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Competitive Inefficiencies in the Presence of Constrained Transactions*

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1. INTRODUCTION

Much of the interest in the Arrow–Debreu model, which has been the benchmark in general equilibrium theory for the last two decades, can probably be traced to its normative implications—specifically, the relationship between its equilibria, the core, and the set of Pareto optima. In this paper, we distinguish two basic types of efficiency criteria, both of which are satisfied by the Arrow–Debreu model. We then proceed to analyze one of these in further detail—with largely negative results—in models with transaction costs.

First, there is efficiency in the traditional sense, namely, *within* an institutional structure. The principal question studied here is whether the hypothesis of competitive behavior would lead to socially optimal results or whether some intervention could improve welfare. Often, though not always, lump-sum redistributions are considered possible reallocative schemes. Second, we introduce the concept of efficiency *among* systems—that is, which set of institutions produces the best points *as competitive equilibria.* In particular, we ask whether a system with more institutional possibilities always has equilibria which dominate those of more restrictive systems.

In its treatment of uncertainty, the Arrow–Debreu model presupposes the existence of markets for claims to contingent delivery of commodities. Because such a multiplicity of markets is not observed in the real world, attention has been focused on transaction costs to explain their absence. Diamond [1], Stiglitz [11], Drèze [2], Mossin [10], and Leland [9], among others, have treated the case in which ownership of shares in firms can

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provide a partial substitute for direct trade in contingent commodities. These studies consider the efficiency of the competitive equilibria generated by this set of markets, relative to a set of possible allocations constrained so as to reflect the implicitly assumed transaction costs. However, they assumed that transactions in shares were costless. We introduce such costs explicitly and reevaluate the optimality of the competitive equilibria of this system of markets.

Transaction costs are often characterized by nonconvexities. Resulting mathematical problems often prevent the existence of a general equilibrium in this instance.[1] However, given that a competitive equilibrium exists, nonconvexities usually do not adversely affect results about its optimality. Drèze [2], in studying a model similar to ours, found that the inefficiency of a certain type of competitive equilibrium was traceable to a nonconvexity arising from the bilinearity of profit shares in the production plan and the share holding. We reexamine the Drèze example in a somewhat different context. It is shown that the inefficiency depends crucially on an extreme ownership pattern of firms across individuals as well as this nonconvexity. In the absence of such a pattern, however, we show that nonconvexities in the transaction costs for share trading can give rise to an inefficiency *among* systems. Thus, a combination of nonconvexities may give rise to an inefficiency although individually they could not produce this phenomenon.

Section 2 specifies more formally the concepts mentioned above and develops some notation. In Section 3 we prove a theorem that gives sufficient conditions for instutional efficiency in a particular model. The hypotheses of this theorem are considered in Section 4, where a series of counterexamples shows that the theorem cannot be strengthened. Since the definition of institutional efficiency depends on the concepts of equilibrium employed in each model, Section 5 treats a variation on the definition we chose in Sections 3 and 4. Under these conditions it is shown that the maximal set of markets is the efficient set even when nonconvexities in transactions costs are present. A conclusion follows in Section 6.

2. Notation and Basic Concepts

In the context of this paper, institutions will be combinations of economic contracts that are tradeable among the individuals in a market

[1] See Hahn [6], Foley [4], and Kurz [8], but note also Heller and Starr [7] who successfully use a large numbers argument to establish approximate equilibria.

equilibrium. Naturally, for each set of institutions there will be different equilibria that result from the optimizing behavior of economic agents. Any behavior mode—for example, perfect competition—will be said to be efficient within an institution if its equilibria are not dominated by any possibilities attainable using only those contracts specified by the institution.

In choosing among institutions, one will be superior to another if *its equilibria* dominate those of the other. Efficiency among institutions depends on the underlying data of the economy in question because these determine the equilibria. An efficient collection of institutions, among those in a given class, is one which is superior to all other collections. The possibility of multiple equilibria may make alternative sets of institutions noncomparable, and "superiority" will have alternative meanings according to the quantifier used. We will not explore such difficulties here.[2]

We are concerned with an economic model in which there are two dates and a variety of contingencies which may occur at the later date. There is a single commodity at the present date and another at the future date whose availability may depend on the state of nature that is realized.

We denote *individuals* by $i = 1,..., I$, *firms* by $j = 1,..., J$, and *states of nature by* $s = 1,..., S$. A subscript 0 denotes the present commodity.

The *consumption plan* of the ith individual is written $x^i = (x_0^i, x_1^i,..., x_S^i) \in \mathbf{R}_+^{S+1}$. Individuals rank alternative consumption plans by their *utility*

$$u : \mathbf{R}_+^{S+1} \to \mathbf{R}.$$

The set of all *joint consumption plans* is $\mathbf{R}_+^{I(S+1)}$ and $x = (x^1,..., x^I)$ will denote a typical point in this space.

Each firm, j, has a set of *alternative production possibilities* Y^j contained in $\mathbf{R}^{S+1}$ that is closed, convex, contains zero, and exhibits free disposal. A typical *production plan* for firm j is written $y^j = (y_0^j, y_1^j,..., y_S^j)$, with the convention that $y_0^j \leqslant 0$, $y_s^j \geqslant 0$ for all s.

A *joint production plan* will be written $y = (y^1,..., y^J) \in \mathbf{R}^{J(S+1)}$ where $y^j \in Y^j$ for all j; the set of all such plans is denoted Y.

Each individual, i, is *endowed* with a quantity of present commodities $\bar{x}_0^i$. We denote the *total quantity of these endowments* by $\bar{x}_0 = \sum_i \bar{x}_0^i$. It is possible to include endowments of future, contingent commodities in the model, but they will play no role in our analysis.

We will be presenting and discussing various concepts of feasibility, equilibrium, and optimality. Certainly, economic activities which are not

[2] See Green and Polemarchakis [5].

physically possible cannot be realized by any conceivable system or institution. Thus the broadest concept of feasibility is that of *technological feasibility*, which gives rise to the set of joint consumption plans given by

$$F^T = \left\{ x \in \mathbf{R}_+^{I(S+1)} \;\middle|\; \begin{array}{l} \exists y \in Y \text{ such that } \bar{x}_0 \geqslant \sum_i x_0^i - \sum_j y_0^j \text{ and} \\ \sum_i x_s^i \leqslant \sum_j y_s^j \text{ for all } s \end{array} \right\}.$$

That is, a set of consumption plans is feasible if it can be provided for through the productive system.

Now we will consider various sets of institutions through which the system attains points in F^T. First is that of having markets in all commodities, current and future with contingent deliveries. When these markets are perfectly competitive, this is known as the *Arrow–Debreu model.*

We denote by $p = (p_0, p_1, ..., p_s) \in \mathbf{R}^{S+1}$ a *price system* (for the Arrow–Debreu model). A nonnegative, $(I \times J)$ matrix, θ, such that $\sum_i \theta_{ij} = 1$ for all j is called an *ownership matrix.* The set of all ownership matrices is denoted Θ.

A joint consumption plan is feasible given the institution of all markets if there are prices, initial endowments, and an ownership matrix such that all individuals satisfy their budget constraints and the activities are technologically possible.

Formally, we denote the *feasible joint consumption plans for the institution of markets* by

$$F^M = \left\{ x \in \mathbf{R}_+^{I(S+1)} \;\middle|\; \begin{array}{l} \exists p \in \mathbf{R}^{S+1},\ \tilde{x}_0^i \in \mathbf{R}_+ \text{ for each } i \text{ with} \\ \sum_i \tilde{x}_0^i = \bar{x}_0,\ \theta \in \Theta \text{ and } y \in Y \text{ such that} \\ \bar{x}_0 \geqslant \sum_i x_0^i - \sum_j y_0^j,\ \sum_i x_s^i \leqslant \sum_j y_s^j, \text{ for all } s, \text{ and,} \\ \text{for each } i,\ p \cdot x^i \leqslant p_0 \tilde{x}_0^i + \sum_j \theta_{ij}(p \cdot y^j) \end{array} \right\}.$$

Clearly, $F^M \subseteq F^T$ since it requires that some additional institutional restrictions be satisfied. In fact, these sets are identical ($F^M = F^T$).

This can be seen as follows: Take $x \in F^T$. We must show that there exist $y \in Y$, $p \in \mathbf{R}^{S+1}$, $\tilde{x}_0^i \in \mathbf{R}_+$ for all i, $\theta \in \Theta$, satisfying the conditions in F^M for this x. Let $p = (1, ..., 1) \in \mathbf{R}^{S+1}$, and choose $y \in Y$ to be the same as that which was sufficient for x in the definition of F^T. Then the budget equations in the definition of F^M are

$$x_0^i + \sum_s x_s^i \leqslant \tilde{x}_0^i + \sum_j \theta_{ij} \left(\sum_s y_s^j + y_0^j \right) \qquad \text{for all } i.$$

We can solve the equation

$$\sum_s x_s^i = \sum_j \theta_{ij} \sum_s y_s^j \qquad \text{for} \quad \theta \in \Theta$$

since $\sum_i \sum_s x_s^i = \sum_j \sum_s y_s^j$. Then this θ can be substituted into the equation

$$\tilde{x}_0^i = x_0^i - \sum_j \theta_{ij} y_0^j,$$

and $\tilde{x}_0^i$ will be nonnegative.

It is easily seen that the $\{\tilde{x}_0^i\}$ and θ so constructed satisfy the budget constraints in the definition of F^M.

It may be that the existence of markets for all commodities is impractical, perhaps for reason of transaction costs or imperfect information. One is therefore led to consider other sets of institutions. A common and natural possibility is to replace markets for contingent futures contracts with markets for trading ownership shares in the firms.

This gives rise to the set of feasible allocations *for the institution of share trading* given by

$$F^S = \left\{ x \in \mathbf{R}^{I(S+1)} \;\middle|\; \begin{array}{l} \exists \theta \in \Theta,\ y \in Y \text{ such that } \bar{x}_0 \geqslant \sum_i x_0^i - \sum_j y_0^j \text{ and} \\ x_s^i \leqslant \sum_j \theta_{ij} y_s^j \text{ for all } s \end{array} \right\}.$$

Clearly, F^S is contained in F^T and therefore also in F^M. Further, if $J < S$ so that $\{y^j\}_{j=1,\dots,J}$ does not span $\mathbf{R}^{S+1}$, there will be allocations in F^T that cannot be realized by share trading. In the case in which $J \geqslant S$, these institutions will not have the same feasible set unless negative values for the θ_{ij} are allowed, corresponding to short sales, and further $\{y^j\}_{j=1,\dots,J}$ spans $\mathbf{R}^{S+1}$.

One may also envision a system in which the institutions are augmented to include the financing of inputs by firms through the sale of claims to uncontingent payment in the second period, which we shall refer to as bonds. Thus we can define the profits of each firm contingent on the occurrence of a state by the output it produces minus any claims on it due to bond holders. It is this profit which is divided according to the ownership shares.

We denote by $r \geqslant 0$ the ratio of sure future payments to current costs of a bond. We write the set of all possible *joint consumption plans given the institution that both stocks and bonds can be traded* by

$$F^{S+B} = \left\{ x \in \mathbf{R}^{I(S+1)} \;\middle|\; \begin{array}{l} \exists \theta \in \Theta,\, r \geqslant 0,\, y \in Y \quad \text{and} \quad \tilde{x}_0{}^i,\, i = 1,\ldots, I \\ \text{such that } \sum_i \tilde{x}_0{}^i = \bar{x}_0 \text{ and such that} \\ \bar{x}_0 \geqslant \sum_i x_0{}^i - \sum_j y_0{}^j \text{ and } x_s{}^i \leqslant \sum_j \theta_{ij}(y_s{}^j + r y_0{}^j) \\ + r(\tilde{x}_0{}^i - x_0{}^i), \text{ for all } i \text{ and } s \end{array} \right\}.$$

Alternatively, we may consider an institution with bonds and ownership shares, but in which ownership shares are not transferable. For a fixed ownership matrix θ, the corresponding set of *feasible joint consumption plans with bonds given* θ is given by

$$F_\theta{}^B = \left\{ x \in \mathbf{R}^{I(S+1)} \;\middle|\; \begin{array}{l} \exists r \geqslant 0,\, y \in Y \text{ and } \{\tilde{x}_0{}^i\} \text{ such that} \\ \sum_i \tilde{x}_0{}^i = \bar{x}_0 \text{ and such that } \bar{x}_0 \geqslant \sum_i x_0{}^i - \sum_j y_0{}^j \\ \text{and } x_s{}^i \leqslant \sum_j \theta_{ij}(y_s{}^j - r y_0{}^j) + r(\tilde{x}_0{}^i - x_0{}^i) \end{array} \right\}.$$

Clearly, $F_\theta{}^B \subseteq F^{S+B}$ and $\bigcup_{\theta \in \Theta} F^B = F^{S+B}$.

It is interesting to note that the Arrow–Debreu model has both types of efficiencies since its competitive equilibria are on the Pareto frontier of F^T. Thus, no alternative system could possibly be superior even if it were costless to operate given the technological constraints, and perfect competition is efficient given the assumptions of the model. This coincidence of efficiencies, we feel, has blurred the distinction between these phenomena. More general models require that they be treated separately, which is the undertaking that we have begun herein.

The first concept of efficiency takes into account the costs of operating within a system but does not depend on the costs of operating the system itself. The main result of the next section is that in the presence of nonconvex costs, which is a plausible circumstance, the efficient set of active institutions is smaller than the set of all possible institutions. That is, an equilibrium of an economy with a particular set of active institutions may be Pareto inferior to the equilibrium in an economy with a more restricted set of institutional possibilities. Section 4 is devoted to delineating the circumstances in which such a counterintuitive inefficiency may arise. It is demonstrated that necessary conditions for this model to exhibit the inefficiency are either nonconvexity of transaction costs or ownership patterns in which the holdings of firms are highly concentrated.

3. A Theorem on Institutional Efficiency

As mentioned in Section 2, the question of superiority of institutions depends on the behavior of individuals—that is to say, on the concept of equilibrium that defines consistent individual actions.

An equilibrium of the economy for the institutions of stock and bond trading is a collection ($y^S \in Y$, $\theta^S \in \Theta$, $r^S \geqslant 0$, $b^S \in \mathbf{R}_+{}^1, p^S \in \mathbf{R}^J$), such that (i) for each i, y^S is optimal in Y given θ^S, r^S and p^S, (ii) for each i, $(\theta^S_{i1},..., \theta^S_{iJ}, b^{i^S})$ is optimal given y^S, r^S, and p^S over all $(\theta_{i1},..., \theta_{iJ}, b^i)$ satisfying

$$\bar{x}_0{}^i + p \cdot \bar{\theta}_i \geqslant b^i + p \cdot \theta_i + g^i(\Delta\theta_i),$$

and (iii) $\sum_j y^{j^S} + \sum_i b^{i^S} \geqslant 0$ and $\sum_i b^{i^S} + \sum_i g^i(\Delta\theta_i) \leqslant \sum_i \bar{x}_0{}^i$.

Similarly, an *equilibrium of the economy for the institution of bond trading* is a collection ($y^B \in Y, r^B \geqslant 0$) such that (i) for each i y^B is optimal in Y given $\bar{x}_0{}^i$ and r^B and (ii) $\sum_j y_0^{j^B} + \sum_i \bar{x}_0{}^i \geqslant 0$.

An ownership matrix θ is said to be *connected* if for any two individuals i and i' there is a sequence of individuals $i_0,..., i_k$ and a sequence of firms $j_0,..., j_{k-1}$ such that $i_0 = i$, $i_k = i'$, and $\theta_{i_n j_n} \cdot \theta_{i_{n+1} j_n} > 0$ for $n = 0,..., k-1$. That is, it is impossible to divide the individuals into two groups that collectively have no ownership claims on any firm in common.

We now prove the following.

Theorem. *Let g^i, the transaction costs function for the ith individual, be convex. Let $(y^S, \theta^S, r^S, b^S, p^S)$ be an equilibrium of the economy with share and bond trading such that θ^S is connected, giving rise to the consumptions x^{i^S}, $i = 1,..., I$. Let (y^B, r^B) be an equilibrium of the same economy with bond trading only, giving rise to the consumptions x^{i^B}, $i = 1,..., I$. Then $\{x^{i^S}\}$ is not Pareto dominated by $\{x^{i^B}\}$.*

Proof. Since θ^S is connected, there exists $\tilde{\rho} = (\rho_1,..., \rho_S)$ such that x^i preferred to x^{i^S} implies $\tilde{\rho} \cdot x^i > \tilde{\rho} \cdot x^{i^S}$ for all i and such that $\tilde{\rho} \cdot \pi^j$ is maximized over Y^j at y^{j^S} for all j, where $\pi_s{}^j = y_s{}^j + r^S y_0{}^j$ for $s = 1,..., S$. We can choose $\sum_s \rho_s = 1$. Thus, letting $\rho = (r^S, \rho_1,..., \rho_S)$, we have that $\rho \cdot y^j$ is maximized over Y^j at y^{j^S} and, therefore, that $\rho \cdot y$ is maximized over Y at y^S.

If $\{x^{i^B}\}$ Pareto dominates $\{x^{i^S}\}$, then, for each i,

$$\sum_{s=1}^{S} \rho_s x_s^{i^B} = \sum_{s=1}^{S} \rho_s \sum_j \bar{\theta}_{ij}(y_s^{j^B} + r^B y_0^{j^B}) + r^B \bar{x}_0{}^i$$

is greater than or equal to

$$\sum_{s=1}^{S} \rho_s x_s^{iS} = \sum_{s=1}^{S} \rho_s \sum_j \theta_{ij}^S (y_s^{jS} + r^S y_0^{jS}) + r^S b^{iS}$$

with strict inequality for at least one i.

Consider the individual opportunity set

$$\left\{ x^i \in \mathbf{R}^S \;\middle|\; \begin{array}{l} \exists \theta_i, b^i \text{ such that } x_s^{\,i} \leqslant \sum_j \theta_{ij}(y_s^{jS} + r^S y_0^{jS}) + r^S b^i \text{ and} \\ \bar{x}_0^{\,i} + p \cdot \bar{\theta}_i \geqslant b^i + p \cdot \theta_i + g^i(\Delta\theta_i) \end{array} \right\}.$$

Since g^i is convex, this set is convex and therefore, for each i,

$$\sum_{s=1}^{S} \rho_s x_s^{iS} \geqslant \sum_{s=1}^{S} \rho_s \left(\sum_j \bar{\theta}_{ij}(y_s^{jS} + r^S y_0^{jS}) + r^S \bar{x}_0^{\,i} \right).$$

Summing over i, using the earlier inequality and recalling $\sum_{s=1}^{S} \rho_s = 1$ and the feasibility conditions of equilibria, we have

$$\sum_{s=1}^{S} \rho_s \sum_j y_s^{jB} > \sum_{s=1}^{S} \rho_s \sum_j y_s^{jS} + r^S \left(\sum_j y_0^{j\,S} + \sum_i \bar{x}_0^{\,i} \right).$$

Subtracting $r^S \sum \bar{x}_0^{\,i} = -r^S \sum y_0^{jB}$ from both sides, we obtain

$$\sum_{s=1}^{S} \rho_s \sum_j y_s^{jB} + r^S \sum_j y_0^{jB} > \sum_{s=1}^{S} \rho_s \sum_j y_s^{jS} + r^S \sum_j y_0^{jS},$$

which contradicts the fact that $(\sum_j y_0^{jS}, \sum_{s=1}^{S} y_s^{jS}, s = 1,..., S)$ maximizes $\rho \cdot y$ over all $y \in Y$.

4. Examples of Institutional Inefficiency

This section reexamines the theorem above. We will present examples that show how institutional inefficiency is possible if either the convexity of transaction costs or the connectedness of the equilibrium ownership matrix is violated.

Consider an economy with two individuals (indexed by $i = 1, 2$) and two firms (indexed by $j = 1, 2$), in which there is a single commodity in each of two periods. In the second period, one of two possible states of nature will arise. There are therefore three economic commodities: goods in period one and goods contingently deliverable in either of the states in period two.

Let us suppose that, perhaps for reasons of transactions costs, there are no markets for contingent delivery in either of the possible events. There is, however, the possibility of trades in which period one commodities are exchanged for uncontingent repayment in period two. That is, these trades are constrained so that the quantities relating to the period two (contingent) commodities are equal.

Further, individuals may trade the shares of the firms. Owning shares in a firm entitles the individual to the profit made by the firm in period two in proportion to his ownership of shares. To the extent that the firm's profits vary over the states of nature, shares of the firm are risky. To the extent that different firms have different patterns of profit over the states of nature, trading in shares can substitute for trading in contingent contracts in that it will allow individuals to avoid risk.

The technologies of the firms are

$$T^1 = \left\{ (y_0^1, y_1^1, y_2^1) \,\middle|\, \begin{array}{l} y_0^1 = -z, y_1^1 = (1 + v)\lambda z, y_2^1 = (1 - \lambda) z, \\ \lambda \in [0, 1], z \geqslant 0 \end{array} \right\}$$

and

$$T^2 = \left\{ (y_0^2, y_1^2, y_2^2) \,\middle|\, \begin{array}{l} y_0^2 = -z, y_1^2 = \lambda z, y_2^2 = (1 + v)(1 - \lambda) z, \\ \lambda \in [0, 1], z \geqslant 0 \end{array} \right\},$$

where v is a (small) positive number. That is, the firms can choose any point in their technologies and execute this production plan by borrowing an amount z in the market for loans (repayable uncontingently) and investing. In the second period they repay this loan by returning rz, where r was determined in the first period equilibrium. Thus, their profits from following a production plan with parameters λ, z are $((1 + v)\lambda z - rz, (1 - \lambda)z - rz)$ in the two states, respectively; we denote this by (π_{11}, π_{12}), and similarly (π_{21}, π_{22}) will be the profits of firm 2.

Individuals have initial endownments of one unit of the commodity in period one, but they have no endowments of period two commodities. They have wealth given by the sum of the value of the shares they own plus this unit (period one commodities will serve as *numeraire*). Their tastes are assumed to be describable by a concave utility function for consumption in the two contingencies, which is symmetric in these arguments. The period 1 commodity is not desirable for consumption.

The shares of firm j have a price, p_j, in units of the period 1 commodity. The ith individual is assumed to own a share $\bar{\theta}_{ij}$ of firm j, initially. The essential phenomenon of this model is that the trading of shares is costly. To trade any number of shares requires the individual to use a quantity, c, of the period one commodity. The individual treats r, p_1, and p_2 as fixed.

Further, he treats the actions of all firms as given. An action for individual i is a quantity of lending b_i and share ownerships θ_{ij} such that the consumption plan

$$\left(rb_i + \sum_j \theta_{ij}\pi_{j1}\,,\, rb_i + \sum_j \theta_{ij}\pi_{j2}\right)$$

is optimal subject to the constraint

$$b_i + \sum_j \theta_{ij} p_j \leqslant 1 - \delta_i c + \sum_j \bar{\theta}_{ij} p_j\,,$$

where

$$\begin{aligned} \delta_i &= 1 \quad \text{if} \quad \theta_{ij} \neq \bar{\theta}_{ij} \quad \text{for some } j, \\ &= 0 \quad \text{if} \quad \theta_{ij} = \bar{\theta}_{ij} \quad \text{for all } j. \end{aligned}$$

Consider the particular case $\bar{\theta}_{ij} = 0$ for $i \neq j$ and $\bar{\theta}_{ii} = 1$. We will show that for suitable positive values of v and c the system which allows both share and bond trading is inferior for the given initial endowments to the more restricted possibility of trading only in bonds. That is, if we consider the same economy in which share trading is *prohibited a priori*, there exists an *equilibrium* of this economy which is preferred by all individuals to an equilibrium of the economy as defined above.

Consider the actions, plans, and prices given by $r = (1 + v)/2$, $p_1 = 0$, $p_2 = 0$, $\theta_{ij} = \frac{1}{2}$ for all i and j, $z_j = 1 - c$ for all j, $b_i = 1 - c$ for all i, and $\lambda_1 = 1$, $\lambda_2 = 0$.

We will now show that this is an equilibrium at least for some values of v and c and some preferences. This action gives rise to consumption of $[(1 + v)/2](1 - c)$ in the two states for both individuals. First, note that (iii) in the definition of equilibrium for this institution given on p. 7 is obviously satisfied. Condition (i) (p. 7) is satisfied because aggregate profit is zero and it is clear that no share ownership other than equal ownership of both firms will provide zero profits in both states. Similarly, condition (ii) (p. 7) is satisfied since no variation in production patterns could provide any better than zero profit. Thus it remains only to check that this action is better than not participating in stock trading at all; i.e., $b_i = 1$, $\theta_{ii} = 1$, $\theta_{ij} = 0$, $i \neq j$.

For individual 1 we can easily compute that this action gives rise to the consumption

$$\left(1 + v - \frac{(1 + v)c}{2}\,,\, \frac{c(1 + v)}{2}\right).$$

We depict these two allocations in Fig. 1, where the lines represent points of equal total consumption in the two states. We will have $c(1 + v)/2 <$

$(1 + v)(1 - c)/2$ if $c < \frac{1}{2}$, so that the point on the higher line has a lower ordinate.

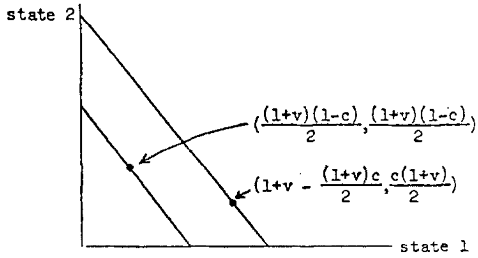

FIGURE 1

Let us consider the allocation that would arise in an equilibrium of this economy in which it is prohibited to trade in shares. It is clear that r is immaterial and that everyone operates his own firm to maximize his satisfaction. That is, $z_j = 1, b_i = 1$, and λ is chosen so that the production point is one at which the marginal rate of substitution in consumption is $1 + v$. Let this point be called A. As $v \to 0$, $A \to ((1 + v)/2, \frac{1}{2})$ (see Fig. 2).

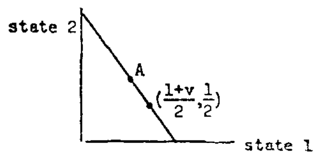

FIGURE 2

Thus we want to construct an example in which $((1 + v)/2, \frac{1}{2})$ and, therefore, A are preferred to $((1 + v)(1 - c)/2, (1 + v)(1 - c)/2)$. The total consumption in the two states is $(2 + v)/2$ and $(1 + v)(1 - c)$, respectively. Therefore, we must have that $(2 + v)/2 > (1 + v)(1 - c)$ or else the no-stock-market equilibrium will be inferior, since $((1 + v)(1 - c)/2, (1 + v)(1 - c)/2)$ is optimal among all consumptions with total consumption $(1 + v)(1 - c)$ by our assumption of symmetric trades. Thus we must have $c > v/2(1 + v)$.

To construct such an example, therefore, let v be very small and c be between $\frac{1}{2}$ and $v/2(1 + v)$. Then we will have the situation depicted in Fig. 3. It is clear that there exist preferences such as those shown below which lead to the indicated paradox.

In a model without transaction costs, Drèze [2] has discovered an inefficiency related to the one above. (His equilibrium concept, however,

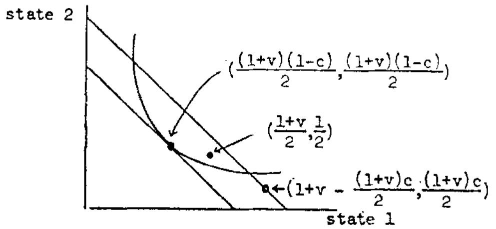

FIGURE 3

is not the same as ours.) A modified version of his example that fits into our context follows.

As before, there are two individuals, two firms, and two states. Technological possibilities are given by

$$T^1 = \left\{ (y_0^1, y_1^1, y_2^1) \in \mathbf{R}^3 \;\middle|\; \begin{array}{l} y_0^1 = -z,\ y_1^1 = 2\lambda z,\ y_2^1 = (1-\lambda) z, \text{ where} \\ z \geqslant 0,\ 0 \leqslant \lambda \leqslant 1 \end{array} \right\}$$

and

$$T^2 = \left\{ (y_0^2, y_1^2, y_2^2) \in \mathbf{R}^3 \;\middle|\; \begin{array}{l} y_0^2 = -z,\ y_1^2 = \lambda z,\ y_2^2 = 2(1-\lambda) z, \text{ where} \\ z \geqslant 0,\ 0 \leqslant \lambda \leqslant 1 \end{array} \right\}$$

with $\bar{x}_0^i = 1$, $i = 1, 2$, $\bar{\theta}_{11} = \bar{\theta}_{22} = 1$, $\bar{\theta}_{12} = \bar{\theta}_{21} = 0$. Consider the feasible production plan given by

$$y^1 = (-1, 0, 1) \qquad \text{and} \qquad y^2 = (-1, 1, 0)$$

with $\theta_{ij} = 1$, $i \neq j$, $\theta_{ii} = 0$, $i = 1, 2$, $r = \frac{1}{2}$, $p = 0$, $b^i = 1$, $i = 1, 2$. The resulting situation may be depicted by Fig. 4.

If preferences of the two individuals are given by indifference curves I^1 and I^2 as shown in Fig. 4, it can easily be verified that this allocation is an equilibrium according to the definition of Section 3. However, the allocation

$$y^{1'} = (-1, 2, 0) \qquad \text{and} \qquad y^{2'} = (-1, 0, 2)$$

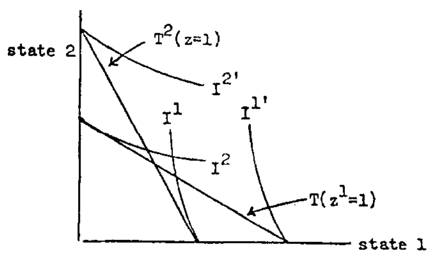

FIGURE 4

with $\theta' = \bar{\theta}$, $b^{i\prime} = 1$, $i = 1, 2$, $p' = 0$, $r' = 1$ allows the individuals to reach $I^{1\prime}$, $I^{2\prime}$, and it is also an equilibrium. Furthermore, it is also an equilibrium of the system with bonds only.

Although the first example in this section would not have exhibited inefficiency if transaction costs were zero, this example shows that these costs do not lie at the heart of the issue. However, in comparing these examples one notices that the extreme pattern of ownership in the last one made the equilibrium production plans very inefficient. Furthermore, they would be drastically altered by slight changes in the equilibrium shares. This was not the case in the first example, where production was efficient and ownership was equally distributed.

5. An Alternative Definition of Equilibrium

A change in the concepts of rationality and, hence, of equilibrium would alter the relative efficiency of alternative sets of institutions. Another concept of equilibrium has been proposed by Ekern and Wilson [3]. In the model with trade in ownership shares and bonds, their concept would correspond to the following definition.

An *alternative definition of equilibrium for the institution of share and bond trading* is a combination of actions $((\theta_{i1}, \ldots, \theta_{iJ}), b^i)$ for each $i = 1, \ldots, I$, a combination of production plans $y^j \in Y^j$ for each $j = 1, \ldots, J$, and $(r, p^1, \ldots, p^J) \in \mathbf{R}^{S+1}$ such that (i) $\max u^i(\sum_j \theta'_{ij}(y_s^{j\prime} + ry_0^{j\prime}) + rb^{i\prime}$, $s = 1, \ldots, S)$ is attained at $((\theta_{i1}, \ldots, \theta_{iJ}), (y^1, \ldots, y^J), b^i)$ over all $((\theta'_{i1}, \ldots, \theta'_{iJ}), (y^{1\prime}, \ldots, y^{J\prime}), b^{i\prime})$ satisfying

$$\sum_i \theta'_{ij} p^j + b^{i\prime} + g(\Delta\theta_i{}') \leqslant \sum_j \bar{\theta}_{ij} p^j + \bar{x}_0{}^i \quad \text{for } i = 1, \ldots, I,$$

and $y^j \in Y^j$ for $j = 1, \ldots, J$, where $\Delta\theta_i{}' = (\theta'_{i1}, \ldots, \theta'_{iJ}) - (\bar{\theta}_{i1}, \ldots, \bar{\theta}_{iJ})$, and

(ii)

$$\sum_i \theta_{ij} = 1 \quad \text{for all } j, \qquad \sum_i g(\Delta\theta_i) - \sum y_0{}^j \leqslant \sum \bar{x}_0{}^i, \qquad \sum_i b^i = -\sum_j y_0{}^j.$$

This broadens the concept of rationality to include the thought experiment of simultaneous variations in ownership *and* production plans for all individuals.

We can show that the inefficiencies demonstrated above cannot arise for any $g(\cdot)$ if this definition of equilibrium were adopted.

An *alternative definition of equilibrium for the institution of bond trading* is defined by a combination of production plans $y^j \in Y^j$ for each j and $r \in \mathbf{R}$

such that (i) $\max u^i(\sum \bar{\theta}_{ij}(y_s^{j'} + ry_0^{j'}) + r\bar{x}_0^{i'}, s = 1,..., S)$ is attained at y^j over all $y^{j'} \in Y^j$ for each individual and

(ii)

$$\sum_i \bar{x}_0{}^i + \sum_j y_0{}^j = 0.$$

THEOREM. *Let $\theta^S \in \Theta$, $y^S \in Y$, b^{i^S}, $i = 1,..., I$, $(p^S, r^S) \in \mathbf{R}^{J+1}$ be an equilibrium for the institution of share and bond trading and let $y^B \in Y$, $r^B \in \mathbf{R}$ be an equilibrium for the institution of bond trading. Then the consumption pattern arising in the latter equilibrium is not Pareto superior to the consumption pattern in the former equilibrium.*

Proof. In the economy with share trading, $\theta = \bar{\theta}$, $y = y^B$, $b^i = \bar{x}_0{}^i$ for all i is a feasible alternative for each individual since $g(0) = 0$ and $y^B \in Y$. Thus it suffices to show that the consumption plans attainable through these actions are not dominated by those arising in the equilibrium with only bond trading. For a typical individual i, his consumption with these alternative actions will be

$$\sum_j \bar{\theta}_{ij}(y_s^{jB} + r^S y_0^{jB}) + r^S \bar{x}_0{}^i = \sum \bar{\theta}_{ij} y_s^{jB} + r^S \left(\sum_j \bar{\theta}_{ij} y_0^{jB} + \bar{x}_0{}^i\right)$$

in each state s.

In the bond market equilibrium, the consumptions are

$$\sum_j \bar{\theta}_{ij}(y_s^{jB} + r^B y_s^{jB}) + r^B \bar{x}_0{}^i = \sum_j \bar{\theta}_{ij} y_s^{jB} + r^B \left(\sum_j \bar{\theta}_{ij} y_0^{jB} + \bar{x}_0{}^i\right).$$

Note that the second term in each of these expressions is independent of the state s and that their sum over all individuals is zero. If $r^S > r^B$, those individuals for whom $\sum_j (\bar{\theta}_{ij} y_0^{jB} + \bar{x}_0{}^i)$ is positive will have higher consumptions in all states under the stock and bond trading institution. If $r^B > r^S$, the same conclusion follows for the other individuals. If $r^B = r^S$ or $\sum_j (\bar{\theta}_{ij} y_0^{jB} + \bar{x}_0{}^i) = 0$ for all i, these consumption patterns are the same. Hence, under no circumstances can the bond market equilibrium be Pareto superior to the one with stock and bond trading.

6. CONCLUSIONS

The main results of this paper may be restated as follows. We compare the efficiency of alternative sets of active markets in a situation in which a full set of contingent commodity markets is not available. Presumably, prohibitive transaction costs account for the absence of these markets. It is therefore natural to suppose that such costs are present in other, potentially active markets. Thus the focus of our study is, in a sense, a second-best question. Is it best to have the maximal number of active markets, or it might be better to artificially restrict the class of available markets when the full set is not available? We prove that, if transaction costs are convex, the maximal number is best and, if nonconvexities are present, second-best policies may involve further restrictions.

The analysis was carried out using a model with share trading and uncontingent borrowing and lending. Extending our results to more general situations remains an open question. Our fragmentary findings suggest the possibility of a more systematic study of efficient institutional environments for decentralized resource allocation mechanisms.

References

1. P. A. Diamond, The role of the stock market in a general equilibrium model with technological uncertainty, *Amer. Econ. Rev.* **57** (1967), 759–776.
2. J. H. Drèze, Investment under private ownership: optimality, equilibrium and stability, Discussion Paper No. 7224, Center for Operations Research and Econometrics, Louvain, 1972.
3. S. Ekern and R. Wilson, On the theory of the firm in an economy with incomplete markets, Working Paper No. 19, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1973.
4. D. K. Foley, Economic equilibrium with costly marketing, *J. Econ. Theory* **2** (1970), 276–291.
5. J. Green and H. Polemarchakis, A brief note on the efficiency of equilibria with costly transactions, unpublished.
6. F. H. Hahn, Equilibrium with transaction cost, *Econometrica* **39** (1971), 417–439.
7. W. P. Heller and R. Starr, Equilibrium in a monetary economy with non convex transaction costs, *Rev. Econ. Studies*, forthcoming.
8. M. Kurz, Equilibrium with transaction cost and money in a single-market exchange economy, Technical Report No. 51, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1972; *Econometrica*, forthcoming.
9. H. Leland, Stockholder's interests, value maximization with optimality of stock markets, *Bell J. Econ. Management Sci.*, forthcoming.
10. J. Mossin, "Theory of Financial Markets," Prentice-Hall, Englewood Cliffs, NJ 1973.
11. J. Siglitz, On the optimality of the stock market allocation of investment, *Quart. J. Econ.* **86** (1972), 25–60.