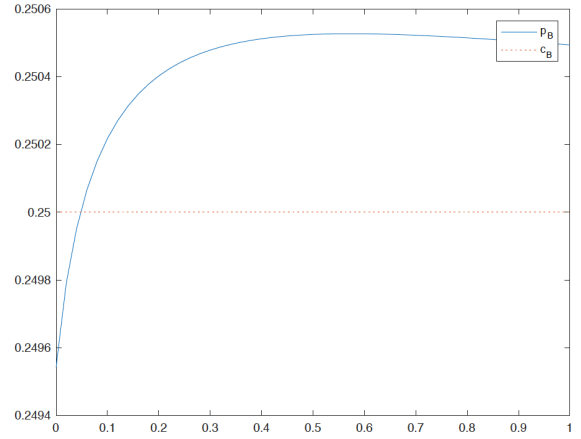
Note: Figure 4(i) shows the equilibrium values of p_A (y-axis) for different values of the outside option, u_0. Similarly, Figure 4(ii) shows equilibrium values of p_B for different values of u_0.

6   Concluding Remarks

In this paper, we study competitive two-part tariffs between asymmetric firms in a general model of multidimensional consumer heterogeneity. The model assumes that consumers have elastic demands and private information regarding their horizontal brand preferences and tastes for product quality. Using both the Hotelling and general discrete choice approaches to horizontal differentiation, we find that two major factors determine the sign of the markup for each firm, as well as the marginal-cost pricing that arises in equilibrium. The first is the average expected demand among all customers who choose the firm's product. The second is the marginal rate of substitution of the demand for access or participation between the marginal price and fixed fee, which provides a more general description of the consumers' participation incentives than the demand of the marginal consumer that has been identified in the literature regarding monopoly two-part tariffs in the context of one-dimensional consumer heterogeneity.

Moreover, in the model with consumer horizontal preferences described à la Hotelling, we show that when firms have asymmetric marginal costs but symmetric demand, the equilibrium strategy involves cross-subsidization between the marginal price and fixed fee for the less efficient firm, with the efficient firm always offering a marginal price above its marginal cost. A similar pattern of cross-subsidization holds when both firms have identical marginal costs but asymmetric product demands. When consumers' horizontal preferences are represented by logit with an outside option, our results indicate that marginal-cost pricing, even in the symmetric setting, is not an equilibrium, and the equilibrium in two-part tariffs involves both firms pricing above the marginal cost, resulting in inefficiency.

A direct extension of our analysis would be to consider a setting in which firms offer more than one product. Furthermore, we could consider more than two firms, using a general discrete choice model of random utility maximization. It would also be interesting to understand the impact of mergers on equilibrium two-part tariffs and consumer welfare. Since firms compete aggressively with their marginal prices and set them close to their marginal costs, mergers between the efficient and less efficient firms may have a substantial effect on other competing firms that also offer 2PTs and hence on consumer welfare.

Appendix: Proofs

Proof of Proposition 2. We split the proof in two steps: In (i) we show that the objective function of firm i is single-peaked so that it has a global maximum if for any $p_i, p_j \in \mathcal{P}$,[44]

$$\text{Cov}\left(v_i\left(p_i, \boldsymbol{\theta}\right) - v_j\left(p_j, \boldsymbol{\theta}\right), q_i\left(p_i, \boldsymbol{\theta}\right)\right) = 0. \tag{A.17}$$

In (ii) we show that for $p_i^* = c_i$ and $p_j^* = c_j$, there do not exist $p_i, p_j \in \mathcal{P}$ with $p_i \neq p_i^*$ and $p_j \neq p_j^*$, such that the first order conditions are satisfied.

(i) We solve the two-variable optimization problem of firm i sequentially. First, note that from the first order condition with respect to F_i, (5), it follows that the profit function is quadratic and strictly concave in F_i, thus, for any $p_i \in \mathcal{P}$, F_i is uniquely defined by (5). Next, firm i chooses p_i to maximize its profits (we substitute $F_i^*\left(p_i\right)$ in the objective function of firm i)

$$E\left[s_i\left(p_i, F_i^*\left(p_i\right), p_j, F_j; \boldsymbol{\theta}\right)\left(\pi_i\left(p_i, \boldsymbol{\theta}\right) + F_i^*\left(p_i\right)\right)\right], \tag{A.18}$$

where $4t \cdot s_i\left(p_i, F_i^*\left(p_i\right), p_j, F_j; \boldsymbol{\theta}\right) = t + v_i\left(p_i, \boldsymbol{\theta}\right) - v_j\left(p_j, \boldsymbol{\theta}\right) + F_j + \pi_i\left(p_i, \boldsymbol{\theta}\right)$. The derivative of (A.18) with respect to p_i, after using the envelope theorem, (5), and (A.17), is

$$-\left(p_i - c_i\right)\underbrace{\left\{\text{Var}\left[q_i\left(p_i, \boldsymbol{\theta}\right)\right] - 2tE\left[q_i'\left(p_i, \boldsymbol{\theta}\right)s_i\left(p_i, F_i^*(p_i), p_j, F_j; \boldsymbol{\theta}\right)\right]\right\}}_{>0}. \tag{A.19}$$

Given $p_j \in \mathcal{P}$ and $F_j \geq 0$, for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, $s_i\left(p_i, F_i^*\left(p_i\right), p_j, F_j; \boldsymbol{\theta}\right)$, is strictly decreasing with respect to p_i for any $p_i > c_i$. Note that if there exists a $\tilde{p}_i \in \mathcal{P}$ and a $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, such that $s_i\left(\tilde{p}_i, F_i^*\left(\tilde{p}_i\right), c_j, F_j^*; \boldsymbol{\theta}\right) = 0$, then for any $p_i \geq \tilde{p}_i\left(p_j, F_j\right) \equiv \sup_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} \tilde{p}_i\left(p_j, F_j; \boldsymbol{\theta}\right)$ the profit is zero. Then, for any $p_j \in \mathcal{P}$ and $F_j \geq 0$, note that (A.19) is positive for $p_i < c_i$ and it is negative for any $p_i \in \left(c_i, \tilde{p}_i\left(p_j, F_j\right)\right)$. Thus, (A.18) is single-peaked in p_i and reaches a unique maximum at $p_i = c_i$. Analogously, the profit function for firm $j \neq i$ in terms of p_j is single-peaked in p_j and reaches a unique maximum at $p_j = c_j$.

(ii) We now show that if (A.17) is satisfied, there do not exist $p_i, p_j \in \mathcal{P}$ with $p_i^* \neq c_i$ and $p_j^* \neq c_j$, such that the first order conditions are satisfied. From equation (6) and (A.17) we get

$$\left(p_i - c_i\right)\left\{2t \cdot E\left[q_i'\left(p_i, \boldsymbol{\theta}\right)s_i\right] - \text{Var}\left[q\left(p_i, \boldsymbol{\theta}\right)\right]\right\} = 0. \tag{A.20}$$

where s_i is given by (7). If we take the derivative to both sides of (A.17) with respect to p_i, multiply by $p_i - c_i$ and substitute in (A.20) we get

$$E\left[q_i'\left(p_i, \boldsymbol{\theta}\right)\left(p_i - c_i\right)\right]\left\{t + \frac{1}{3}E\left[v_i\left(p_i, \boldsymbol{\theta}\right) + \pi_i\left(p_i, \boldsymbol{\theta}\right) - v_j\left(p_j, \boldsymbol{\theta}\right) - \pi_j\left(p_j, \boldsymbol{\theta}\right)\right]\right\} = 0. \tag{A.21}$$

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[44]Note that the necessary condition (without second-order conditions) is implied by Proposition 1 and Corollary 1.

Note that if $p_i \neq c_i$ the left-hand side of (A.21) implies that $E\left[\pi_i\left(p_i, \boldsymbol{\theta}\right)\right]+F_i = 0$ for $i \in \{A, B\}$. However, both firms have a profitable deviation by setting marginal prices equal to marginal costs and the fixed fee equal to $F_i^* = t + \frac{E[v_i(c_i,\boldsymbol{\theta})-v_j(c_j,\boldsymbol{\theta})]}{3}$. If both firms have strictly positive profits, the second term of the left side of (A.21) is strictly positive, then $p_i = c_i$ for $i \in \{A, B\}$. Thus, marginal-cost pricing is a unique equilibrium.

Proof of Corollary 2. We solve the two-variable optimization problem of firm i sequentially. First we show that for any $p_i \in \mathcal{P}$, firm i chooses F_i to maximize its profits. The first order condition with respect to F_i yields

$$s_i\left(p_i, F_i, p_j, F_j\right) - \frac{1}{2t}\left[\pi_i\left(p_i\right) + F_i\right] = 0. \tag{A.22}$$

The profit is quadratic and strictly concave in F_i, and the unique solution is given by

$$2F_i^* = t + v_i\left(p_i\right) - v_j\left(p_j\right) + F_j - \pi_i\left(p_i\right). \tag{A.23}$$

Next, firm i chooses p_i to maximize its maximum profits (we substitute $F_i^*\left(p_i\right)$ in the objective function of firm i)

$$s_i\left(p_i, F_i^*\left(p_i\right), p_j, F_j\right)\left[\pi_i\left(p_i\right) + F_i^*\left(p_i\right)\right]. \tag{A.24}$$

The derivative of (A.24) with respect to p_i, after using the Envelope Theorem, is

$$q'\left(p_i\right)\left(p_i - c_i\right)s_i\left(p_i, F_i^*\left(p_i\right), p_j, F_j\right) = 0. \tag{A.25}$$

Given $p_j \in \mathcal{P}$ and $F_j \geq 0$, $s\left(p_i, F_i^*\left(p_i\right), p_j, F_j\right) = 1/4t\left\{t + v_i\left(p_i\right) - v_j\left(p_j\right) + F_j + \pi_i\left(p_i\right)\right\}$, is strictly decreasing with respect to p_i for any $p_i > c_i$. Thus, if there exists a $\tilde{p}_i\left(p_j, F_j\right) \in \mathcal{P}$, such that $s\left(\tilde{p}_i, F_i^*\left(\tilde{p}_i\right), p_j, F_j\right) = 0$, then, for any $p_i \geq \tilde{p}_i$ the profit is zero. Then, for any $p_j \in \mathcal{P}$ and $F_j \geq 0$, note that (A.25) is positive for $p_i < c_i$ and it is negative for any $p_i \in \left(c_i, \tilde{p}_i\left(p_j, F_j\right)\right)$. Thus, (A.24) is single-peaked in p_i and reaches a unique maximum at $p_i = c_i$. Analogously, the profit function for firm $j \neq i$ in terms of p_j is single-peaked in p_j and reaches a unique maximum at $p_j = c_j$. Thus the equilibrium is unique, and it follows that $F_i^* = t + \frac{v_i(c_i)-v_j(c_j)}{3}$ for $j \neq i$ and $i, j \in \{A, B\}$.

Proof of Corollary 3. It follows directly from Proposition 1.

Proof of Corollary 4. It follows directly from Proposition 1 and Corollary 1.

Proof of Proposition 3.

(i) We first show that for every $p_A, p_B \in \mathcal{P}$ that satisfies (10) for $i = A$, if $p_A > c_A$ then $p_A < p_B$. Second we show that for every $p_A, p_B \in \mathcal{P}$ that satisfy (10) for $i = B$, if $p_B \geq c_B$, then $p_B < p_A$ which is a contradiction. Thus we conclude that no equilibrium exists for values of $p_B \geq c_B$. (iii) We show that for every $p_A, p_B \in \mathcal{P}$ satisfying (10) for $i \in \{A, B\}$, $p_A > c_A$. Finally we show that the two curves defined by (10) for $i \in \{A, B\}$ cross each other at least once in the set $\left[c_A, c_B\right]^2$.

Let us first show that for every $p_A, p_B \in \mathcal{P}$ that satisfy (10) for $i = A$, if $p_A > c_A$ then $p_A < p_B$.

From (10)

$$0 < p_A - c_A = \omega_A\left(p_A\right) \mathrm{Cov}\left(v\left(p_A, \boldsymbol{\theta}\right) - v\left(p_B, \boldsymbol{\theta}\right), q\left(p_A, \boldsymbol{\theta}\right)\right).$$

Since $\boldsymbol{\theta}$ is associated and $\omega_A(p_A) > 0$, it follows that $p_A \leq p_B$. In fact, $p_A < p_B$, since $p_A = p_B$ gives us the contradiction $c_A = c_B$. Thus we conclude that if $p_A, p_B \in \mathcal{P}$ satisfy (10) for $i = A$, if $p_A > c_A$ has to be true that $p_A < p_B$. Similarly, we can show that for every $p_A, p_B \in \mathcal{P}$ that satisfy (10) for $i = B$, if $p_B \geq c_B$, then $p_B < p_A$, which is a contradiction. Thus we conclude that no equilibrium exists for values of $p_B \geq c_B$. Also, p_A cannot be smaller than c_A, because if $p_A < c_A$, by equation (10) for $i = A$, $p_A > p_B$, and then by equation (10) for $i = B$, $p_B > c_B$, which gives us the contradiction $c_A > c_B$. If $p_A = c_A$, we get the contradiction $c_A = c_B$. Thus, for every $p_A, p_B \in \mathcal{P}$ satisfying (10) for $i \in \{A, B\}$, $p_A > c_A$.

Now let us prove existence. Note that as $p_A \to c_A$ in (10) for $i = A$, we have that $p_B \to c_A$, and as $p_A \to c_B$ (from the previous paragraph), we know that $p_B \to \alpha_A > c_B$. Similarly, from (10) for $i = B$, note that as $p_A \to c_A$

$$0 \geq p_B - c_B = \omega_B\left(p_B\right) \mathrm{Cov}\left(v\left(p_B, \boldsymbol{\theta}\right) - v\left(c_A, \boldsymbol{\theta}\right), q\left(p_B, \boldsymbol{\theta}\right)\right),$$

which implies that $p_B \to \alpha_B > c_A$ since the term $\mathrm{Cov}\left(v\left(p_B, \boldsymbol{\theta}\right) - v\left(c_A, \boldsymbol{\theta}\right), q\left(p_B, \boldsymbol{\theta}\right)\right)$ must be negative. Finally, from (10) for $i = B$, as $p_A \to c_B$, $p_B \to c_B$. Note that the *quasi* best-response functions, (10) for $i \in \{A, B\}$, are differentiable and therefore continuous for every $(p_A, p_B) \in [c_A, c_B]^2$ (see also Lemma 1).

(*ii*) It follows from substituting F_i^* for $i \in \{A, B\}$ in the expected market share and the fact that in any pure-strategy Nash equilibrium, $c_A < p_A^* < p_B^* < c_B$.

Lemma 1. *Suppose that (A1)-(A4) are satisfied. Then the slope of the implicit functions defined by (10), $\frac{\partial R^i(p_A)}{\partial p_A}$ for $i \in \{A, B\}$, is positive for $(p_A, p_B) \in [c_A, c_B]^2$.*

Proof of Lemma 1. Note that (A4) allows us to express (10) as a function of $\sigma \equiv \mathrm{Var}\left[h\left(\boldsymbol{\theta}\right)\right]$ and $\bar{\theta} \equiv E\left[h\left(\boldsymbol{\theta}\right)\right]$, in the sense that (10) is equivalent to the equation $\xi^i\left(\boldsymbol{p}\right) = 0$, where

$$\xi^i\left(\boldsymbol{p}\right) := \left(v\left(p_i\right) - v\left(p_j\right) - \phi_i\left(p_i\right)\right)\sigma - h^i\left(p_i\right)\left(p_i - c_i\right)\bar{\theta}\left\{t + \bar{\theta}\frac{TS_i\left(p_i\right)}{3} - \bar{\theta}\frac{TS_j\left(p_j\right)}{3}\right\}, \quad (\mathrm{A.26})$$

$\boldsymbol{p} \equiv (p_i, p_j)$, $h^i\left(p\right) \equiv -\frac{q'(p)}{\pi_i'(p)}$, $\phi_i\left(p\right) \equiv \frac{q(p)^2(p - c_i)}{\pi_i'(p)}$, $q\left(p\right) \equiv -\frac{\partial v(p)}{\partial p}$, $\pi_i\left(p\right) \equiv q\left(p\right)\left(p - c_i\right)$, and $TS_i\left(p\right) \equiv v\left(p\right) + \pi_i\left(p\right)$ for $i \in \{A, B\}$ and $j \neq i$.

(*i*) $\frac{\partial R^A(p_A)}{\partial p_A} > 0$: In Proposition 4 we show that $-\frac{\partial \xi^A}{\partial p_A} > \frac{\partial \xi^B}{\partial p_A}$, and in part (*ii*) of this proposition we show that $\frac{\partial \xi^B}{\partial p_A} > 0$. Thus, we just need to show that $\frac{\partial \xi^A}{\partial p_B} > 0$. Note that

$$\frac{\partial \xi^A}{\partial p_B} = q\left(p_B\right)\sigma + \underbrace{h^A\left(p_A\right)\left(p_A - c_A\right)\bar{\theta}^2 \cdot \frac{TS'\left(p_B\right)}{3}}_{>0} > 0,$$

since $TS'(p_B) > 0$ for $p_B < c_B$. Then from the Implicit Function Theorem we conclude that $\frac{\partial R^A(p_A)}{\partial p_A} > 0$ for $(p_A, p_B) \in [c_A, c_B]^2$.

(ii) $\frac{\partial R^B(p_A)}{\partial p_A} > 0$: In Proposition 4 we show that $\frac{\partial \xi^A}{\partial p_B} < -\frac{\partial \xi^B}{\partial p_B}$. In part (i) we showed that $\frac{\partial \xi^A}{\partial p_B} > 0$. Thus, we just need to show that $\frac{\partial \xi^B}{\partial p_A} > 0$ for $(p_A \times p_B) \in [c_A, c_B]^2$. Note that

$$\frac{\partial \xi^B}{\partial p_A} = q(p_A)\sigma + \underbrace{h^B(p_B)(p_B - c_B)\bar{\theta}^2 \cdot \frac{TS'(p_A)}{3}}_{>0} > 0,$$

since $TS'(p_A)(p_B - c_B) > 0$ for any $(p_A, p_B) \in [c_A, c_B]^2$. Then from the Implicit Function Theorem we conclude that $\frac{\partial R^B(p_A)}{\partial p_A} > 0$ for $(p_A \times p_B) \in [c_A, c_B]^2$.

Before we show the proof for Proposition 4, first, we introduce the following assumption.

Assumption A1.

$$c_B - c_A < -\frac{q(c_B)}{3q'(c_A)}.$$

Proof of Proposition 4. We split the proof in two steps: (i) using the result in (ii), we show that the objective function of firm i is single-peaked so that it has a global maximum. (ii) For p_A^* and p_B^* defined by (10) for $i \in \{A, B\}$,[45] we show that there do not exist $(p_A, p_B) \in (c_A, c_B)^2$ with $p_A \neq p_A^*$ and $p_B \neq p_B^*$, such that the first order conditions are satisfied.

(i) We show that the objective function of firm i is single-peaked so that it has a global maximum. We fix firm j's strategy to $F_j = F_j^*$ and $p_j = p_j^*$, and solve the two-variable optimization problem of firm i sequentially. First we show that for any $p_i \in \mathcal{P}$, firm i chooses F_i to maximize its profits. The first order condition with respect to F_i yield

$$2F_i^* = t + E\left[v(p_i, \boldsymbol{\theta}) - v(p_j^*, \boldsymbol{\theta})\right] + F_j^* - E\left[\pi_i(p_i, \boldsymbol{\theta})\right]. \tag{A.27}$$

The profit is quadratic and strictly concave in F_i and the unique solution is given by (A.27). Next, firm i chooses p_i to maximize its profits (we substitute $F_i^*(p_i)$ in the objective function of firm i)

$$E\left[s\left(p_i, F_i^*(p_i), p_j^*, F_j^*; \boldsymbol{\theta}\right)\left(\pi_i(p_i, \boldsymbol{\theta}) + F_i^*(p_i)\right)\right], \tag{A.28}$$

where $4t \cdot E\left[s\left(p_i, F_i^*(p_i), p_j^*, F_j^*; \boldsymbol{\theta}\right)\right] = t + E\left[v(p_i, \boldsymbol{\theta}) - v(p_j^*, \boldsymbol{\theta})\right] + F_j^* + E\left[\pi_i(p_i, \boldsymbol{\theta})\right]$. The derivative of (A.28) with respect to p_i, after using the envelope theorem and (A.27), is

$$\underbrace{\text{Cov}\left(v(p_i, \boldsymbol{\theta}) - v(p_j^*, \boldsymbol{\theta}), q(p_i, \boldsymbol{\theta})\right)}_{=:T_1(p_i)} - (p_i - c_i)\left\{\text{Var}\left[q(p_i, \boldsymbol{\theta})\right] - 2tE\left[q'(p_i, \boldsymbol{\theta})s_i(p_i, F_i, p_j, F_j; \boldsymbol{\theta})\right]\right\},$$

$$\tag{A.29}$$

---

[45]We showed that such a pair exists in the proof of Proposition 3.

Note that for any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, $s\left(p_i, F_i^*\left(p_i\right), c_j, F_j^*; \boldsymbol{\theta}\right)$ is strictly decreasing with respect to p_i for any $p_i > c_i$. Note that if there exists a $\tilde{p}_i \in \mathcal{P}$ and a $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, such that $s\left(\tilde{p}_i, F_i^*\left(\tilde{p}_i\right), c_j, F_j^*; \boldsymbol{\theta}\right) = 0$, then from (A3), it follows that for any $p_i \geq \tilde{p}_i$ the profit is zero for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. We show that (A.29) is single peaked for each firm $i \in \{A, B\}$.

1. Suppose $i = A$ and $j = B$. Note that $T_1\left(p_A\right) > (<) \, 0$ if $p_A < (>) \, p_B^*$, by the increasing difference property and the fact that $\boldsymbol{\theta}$ is strictly associated. Finally, note that the second term of (A.29) is negative for $p_A \in (c_A, \tilde{p}_A)$. So if $p_A > p_B^*$, (A.29) is negative. From the second part of the proof, we know that there do not exist $(p_A, p_B) \in (c_A, c_B)^2$ with $p_A \neq p_A^*$ and $p_B \neq p_B^*$ such that (A.29) is zero; by continuity, we conclude that (A.29) is negative for $p_A > p_A^*$.[46] To show that (A.29) is positive for any $p_A < p_A^*$, note that (A.29) is positive for $p_A = c_A$ and we know that there do not exist $(p_A, p_B) \in (c_A, c_B)^2$ with $p_A \neq p_A^*$ and $p_B \neq p_B^*$ such that (A.29) is zero, by continuity, (A.29) is positive for any $p_A < p_A^*$.

2. Suppose $i = B$ and $j = A$. Note that (A.29) is negative for all $p_B > c_B$. Since there do not exist $(p_B, p_A) \in (c_A, c_B)^2$ with $p_B \neq p_B^*$ and $p_A \neq p_A^*$ such that (A.29) is zero, by continuity, it follows that (A.29) is negative for all $p_B > p_B^*$. Similarly, note that (A.29) is positive for $p_B \leq p_A^*$, and since there do not exist $(p_B, p_A) \in (c_A, c_B)^2$ with $p_B \neq p_B^*$ and $p_A \neq p_A^*$ such that (A.29) is zero, (A.29) is positive for $p_B \leq p_B^*$.

We conclude that (A.28) is single-peaked in p_i for $p_i \in [\min\{c_i, c_j\}, \tilde{p}_i)$ and hence has a unique maximum at $p_i = p_i^*$.

(ii) We now show that there do not exist $(p_A, p_B) \in (c_A, c_B)^2$ with $p_A \neq p_A^*$ and $p_B \neq p_B^*$ such that (A.29) is zero. To prove this claim we show that $\frac{\partial R^A(p_A)}{\partial p_A} > \frac{\partial R^B(p_A)}{\partial p_A}$, i.e.,

$$-\frac{\partial \xi^A / \partial p_A}{\partial \xi^A / \partial p_B} > -\frac{\partial \xi^B / \partial p_A}{\partial \xi^B / \partial p_B}. \tag{A.30}$$

From Lemma 1 (upward sloping *quasi* best-response functions for any $(p_A, p_B) \in (c_A, c_B)^2$) and (A.30) it follows that there do not exist $(p_A, p_B) \in (c_A, c_B)^2$ with $p_A \neq p_A^*$ and $p_B \neq p_B^*$ such that (A.29) is zero.

We first show that $-\partial \xi^A / \partial p_A > \partial \xi^B / \partial p_A$. Next we show that $-\partial \xi^B / \partial p_B > \partial \xi^A / \partial p_B$.

$(ii.a)$ $-\partial \xi^A / \partial p_A > \partial \xi^B / \partial p_A$. We need to show that

$$q\left(p_A\right) \sigma + h^B\left(p_B\right)\left(p_B - c_B\right) \bar{\theta}^2 \cdot \frac{TS'(p_A)}{3} < \left(q\left(p_A\right) + \phi_A'\left(p_A\right)\right) \sigma +$$

$$\left[\frac{\partial}{\partial p_A} h^A\left(p_A\right)\left(p_A - c_A\right)\right] \bar{\theta} \cdot TR_A + h^A\left(p_A\right)\left(p_A - c_A\right) \bar{\theta}^2 \cdot \frac{TS'(p_A)}{3},$$

where $TR_A \equiv \left\{t + \bar{\theta} \frac{TS(p_A)}{3} - \bar{\theta} \frac{TS(p_B)}{3}\right\}$.

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[46]Note that continuity follows from Lemma 1.

Note that $-h^B (p_B) (p_B - c_B) < 1$ and that $TS'(p_A) = q'(p_A)(p_A - c_A)$ thus we need to show that

$$0 < \frac{\bar{\theta}^2 \cdot q(p_A)}{3} \left(\frac{q'(p_A)(p_A - c_A)}{\pi'_A(p_A)} \right) + \phi'_A(p_A)\sigma + \left[\frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) \right] \bar{\theta} \cdot TR_A.$$

In Lemma A1 we show that $\frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) > 0$. Given that $q'(p_A)(p_A - c_A) > -q(p_A)$ and $TR_A > 0$, it is enough to show that

$$-\bar{\theta}^2 \frac{q(p_A)^2}{3\pi'_A(p_A)} + \phi'_A(p_A)\sigma > 0,$$

which follows from Lemma A2.

$(ii.b)$ $-\partial \xi^B / \partial p_B > \partial \xi^A / \partial p_B$. We need to show that

$$0 < \bar{\theta}^2 \frac{q'(p_A)(p_A - c_A)}{\pi'_A(p_A)} \frac{q'(p_B)(p_B - c_B)}{3} + \phi'_B(p_B)\sigma + \left[\frac{\partial}{\partial p_B} h^B(p_B)(p_B - c_B) \right] \bar{\theta} \cdot TR_B$$

$$- \frac{q'(p_B)(p_B - c_B)}{\pi'_B(p_B)} \bar{\theta}^2 \cdot \frac{q'(p_B)(p_B - c_B)}{3},$$

where $TR_B \equiv \left\{ t + \bar{\theta} \frac{TS(p_B)}{3} - \bar{\theta} \frac{TS(p_A)}{3} \right\}$.

In Lemma A3 we show that if c_B satisfies (A-A1), $\frac{\partial}{\partial p_B} h^B(p_B)(p_B - c_B) > 0$. Next, note that by (A-A1) it follows that $-\frac{q'(p_A)(p_A - c_A)}{\pi'_A(p_A)} < \frac{1}{2}$ and $-\frac{q'(p_B)(c_B - p_B)}{\pi'_B(p_B)} < \frac{1}{2}$ for $p_A \in (c_A, c_B)$ if c_B satisfies (A-A1). Thus, it is enough if we show that

$$0 < -\frac{\bar{\theta}^2 q'(p_B)(p_B - c_B)}{3} + \phi'_B(p_B)\sigma,$$

which follows from Lemma A2 and $3\sigma > \bar{\theta}^2$.

Lemma A1. $\frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) > 0$.
Proof of Lemma A1. The proof of this lemma follows from the observation that

$$\frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) = \frac{-q'(p_A)\pi'_A(p_A) + (p_A - c_A) \left\{ 2q'(p_A)^2 - q''(p_A)q(p_A) \right\}}{\pi'_A(p_A)^2}.$$

From (A2) it follows that $2q'(p_A)^2 - q''(p_A)q(p_A) > 0$. Thus, $\frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) > 0$.

Lemma A2. $\phi'_A(p_A) - \frac{q(p_A)^2}{\pi'_A(p_A)} > 0$.
Proof Lemma A2. Note first that

$$\phi'_A(p_A) - \frac{q(p_A)^2}{\pi'_A(p_A)} = \frac{\pi'_A(p_A) q(p_A) \pi'_A(p_A) + \pi_A(p_A) q'(p_A) \pi'_A(p_A)}{\pi'_A(p_A)^2} \tag{A.31}$$

$$\frac{-\pi_A(p_A) q(p_A) \pi''_A(p_A)}{\pi'_A(p_A)^2} - \frac{q_A(p_A)^2}{\pi'_A(p_A)}.$$

Substituting $\pi'_A(p_A) = q(p_A) + q'(p_A)(p_A - c_A)$ and $\pi''_A(p_A) = 2q'(p_A) + q''(p_A)(p_A - c_A)$ in (A.31), gives us

$$\phi'_A(p_A) - \frac{q(p_A)^2}{\pi'_A(p_A)} = \frac{q(p_A)(p_A - c_A)^2}{\pi'_A(p_A)^2}\left\{2q'(p_A)^2 - q''(p_A) q(p_A)\right\} > 0,$$

by (A2).

Lemma A3. $\frac{\partial}{\partial p_B} h^B(p_B)(p_B - c_B) > 0.$

Proof of Lemma A3. Note that

$$\frac{\partial}{\partial p_B} h^B(p_B)(p_B - c_B) = -\frac{\partial}{\partial p_B} \frac{q'(p_B)(p_B - c_B)}{\pi'_B(p_B)}$$

$$= \frac{-q'(p_B)(q(p_B) - q'(p_B)(p_B - c_B)) + \underbrace{(c_B - p_B) q''(p_B) q(p_B)}_{>0}}{\pi'_B(p_B)^2}.$$

Now, $(q(p_B) - q'(p_B)(p_B - c_B)) > 0$ if c_B satisfies (A-A1) and $p_B \in (c_A, c_B)$. Thus, we conclude that $\frac{\partial}{\partial p_B} h^B(p_B)(p_B - c_B) > 0.$

Proof of Corollary 5. From the proof of Proposition 3 we know that as $p_A \to c_A$ in (10) for $i = A$, we have that $p_B \to c_A$ and as $p_A \to c_B$, $p_B \to \alpha_A > c_B$. Similarly, from (10) for $i = B$, as $p_B \to c_B$ we have that $p_A \to c_B$, while as $p_A \to c_A$, $p_B \to \alpha_B > c_A$. As c_A, c_B tends to c, α_A and α_B also tend to c. Thus, the two best response functions intersect each other only at $p_i = c_i$.

Proof of Corollary 6. It follows from the proof of Lemma 1, specifically from equation (A.26), from the proof of Proposition 4, and by the monotonicity of the *quasi* best-response functions with respect to the unit prices for both firms.

Before we show the proof for Proposition 5, we introduce the following assumption.

Assumption A2.

$$\alpha_A - \gamma_B < -\frac{q(\alpha_A)}{3q'(\gamma_B)}.$$

Proof of Proposition 5. We split the proof in five steps. (*i*) We first show that no solution exist for (p_A, p_B) outside the set Ω, where $\Omega \equiv \{p_A, p_B \in \mathcal{P} \,|\, (p_A, p_B) \in [c, \alpha_A] \times [\gamma_B, c]\}$. (*ii*) We

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show existence of $(p_A^*, p_B^*) \in \Omega$ solving (13) for $i \in \{A, B\}$. (iii) Using the result in (v), we show that the objective function of firm i is single-peaked so that it has a global maximum. (iv) We show that the slope of the implicit functions defined by (13) is positive. (v) We show that for p_A^* and p_B^* defined by (13) for $i \in \{A, B\}$, there do not exist $(p_A, p_B) \in \Omega$ with $p_A \neq p_A^*$ and $p_B \neq p_B^*$, such that the first order conditions are satisfied. The proof of part (iii) is very similar to the Proof of Proposition 4 so we omit it.

(i) We assume that $p_A, p_B \in \mathcal{P}$ satisfy (13) for $i \in \{A, B\}$. We first show that $p_B \leq c$. Assume $p_B > c$. From (13) for $i = B$, it follows that

$$0 < p_B - c = \omega_B(p_B) \operatorname{Cov}(\alpha v(p_B, \boldsymbol{\theta}) - v(p_A, \boldsymbol{\theta}), q_B(p_B, \boldsymbol{\theta})).$$

Since $\omega_B(p_B) > 0$, from (A4) it follows that

$$q_B(p_B)(\alpha v(p_B) - v(p_A))\operatorname{Var}[h(\boldsymbol{\theta})] > 0.$$

Since $\alpha \in (0, 1)$, then $v(p_B) > v(p_A)$, which implies that $p_A > p_B \; (> c)$. Thus, from (13) for $i = A$, we get a contradiction, $v(p_A) - \alpha v(p_B) > 0$. Similarly we show that $p_A \geq c$. Suppose that $p_A < c$. From (13) for $i = A$ it follows that $p_A < c < p_B$, which from (13) for $i = B$ would be a contradiction, $p_A > p_B$.

Now we show that $p_B \geq \gamma_B$. By contradiction; assume $p_B < \gamma_B$, then $\alpha v(p_B) > \alpha v(\gamma_B) = v(c) \geq v(p_A)$. We showed that $p_A \geq c$, then, it follows that $\alpha v(p_B) - v(p_A) > 0$ implying that $p_A < c$, which is a contradiction. Finally, we show that $p_A \leq \alpha_A$. Suppose $p_A > \alpha_A$, then, $v(p_A) < v(\alpha_A) = \alpha v(c) \leq \alpha v(p_B)$ (recall $p_B \leq c$), which implies that $p_A < c$, which is a contradiction.

Thus, we conclude that no equilibrium exists outside the set Ω.

(ii) Let $\varphi(p_A, p_B; \boldsymbol{\theta}) \equiv v(p_A, \boldsymbol{\theta}) - \alpha v(p_B, \boldsymbol{\theta})$. Note that as $p_A \to c$ in (13) for $i = A$,

$$\tilde{\xi}^A(c, p_B) = (v(c) - \alpha v(p_B)) \cdot q(c) \cdot \operatorname{Var}[h(\boldsymbol{\theta})] < 0. \tag{A.32}$$

It is not true that $p_B \to c$, since $\varphi(c, c; \boldsymbol{\theta}) > 0$ for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$. Thus, from part (ii),[47] we conclude that $p_B \to \tilde{\gamma}_B < c$, as $p_A \to c$, where $\tilde{\gamma}_B$ is such that $\tilde{\xi}^A(c, \gamma_B) = 0$, that is

$$E[\varphi(c, \gamma_B; \boldsymbol{\theta})] = \frac{E[q(c, \boldsymbol{\theta}) \varphi(c, \gamma_B; \boldsymbol{\theta})]}{E[q(c, \boldsymbol{\theta})]}.$$

Similarly, as $p_B \to c$ in (13) for $i = B$,

$$\tilde{\xi}^B(p_A, c) = (\alpha v(c) - v(p_A)) \cdot q(c) \cdot \operatorname{Var}[h(\boldsymbol{\theta})] > 0. \tag{A.33}$$

From part (ii) we conclude that $p_A \to \alpha_A > c$, with $\tilde{\xi}^B(\alpha_A, c) = 0$.[48] Likewise, as $p_A \to c$ in (13) for $i = B$, we have

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[47]Note that $\tilde{\xi}^A(c, p_B) > 0$ if $p_B = c$.

[48]Note that $\tilde{\xi}^B(p_A, c) < 0$ if $p_A = c$.

$$\tilde{\xi}^B(c,p_B) = \underbrace{(\alpha v(p_B) - v(c)) \cdot q(p_B) \cdot \mathrm{Var}[h(\boldsymbol{\theta})]}_{>0}$$

$$+ \underbrace{(p_B - c)\left\{2t \cdot E\left[s_B\left(p_B, F_B^*, c, F_A^*, \boldsymbol{\theta}\right) q'\left(p_B, \boldsymbol{\theta}\right)\right] - \alpha \mathrm{Var}\left[q\left(p_B, \boldsymbol{\theta}\right)\right]\right\}}_{>0} > 0.$$

Note that if $p_B = c$, $\tilde{\xi}^B(c,c) < 0$, so from part (ii) we know that as $p_A \to c$ in (13) for $i = B$, $p_B \to \alpha_B < c$, with $\tilde{\xi}^B(c,\alpha_B) = 0$. It is easy to show that $\alpha_B > \gamma_B$. If we suppose that is not true, i.e., $\gamma_B \geq \alpha_B$ then

$$\tilde{\xi}^B(c,\gamma_B) = E\left[q\left(\tilde{\gamma}_B, \boldsymbol{\theta}\right)\right] E\left[\varphi\left(c, \gamma_B; \boldsymbol{\theta}\right)\right] - E\left[q\left(\gamma_B, \boldsymbol{\theta}\right) \varphi\left(c, \gamma_B; \boldsymbol{\theta}\right)\right]$$

$$+ \underbrace{(\gamma_B - c)\left\{2t \cdot E\left[s_B\left(\gamma_B, F_B^*, c, F_A^*, \boldsymbol{\theta}\right) q'\left(\gamma_B, \boldsymbol{\theta}\right)\right] - \alpha \mathrm{Var}\left[q\left(\gamma_B, \boldsymbol{\theta}\right)\right]\right\}}_{>0}.$$

From (A.32), γ_B is such that

$$E\left[\varphi\left(c, \gamma_B; \boldsymbol{\theta}\right)\right] = \frac{E\left[q\left(c, \boldsymbol{\theta}\right) \varphi\left(c, \gamma_B; \boldsymbol{\theta}\right)\right]}{E\left[q\left(c, \boldsymbol{\theta}\right)\right]}. \tag{A.34}$$

Thus,

$$\tilde{\xi}^B(c,\gamma_B) > \frac{E[q(\gamma_B, \boldsymbol{\theta})]}{E[q(c, \boldsymbol{\theta})]} E[q\left(c, \boldsymbol{\theta}\right) \varphi\left(c, \gamma_B; \boldsymbol{\theta}\right)] - E\left[q\left(\gamma_B, \boldsymbol{\theta}\right) \varphi\left(c, \gamma_B; \boldsymbol{\theta}\right)\right] > 0,$$

since $\gamma_B < c$. So from part (ii) we conclude that $\alpha_B > \gamma_B$. Finally, note that as $p_B \to c$ in (13) for $i = A$, $p_A \to \gamma_A > c$, since

$$\tilde{\xi}^A(p_A,c) = \underbrace{(v(p_A) - \alpha v(c)) \cdot q(p_A) \cdot \mathrm{Var}[h(\boldsymbol{\theta})]}_{<0}$$

$$+ \underbrace{(p_A - c)\left\{2t \cdot E\left[s_A\left(p_A, F_A^*, c, F_B^*, \boldsymbol{\theta}\right) q'\left(p_A, \boldsymbol{\theta}\right)\right] - \alpha \mathrm{Var}\left[q\left(p_A, \boldsymbol{\theta}\right)\right]\right\}}_{<0} < 0,$$

and $\tilde{\xi}^A(c,c) > 0$, where γ_A is such that

$$\tilde{\xi}^A(\gamma_A,c) = E\left[q\left(\gamma_A, \boldsymbol{\theta}\right) \varphi\left(\gamma_A, c; \boldsymbol{\theta}\right)\right] - E\left[q\left(\gamma_A, \boldsymbol{\theta}\right)\right] E[\varphi\left(\gamma_A, c; \boldsymbol{\theta}\right)] \tag{A.35}$$

$$+ (\tilde{\gamma}_A - c)\underbrace{\left\{2t \cdot E\left[s_A\left(\gamma_A, F_A^*, c, F_B^*, \boldsymbol{\theta}\right) q'\left(\gamma_A, \boldsymbol{\theta}\right)\right] - \alpha \mathrm{Var}\left[q\left(\gamma_A, \boldsymbol{\theta}\right)\right]\right\}}_{<0} = 0.$$

Note that as $p_A \to \gamma_A$ in (13) for $i = B$ and $p_B = c$,

$$\tilde{\xi}^B(\gamma_A,c) = E\left[q\left(c, \boldsymbol{\theta}\right)\right] E[\varphi\left(\gamma_A, c; \boldsymbol{\theta}\right)] - E\left[q\left(c, \boldsymbol{\theta}\right) \varphi\left(\gamma_A, c; \boldsymbol{\theta}\right)\right], \tag{A.36}$$

substituting (A.35) in (A.36)

$$\tilde{\xi}^B(\gamma_A, c) < \frac{E[q(c, \boldsymbol{\theta})]}{E[q(\gamma_A, \boldsymbol{\theta})]} E\left[q(\gamma_A, \boldsymbol{\theta}) \varphi(\gamma_A, c; \boldsymbol{\theta})\right] - E\left[q(c, \boldsymbol{\theta}) \varphi(\gamma_A, c; \boldsymbol{\theta})\right] < 0,$$

since $\gamma_A > c$. Thus we conclude that as $p_A \to \gamma_A$ in (13) for $i = B$, $p_B \to \delta_B < c$. Thus, both curves cross each other at least once in the set Ω.

(iv) Next, we show that for $(p_A, p_B) \in \Omega$ the slope of the implicit functions defined by (13) for $i \in \{A, B\}$, $\tilde{R}^i(p) : \mathcal{P} \to \mathcal{P}$, where $\tilde{R}^i\left(\tilde{p}_A^i\right) = \tilde{p}_B^i$ is such that $\tilde{p}_A^i$ and $\tilde{p}_B^i$ satisfy (13) for $i \in \{A, B\}$, is positive. Note that from (A4) and part(ii), every claim in the proof of Lemma 1 holds. It follows that if the following inequalities are satisfied (we prove them in part (v)),

$$-\frac{\partial \tilde{\xi}^A}{\partial p_A} > \frac{\partial \tilde{\xi}^B}{\partial p_A} \quad \text{and} \quad \frac{\partial \tilde{\xi}^A}{\partial p_B} < -\frac{\partial \tilde{\xi}^B}{\partial p_B},$$

the slope of the implicit functions defined by (13) is positive, i.e.,

$$\frac{\partial \tilde{R}^A(p_A)}{\partial p_A} > 0 \quad \text{and} \quad \frac{\partial \tilde{R}^B(p_A)}{\partial p_A} > 0.$$

(v) Finally, we show that the slope of the implicit function defined by (13) for $i = A$ is greater than the slope defined for $i = B$, that is, $\frac{\partial \tilde{R}^A(p_A)}{\partial p_A} > \frac{\partial \tilde{R}^B(p_A)}{\partial p_A}$ for any $(p_A, p_B) \in \Omega$.

- Inequality $-\partial \tilde{\xi}^A / \partial p_A > \partial \tilde{\xi}^B / \partial p_A$ holds if Lemma A1, Lemma A2, $-h(p_B)(p_B - c) < 1$, and $3\sigma > \overline{\theta}^2$ hold. Note that Lemma A1 follows from $p_A > c$ and (A2). Also, Lemma A2 holds whenever (A2) holds. Finally, inequality $-h(p_B)(p_B - c) < 1$ holds for any $p_B < c$.

- Inequality $-\partial \tilde{\xi}^B / \partial p_B > \partial \tilde{\xi}^A / \partial p_B$ holds if $\frac{\partial}{\partial p_B} h^B(p_B)(p_B - c) > 0$, $-\frac{q'(p_A)(p_A - c)}{\pi'_A(p_A)} < \frac{1}{2}$, $-\frac{q'(p_B)(c - p_B)}{\pi'_B(p_B)} < \frac{1}{2}$, and $3\sigma > \overline{\theta}^2$ hold. The previous claims follow from (A2) and the fact that $p_B < c < p_A$.

Then, in equilibrium

$$-\frac{\partial \tilde{\xi}^A / \partial p_A}{\partial \tilde{\xi}^A / \partial p_B} > -\frac{\partial \tilde{\xi}^B / \partial p_A}{\partial \tilde{\xi}^B / \partial p_B}.$$

Proof of Proposition 6. The first order conditions for firm $i \in \{A, B\}$ are

$$[p_i]: \quad E\left[s\left(v_i(p_i, \boldsymbol{\theta}) - F_i, v_j(p_j, \boldsymbol{\theta}) - F_j\right) \pi'_i(p_i, \boldsymbol{\theta})\right. \tag{A.37}$$
$$\left. - s_1\left(v_i(p_i, \boldsymbol{\theta}) - F_i, v_j(p_j, \boldsymbol{\theta}) - F_j\right) q(p_i, \theta)\left(\pi_i(p_i, \boldsymbol{\theta}) + F_i\right)\right] = 0$$

and

$$[F_i]: \quad E\left[s\left(v_i(p_i, \boldsymbol{\theta}) - F_i, v_j(p_j, \boldsymbol{\theta}) - F_j\right) - \right. \tag{A.38}$$
$$\left. s_1\left(v_i(p_i, \boldsymbol{\theta}) - F_i, v_j(p_j, \boldsymbol{\theta}) - F_j\right)\left(\pi_i(p_i, \boldsymbol{\theta}) + F_i\right)\right] = 0.$$

Let $v_i^*(p_i, \boldsymbol{\theta}) \equiv v_i(p_i, \boldsymbol{\theta}) - F_i^*$ for $i \in \{A, B\}$. From (A.38), for each $p_i \in \mathcal{P}$, F_i^* is implicitly defined by

$$F_i^* = \frac{E\left[s\left(v_i^*(p_i, \boldsymbol{\theta}), v_j^*(p_j, \boldsymbol{\theta})\right) - s_1\left(v_i^*(p_i, \boldsymbol{\theta}), v_j^*(p_j, \boldsymbol{\theta})\right) \pi_i(p_i, \boldsymbol{\theta})\right]}{E\left[s_1\left(v_i^*(p_i, \boldsymbol{\theta}), v_j^*(p_j, \boldsymbol{\theta})\right)\right]}. \tag{A.39}$$

From (A.39) and (A.37), it follows that

$$p_i - c_i = \frac{E[s] E[s_1 \cdot q_i(p_i, \boldsymbol{\theta})] - E[s_1] E[s \cdot q_i(p_i, \boldsymbol{\theta})]}{E[s \cdot q_i'(p_i, \boldsymbol{\theta})] E[s_1] - E[s_1 \cdot q_i^2(p_i, \boldsymbol{\theta})] E[s_1] + E[s_1 \cdot q_i(p_i, \boldsymbol{\theta})]^2} \tag{A.40}$$

where s and s_1 are evaluated at $(v_i^*(p_i, \boldsymbol{\theta}), v_j^*(p_j, \boldsymbol{\theta}))$. Given that $s \geq 0$, $s_1 \geq 0$ and $q_i'(p_i, \boldsymbol{\theta}) < 0$ for all $p_i \in \mathcal{P}$ and for all $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, it follows that the denominator of the right-hand side of (A.40) is negative if

$$-E\left[s_1 q_i^2(p_i, \boldsymbol{\theta})\right] E[s_1] + E\left[s_1 q_i(p_i\boldsymbol{\theta})\right]^2 \leq 0,$$

which is equivalent to

$$E\left[\frac{s_1}{E[s_1]} q_i(p_i, \boldsymbol{\theta})\right]^2 \leq E\left[\frac{s_1}{E[s_1]} q_i^2(p_i, \boldsymbol{\theta})\right]. \tag{A.41}$$

Inequality (A.41) follows from Kimball's inequality (Lemma 2.2.1 in Tong (1980)).[49] Thus from (A.40), $p_i - c_i$ has the same sign as

$$\frac{E[s q_i(p_i, \boldsymbol{\theta})]}{E[s]} - \frac{E[s_1 q_i(p_i, \boldsymbol{\theta})]}{E[s_1]}.$$

Proof of Corollary 7. The proof of this corollary immediately follows from Proposition 6.

Proof of Corollary 8. The problem of each firm is

$$\max_{p_i, F_i} \quad s\left(u_i(p_i, F_i), u_j(p_j, F_j)\right)[\pi_i(p_i) + F_i].$$

where the utility offered by firm $i \in \{A, B\}$ is $u_i(p_i, F_i) = v_i(p_i) - F_i$. We solve the two-variable optimization problem of firm i sequentially. First we show that for any $p_i \in \mathcal{P}$, firm i chooses F_i to maximize its profits. The first order condition with respect to F_i yields

$$\pi_i(p_i) + F_i^* = \frac{s\left(u_i(p_i, F_i^*), u_j(p_j, F_j)\right)}{s_1\left(u_i(p_i, F_i^*), u_j(p_j, F_j)\right)}. \tag{A.42}$$

The profit is strictly log-concave in F_i, and the unique solution is implicitly determined by (A.42). Next, firm i chooses p_i to maximize its maximum profits (we substitute $F_i^*(p_i)$ in the

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[49]Notice that we are applying Kimball's inequality with respect to the positive finite measure $\frac{s_1}{E[s_1]} d\boldsymbol{G}(\boldsymbol{\theta})$.

objective function of firm i)

$$s\left(p_i, F_i^*\left(p_i\right), p_j, F_j\right)\left[\pi_i\left(p_i\right) + F_i^*\left(p_i\right)\right]. \tag{A.43}$$

The derivative of (A.43) with respect to p_i, after using the Envelope Theorem, is

$$q'\left(p_i\right)\left(p_i - c_i\right)s\left(p_i, F_i^*\left(p_i\right), p_j, F_j\right) \tag{A.44}$$

Given $p_j \in \mathcal{P}$ and $F_j \geq 0$, it follows from (A.42) and the Implicit Function Theorem that $s\left(p_i, F_i^*\left(p_i\right), p_j, F_j\right)$, is strictly decreasing with respect to p_i for any $p_i > c_i$. Thus, if there exists a $\tilde{p}_i\left(p_j, F_j\right) \in \mathcal{P}$, such that $s\left(\tilde{p}_i, F_i^*\left(\tilde{p}_i\right), p_j, F_j\right) = 0$, then, for any $p_i \geq \tilde{p}_i$ the profit is zero. Then, for any $p_j \in \mathcal{P}$ and $F_j \geq 0$, note that (A.44) is positive for $p_i < c_i$ and it is negative for any $p_i \in \left(c_i, \tilde{p}_i\left(p_j, F_j\right)\right)$. Thus, (A.43) is single-peaked in p_i and reaches a unique maximum at $p_i = c_i$. Analogously, the profit function for firm $j \neq i$ in terms of p_j is single-peaked in p_j and reaches a unique maximum at $p_j = c_j$. Thus the equilibrium is unique, and it follows that F_i^* is implicitly determined by:

$$F_i^* = \frac{s\left(v_i^*, v_j^*\right)}{s_1\left(v_i^*, v_j^*\right)}, \tag{A.45}$$

where $v_i^* \equiv v_i\left(c_i\right) - F_i^*$.

Next, we show that F_i^*, in equilibrium, is well defined. Using the dual approach, we show that there exist F_i^*, F_j^* that satisfy (A.45) for $j \neq i$ and $i, j \in \{A, B\}$.

To simplify the notation, let $u_i \equiv v_i\left(c_i\right) - F_i$. Then we show that there exist $u_i = v_i^*$ and $u_j = v_j^*$ that satisfy

$$\frac{s\left(u_i, u_j\right)}{s_1\left(u_i, u_j\right)} - v_i\left(c_i\right) + u_i = 0, \tag{A.46}$$

for $u_i, u_j \in \mathcal{U}$ and $i \in \{A, B\}$.[50] First, note that from the analogue of (A.46) for firm j and the Implicit Function Theorem there exists a function $R^j\left(u_i\right) : \mathcal{U} \to \mathcal{U}$, where $R^j\left(u_i\right) = u_j$ is such that u_i and u_j satisfy (A.46) for $j \neq i$. Note that

$$\frac{\partial R^j\left(u_i\right)}{\partial u_i} = \frac{-\gamma_2\left(R^j\left(u_i\right), u_i\right)}{1 + \gamma_1\left(R^j\left(u_i\right), u_i\right)} > 0, \tag{A.47}$$

where $\gamma\left(u_i, u_j\right) \equiv \frac{s\left(u_i, u_j\right)}{s_1\left(u_i, u_j\right)}$, $\gamma_2\left(u_i, u_j\right) < 0$ and $\gamma_1\left(u_i, u_j\right) > 0$ for $u_i, u_j \in \mathcal{U}$. We show in two steps that there exists a unique $u_i = v_i^*$ that satisfies (A.46).

1. First note that as $u_i \to 0$, $R^j\left(u_i\right) \to \alpha > 0$: For $u_i = 0$, $R^j\left(0\right)$ is implicitly defined by u_j such that

$$v_j\left(c_j\right) - u_j = \gamma\left(u_j, 0\right). \tag{A.48}$$

Note that the RHS of (A.48) is increasing with respect to u_j, and as $u_j \to 0$, the RHS of

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[50]$\mathcal{U}$ is the set of feasible utility offered to consumers defined by $\mathcal{U} = [0, v\left(c\right)]$.

$(A.48) \to \gamma(0,0) > 0$. Similarly, note that the LHS of $(A.48)$ is decreasing with respect to u_j, and as $u_j \to 0$, the LHS of $(A.48) \to v_j(c_j) > \gamma(0,0)$, and as $u_j \to v_j(c_j)$, the LHS of $(A.48) \to 0$. Thus, as $u_i \to 0$, from $(A.47)$ and continuity we conclude that there is a unique value $R^j(u_i) > 0$ that satisfies

$$v_j(c_j) - R^j(u_i) = \gamma(R^j(u_i), u_i).$$

2. By assumption, we know that $v_i(c_i) > \gamma(0,0) > \gamma(0, R^j(0))$. Then, as $u_i \to 0$, the LHS of $(A.46) \to \gamma(0, R^j(0)) - v_i(c_i) < 0$. Similarly, as $u_i \to v_i(c_i)$, the LHS of $(A.46) \to s(v_i(c_i), R^j(v_i(c_i))) > 0$. Thus existence follows.

Remember that the profit is strictly log-concave in F_i, thus, we conclude that there exist F_i^*, F_j^* implicitly determined by $(A.45)$.

Proof of Proposition 7. Note that for any $u \in \mathbb{R}$

$$s(u,u) \equiv \frac{e^u}{2e^u + e^{u_0}} \quad \text{and} \quad s_1(u,u) = s(u,u)(1 - s(u,u)). \tag{A.49}$$

Let $u^* \equiv v(p^*, \boldsymbol{\theta}) - F^*$. From Corollary 7, $p^* > c$ if

$$\text{Cov}\left(\underbrace{\frac{s(u^*, u^*)}{E[s(u^*, u^*)]} - \frac{s_1(u^*, u^*)}{E[s_1(u^*, u^*)]}}_{\equiv \phi(u^*, \boldsymbol{\theta})}, q(p^*, \boldsymbol{\theta})\right) > 0. \tag{A.50}$$

Since $\boldsymbol{\theta}$ is strictly associated, $(A.50)$ holds if $\phi(u^*, \boldsymbol{\theta})$ is strictly increasing in $\boldsymbol{\theta}$. Note that from $(A.49)$

$$\frac{s(u^*, u^*)}{E[s(u^*, u^*)]} - \frac{s_1(u^*, u^*)}{E[s_1(u^*, u^*)]} = \frac{s(u^*, u^*)\left(s(u^*, u^*)E[s(u^*, u^*)] - E\left[s(u^*, u^*)^2\right]\right)}{E[s(u^*, u^*)]E[s_1(u^*, u^*)]}$$

which is strictly increasing in $\boldsymbol{\theta}$. Thus, in equilibrium $p^* > c$.

Proof of Proposition 8. For completeness of the proof we first show that marginal-cost-based 2PT is not an equilibrium. Next, we show that in any equilibrium, $c_A < p_A^* < p_B^* < c_B$. Finally, we show existence. The first order conditions are

$$[p_i] \quad \sum_{k \in \{L,H\}} \lambda_k s\left(v(p_i, \theta_k) - F_i, v(p_j, \theta_k) - F_j\right) \pi_i'(p_i, \theta_k) \tag{A.51}$$

$$- \sum_{k \in \{L,H\}} \lambda_k s_1\left(v(p_i, \theta_k) - F_i, v(p_j, \theta_k) - F_j\right) q(p_i, \theta_k)(\pi_i(p_i, \theta_k) + F_i) = 0$$

and

$$[F_i] \qquad \sum_{k \in \{L,H\}} \lambda_k s \left(v\left(p_i, \theta_k\right) - F_i, v\left(p_j, \theta_k\right) - F_j \right) \tag{A.52}$$

$$- \sum_{k \in \{L,H\}} \lambda_k s_1 \left(v\left(p_i, \theta_k\right) - F_i, v\left(p_j, \theta_k\right) - F_j \right) \left(\pi_i\left(p_i, \theta_k\right) + F_i \right) = 0.$$

Let

$$\mathbf{s}\left(u\left(p_i, F_i\right), u\left(p_j, F_j\right); \boldsymbol{\theta}\right) \equiv \left[\begin{array}{c} \lambda_L s\left(u\left(p_i, F_i, \theta_L\right), u\left(p_j, F_j, \theta_L\right)\right) \\ \lambda_H s\left(u\left(p_i, F_i, \theta_H\right), u\left(p_j, F_j, \theta_H\right)\right) \end{array} \right],$$

where $u\left(p_i, F_i, \theta_k\right) \equiv v\left(c_i, \theta_k\right) - F_i$, and similarly for $\mathbf{s}_1\left(u\left(p_i, F_i\right), u\left(p_j, F_j\right); \boldsymbol{\theta}\right)$. Let also

$$q\left(p_i; \boldsymbol{\theta}\right) = \left[\begin{array}{c} q\left(p_i, \theta_L\right) \\ q\left(p_i, \theta_H\right) \end{array} \right]$$

and similarly for $q'\left(p_i; \boldsymbol{\theta}\right)$ and $q\left(p_i; \boldsymbol{\theta}\right)^2$. Finally, let

$$\gamma\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \theta_H\right) \equiv \gamma\left(u\left(c_i, F_i, \theta_H\right), u\left(c_j, F_j, \theta_H\right)\right).$$

(i). Here, we show that marginal-cost-based 2PT is not an equilibrium. Let us suppose is not true, i.e., suppose cost-based 2PT is an equilibrium. Then, from (A.51),

$$F_i = \frac{\mathbf{s}\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \boldsymbol{\theta}\right)' q\left(c_i; \boldsymbol{\theta}\right)}{\mathbf{s}_1\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \boldsymbol{\theta}\right)' q\left(c_i; \boldsymbol{\theta}\right)}. \tag{A.53}$$

Note that from (A.52) and (A.53),

$$\lambda\left(1 - \lambda\right) \frac{s_1\left(u\left(c_i, F_i, \theta_L\right), u\left(c_j, F_j, \theta_L\right)\right) s_1\left(u\left(c_i, F_i, \theta_H\right), u\left(c_j, F_j, \theta_H\right)\right)}{A} \left(q\left(c_i, \theta_H\right) - q\left(c_i, \theta_L\right) \right)$$

$$\times \left\{ \gamma\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \theta_H\right) - \gamma\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \theta_L\right) \right\} = 0,$$

where

$$A \equiv \mathbf{s}_1\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \boldsymbol{\theta}\right)' q\left(c_i; \boldsymbol{\theta}\right),$$

which is a contradiction since

$$\gamma\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \theta_H\right) > \gamma\left(u\left(c_i, F_i\right), u\left(c_j, F_j\right); \theta_L\right)$$

for $c_i < c_j$, given that $\frac{s(\cdot)}{s_1(\cdot)}$ is increasing (following a similar strategy as in Quint, 2014, Theorem 1), and by the increasing differences property, $v\left(c_i, \theta_H\right) - F_i - v\left(c_j, \theta_H\right) + F_j > v\left(c_i, \theta_L\right) - F_i - v\left(c_j, \theta_L\right) + F_j$.

(ii). Note that from (A.51) and (A.52), in equilibrium we have

$$\left(p_i - c_i\right) \left\{ \mathbf{s}\left(u_i, u_j; \boldsymbol{\theta}\right)' q'\left(p_i; \boldsymbol{\theta}\right) - \mathbf{s}_1\left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right)^2 \tag{A.54}$$

$$+\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right) \times \left[\frac{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right)}{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}}\right]\Bigg\} +$$

$$+\left\{\mathbf{s}\left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right) - \mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right) \times \frac{\mathbf{s}\left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}}{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}}\right\},$$

where $u_i = u\left(p_i, F_i\right)$ and $u_j = u\left(p_j, F_j\right)$. We first show that the expression inside the first big bracket is negative. Thus, for $p_i > c_i$, the expression in the second big bracket must be positive. Now let's analyze the first big bracket

$$\Bigg\{\underbrace{\mathbf{s}\left(u_i, u_j; \boldsymbol{\theta}\right)' q'\left(p_i; \boldsymbol{\theta}\right)}_{<0} - \underbrace{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right)^2}_{A}$$

$$+\underbrace{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right)}_{B} \times \underbrace{\left[\frac{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right)}{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}}\right]}_{C}\Bigg\},$$

where

$$-A + B \times C =$$

$$-D \cdot \left[q\left(p_i, \theta_H\right) - q\left(p_i, \theta_L\right)\right]^2 < 0$$

and

$$D \equiv \frac{\lambda\left(1-\lambda\right) s_1 \left(u\left(p_i, F_i, \theta_H\right), u\left(p_j, F_j, \theta_H\right)\right) s_1 \left(u\left(p_i, F_i, \theta_L\right), u\left(p_j, F_j, \theta_L\right)\right)}{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}} > 0.$$

Now we show that the second big bracket is positive only if $p_i < p_j$. That is,

$$\left\{\mathbf{s}\left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right) - \mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' q\left(p_i; \boldsymbol{\theta}\right) \times \frac{\mathbf{s}\left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}}{\mathbf{s}_1 \left(u_i, u_j; \boldsymbol{\theta}\right)' \cdot \mathbf{1}_{[2,1]}}\right\}$$

$$= D \cdot \left[q\left(p_i, \theta_H\right) - q\left(p_i, \theta_L\right)\right]$$

$$\times \left[\gamma\left(u\left(p_i, F_i\right), u\left(p_j, F_j\right); \theta_H\right) - \gamma\left(u\left(p_i, F_i\right), u\left(p_j, F_j\right); \theta_L\right)\right],$$

since $\gamma\left(u_i, u_j\right)$ is increasing with respect to u_i, and by the increasing differences property, $p_i < p_j$. Thus we conclude that there is no equilibrium in which $p_B > c_B$. Moreover, in any equilibrium, $c_A < p_A^* < p_B^* < c_B$.

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