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Budget Displacement Effects of Inflationary Finance

By JERRY GREEN AND EYTAN SHESHINSKI*

When inflation is caused by an increase in the rate of issuance of real money balances, less recourse to other sources of government revenue is necessary. This policy will therefore influence the equilibrium growth path through both the induced additional capital losses that individuals bear on their money balances and the necessary changes in fiscal policy required to balance the government budget. Though the first of these effects has received wide treatment in the literature, the second has been largely neglected.

We analyze this issue through a variety of simple monetary growth models, using alternative specifications of the government budget relation and individual savings functions. The central conclusion of all of these models is the tendency for inflation to increase capital intensity in the absence of any effects on portfolio proportions or the savings rate induced by changes in interest rates. We also present some numerical results for these models using parameters related to the current U.S. situation. Typically, a 1 percent increase in the permanent rate of inflation will produce a capital stock that is 2–4 percent larger.

The following symbols will be used:

Tax Parameters:

- τ_1 = corporate tax rate on real profits
- τ_2 = corporate tax rate on inflation induced profits
- τ = corporate tax rate (used when $\tau_1 = \tau_2$ is assumed)

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- ρ = tax rate on labor income
- T = total real taxes collected, per capita

Rates and Proportions

- n = population growth rate
- γ = government budget as proportion of real output per capita
- σ = savings rate out of disposable income
- Λ = fraction of wealth held in money balances

Commodities

- Y, F = total national income (F is used as function of capital and population)
- K = total real capital stock
- N = total population
- y, f = per capita national income (f is used as function of capital per capita)
- k = per capita real capital stock
- m = per capita real money balances
- l = per capita real government bonds
- e = per capita government real expenditures
- d = per capita disposable income

Prices

- i = nominal interest rate
- π = rate of inflation
- r = real interest rate, real before tax return to capital
- w = real wage
- P = price of output
- W = nominal wage

I. Introduction

A. Models of Taxation and Inflation

The literature of monetary growth theory has been consistently concerned with the ef-

fects of inflation on long-run economic equilibrium. Nonmonetary one-sector models¹ have been extended to treat the case in which governments use lump sum taxation and inflationary monetary policy to finance their expenditures.² In these models, inflation influences capital accumulation by changing the desired composition of portfolios as the rates of return to holding monetary and nonmonetary assets diverge. Recently, Martin Feldstein has generalized these models to include other types of taxation and a savings behavior derivable from the life cycle hypothesis.

He sets up a full-employment model of the standard type in which there is a single good, and money is the only nonphysical asset. Competitive behavior is assumed throughout. Labor grows at a constant rate n , and is supplied inelastically. Production is assumed to be in accordance with a neo-classical constant returns to scale technology. The corporate tax enters the profit-maximization calculus as follows.

A firm that is employing N units of labor at wage rate W , and K units of capital produces a flow $Y = F(N, K)$ of output, measured in the same units as capital. All capital is financed by debt denominated in monetary units, which pays a nominal interest rate i . Let P be the price of output. Therefore, at any instant of time, the firm's nominal cash flow is equal to $PY - WN - iPK$. In addition, the stock of capital, which is assumed not to depreciate, is increasing in nominal value at the rate of price increase π . These two components of profit can be taxed at different rates: τ_1 for cash flow and $\tau_1 - \tau_2$ for the inflationary inventory revaluation.³ Therefore, after-tax profit is

$$(1) \quad (PF(N, K) - WN - iPK)(1 - \tau_1) + (\pi PK)(1 - \tau_1 + \tau_2)$$

Defining the real wage as $w = W/P$, we

¹See Robert Solow.

²See James Tobin, David Levhari and Don Patinkin, Duncan Foley and Miguel Sidrauski, and Jerome Stein.

³Feldstein writes net profit as $(1 - \tau_1)(y - wl) - (1 - \tau_1)rk - (1 - \tau_1)\pi k + \tau_2\pi k$ inventory profit. This is equivalent to our formulation.

have that maximal profit is attained when

$$(2) \quad F_N = w$$

$$(3) \quad (F_K - i)(1 - \tau_1) + \pi(1 - \tau_1 + \tau_2) = 0$$

Using the assumption of constant returns to scale, and letting $f = F/N$ and $k = K/N$, we have

$$(4) \quad f - kf' = w$$

$$(5) \quad f' = i - \frac{1 - \tau_1 + \tau_2}{1 - \tau_1} \pi$$

Feldstein examines the dependence of the steady state on the two components of the tax rate. To simplify the analysis and help us focus on some other issues, we will assume that the nominal capital gains are untaxed. Thus $\tau_1 = \tau_2$, and (dropping the subscript on τ)

$$(6) \quad f' = i - \frac{1}{1 - \tau} \pi$$

Corporate after-tax profits in the competitive equilibrium are therefore

$$(7) \quad (f - w - ik)(1 - \tau) + \pi k$$

which is equal to zero, using (4) and (5), as one would expect by the constant returns postulate. However, it is interesting to note that before tax, the profits

$$(8) \quad f - w - ik + \pi k$$

are negative. They are, in fact,

$$(9) \quad \frac{-\tau \pi k}{1 - \tau}$$

which is the *subsidy* received by firms because the "taxable" component of "profit," $f - w - ik$, is actually a loss.

Personal income in the Feldstein model exceeds total product by exactly the amount of these losses. It is $w + rk$ on a per capita basis. Individuals are assumed to hold money according to the relation

$$(10) \quad m = \Lambda k$$

where m is the real money stock per capita. Feldstein assumed that Λ varies with the real interest rate.

The government is assumed to spend a

fixed proportion γ of real output. The total government budget is therefore

$$\gamma f + \frac{\tau \pi k}{1 - \tau}$$

since firms must be subsidized from general revenues. The sources of these revenues are a personal interest income tax,⁴ a lump sum tax, and inflationary finance. Thus the government's budget equation is⁵

$$(11) \quad \gamma f = T - \frac{\tau \pi k}{1 - \tau} + (\pi + n)m$$

since inflation is caused by the issuance of real money balances in excess of the rate of population growth in the steady state.

Real disposable income is personal income net of tax minus the losses on money holdings due to inflation, πm . This is

$$(12) \quad (1 - \gamma)f + mn$$

It is assumed that a fraction, σ , of this is saved and invested according to (10) in the two assets. Feldstein allowed σ to depend in a general way on the real net interest rate.

The steady-state equation is

$$(13) \quad \sigma((1 - \gamma)f + n\Lambda k) = n(1 + \Lambda)k$$

It is important to note that τ and π affect the steady-state value of k only through the values of σ and Λ which depend on the net rates of return. Therefore, as Feldstein derives, the effects of changes in inflation on the steady-state capital intensity and real money balances occur only because savings and liquidity preference are interest responsive to these parameters. His analysis suggests that the effects of savings are likely to dominate those of liquidity preference since the latter operate only through money holdings which empirically form a small proportion of total wealth.

With $\sigma = .1$, $\pi = .05$, $\Lambda = 1/40$, $\gamma = .24$, $n = .01$, and a Cobb-Douglas technology with capital's share at .25, the equilibrium is characterized by a real rate of

⁴ θ_1 and θ_2 in Feldstein.

⁵Here T is total personal taxes. Feldstein's T is net taxes inclusive of subsidies to firms. But he wrote personal income directly as $f - T$, which is $w + rk - T$ in our notation.

interest of 3.28 percent. At $\tau = .5$ total personal tax collections are 60 percent of total output, although real government expenditures are only 24 percent. The difference consists of 38 percent of total output, representing subsidies to firms minus 20 percent of output which is inflationary finance.

In the absence of savings or portfolio sensitivity to interest rates, k will be a constant. The change in lump sum taxation with respect to the inflation rate can be seen from (11) to be

$$(14) \quad \frac{dT}{d\pi} = \frac{\tau k}{1 - \tau} - m$$

At a 50 percent corporate tax rate which corresponds to the current *U.S.* situation, and with $\Lambda \approx 1/40$, this is clearly positive. Thus rather than acting as a substitute for other sources of government revenues, inflationary finance creates the need for higher personal taxes!

However, higher inflation rates increase pretax personal income because of the higher level of i given by (6). But it is nevertheless true that tax collections take an increasing share of personal income net of inflationary capital losses. At $\tau = .5$ we have equation (15). Since $\Lambda \approx 1/40$, $n \approx .01$, and $\gamma \ll .5$, the derivative in (15) is clearly positive.

Because these effects of inflation on taxation are somewhat perverse, we are led to study models in which the revenue-generating aspect of inflationary finance can be separated from its other impacts. In these models it will be seen that inflation is not neutral even when savings and portfolio effects are neglected. We introduce the possibility of government borrowing and taxation on all forms of income at constant marginal rates, instead of lump sum personal taxation and interest income taxes only.

Section IB considers a model directly analogous to Feldstein's in which the savings base is broadened to include real government expenditures net of subsidies to firms, and in which total government spending rather than net expenditures only

$$(15) \quad \frac{d\left(\frac{T}{w + ik - \pi m}\right)}{d\pi} = \frac{f(1 - 2\gamma - \Lambda(1 - \gamma)) + \pi k(1 - \Lambda) + (2 - \Lambda)(\pi - \Lambda(\pi + n))}{(w + ik - \pi m)^2}$$

are held at a fixed fraction of total output. We show that this model has all of the qualitative properties of Feldstein's model.

Section II treats the two possibilities that the savings base can be broadened without changing the government expenditure rule, and vice versa. These models still have the feature that lump sum taxation is used to balance the government budget. They are not exactly symmetric in their properties. These differences are explored, and the magnitudes of the inflationary effects are studied using numerical examples with reasonable parameter values.

In Section III we treat a variety of models in which government borrowing replaces lump sum taxation. The results of these are compared, both analytically and numerically. Under reasonable hypotheses about the parameters of the system, inflation will tend to increase capital intensity and decrease the steady-state value of government debt per capita in the absence of life cycle savings variation or changes in liquidity preference.

In Section IV we study a model in which firms hold money. Section V covers a model with neither borrowing nor lump sum taxation, in which the rate of inflation is endogenously determined by the tax and spending parameters.

B. A Broader Savings Base and Proportionality of Total Government Spending to Output

If real government purchases are used to provide private goods on a public basis, we might expect that savings out of net disposable income will respond positively to this activity. The simplest hypothesis is that real government purchases are perfectly substitutable for net disposable income in

the savings base, and that the fraction of their sum saved is a constant.

Total government spending is divided into real purchases e , and subsidies to firms $\tau\pi k/(1 - \tau)$. The sum of these is a fixed fraction of real output. The government budget equation is therefore

$$(16) \quad \gamma y = e + \frac{\tau}{1 - \tau} \pi k = T + (\pi + n)m$$

The savings base can be written as

$$(17) \quad \begin{aligned} d + e &= w + rk - \pi m - T \\ &= w + rk - \frac{\tau}{1 - \tau} \pi k + nm \\ &\quad \text{(using (16))} \\ &= f + n\Lambda k \quad \text{(using (10), (4))} \end{aligned}$$

Therefore the condition for a steady state is

$$(18) \quad \sigma(f + n\Lambda k) = n(1 + \Lambda)k$$

Hence, when σ and Λ are constant, as we shall be supposing throughout, k is independent of both the rate of inflation and corporate taxation.

Equation (18) is not exactly the same as (13). However, due to the broader savings base we might expect σ to be lower in this case. Since savings are about 10 percent of disposable income, the comparable figure is 7.8 percent of disposable income plus government purchases. Using the other parameters as given in Section IA above, the equilibrium level of the capital stock is consistent with a real rate of interest of 3.28 percent. At a zero-corporate tax rate, the share of the savings base attributable to government spending is 24 percent. As τ rises, firms' losses increase and therefore the share of total government revenues going into real purchases falls. Since the savings base is constant this change induces an

increase in real disposable income. Personal interest income is rising since r increases with τ and all other components of disposable income depend only on k , which is a constant.

II. Budget Displacement Effects with Lump Sum Taxation

A. Government Purchases Proportional to Output—Broad Savings Base

In the models of the last section it was shown that inflation could not affect capital intensity, even in the presence of corporate taxation, unless savings or liquidity preference depend on the rate of return. This is due to the fact that the savings base is independent of the rate of inflation and that steady-state capital intensity is determined completely by the equality of savings and investment. In Feldstein's model, savings depend on real output plus the rate of increase in the real money stock minus real government expenditures. In our model with a broader base, the last term is omitted and one would expect a correspondingly lower average savings rate. The independence of the savings base in both models is due to the compensating changes in lump sum taxation.

In the first case, an increase in the rate of inflation increases losses made by firms, which are subsidized through higher lump sum taxes. However, perceived personal interest income increases by exactly the increase in taxation plus inflationary losses on real money balances, since the real rate of interest on the fixed capital stock increases by more than the rate of inflation. In the second case the gain in personal interest income is offset instead by the fall in real government expenditures, which are assumed to be a perfect substitute in the savings base. The level of taxation necessary to maintain a balanced budget is a constant. One can then see that by using a definition of the savings base that is compatible with the government's expenditure rule, the independence property would be maintained.

Therefore, one is led to consider other cases. For example, we could include government expenditure in the savings base, and assume that they are a constant fraction of real output. That is, we use Feldstein's specification of the government budget equation, but enlarge the savings base as in the second model of Section I.

Here, disposable income is defined by

$$(19) \quad d = w + rk - \pi m - T$$

and the government's budget equation is

$$(20) \quad \gamma y = e = T + (\pi + n)m - \frac{\tau}{1 - \tau} \pi k$$

With a savings base of $d + e$, the steady-state equation becomes

$$(21) \quad \sigma \left(w + rk + mn - \frac{\tau}{1 - \tau} \pi k \right) = n(m + k)$$

Using the relations for the firm's equilibrium (4) and (6) and the liquidity preference relation (10), this becomes

$$(22) \quad \sigma(f(k) + n\Delta k) = nk(1 + \Lambda)$$

which is identical to the equilibrium condition in the second model of Section I.

This reflects the fact that real government expenditures are equal to taxes plus inflationary finance minus subsidies to firms in both cases. The broader savings base with complete substitutability between government and personal disposable income removes the influence of the government expenditures on savings.

B. Government Budget Proportional to Output—Narrow Savings Base

However, the situation would be markedly different if we were to take the narrower savings base of Feldstein's paper together with a government budget equation in which total expenditures, including subsidies to firms, are proportional to real output.

The government's budget equation is

$$(23) \quad \gamma y = T + (\pi + n)m$$

which, when combined with (14), (16), and (10), gives the steady-state relation

$$(24) \quad \sigma(1 - \gamma)f + \frac{\tau}{1 - \tau}\pi k + n\Lambda k = n(1 + \Lambda)k$$

Therefore, as long as the corporate tax rate is different from zero, the rate of inflation will affect the real variables in the steady state. A higher rate of inflation will decrease the level of lump sum taxes necessary to balance the budget at the same level, the magnitude of this effect being exactly equal to the inflationary finance created. Disposable income is therefore increased by exactly the increase in personal interest income as can be seen from the above remarks and equation (19). Savings increase because of this, although real government purchases are lower. The new equilibrium will therefore occur at a higher level of capital per head, and a higher real output. On the other hand, since the share of output going to government expenditures falls, no firm welfare conclusions can be drawn without a specification of individuals' tastes for alternative forms of income.

This can be seen from differentiating (24) to obtain

$$(25) \quad \frac{dk}{d\pi} = \frac{-\sigma\tau k}{1 - \tau} \cdot \left(\frac{1}{\sigma(1 - \gamma)f' + \frac{\sigma\tau\pi}{1 - \tau} + \sigma n\Lambda - n(1 + \Lambda)} \right)$$

Combined with the equilibrium condition, this is

$$(26) \quad \frac{dk}{d\pi} = \frac{-\tau k}{1 - \tau} \left(\frac{1}{(1 - \gamma) \left(f' - \frac{f}{k} \right)} \right)$$

which is positive by the concavity of f .

Differentiating (19) and using (26), it is easy to see that the steady-state aggregate consumption level will be increased. Differentiating (20) it can be shown that

$$(27) \quad \frac{de}{d\pi} = \frac{-\tau k}{(1 - \gamma)(1 - \tau)} \left(1 + \frac{e/k}{\left(f' - \frac{f}{k} \right)} \right)$$

which can have either sign. Initial forces tend to decrease e but if the elasticity of output with respect to input is sufficiently small, the resulting increase in k may offset the budget displacement effect. Even in this case, however, welfare is not necessarily improved in the new steady state because the intergenerational distribution of output is altered due to changes in the real rate of interest.

Using a Cobb-Douglas production function with capital's share set at .25 and parameter values of $\sigma = 1$, $\gamma = .24$, $\pi = .05$, $\tau = .5$, $\Lambda = .025$ (which roughly correspond to the current U.S. experience), the change in the capital-labor ratio induced by a 1 percent increase in the inflation rate can be computed from (26) to be 15.4 percent of its previous equilibrium value. The change in the real rate of interest is given by

$$(28) \quad \frac{dr}{d\pi} = f'' \frac{dk}{d\pi} + \frac{\tau}{1 - \tau}$$

which is 0.8 percent per 1 percent increase in π . This should be compared with the comparable expression obtained by Feldstein, which yields a 1 percent increase (in the absence of savings and portfolio effects) for the same parameter values. The budget displacement effect therefore mitigates the induced increase in the real interest rate found by Feldstein. If, however, government follows a policy through which real purchases are not decreased to the full extent of the additional subsidy required for firms, this effect will be correspondingly smaller. In any case it is likely to be much larger than a pure liquidity preference effect (see Tobin), which is probably less than 0.01 percent.

In the models of this section changes in the rate of inflation will not effect any real variables in the absence of corporate taxation. This is because disposable income varies only due to changes in the real rate of

interest, which remains unaffected without corporate taxation. Lump sum taxation offsets the losses on real money balances due to the increased inflation. It is therefore of interest to study such models in cases where the government issues debt, rather than changes taxes, to balance the budget when the rate of inflation is altered. The private sector then bears the burden of inflationary finance immediately. Only over time will these forces cause a compensating change in disposable income through the effect of debt service costs in the government budget. Introducing an additional asset in this way allows us to study models in which monetary policy can be analyzed without making other compensating changes in the government's actions. This is the topic of the next section.

III. Budget Displacement Effects with Public Debt

A. Government Purchases Proportional to Real Output

In this section we study models in which public debt is a perfect substitute for the obligations of firms in individual's portfolios. We neglect corporate taxation for simplicity. We will demonstrate that the real economic variables of the system are affected by inflation, even in the absence of a corporate tax—which was not the case in the previous models studied.

We assume that the government taxes wage income and interest income from both corporate and public debt at the same rate, ρ . Let l represent the real value of the stock government debt per capita, and \dot{L} be the rate of issuance of new government debt at any instant of time, denominated in units of money.

In the presence of inflation the real value of government debt held by any individual is falling at any instant of time. We assume in this section that the tax laws allow a full deduction of these losses.⁶ Since individuals

regard corporate and public debt as perfect substitutes, the rate of return on these assets is equal. Disposable income is therefore

$$(29) \quad d = [w + r(k + l)](1 - \rho) - \pi m$$

Maintaining the spirit of the portfolio condition of previous sections, we assume

$$(30) \quad m = \Lambda \cdot (l + k)$$

where Λ is a constant.

If we suppose that the government purchases a constant share of real output, the government's budget equation is

$$(31) \quad \gamma y = \rho(w + r(k + l)) - (r + \pi) + \frac{\dot{L}}{PN} + \frac{\dot{M}}{PN}$$

The first term on the right-hand side is real tax collections, noting that a loss-offset on both types of debt is allowed. The second term is the real debt-service paid by the government. The rate of interest is the real rate plus the rate of inflation. The fall in the value of government debt allows further borrowing at every instant, to keep the real stock of debt per capita constant. The final two terms are the real values of currently issued bonds and outside money.

In the steady state

$$(32) \quad \frac{\dot{L}}{L} = n + \pi \quad \frac{\dot{M}}{M} = n + \pi$$

so that the real levels of debt and money are constant in per capita terms. Substituting these relations in (31) we have

$$(33) \quad \gamma y = \rho(w + r(k + l)) - (r + \pi)l + (\pi + n)l + (\pi + n)\Lambda(l + k)$$

or

$$(34) \quad \gamma y = \rho(w + rk) - ((1 - \rho)r - n)l + (\pi + n)\Lambda(l + k)$$

In the absence of corporate taxation or subsidization, firms will borrow up to the

⁶We have also recomputed our results under the assumption that these losses can be deducted from the

tax base. This lowers the steady-state capital-labor ratio, but has no significant effects on the comparative statics of the system.

point where

$$(35) \quad f'(k) = r$$

and will hire labor as before, so that

$$(36) \quad f(k) - kf'(k) = w$$

Therefore, the government's budget equation becomes

$$(37) \quad \gamma y = \rho y - ((1 - \rho)r - n)l \\ + (\pi + n)\Lambda(l + k)$$

Together with the steady-state condition

$$(38) \quad n(1 + \Lambda)(l + k) \\ = \sigma\{w + r(k + l)\}(1 - \rho) - \pi m\}$$

This determines the behavior of the steady states of this system as the rate of inflation is varied.

Unfortunately this system is likely to be unstable for typical values of the parameters, although a complete analysis would require a specification of nonsteady-state behavior in the commodity and asset markets in the presence of inflation, which is beyond the scope of this paper. The potential for instability can be seen as follows: The initial increase in π causes the issuance of new bonds \dot{L} to fall, as can be seen from (31), since at that instant both budget and debt service levels are fixed by the historically given stocks. The magnitude of this decrease is 1 percent of the nominal money stock for each percent of additional inflation. Nominal disposable income goes down by 1 percent of the nominal money stock, which can be verified by multiplying (29) by the price level. Savings decrease by less than this, since $\sigma < 1$. Since the government supplies bonds inelastically at the market rate of interest, the fall in supply is greater than that in total savings so that the quantity of real output channeled into capital formation initially increases with the higher inflation rate. On the other hand, the budget equation (37) can be rewritten as

$$(39) \quad 0 = (\rho - \gamma)f(k) \\ - ((1 - \rho)r - n - (\pi + n)\Lambda)l \\ + (\pi + n)\Lambda k$$

from which we see that if the coefficient of l is positive, a *higher* value of debt will have to be maintained at each level of capital intensity to balance the budget when inflation increases. Moreover, since the system begins to accumulate capital at a higher rate when population is growing, instability will surely result whenever $\rho > \gamma$ as well. This condition is equivalent to the fact that debt service net of taxes is greater than the level of deficit finance. The current *U.S.* data do not give direct evidence on this matter but it must be remembered that the deficit should be calculated on a full-employment basis in a steady-state situation. Taking this into account, the indicated inequality is almost surely valid. The coefficient of l , $(1 - \rho)r - n - (\pi + n)\Lambda$, is positive in any steady state with relatively moderate inflation and a savings propensity 10 or 15 times the rate of population growth. For these reasons, due to the instability of the system we believe that the comparative statics of this case are likely to be misleading. We will therefore concentrate on an alternative specification of the government budget equation related to that studied previously.

B. Budget Proportional to Real Output-Broad Savings Base

We will assume that instead of controlling purchases of real output, the government policy is to keep total spending, including net debt service, at a constant proportion of national product. Specifically, letting e represent government purchases of output per capita we have

$$(40) \quad \gamma y = e + (1 - \rho)rl \\ = \rho f + nl + (\pi + n)\Lambda(l + k)$$

We will assume further that savings are proportional to disposable income plus government purchases of real output. Thus the steady-state condition becomes (using (29), (36), (38), and (40))

$$(41) \quad n(l + k)(1 + \Lambda) = \sigma(d + e) \\ = \sigma(f + nl + n(l + k)\Lambda)$$

This savings assumption is compatible with

$$(42) \quad \frac{dk}{d\pi} = \frac{\Lambda(l+k)(n-f/k)}{n(f'-f/k)(l-\rho+\gamma)}$$

$$(43) \quad \frac{dl}{d\pi} = \frac{\Lambda(l+k)^2(\sigma(f'+nL)-n(1+L))}{n\sigma(f'-f/k)k(1-\rho+\gamma)}$$

the inclusion of net debt service in the government budget relation. The case of a narrower savings base will be treated later in this section.

Differentiating totally with respect to π , l , and k , and substituting the solutions of the equilibrium equations, we obtain (42) and (43). The denominators are negative by the concavity of the production function. The numerator of (42) is negative and that of (43) will be negative provided that

$$(44) \quad n > \sigma f'$$

which is assured in the steady state of a Solow-type one-sector model and is valid for our typical parameter values as well.

For example, using the parameters of Section II,⁷ taking a Cobb-Douglas production function with capital's share equal to .25, $\sigma = .078$, $\gamma = .24$, $\pi = .05$, $\Lambda = .025$, and assuring further that $\rho = .22$ (instead of the lump sum taxation of Section II), we can calculate that the steady state is characterized by an interest rate of 3.6 percent.

When the inflation rate increases by 1 percent, the equilibrium level of the real capital stock increases by 3.44 percent (using (42)) and the equilibrium level of real bond holdings decreases by 0.4 percent (using (43)). This change in the capital stock reduces the equilibrium interest rate by 0.01 percent.

These are in contrast to the model of the end of Section I which was identical except for the presence of lump sum taxes instead

⁷We have assumed $\sigma = .078$ to maintain comparability with other sections. We have also assumed

$$\frac{d}{d+e} = \frac{1.508}{1.918} \approx .78$$

so that

$$\sigma_{wide} = \frac{d \cdot \sigma_{narrow}}{d+e} \approx .078$$

of bond sales in the government budget equation. There we found no influence of inflation on the steady-state real variables, and an equilibrium real interest rate of 3.28 percent. Thus the real capital stock in a model with lump sum taxation under the budgetary and savings assumption we are using is equivalent to that in a model with debt finance under a much higher inflation rate.

C. Budget Proportional to Real Output-Narrow Savings Base

Section IIb discussed a model comparable to that of Section IA, in the sense that the savings base and the government's budget equation were the same, but government debt replaced lump sum taxes as the residual variable in the budget equation. In this part we analyze a model with the same budget specifications, but disposable income is now the only component of the savings base. Disposable income is defined as in (29) so that with total savings equal to σ times this, the steady-state equation is

$$(45) \quad n(1+\Lambda)(l+k) = \sigma(1-\rho)f + (1-\rho)f'l - \pi\Lambda(l+k)$$

We repeat the government's budget equation for convenience:

$$0 = (\rho - \gamma)f + nl + (\pi + n)\Lambda(l+k)$$

Differentiating this system totally and substituting the equilibrium expressions obtained by eliminating l from the government budget equation and using (45), we obtain (46) and (47).

$$(46) \quad \frac{dk}{d\pi} = [\Lambda(l+k)^2 f'] \div [(f' - f/k)k(\sigma n - f'(\rho - \gamma)) + f''kl(n - f(\rho - \gamma)/k)]$$

$$(47) \quad \frac{dl}{d\pi} = -\Lambda(l+k)^2 \left[-\frac{n}{1-\rho} + \left(\sigma + \frac{\rho-\gamma}{1-\rho} \right) f' + f''l \right] \div [(f' - f/k)k(\sigma n - f'(\rho - \gamma)) + f''kl(n - f(\rho - \gamma)/k)]$$

The denominator can be shown to be negative whenever

$$(48) \quad \gamma > \rho$$

Even if this were violated, the negativity would be preserved unless σ were unrealistically low. Thus $dk/d\pi < 0$ under our assumptions.

The numerator of (47) will be positive under the same conditions provided

$$(49) \quad n > \sigma f'$$

which is surely valid for economies in which we are interested. Therefore $dl/d\pi < 0$.

Using, for illustrative purposes, the same parameters as those taken in Section IIIb, we find that the equilibrium capital-labor ratio produces a real rate of interest of 3.6 percent and the equilibrium ratio of real capital to holdings of government bonds is 8.2:1. The value of $dk/d\pi$ is -6.9 percent per 1 percent increase in the inflation rate. Real bond holdings decrease by 10.2 percent of their equilibrium value per 1 percent increase in inflation. This means that the real interest rate will respond to a 1 percent increase in inflation by rising .2 percent.

Comparing this to the model at the end of Section II, which was identical except for the presence of lump sum taxation instead of government borrowing, we see that the equilibrium interest rate is much higher here due to the fact that some wealth is channeled away from real capital formation. The former model had an interest rate of 1.7 percent at a corporate tax rate of 50 percent. At a zero-corporate tax rate the former model would have had an interest rate of 3.3 percent. Here the equilibrium interest rate is 3.6 percent.

These figures can be explained as follows: since firms make losses in the presence of corporate taxation, a lower tax rate induces lower subsidies at each fixed rate of infla-

tion. The loss of these subsidies forces some firms out of business reducing the capital stock. Moreover, in the presence of debt finance, part of wealth (about 1/9 with these parameters) is held in this form, further reducing the equilibrium stock of productive capital.

IV. A Model with Firms Holding Money

One of the most striking disparities between the monetary growth models presented above and the real world is the fact that most money is in reality held by firms. We will show that the basic results of these models are preserved in a model analogous to that of Section IIIc, with such a modification. The simplest assumption paralleling that used above is that firms must keep real money balances proportional to capital according to

$$(50) \quad m = \Lambda k$$

and individuals hold government debt, on which a full offset for inflation produced capital losses is allowed. We will use the narrow savings base—but the results would be essentially unchanged with the wide base and corresponding modifications in the levels of the assumed parameters. For simplicity, and to isolate the effects of the change in the ownership of money balances on the system, we will assume no corporate taxation.

The rate of inflation affects firms' choices of capital intensity through the fact that capital losses on real money balances are causing them to economize on money and hence on capital, which is a complementary input. In fact, one can regard these losses as a type of depreciation since the technological justification for (50) is presumably a production function of the form $y = f(\min(m\Lambda, k))$.

Firms' profits are given by

$$(51) \quad y - w - ((1 + \Lambda)i - \pi)k$$

in per capita terms. The first-order condition for a maximum is

$$(52) \quad f' = r(1 + \Lambda) + \pi\Lambda$$

We assume that the government budget is proportional to $(y - \pi\Lambda k)$, which yields

$$(53) \quad \gamma(y - \pi \Delta k) = \rho(f - \pi \Delta k) + nl + (\pi + n)\Delta k$$

The justification for this is that the net output of the economy is really the gross output y minus the real savings channeled into money that would be necessary to maintain a constant output level—recall the technological specification made above. This parallels the budget justification of Sections IB, IIB, and IIIB and c. Disposable income includes wages plus interest on government and corporate debt. The latter is proportional to $m + k$ since firms must finance their money holdings as well as their capital through borrowing. Thus using (51),

$$(54) \quad d = (1 - \rho) \cdot \left(f - \pi \Delta k - \left(\frac{f' - \pi \Delta}{1 + \Delta} \right) l \right)$$

Finally, the steady-state equation is

$$(55) \quad \sigma d = n(k + m + l)$$

Substituting (54) into (55), then differentiating the result and (52) totally with respect to π , l , and k at the equilibrium values, we find (56) and (57).

$$(56) \quad \frac{dk}{d\pi} = \left[(1 + \gamma - \rho)\Delta kn + (1 + \gamma - \rho)\Delta k\sigma(1 - \rho)r - n\sigma(1 - \rho)\Delta \left(k - \frac{l}{1 + \Delta} \right) \right] \div \left[\frac{f}{k} - f' \right] (\gamma - \rho)(n - \sigma(1 - \rho)r) - n\sigma(1 - \rho) + \frac{n\sigma(1 - \rho)f''}{1 + \Delta}$$

$$(57) \quad \frac{dl}{d\pi} = \left[(1 + \gamma - \rho)\Delta k(1 + \Delta)(-n + \sigma r(1 - \rho)) + (1 - \rho)\sigma\Delta(-k(1 + \Delta) + l) \left((\gamma - \rho)r - \frac{(\pi + n)\Delta}{1 + \Delta} \right) - \sigma(1 - \rho) \frac{f''l}{1 + \Delta} (1 + \gamma - \rho)k\Delta \right] \div \left[\frac{f}{k} - f' \right] (\gamma - \rho)(n - \sigma(1 - \rho)r) - n\sigma(1 - \rho) + \frac{n\sigma(1 - \rho)f''}{1 + \Delta}$$

One can show, using the equilibrium conditions, that sufficient conditions for the denominator to be negative are

$$(58) \quad \gamma - \rho > 0$$

$$(59) \quad \sigma > \frac{\gamma - \rho}{1 - \rho}$$

which are analogous to the conditions of Section III.

Numerically, using the same parameter values as in Section III, we find that the original steady-state real interest rate is 3.65 percent. The induced change in capital per 1 percent change in the inflation rate is -0.19 percent of the original capital stock.

V. Endogenous Inflation: Effects of Tax and Budget Changes

Suppose that the government finances its budget by nonlump sum taxes and without any borrowing. If real government purchases are a fixed proportion of real output, then the rate of inflation is endogenously determined in the system so as to satisfy the government's budgetary needs. This can be seen as follows.

We assume for simplicity that there are no corporate taxes, and that a full loss offset on capital losses due to inflation is permitted. Disposable income is therefore as in (29) with $l = 0$:

$$(60) \quad d = (w + rk)(1 - \rho) - \pi m$$

while the government's budget equation is similarly

$$(61) \quad \gamma y = (w + rk)\rho + (\pi + n)m$$

Using the firm's equilibrium conditions (4), (6) and the portfolio condition $m = \Delta k$, the steady-state equations corresponding to the above model can be written

$$(62) \quad n(1 + \Delta)k - \sigma[f(1 - \rho) - \pi \Delta k] = 0$$

$$(63) \quad (\gamma - \rho)f - (\pi + n)\Delta k = 0$$

Equations (62) and (63) determine the endogenous variables k and π .

We can now find the effects on these variables of changes in the tax rate ρ , and in

the government's budget proportion γ . Differentiating (62) and (63) totally, substituting for the equilibrium conditions, we get

$$(64) \quad \frac{dk}{d\rho} = 0$$

$$(65) \quad \frac{d\pi}{d\rho} = \frac{-f}{\Lambda k}$$

The effects of changes in the government's budget proportion are similarly found:

$$(66) \quad \frac{dk}{d\gamma} = \frac{-\sigma f}{(f/k - f')\sigma(1 - \gamma) + (1 - \sigma)\pi\Lambda}$$

$$(67) \quad \frac{d\pi}{d\gamma} = \left[\Lambda^{-1} \frac{f}{k} \right. \\ \left. (f/k - f')\sigma(1 - \rho) - (1 - \sigma)\pi\Lambda \right] \div \\ [(f/k - f')\sigma(1 - \gamma) + (1 - \sigma)\pi\Lambda]$$

Under these conditions, a decreased tax rate can be fully compensated by a change in the rate of inflation without affecting the government's budget equation. This is because no substitution of real capital for money balances will take place under these

extreme conditions. The full effect of tax changes falls on the rate of inflation.

On the other hand, expenditure changes necessitate a faster inflation rate. This lowers savings and hence capital intensity in the long run.

These results would basically not change if we allowed for only a partial inflationary loss offset to individuals or by the introduction of corporate taxes.

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