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Problem Definition: Farmers face several challenges in agricultural supply chains in emerging economies that contribute to extreme levels of poverty. One common challenge is that farmers only have access to one channel, often an auction, for which to sell their crops. Recently, e-intermediaries have emerged as alternate, technology-driven posted-price channels. We aim to develop insights into the structural drivers of farmer and supply chain profitability in emerging markets and understand the impact of e-intermediaries.
Academic / Practical Relevance: In practice, much attention has been given to e-intermediaries and they have often been touted as for-profit social enterprises that improve farmers' welfare. Yet, studies in the operations literature that systematically analyze the impact of e-intermediaries are lacking. Our work fills this gap and answers practical questions regarding the responsible operations of e-intermediaries.
Methodology: We develop an analytical model of a supply chain that allows us to study several key features of intermediated supply chains. We complement the model's insights with observations from a numerical study.
Results: In the absence of an e-intermediary, auctions cause farmers to either overproduce or underproduce compared to their ideal production levels in a vertically integrated chain. The presence of an e-intermediary with limited market share improves farmers' profits; however, if the e-intermediary grows too large, it negatively impacts both farmers' and supply chain profits. Finally, as the number of farmers increases, farmers' profits approach zero, *irrespective of the e-intermediary's presence.*
Managerial Implications: Our results provide a balanced perspective on the value of e-intermediation, compared to the generally positive views advanced by case studies. For-profit e-intermediaries that also aim to improve farmers' livelihoods cannot blindly operate as pure profit-maximizers, assuming that market forces alone will ensure that farmers benefit. Even when e-intermediation benefits farmers, it is insufficient to mitigate the negative effects of supply fragmentation, suggesting that for farmers, market power is more important than market access.
Keywords: Developing countries, agricultural supply chains, intermediation, multiple channels, Walrasian auction

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1. Introduction

Approximately two billion people live on 475 million small farms in developing countries, where each farm is no more than two hectares and is typically family operated (Rapsomanikis 2015). These small farmers have a large footprint on a global scale: Studies have estimated that in aggregate, these farmers account for approximately one third of the total food supply in the world (Wegner and Zwart 2011). However, these farmers are often very poor, in many cases earning less than 1,000 USD per year; for example, each small cashew farm in Africa earns an average of only 500 USD per year (Graham et al. 2012). Farmers in such settings typically face several common challenges that make it difficult for them to improve their profits. The first challenge arises from the timing of the farmers' decisions. It is common for farmers to decide upon production quantities and harvest their crops before knowing how much their crops can be sold for. This price uncertainty makes it difficult for farmers to make optimal production and harvesting decisions. Second, farmers often have little to no choice of where to sell their crop: some farmers only sell to traders who stop by their farm gates, others might only sell through their local marketplace. Although these marketplaces are usually run as auctions, evidence suggests that cheating is a common practice (see, e.g., Upton and Fuller (2004)) that further reduces how much farmers can earn for their crop. Interestingly, in a survey of farmers in Nigeria, farmers cite price fluctuations (coupled with the lack of price transparency) and market failures as the top two risks that they face, and rank these factors even above weather-related risk factors (Salimonu and Falusi (2009)).

The dire plight and vast numbers of these struggling rural farmers has gained the attention of private charitable foundations and governments across the globe, and many have funded initiatives to improve the income of farmers, often through social enterprises that leverage digital or mobile technology. A recent survey conducted by researchers at the World Bank (Qiang et al. 2012) identified "providing better market links and distribution networks" (p. vi) as one of the key categories of functions provided by such initiatives. The implementation of this function varies with the specific context: some implementations provide matches between buyers and sellers, as well as support on logistics and order fulfillment, while others purchase from farmers and resell to buyers directly. What is common among these implementations is that they provide a new, digitally-enabled channel through which farmers can sell their product to buyers at known, posted prices. For brevity, throughout this paper, we refer to the entity that operates this new sales channel as an *e-intermediary*, where the "e-" prefix conveys the reliance on digital technology, and the term "intermediary" is used to highlight the role played by these entities in the agricultural supply chain. By providing price transparency and by offering an alternate sales channel for the farmers, at least in principle, e-intermediaries alleviate some of the key challenges faced by farmers in traditional agricultural supply chains.

One of the most well-known examples of an e-intermediary is ITC's e-Choupal initiative in the soybean supply chain in India, which has been the subject of several studies (e.g., Upton and Fuller 2004, Anupindi and Sivakumar 2007, Goyal 2010, Devalkar et al. 2011). This e-intermediary provides a computer kit to each farming village, which gives farmers in the village access to a variety of information: ITC's daily crop prices, weather forecasts, farming best practices, government news related to farming, and a Q & A section. Another example of an e-intermediary is a company called EasyFlower (www.easyflower.com) in the flower supply chain in China. This e-intermediary has developed a mobile application that announces daily flower prices to the farmers. Our work was initially motivated by several discussions with EasyFlower's leadership team. Both ITC e-Choupal and EasyFlower give farmers the additional option to sell their crops to the e-intermediary at a fixed price known in advance, as opposed to the farmers being restricted to selling their crops at local auctions. These and other examples of e-intermediaries are listed in Table 1. We note that all of the examples displayed are private, for-profit organizations except for ECX, which is a public-private partnership.

Despite the emergence of these and other similar e-intermediaries in developing economies and some preliminary accounts of positive impact (see, e.g., Upton and Fuller 2004, Qiang et al. 2012), we are not aware of any studies in the operations and supply chain literature that systematically analyze the role and impact of e-intermediaries in developing markets. We argue that this gap in the literature warrants scholarly attention for at least two reasons. First, from an academic perspective, the study of supply chain issues in developing economies is currently a nascent field with limited, context-specific results. Assumptions that underlie the models, theories, and insights that have been established for supply chains in developed markets may fail to hold in developing countries. It is therefore necessary to tailor such models to developing economies in order to obtain insights that are relevant to the issues faced in such settings. As an example, although the literature on supply chain contracting is large and well-developed (e.g., Taylor 2002, Cachon and Lariviere 2005), contracts are often unenforceable in developing countries due to weak regulatory environments and would not be a viable solution to help farmers improve their livelihood.

Second, from an applied perspective, because interventions in a supply chain may have unintended consequences on other stakeholders in the chain, well-intentioned interventions such as e-intermediation could backfire and lead to poor outcomes. Moreover, reports of social impacts on farmers from e-intermediaries (such as those listed in Table 1) are usually anecdotal and typically self-reported. Therefore, a rigorous study of how e-intermediaries work, as well as understanding the extent to which farmers and supply chains benefit from their presence, can serve to guide charitable foundations and governments in their decisions about whether and how to direct scarce resources to support e-intermediation efforts.

In this paper, we have two primary objectives. First, we aim to develop insights into the structural drivers of farmer and supply chain profitability in emerging market supply chains. Second, we aim to develop an understanding of the impact of an e-intermediary on the profitability of farmers and supply chains. To fulfill these objectives, we develop a model of a supply chain, described in

Table 1: Examples of E-Intermediaries

E-Intermediary	**Country**	**Crop**	**Price Communication**	**Operations**	**Reach**	**Reported Impact**
ITC e-Choupal	India	Soybean & other agricultural products	Internet kiosk	Network of warehouses and collection points. Farmer is reimbursed for travel.	Present in 10 states, 40,000 villages, and covers 4 million farmers.[a]	Estimated 33% increase in farmers' revenue.[b]
EasyFlower	China	Fresh flowers	Mobile app	Network of warehouses and collection points	Present in the Yunnan province in southern China, and covers approx. 10,000 farmers. Flowers are distributed to retail outlets in approx. 40 cities in three major metropolitan regions.[c]	Estimated 200 million yuan increase in aggregate farmers' revenue.[c]
KACE[d]	Kenya	Multiple agricultural products	Mobile SMS, radio program with live auction	Network of Market Resource Centers (MRCs) that provide supply chain services (transportation, warehousing, quality control).	10 MRCs in operation in four provinces. Approximate reach of 1 million farmers and 250,000 small-scale traders.[e]	A case study reported 22% higher prices for maize from using KACE.[f]
ECX[g]	Ethiopia	Multiple agricultural products	Mobile SMS	Network of warehouses that provide storage, quality control, and payment fulfillment services.	Reaches approx. 2.4 million small-scale farmers.[h]	Farmers received 70% of final price of coffee, up from 38%.[h]
TradeGhana.co	Ghana	Maize	Mobile SMS	Order placement via SMS or web. Partner with logistics providers for delivery of crops directly to buyers.[i]	Current Grand Challenge Explorations Grant from the Bill and Melinda Gates Foundation.[i]	

[a] http://www.itcportal.com/sustainability/embedding-sustainability-in-business.aspx
[b] Goyal (2010)
[c] https://www.easyflower.com/officialStory/index.html
[d] KACE: Kenya Agricultural Commodity Exchange
[e] Karugu (2010)
[f] Mukhebi (2004)
[g] ECX: Ethiopia Commodity Exchange
[h] Gabre-Madhin (2012)
[i] http://www.tradeghana.co

§2, which captures several key features of traditional agricultural supply chains that operate in developing economies. We analyze the model with and without an e-intermediary in §3 and §4, respectively, and complement the insights gained with observations from a numerical study in §5. All proofs and supporting lemmas are in the electronic companion.

One advantage of using a modeling approach is that it allows us to isolate effects of interest that can be difficult to disentangle using other research approaches, e.g., empirical methods or case studies. As is with ITC's e-Choupal initiative, many real-world e-intermediaries not only provide an alternate sales channel, but also provide additional useful information (e.g., weather forecasts) that could help farmers make better decisions. Case studies which point to the positive impact of e-intermediaries may be simply picking up improvements stemming from better information, rather than providing direct evidence that a new e-intermediated channel is beneficial to farmers. In our model, we abstract away from such information asymmetry issues and focus on what we think is the more fundamental issue of understanding the impact of the new e-intermediated channel itself.

Our key findings are summarized below:

1. Auction mechanisms that intermediate traditional supply chains introduce sub-optimality to the overall supply chain, in that they cause farmers to either overproduce or underproduce compared to their ideal production levels had they been vertically integrated with the buyers.
2. The presence of an e-intermediary with limited market share tends to improve farmers' profits compared to the traditional supply chain. However, if the e-intermediary grows too large, it has a negative impact on both farmers' and total supply chain profits.
3. As the number of farmers increases, the total profits of all farmers converge to zero in the limit, *irrespective of the e-intermediary's presence*. Thus, a more impactful way to improve the livelihood of farmers in developing economies is to consolidate farmers into larger farming collectives to improve their market power.

1.1. Related Literature

We structure our review of related work into three groups that progressively broaden in terms of their contextual similarity to our work. The first group comprises case studies of e-intermediaries and empirical evaluations of specific e-intermediaries' impact on farmers. The second group comprises modeling papers that fall in the broader context of agricultural supply chains in developing economies. The third group comprises papers that may be in a different context from our work, but share methodological similarities.

The first group of papers are case studies and empirical studies that focus specifically on e-intermediaries operating in developing countries. As summarized in Table 1, there are numerous examples of case studies detailing such e-intermediaries across the world. More in-depth case studies of e-intermediaries can be found for ITC e-Choupal (Upton and Fuller 2004), ECX (Alemu and Meijerink 2010), and KACE (Mukhebi and Kundu 2014). While these case studies offer rich descriptions about how these e-intermediaries operate, we caution they are also not typically peer-reviewed. In particular, reports of positive impact on farmers from these case-studies (some of which are summarized in Table 1) are usually obtained from the e-intermediaries themselves, who have vested interests in what information they present. Independent, academic evaluations of the impact of e-intermediaries on farmers are less prevalent, and we are only aware of two such studies, with mixed findings. Goyal (2010) empirically studies the impact of ITC e-Choupal when it was first introduced as an alternate channel to the traditional auction channel that had operated for decades. Specifically, Goyal (2010) measures the e-intermediary's impact on the auction price, farmers' production quantity, and farmers' revenue, and finds that the introduction of the e-intermediary was beneficial to farmers. In contrast, a recent study by Hernandez et al. (2017) studies the impact of the ECX e-intermediary on coffee prices in Ethiopia; they find that the introduction of ECX did not have a substantial impact on coffee prices and farmers' profits. A result in our paper could offer a potential explanation to these divergent empirical findings. We find that the e-intermediary can be beneficial to farmers when its market share is small - as was likely the case when ITC e-Choupal was first introduced. However, we also find that the reverse occurs when the e-intermediary grows large, which reflects the setting studied by Hernandez et al. (2017).

The second group of papers share a similar context with our paper, namely, that of agricultural supply chains in developing economies. The paper by Devalkar et al. (2011) is motivated by ITC's e-Choupal operations, and employs a stochastic dynamic program to characterize the firm's optimal operating decisions in a multi-period setting, specifically, the optimal quantities that the firm should procure, process, and sell in each period. Whereas this paper focuses on the operations of a single firm over multiple periods, our paper focuses on the joint decisions of multiple players in the supply chain (e-intermediary and farmers) in order to study the impact that a two-channel setting has on farmers, but we do not consider a multi-period model for tractability.

Two recent papers use models to study the impact of government interventions on agricultural supply chains. Akkaya et al. (2016) study three such interventions: price support (i.e. instituting a minimum selling price per unit), cost support (i.e. subsidizing input cost), and yield enhancement (i.e. educating farmers on best practices). They find that in some cases, price support benefits consumers more than farmers. Alizamir et al. (2015) study two potential interventions: price support and revenue support (i.e. instituting a minimum revenue for each farmer). Their results are positive for price support: farmer and consumer welfare are typically improved, and the intervention cost to the government is less than that of revenue support. Our present work is distinct from these studies of government intervention because we study the impact of *for-profit intermediation.*

Several papers that study interactions between for-profit firms and farmers focus on the role and impact of supply contracts on various players. For example, both Hu et al. (2016) and Federgruen et al. (2015) consider a setting where a for-profit firm offers procurement contracts to a subset of farmers, and derive the profit-maximizing contract(s) to offer farmers. Hu et al. (2016) show that such a contract benefits all farmers, not just those who were offered the contract. Our paper differs from these works because we consider price-based intermediated procurement (i.e., without contracts). As we argued in the introduction and also noted by Federgruen et al. (2015), contracting approaches, which are common in developed economies, may not be feasible in developing countries when the regulatory environment is weak.

Chen and Tang (2015) also study a supply chain intervention – information provision – in the context of developing economies, but unlike the studies above who consider the objectives of the implementor of the intervention, their study is focused primarily on understanding the mechanisms through which farmers' welfare is impacted by the intervention. The authors are motivated by a growing effort by numerous organizations (including e-intermediaries like ITC e-Choupal) to provide rural farmers with access to better market and farming information, and use a signaling game to study the value of information on farmers' welfare. One of their main findings is that the provision of such "private signals" typically have a positive impact on farmers' welfare. A similar paper by Tang et al. (2015) studies whether farmers will act upon such information to increase their profits and find that farmers do so in equilibrium. Similar to these papers, our paper shares the same broad goal of understanding how farmers' livelihood can be improved. However, it differs because we focus on the role of a new supply chain channel as opposed to the role of information.

The third group of papers are those that share similar methodologies or modeling elements with our paper, even though they could be motivated by firms in different industries than ours. Caldentey and Vulcano (2007) study the relationship between an auction channel and a posted-price channel (the e-intermediary's pricing mechanism in our model) in the context of eBay. Consumers have the option to buy the product in either channel, and the authors study the consumers' optimal channel choice and how the firm should structure the auction as a function of the posted price. Although their model is similar to ours in that it studies a two-channel setting with similar pricing mechanisms, a key distinction between our work and theirs is that we consider the channels' interactions in a two-sided market with both suppliers and buyers, whereas they focus on the channels' interactions with only one side of the market (buyers). Our model is also tailored to study supply chain issues (including production decisions of farmers), whereas theirs is tailored to study purchases made in an online setting, featuring stochastically arriving customers.

Our paper also shares some modeling elements with that of Lee and Whang (2002), who consider the impact of a secondary market formed by a network of retailers that face stochastic demands over two periods. In the first period, retailers purchase products from a single manufacturer and face stochastic demand; in the second period, the same retailers form a secondary market to rebalance their inventories before again facing stochastic demand. Although Lee and Whang (2002) do not explicitly note this in their paper, their model of a secondary market is a Walrasian auction, which is also the basis of the auction mechanism used in this paper. However, there are several important key differences between the operations of the auctions in our papers. First, demand in their model is realized over two periods. In our model it is realized only in a single period. Second, in their model, retailers are both sellers and buyers at the auction, and are also the same players who face demand. In our auction model, the sellers (farmers) are distinct from the buyers (retailers), and it is only the latter who face demand. Third, in their model, the entities (retailers) who make procurement decisions in period 1 are also the participants in the auction in period 2; in contrast, the entities (farmers) in our model that make production decisions in period 1 are distinct from the buyers in the auction in period 2.

In summary, we have not found work in the literature on supply chain management that models the interaction of farmers and buyers through a combination of an auction-based sales channel and a posted-price intermediating channel that furthermore adds considerations that are relevant to issues faced in developing countries (e.g., supply fragmentation). Our paper aims to fill this gap in the literature, with an overarching objective of developing a deeper understanding of how to effect positive change in the welfare of farmers in these settings.

2. Model Description and Preliminary Analysis

We first present our model of the *traditional* supply chain (without an e-intermediary) in §2.1. Then we introduce the e-intermediary in §2.2 and outline our *e-intermediated* supply chain model. Before proceeding to our main results, in §2.3 we offer some preliminary analyses and summarize the notation that we will use throughout the rest of the paper.

2.1. Traditional Supply Chain Model

Our model of the traditional supply chain comprises two groups of players: N farmers (indexed by i) and M retailers (indexed by j). Although we refer to the second group as "retailers", depending on the context and product, these retailers could in fact be disjoint groups of retailers, wholesalers, food processors, or end-consumers; we simply use the word "retailer" for consistency. These retailers drive stochastic demand for the product.

At a high level, the model operates as follows: farmers first make investments to each produce some quantity of a homogenous crop. Farmers can then sell their crop to retailers through a market, which we model as a Walrasian auction (described in detail below), who in turn sell the product to their customers at a fixed price. We assume that all exogenous parameters to the model are common knowledge to all parties. At the end of §2, Table 2 summarizes each of these parameters and decision variables for both the traditional and e-intermediated supply chains for quick reference.

The following timeline describes the order of events in the traditional supply chain, summarized in Figure 1:

Period 1: Farmers decide upon their production quantity, q_i, and incur a growing cost of cq_i where c is the per unit production cost.

Period 2: Farmers choose the quantity of their product to send to the auction, $q_i^{\mathbf{a}} \leq q_i$, where the produce from all farmers is aggregated and distributed according to a Walrasian auction. In a Walrasian auction, each of the M retailers participate by declaring their demand functions (i.e., the quantity they are willing to purchase at any given price). The auctioneer clears the market by setting a single clearing price for the goods, $p^{\mathbf{a}}$, such that (i) the total supply matches the total demand, and (ii) the quantity received by each retailer matches what it demands at that price. We formally define the equilibrium outcome for a Walrasian auction in Definition 1 at the end of this subsection.

Period 3: (Stochastic) demand at each retailer is realized. Demand is independent across retailers, and for retailer j, is normally distributed as $N\left(\mu_j, \sigma_j^2\right)$. Sales prices are fixed and exogenous, set at π per unit. Each retailer either sells how much product it has on hand, or the realized demand that it faces, whichever is lower. Any leftover product is discarded at no value.

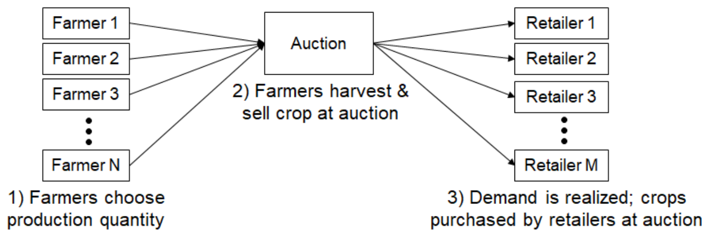

Figure 1 **Traditional Supply Chain Model Summary**

Periods 1 and 2 in our model can be mapped to distinct seasons in agricultural supply chains. Namely, period 1 represents the *growing season* and typically takes place over three to six months, depending on the crop. Period 2 represents the *harvest/selling season* which typically takes place over about one month. Period 3 is not intended to represent a distinct period of physical time (although this interpretation may be appropriate in certain settings), but is simply an abstraction that captures random demand. Along the lines of such an abstraction, we make the simplifying assumption that the sales price of crops to the end-customer is fixed (π per unit). Although we acknowledge that this is one of the limitations of our model, we think that it is not too far from reality. Consumer and wholesale prices of commodity agricultural products tend to be relatively stable over time with few exceptions (Gilbert and Morgan 2010). Even if these prices exhibit seasonal variations (e.g., fresh flowers, fruit), these variations are typically quite predictable.

It is worth noting that in reality, farmers grow and harvest different types of crops. For the purpose of our study, we focus on just one of those crops that they might produce and its corresponding supply chain structure; the per unit cost of the crop, c, can thus be interpreted as incorporating the opportunity cost of planting a different crop instead of the crop of interest in our study. Similarly, the farmer's choice of production quantity in our model translates to a decision of how many resources (e.g., land, labor, etc.) he chooses to allocate to this crop of interest relative to a different crop. Using this interpretation, our model of each farmer's decision is similar to what other authors have considered in recent work (e.g., Akkaya et al. 2016).

However, primarily for analytical tractability, our model abstracts from reality by assuming that there is no supply uncertainty (i.e., farmers can choose production quantities exactly), no capacity limits on production, and no fixed costs or other forms of economies of scale. We believe that these model elements would not fundamentally alter the direction of our key findings, but would add more notation and technicalities to the paper that would likely be distracting.

Moreover, in reality, the exact auction mechanism used for many agricultural products in emerging markets are variants on first-price auctions of individual units of products. Because there are invariably multiple units of products being sold at the auction, one option might be to directly analyze this as a sequence of first-price auctions. Unfortunately, this turns out to be analytically intractable. We use a Walrasian model as an approximation not only for the sake of tractability, but also because it is an idealized model of an exchange. The latter, in particular, removes inefficiencies due to a sub-optimal auction implementation and allows our analysis to focus on the impact that the supply chain's structure has on its overall profits and the profits accrued to the players in the chain. For an in-depth review of Walrasian auctions, we refer the reader to Joyce (1984). The following definition describes the equilibrium outcome for the Walrasian auction.

Definition 1. Let $Q^{\mathbf{a}} = \sum_i^N q_i^{\mathbf{a}}$ represent the total quantity available for distribution in a Walrasian auction with m retailers. Suppose that $y_j(p)$ represents the demand function submitted by retailer j. Then, the *equilibrium of the Walrasian auction* is $(p^{\mathbf{a}}, x_1, \ldots, x_m)$, where $p^{\mathbf{a}}$ represents the clearing price of the auction, and $(x_1, \ldots, x_m)$ represents the allocation of goods to retailers, such that (i) each retailer receives what it demands at price $p^{\mathbf{a}}$, i.e., $x_j = y_j(p^{\mathbf{a}})$, and (ii) the market clears with total supply matching total demand, i.e., $Q^{\mathbf{a}} = \sum_{j=1}^m x_j$.

2.2. E-Intermediated Supply Chain Model

Our model of the e-intermediated supply chain is similar to that of the traditional supply chain except in that it allows farmers to sell their crops through an alternate channel - the e-intermediary - as opposed to solely an auction channel. We interpret the e-intermediary as a third-party who has access to the retailers through the use of digital technology - either on a personal computer, mobile application, or Internet kiosk - and sells crops that it purchases from these farmers to some fraction of the retailers. On the supply side, the e-intermediary chooses a fixed price per unit, p^{e}, to procure the crop from farmers. Using fixed pricing as an operational lever to obtain supply is broadly consistent with what we have observed from the operations of existing e-intermediaries in emerging markets. On the demand side, the e-intermediary also sells the crop at the exogenous price per unit π, but captures only some portion of the overall retailer demand. Namely, $m \in \{1, \ldots, M\}$ retailers participate in the auction in the e-intermediated supply chain, and the e-intermediary serves the remaining retailers $m+1, \ldots, M$ or their end-customers by taking on their total aggregated demand. We assume that there are sufficient barriers to entry such that the e-intermediary does not have the option to participate in the auction, and the m retailers who do participate in the auction do not have the ability to become e-intermediaries.

There are several plausible mechanisms that motivate why this is an appropriate model, which we describe below. Our goal in having this discussion is not to argue in favor of one mechanism over the other, but rather to motivate that our abstract model has plausible practical roots. One plausible mechanism is that $M - m$ retailers decide to consolidate their sourcing operations for the crop, and acquire or invest in technology that allows them to purchase directly from farmers without going through the auction. A variant of this mechanism is that a government, charitable organization, or corporate entity could be a driving force behind the development of the technology, which is then successfully marketed to $M - m$ of the retailers. ITC e-Choupal is an example of an e-intermediary that fits this mechanism. Another plausible mechanism for this model is that the e-intermediary is able to supply end-customers at lower frictions (i.e., transaction costs) than the $M - m$ retailers that it displaces. For example, these retailers may serve customers within a certain geographic region and the e-intermediary might possess some geographic advantages (e.g., better relationships with local logistics providers), which enables it to provide a better quality of service to customers and displace the retailers in selling the crop. EasyFlower, KACE, ECX, and TradeGhana.co are examples of e-intermediaries that fit this mechanism. Nevertheless, in either mechanism, our abstract model is a reasonable representation of the key economic tension introduced by the e-intermediary, allowing us to study its impact on the supply chain. In particular, using our model, we can compare the impact of an e-intermediary in various stages of its growth, e.g., in its early growth phase when it captures a relatively small portion of the market ($M - m$ is small), or in more mature stages when it captures a larger fraction of the market ($M - m$ is large).

On the surface, our model appears to match the operations of procurement-based e-intermediaries (e.g., ITC e-Choupal, EasyFlower) more so than that of the e-intermediaries that operate as commodity exchanges (e.g., KACE, ECX, TradeGhana.co). However, we argue that this is in fact not the case. These commodity exchanges operate by charging a transaction fee on purchases made through their channel, which, analogous to procurement prices set by ITC e-Choupal or

EasyFlower, affects the quantity of crop that flows through the e-intermediary. Moreover, because these exchanges provide transportation and/or warehousing services, they effectively bear inventory risk as do ITC e-Choupal and EasyFlower. Therefore, even though the e-intermediary in our model is described as a firm who "purchases" crops from farmers, we argue that the model and ensuing insights are applicable for exchange-type e-intermediaries, who face similar tradeoffs.

The following timeline describes the order of events in the e-intermediated supply chain, summarized in Figure 2:

Period 1: As in the traditional supply chain, farmers decide upon their production quantity, q_i, and incur a growing cost of cq_i.

Period 2: The e-intermediary observes the total production quantity, $Q = \sum_{i=1}^{N} q_i$, and chooses a fixed price per unit, $p^{\mathbf{e}}$, to purchase crops from farmers. Farmers observe this price and decide how much of their produce to sell to the e-intermediary, $q_i^{\mathbf{e}}$, and how much to sell at the Walrasian auction, $q_i^{\mathbf{a}}$, which is run as before; clearly we must have $q_i^{\mathbf{e}} + q_i^{\mathbf{a}} \leq q_i$. Although farmers do not observe $p^{\mathbf{e}}$ in period 1, they rationally infer its value to inform their production quantity decision.

Period 3: (Stochastic) demand at each retailer and the e-intermediary are realized. Demand is independent across retailers, and for retailer $j = 1, ..., m$, is normally distributed as $N\left(\mu_j, \sigma_j^2\right)$, just as in the traditional supply chain. The demand at the e-intermediary is pooled across the $M - m$ retailers that it represents/displaced and is normally distributed, i.e. $F_e \sim N\left(\sum_{j=m+1}^{M} \mu_j, \sum_{j=m+1}^{M} \sigma_j^2\right)$. As in the traditional supply chain, sales prices are fixed and exogenous at π per unit, both for the e-intermediary and all retailers participating in the auction. Each retailer and e-intermediary either sells how much product it has on hand, or the realized demand that it faces, whichever is lower; any leftover product is discarded at no value.

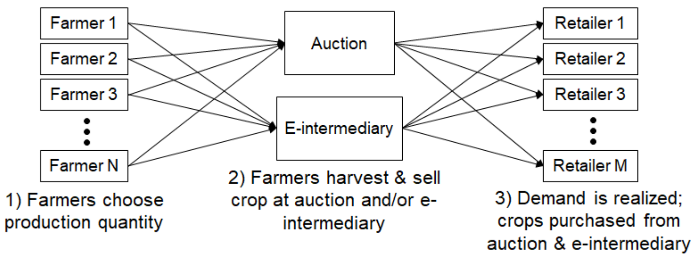

Figure 2 **E-Intermediated Supply Chain Model Summary**

Note that the traditional supply chain can be viewed as a special case of the e-intermediated supply chain where $m = M$.

2.3. Preliminary Analysis

Here, we present some preliminary analyses to establish foundational results used throughout the paper.

In the final period, each retailer participating in the auction faces a newsvendor problem, where stochastic demand for the product, which is sold at price π, is fulfilled with inventory purchased at price $p^{\mathbf{a}}$. This allows us to derive retailer j's demand function, $y_j(p^{\mathbf{a}})$, using standard newsvendor analysis. Namely, for a fixed $p^{\mathbf{a}}$, retailer j's optimal inventory level as a function of $p^{\mathbf{a}}$ (equivalently, demand function), is

$$y_j(p^{\mathbf{a}}) = \mu_j + \Phi^{-1}\left(\frac{\pi - p^{\mathbf{a}}}{\pi}\right)\sigma_j, \tag{1}$$

where Φ^{-1} is the inverse cdf of the standard normal distribution.

In the following lemma, we use Definition 1 and the retailers' demand functions from (1) to formally describe the equilibrium outcome of the auction. We are particularly interested in characterizing how the (equilibrium) auction price, $p^{\mathbf{a}}$, varies with the total quantity for sale at the auction, $Q^{\mathbf{a}}$. For notational consistency, we will denote the number of retailers participating in the auction as $m \in \{1, \ldots, M\}$. In the context of the traditional supply chain, we only consider $m = M$ without loss of generality; in the context of the e-intermediated supply chain, we can have the more general case where $m \leq M$.

Lemma 1. *Let $m \in \{1, \ldots, M\}$ represent the total number of retailers participating in the auction (indexed by $j = 1, ..., m$), and let $\bar{F}_{\mathbf{a}}(\cdot)$ represent the tail cdf of a $N(\sum_{j=1}^m \mu_j, (\sum_{j=1}^m \sigma_j)^2)$ random variable. Letting $Q^{\mathbf{a}}$ represent the total quantity for sale at the auction, the equilibrium auction price, $p^{\mathbf{a}}$, is given by $p^{\mathbf{a}} = \pi \bar{F}_{\mathbf{a}}(Q^{\mathbf{a}})$.*

Notice that $N(\sum_{j=1}^m \mu_j, (\sum_{j=1}^m \sigma_j)^2)$ is not the total demand distribution faced by the auction. Rather, $N(\sum_{j=1}^m \mu_j, \sum_{j=1}^m \sigma_j^2)$ is the total demand distribution faced by the auction, which has significantly smaller variance. This observation will be helpful in understanding some of our key results. Our subsequent analysis will also leverage the following technical lemma.

Lemma 2. *For any $\mu, \sigma \in \mathbb{R}$ such that $\sigma > 0$, let f and $\bar{F}(\cdot)$ represent, respectively, the density and tail distribution of a $N(\mu, \sigma^2)$ random variable. Let $s \in \mathbb{R}$, $\nu > 0$ be given and consider the function $L_{s,\nu} : \mathbb{R} \to \mathbb{R}$, defined as*

$$L_{s,\nu}(z) := \bar{F}(z + s) - \nu z f(z + s).$$

Define $\mathcal{L}_{s,\nu} := \{z \in \mathbb{R}_+ : L_{s,\nu}(z) \geq 0\}$. Then,

(a) For every $z \in \mathcal{L}_{s,\nu}$, we have $L'_{s,\nu}(z) < 0$.

(b) $\mathcal{L}_{s,\nu}$ may be represented as $\mathcal{L}_{s,\nu} = [0, z^*_{s,\nu}]$ for some $z^*_{s,\nu} \in \mathbb{R}_+$.

Notation	Type	Description
N	exogenous parameter	total number of farmers
M	exogenous parameter	total number of retailers
c	exogenous parameter	farmers' production cost per unit
π	exogenous parameter	retailers' selling price per unit
$F_j(\cdot) =$ cdf of $N(\mu_j, \sigma_j^2)$ random variable	exogenous parameter	demand distribution for retailer j
q_i	decision variable	farmer i's production quantity
Q	endogenous parameter	total production quantity
$q_i^{\mathbf{a}}$	decision variable	farmer i's quantity sold in auction
$Q^{\mathbf{a}}$	endogenous parameter	total quantity sold in auction
$p^{\mathbf{a}}$	endogenous parameter	auction clearing price
m	exogenous parameter	number of retailers participating in the auction
$F_{\mathbf{a}}(\cdot) =$ cdf of $N(\sum_{j=1}^{m} \mu_j, (\sum_{j=1}^{m} \sigma_j)^2)$ random variable	exogenous parameter	distribution prevalent in auction pricing; note implicit dependence on m
$F_{\mathbf{e}}(\cdot) =$ cdf of $N(\sum_{j=m+1}^{M} \mu_j, \sum_{j=m+1}^{M} \sigma_j^2)$ random variable	exogenous parameter	demand distribution for e-intermediary; note implicit dependence on m
$q_i^{\mathbf{e}}$	decision variable	farmer i's quantity sold to e-intermediary
$Q^{\mathbf{e}}$	endogenous parameter	total quantity sold to e-intermediary
$p^{\mathbf{e}}$	endogenous parameter	fixed price at e-intermediary

Table 2 Summary of Notation

Throughout the paper, we make the following assumption about the model parameters. This is a regularity condition that ensures that positive quantities will be produced in the absence of an e-intermediary and/or auction. This is similar in spirit to assumptions in many other papers that use normally distributed demands (see, e.g., Eppen (1979) and Cho and Wang (2017)).

ASSUMPTION 1. *Assume that* $c < \pi \min\left\{\bar{F}_{\mathbf{a}}(0), \bar{F}_{\mathbf{e}}(0)\right\}$.

To conclude this section, we provide a summary of the notation introduced thus far and used throughout the paper in Table 2. For quantities, we generally use lower case q when we refer to quantities from individual farmers, and upper case Q to refer to quantities that are aggregated across farmers.

3. Analysis of the Traditional Supply Chain

In order to study the impact of an e-intermediary on the agricultural supply chain, we first study the traditional supply chain where no e-intermediary exists; recall that the traditional supply chain can be considered as a special case of the e-intermediated supply chain with $m = M$. In §3.1, we characterize the optimal solution of the key decision variables in our model: each farmer's production quantity and the quantity the farmer chooses to sell in the auction. In §3.2, we pay particular attention to the case where supply is highly fragmented - i.e. where the number of farmers is large - which is commonplace in emerging markets. We compare the highly fragmented traditional supply chain with a hypothetical, fully-integrated supply chain where farmers and retailers are vertically integrated into a single entity which has direct access to market demand; we refer to this as the *integrated* supply chain. Note that because farmers and retailers are fully integrated, there is no longer any auction that intermediates transactions. The integrated supply chain serves as a best-case scenario - albeit unrealistic itself - for which to compare and better understand the auction's impact on production quantities and profits throughout the entire supply chain.

3.1. Farmers' Optimal Decisions

Recall the two decisions that farmers face in the traditional supply chain: in period 1, farmers must decide their production quantity, and in period 2, they decide how much to send to the auction. Since farmers do not coordinate their decisions, they can be viewed as players in a noncooperative game, which we will call the *traditional game*. In this game, each farmer's strategy can be represented by a pair $(q_i, q_i^{\mathbf{a}}(\cdot))$, where the second element of the pair, $q_i^{\mathbf{a}}(\cdot)$ maps the farmer's production quantity to the quantity that he sends to the auction. Since farmers are homogeneous, our analysis focuses on defining and characterizing symmetric equilibria of the game, where the farmers' decisions are identical. Throughout this paper, whenever we refer to "equilibria", unless explicitly stated otherwise, we will implicitly mean Nash equilibria that are subgame perfect. We specify conditions for such equilibria below.

Definition 2. For a given scalar $\widehat{q} \geq 0$ and function $\widehat{q}^{\mathbf{a}} : \mathbb{R}_+ \to \mathbb{R}$, consider the optimization problem

$$\max_{q\geq 0}\left\{-cq + \max_{0\leq q^{\mathbf{a}}\leq q}\left\{\pi q^{\mathbf{a}}\bar{F}_{\mathbf{a}}\left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}(\widehat{q})\right)\right\}\right\}. \tag{2}$$

We call $(\widehat{q}, \widehat{q}^{\mathbf{a}})$ a *symmetric equilibrium of the traditional game* if the following conditions hold:

(a) $\widehat{q}^{\mathbf{a}}(q)$ is an optimal solution to the inner problem of (2) for any q, and

(b) $\widehat{q}$ is an optimal solution to the outer problem of (2).

In Definition 2, we have implicitly invoked the auction price formula from Lemma 1, and the term $(N-1)\widehat{q}^{\mathbf{a}}(\widehat{q})$ represents the sum of quantities that the other $N-1$ farmers send to the auction. In other words, this equilibrium definition means that each farmer plays a best response to the decisions of all the other farmers. Using Definition 2, we can characterize the decisions made by farmers in a symmetric equilibrium. To be precise, we characterize the *equilibrium outcome* $(\widehat{q}, \widehat{q}^{\mathbf{a}}(\widehat{q}))$ of the game in the following proposition. Although we could, in principle, characterize the entire equilibrium strategy (i.e., specify $\widehat{q}^{\mathbf{a}}(q)$ for an arbitrary q), we refrain from doing so because it would involve additional notation and technicalities that distract from the key thrust of our analysis.

Proposition 1. *Let $Q_N^{\mathbf{T}}$ represent the unique value of $z \in \mathbb{R}_+$ that solves*

$$\bar{F}_{\mathbf{a}}(z) - \frac{z f_{\mathbf{a}}(z)}{N} = \frac{c}{\pi}. \tag{3}$$

Then, in equilibrium, each farmer produces $\frac{1}{N}Q_N^{\mathbf{T}}$ and sells everything that he produces at the auction. Moreover, each farmer's profit in equilibrium is given by $\Pi_N^{\mathbf{T}} = \pi\left(\frac{Q_N^{\mathbf{T}}}{N}\right)^2 f_{\mathbf{a}}(Q_N^{\mathbf{T}})$.

Note that $Q_N^{\mathbf{T}}$ refers to the total production quantity of all N farmers, where the superscript T refers to the traditional supply chain; later in the paper, we will use superscript I to refer to the integrated supply chain and superscript E to refer to the e-intermediated supply chain.

One implication of Proposition 1 is that in equilibrium, farmers only produce what they eventually sell. That is, there is no "wasted" production. Although this may seem obvious, the intuition behind this is rather nuanced. In particular, if we consider the second-period subgame alone and study the farmers' decisions as a function of the quantity produced in the first period, it is easy to see that in regimes where the first-period production quantity is sufficiently high, the symmetric equilibrium of this subgame will entail wastage. It is therefore only in the equilibrium outcome of the combined game that the "no wastage" property holds.

Having characterized the equilibrium for a fixed number of farmers, N, we now analyze how these equilibrium quantities vary with N in the following proposition.

PROPOSITION 2. *As the number of farmers increases, the equilibrium production of the chain increases. That is, $Q_N^{\mathbf{T}}$ increases in N.*

3.2. Impact of Supply Fragmentation

In emerging markets, agricultural supply chains are often characterized by a high level of supply fragmentation, i.e. a very large number of farmers. As stated earlier, there are approximately 475 million farms in developing countries that are less than two hectares; furthermore, in many of these countries, there is a trend towards a greater number of even smaller farms (Rapsomanikis (2015)). Because of this, we are particularly interested in the optimal production quantity and supply chain profits in the asymptotic limit of high levels of supply fragmentation where $N \to \infty$.

We first use Proposition 2 to characterize the optimal production quantity, $Q_N^{\mathbf{T}}$, when $N \to \infty$.

PROPOSITION 3. *As $N \to \infty$, in the symmetric equilibrium, the total production quantity of all farmers converges to $Q_\infty^{\mathbf{T}}$, where $Q_\infty^{\mathbf{T}}$ is the unique value of z on $\mathbb{R}_+$ that solves $\pi \bar{F}_{\mathbf{a}}(z) = c$, i.e.,*

$$Q_\infty^{\mathbf{T}} = \sum_{j=1}^{M} \mu_j + \Phi^{-1}\left(\frac{\pi - c}{\pi}\right) \sum_{j=1}^{M} \sigma_j. \tag{4}$$

The idea underlying Proposition 3 is that as N grows, the marginal impact on the auction price from each individual farmer diminishes. Intuitively, if $\pi \bar{F}_{\mathbf{a}}(Q_\infty^{\mathbf{T}}) > c$, then farmers have an incentive to produce an infinite quantity, which cannot be sustained in a symmetric equilibrium. Similarly, if $\pi \bar{F}_{\mathbf{a}}(Q_\infty^{\mathbf{T}}) < c$, then farmers have an incentive to produce nothing, which again cannot be sustained in a symmetric equilibrium. Consequently, in equilibrium, we require $\pi \bar{F}_{\mathbf{a}}(Q_\infty^{\mathbf{T}}) = c$, and each farmer produces an infinitesimal and equal proportion of $Q_\infty^{\mathbf{T}}$. The following corollary describes an implication of Proposition 3 on farmers' profits.

Corollary 1. *As $N \to \infty$, in the symmetric equilibrium, the total profits of all farmers converge to zero.*

This is a result of the fact that $\pi \bar{F}_{\mathbf{a}}(Q_{\infty}^{\mathbf{T}}) = c$, which implies that when $N \to \infty$, the farmers' revenue per unit equals the production cost per unit; therefore, the farmers' profit converges to zero. This is a troubling result for agricultural supply chains in emerging economies, given their high levels of supply fragmentation. In the remainder of this section, we study the impact of the auction mechanism itself on the optimal production quantity and profits, given high levels of supply fragmentation. Then in §4 and via numerical studies in §5, we will determine to what extent, if any, the presence of an e-intermediary helps mitigate this negative impact on farmers.

Comparison with an Integrated Supply Chain

To better understand the impact that the auction has on the optimal production quantity in the case of high levels of supply fragmentation, we compare the traditional supply chain when $N \to \infty$ with a hypothetical, fully-integrated supply chain. Specifically, we assume that farmers and retailers are vertically integrated into a single entity (we will refer to this entity as "the firm"), which has direct access to market demand. In particular, in the integrated firm, because farmers and retailers are fully integrated, there is no longer any auction that intermediates transactions.

The firm faces a newsvendor problem, that is, $\max_Q \Pi^{\mathbf{I}}(Q)$, where $\Pi^{\mathbf{I}}$ represents the profit of the integrated chain as a function of the total quantity produced by the farmers, Q, and is defined as $\Pi^{\mathbf{I}}(Q) := \pi \mathbb{E}\left(D^{\mathbf{I}} \wedge Q\right) - cQ$, where $D^{\mathbf{I}} \sim N\left(\sum_{j=1}^{M} \mu_j, \sum_{j=1}^{M} \sigma_j^2\right)$ and can be interpreted as the integrated firm's total distribution. Standard newsvendor analysis gives us the optimal production quantity of the integrated firm, outlined in the following proposition.

Proposition 4. *In the integrated supply chain, the optimal production quantity of the firm is $Q^{\mathbf{I}}$, where*

$$Q^{\mathbf{I}} := \sum_{j=1}^{M} \mu_j + \Phi^{-1}\left(\frac{\pi - c}{\pi}\right) \sqrt{\sum_{j=1}^{M} \sigma_j^2} \tag{5}$$

Note that $Q_{\infty}^{\mathbf{T}}$ in Proposition 3 is identical to $Q^{\mathbf{I}}$ except that $\sum_{j=1}^{M} \sigma_j$ replaces $\sqrt{\sum_{j=1}^{M} \sigma_j^2}$. Given this difference, we can see that an alternative representation of the traditional supply chain when $N \to \infty$ is that of an integrated supply chain where the total retailer demand is $D^{\mathbf{T}} \sim N\left(\sum_{j=1}^{M} \mu_j, (\sum_{j=1}^{M} \sigma_j)^2\right)$ instead of $D^{\mathbf{I}} \sim N\left(\sum_{j=1}^{M} \mu_j, \sum_{j=1}^{M} \sigma_j^2\right)$, i.e. an integrated chain with larger effective variance of retailer demand. Thus, for high levels of supply fragmentation (as $N \to \infty$), the auction in the traditional supply chain is responsible for inflating the total retailer demand variance that the farmers face when making their production decisions. This increase in variance results in a sub-optimal production quantity in the highly fragmented traditional supply chain compared to the integrated chain, as outlined in the following proposition.

PROPOSITION 5. *The inefficiency of total production of the highly fragmented traditional supply chain depends on the value of* c *relative to* π*. If* $c < \pi/2$*, the traditional chain overproduces relative to the integrated chain, i.e.,* $Q^{\mathbf{T}}_{\infty} \geq Q^{\mathbf{I}}$*. Conversely, if* $c > \pi/2$*, the traditional chain underproduces, i.e.,* $Q^{\mathbf{T}}_{\infty} \leq Q^{\mathbf{I}}$*. Finally, if* $c = \pi/2$*, the traditional chain produces exactly what the integrated chain would produce, i.e.,* $Q^{\mathbf{T}}_{\infty} = Q^{\mathbf{I}}$*.*

Proposition 5 shows that the highly fragmented traditional supply chain, in equilibrium, generally produces a different quantity than the integrated supply chain. One interpretation of this discrepancy in production quantity is that it is a manifestation of distorted production incentives among farmers in the traditional supply chain. Specifically, this distortion takes on the form of variance inflation: although the auction accurately aggregates mean total demand from retailers, it inflates the demand variance in the traditional chain relative to the integrated chain. Farmers produce according to this inflated variance, which is sub-optimal from the perspective of the overall chain. It is noteworthy that the traditional supply chain may *overproduce* relative to the integrated chain; other studies have found that supply chains that are intermediated typically tend to *underproduce* relative to comparable integrated supply chains (see, e.g., Tang and Kouvelis 2014, Arya and Mittendorf 2006, Gal-Or 1991, and references therein).

With a deeper understanding of how the auction impacts the optimal production quantity in the highly fragmented traditional supply chain, we next study the impact that the auction has on the farmers' profit, retailers' profit, and overall supply chain profit. In general, we expect the integrated supply chain to have higher profits than the traditional chain. Nonetheless, it is instructive to delve deeper to better understand the sources of inefficiency that the auction introduces in the market. We first do so by considering a decomposition of the integrated supply chain's profit into parts, given a total production quantity Q. For every $j = 1, \ldots, M$, define $Q_j := \mu_j + \Phi^{-1}\left(\frac{\pi - p^{\mathbf{a}}}{\pi}\right)\sigma_j$, where $p^{\mathbf{a}}$ represents the equilibrium price from the Walrasian auction with total quantity Q, and is calculated as $p^{\mathbf{a}} = \pi \bar{F}_{\mathbf{a}}(Q)$ by Lemma 1. Here, Q_j represents the quantity allocated to retailer j at the auction in the traditional supply chain, and because the auction distributes all the quantity of goods available (Definition 1, part (ii)) , we have $\sum_{j=1}^{M} Q_j = Q$. Based on this definition of Q_j, we may decompose the total supply chain profit of the integrated supply chain as

$$\Pi^{\mathbf{I}}(Q) = \underbrace{\pi Q \bar{F}_{\mathbf{a}}(Q) - cQ}_{(i)} + \underbrace{\pi \sum_{j=1}^{M} \mathbb{E}(D_j \wedge Q_j) - \pi Q \bar{F}_{\mathbf{a}}(Q)}_{(ii)} + \underbrace{\pi \mathbb{E}(D^{\mathbf{I}} \wedge Q) - \pi \sum_{j=1}^{M} \mathbb{E}(D_j \wedge Q_j)}_{(iii)}, \quad (6)$$

where $D_j \sim N(\mu_j, \sigma_j^2)$. In (6), elementary analysis reveals that each of the three terms (i), (ii), and (iii) are nonnegative. Furthermore, term (i) represents the total farmers' profits in the traditional supply chain and term (ii) represents the total retailers' profits in the traditional supply chain. The sum of terms (i) and (ii) is thus the total supply chain profit in the traditional supply chain.

This leaves us with additional profit in the integrated supply chain compared to the traditional supply chain, which is quantified in term (iii). Term (iii) can be interpreted as the profit loss in the traditional supply chain due to retailer fragmentation; if there had only been a single retailer (i.e., $M=1$), term (iii) would be zero. It is worth noting that such retailer fragmentation is intimately connected to the auction, and is a prerequisite for having an auction in the first place.

Combining our preceding results with decomposition (6) enables us to analyze the impact of supply fragmentation on the traditional supply chain's profits, as outlined in the following theorem.

THEOREM 1. *As supply fragmentation (i.e., N) increases in the traditional supply chain and farmers produce their equilibrium production quantity,*

(a) the total profit of all farmers decreases,

(b) the total profit of all retailers increases,

(c) the total profit loss due to retailer fragmentation decreases if $c<\pi/2$, and

(d) the total supply chain's profit increases if $c>\pi/2$.

One implication of part (a) of Theorem 1 is that as the number of farmers increases, individual farmers are not only impacted by having to spread profits across more farmers, unfortunately they are further impacted by the decrease in total farmers' profits. In other words, not only do the farmers have to "cut the (profit) pie" in more ways, they also face a shrinking "pie". This means that as supply fragmentation increases, farmers' profits take a hit from multiple directions. In contrast, part (b) of the theorem shows that the retailers benefit from supply fragmentation with increased profits. When $c>\frac{\pi}{2}$, we can further conclude from part (d) that the increase in retailers' profits outweighs the decrease in farmers' profits, and thus the total supply chain profits increase with more supply fragmentation. This suggests that supply fragmentation actually helps the overall supply chain, yet the auction allocates those additional profits in favor of the retailers instead of the farmers. Finally, we are unable to conclude whether or not the total supply chain profit increases with supply fragmentation when $c<\frac{\pi}{2}$, but part (c) tells us that under this condition and as supply fragmentation increases, the total supply chain profit of the traditional chain approaches the total supply chain profit of the integrated chain, i.e., the negative impact of the retailer fragmentation on total supply chain profits is mitigated by supply fragmentation when $c<\frac{\pi}{2}$.

3.3. Discussion

Our results show that even though the idealized Walrasian auction mechanism is efficient in a local sense, in that it finds a price for the product that perfectly matches supply and demand between farmers and retailers, it nonetheless introduces sub-optimality to the overall supply chain. Specifically, farmers either overproduce or underproduce in the traditional supply chain compared to the integrated chain, which results in a sub-optimal total supply chain profit. It is striking that this sub-optimality occurs even for such an idealized auction mechanism as an intermediary.

We argue that this insight is more salient than it may appear at first glance. The dominant narrative about investments in emerging economies (particularly in the academic and popular discourse on microfinancing) has largely revolved around the issue of developing more efficient markets by providing liquidity to farmers (or small business owners in other settings) and reducing transaction costs (see, e.g., Morduch 1999). Our result in Proposition 3 does not contravene this narrative, but does provide an important caveat that a singular focus on local market creation alone is inadequate, and that a holistic perspective of the supply chain is necessary.

Second, one might interpret this as a form of *double marginalization*, which has been extensively studied in various contexts in both the operations and marketing literature (Jeuland and Shugan 1983, Pasternack 1985, Iyer 1998, Iyer et al. 2007). Generally speaking, double marginalization refers to the phenomenon where supply chains perform sub-optimally as a consequence of intermediaries who optimize their operations for their own benefit. Our setting shows that a form of double marginalization persists even when both prices and quantities are determined via an auction-based exchange, in contrast to other settings where prices (and/or quantities) are the direct decisions of supply chain participants. Moreover, the way that this suboptimality manifests is noteworthy. Our results show that the supply chain distortion can manifest itself either as overproduction or underproduction. This differs from typical findings of several variants of the classical setting where double marginalization usually leads to underproduction (see, e.g., Tang and Kouvelis 2014, Arya and Mittendorf 2006, Gal-Or 1991, and references therein).

4. Analysis of the E-Intermediated Supply Chain

The analysis of the traditional supply chain showed us that the auction inflates demand variance compared to the integrated chain, which causes under/over-production and suboptimal profits. In this section as well as the next, we study whether or not these negative effects can be mitigated by the introduction of an e-intermediary as an alternate, fixed-price channel for which the farmers have the option to sell their crop. Specifically, in the second period, each farmer must decide how much product to send (a) to the auction and (b) to the e-intermediary. In §4.1, we characterize the optimal solution of the key decision variables in our model: each farmer's production quantity and the quantities the farmer chooses to sell at the auction and e-intermediary, as well as the price offered by the e-intermediary. In §4.2, we study the impact of supply fragmentation on the e-intermediated supply chain and compare its performance to the traditional chain.

4.1. Farmers' and E-Intermediary's Optimal Decisions

For the auction, recall from Lemma 1 that the equilibrium auction price, $p^{\mathbf{a}}$, can be expressed as a function of the total quantity available for distribution at the auction $Q^{\mathbf{a}}$, i.e., $p^{\mathbf{a}} = \pi \bar{F}_{\mathbf{a}}(Q^{\mathbf{a}})$, where $\bar{F}_{\mathbf{a}}(\cdot)$ represents the tail cdf of a $N\left(\sum_{j=1}^{m}\mu_j, \left(\sum_{j=1}^{m}\sigma_j\right)^2\right)$ random variable. In contrast, the e-intermediary sets a price, $p^{\mathbf{e}}$, that it will purchase the product for. As described in §2.2, we assume that the e-intermediary takes on the aggregate demand of retailers $m+1$ through M, such that its demand is a $N\left(\sum_{j=m+1}^{M}\mu_j, \sum_{j=m+1}^{M}\sigma_j^2\right)$ random variable. For notational brevity, we use $\bar{F}_{\mathbf{e}}(\cdot)$ to denote the tail cdf of this random variable.

We now proceed to characterize key metrics of this e-intermediated supply chain. For ease of reference, we will refer to the game comprising of the decisions of the farmers and the e-intermediary in periods 1 and 2 as the *e-intermediated game.* Again, since farmers are homogeneous, our analysis focuses on characterizing symmetric equilibria of the game, defined below.

Definition 3. Given a scalar $\widehat{q} \geq 0$, functions $\widehat{p}^{\mathbf{e}}: \mathbb{R}_+ \to \mathbb{R}$, $\widehat{q}^{\mathbf{a}}: \mathbb{R}_+^2 \to \mathbb{R}$, and $\widehat{q}^{\mathbf{e}}: \mathbb{R}_+^2 \to \mathbb{R}$, we call $(\widehat{q}, \widehat{p}^{\mathbf{e}}, \widehat{q}^{\mathbf{a}}, \widehat{q}^{\mathbf{e}})$ a *symmetric equilibrium of the e-intermediated game* if the following conditions hold simultaneously:

(a) For every given $q \geq 0$ and $p \geq 0$, $\widehat{q}^{\mathbf{a}}(q,p), \widehat{q}^{\mathbf{e}}(q,p)$ are optimizers of the problem

$$\begin{aligned} \Pi(q,p) := \max_{q^{\mathbf{a}}\geq 0, q^{\mathbf{e}}\geq 0} \; & pq^{\mathbf{e}} + \pi q^{\mathbf{a}} \bar{F}_{\mathbf{a}}\left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}(\widehat{q},p)\right) \\ \text{s.t. } & 0 \leq q^{\mathbf{a}} + q^{\mathbf{e}} \leq q. \end{aligned} \tag{7}$$

(b) For any $q \geq 0$, $\widehat{p}^{\mathbf{e}}(Nq)$ is an optimizer of the problem

$$\max_{p\geq 0} \left\{\pi \mathbb{E}\left(D^{\mathbf{e}} \wedge N\widehat{q}^{\mathbf{e}}(q,p)\right) - pN\widehat{q}^{\mathbf{e}}(q,p)\right\}, \tag{8}$$

where $D^{\mathbf{e}} \sim N\left(\sum_{j=m+1}^{M}\mu_j, \sum_{j=m+1}^{M}\sigma_j^2\right)$.

(c) $\widehat{q}$ satisfies

$$\widehat{q} \in \arg\max_{q\geq 0} \left\{-cq + \Pi(q, \widehat{p}^{\mathbf{e}}(q + (N-1)\widehat{q}))\right\}, \tag{9}$$

where $\Pi(q,p)$ is as defined in part (a).

In Definition 3, note that we have suppressed denoting the dependence of the equilibrium quantities with model parameters (e.g., N, M, m) for notational brevity. Intuitively, part (a) requires that each farmer allocates his products to channels optimally to maximize period 2 profits, as a best response to the actions taken by all other farmers. Recall that $q^{\mathbf{a}}$ and $q^{\mathbf{e}}$ represent the quantities allocated to the auction and e-intermediary, respectively; we have implicitly invoked Lemma 1 to characterize the equilibrium price for the auction. Part (b) states that the price chosen by the e-intermediary should maximize its expected profit. It turns out to be more convenient to (equivalently) represent the e-intermediary's price as a function of the total production of all farmers; this is reflected in the notation for (b), where $\widehat{p}^{\mathbf{e}}$ is defined as a function of Nq instead of q. Part (c) states that each farmer chooses his first period quantity optimally.

Based on Definition 3, we can now characterize the symmetric equilibrium outcome of the e-intermediated game. As in §3, we do not characterize off-equilibrium outcomes. We first establish some preliminary definitions in order to streamline the presentation of our characterization. First, define the function $\psi_N : \mathbb{R}_+ \to \mathbb{R}$ as

$$\psi_N(z) := \bar{F}_{\mathbf{a}}(z) - \frac{z f_{\mathbf{a}}(z)}{N}. \tag{10}$$

By Lemma 2, there exists a unique value of $z \geq 0$ such that $\psi_N(z) = 0$, and, for convenience, we denote this as $\bar{z}_N$. Further, define $\zeta_N : \mathbb{R}_{++} \to \mathbb{R}$ as follows:

$$\zeta_N(Q) := \begin{cases} 0 & \text{if } \psi_N(0) - Q\psi_N'(0) \leq \bar{F}_{\mathbf{e}}(Q) \\ Q & \text{if } \psi_N(Q) \geq \bar{F}_{\mathbf{e}}(0) \\ z_N(Q) \wedge \bar{z}_N & \text{otherwise,} \end{cases} \tag{11}$$

where $z_N(Q)$ is the unique value of $z \in (0, Q)$ that solves

$$\psi_N(z) - (Q - z)\psi_N'(z) = \bar{F}_{\mathbf{e}}(Q - z). \tag{12}$$

It can be shown that z_N, ζ_N are well-defined. We relegate the formal statement and verification of technical lemmas involving these definitions to §EC.1 of the electronic companion.

The first mapping, ψ_N, will be used to connect the e-intermediary's price and the total equilibrium quantity sent to the auction. Specifically, if z satisfies $\pi\psi_N(z) = p$ for a given p, then z will represent the total equilibrium quantity at the auction when the e-intermediary offers price p to farmers. The inverse mapping of ψ_N can thus be viewed as the impact of the e-intermediary's price on the total auction quantity. The second mapping, ζ_N, will be used to capture the impact of the farmers' total equilibrium production quantity on the total equilibrium quantity sent to the auction. It is important to note that our construction of ζ_N will implicitly capture the e-intermediary's optimal pricing decision.

THEOREM 2. *For a fixed $\widehat{q} \geq 0$, consider the optimization problem*

$$\begin{aligned} \max_{q,z} \; & \left(\psi_N(z) - \frac{c}{\pi}\right) q + \frac{z^2 f_{\mathbf{a}}(z)}{N^2} \\ \text{s.t. } & z = \zeta_N(q + (N-1)\widehat{q}) \\ & q \geq 0. \end{aligned} \tag{13}$$

and suppose that there is an optimal (q^*, z^*) of (13) such that $q^* = \widehat{q}$. Then,

(a) The total amount produced by farmers is $Q_N^{\mathbf{E}} := N\widehat{q}$.

(b) The total quantity sent to the auction is $Q_N^{\mathbf{a}} := \zeta_N(Q_N^{\mathbf{E}})$,

(c) The total quantity sent to the e-intermediary is $Q_N^{\mathbf{e}} := Q_N^{\mathbf{E}} - Q_N^{\mathbf{a}}$.

(d) The e-intermediary chooses the purchase price $\widehat{p}^{\mathbf{e}} := \pi\psi_N(Q_N^{\mathbf{a}})$ for the product.

In Theorem 2, problem (13) may be interpreted as the first-stage problem of farmers, who are choosing their individual production quantity (q) as a best response to the symmetrically chosen production quantities of the other farmers ($\widehat{q}$). We have included an auxiliary variable z to simplify the presentation, which represents the total quantity sent by all farmers to the auction in the second period. Although we do not have an explicit, closed-form characterization of the equilibrium of this game, we can nonetheless derive certain properties about the equilibrium, which we state in the following corollary.

COROLLARY 2. *In a symmetric equilibrium, it is necessary that*

(a) The auction price is weakly greater than the e-intermediary's price, $\widehat{p}^{\mathbf{e}}$*, that is,* $\widehat{p}^{\mathbf{a}} \geq \widehat{p}^{\mathbf{e}}$*.*

(b) If $\widehat{q}^{\mathbf{e}} > 0$*, the e-intermediary's price is weakly greater than the farmer's marginal production cost* c*, that is,* $\widehat{p}^{\mathbf{e}} \geq c$*.*

4.2. Impact of Supply Fragmentation

As described earlier, given that agricultural supply chains are often characterized by a very large number of farmers, we pay particular attention in this section to the impact of supply fragmentation on the e-intermediated supply chain and make comparisons to its impact on the traditional supply chain. In the limit as $N \to \infty$, we can characterize the symmetric equilibrium of the e-intermediated game more precisely with the following proposition.

PROPOSITION 6. *Define* $Q^{\mathbf{a}}_{\infty}$ *as*

$$Q^{\mathbf{a}}_{\infty} := \sum_{j=1}^{m} \mu_j + \Phi^{-1}\left(\frac{\pi - c}{\pi}\right) \sum_{j=1}^{m} \sigma_j .$$

Furthermore, let $Q^{\mathbf{e}}_{\infty}$ *represent the unique value of* z *that solves*

$$\bar{F}_{\mathbf{e}}(z) - z f_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty}) = \frac{c}{\pi}. \tag{14}$$

Then, in the limit as $N \to \infty$*, the following hold in equilibrium:*

(a) The e-intermediary's purchase price converges to c*.*

(b) The auction price converges to c*.*

(c) The total quantity sent to the auction converges to $Q^{\mathbf{a}}_{\infty}$*, the total quantity sent to the e-intermediary converges to* $Q^{\mathbf{e}}_{\infty}$*, and the total quantity produced by all farmers converges to* $Q^{\mathbf{E}}_{\infty} = Q^{\mathbf{a}}_{\infty} + Q^{\mathbf{e}}_{\infty}$*.*

Parts (a) and (b) together imply that the total farmers' profits vanish in the limit of high supply fragmentation, even in the e-intermediated chain. This can be viewed as an analog to Corollary 1 in the setting of a traditional chain. This result suggests a certain "hierarchy" of problems faced by farmers in developing economies: The first-order problem is that of supply fragmentation, which e-intermediation does not overcome. Through our numerical study in §5, we evaluate the possible beneficial or detrimental second-order effects of e-intermediation on farmers' profits for finite values of N.

The following proposition presents results on the relative impact of the e-intermediary on the overall production quantity and profits of the supply chain when supply is highly fragmented.

PROPOSITION 7. *Define* $\sigma_{\mathbf{I}}, \sigma_{\mathbf{a}}, \sigma_{\mathbf{e}}, \sigma_{\mathbf{T}}$ *as follows:*

$$\sigma_{\mathbf{I}} := \sqrt{\sum_{j=1}^{M} \sigma_j^2}, \qquad \sigma_{\mathbf{a}} := \sum_{j=1}^{m} \sigma_j, \qquad \sigma_{\mathbf{e}} := \sqrt{\sum_{j=m+1}^{M} \sigma_j^2}, \qquad \sigma_{\mathbf{T}} := \sum_{j=1}^{M} \sigma_j.$$

If $c < \pi/2$*, and in addition,*

$$\sum_{j=m+1}^{M} \mu_j \leq \Phi^{-1}\left(\frac{\pi - c}{\pi}\right)\left(\sigma_{\mathbf{a}} - \frac{(\sigma_{\mathbf{I}} - \sigma_{\mathbf{a}})(\sigma_{\mathbf{a}} + \sigma_{\mathbf{e}})}{\sigma_{\mathbf{e}}}\right) \qquad \text{and} \qquad \sigma_{\mathbf{I}} \geq \sigma_{\mathbf{a}}, \tag{15}$$

we have $Q^{\mathbf{I}} \leq Q^{\mathbf{E}}_{\infty} \leq Q^{\mathbf{T}}_{\infty}$.

Conversely, if $c > \pi/2$*, and, in addition*

$$\sum_{j=m+1}^{M} \mu_j \geq \Phi^{-1}\left(\frac{\pi - c}{\pi}\right)\left(\sigma_{\mathbf{a}} - \frac{(\sigma_{\mathbf{T}} - \sigma_{\mathbf{a}})(\sigma_{\mathbf{a}} + \sigma_{\mathbf{e}})}{\sigma_{\mathbf{e}}}\right) \tag{16}$$

we have $Q^{\mathbf{E}}_{\infty} \leq Q^{\mathbf{T}}_{\infty} \leq Q^{\mathbf{I}}$.

First, note that $\sigma_{\mathbf{I}}, \sigma_{\mathbf{a}}, \sigma_{\mathbf{e}}$, and $\sigma_{\mathbf{T}}$ represent the standard deviation of retailer demand faced by the integrated chain, the auction in the e-intermediated chain, the e-intermediary in the e-intermediated chain, and the traditional supply chain, respectively. The interpretation of the proposition above is that the impact of the e-intermediary on the supply chain's overall profitability is mixed. When $c < \pi/2$ and under some technical conditions, the result shows that the effect of the e-intermediary relative to the traditional supply chain is that it pulls the total production quantity *toward* the system optimal $Q^{\mathbf{I}}$, which implies that supply chain profits are improved relative to the traditional chain. The technical condition is an upper bound on the total average demand experienced by the e-intermediary, which can be interpreted as the e-intermediary's market share. Therefore, this condition will tend to hold when the e-intermediary is small or in an early stage of growth. When $c > \pi/2$, the converse holds. Under some technical conditions, the e-intermediary now pulls the total production quantity of the chain *away* from the system optimal $Q^{\mathbf{I}}$, diminishing the profits of the chain. This condition tends to hold when the e-intermediary is large, or in a mature phase of growth. The impact of the e-intermediary's market share on the e-intermediated supply chain's profits, as well as on farmers' profits, is further explored in §5.

5. Numerical Study

We conducted a numerical simulation study in order to gain further insights into how farmers' and supply chain profits are influenced by the e-intermediary's presence and size, the extent of supply fragmentation, and other parameters. Throughout our numerical analysis, we set $\pi = 1$ and fixed total mean demand at 1000 units. We let κ represent the coefficient of variation of demand, which we varied across simulations such that the standard deviation was 1000κ. We assumed that when there were M retailers, these retailers were symmetric and had independent demands, so that the sum of their demands would be equal in distribution to a $N(1000,(1000\kappa)^2)$ random variable. That is, for a fixed M, the demand at retailer j was distributed according to $N(1000/M,(1000\kappa)^2/M)$. Using these assumptions, the ratio $(M-m)/M$ captures the fraction of average demand faced by the e-intermediary, which we will refer to as the e-intermediary's *market share*. We will only present results for $c=0.2$ and $\kappa=0.2$ for brevity. We ran our numerical studies on numerous combinations of other parameters and the general insights and findings were unchanged.

5.1. Impact of Supply Fragmentation

Due to the prevalence of supply fragmentation in emerging market agricultural supply chains, we supplement our analysis from §3.2 and §4.2 with a numerical analysis of the relationship between the farmers' aggregate profit and the number of farmers. Figures 3 and 4 show total farmers' profits converging to zero as $N\to\infty$ for both the traditional and e-intermediated supply chains. These figures confirm our theoretical findings in Corollary 1 and Proposition 6: The first-order problem faced by farmers in developing economies is that of supply fragmentation, which e-intermediation does not overcome. What these numerical results add to our understanding is how quickly aggregate farmers' profits diminish to zero. This insight suggests that a more impactful way to improve the livelihood of farmers in developing economies is to consolidate farmers into larger farming collectives (i.e., find ways to reduce N) to improve their market power. Our insight is consistent with empirical studies that have found evidence of positive impacts of collective organization on farmers in developing economies, for example, in Ethiopia (Francesconi and Heerink 2010, Chagwiza et al. 2016), Kenya (Fischer and Qaim 2012), and Central America (Hellin et al. 2009).

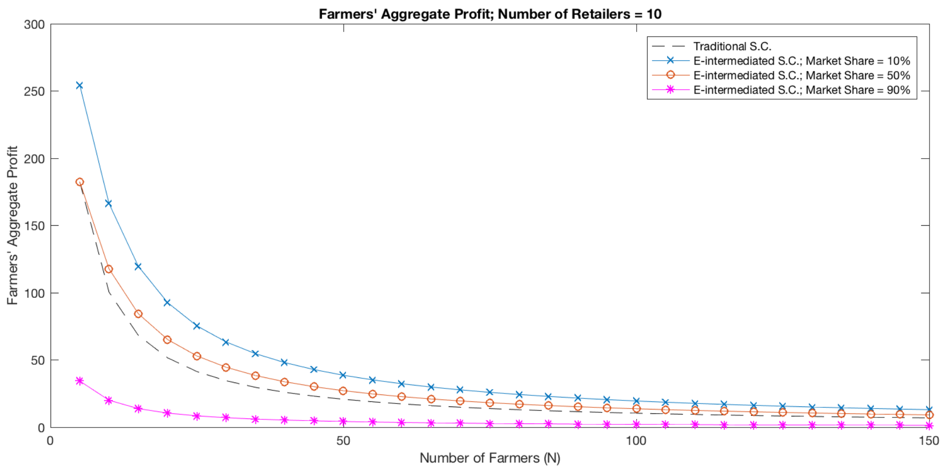

Figure 3 Impact of Supply Fragmentation on Farmers' Aggregate Profit for $M=10$ Retailers

Interestingly, Figures 3 and 4 illustrate that the size of the e-intermediary plays a role with respect to whether or not the e-intermediary is beneficial to farmers. Specifically, when the e-intermediary's market share is small, the presence of the e-intermediary benefits the farmers. In

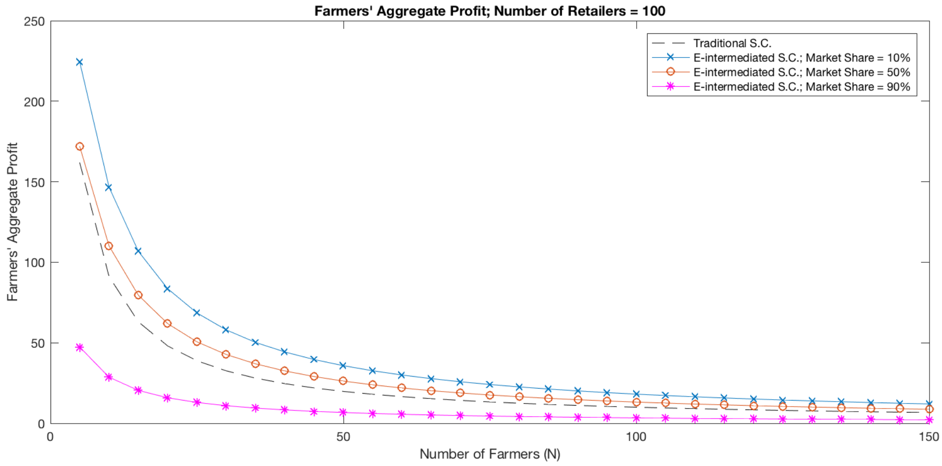

Figure 4 Impact of Supply Fragmentation on Farmers' Aggregate Profit for $M = 100$ Retailers

contrast, when the e-intermediary's market share is large, the presence of the e-intermediary hurts the farmers. Next we take a deeper dive into the relationship between the e-intermediary's market share and total farmer and supply chain profits.

5.2. Impact of E-Intermediary's Market Share

Figure 5 illustrates that as the e-intermediary increases its market share, farmers see their aggregate profits drop, albeit slowly at the start. However, this decline tends to become more rapid as the e-intermediary grows, and can even fall below their profits in the traditional supply chain, as seen in Figures 3 and 4. This result can be explained by the dual role played by the e-intermediary: On the one hand, it is a source of competition with the auction, providing farmers with greater choice over where they can send their produce; on the other hand, it is also a profit-maximizing entity, which can act as a monopolist and squeeze farmers' profits as it grows large.

Another way that this phenomenon manifests is in the variation of supply chain profits with e-intermediary market share. For the e-intermediated supply chain, total supply chain profit is the sum of three terms: farmers' aggregate profit, profit of retailers participating in the auction, and profit of retailers participating in the e-intermediary. Figure 6 illustrates that when the e-intermediary is small, growth in the e-intermediary tends to improve overall supply chain profits; conversely, when the e-intermediary becomes too large, further growth tends to diminish supply chain profits. One value from this insight is that it makes predictions that can help to inform future policy. At present, the vast majority of e-intermediaries are small players and as such, governments or NGOs should encourage their growth to increase the value generated by such supply chains. However, our insight suggests that as e-intermediaries take on larger portions of the market share, policies that encourage further growth can ultimately be detrimental to the supply chain.

This finding is consistent with a recent study by Hernandez et al. (2017) that offered sobering analyses about an e-intermediation effort in Ethiopia. The authors studied a single crop, coffee,

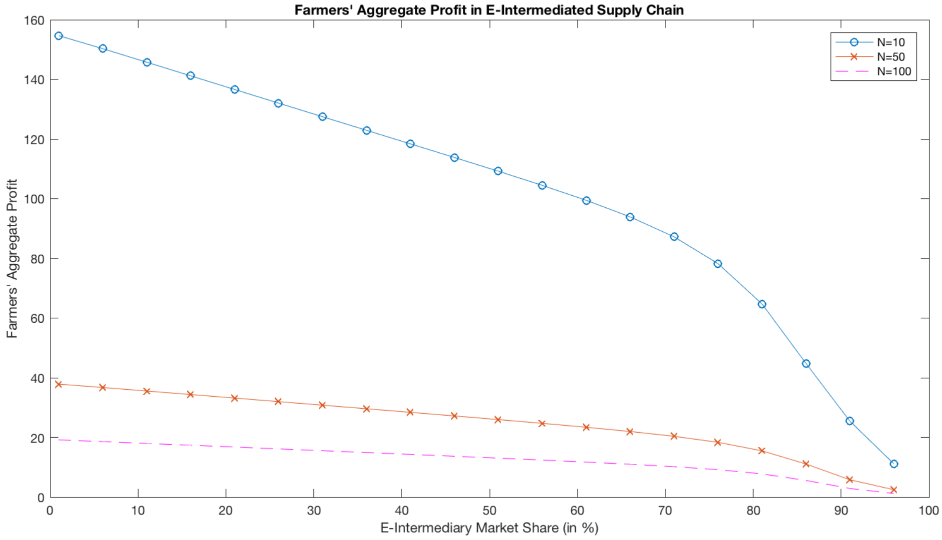

Figure 5 **Impact of E-Intermediary's Market Share on Farmers' Aggregate Profit for $M = 100$ Retailers**

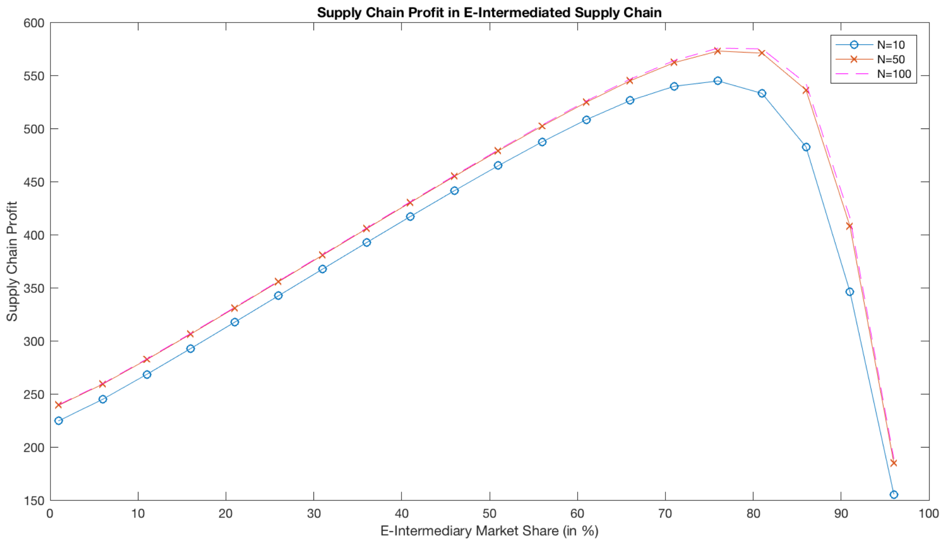

Figure 6 **Impact of E-Intermediary's Market Share on Total Supply Chain Profits for $M = 100$ Retailers**

and performed an econometric analysis of the impact of the e-intermediary ECX on coffee prices. They found that the introduction of ECX had little impact on coffee prices, and, by extension, farmers' profits. The authors suggest that this was a consequence of a policy action taken by the Ethiopian government, who "suspended the age-old coffee auction floor in Addis Ababa and made it mandatory to trade all coffee through the ECX in December 2008" (Hernandez et al. 2017, p.3). In the context of our model, this policy change effectively caused ECX to enjoy a large market share. In such a setting, our results suggest that e-intermediation can reduce farmers' profits, and contribute a potential explanation for why coffee farmers' profits did not improve with ECX.

5.3. Comparison of Farmers' Profits in E-Intermediated vs. Traditional Supply Chains for Small E-Intermediaries

Given that e-intermediaries with small market share tend to benefit farmers, our final study compares farmers' profits in the e-intermediated vs. traditional supply chains for particularly small e-intermediaries. Specifically, we focus on e-intermediaries who only have a market share of $\frac{100}{M}\%$ and study the impact of the e-intermediary as the number of retailers M grows large. Figure 7 corroborates our results from Figures 3 and 4 that the small e-intermediary tends to improve farmers' profits relative to traditional supply chains. Furthermore, the extent of this improvement tends to increase with the number of retailers; we observe that farmers' profits can be potentially increased by nearly 100% relative to the traditional supply chain.

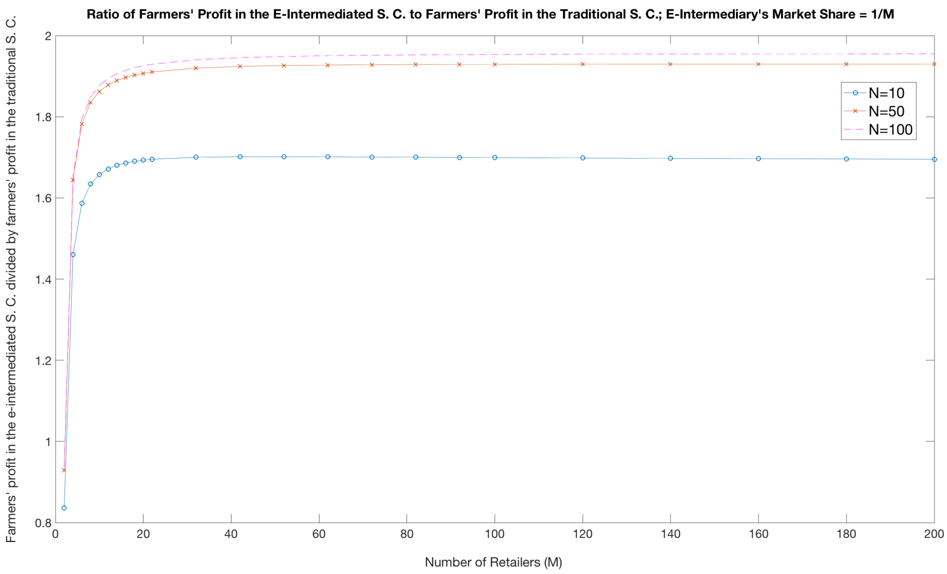

Figure 7 Comparison of Farmers' Profits in E-Intermediated vs. Traditional Supply Chains

What is striking is that this improvement in profits occurs even when the e-intermediary has very small market share. This might seem counterintuitive, because one might expect a very small e-intermediary to have a negligible impact on farmers. However, this observation can be explained intuitively as follows: If the e-intermediary had zero market share, the total quantity produced (and traded at the auction) in the e-intermediated chain would be equal to that of the traditional chain at some price $p^{\mathbf{a}}_{trad}$, and farmers would earn a margin of $p^{\mathbf{a}}_{trad} - c$. Because the e-intermediary has a small market share, it cannot set a price $p^{\mathbf{e}} << p^{\mathbf{a}}_{trad}$ because doing so would have negligible impact on the auction price, and farmers would rather trade at the auction to get a higher price; therefore, the e-intermediary has to set $p^{\mathbf{e}} \approx p^{\mathbf{a}}_{trad}$. Note that if the e-intermediary had a greater market share, farmers could potentially sell higher quantities to the e-intermediary even at a low price in order to reduce the quantity traded at the auction, thereby increasing the auction price.

Essentially, the price offered by the small e-intermediary, $p^{\mathbf{e}} \approx p^{\mathbf{a}}_{trad}$, serves as a reserve price on the auction, and the actual auction price $p^{\mathbf{a}}_{e-int}$ would be higher. Since the e-intermediary purchases a negligible amount, the quantity traded at this auction is again similar to the quantity traded in an auction in a traditional supply chain, where the effective reserve price is c. Therefore, the farmers' margins in these two auctions - differing only by their effective reserve price - should intuitively be similar, such that $p^{\mathbf{a}}_{e-int} - p^{\mathbf{a}}_{trad} \approx p^{\mathbf{a}}_{trad} - c$. In other words, farmers' margins in an e-intermediated chain with a negligibly small e-intermediary should be roughly twice that of farmers' margins in the traditional supply chain with no e-intermediary (because $p^{\mathbf{a}}_{e-int} - c \approx 2(p^{\mathbf{a}}_{trad} - c)$). Since the quantity traded in both these settings is also similar, farmers' should earn roughly twice the profits in the e-intermediated chain relative to the traditional chain. This insight parallels the anecdotal findings from examples summarized in Table 1 that e-intermediation efforts tend to improve farmers' profits.

6. Discussion and Conclusion

In our analysis of the traditional supply chain with no e-intermediary, we found that even when the trade of produce was governed by a highly idealized model of a Walrasian auction, this nonetheless introduced distorted incentives in the supply chain through a form of variance inflation. This can be viewed as a manifestation of the bullwhip effect in supply chains (Lee et al. 1997). To the best of our knowledge, this particular mechanism of generating the bullwhip effect has not been identified in the literature before.

By analyzing an analogous supply chain with an e-intermediary, we found that e-intermediaries with a small market share tend to increase farmers' profits, potentially boosting their profits by a factor of almost 2, but large e-intermediaries tend to diminish the profits of both farmers and the supply chain. These results provide a more balanced perspective on the value of e-intermediation, compared to the generally positive view that is advanced by case studies (see, e.g., references in the notes of Table 1). One way to interpret this result is that for-profit e-intermediaries who also aim to improve farmers' livelihoods cannot blindly operate as pure profit-maximizers and assume that the market will ensure that farmers benefit. Instead, they should also be conscious of their social objectives, particularly as they mature and grow.

Finally, from our analyses of both the traditional and e-intermediated supply chains, we observed that supply fragmentation seems to be the first-order issue that suppresses farmers' profits. Even in cases when e-intermediation has beneficial effects on farmers' profits, it is insufficient to mitigate the negative effects of supply fragmentation.

The broad implication of these results, viewed collectively, is a more nuanced interpretation of how such market interventions work. Even with a casual perusal of the popular and academic literature on developmental economics, one invariably encounters "improving market access" as one of the mechanisms through which rural farmers improve their livelihood (see, e.g., Qiang et al. 2012, and references therein). Our results, however, suggest that the central issue is not market "access" per se, but rather market "power". After all, in our model, farmers in the traditional chain have access to an idealized, frictionless market, but it is a lack of market power that diminishes their profits when they are highly fragmented. Similarly, if "access" was the central issue, then the size of the e-intermediary shouldn't matter, since farmers have the same amount of "access" regardless of the size (i.e., market share) of the e-intermediary. However, as the e-intermediary grows larger, the farmers' overall market "power" in the chain diminishes, and so do their profits.

In choosing to focus our study on these issues described above, we had to make some trade-offs in the conception of the model to sacrifice some realism. Below, we describe the main limitations of our model, in hopes that they can be addressed by future research. First, the two-period nature of our model was necessary for tractability, but represents an abstraction from reality, where auctions often take place daily over several weeks. Second, for tractability and in order to focus on broader supply chain issues, we focus on farmers' optimal production decisions for a single crop. Although this is a common simplification adopted by many supply chain papers (e.g., Lee et al. (1997), Taylor (2002), and Cachon and Lariviere (2005)), we acknowledge that this is nonetheless a simplification from reality. Third, our model assumes no fixed costs, transaction frictions, or economies of scale in production, all of which exist in reality. Again, these assumptions were made for tractability. Finally, our model does not incorporate considerations of crop quality/yield; in practice, this too is a likely driver of farmers' profits, which may be affected positively or negatively by intermediation efforts. Nevertheless, despite these limitations, we believe that our model broadly captures the main tradeoffs faced by farmers, retailers, and e-intermediaries in such supply chains, and the insights generated from the model are valuable to key stakeholders in such chains.

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Electronic Companion

EC.1. Statements and Proofs of Technical Lemmas for Section 4.1

LEMMA EC.1. *For a given $Q>0$, define $\ell_N(z) := \psi_N(z) - (Q-z)\psi_N'(z)$, where $\psi_N(z)$ is as defined in (10). It has the following properties:*

(a) $\ell_N(z)$ has at most two turning points in $[0,Q]$.

(b) If, for some $s \in (0,Q)$, $\ell_N'(s) = 0$ and $\ell_N''(s) \geq 0$, then $\ell_N'(z) > 0$ for all $s < z \leq Q$.

(c) If, for some $s \in (0,Q)$, $\ell_N'(s) = 0$ and $\ell_N''(s) \leq 0$, then $\ell_N'(z) > 0$ for all $0 \leq z < s$.

(d) If $\ell_N(0) < 1$, then $\ell_N'(0) < 0$.

(e) If $\ell_N(Q) > 0$, then $\ell_N'(Q) < 0$.

Proof. Fix $Q > 0$ throughout. We prove part (a) first, and begin with some preliminaries. Throughout the proof, define $\mu_{\mathbf{a}} := \sum_{j=1}^{m} \mu_j$ and $\sigma_{\mathbf{a}} := \sum_{j=1}^{m} \sigma_j$. Because

$$\frac{df_{\mathbf{a}}(z)}{dz} = -\frac{z-\mu_{\mathbf{a}}}{\sigma_{\mathbf{a}}^2} f_{\mathbf{a}}(z), \tag{EC.1}$$

we can rewrite (12) as

$$\bar{F}_{\mathbf{a}}(z) + \left[(Q-z)\left(1+\frac{1}{N}-\frac{z(z-\mu_{\mathbf{a}})}{N\sigma_{\mathbf{a}}^2}\right) - \frac{z}{N}\right] f_{\mathbf{a}}(z) = \bar{F}_{\mathbf{e}}(Q-z). \tag{EC.2}$$

Moreover, after some algebraic manipulations, we may express $\ell_N'(z) = (c_4(z)-1)\, f_{\mathbf{a}}(z)$, where c_4 represents a quartic polynomial with a negative leading coefficient, such that the turning points of ℓ_N may be obtained as the roots of the equation $c_4(z) = 1$.

Note that the equation $c_4(z) = 1$ either has 0, 2, or 4 real roots on $\mathbb{R}$. In the first two cases, (a) holds immediately. Thus, we focus on the final case where $c_4(z) = 1$ has 4 real roots. We will denote these roots as $y_j, j = 1,\ldots,4$ and assume WLOG that $y_1 \leq y_2 \leq y_3 \leq y_4$. Moreover, it is straightforward to see that y_1, y_3 represent local minima of ℓ_N and y_2, y_4 represent local maxima.

Suppose for a contradiction that $y_1 \geq 0$. From basic properties of polynomials, this implies that $\ell_N'(z) < 0$ for all $z < y_1$. Now consider the point, x_1, defined as $x_1 := \frac{\mu_{\mathbf{a}}}{2} - \sqrt{\frac{\mu_{\mathbf{a}}^2}{4} + (N+1)\sigma_{\mathbf{a}}^2}$. Note that $x_1 < 0 \leq y_1$. Further, it can be easily verified that x_1 is a local maximizer of ψ_N on $\mathbb{R}$ such that $\psi_N'(x_1) = 0$ and $\psi_N''(x_1) \leq 0$. However, this implies that $\ell_N'(x_1) = 2\psi_N'(x_1) - (Q-x_1)\psi_N''(x_1) \geq 0$, establishing a contradiction.

Next, suppose for a contradiction that $y_4 \leq Q$. Again, by basic polynomial properties, $\ell_N'(z) < 0$ for all $z > y_4$. In particular, this implies that $\ell_N'(Q) = \psi_N'(Q) \leq 0$. Using the definition of ψ_N and computing its derivative, we obtain that $\psi_N'(Q) \leq 0$ is satisfied when $Q \leq \frac{\mu_{\mathbf{a}}}{2} + \sqrt{\frac{\mu_{\mathbf{a}}^2}{4} + (N+1)\sigma_{\mathbf{a}}^2}$. Now consider the point, x_2, defined as $x_2 := \frac{\mu_{\mathbf{a}}}{2} + \sqrt{\frac{\mu_{\mathbf{a}}^2}{4} + (N+1)\sigma_{\mathbf{a}}^2}$. Note that $x_2 \geq Q > y_4$ by construction, and it can further be verified that x_2 is a local minimizer of ψ_N on $\mathbb{R}$ such that

$\psi_N'(x_2) = 0$ and $\psi_N''(x_2) \geq 0$. Moreover, we have $\ell_N'(x_2) = 2\psi_N'(x_2) - (Q - x_2)\psi_N''(x_2) \geq 0$, establishing a contradiction.

Hence, we have shown that when $c_4(z) = 1$ has 4 roots, it is necessary that $y_1 < 0$ and $y_4 > Q$, so that ℓ_N has at most 2 turning points in $[0, Q]$, thus proving part (a).

To prove (b), suppose that for some $s \in (0, Q)$, $\ell_N'(s) = 0$ and $\ell_N''(s) \geq 0$. Then s is a local minimum of ℓ_N in $(0, Q)$, i.e., $s = y_3$. Suppose for a contradiction that for some $s_1 > s$, $\ell_N'(s_1) \leq 0$. Then there must exist some local maximum s_2 of ℓ_N such that $y_3 < s_2 \leq s_1 < Q$. However, by part (a), there is at most one local maximum of ℓ_N in $(0, Q)$, $y_2 < y_3$, establishing a contradiction.

To prove (c), suppose that for some $s \in (0, Q)$, $\ell_N'(s) = 0$ and $\ell_N''(s) \leq 0$. Then s is a local maximum of ℓ_N in $(0, Q)$, i.e., $s = y_2$. Suppose for a contradiction that for some $s_1 < s$, $\ell_N'(s_1) \leq 0$. Then there must exist some local minimum s_2 of ℓ_N such that $0 < s_2 \leq s_1 < y_2$. However, by part (a), there is at most one local minimum of ℓ_N in $(0, Q)$, $y_3 > y_2$, establishing a contradiction.

To prove (d), suppose that $\ell_N(0) < 1$. This implies that

$$Q < \frac{N F_{\mathbf{a}}(0)}{(N+1) f_{\mathbf{a}}(0)}. \tag{EC.3}$$

To complete the proof of (d), we use the following well-known bound that was first proven by Gordon (1941) for the case of the standard normal distribution and adapted to our setting:

$$\frac{f_{\mathbf{a}}(z)}{\bar{F}_{\mathbf{a}}(z)} \geq \frac{z - \mu_{\mathbf{a}}}{\sigma_{\mathbf{a}}^2}. \tag{EC.4}$$

Therefore,

$$\frac{F_{\mathbf{a}}(0)}{f_{\mathbf{a}}(0)} = \frac{\bar{F}_{\mathbf{a}}(2\mu_{\mathbf{a}})}{f_{\mathbf{a}}(2\mu_{\mathbf{a}})} \leq \frac{\sigma_{\mathbf{a}}^2}{\mu_{\mathbf{a}}}. \tag{EC.5}$$

By direct computation,

$$\ell_N'(0) = \left[\frac{Q\left(1 + \frac{2}{N}\right)\mu_{\mathbf{a}}}{\sigma_{\mathbf{a}}^2} - 2\left(1 + \frac{1}{N}\right)\right] f_{\mathbf{a}}(0).$$

Consider

$$\begin{aligned}
\ell_N'(0)/f_{\mathbf{a}}(0) &= Q\left(1 + \frac{2}{N}\right)\frac{\mu_{\mathbf{a}}}{\sigma_{\mathbf{a}}^2} - 2\left(1 + \frac{1}{N}\right) \\
&< \frac{(N+2) F_{\mathbf{a}}(0)}{(N+1) f_{\mathbf{a}}(0)} \frac{\mu_{\mathbf{a}}}{\sigma_{\mathbf{a}}^2} - 2\left(1 + \frac{1}{N}\right) \quad \text{[By (EC.3)]} \\
&\leq \left(1 + \frac{1}{N+1}\right) - 2\left(1 + \frac{1}{N}\right) \quad \text{[By (EC.5)]} \\
&< 0
\end{aligned}$$

To prove part (e), note that $\ell_N(Q) = \psi_N(Q)$, and the result follows from Lemma 2(a). Q.E.D.

Lemma EC.2. *For a given $Q > 0$, define $r_N(z) := \bar{F}_{\mathbf{e}}(Q - z)$ and let $\ell_N(z)$ be as defined in Lemma EC.1, and $\bar{z}_N$ be as defined in (10). Then,*

(a) If $\ell_N(0) \leq r_N(0)$, then $\ell_N(z) \leq r_N(z)$ for all $z \in [0, Q]$.

(b) If $\ell_N(Q) \geq r_N(Q)$, then $\ell_N(z) \geq r_N(z)$ for all $z \in [0,Q]$.

(c) If $\ell_N(0) > r_N(0)$ and $\ell_N(Q) < r_N(Q)$, then there exists a unique value of $z := z(Q) \in (0,Q)$ such that $\ell_N(z) = r_N(z)$.

(d) Consider the optimization problem $\max_{z\in[0,\bar{z}_N \wedge Q]} g(z)$, such that $g'(z) = \ell_N(z) - r_N(z)$. Then, $\zeta_N(Q)$, as defined in (11), is optimal for this problem.

Proof. To prove (a), suppose that $\ell_N(0) \leq r_N(0)$. Suppose for a contradiction that for some $s \in [0,Q]$, $\ell_N(s) > r_N(s)$. Thus, $\ell_N(0) \leq r_N(0) \leq r_N(s) < \ell_N(s)$. Moreover, since $r_N(0) < 1$, Lemma EC.1(d) implies that $\ell_N'(0) < 0$. Together, these imply that there exists some $s_1 \in (0,s)$ such that s_1 is a local minimum of ℓ_N. By Lemma EC.1(b), $\ell_N'(z) > 0$ on $(s_1,Q]$. This implies that $\ell_N(Q) \geq \ell_N(s) > r_N(0) > 0$. By Lemma EC.1(e), this implies that $\ell_N'(Q) < 0$, establishing a contradiction. Therefore, $\ell_N(z) \leq r_N(z)$ for all $z \in [0,Q]$.

To prove (b), suppose that $\ell_N(Q) \geq r_N(Q)$. Suppose for a contradiction that there exists some $s \in [0,Q]$ such that $\ell_N(s) < r_N(s)$. Thus, $\ell_N(Q) \geq r_N(Q) \geq r_N(s) > \ell_N(s)$. Moreover, since $r_N(Q) > 0$, Lemma EC.1(e) implies that $\ell_N'(Q) < 0$. Together, these imply that there must exist some $s_1 \in (s,Q)$ such that s_1 is a local maximum of ℓ_N. By Lemma EC.1(c), $\ell_N'(z) > 0$ on $[0,s_1)$, and in particular, $\ell_N(0) \leq \ell_N(s) < r_N(s) < 1$. However, by Lemma EC.1(d), this implies that $\ell_N'(0) < 0$, establishing a contradiction. Therefore, in this case, $\ell_N(z) \geq r_N(z)$ for all $z \in [0,Q]$.

To prove (c), suppose that $\ell_N(0) > r_N(0)$ and $\ell_N(Q) < r_N(Q)$. Hence, there is some $z^* \in (0,Q)$ for which $\ell_N(z^*) = r_N(z^*)$. We claim that in order to show that this z^* is unique, it suffices to show that there is some $z^{**} \leq z^*$ for which $\ell_N'(z^{**}) < 0$. Suppose that this were true, and now suppose for a contradiction that there is some other $s > z^*$ (WLOG) such that $\ell_N(s) = r_N(s)$. Then, since r_N is strictly increasing, we must have $\ell_N(s) = r_N(s) > r_N(z^*) = \ell_N(z^*)$. Since $\ell_N'(z^{**}) < 0$, there must exist some $s_1 \in (z^{**}, s)$ that is a local minimum of ℓ_N. By Lemma EC.1(b), this implies that $\ell_N'(z) > 0$ on $(s_1,Q]$. In particular, this means that $\ell_N'(Q) > 0$, and furthermore, $\ell_N(Q) \geq \ell_N(s) = r_N(s) > 0$. By Lemma EC.1(e), $\ell_N(Q) > 0$ implies that $\ell_N'(Q) < 0$, establishing a contradiction. Hence, z^* must be unique.

It remains to show the existence of such a point z^{**}. We consider two separate cases:

Case 1: $\ell_N'(0) < 0$. Here, we can choose $z^{**} = 0$.

Case 2: $\ell_N'(0) \geq 0$. We need only consider the sub-case in which $\ell_N(0) \geq 1$ since the converse sub-case is ruled out by Lemma EC.1(d). Since $\ell_N(0) \geq 1 > r_N(Q) > \ell_N(Q)$, there must exist some $y_2 \in [0,Q)$ that is a local maximum of ℓ_N. Moreover, $z^* > y_2$, since $\ell_N(z) \geq 1 > r_N(z)$ for all $z \leq y_2$. Hence, any point $z^{**} \in (y_2, z^*)$ would satisfy $z^{**} < z^*$ and $\ell_N'(z^{**}) < 0$, as required.

The proof of part (d) proceeds by direct verification using parts (a)-(c). In the case where $\psi_N(0) - Q\psi_N'(0) \leq \bar{F}_{\mathbf{e}}(Q)$, part (a) implies that g is decreasing over $[0,Q]$, and is thus maximized at $z = 0$. An identical logic applies for the case of $\psi_N(Q) \geq \bar{F}_{\mathbf{e}}(0)$ using part (b) and is omitted for brevity. In the case where $\psi_N(0) - Q\psi_N'(0) > \bar{F}_{\mathbf{e}}(Q)$ and $\psi_N(Q) < \bar{F}_{\mathbf{e}}(0)$, part (c) guarantees that a solution $z_N(Q)$ to (12) on $(0,Q)$ exists, and moreover that g is increasing on $(0, z_N(Q))$ and decreasing on $(z_N(Q), Q)$. Thus, if $\bar{z}_N < z_N(Q)$, $\bar{z}_N$ maximizes g on $[0, \bar{z}_N \wedge Q]$, whereas if $\bar{z}_N \geq z_N(Q)$, then z_N maximizes g on $[0, \bar{z}_N \wedge Q]$, as specified by the definition of ζ_N given in (11).

Q.E.D.

EC.2. Proofs of Main Results

Proof of Lemma 1

Proof. Combining the two conditions from Definition 1, and the retailers' demand functions from (1), the auction price $p^{\mathbf{a}}$ satisfies

$$\sum_{j=1}^{m} \left(\mu_j + \Phi^{-1} \left(\frac{\pi - p^{\mathbf{a}}}{\pi} \right) \sigma_j \right) = q^{\mathbf{a}}.$$

Re-arranging, this gives us $p^{\mathbf{a}} = \pi \left(1 - \Phi \left(\frac{q^{\mathbf{a}} - \sum_{j=1}^{m} \mu_j}{\sum_{j=1}^{m} \sigma_j} \right) \right) = \pi \bar{F}_{\mathbf{a}}(q^{\mathbf{a}})$. Q.E.D.

Proof of Lemma 2

Proof. To prove (a), define $h(z) := f(z)/\bar{F}(z)$. Noting that we may write $L_{s,\nu}(z) = \bar{F}(z+s)[1 - \nu z h(z+s)]$, for any $z \in \mathcal{L}_{s,\nu}$, we have

$$L_{s,\nu}'(z) = -f(z+s)[1 - \nu z h(z+s)] + \nu \bar{F}(z+s)[-h(z+s) - z h'(z+s)].$$

In the display above, $1 - \nu z h(z+s) \geq 0$ since $z \in \mathcal{L}_{s,\nu}$. Also, $h(z+s) > 0$ and $zh'(z+s) \geq 0$ because $z \geq 0$ and $h'(z+s) > 0$ by the IFR property of the normal distribution (Lariviere 2006, Banciu and Mirchandani 2013). Hence, $L_{s,\nu}'(z) < 0$ for all $z \in \mathcal{L}_{s,\nu}$.

To prove (b), note that $\mathcal{L}_{s,\nu} \neq \emptyset$ since $\bar{F}(s) > 0$, i.e., $0 \in \mathcal{L}_{s,\nu}$ trivially. Moreover, from part (a), since $L_{s,\nu}(z)$ is strictly decreasing where it is positive, it follows that $\mathcal{L}_{s,\nu} = [0, z_{s,\nu}^*]$ for some $z_{s,\nu}^*$, where $z_{s,\nu}^*$ is potentially infinite. We conclude the proof by showing that $z_{s,\nu}^*$ is finite. To see this, note that the condition $L_{s,\nu}(z) = 0$ is equivalent to $\nu h(z+s) = 1/z$. It is well known (see, e.g., Gordon 1941) that $h(z+s) \to \infty$ as $z \to \infty$, and since $1/z \to 0$ as $z \to \infty$, there exists some $z_{s,\nu}^* < \infty$ for which $h(z_{s,\nu}^* + s) = 1/z_{s,\nu}^*$. Moreover, since $h(z+s)$ is strictly increasing in z and $1/z$ is strictly decreasing in z, $z_{s,\nu}^*$ is unique. Q.E.D.

Proof of Proposition 1

Proof. First, we show that (3) has a unique solution on $\mathbb{R}_+$. By Assumption 1, the LHS of (3) exceeds the RHS at $z = 0$. Moreover, by Lemma 2 (for the case $s = 0$ and $\nu = 1/N$), the LHS eventually crosses zero as $z \to \infty$. By the same Lemma, in the region where the LHS is positive, it is strictly decreasing. Hence, (3) has a unique solution on $\mathbb{R}_+$.

To prove the result, let $(\widehat{q}, \widehat{q}^{\mathbf{a}}(\cdot))$ be arbitrary. We will use the specification of a symmetric equilibrium from Definition 2 to find necessary conditions on these quantities. Observe that each farmer's problem can be equivalently written as a single optimization problem

$$\max_{q\geq 0, 0\leq q^{\mathbf{a}}\leq q} \left\{-cq + \pi q^{\mathbf{a}} \bar{F}_{\mathbf{a}}\left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}(\widehat{q})\right)\right\}.$$

We observe that the objective of this problem is decreasing in q, which implies that a necessary condition for optimality is $q = q^{\mathbf{a}}$. From this condition, and writing $\widehat{q}^{\mathbf{a}} = \widehat{q}^{\mathbf{a}}(\widehat{q})$ for brevity, the problem above simplifies to

$$\max_{q^{\mathbf{a}}\geq 0} \left\{-cq^{\mathbf{a}} + \pi q^{\mathbf{a}} \bar{F}_{\mathbf{a}}\left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}\right)\right\} \tag{EC.6}$$

The derivative of the objective of (EC.6) at an arbitrary point $q^{\mathbf{a}} \geq 0$ may be expressed as $g(q^{\mathbf{a}}) - c$, where

$$g(q^{\mathbf{a}}) := \pi \bar{F}_{\mathbf{a}}\left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}\right) - \pi q^{\mathbf{a}} f_{\mathbf{a}}\left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}\right). \tag{EC.7}$$

We note that by Lemma 2(a), $g(q^{\mathbf{a}})$ is strictly decreasing in the region where $g(q^{\mathbf{a}}) \geq 0$. Since $c > 0$, this implies that there exists a unique value of $q^{\mathbf{a}}$ such that $g(q^{\mathbf{a}}) = c$, and moreover, this choice of $q^{\mathbf{a}}$ maximizes the objective of (EC.6).

By Definition 2, for $\widehat{q}^{\mathbf{a}}$ to be an equilibrium, each farmer choosing $q^{\mathbf{a}} = \widehat{q}^{\mathbf{a}}$ has to be a best response. First, we note that $\widehat{q}^{\mathbf{a}} = 0$ cannot hold in equilibrium, because (EC.7) gives $\pi \bar{F}_{\mathbf{a}}(0) - c > 0$ by Assumption 1, implying that any given farmer would have higher profits by deviating and choosing some small $q^{\mathbf{a}} > 0$. Therefore, for $\widehat{q}^{\mathbf{a}}$ to represent an equilibrium, it is necessary that $g(\widehat{q}^{\mathbf{a}}) = c$. Changing variables such that $\widehat{q}^{\mathbf{a}} = \frac{1}{N} Q_N^{\mathbf{T}}$ yields (3) as required. Uniqueness of the equilibrium follows from the fact that (3) has a unique solution.

From this, each farmer's profit, $\Pi_N^{\mathbf{T}}$, is given by $\Pi_N^{\mathbf{T}} = \frac{Q_N^{\mathbf{T}}}{N}\left(\pi \bar{F}_{\mathbf{a}}(Q_N^{\mathbf{T}}) - c\right) = \pi \left(\frac{Q_N^{\mathbf{T}}}{N}\right)^2 f_{\mathbf{a}}(Q_N^{\mathbf{T}})$, where the last equation is by (3). Q.E.D.

Proof of Proposition 2

Proof. Fix some N. Rewrite (3) as

$$1 = \frac{c}{\pi \bar{F}_{\mathbf{a}}(z)} + \frac{z f_{\mathbf{a}}(z)}{N \bar{F}_{\mathbf{a}}(z)} \tag{EC.8}$$

Suppose for a contradiction that $Q_N^{\mathbf{T}} > Q_{N+1}^{\mathbf{T}}$. For notational convenience, define the functions $g, h : \mathbb{R}_+ \to \mathbb{R}$ as $g(z) := \frac{c}{\pi \bar{F}_{\mathbf{a}}(z)}$ and $h(z) := \frac{z f_{\mathbf{a}}(z)}{\bar{F}_{\mathbf{a}}(z)}$. Note that g, h are strictly increasing functions; the latter is a consequence of the IGFR property of normal distributions (Lariviere 2006, Banciu and Mirchandani 2013). Now, (EC.8) implies that we must have

$$g(Q_N^{\mathbf{T}}) + \frac{1}{N} h(Q_N^{\mathbf{T}}) = g(Q_{N+1}^{\mathbf{T}}) + \frac{1}{N+1} h(Q_{N+1}^{\mathbf{T}}). \tag{EC.9}$$

However, by our assumption that $Q_N^{\mathbf{T}} > Q_{N+1}^{\mathbf{T}}$, we have $g(Q_N^{\mathbf{T}}) > g(Q_{N+1}^{\mathbf{T}})$, and $\frac{1}{N} h(Q_N^{\mathbf{T}}) > \frac{1}{N} h(Q_{N+1}^{\mathbf{T}}) > \frac{1}{N+1} h(Q_{N+1}^{\mathbf{T}})$. Together, these imply that (EC.9) cannot be satisfied, establishing the contradiction. Q.E.D.

Proof of Proposition 3

Proof. This follows from (3) by observing that the function $zf_{\mathbf{a}}(z)$ is bounded over all $z \in \mathbb{R}$ and by taking $N \to \infty$. Q.E.D.

Proof of Proposition 5

Proof. Consider the difference:

$$\begin{aligned}\Delta &:= Q^{\mathbf{T}}_{\infty} - Q^{\mathbf{I}} \\ &= \Phi^{-1}\left(\frac{\pi - c}{\pi}\right)\left(\sum_{j=1}^{M}\sigma_j - \sqrt{\sum_{j=1}^{M}\sigma_j^2}\right) \text{ [From (4), (5)].}\end{aligned}$$

First note that $\sum_{j=1}^{M}\sigma_j \geq \sqrt{\sum_{j=1}^{M}\sigma_j^2}$. Hence, if $c < \pi/2$, we have $\Phi^{-1}\left(\frac{\pi-c}{\pi}\right) > 0$, and the difference $\Delta \geq 0$. By a similar argument, the converse result holds for $c > \pi/2$, and $\Delta = 0$ for $c = \pi/2$. Q.E.D.

Proof of Theorem 1

Proof. To prove parts (a) and (b), note that term (i) in (6), which represents the total farmers' profits, decreases in Q. By Proposition 2, as N increases, in equilibrium, the total production quantity increases, which implies that the total farmer surplus decreases. Conversely, term (ii) in (6), which represents the total retailers' profits, increases in Q, and by an analogous argument, now increases as N increases.

To prove part (c), first recall that the total profit loss due to retailer fragmentation is quantified in term (iii) of decomposition (6), and suppose that $c < \pi/2$ and fix some Q. Let $\bar{F}_{\mathbf{I}}(\cdot)$ represent the tail cdf of the integrated supply chain's total retailer demand distribution, i.e., $N\left(\sum_{j=1}^{M}\mu_j, \sum_{j=1}^{M}\sigma_j^2\right)$. The derivative of term (iii) is

$$\begin{aligned}\pi\bar{F}_{\mathbf{I}}(Q) - \pi\sum_{j=1}^{M}\bar{F}_{\mathbf{j}}(Q_j) &= \pi\bar{F}_{\mathbf{I}}(Q) - \pi\sum_{j=1}^{M}\bar{F}_{\mathbf{a}}(Q) \text{ [Definition 1, Lemma 1]} \\ &= \pi\bar{F}_{\mathbf{I}}(Q) - M\pi\bar{F}_{\mathbf{a}}(Q).\end{aligned}$$

We aim to show that this expression is negative at $Q = Q^{\mathbf{T}}_{\infty}$. To do so, it suffices to show that $\bar{F}_{\mathbf{I}}(Q^{\mathbf{T}}_{\infty}) \leq \bar{F}_{\mathbf{a}}(Q^{\mathbf{T}}_{\infty})$. To this end, note that

$$\begin{aligned}\bar{F}_{\mathbf{I}}(Q^{\mathbf{T}}_{\infty}) &\leq \bar{F}_{\mathbf{I}}(Q^{\mathbf{I}}) \quad \text{[Proposition 5 and } \bar{F}_{\mathbf{I}}(\cdot) \text{ is decreasing]} \\ &= \frac{c}{\pi} \quad \text{[From (5)]} \\ &= \bar{F}_{\mathbf{a}}(Q^{\mathbf{T}}_{\infty}), \text{ [From (4)]}\end{aligned}$$

which concludes the proof of part (c).

To prove part (d), suppose that $c > \pi/2$, and fix an arbitrary N. Note that we have

$$Q^{\mathbf{T}}_{N} \leq Q^{\mathbf{T}}_{N+1} \leq Q^{\mathbf{T}}_{\infty} \leq Q^{\mathbf{I}},$$

where the last inequality above is by Proposition 5 when $c > \pi/2$ and the other inequalities are by Proposition 2. Since $\Pi^{\mathbf{I}}(Q)$ is concave, it is increasing for every $Q \leq Q^{\mathbf{I}}$, which therefore implies that $\Pi^{\mathbf{I}}(Q_N^{\mathbf{T}}) < \Pi^{\mathbf{I}}(Q_{N+1}^{\mathbf{T}})$, concluding the proof. Q.E.D.

Proof of Theorem 2

We characterize the equilibria of this game by backward induction. First, we will characterize each farmer's actions in period 2 under a symmetric equilibrium. We do so by solving problem (7) for fixed values of q and p and finding conditions on $\widehat{q}^{\mathbf{a}}$ as a function of q,p such that farmers optimally choose $\widehat{q}^{\mathbf{a}}$ in a symmetric equilibrium. Note that in (7), for any $q,p \geq 0$, the objective is increasing in $q^{\mathbf{e}}$, which implies that at optimality, we necessarily have $q^{\mathbf{a}} + q^{\mathbf{e}} = q$. This means that we can eliminate $q^{\mathbf{e}}$ in (7), and write the optimization problem equivalently as

$$\begin{aligned} \Pi(q,p) := pq + \max_{q^{\mathbf{a}}} & \left\{ -pq^{\mathbf{a}} + \pi q^{\mathbf{a}} \bar{F}_{\mathbf{a}} \left(q^{\mathbf{a}} + (N-1)\widehat{q}^{\mathbf{a}}(\widehat{q},p) \right) \right\} \\ \text{s.t. } & 0 \leq q^{\mathbf{a}} \leq q. \end{aligned} \tag{EC.10}$$

Following a similar analysis to Proposition 1, it follows that a necessary and sufficient condition for $\widehat{q}^{\mathbf{a}}(q,p)$ to be an equilibrium is that it satisfies

$$\widehat{q}^{\mathbf{a}}(q,p) := \begin{cases} 0 & \text{if } p \geq \pi \bar{F}_{\mathbf{a}}(0) \\ \frac{z_{\mathbf{a}}(p)}{N} \wedge q & \text{if } p < \pi \bar{F}_{\mathbf{a}}(0) \end{cases} \tag{EC.11}$$

where in the latter case $z_{\mathbf{a}}(p)$ represents the unique value of $z \in \mathbb{R}_+$ that solves

$$\bar{F}_{\mathbf{a}}(z) - \frac{z f_{\mathbf{a}}(z)}{N} = \frac{p}{\pi}. \tag{EC.12}$$

Therefore, in equilibrium, the second-stage profit of each farmer may be written as

$$\Pi(q,p) = pq + \left(\frac{z_{\mathbf{a}}(p) \wedge Nq}{N} \right) \left(\pi \bar{F}_{\mathbf{a}} \left(z_{\mathbf{a}}(p) \wedge Nq \right) - p \right). \tag{EC.13}$$

We note that Lemma 2 implies that $z_{\mathbf{a}}(p)$ is strictly decreasing for $p \in (0, \pi \bar{F}_{\mathbf{a}}(0))$, and therefore possesses an inverse. In particular, recalling the definition of $\bar{z}_N$ from (10), we have $z_{\mathbf{a}}^{-1}(z) = \pi \psi_N(z)$ for all $z \in [0, \bar{z}_N]$.

Next, we solve the e-intermediary's pricing problem, i.e., problem (8). Fix $q \geq 0$, and define $Q = Nq$ for notational brevity. We now claim that if $\psi_N(Q) \geq 0$, then it is without loss that $p \geq \pi \psi_N(Q)$. Consider any p such that $0 \leq p \leq \pi \psi_N(Q)$. Then, for such a choice of p, by (EC.12) and (10), we would have $z_{\mathbf{a}}(p) < Nq = Q$, so that from (EC.11), we get $N\widehat{q}^{\mathbf{a}}(q,p) = Q$, and therefore $N\widehat{q}^{\mathbf{e}}(q,p) = Q - N\widehat{q}^{\mathbf{a}}(q,p) = 0$. Thus the objective of (8) at this point would be $\pi \mathbb{E}(D^{\mathbf{e}} \wedge 0)$ for any such p. In particular, at the boundary point, $p = \pi \psi_N(Q)$ has the same objective value of $\pi \mathbb{E}(D^{\mathbf{e}} \wedge 0)$. Therefore, it is without loss that $p \geq \pi \psi_N(Q)$. By a similar logic, it is without loss that $p \leq \pi \bar{F}_{\mathbf{a}}(0)$.

Now, for p such that $\max\{\pi\psi_N(Q),0\} \le p \le \pi\bar{F}_{\mathbf{a}}(0)$, we may simplify (EC.11) to get

$$N\widehat{q}^{\mathbf{a}}(q,p) = z_{\mathbf{a}}(p) \qquad \text{and} \qquad N\widehat{q}^{\mathbf{e}}(q,p) = Q - z_{\mathbf{a}}(p). \tag{EC.14}$$

Therefore, after dividing (8) by the constant π, we get

$$\begin{aligned}
&\max_{p\ge 0}\left\{\mathbb{E}\left(D^{\mathbf{e}}\wedge N\widehat{q}^{\mathbf{e}}(q,p)\right) - \frac{p}{\pi}N\widehat{q}^{\mathbf{e}}(q,p)\right\} \\
&= \max_{\max\{\pi\psi_N(Q),0\}\le p\le \pi\bar{F}_{\mathbf{a}}(0)}\left\{\mathbb{E}\left(D^{\mathbf{e}}\wedge N\widehat{q}^{\mathbf{e}}(q,p)\right) - \frac{p}{\pi}N\widehat{q}^{\mathbf{e}}(q,p)\right\} && \text{[No loss of optimality in restriction]} \\
&= \max_{\max\{\pi\psi_N(Q),0\}\le p\le \pi\bar{F}_{\mathbf{a}}(0)}\left\{\mathbb{E}\left(D^{\mathbf{e}}\wedge (Q - z_{\mathbf{a}}(p))\right) - \frac{p}{\pi}(Q - z_{\mathbf{a}}(p))\right\} && \text{[By (EC.14)]} \\
&= \max_{0\le z_{\mathbf{a}}(p)\le \min\{\bar{z}_N,Q\}}\left\{\mathbb{E}\left(D^{\mathbf{e}}\wedge (Q - z_{\mathbf{a}}(p))\right) - \psi_N(z_{\mathbf{a}}(p))(Q - z_{\mathbf{a}}(p))\right\} && \text{[Change of optimization variable]} \\
&= \max_{0\le z\le \min\{\bar{z}_N,Q\}}\left\{\mathbb{E}\left(D^{\mathbf{e}}\wedge (Q - z)\right) - \psi_N(z)(Q - z)\right\} && \text{[Eliminate variable } p\text{]}
\end{aligned}$$

Note that in the third equality, we have used the fact that $\pi\psi_N(z_{\mathbf{a}}(p)) = p$ for any feasible p.

We also note that the final form of the optimization problem above can be verified to satisfy the conditions of Lemma EC.2(d), and hence its optimal solution is given by $\zeta_N(Q)$ as defined in (11). The e-intermediary's optimal pricing policy is obtained by transforming the optimal value of $\zeta_N(Q)$, and is expressed as $\widehat{p}^{\mathbf{e}}(Q) = \pi\psi_N(\zeta_N(Q))$. Furthermore, substituting this into (EC.14), we obtain

$$N\widehat{q}^{\mathbf{a}}(q,\widehat{p}^{\mathbf{e}}(Q)) = z_{\mathbf{a}}(\widehat{p}^{\mathbf{e}}(Q)) = z_{\mathbf{a}}(\pi\psi_N(\zeta_N(Q))) = \zeta_N(Q), \tag{EC.15}$$

which represents the quantity traded at the auction when the e-intermediary prices optimally.

Finally, we will show that problem (13) represents a given farmer's first-stage problem of choosing his own production quantity, q, given the symmetric production quantities $\widehat{q}$ of each of the $N-1$ other farmers (i.e., total production quantity is $Q = q + (N-1)\widehat{q}$), taking into account the optimal pricing from the e-intermediary and equilibrium decisions from all farmers in the second stage. Using the derived expression for each farmer's second-stage profit in (EC.13), substitute this into (9), and also substitute the optimal pricing function of the e-intermediary, $\widehat{p}^{\mathbf{e}}(Q) = \pi\psi_N(\zeta_N(Q))$ and equilibrium quantity at the auction (EC.15), so that (9) simplifies to

$$\begin{aligned}
\max_{q,Q}\ & \left(\psi_N(\zeta_N(Q)) - \frac{c}{\pi}\right)q + \frac{\zeta_N(Q)}{N}\left(\bar{F}_{\mathbf{a}}\left(\zeta_N(Q)\right) - \psi_N(\zeta_N(Q))\right) \\
\text{s.t. } & Q = q + (N-1)\widehat{q} \\
& Q, q \ge 0.
\end{aligned}$$

Using (10) to simplify the expression above, adding the auxiliary variable $z = \zeta_N(Q)$ and eliminating the auxiliary variable Q yields (13).

Therefore, if supposing (z^*, q^*) is optimal in (13) such that $q^* = \widehat{q}$, then, $N\widehat{q}$ represents the (symmetric) equilibrium total production of all farmers, giving us part (a). From (EC.15), the total quantity sent to the auction is $\zeta_N(N\widehat{q})$ and the total quantity sent to the e-intermediary is $N\widehat{q} - \zeta_N(N\widehat{q})$, giving us parts (b) and (c). Finally, from the optimal e-intermediary's pricing policy, the e-intermediary's equilibrium price is $\widehat{p}^{\mathbf{e}} = \pi\psi_N(\zeta_N(N\widehat{q}))$, establishing part (d). Q.E.D.

Proof of Corollary 2

Proof. To prove (a), suppose for a contradiction that in equilibrium, each farmer sends some $\widehat{q}^{\mathsf{a}}$ to the auction such that the equilibrium auction price is $\pi\bar{F}_{\mathbf{a}}(N\widehat{q}^{\mathsf{a}}) < \widehat{p}^{\mathsf{e}}$. If $\widehat{q}^{\mathsf{a}} > 0$, then any given farmer would increase his profit by reducing $\widehat{q}^{\mathsf{a}}$ by some $\epsilon > 0$ and sending this ϵ to the e-intermediary. If $\widehat{q}^{\mathsf{a}} = 0$, since the objective of (8) is decreasing in p, then the e-intermediary can increase his profit by deviating and reducing $\widehat{p}^{\mathsf{e}}$ by some small amount. Therefore, $\pi\bar{F}_{\mathbf{a}}(N\widehat{q}^{\mathsf{a}}) \geq \widehat{p}^{\mathsf{e}}$ is necessary in equilibrium.

To prove (b), suppose for a contradiction that in equilibrium, $\widehat{p}^{\mathsf{e}} < c$. If $\widehat{q}^{\mathsf{e}} > 0$, then any farmer i can increase his profit by simultaneously reducing both q^{e} and q by some small $\epsilon > 0$. Q.E.D.

Proof of Proposition 6

Proof. Throughout, we subscripted N to explicitly denote the dependence of various equilibrium quantities on N. For example, we will use $\widehat{p}^{\mathsf{e}}_N$ to denote the equilibrium price offered by the e-intermediary.

We begin by proving part (a). To do so, let $\widehat{q}_N$ represent the equilibrium production of each farmer as a function of N. We will first assert that

$$\limsup_{N\to\infty} N\widehat{q}_N < \infty. \tag{EC.16}$$

Suppose for a contradiction that $\limsup_{N\to\infty} N\widehat{q}_N = \infty$. We note that from Corollary 2(b), we necessarily have $\limsup_{N\to\infty} N\widehat{q}^{\mathsf{e}}_N < \infty$, since either $\widehat{q}^{\mathsf{e}}_N = 0$ or $\widehat{p}^{\mathsf{e}}_N \geq c > 0$, and in the latter case, the e-intermediary would make negative profit for large N. Since each farmer's production is fully allocated either to the e-intermediary or the auction, it is necessary that $\limsup_{N\to\infty} N\widehat{q}^{\mathsf{a}}_N = \infty$. Therefore, this implies that there exists large enough N, such that the equilibrium auction price, $\pi\bar{F}_{\mathbf{a}}(N\widehat{q}^{\mathsf{a}}_N) < c$, contradicting Corollary 2(a).

We now proceed to prove (a), that is, we claim that $\lim_{N\to\infty} \widehat{p}^{\mathsf{e}}_N = c$. We do so in two steps. First, we will prove that each farmer's equilibrium profit converges to zero. By (EC.16), we necessarily have $\widehat{q}_N \to 0$. Thus, for any $\epsilon > 0$, there exists large enough N such that $\widehat{q}_N < \epsilon$. Since the objective of the optimization problem in (EC.10) is bounded above by πq, this implies that each farmer's equilibrium profit is bounded above by $(\widehat{p}^{\mathsf{e}}_N - c)\epsilon + \pi\epsilon$ for sufficiently large N. If $\widehat{q}^{\mathsf{e}}_N > 0$, since $\widehat{p}^{\mathsf{e}}_N \geq c$ (by Corollary 2) and $\widehat{p}^{\mathsf{e}}_N \leq \pi$ (obvious), and since ϵ is arbitrary, this means that each farmer's equilibrium profit converges to zero as $N \to \infty$. If $\widehat{q}^{\mathsf{e}}_N = 0$, each farmer's equilibrium profit is bounded above by $\pi\epsilon$ for sufficiently large N, and since ϵ is arbitrary, this means that each farmer's equilibrium profit converges to zero as $N \to \infty$.

Second, suppose for a contradiction that $\limsup_{N\to\infty} \widehat{p}^{\mathsf{e}}_N > c$. Because $q^{\mathsf{a}} = 0$ is feasible in (EC.10), each farmer's profit is bounded from below by $(\widehat{p}^{\mathsf{e}}_N - c)\widehat{q}_N$. Now we claim that we can establish a profitable deviation for any given farmer, for sufficiently large N, by violating (EC.16), thereby establishing a contradiction. To see this, note that there must exist a subsequence N_j such that $\inf_j \widehat{p}^{\mathbf{e}}_{N_j} > c$, and define $\delta := \inf_j \left\{\widehat{p}^{\mathbf{e}}_{N_j} - c\right\} > 0$. By choosing $\widehat{q}_{N_j} = 1$ for all N_j, each farmer's profit is bounded from below by δ along this entire subsequence, contradicting the assertion that each farmer's profit converges to zero. Therefore, part (a) must hold.

Part (b) uses the equilibrium auction price in Lemma 1 and follows from (a) and (EC.12), noting that $z f_{\mathbf{a}}(z)$ is bounded on $\mathbb{R}$.

To prove (c), first define define $g_N : \mathbb{R}_+ \to \mathbb{R}$ as follows:

$$g_N(y) := \psi_N(Q^{\mathbf{a}}_\infty) - y\psi'_N(Q^{\mathbf{a}}_\infty) - \bar{F}_{\mathbf{e}}(y).$$

Suppose that for a given N, a given y satisfies $g_N(y) = 0$. Then, by (11) and (12), y is the quantity that is sent to the e-intermediary when $Q^{\mathbf{a}}_\infty$ is sent to the auction when there are N farmers. Since we know that $Q^{\mathbf{a}}_\infty$ is sent to the auction in the limit as $N \to \infty$, if some given $Q^{\mathbf{e}}_\infty$ satisfies $\lim_{N\to\infty} g_N(Q^{\mathbf{e}}_\infty) = 0$, then $Q^{\mathbf{e}}_\infty$ would represent the limiting quantity sent to the e-intermediary.

In what follows, we show that the condition $\lim_{N\to\infty} g_N(Q^{\mathbf{e}}_\infty) = 0$ can be simplified to (14). For any $z \geq 0$, using the definition of ψ_N, we obtain $\lim_{N\to\infty} \psi_N(z) = \bar{F}_{\mathbf{a}}(z)$ and $\lim_{N\to\infty} \psi'_N(z) = -f_{\mathbf{a}}(z)$, such that for any $y \geq 0$,

$$\lim_{N\to\infty} g_N(y) = \bar{F}_{\mathbf{a}}(Q^{\mathbf{a}}_\infty) + y f_{\mathbf{a}}(Q^{\mathbf{a}}_\infty) - \bar{F}_{\mathbf{e}}(y) = \frac{c}{\pi} + y f_{\mathbf{a}}(Q^{\mathbf{a}}_\infty) - \bar{F}_{\mathbf{e}}(y).$$

Hence, $Q^{\mathbf{e}}_\infty$ that satisfies $\lim_{N\to\infty} g_N(Q^{\mathbf{e}}_\infty) = 0$ will satisfy (14) and vice-versa.

It remains to show that (14) has a unique solution. Note that at $z = 0$, the LHS of (14) is $\bar{F}_{\mathbf{e}}(0) > c/\pi$ by Assumption 1. Moreover, as $z \to +\infty$, the LHS is strictly decreasing in z and diverges to $-\infty$. Hence, a unique solution to (14) exists, completing the proof. Q.E.D.

Proof of Proposition 7

Proof. We begin with several preliminaries. First, for notational brevity, throughout the proof, we will let $K := \Phi^{-1}\left(\frac{\pi - c}{\pi}\right)$. Note that K can be positive or negative depending on whether $c < \pi/2$ or otherwise. Second, from (14), it is easy to see that $Q^{\mathbf{e}}_\infty \leq Q^{\mathbf{e}}_*$, where we define $\mu_{\mathbf{e}} := \sum_{j=m+1}^{M} \mu_j$, and $Q^{\mathbf{e}}_* := \mu_{\mathbf{e}} + K\sigma_{\mathbf{e}}$. Third, by direct computation, it is easy to see that $\bar{F}_{\mathbf{e}}(Q)$ is concave for $Q < \mu_{\mathbf{e}}$ and convex for $Q > \mu_{\mathbf{e}}$. Finally, by the triangle inequality and respective definitions of $\sigma_{\mathbf{I}}, \sigma_{\mathbf{a}}, \sigma_{\mathbf{e}}$, and $\sigma_{\mathbf{T}}$, we have, for any $m = 1, \ldots, M$,

$$\sigma_{\mathbf{I}} \leq \sigma_{\mathbf{a}} + \sigma_{\mathbf{e}} \leq \sigma_{\mathbf{T}}. \tag{EC.17}$$

We now prove the first part of the proposition. Suppose that $c < \pi/2$, i.e., $K > 0$. Then, we claim that we necessarily have

$$\mu_{\mathbf{e}} \leq Q^{\mathbf{e}}_* \frac{\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}} + \sigma_{\mathbf{a}}}. \tag{EC.18}$$

This is because

$$Q^{\mathbf{e}}_{*}\frac{\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}-\mu_{\mathbf{e}}=\frac{\sigma_{\mathbf{e}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}\left(K\sigma_{\mathbf{a}}-\mu_{\mathbf{e}}\right)\geq 0,$$

where the final inequality is because $K>0$ and because (15) implies that $\mu_{\mathbf{e}}\leq K\sigma_{\mathbf{a}}$. Now, since $K>0$, we have $Q^{\mathbf{e}}_{*}>\mu_{\mathbf{e}}$ by definition. Hence, using a Taylor expansion for $\bar{F}_{\mathbf{e}}(Q)$ in the region $\mu_{\mathbf{e}}<Q<Q^{\mathbf{e}}_{*}$, we obtain

$$\bar{F}_{\mathbf{e}}(Q)-Qf_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty})\geq\bar{F}_{\mathbf{e}}(Q^{\mathbf{e}}_{*})-(Q-Q^{\mathbf{e}}_{*})f_{\mathbf{e}}(Q^{\mathbf{e}}_{*})-Qf_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty}).$$

The inequality is a consequence of the convexity of $\bar{F}_{\mathbf{e}}(Q)$ on $Q>\mu_{\mathbf{e}}$. From this inequality, the value of Q that sets the RHS of the above inequality to c/π must therefore bound $Q^{\mathbf{e}}_{\infty}$ from below, that is,

$$Q^{\mathbf{e}}_{\infty}\geq Q^{\mathbf{e}}_{*}\frac{f_{\mathbf{e}}(Q^{\mathbf{e}}_{*})}{f_{\mathbf{e}}(Q^{\mathbf{e}}_{*})+f_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty})}=Q^{\mathbf{e}}_{*}\frac{\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}},\tag{EC.19}$$

where the final equality is by direct computation using the definitions of $Q^{\mathbf{e}}_{*}$ and $Q^{\mathbf{a}}_{\infty}$. Therefore,

$$\begin{aligned}Q^{\mathbf{E}}_{\infty}-Q^{\mathbf{I}}&\geq Q^{\mathbf{e}}_{*}\frac{\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}-\mu_{\mathbf{e}}+K(\sigma_{\mathbf{a}}-\sigma_{\mathbf{I}}) && \text{[By (EC.19)]}\\&=-\mu_{\mathbf{e}}\frac{\sigma_{\mathbf{e}}}{\sigma_{\mathbf{a}}+\sigma_{\mathbf{e}}}+K\left(\frac{\sigma_{\mathbf{e}}\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}+\sigma_{\mathbf{a}}-\sigma_{\mathbf{I}}\right) && \text{[By definition of } Q^{\mathbf{e}}_{*}\text{]}\\&\geq 0 && \text{[By (15)],}\end{aligned}$$

thus proving the lower bound on $Q^{\mathbf{E}}_{\infty}$. To prove the upper bound, consider

$$\begin{aligned}Q^{\mathbf{E}}_{\infty}-Q^{\mathbf{T}}_{\infty}&\leq Q^{\mathbf{a}}_{\infty}+Q^{\mathbf{e}}_{*}-Q^{\mathbf{T}}_{\infty} && \text{[Because } Q^{\mathbf{e}}_{\infty}\leq Q^{\mathbf{e}}_{*}\text{]}\\&=K(\sigma_{\mathbf{a}}+\sigma_{\mathbf{e}}-\sigma_{\mathbf{T}})\\&\leq 0. && \text{[By (EC.17)]}\end{aligned}$$

To prove the second part of the proposition, suppose that $c>\pi/2$, i.e., $K<0$. In this case, we have $Q^{\mathbf{e}}_{*}<\mu_{\mathbf{e}}$. Using a Taylor expansion for $\bar{F}_{\mathbf{e}}(Q)$ in the region $Q^{\mathbf{e}}_{*}<Q<\mu_{\mathbf{e}}$, we obtain

$$\bar{F}_{\mathbf{e}}(Q)-Qf_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty})\leq\bar{F}_{\mathbf{e}}(Q^{\mathbf{e}}_{*})-(Q-Q^{\mathbf{e}}_{*})f_{\mathbf{e}}(Q^{\mathbf{e}}_{*})-Qf_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty}),$$

The inequality is a consequence of the concavity of $\bar{F}_{\mathbf{e}}(Q)$ on $Q<\mu_{\mathbf{e}}$. From this inequality, the value of Q that sets the RHS of the above inequality to c/π must therefore bound $Q^{\mathbf{e}}_{\infty}$ from above, that is,

$$Q^{\mathbf{e}}_{\infty}\leq Q^{\mathbf{e}}_{*}\frac{f_{\mathbf{e}}(Q^{\mathbf{e}}_{*})}{f_{\mathbf{e}}(Q^{\mathbf{e}}_{*})+f_{\mathbf{a}}(Q^{\mathbf{a}}_{\infty})}=Q^{\mathbf{e}}_{*}\frac{\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}.\tag{EC.20}$$

Hence,

$$\begin{aligned}Q^{\mathbf{E}}_{\infty}-Q^{\mathbf{T}}_{\infty}&\leq Q^{\mathbf{e}}_{*}\frac{\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}-\mu_{\mathbf{e}}+K(\sigma_{\mathbf{a}}-\sigma_{\mathbf{T}}) && \text{[From (EC.20)]}\\&=-\mu_{\mathbf{e}}\frac{\sigma_{\mathbf{e}}}{\sigma_{\mathbf{a}}+\sigma_{\mathbf{e}}}+K\left(\frac{\sigma_{\mathbf{e}}\sigma_{\mathbf{a}}}{\sigma_{\mathbf{e}}+\sigma_{\mathbf{a}}}+\sigma_{\mathbf{a}}-\sigma_{\mathbf{T}}\right) && \text{[Definition of } Q^{\mathbf{e}}_{*}\text{]}\\&\leq 0. && \text{[By (16)]}\end{aligned}$$

This completes the proof because $Q^{\mathbf{T}}_{\infty}\leq Q^{\mathbf{I}}$ has already been established in Proposition 5. Q.E.D.