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Connecting Expected Stock Returns to Accounting Valuation Multiples: A Primer

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Abstract

We outline a framework in which accounting "valuation anchors" could be connected to expected stock returns. Under two general conditions, expected log returns is a log-linear function of a valuation (market value-to-accounting) multiple and the expected growth in the valuation anchor. We show that the framework can: 1) allow for expected enterprise returns, 2) correct for the use of stale accounting data in estimation, and 3) accommodate differences in information quality. This analytical formulation is tractable and flexible, and provides building blocks for further innovations in accounting valuation research.

Keywords: Expected returns; Present value; Valuation multiples.

JEL: D83, G12, G14, M41

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1 Introduction

In recent years, there have been growing interest in the connection between accounting information/data and expected returns. In particular, the development of present-value-based "implied cost of capital" (ICC) measures, beginning with Botosan (1997) and followed by a body of methodological work (e.g., Claus and Thomas, 2001; Gebhardt, Lee, and Swaminathan, 2001; Gode and Mohanram, 2003; Easton, 2004), have been instrumental in engendered a burgeoning literature that refined these methodological approaches or used these measures in studies.

Recent work in this area has shown that expected return proxies (ERPs) derived from a log-linear present value (LPV) framework perform well in forecasting future returns not only in the U.S. (Lyle and Wang, 2015; Lee, So, and Wang, 2020) but also in a large number of stock markets worldwide (Chattopadhyay, Lyle, and Wang, 2021). These empirical findings suggest that the LPV framework provides a tractable analytical formulation for helping to understand the connection between expected returns, market values, and accounting data.

In this document, we provide an introduction to the LPV framework for scholars and researchers interested in accounting-based valuation. We provide the theoretical underpinnings of LPV ERPs, in particular the general conditions under which a connection between an accounting valuation anchor and expected returns can be established. Under this formulation, ERPs are a log-linear combination of a valuation multiple, the ratio of the valuation anchor and the market price, and the expected growth in the valuation anchor. The intuition for these ERPs is similar to that of ICCs: the valuation anchor and its expected growth provide accounting estimates of expected future cash flows; combining this information with market prices uncovers information about the discount rate for these cash flows.

Unlike ICCs, the log-linear nature of LPV ERPs makes them more tractable and allows for a variety of extensions. We provide three examples for how the LPV formulation can be usefully extended to yield additional insights and analyses. We show that the framework can be easily generalized to analyze expected returns on the enterprise. We also show how the framework can be adapted to correct for the use of stale fundamental data. Finally, we show how differences in information quality can be incorporated into the framework. Our hope is that this primer helps to stimulate further innovations in this area of work.

2 Analytical Underpinnings of LPV ERPs

This section outlines the basic steps in formulating a connection between market value of equity and accounting values. We then derive a log-linear relation among expected (log) equity returns, accounting value (A), and market prices (M). The framework outlined below is a generalized version of that introduced in Lyle and Wang (2015); it characterizes a class of accounting-based ERPs based on a positive valuation anchor and its expected growth.

2.1 Log-Linear Present-Value Relation

We first derive a log-linear present value relation between market prices and a positive accounting-valuation anchor for a firm i. We begin with the definitions of gross realized growth in these quantities:

$$G^M_{i,t+1} = \frac{M_{i,t+1}}{M_{i,t}}, \tag{1}$$

$$G^A_{i,t+1} = \frac{A_{i,t+1}}{A_{i,t}}, \tag{2}$$

where $M_{i,t}$ and $A_{i,t}$ are, respectively, firm i's market value of equity and its value in the valuation anchor at the end of period t. Defining the ratio between the valuation anchor and market price,

$$\frac{A_{i,t}}{M_{i,t}} = \frac{A_{i,t+1}G^M_{i,t+1}}{M_{i,t+1}G^A_{i,t+1}}, \tag{3}$$

then taking logs of both sides, yields a linear expression for $am_{i,t}$:

$$am_{i,t} = am_{i,t+1} + g^m_{i,t+1} - g^a_{i,t+1}, \tag{4}$$

where $g^m_{i,t+1} \equiv \log\left(G^M_{i,t+1}\right)$ and $g^a_{i,t+1} \equiv \log\left(G^A_{i,t+1}\right)$. Iterating (4) forward obtains

$$am_{i,t} = am_{i,t+T} + \sum_{\tau=1}^{T}[g^m_{i,t+\tau} - g^a_{i,t+\tau}]. \tag{5}$$

Taking conditional expectations on both sides with respect to information known at time t, and assuming that am_t satisfies the following "value-relevance" assumption,

$$\lim_{T\to\infty} \mathbb{E}_t[am_{i,t+T}] = \overline{am}_i \tag{6}$$

for $\overline{am}_i$ bounded and time-invariant, we obtain:

$$am_{i,t} = \overline{am}_i + \sum_{\tau=1}^{\infty} \mathbb{E}_t[g^m_{i,t+\tau} - g^a_{i,t+\tau}]. \tag{7}$$

If we make the additional assumption that the expected growth in market values (denoted $\mu_{i,t} = \mathbb{E}_t[g^m_{i,t+1}]$) and the expected growth in the accounting-valuation anchor (denoted $h_t = \mathbb{E}_t[g^a_{i,t+1}]$) are mean-reverting and follow AR(1) processes, i.e.,

$$\mu_{i,t+1} = \mu_i + \kappa_i(\mu_{i,t} - \mu_i) + \xi_{i,t+1}, \text{ and} \tag{8}$$

$$h_{i,t+1} = \mu_i + \omega_i(h_{i,t} - \mu_i) + \epsilon_{i,t+1}, \tag{9}$$

we can reorganize (7) into following log-linear present-value relation:

$$am_{i,t} = \overline{am}_i + \frac{1}{1-\kappa_i}(\mu_{i,t} - \mu_i) - \frac{1}{1-\omega_i}(h_{i,t} - \mu_i). \tag{10}$$

2.2 Expected Log Returns

Using the present-value relation in Eq. (10), we can derive the following linear relation among the expected growth in market prices, the valuation-anchor-to-market ratio, and the expected growth in the valuation anchor:

$$\mu_{i,t} = \left[\left(\frac{\kappa_i - \omega_i}{1-\omega_i}\right)\mu_i - (1-\kappa_i)\overline{am}_i\right] + (1-\kappa_i)am_{i,t} + \left(\frac{1-\kappa_i}{1-\omega_i}\right)h_{i,t}. \tag{11}$$

Eq. (11) is a representation of expected log ex-dividend returns as a linear combination of firm characteristics; a more accurate representation should also take into account dividend payouts. Doing so requires us to make an additional assumption that dividends are proportional to the accounting anchor over the interval t to $t+1$, i.e., $D_{i,t+1} = \delta A_{i,t+1}$. Applying the proportionality assumption to the definition of expected stock returns, $\mathbb{E}_t[r_{i,t+1}] = \mu_{i,t} + \mathbb{E}_t\left[\log\left(1+\frac{D_{i,t+1}}{M_{i,t+1}}\right)\right]$, we obtain an expression for expected log returns after a first-order linearization around the unconditional mean of $am_{i,t}$ ($\overline{am}_i$):

$$\begin{aligned}\mathbb{E}_t[r_{i,t+1}] &= \mu_{i,t} + \log\left(1+\delta\frac{A_{i,t+1}}{M_{i,t+1}}\right) = \mu_{i,t} + \log\left(1+\delta\exp(am_{i,t+1})\right), &(12)\\ &\approx \mu_{i,t} + K_i + \rho_i\left[\frac{\kappa_i}{1-\kappa_i}(\mu_{i,t}-\mu_i) - \frac{\omega_i}{1-\omega_i}(h_{i,t}-\mu_i)\right], &(13)\end{aligned}$$

where $K_i = \log(1+\delta\exp(\overline{am}_i))$, $\rho_i = \frac{\delta\exp(\overline{am}_i)}{1+\delta\exp(\overline{am}_i)}$. Reorganizing terms, we obtain the following log-linear relation for a firm's expected log returns:

$$\mathbb{E}_t[r_{i,t+1}] = A_{i,0} + A_{i,1}am_{i,t} + A_{i,2}h_{i,t}, \tag{14}$$

where

$$\begin{aligned}A_{i,0} &= K_i - A_{i,1}\overline{am}_i + \frac{(\kappa_i-\omega_i)(1-\rho_i)}{1-\omega_i}\mu_i,\\ A_{i,1} &= 1-\kappa_i+\rho_i\kappa_i,\\ A_{i,2} &= \frac{A_{i,1}-\rho_i\omega_i}{1-\omega_i}.\end{aligned}$$

Note that, in general, the weights on characteristics are functions of persistence in the growth of firm i's market values and in the growth of its accounting-valuation anchor. Further, incorporating dividends into Eq. (10) does not change the inputs into the log-linear relation for expected returns, only the specifics of the weights. Finally, for a non-dividend-paying firm, expected return on the stock is simply the expected growth in its market value, and Eq. (14) collapses to Eq. (10).

2.3 Application: Book Value of Equity as the Valuation Anchor

Eq. (14), above, characterizes a class of ERPs formed by combining a positive accounting-valuation anchor (satisfying the value-relevance and mean-reversion conditions) and its expected growth with market prices. A natural accounting-valuation anchor to benchmark against market value of equity is the book value of equity.

Using Eq. (10) and the relation $h_{i,t} = \mathbb{E}_t[roe_{i,t+1}] - \log(1+\delta)$, expected log returns can be expressed as a linear combination of the log of book-to-market ratio ($bm_{i,t}$) and the expected log of gross future return on equity ($\mathbb{E}_t[roe_{i,t+1}]$):

$$\mathbb{E}_t[r_{i,t+1}] = A_{i,0} + A_{i,1} bm_{i,t} + A_2 \mathbb{E}_t[roe_{t+1}], \tag{15}$$

where

$$\begin{aligned} A_{i,0} &= K_i - A_{i,1}\overline{bm}_i + \frac{(\kappa_i - \omega_i)(1-\rho_i)}{1-\omega_i}\mu_i - A_{i,2}\log(1+\delta), \\ A_{i,1} &= 1 - \kappa_i(1-\rho_i), \\ A_{i,2} &= \frac{A_{i,1} - \omega_i \rho_i}{1-\omega_i}. \end{aligned}$$

Here, only the expression of the constant term differs from Eq. (14).

2.4 Discussion of Assumptions

The above derivation is based on two main assumptions. The first assumption—that the accounting-valuation anchor and market prices are expected to converge—is an assertion about the accounting system. The valuation anchor of interest is expected to be "value-relevant"(that is, book values are expected to be tied to market values). This assumption is fairly general and likely to be satisfied by most accounting systems. To build intuition, consider an accounting system in which this assumption fails. For example, if bm is expected to diverge to $+\infty$, book values must be expected to grow faster than market values indefinitely. Such an outcome would imply an accounting system that aggressively books assets or rarely writes them off, such that book values eventually become detached from market values and thus lose relevance. Because accounting regimes around the world tend to be conservative in nature, it is difficult to envision an accounting system that would imply that investors expect bm_{t+j} to diverge to $+\infty$ in the long run.[1] Conversely, if bm is expected to diverge to $-\infty$, market values are expected to grow faster than book values indefinitely. Such an outcome is consistent with an accounting system that is extremely conservative about booking assets or prone to very quickly writing off assets, such that book values become detached from, and irrelevant for to, market values. Not only is this divergence unlikely; it is also inconsistent with the data. For example, Pástor and Veronesi (2003) show that, though the market-to-book ratio does decline on average as firms age, it tends to settle at a stable value above 1. Because this value-relevance assumption implies an accounting system that is neither very conservative nor very aggressive, we view this assumption as applicable to the various accounting regimes—i.e., variants of GAAP or IFRS—around the world.

The second main assumption is that the stochastic processes governing the expected growth of market values and the accounting-valuation anchor follow an AR(1). Although statistical in nature, this assumption captures empirical regularities and economic intuition. The AR(1) assumption for expected market growth captures the possibility, consistent with a growing body of empirical evidence, that expected returns can be time-varying (e.g., Cochrane, 2011; Ang and Liu, 2004; Fama and French, 2002; Jagannathan, McGrattan, and Scherbina, 2001) and persistent (e.g., Fama and French, 1988; Campbell and Cochrane, 1999; Pástor and Stambaugh, 2009). The assumption about expected growth in the accounting-valuation anchor captures the idea that period performance tends to mean-revert over time due to competitive forces. As applied to the growth in book value, this assumption captures

[1] Under mark-to-market accounting, which can be considered an aggressive regime, book values and market values are guaranteed to converge by the definition of marking to market.

the intuition that, though firms can experience periods of unusually high or low profitability, competitive forces drive a tendency for accounting rates of return to revert to a steady-state mean (e.g., Beaver, 1970; Penman, 1991; Pástor and Veronesi, 2003; Healy, Serafeim, Srinivasan, and Yu, 2014).

In addition, the combination of the value-relevance and AR(1) assumptions implies that the expected market growth and valuation-anchor growth revert to a common mean. As applied to the book value of equity, this implication is consistent with the economic intuition that, in the long run, abnormal growth in book values is expected to converge due to competition (similar to the assumed dynamics in Ohlson, 1995). Overall, the two assumptions that we rely on are fairly general, applicable to a wide range of accounting systems, and consistent with empirical data and economic intuition.

Although the log-linear form is similar to the derivation presented in Vuolteenaho (1999), Vuolteenaho (2002), and Lyle and Wang (2015), and with a valuation equation similar to the popular "Levels" model of Ohlson (1995), which is based on residual income, the model presented here is more general. The assumptions are less restrictive and allow for a variety of accounting valuation anchors.

3 Extensions

3.1 Expected Returns on the Enterprise

The expected returns on the enterprise can be derived easily under this framework. This is a straightforward extension of the above, which can be done by re-defining $M_{i,t}$ as the market value of the enterprise and $A_{i,t}$ as the a positive accounting valuation anchor that serves as a benchmark against enterprise value. Making an enterprise version of the "value-relevance" and AR(1) assumptions, and under the assumption of proportional payouts (debt and equity in this case), the expected (log) returns on the enterprise can be written as a linear combination of: 1) the log of the ratio between the accounting anchor and enterprise value and 2) the expected growth in the accounting anchor.

For example, the closest to our book-based model would be to use total assets as an accounting anchor for enterprise value. Then, under the above assumptions, the expected (log) returns of the enterprise ($\mathbb{E}_t[r^{ev}_{i,t+1}]$) is a linear combination of the log of asset-to-enterprise value ($aev_{i,t}$), or Tobin's Q, and the expected growth in total assets ($\mathbb{E}_t[h^a_{i,t+1}]$):

$$\mathbb{E}_t[r^{ev}_{i,t+1}] = \theta^a_{i,0} + \theta^a_{i,1} aev_{i,t} + \theta^a_{i,2}\mathbb{E}_t[h^a_{i,t+1}]. \tag{16}$$

Similarly, if we use sales as an accounting anchor for enterprise value, under the above assumptions the expected (log) returns of the enterprise is a linear combination of the log of the sales-to-enterprise value multiple ($sev_{i,t}$) and the expected growth in sales ($\mathbb{E}_t[h^s_{i,t+1}]$):

$$\mathbb{E}_t[r^{ev}_{i,t+1}] = \theta^s_{i,0} + \theta^s_{i,1} sev_{i,t} + \theta^s_{i,2}\mathbb{E}_t[h^s_{i,t+1}]. \tag{17}$$

3.2 Correcting for the Use of Stale Fundamental Data

In practice, market values and accounting data are not updated in lock-step, thus application of the LPV framework to ERPs needs to take into account the timing of fundamental information. In some contexts, the econometrician only observes stale fundamental information, creating measurement error in estimating expected returns. We present a general approach to dealing with stale fundamental data that is particularly salient in international research contexts Chattopadhyay et al. (2021).

By the decomposition property of conditional expectations, an arbitrary stochastic process y_{t+1} can be written as $y_{t+1} = \mu_t + \epsilon_{t+1}$, where $\mathbb{E}_t[y_{t+1}] = \mu_t$ is a function of time-t observable variables, e.g., bm_t and roe_t, and ϵ_{t+1} is mean 0 and uncorrelated with μ_t. If μ_t is not observable at time t, for example because up-to-date book value and roe are not available, then

$$\mathbb{E}_t[y_{t+1}] = E_t[\mu_t|\mu_{t-1}, y_t]. \tag{18}$$

That is, the time-t expectation of y_{t+1} is a "best guess" about what μ_t would be given prior expected returns, μ_{t-1} and the realization of y_t itself.

Moreover, assuming that μ_t is persistent and follows an AR(1) process, e.g., $\mu_{t+1} = a + b \times (\mu_t - a) + w_{t+1}$, then given $\mu_{t-\tau}$ and $\tau > 1$ we have

$$y_{t+1} = a + b^{\tau}(\mu_{t-\tau} - a) + \sum_{j=1}^{\tau} b^{\tau-j} w_{t-j+1} + \epsilon_{t+1}. \tag{19}$$

Here, $\sum_{i=1}^{\tau} b^{\tau-i} w_{t-i+1}$ represents a moving-average term that arises due to the timing mismatch between updating the predictor variable ($\mu_{t-\tau}$) and updating the response variable (y_{t+1}). Empirically, this mismatch can show up as autocorrelation in returns. Though imperfect, a simple and common solution in statistics is to include observable lags in the dependent variable (e.g., include y_t, y_{t-1},..., and $y_{t-\tau+1}$ in the set of predictors) to capture this autocorrelation (see, for example, Wooldridge 2003 or other text books on approaches for dealing with autocorrelation).

To understand the correction more rigorously, note that a general Bayesian solution for obtaining an estimate of μ_t with 1-period-stale data (i.e., μ_{t-1}) is given by the following:

$$\begin{aligned} \bar{\mu}_t &= a + b \times \mu_{t-1} + q(y_t - a - b \times \mu_{t-1}) \qquad (20) \\ &= (1-q) \times (a + b \times \mu_{t-1}) + q \times y_t, \end{aligned}$$

where $q \in (0, 1)$ represents the weight that investors place on y_t, and is known in the filtering literature as the "Kalman Gain," which we assume for purposes of exposition to be constant over time. Given τ-period-stale data, therefore, the Bayesian solution becomes

$$\mathbb{E}_t[y_{t+1}] = \bar{\mu}_t = (1-q)^{\tau}(a + b \times \mu_{t-\tau}) + q \sum_{i=1}^{\tau} (1-q)^{\tau-i} y_{t-i+1}. \tag{21}$$

Thus, when the expectation estimate is formed using stale data, including lags in the response variables corrects for measurement error by adding information that would not otherwise be available.

3.3 Information Quality and Expected Returns

We also examine analytically how the above model of expected returns is affected by the quality of accounting information provided to investors. We extend the baseline model of expected returns to a setting where information is imperfect. For ease of exposition, we assume the use of book value of equity as the accounting valuation anchor, but the framework is sufficiently flexible to accommodate other anchors.

To introduce the concept of imperfections in the accounting system vis-à-vis return prediction, we assume that investors do not directly observe expected growth in book value, h_t, but learn about it dynamically over time using realized accounting reports.[2]

In the spirit of Dechow, Ge, and Schrand (2010), we assume that investors observe financial reports of book growth, $g_{b,t+1}$, which reflects both "true" firm performance ($g_{true,t+1}$) and noise (ξ^r_{t+1}) from the accounting system (we drop the firm i notation for convenience):

$$g_{b,t+1} = g_{true,t+1} + \xi^r_{t+1}, \text{ and} \tag{22}$$

$$g_{true,t+1} = h_t + \xi^{true}_{t+1}. \tag{23}$$

It follows that observed reports of book growth ($g_{b,t+1}$) have two sources of noise: (1) true "fundamental" or "innate" noise (ξ^{true}_{t+1}) and (2) measurement errors from the accounting system (ξ^r_{t+1}). We assume that the noise in the reports is captured by two independent error terms, $\xi^{true}_{t+1} \sim N(0, \sigma^2)$ and $\xi^r_{t+1} \sim N(0, \sigma^2_r)$.[3] Mapping this back into the assumptions about

[2]Unlike related studies in the literature (e.g., Van Binsbergen and Koijen, 2010), we do not assume that investors need to filter expected returns. Our rationale is that since investors set prices, given their expectations of book growth, they must also set expected market returns. Our setting is thus closely related to that of Pástor and Veronesi (2003, 2006), except that we do not assume an exogenous discount factor but instead assume a specific stochastic process for expected returns.

[3]While the assumption of Gaussian error terms is common and somewhat restrictive, the assumption of independence is without loss of generality.

growth in book value (9), we have:

$$g_{b,t+1} = h_t + \xi_{t+1}^{true} + \xi_{t+1}^{r}. \quad (24)$$

Since investors observe only realized growth in book values, they form expectations of book growth by making inferences (or learning) about the unobserved h_t using relevant information to optimally update their beliefs over time. We denote $f_t = \mathbb{E}[h_t|\mathcal{F}_t]$ as investors' beliefs about mean book growth given $\mathcal{F}_t$, where $\mathcal{F}_t = \{g_{b,\tau}\}_{\tau\in\{0,1,\ldots,t\}}$ represents the history of accounting reports available to investors. Assuming that h_t is also conditionally Gaussian, it can be shown that investors' optimal dynamic updates to their beliefs take the following form:

$$f_{t+1} = \mu + \omega(f_t - \mu) + \frac{\omega v_t}{\sigma^2 + \sigma_r^2 + v_t}(g_{b,t+1} - f_t), \quad (25)$$

where $v_t = \mathbb{E}[(h_t - \mathbb{E}[f_t|\mathcal{F}_t])^2|\mathcal{F}_t]$ is the conditional variance of f_t with respect to investors filtration $\mathcal{F}_t$, or the dispersion in investors' prior beliefs; σ_h^2 is the conditional variance of h_t; and $v_{t+1} = \omega^2 v_t + \sigma_h^2 - \frac{\omega^2 v_t^2}{\sigma^2+\sigma_r^2+v_t}$.[4]

To see how to apply this update rule, we need to first derive expected returns under imperfect information. We return to the valuation equation:

$$bm_t \;=\; \overline{bm} + \frac{1}{1-\kappa}(\mu_t - \mu) - \frac{1}{1-\omega}(f_t - \mu). \quad (26)$$

Log-linearizing as above, we have:

$$\log\left(1 + \frac{D_{t+1}}{M_{t+1}}\right) \approx \log(1 + \delta\exp(\overline{bm})) + \frac{\delta\exp(\overline{bm})}{1+\delta\exp(\overline{bm})}(bm_{t+1} - \overline{bm}), \quad (27)$$

$$= \log(1 + \delta\exp(\overline{bm})) + \frac{\delta\exp(\overline{bm})}{1+\delta\exp(\overline{bm})}\left[\frac{1}{1-\kappa}(\mu_{t+1} - \mu) - \frac{1}{1-\omega}(f_{t+1} - \mu)\right]. \quad (28)$$

[4]This follows directly from Theorem 13.4 of Liptser and Shiryaev (1977).

Taking expectations conditional on $\mathcal{F}_t$, we have:

$$\mathbb{E}\left[\log\left(1+\frac{D_{t+1}}{M_{t+1}}\right)\Big|\mathcal{F}_t\right] \approx K+\rho\left(\frac{\kappa}{1-\kappa}(\mu_t-\mu)-\frac{\omega}{1-\omega}(f_t-\mu)\right), \tag{29}$$

where K and ρ are given in the prior subsection. This implies that expected returns are given as follows:

$$\mathbb{E}[r_{t+1}|\mathcal{F}_t] = A_1+A_2 bm_t+A_3(f_t+y). \tag{30}$$

Here $A_1=K-\rho\kappa\overline{bm}-(1-\kappa)\overline{bm}-A_3 y+\frac{(\kappa-\omega)(1-\rho)}{(1-\omega)}\mu$, $A_2=\rho\kappa+(1-\kappa)$, and $A_3=\frac{\rho(\kappa-\omega)+1-\kappa}{1-\omega}$; $y=\log(1+\delta)$

From the updating rule (Eq. (25)), we can express expected returns in terms of current roe by noting that:

$$f_t-\mu=\omega(f_{t-1}-\mu)+\frac{\omega v_{t-1}}{\sigma^2+\sigma_r^2+v_{t-1}}(roe_t-y-f_{t-1}). \tag{31}$$

Plugging this into the expected-returns Eq. (30), we have:

$$\begin{aligned}\mathbb{E}[r_{t+1}|\mathcal{F}_t] &= A_1+A_2 bm_t+ \\ &\quad A_3\left[\mu+y+\omega(f_{t-1}-\mu)+\frac{\omega v_{t-1}}{\sigma^2+\sigma_r^2+v_{t-1}}(roe_t-y-f_{t-1})\right].\end{aligned} \tag{32}$$

Re-arranging, we obtain:

$$\mathbb{E}[r_{t+1}|\mathcal{F}_t] = C_1(t)+C_2(t)f_{t-1}+C_3 bm_t+C_4(t)roe_t, \tag{33}$$

where

$$\begin{aligned}
C_1(t) &= A_1 + A_3\left[\mu(1-\omega) + y\left(1 - \frac{\omega v_{t-1}}{\sigma^2 + \sigma_r^2 + v_{t-1}}\right)\right], \\
C_2(t) &= A_3\left(\omega - \frac{\omega v_{t-1}}{\sigma^2 + \sigma_r^2 + v_{t-1}}\right), \\
C_3 &= A_2, \\
C_4(t) &= A_3\frac{\omega v_{t-1}}{\sigma^2 + \sigma_r^2 + v_{t-1}}.
\end{aligned}$$

This model provides the key insight that, all else equal, better accounting information quality elevates the importance of profitability in forecasting future returns. Specifically, conditional on the dispersion of investors' prior beliefs (v_{t-1}), the volatility of the underlying fundamentals (σ^2), and the persistence in expected returns and expected profitability, the coefficient on *roe* is increasing with accounting information quality (or decreasing in σ_r^2).

The above also allows a reconciliation of the baseline LPV model of expected returns with the various alternative firm characteristics and signals (e.g., valuation ratios and accounting data) that relate to future stock returns. In particular, generalizing the above to include multiple information sources in investors' information set, if a signal systematically forecasts future profitability (i.e., future *roe*), conditional on *bm*, it follows that such a variable is also systematically associated with expected returns.

4 Conclusion

This primer provides an overview of a log-present-value framework and the conditions under which accounting information can be connected to expected returns. We show that this framework is fairly general, flexible, and can be extended to accommodate the effect of various attributes in accounting information on expected returns. We believe this framework can help to stimulate further innovations in the accounting valuation and empirical asset pricing literature.

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