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ON THE REVELATION OF PREFERENCES FOR PUBLIC GOODS

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1. Introduction

The problem of ascertaining tastes for public goods is one of serious practical as well as theoretical concern. Recently, attention has been devoted to the design of mechanisms to elicit the true tastes of the population and to act in accordance with the information revealed [Groves (1973), Groves and Loeb (1975), Kurz (1974)]. We shall be concerned with the simplest of such problems – the determination of the desire of the population for a single specific public project. We require that the procedure be known to all participants, and that the government be bound to act in accordance with the announced preferences.

Groves (1973) has studied one such process which has the following desirable properties: it is in every individual's interest to announce his true preference for the project independent of the announcements of others, and the project is undertaken whenever the announced value to society exceeds its cost. Such procedures we call *individually incentive compatible and successful.*

Specifically, it is supposed that individual i's willingness-to-pay for the project is v_i and his announced valuation is w_i. Without loss of generality, we take the cost of the project to be zero. Costly projects can be treated by subtracting the per capita cost from everyone's evaluation.[1] The project is undertaken whenever

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[1]An alternative procedure is to introduce the government as an artificial player whose expressed preferences for the project are minus its cost. If the decision is then made on the basis of the sign of the sum of willingnesses-to-pay, including the government's, the project will be accepted only if the value exceeds the cost. Transfer payments must also be calculated including the government's statement as well, but, if the costs are assumed to be proportional to the number of individuals served by the project, all of the results of this paper are preserved. Subtracting the per capita cost from individuals' statements, as suggested in the text, leads to a system that is exactly analogous to this one as well.

$\Sigma_i w_i \geqq 0$, and the ith individual receives a subsidy of $\Sigma_{j \neq i} w_j$ in this case. If the project fails, no subsidy is paid. Whatever the values for w_j, $j \neq i$, the ith individual cannot do better than to set $w_i = v_i$, for with this choice he will receive

$$v_i + \sum_{j \neq i} w_j \quad \text{if} \quad \sum_i w_i = v_i + \sum_{j \neq i} w_j > 0,$$

and zero otherwise. With other choices of w_i, the individual runs the risk of either a negative payoff (if $w_i > v_i$) or a zero payoff when the truth would have led to a positive result (if $w_i < v_i$), and there is no potential gain.

Since the strategy of 'telling the truth' is dominant for each individual, the Nash equilibrium in which everyone follows this policy has a strong claim on our attention. We want to ascertain if the costs of attaining it through the Groves procedure are warranted. If a project is accepted a total of $\Sigma_i \Sigma_{j \neq i} w_j$ must be paid in subsidies. If it can be recouped through lump sum mechanisms so as not to distort the incentives, then the cost is purely the adverse effect on the distribution of income. An upper bound on these costs can therefore be obtained by treating the total subsidy payment as a dead weight loss.[2]

One way of attempting to mitigate these losses is taking a random sample from the population to estimate tastes, and acting upon this estimate. This introduces the obvious trade-off – sampling error vs. the cost of the subsidies.[3] This procedure does not insure that a Pareto optimum relative to the full information situation will be found. The potential for nonoptimal decisions is one manifestation of the costs of the government's imperfect information. The distribution of the mean preference of the sample is, of course, independent of the size of the population. But when an error is made, it affects all the people who were omitted from the sample. The risks therefore increase with the size of the population while the cost of the procedure grows at the rate of the square of the sample size. It is natural to ask how the optimal sample size depends on the size of the entire population and in particular to focus on the asymptotic

[2]An additional problem is that, although the mechanism induces honest responses on an individual level, it is not immune to cooperative behavior. In fact, any two individuals can guarantee each other a highly desirable outcome by both announcing a very large, fictitious, evaluation of the project. Furthermore, on the individual level, the incentive to reveal one's true tastes decreases with the size of the population. The only instance in which the announcement affects an individual's payoff is when it changes the sign of the aggregate. The likelihood of this clearly decreases as the population grows. Thus, although the incentive to tell the truth still exists, it is greatly weakened in large group situations. In a separate paper we take up the question of choosing an individually incentive compatible mechanism with desirable properties relative to these problems as well.

[3]Related to the problems mentioned in footnote 2, this idea has some subsidiary advantages. Sampling may tend to make cooperative behavior more costly as members of potential coalitions may have trouble seeking each other out. Moreover, by keeping the set of individuals smaller, the strength of the incentives to tell the truth (or the potential regret associated with waking an erroneous response), will increase.

properties of this dependence. To make this precise, we assume that the central planner is subjectively uncertain about the distribution of tastes in the population, and revises his beliefs in a Bayesian way after the sample is taken. However, he is bound by the rules of the Groves procedure to act in accordance with the sample, for otherwise its incentive compatibility would be destroyed.

The mechanism will be *informationally useful* if the expected *post sample* sum of utilities exceeds the expected outcome based on the prior beliefs alone. We can think of the informational value of the procedure as the excess of the former over the latter. We study the asymptotic properties of this information value per capita as the size of the population grows. In cases in which this is positive in the limit it would be justifiable to say that the elicitation mechanism has made a definite contribution towards the solution of the 'free rider' problem, where, in the extreme case, the best action would be to follow prior beliefs.

In the next section the problem is set out in the case of normally distributed tastes in the population and a normal prior on its unknown mean. The value of the Groves procedure is computed as it depends on the sample size and the other parameters. In section 3 we show that the optimal sample size grows *not faster* than the cube root of the population size. A more precise analytical result is *not* attainable due to inherent nonconvexities. Computational results seem to indicate the tightness of this upper bound. Section 4 is concerned with the excess of the value of this procedure, minus the necessary subsidies, over the level of welfare attainable by the government on the basis of its a priori beliefs alone. We refer to this as the informational value of the Groves mechanism. It is shown that this value is always nonnegative and a characterization of its behavior with respect to the parameters is given. A final section presents some computational results that tend to confirm the properties indicated above.

2. Value of the Groves experiment

We consider the decision-making process concerning a potential public project from which exclusion will not be possible. The distribution of willingness to pay for the project by the members of the population can be approximated by a normal distribution with unknown mean m and known variance s^2. The actual size of the population is N. Moreover, the decision-maker has a prior distribution on m which is a normal distribution with mean μ and variance σ^2. Let $k = \sigma^2/s^2$ be the ratio of the prior variance to the sampling variance.

A sample of size n is drawn from the population; then, the true willingness to pay of agents in the sample is elicited by the Groves experiment. Let w be the sample mean. A Bayesian revision of the prior distribution on m leads then to the following posterior distribution,

$$\mathcal{N}\left(\frac{\mu+nkw}{1+nk}, \frac{\sigma^2}{1+nk}\right). \tag{1}$$

For each value of the sample mean w, the expected post sample utility of the project is

$$(N-n)\left(\frac{\mu+nkw}{1+nk}\right)+nw. \tag{2}$$

In the social evaluation of the project we assume that agents have the same marginal utilities of income.

The cost of the mechanism is assumed to be the total amount of transfers necessary in the Groves experiment,[4]

$$c(w,n) = n(n-1)w. \tag{3}$$

This pessimistic evaluation of costs gives us a lower bound for the post sample value of the experiment when the project is performed, i.e. $w > 0$:

[4]Actually, using $c(w,n)$ as an upper bound on the costs of the mechanism involves a slight underestimate in some cases. The sum of transfers may be written

$$\sum_i \sum_{j\neq i} w_j,$$

whenever $\Sigma_i w_i \geqq 0$. In some situations, $\Sigma_{j\neq i} w_j$ may be negative for some i even though $\Sigma_i w_i \geqq 0$. These individuals are taxed instead of being subsidized by the mechanism. In computing an upper bound on the cost these transfers impose on the system, these taxes should not be counted as a net benefit to the economy. To obtain a bound, we should treat taxes as a dead-weight loss and not count subsidies at all. Therefore, the sum of the taxes imposed on the sampled group is

$$-\sum_i \min\,(0, \sum_{j\neq i} w_j),$$

while the unsampled group must pay a tax of

$$\sum_i \max\,(0, \sum_{j\neq i} w_j).$$

Therefore total taxes are

$$\sum_i |\sum_{j\neq i} w_j|,$$

which differs from $c(w,n)$ by

$$-2\sum_i \min\,(0, \sum_{j\neq i} w_j).$$

Incorporating this modification in our analysis would greatly complicate matters. However, it would not change the asymptotic results in any way. Space precludes a full analysis of this matter in the present article. The interested reader should consult Green, Kohlberg and Laffont (1976) where it is shown that the expected value of the last expression, under the normality assumptions we have made, grows at the rate of $n^{1/2}$ or less. Since $c(w, n)$ grows like n^2, this correction term does not have a significant influence on the asymptotic value of the procedure and does not affect the rate of growth of the optimal sample size. Small sample results would display slightly smaller sample sizes, but the effect is extremely small for all parameter values we have explored.

$$v(w, n, N) = \left(\frac{(N-n)nk}{1+nk} + 2n - n^2\right) w + \left(\frac{N-n}{1+nk}\right) \mu. \quad (4)$$

Let $f(w)$ be the ex ante law of w; it is clearly a normal distribution with mean μ and variance $s^2\ ((1/n)+k)$. Therefore, the pre-sample expected value of the experiment is

$$V(n, N) = \left(\frac{(N-n)nk}{1+nk} + 2n - n^2\right) \int_0^\infty wf(w)\mathrm{d}w + \frac{(N-n)}{1+nk}\,\mu \int_0^\infty f(w)\mathrm{d}w, \quad (5)$$

since the project is undertaken only if $w \geqq 0$.[5]

[5]One can show that this criterion is not strictly optimal in the sampling context, although it is the only way to produce a Pareto optimum relative to the preferences of the sample. Superior results could be attained by using an acceptance criterion of

$$w > -\frac{(N-n)\mu}{Nnk+n},$$

which will result in acceptance iff the posterior evaluation of the government is nonnegative. In order to maintain the incentive compatibility of the mechanism, an additional subsidy of

$$\frac{(N-n)\mu}{N\mu k+n}$$

would have to be given to each member of the sampled group in the event that the project is accepted. This modification would, therefore, affect the choice of the sample size. In general, the sample size and cutoff point would have to be jointly determined at an optimum. One can write down the expected value of the mechanism in this case, paralleling the method of this section, as a function of these two variables. It can be shown that the cutoff point converges to zero with the population size and that the asymptotic rate of growth of the sample size is unchanged. Variations in the small sample results depend on the parameters used, but the magnitude of the modification is very minor in all cases. Precise results are available from the authors on request.

Intuitively what happens is the following: when $\mu > 0$, the cutoff point and subsidies increase. This is equivalent to an artificial stated willingness-to-pay by the government of

$$w_0 = \frac{(N-n)\mu}{Nnk+n},$$

which is then counted along with all the other statements (cf. footnote 1). However, as w_0 increases from zero to the value above, subsidies rise and it can be shown that the optimal value of w_0 is between these points. Since this involves somewhat larger subsidies than the mechanism we have considered, sample sizes will be smaller to economize in this dimension. On the other hand, when $\mu < 0$, moving towards this value of w_0 improves the performance of the decision making mechanism as well as reducing the subsidies. The optimal statement by the government is therefore below this value of w_0, and, since subsidies will be smaller on average, a larger sample will be used for every finite value of N.

The evaluation of the integrals is straightforward and leads to (see Appendix).

$$V(n, N) = \frac{[(N+n-n^2)k+(2-n)]s}{\left(\frac{1}{n}+k\right)^{1/2} \sqrt{(2\pi)}} \exp \left[\quad \left[\frac{(\mu/s)^2}{\left(\frac{1}{n}+k\right)}\right]\right]$$

$$+\mu[N-n(n-1)]\, \rho \left[\frac{\mu/s}{\left(\frac{1}{n}+k\right)^{1/2}}\right], \tag{6}$$

with

$$\rho(x) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{x} \exp\,[-\tfrac{1}{2}z^2]\mathrm{d}z.$$

A naive procedure would be to use only the prior information in deciding whether to do the project. This leads to an ex ante expected value of

$$W(N) = N \max\,[0, \mu]. \tag{7}$$

The expected per capita gain of the experiment is then

$$g(n, N) = \frac{1}{N}\; V(n, N) - \max\,[0, \mu], \tag{8}$$

which represents the value of the information acquired per capita net of the costs of acquisition.

In the next section we study the behavior of the optimal sample size $n^*(N)$ for large values of N.

3. Optimal sample size

Because of the nonconcavity of the function $V(n, N)$, it is not possible to obtain analytically the optimal sample size $n^*(N)$. However, we are able to bound the rate of increase of $n^*(N)$ from above and to find a lower bound for the value of the experiment which is shown to be a good one in the simulation of section 5. This is done by finding the change in the objective function with respect to n, and showing that it is negative in the limit if $n(N) = N^{\delta}$ with $\delta > \frac{1}{3}$.

Consider first the case where $\mu = 0$. The objective function is then

$$V(n, N) = \frac{s}{\sqrt{(2\pi)}} \left[\frac{(Nk+2)+n(k-1)-n^2k}{\left(\frac{1}{n}+k\right)^{1/2}}\right].$$

The derivative of this expression with respect to n is

$$\frac{\partial V}{\partial n}(n, N) = \left[\frac{s}{2\sqrt{(2\pi)}}\left(\frac{1}{n}+k\right)^{-3/2}\right]\frac{1}{n^2}\,[Nk-4k^2n^3$$
$$+(2k^2-7k)n^2+3(k-1)n+2].$$

Let

$$\frac{s}{2\sqrt{(2\pi)}}\left(\frac{1}{n}+k\right)^{-3/2} = K;$$

then

$$\bar{K} = \lim_{n\to\infty} K = \frac{\sigma}{2k^2\sqrt{(2\pi)}}.$$

If $n = N^\delta$, with $\delta > 0$ and N goes to infinity, the dominant terms are $KkN^{1-2\delta}$ among the positive ones and $-4Kk^2N^\delta$ among the negative ones. Since the positive term is of the order of $N^{1-2\delta}$ and the negative one of the order N^δ, we have that the derivative is negative whenever

$$\delta > 1-2\delta \quad \text{or} \quad \delta > \tfrac{1}{3} \quad \text{as } N\to\infty.$$

It is easy to verify that for $\mu = 0$, the objective function is quasi-concave. For N large but finite, an approximation of the zero of the derivative of the objective function is then obtained with:

$$\delta = \tfrac{1}{3}-\frac{\log 4k}{3\log N}.$$

When $\mu \neq 0$, the objective function is more complex. The derivative of $V(n, N)$ is then:

$$[K\exp[-\tfrac{1}{2}\mu^2/[(s^2/n)+\sigma^2]]]\cdot 1/n^2$$
$$\cdot\Bigg[Nk-4k^2n^3+(2k^2-7k)n^2+3(k-1)n+2$$
$$-\left((Nk-kn^2+[k-1]n+2)\frac{\mu^2/s^2}{k+(1/n)}\right)+\frac{\mu^2}{s^2}(N-n^2+n)\Bigg]$$
$$+\mu[1-2n]\rho\left(\frac{\mu}{([s^2/n]+\sigma^2)^{1/2}}\right).$$

We must distinguish two cases according to the sign of μ.

(a) $\mu > 0$. As N goes to infinity, the dominant terms are

$$k\bar{K} \exp [-\tfrac{1}{2}(\mu/\sigma)^2]\, N^{1-2\delta}$$

among the positive ones, and

$$[4k^2\bar{K} \exp [-\tfrac{1}{2}(\mu/\sigma)^2]+2\mu\rho(\mu/\sigma)]\, N^{\delta}$$

among the negative ones. Therefore, as for $\mu = 0$, we have that the derivative is negative whenever

$$\delta > \tfrac{1}{3} \quad \text{as} \quad N \to \infty,$$

and an approximation of the *last zero* of the derivative is obtained with:

$$\delta = \tfrac{1}{3} - \frac{1}{3 \log N} \log [4k+4k\sqrt{(2\pi)}\cdot\mu/\sigma\cdot\rho(\mu/\sigma) \exp [\tfrac{1}{2}(\mu/\sigma)^2]]. \tag{9}$$

(b) $\mu < 0$. As N goes to infinity, the dominant terms are

$$k\bar{K} \exp [-\tfrac{1}{2}(\mu/\sigma)^2]\, N^{1-2\delta} \quad \text{and} \quad 2|\mu|\rho(\mu/\sigma)N^{\delta}$$

among the positive ones, and

$$4k^2\bar{K} \exp [-\tfrac{1}{2}(\mu/\sigma)^2]\, N^{\delta}$$

among the negative ones. Therefore if

$$4k^2\bar{K} \exp [-\tfrac{1}{2}(\mu/\sigma)^2] > 2|\mu|\rho(\mu/\sigma), \tag{10}$$

the result is the same as for $\mu > 0$. The relation (10) can be rewritten

$$\exp [-\tfrac{1}{2}(\mu/\sigma)^2] - \frac{|\mu|}{\sigma} \int_{-\infty}^{\mu/\sigma} \exp [-\tfrac{1}{2}x^2]\mathrm{d}x > 0. \tag{11}$$

Let $y = |\mu|/\sigma$. Condition (11) becomes

$$\phi(y) = \exp [-\tfrac{1}{2}y^2] - y \int_{-\infty}^{-y} \exp [-\tfrac{1}{2}x^2]\, \mathrm{d}x > 0.$$

We have

$$\phi'(y) = -\int_{-\infty}^{-y} \exp\,[-\tfrac{1}{2}x^2]\mathrm{d}x < 0,$$

and $\phi(0) = 1$. Moreover, $\lim_{y\to\infty} \phi(y) = 0$. Therefore it is clear that ϕ can have no zeros, since it is a decreasing function with a zero asymptote. Hence (10) is always satisfied.

To sum up, for $\mu \neq 0$, the derivative is negative whenever $\delta > \frac{1}{3}$ when $N \to \infty$. However, we do not know in this case if $V(n, N)$ is quasi-concave; we know only that an upper bound of the maximum sample size is of the order of $N^{1/3}$ as $N \to \infty$. The simulation shows that the objective function is often quasi-concave and that (9) is a good approximation of the optimal sample size.

4. A lower bound for the per capita gain of the Groves experiment

According to section 3, an obvious lower bound for the per capita value of the Groves experiment when $N \to \infty$ is obtained by replacing n by $N^{1/3}$ in (6):

$$V(N^{1/3}, N) = \frac{(N+N^{1/3}-N^{2/3})k+(2-N^{1/3})}{\frac{1}{s}\left(\frac{1}{N^{1/3}}+k\right)^{1/2}\sqrt{(2\pi)}} \exp\left[-\tfrac{1}{2}\frac{(\mu/s)^2}{(N^{-1/3}+k)}\right]$$

$$+\mu(N+N^{1/3}-N^{2/3})\rho\left[\frac{\mu}{s\left(\frac{1}{N^{1/3}}+k\right)^{1/2}}\right], \qquad (12)$$

or, taking the limit when $N \to \infty$,

$$\lim_{N\to\infty} \frac{1}{N} V(N^{1/3}, N) = \exp\left[-\tfrac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\right]\frac{\sigma}{\sqrt{(2\pi)}} + \mu\rho\left(\frac{\mu}{\sigma}\right), \qquad (13)$$

and

$$\lim_{N\to\infty} g(N^{1/3}, N) = \exp\left[-\tfrac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\right]\frac{\sigma}{\sqrt{(2\pi)}} - \mu\left(1-\rho\left(\frac{\mu}{\sigma}\right)\right), \quad \text{if } \mu > 0,$$

$$= \exp\left[-\tfrac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\right]\frac{\sigma}{\sqrt{(2\pi)}} + \mu\rho\left(\frac{\mu}{\sigma}\right), \quad \text{if } \mu < 0. \qquad (14)$$

Let $B(\mu, \sigma) = \lim_{N\to\infty} g(N^{1/3}, N)$ be this bound on the asymptotic per capita gain. For $\mu > 0$, $B(\mu, \sigma)$ is

$$\frac{1}{\sqrt{(2\pi)}}\left[\sigma \exp\left[-\tfrac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\right]-\mu\int_{\mu/\sigma}^{\infty}\exp\left[\frac{-z^2}{2}\right]\mathrm{d}z\right].$$

Writing $x = \mu/\sigma$, this is proportional to

$$\exp\left[-\frac{x^2}{2}\right]-x\int_{x}^{\infty}\exp\left[-\frac{z^2}{2}\right]\mathrm{d}z,$$

which is greater than or equal to

$$\exp\left[-\frac{x^2}{2}\right]-\int_{x}^{\infty}z\exp\left[-\frac{z^2}{2}\right]\mathrm{d}z = 0,$$

since $z > x$ throughout the range of integration. The case of $\mu < 0$ can be treated in a parallel manner, obtaining the same result.

The loci of '$B(\mu, \sigma) =$ constant' can be analyzed as follows:

$$\frac{\partial B(\mu, \sigma)}{\partial \sigma} = B_\sigma = \frac{1}{\sqrt{(2\pi)}}\exp\left[-\tfrac{1}{2}(\mu/\sigma)^2\right] \geqq 0,$$

$$\frac{\partial B(\mu, \sigma)}{\partial \mu} = B_\mu = \frac{-1}{\sqrt{(2\pi)}}\int_{\mu/\sigma}^{\infty}\exp\left[-\frac{z^2}{2}\right]\mathrm{d}z \leqq 0.$$

Since $B(\mu, \sigma)/\sigma$ is homogeneous in μ/σ, the locus of $B = 0$ in the μ, σ-plane is linear. Therefore, because B_μ and B_σ are one-signed, the locus of $B = 0$ must be the horizontal axis. Furthermore,

$$\frac{\partial^2 B(\mu, \sigma)}{\partial \mu^2} = B_{\mu\mu} = \frac{1}{\sigma\sqrt{(2\pi)}}\exp\left[-\tfrac{1}{2}(\mu/\sigma)^2\right],$$

$$\frac{\partial^2 B(\mu, \sigma)}{\partial \sigma^2} = B_{\sigma\sigma} = \frac{\mu^2}{\sigma^3}\cdot\frac{1}{\sqrt{(2\pi)}}\exp\left[-\tfrac{1}{2}(\mu/\sigma)^2\right],$$

$$\frac{\partial^2 B(\mu, \sigma)}{\partial \mu\,\partial \sigma} = B_{\mu\sigma} = \frac{-\mu}{\sigma^2}\cdot\frac{1}{\sqrt{(2\pi)}}\exp\left[-\tfrac{1}{2}(\mu/\sigma)^2\right].$$

Along $B(\mu, \sigma) = \text{constant} = C/\sqrt{(2\pi)}$,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mu} = -\frac{B_\mu}{B_\sigma} > 0,$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\mu^2} = -\frac{B_\sigma^2 B_{\mu\mu} - 2B_\mu B_\sigma B_{\mu\sigma} + B_\mu^2 B_{\sigma\sigma}}{B_\sigma^3}$$

$$= -\frac{\frac{1}{\sigma}\left(\exp\left[-\frac{1}{2}(\mu/\sigma)^2\right] - \frac{\mu}{\sigma}\int_{\mu/\sigma}^{\infty} \exp\left[-\left(\frac{z^2}{2}\right)\right]\mathrm{d}z\right)^2}{\exp\left[-(\mu/\sigma)^2\right]},$$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\mu^2} = -\frac{C^2}{\sigma^3}\exp\left[-(\mu/\sigma)^2\right] < 0.$$

This gives rise to fig. 1.

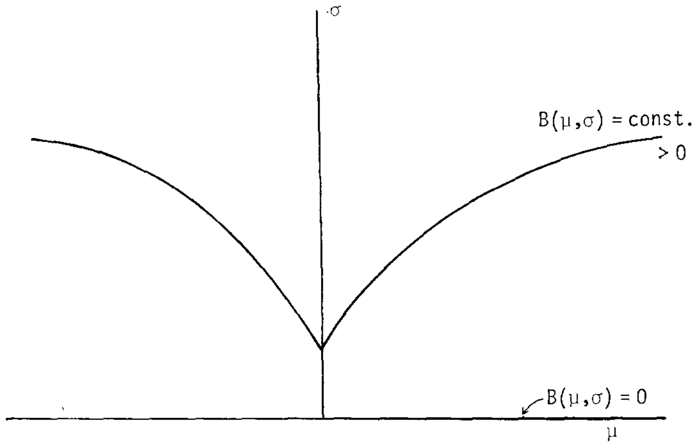

Fig. 1

Thus for every $\sigma > 0$, we have a positive lower bound for the *limiting* per capita value of the optimal elicitation procedure. However, as shown by the simulation, for large (absolute) values of μ, the mechanism may require a very large population to confirm this asymptotic result. When $\sigma = 0$, since the prior has no uncertainty, the optimal sample size is clearly zero, and any utilization of this mechanism can only waste resources.

5. Simulation

The main purpose of the simulation is to check if the upper bound obtained for the optimal sample size and the lower bound obtained for the per capita gain of the Groves experiment are tight bounds which can be used as approximations. This is necessary because of the inherent nonconvexities in the objective function $V(n, N)$ (6).

Table 1

μ	$N = 10^2$ n^*	10^2 $g(n^*, N)$	10^3 n^*	10^3 $g(n^*, N)$	10^4 n^*	10^4 $g(n^*, N)$	10^5 n^*	10^5 $g(n^*, N)$	10^6 n^*	10^6 $g(n^*, N)$	10^7 n^*	10^7 $g(n^*, N)$	10^8 n^*	10^8 $g(n^*, N)$
−0.175	0	*[a]	0	*	0	*	0	*	0	*	1141	0.001	2420	0.001
−0.150	0	*	0	*	0	*	0	*	485	0.000	1048	0.002	2247	0.002
−0.125	0	*	0	*	0	*	0	*	434	0.001	958	0.004	2077	0.004
−0.100	0	*	0	*	0	*	0	*	387	0.004	871	0.006	1911	0.007
−0.075	0	*	0	*	0	*	135	0.000	343	0.007	789	0.010	1749	0.012
−0.050	0	*	0	*	0	*	114	0.005	303	0.013	711	0.016	1593	0.018
−0.025	0	*	0	*	25	0.002	98	0.013	268	0.020	639	0.025	1443	0.027
0.000	1	0.008	4	0.006	22	0.013	85	0.023	238	0.013	573	0.035	1300	0.038
+0.025	0	*	0	*	20	0.001	76	0.011	213	0.019	514	0.024	1165	0.026
+0.050	0	*	0	*	0	*	71	0.001	193	0.010	461	0.015	1039	0.017
+0.075	0	*	0	*	0	*	0	*	177	0.003	414	0.008	923	0.011
+0.100	0	*	0	*	0	*	0	*	0	*	373	0.004	818	0.006
+0.125	0	*	0	*	0	*	0	*	0	*	337	0.001	722	0.003
+0.150	0	*	0	*	0	*	0	*	0	*	0	*	637	0.001
+0.175	0	*	0	*	0	*	0	*	0	3	0	*	560	0.000

[a]An asterisk indicates that the per capita gain is negative for all positive values of the sample size.

We compute the value of $V(n, N)$ for $s = 1$, $\sigma = 0.1$, $k = 0.01$ and different values of μ and N. Then, we identify the optimal sample size and compute the per capita gain in information.

Results are gathered in table 1. Fig. 2 pictures the typical evolution of the optimal sample size. Fig. 3 represents the per capita gain of the Groves procedure.

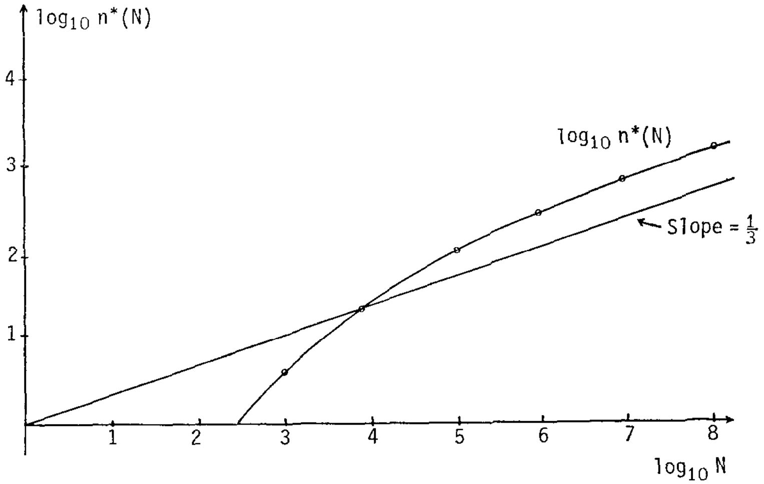

Fig. 2. Optimal sample size (μ = 0), N = size of the population.

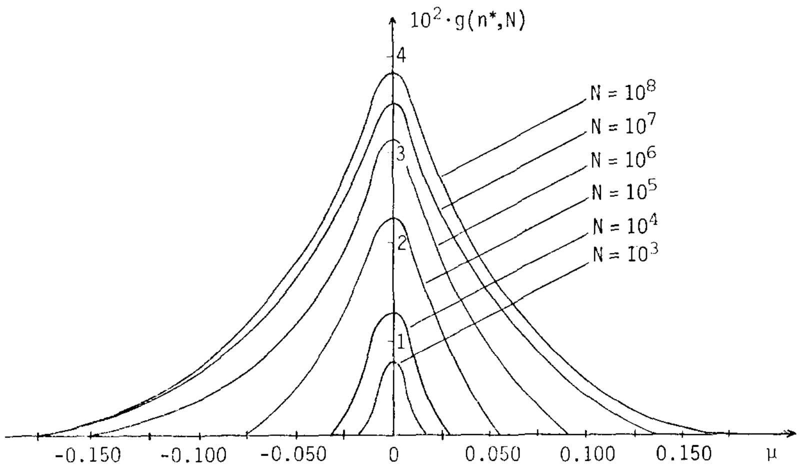

Fig. 3. Per capita gain $g(n^*, N)$, μ = prior mean.

We observe that the optimal sample size converges to $\alpha N^{1/3}$, i.e. that the last zero which was approximated in the analytical derivations corresponds to the optimum (fig. 2).

As expected, fig. 3 shows that the experiment is the most valuable when μ is small in absolute value with respect to σ; this corresponds to very uncertain prior beliefs. For large values of μ, the experiment becomes valuable only for large populations. Note also that the per capita informational gain is almost symmetrical around the value of $\mu = 0$. However, to achieve the same informational gain, the sample size is larger for negative values of μ than for positive values. Therefore, if there were a per unit sample cost, the experiment would be less valuable for negative than positive values of μ. Sufficiently large values of N are checked to verify the finding of section 4 that the asymptotic per capita gain is always positive when $\sigma > 0$.

6. Conclusion

This paper has attempted to ascertain some of the performance characteristics of one particular mechanism for preference elicitation. Our results are largely favorable. They show that the essential difficulties of 'free rider' problem can be overcome even if we assume the worst of all possible circumstances in which the full value of the necessary transfer payments are reckoned as irretrievable costs. Further (though we do not explore this in detail), it is clear that the cost of forming and policing coalitions (if these increase with the size of the coalition) will, in the limit, outweigh the potential benefits as long as there is some upper bound on the size of the evaluation statements that will be believed by the central planner. This is because the proportion of the population sampled decreases to zero, making it prohibitively costly for any individual to search for others in the sampled group with whom he can profitably collude. The characteristics of other individually incentive compatible and successful mechanisms in this regard seem worthy of further study.

A further class of problems of relevance in a more empirical context [see Bohm (1972)], is how people will respond to various incentive structures when they have costs of accuracy in response – for example, their replies may require information gathering before the fate of the project is determined. In such cases the more sharply peaked the individual payoff functions the more accuracy will be induced and one must balance the need for this against the cost of production of potentially useless information.

Appendix

$$\int_0^\infty wf(w)\mathrm{d}w = \frac{1}{\sqrt{(2\pi)}s\left(\frac{1}{n}+k\right)^{1/2}}\int_0^\infty w\exp\left[-\tfrac{1}{2}\,\frac{(w-\mu)^2}{s^2\left(\frac{1}{n}+k\right)}\right]\mathrm{d}w$$

$$= \frac{1}{\sqrt{(2\pi)}s\left(\frac{1}{n}+k\right)^{1/2}}$$

$$\cdot\left[\int_0^{\infty}(w-\mu)\exp\left[-\frac{1}{2}\frac{(w-\mu)^2}{s^2\left(\frac{1}{n}+k\right)}\right]\mathrm{d}w + \int_0^{\infty}\mu\exp\left[-\frac{1}{2}\frac{(w-\mu)^2}{s^2\left(\frac{1}{n}+k\right)}\right]\mathrm{d}w\right]$$

$$= \frac{s\left(\frac{1}{n}+k\right)^{1/2}}{\sqrt{(2\pi)}}$$

$$\cdot\exp\left[-\frac{\mu^2}{2s^2\left(\frac{1}{n}+k\right)}\right]+\mu\rho\left[\frac{\mu/s}{\left(\frac{1}{n}+k\right)^{1/2}}\right]$$

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