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SEQUENTIAL STRUCTURE OF FUTURES MARKETS AND THE VALUE OF IMPROVING INFORMATION
An Example

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In an example, the value of improving information is computed under alternative structures for the timing of futures markets. It is always beneficial if futures markets are active both before and after the information is revealed.

1. The model

We consider a simple model in which futures markets can be used by a producer to reduce the variability in his income. We then evaluate the effect on his welfare resulting from an improvement in the quality of information perceived by this producer and all other participants in the futures markets in common.

From the producers point of view, his output, q, and the equilibrium price of output at the date of production, p, are jointly distributed random variables over which he exerts no influence. He maximizes a von Neumann–Morgenstern utility function that depends only on his income. Therefore the relevant variables from his point of view will be the value of his output, $v = p \cdot q$, and the price, p.

We assume that at some time, before the values of v and p are realized, an observation, s, is received by all participants in the market system. In this example we assume that the joint distribution of the triple (p, v, s) is normal with mean zero [1] and variance–covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{pp} & \sigma_{pv} & \sigma_{ps} \\ \sigma_{pv} & \sigma_{vv} & \sigma_{vs} \\ \sigma_{ps} & \sigma_{vs} & \sigma_{ss} \end{bmatrix} .$$

The decisions to be taken by the producer concern his purchases on futures markets. There are two potential dates at which a futures market can meet: before the

[1] The mean can be normalized arbitrarily. It will be shown below that it plays no role in the analysis.

announcement of s, called the *initial* date, or after this announcement, but before the realization of (p, v), called the *intermediate* date.

The basic assumption of this model is that the equilibrium price on any futures market is the expected final price, p, given the information available. We denote by p^0 the initial futures market equilibrium and by p^1 the intermediate market equilibrium. By virtue of the assumptions made above, p^0 is identically zero and the value of p^1 depends on the observation s. Given s, the conditional joint distribution of (p, v) is normal with mean

$$\left(\frac{\sigma_{ps}}{\sigma_{ss}} \cdot s, \frac{\sigma_{vs}}{\sigma_{ss}} \cdot s\right), \tag{1}$$

and variance–covariance matrix

$$\begin{bmatrix} \sigma_{pp} - \dfrac{\sigma_{ps}^2}{\sigma_{ss}} & \sigma_{pv} - \dfrac{\sigma_{ps}\sigma_{vs}}{\sigma_{ss}} \\ \sigma_{pv} - \dfrac{\sigma_{ps}\sigma_{vs}}{\sigma_{ss}} & \sigma_{vv} - \dfrac{\sigma_{vs}^2}{\sigma_{ss}} \end{bmatrix} .$$

Therefore,

$$p^1 = p^1(s) = \frac{\sigma_{ps}}{\sigma_{ss}} \cdot s . \tag{2}$$

2. Optimal actions

A futures contract entitles the buyer to receive (and obligates the seller to supply) one unit of output at the final date. Let z^0 be the quantity of futures contracts purchased at the initial date, and let $z^1(s)$ be the purchases at the intermediate date, given that s has been observed. Total income of the producer, as a function of (p, v, s), is, therefore

$$w = v + z^0(p - p^0) + z^1(s)(p - p^1(s))$$

$$= v + z^0 p + z^1(s)\left(p - \frac{\sigma_{ps}}{\sigma_{ss}}\, s\right) .$$

The decisions $[z^0, z^1(s)]$ are chosen to maximize

$$E[u(w)] ,$$

where the expectation is taken with respect to the joint distribution of (p, v, s). The von Neumann–Morgenstern utility function, u, is assumed to be strictly concave.

Given that z^0 was the action taken, it is straightforward to verify that the optimal value of $z^1(s)$ for every value of s is given by

$$z^1(s) = \frac{\sigma_{ps}\sigma_{vs} - \sigma_{pv}\sigma_{ss}}{\sigma_{pp}\sigma_{ss} - \sigma_{ps}^2} - z^0 , \tag{3}$$

independent of the utility function u. Note that $z^1(s)$ is, in fact, independent of s. This follows from the constancy of the conditional variance–covariance matrix, which is a property of the joint normality assumption. [2]

The optimal z^0 can be shown to be

$$-\sigma_{vs}/\sigma_{ps} . \tag{4}$$

Therefore, the variance of w at the optimum is given by

$$\sigma_{vv} - \frac{\sigma_{vs}^2}{\sigma_{ss}} - \frac{(\sigma_{ps}\sigma_{vs} - \sigma_{pv}\sigma_{ss})^2}{\sigma_{ss}(\sigma_{pp}\sigma_{ss} - \sigma_{ps}^2)} . \tag{5}$$

3. The value of improved information

Suppose that the observation s is contaminated by a measurement error ϵ that is normally distributed with mean zero and variance $\sigma_{\epsilon\epsilon}$, and that is independent of (p, v, s). The observation actually received is $s' = s + \epsilon$. Smaller values of $\sigma_{\epsilon\epsilon}$ clearly correspond to more accurate information structures. We compare the utility at the optimum as a function of $\sigma_{\epsilon\epsilon}$ under two alternative market structures.

3.1. Intermediate market only

If there were no opportunity to trade futures contracts at the initial date, $z^0 = 0$ is effectively imposed as a constraint. Let Σ' be the variance–covariance matrix of (p, v, s'). (Σ' is just Σ with $\sigma_{s's'} = \sigma_{ss} + \sigma_{\epsilon\epsilon}$.) Let C_{ij} be the cofactor of the ijth element of Σ'. Using the analysis of section 2, the derivative of the variance of w with respect to changes in $\sigma_{\epsilon\epsilon}$, computed at the optimum, can be shown to have the same sign as

$$\sigma_{ps'} C_{pv}(\sigma_{vs'} C_{vv} - \sigma_{s's'} C_{vs'}) . \tag{6}$$

It can be verified that (6) can be of either sign. Therefore, with a futures market at the initial date only, improvements in information can be either beneficial or harmful.

[2] Because $z^1(s)$ is constant, w is normally distributed with mean $E(v)$. Therefore all risk averters would agree on the policy of minimizing the variance. For this reason the precise form of the utility is irrelevant. This is the central simplifying feature of this example.

3.2. Initial and intermediate markets

Using the analysis of section 2, the derivative of the variance of w with respect to $\sigma_{\epsilon\epsilon}$ is

$$-\sigma^2_{s's'}(\sigma^2_{pp}\sigma^2_{vs'} - 2\sigma_{pp}\sigma_{pv}\sigma_{ps'}\sigma_{vs'} + \sigma^2_{pv}\sigma^2_{ps'}) \,, \tag{7}$$

which is obviously negative. Superior information is necessarily beneficial in this case.

4. Conclusion

The contrast between the results in the last section is due to the fact that with only an intermediate market, improved information increases the variability in p^1 at the same time as it increases the conditional correlation of p and v. The former is harmful whereas the latter is beneficial. A futures market at the initial date allows the producer to hedge against the first part of this risk.

The general welfare analysis of improvements in information under alternative market structures is still an open question. Some further results are given in: Green, J., *Value of information with sequential futures markets* (Harvard Institute of Economic Research Discussion Paper no. 631, July 1978), which is available from the author upon request.