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An Exploration of Optimal Stabilization Policy

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Abstract

This paper examines the optimal response of monetary and fiscal policy to a decline in aggregate demand. The theoretical framework is a two-period general equilibrium model in which prices are sticky in the short run and flexible in the long run. Policy is evaluated by how well it raises the welfare of the representative household. While the model has Keynesian features, its policy prescriptions differ significantly from textbook Keynesian analysis. Moreover, the model suggests that the commonly used "bang for the buck" calculations are potentially misleading guides for the welfare effects of alternative fiscal policies.

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1 Introduction

What is the optimal response of monetary and fiscal policy to an economy-wide decline in aggregate demand? This question has been at the forefront of many economists' minds over the past several years. In the aftermath of the 2008-2009 housing bust, financial crisis, and stock market decline, households and firms were less eager to spend. The decline in the aggregate demand for goods and services led to the most severe recession in a generation or more.

The textbook answer to such a situation is for policymakers to use the tools of monetary and fiscal policy to prop up aggregate demand. And, indeed, during this recent episode, the Federal Reserve reduced the federal funds rate—its primary policy instrument—almost all the way to zero. With monetary policy having used up its ammunition of interest rate cuts, economists and policymakers increasingly looked elsewhere for a solution. In particular, they focused on fiscal policy and unconventional instruments of monetary policy.

Traditional Keynesian economics suggests a startlingly simple solution: The government can increase its spending to make up for the shortfall in private spending. Indeed, this was one of the motivations for the stimulus package proposed by President Obama and passed by Congress in early 2009. The logic behind this policy should be familiar to anyone who has taken a macroeconomics principles course anytime over the past half century.

Yet many Americans (including quite a few congressional Republicans) are skeptical that increased government spending is the right policy response. They are motivated by some basic economic and political questions: If we as individual citizens are feeling poorer and cutting back on our spending, why should our elected representatives in effect reverse these private decisions by increasing spending and going into debt on our behalf? If the goal of government is to express the collective will of the citizenry, shouldn't it follow the lead of those it represents by tightening its own belt?

Traditional Keynesians have a standard answer to this line of thinking. According to the paradox of thrift, increased saving may be individually rational but collectively irrational. As individuals try to save more, they depress aggregate demand and thus national income. In the end, saving might not increase at all. Increased thrift might lead only to depressed economic activity, a malady that can be remedied by an increase in government purchases of goods and services.

The goal of this paper is to address this set of issues in light of modern macroeconomic theory. Unlike traditional Keynesian analysis of fiscal policy, modern macro theory begins with the preferences and constraints facing households and firms and builds from there. This feature of modern theory is not a mere fetish for microeconomic foundations. Instead, it allows policy prescriptions to be founded on the basic principles of welfare economics. This feature seems particularly important for the case at hand, because the Keynesian recommendation is to have the government undo the actions that private citizens are taking on their own behalf. Figuring out whether such a policy can improve the well-being of those citizens is the key issue, a task that seems impossible to address without some reliable measure of welfare.

The model we develop to address this question fits solidly in the new Keynesian tradition. That is, the starting point for the analysis is an intertemporal general equilibrium model with prices that are assumed to be sticky in the short run. This temporary price rigidity prevents the economy from reaching an optimal allocation of resources, and it gives a possible role for monetary and fiscal policy to help the economy reach a better allocation through their influence on aggregate demand. The model yields several significant conclusions about the best responses of policymakers under various economic conditions and constraints on the set of policy tools at their disposal.

To be sure, by the nature of this kind of exercise, the validity of any conclusion depends on whether the model captures the essence of the problem being examined. Because all models are simplifications, one can always question whether a conclusion is robust to generalization. Our strategy is to begin with a simple model that illustrates our approach and yields some stark results. We then generalize this baseline model along several dimensions both to check robustness and to examine a broader range of policy issues. Inevitably, policy conclusions from such a theoretical exploration must be tentative. In the final section, we discuss some of the simplifications we make that might be relaxed in future work.

Our baseline model is a two-period general equilibrium model with sticky prices in the first period. The available policy tools are monetary policy and government purchases of goods and services. Like private consumption goods, government purchases yield utility to households. Private and public consumption are not, however, perfect substitutes. Our goal is to examine the optimal use of the tools of monetary and fiscal policy when the economy finds itself producing below potential because of insufficient aggregate demand.

We begin with the benchmark case in which the economy does not face the zero lower bound for nominal interest rates. In this case, the only stabilization tool that is necessary is conventional monetary policy. Once monetary policy is set to maintain full employment, fiscal policy should be determined based on classical principles. In particular, government consumption should be set to equate its marginal benefit with the marginal benefit of private consumption. As a result, when private citizens are cutting back on their private consumption spending, the government should cut back on public consumption as well.

We then examine the complications that arise because nominal interest rates cannot be set below zero. We show that even this constraint on monetary policy does not by itself give a role for traditional fiscal policy as a stabilization tool. Instead, the optimal policy is for the central bank to commit to future monetary policy actions in order to increase current aggregate demand. Fiscal policy continues to be set on classical principles.

A role for countercyclical fiscal policy might potentially arise if the central bank both hits the zero lower bound on the current short-term interest rate and is unable to commit itself to expansionary future policy. In this case, monetary policy cannot maintain full-employment of productive resources on its own. Absent any fiscal policy, the economy would find itself in a non-classical short-run equilibrium. Optimal fiscal policy then looks decidedly Keynesian. If the only instrument of fiscal policy is the level of government purchases, optimal policy is to increase those purchases to increase the demand for idle productive resources, even if the marginal value of the public goods being purchased is low.

This very Keynesian result, however, is overturned once the set of fiscal tools available to policymakers is expanded. Optimal fiscal policy in this situation is the one that tries to replicate the allocation of resources that would be achieved if prices were flexible. An increase in government purchases cannot accomplish that goal: While it can yield the same level of national income, it cannot achieve the same composition of it. We discuss how tax instruments might be used to induce a better allocation of resources. The model suggests that tax policy should aim at increasing the level of investment spending. Something like an investment tax credit comes to mind. In essence, optimal fiscal policy in this situation tries to produce incentives similar to what would be achieved if the central bank were somehow able to reduce interest rates below zero.

A final implication of the baseline model is that the traditional fiscal policy multiplier may well be a poor tool for evaluating the welfare implications of alternative fiscal policies. It is common in policy circles to judge alternative stabilization ideas using "bang-for-the-buck" calculations. That is, fiscal options are judged according to how many dollars of extra GDP are achieved for each dollar of extra deficit spending. But such calculations ignore the composition of GDP and, therefore, are potentially misleading as measures of welfare.

After developing these results in our baseline model, we examine three variations on it. First, we add a third period to the model. We show how the central bank can use long-term interest rates as an additional tool to achieve the flexible-price equilibrium. Second, we add government investment spending to the baseline model. We show that all government expenditure follows classical principles when monetary policy is sufficient to stabilize output. Moreover, even when monetary policy is limited, the model does not point toward government investment as a particularly useful tool for putting idle resources to work. Third, we modify the baseline model to include non-Ricardian, rule-of-thumb households who consume a constant fraction of income. The presence of such households means that the timing of taxes may affect output, and we characterize the optimal policy mix in that setting. We find that the description of the equilibrium closely resembles the traditional Keynesian model, but the prescription for optimal policy can differ substantially from the textbook answer.

2 Introducing the Model

In this section we introduce the elements of the baseline model. Before delving into the model's details, it may be useful to describe how this model is related to a few other models with which readers may be familiar. Our goal is not to provide a completely new model of stabilization policy but rather to illustrate conventional mechanisms in a way that permits an easier and more transparent analysis of the welfare implications of alternative policies.

First, the model is closely related to the model of short-run fluctuations found in most leading undergraduate textbooks. Our students are taught that prices are sticky in the short run but flexible in the long run. As a result, the economy can temporarily deviate from its full-employment equilibrium, yet over time it gravitates toward full employment. Similarly, we will (in a later section) impose a sticky price level in period one, but allow future prices to be flexible.

Second, this model is closely related to the large literature on dynamic, stochastic, general equilibrium (DSGE) models. Strictly speaking, the model is not stochastic: We will solve for the deterministic path of the economy after one (or more) of the exogenous variables changes. But the spirit of the model is much the same. As in DSGE models, all decisions are founded on underlying preferences and technology. Moreover, all decision makers are forward-looking, so their actions will depend not only on current policy but also on policy they expect to prevail in the future.

There is a key methodological difference between our approach and that in the DSGE literature. In recent years, the DSGE literature has evolved in the direction of greater complexity, as researchers have attempted to match various moments of the data more closely. (See, for example, Christiano, Eichenbaum, and Evans 2005 and Smets and Wouters 2003). By contrast, our goal is greater simplicity and transparency so the welfare implications of alternative monetary and fiscal policies can be better illuminated.

Third, the model we examine is related to the older literature on "general disequilibrium" models, such as Barro and Grossman (1971) and Malinvaud (1977). As in these models, we will assume that the price level in the first period is exogenously stuck at a level that is inconsistent with full employment of productive resources. At the prevailing price level, there will be an excess supply of goods. But unlike this earlier literature, the model is explicitly dynamic. That is, we emphasize the role of forward-looking, intertemporal behavior in determining current spending decisions and the impact of policy.

2.1 Households

The economy is populated by a large number of identical households. The representative household has the following objective function:

$$\max \{u(C_1) + v(G_1) + \beta [u(C_2) + v(G_2)]\}, \tag{1}$$

where C_t is consumption, G_t is government purchases, and β is the discount factor. Households choose consumption, but they take government purchases as given.

Households derive all their income from their ownership of firms. Each household's consumption choices are limited by a present value budget constraint:

$$P_1 (\Pi_1 - T_1 - C_1) + \frac{P_2 (\Pi_2 - T_2 - C_2)}{(1 + i_1)} = 0, \tag{2}$$

where P_t is the price level in period t, Π_t are profits of the firm, T_t are tax payments, and i_1 is the nominal interest rate between period one and two. Implicit in this budget constraint is the assumption of a bond market in which households can borrow or lend at the market interest rate.

2.2 Firms

Firms do all the production in the economy and provide all household income. It is easiest to imagine that the number of firms is the same as the number of households and that each household owns one firm.

For simplicity, we assume that capital K is the only factor of production. Each period, the firm produces output with an AK production function, where A is an exogenous technological parameter. The firm begins with an endowment of capital K_1 and is able to borrow and lend in financial markets to determine the future capital stock K_2. Without loss of generality, we assume capital fully depreciates each period, so investment in period one equals the capital stock in period two.

The parameter A plays a key role in our analysis. In particular, we are interested in studying the optimal policy response to a decline in aggregate demand. In our model, the most natural cause of a decline in aggregate demand is a decrease in the future value of A. Such an event can be described as a decline in expected growth, a fall in confidence, or a pessimistic shock to "animal spirits." In any event, in our model, it will tend to reduce wealth and current aggregate demand, as well as reducing the natural rate of interest (that is, the real interest rate consistent with full employment). A similar set of events would unfold if the shock were to households' discount factor β, but it seems more natural to assume stable household preferences and changes in the expected technology available to firms.

Before proceeding, it might be worth commenting on the absence of a labor input in the model. That omission is not crucial. As we will describe more fully later, it could be remedied by giving each household an endowment of labor in each period and by making the simplifying assumption that capital and labor are perfect substitutes in production. That somewhat more general model yields identical results regarding monetary and fiscal policy. Therefore, to keep the results as clean and easily interpretable as possible, we will focus on the one-factor case.

Firms choose the second period's capital stock to maximize the present value of profits:

$$\max_{K_2} \left[P_1 \Pi_1 + \frac{P_2 \Pi_2}{(1+i_1)} \right],$$

where the second period's nominal profit is discounted by the nominal interest rate. Profits are:

$$\Pi_t = Y_t - I_t, \tag{3}$$

where Y_t is equilibrium aggregate output and I_t is investment. Because capital fully depreciates each period, investment in period one becomes the capital stock in period two:

$$K_2 = I_1. \tag{4}$$

Recall that the initial capital stock K_1 is given. Also, because there is no third period, there is no investment in the second period ($I_2 = 0$).

As noted above, the production function is

$$F(A_t, K_t) = A_t K_t,$$

with $A_t > 0$.

Finally, it is important to note an assumption implicit in this statement of the firm's optimization problem: The firm is assumed to sell all of its output at the going price, and it is assumed to buy investment goods at the going price. In particular, the firm is not permitted to produce capital for itself, nor is it allowed to produce consumption goods directly for the household that owns it. This restriction is irrelevant in the case of fully flexible prices, but it will matter in the case of sticky prices, where firms may be demand-constrained. In that case, this assumption prevents the firm from directly circumventing the normal inefficiencies that arise from sticky prices. In practice, such a restriction arises naturally because firms are specialists in producing highly differentiated goods. Because we do not formally incorporate product differentiation in our analysis, it makes sense to impose this restriction as an additional constraint on the firm's behavior.

2.3 The Money Market and Monetary Policy

Households are required to hold money to purchase consumption goods. The money market in this economy is assumed to be described by the quantity equation:

$$\mathbb{M}_t = \phi P_t C_t.$$

That is, money holdings are proportional to nominal consumer spending. The parameter ϕ reflects the efficiency of the monetary system; a small ϕ implies a large velocity of money. We tend to think of ϕ as being very small, which is why we ignore the cost of holding money in the households' budget constraint above. The limiting case as ϕ approaches zero is sometimes called a "cashless" economy.

Hereafter, it will prove useful to define

$$M_t = \frac{\mathbb{M}_t}{\phi},$$

which implies the conventional money market equilibrium condition

$$M_t = P_t C_t.$$

M can be interpreted either as the money supply adjusted for the money demand parameter ϕ or as the determinant of nominal consumer spending.

Money earns a nominal rate of return of zero. When the nominal interest rate on bonds is positive, money is a dominated asset, and households will hold only what is required for transactions purposes, as determined above. However, they could choose to hold more (in which case $M_t > P_t C_t$). This possibility prevents the nominal interest rate in the bond market from falling below zero.

Because there are two periods, there are two policy variables to be set by the central bank. In the first period, the central bank is assumed to set the nominal interest rate i_1, subject to the zero lower bound. It allows that period's money supply M_1 to adjust to whatever is demanded in the economy's equilibrium. In the second period, the central bank sets the money supply M_2. (Recall that there is no interest rate in period two, because there is no period three.) We can think of the current interest rate i_1 as the central bank's short-run policy instrument and the future money supply M_2 as the long-run nominal anchor.

2.4 Fiscal Policy

Fiscal policy in each period is described by two variables: G_t is government purchases in period t, and T_t is lump-sum tax revenue. (In a later section, we introduce an investment subsidy as an additional fiscal policy tool.) It will prove useful to define g_t, the share of government purchases in full-employment output:

$$g_t = \frac{G_t}{A_t K_t}. \tag{5}$$

Any deficits are funded by borrowing in the bond market at the market interest rate. The government's budget constraint is:

$$P_1 (T_1 - G_1) + \frac{P_2 (T_2 - G_2)}{1 + i_1} = 0. \tag{6}$$

Note that because households are forward-looking and have the same time horizon at the government, this model will be fully Ricardian: the timing of tax payments is neutral. In a later section, we will generalize the model to include some non-Ricardian behavior.

2.5 Aggregate Demand and Aggregate Supply

Output is used for consumption, investment, and government purchases:

$$Y_t = C_t + I_t + G_t. \tag{7}$$

Equilibrium aggregate output is also constrained by potential output:

$$Y_t \leq A_t K_t. \tag{8}$$

In the full-employment equilibrium, this last equation holds with equality. However, we are particularly interested in cases in which this equation holds as a strict inequality. In these cases, aggregate demand is insufficient to employ all productive resources, and monetary and fiscal policy can potentially remedy the problem. The key issue is the optimal use of these policy tools.

3 The Equilibrium under Flexible Prices

The natural place to start in analyzing the model is to consider the behavior of the firms and households, as well as optimal policy, for the case of flexible prices. The flexible-price equilibrium will provide the benchmark when we impose sticky prices in the next section.

3.1 Firm and Household Behavior

We first derive the equations characterizing the equilibrium decisions of the private sector (households and firms), taking government policy as given.

Let's start with firms. In this setting, prices adjust to guarantee full employment in each period. Therefore,

$$Y_t = A_t K_t \quad \text{for all } t. \tag{9}$$

The firm's profit maximization problem can be restated, using the full-employment condition (9) and the investment equation (4), as:

$$\max_{K_2} \left[P_1 \left(A_1 K_1 - K_2 \right) + \frac{P_2}{(1+i_1)} A_2 K_2 \right].$$

This yields the following first-order condition:

$$(1+i_1) = A_2 \frac{P_2}{P_1}. \tag{10}$$

Expression (10) is similar to a conventional Fisher equation: the nominal interest rate reflects the marginal productivity of capital and the equilibrium inflation rate.

The household's utility maximization yields the standard intertemporal Euler equation:

$$\frac{u'\left(C_1\right)}{\beta u'\left(C_2\right)} = (1+i_1) \frac{P_1}{P_2}. \tag{11}$$

The full-employment condition (9) and accounting identity (7) imply the following values for consumption:

$$C_1 = A_1 K_1 - K_2 - G_1, \tag{12}$$

$$C_2 = A_2 K_2 - G_2. \tag{13}$$

The four equations (10) through (13) simultaneously determine the equilibrium for four endogenous variables: C_1, C_2, K_2, and P_2/P_1. The second-period money-market equilibrium condition ($M_2 = P_2 C_2$) then pins down P_2 and thereby P_1.

To derive explicit solutions for the economy's equilibrium, we specify the household's utility function as isoelastic:

$$u\left(C_t\right) = \frac{C_t^{\left(1-\frac{1}{\sigma}\right)} - 1}{1 - \frac{1}{\sigma}},$$

where σ is the elasticity of intertemporal substitution.

The equilibrium real quantities are:

$$C_1 = \frac{\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)} \left(A_1 K_1 - G_1\right), \tag{14}$$

$$C_2 = \frac{A_2 \left(1-g_2\right)}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)} \left(A_1 K_1 - G_1\right), \tag{15}$$

$$I_1 = \frac{1}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)} \left(A_1 K_1 - G_1\right), \tag{16}$$

$$Y_1 = A_1 K_1, \tag{17}$$

$$Y_2 = \frac{A_2}{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)} \left(A_1 K_1 - G_1\right). \tag{18}$$

The equilibrium nominal quantities are:

$$P_1 = \frac{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)}{\left(1-g_2\right)\left(A_1 K_1 - G_1\right)} \frac{M_2}{\left(1+i_1\right)}, \tag{19}$$

$$P_2 = \frac{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left(1-g_2\right)}{A_2\left(1-g_2\right)\left(A_1 K_1 - G_1\right)} M_2, \tag{20}$$

$$M_1 = \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{\left(1+i_1\right)}. \tag{21}$$

Note that the economy exhibits monetary neutrality. That is, the monetary policy instruments do not affect any of the real variables. Expansionary monetary policy—as reflected in either lower i_1 or higher M_2—implies a higher price level P_1.

As already mentioned, we are interested in studying the effects of a decline in aggregate demand. Most naturally, such a shock can be thought of as some exogenous event leading to a decline in the private sector's desire to spend. There are various ways in which such a shock can be incorporated into this kind of model. One often used option is to assume a shock to the intertemporal discount rate (which here would be an increase in β). Alternatively, a decline in spending desires can arise because of a decrease in A_2, the productivity of technology projected to prevail in the future. The impact of A_2 on current demand depends crucially on σ, which in turn governs the relative size of income and substitution effects from a change in the rate of return. If $\sigma < 1$, income effects dominate substitution effects, and a lower A_2 primarily causes households to feel poorer, inducing a reduction in desired consumption. Hereafter, we focus on the case of a decline in A_2 together with the maintained assumption that $\sigma < 1$. This is, of course, not the only way one might model shocks to aggregate demand, but we believe it is the closest approximation in this model to what one might call a decline in confidence or an adverse shift in "animal spirits."

Equations (14) to (21) above show what a decline in A_2 does to all the endogenous variables in the flexible-price equilibrium. Consumption falls because households are poorer. Higher saving translates into higher investment. Output in the first period remains the same. The flexibility of the price level is crucial for this result. Equation (19) shows that a fall in A_2 leads to a fall in the price level P_1. In a later section, we will examine the case in which the price level is sticky and thus unable to respond to this shock.

3.2 Optimal Fiscal Policy under Flexible Prices

Optimal fiscal policy follows classical principles. We state the government's optimization problem formally in a later section, but in words it chooses public expenditure G_t and taxes T_t to maximize household utility subject to the economy's feasibility and the government's budget constraints. The following conditions define optimal government purchases:

$$v'\left(G_1\right) = \beta A_2 v'\left(G_2\right), \tag{22}$$

$$u'\left(C_t\right) = v'\left(G_t\right) \text{ for all } t. \tag{23}$$

Result (23) shows that optimal fiscal policy has government purchases move in the same direction as private consumption, unless there is a change in preferences for government services.

To derive explicit solutions, we assume that the utility from government purchases takes a similar form as that from consumption:

$$v\left(G_t\right) = \theta^{\frac{1}{\sigma}} \frac{G_t^{\left(1-\frac{1}{\sigma}\right)} - 1}{1 - \frac{1}{\sigma}},$$

where θ is a taste parameter. These expressions imply optimal government purchases:

$$G_1 = \theta C_1,$$

$$G_2 = \theta C_2,$$

and therefore the equilibrium quantities in closed form:

$$C_1 = \frac{\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{(1+\theta)\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1, \tag{24}$$

$$C_2 = \frac{A_2}{(1+\theta)\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1, \tag{25}$$

$$I_1 = \frac{1}{\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1, \tag{26}$$

$$G_1 = \frac{\theta\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{(1+\theta)\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1, \tag{27}$$

$$G_2 = \frac{\theta A_2}{(1+\theta)\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1. \tag{28}$$

$$Y_1 = A_1 K_1, \tag{29}$$

$$Y_2 = \frac{A_2}{\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1, \tag{30}$$

This flexible-price equilibrium with optimal fiscal policy will be a natural benchmark in the analysis that follows.

3.3 An Aside on Labor

As mentioned earlier, it is possible to incorporate labor as an additional factor of production without affecting the key results of the model. Suppose that the production function is

$$Y_t = A_t \left(K_t + \omega_t L_t\right),$$

where ω_t is an exogenous labor productivity parameter and L_t is the exogenous level of labor supplied inelastically to the firm by the representative household. With this production function, the baseline model is more cumbersome but little changed. In essence, current and future labor inputs serve as additions to the initial productive endowment of the household, funding consumption and government purchases just as does K_1. None of the policy analysis would be altered by adding labor input in this way. Interested readers are referred to a technical appendix available both at the Brookings' Papers website and at the authors' personal websites.

If labor and capital were not perfect substitutes in production, contrary to what is assumed in the above production function, more details about factor markets would need to be specified. In particular, firms facing insufficient demand would have to choose between idle labor and idle capital in some way. We suspect that this issue is largely unrelated to the topics at hand, so we avoid these additional complexities. Hereafter, we maintain the assumption of a single input into production.

4 The Equilibrium under Short-run Sticky Prices

So far we have introduced a two-period general equilibrium model with monetary and fiscal policy and solved for the equilibrium under the assumption that prices are flexible in both periods. In this section, we use the model to analyze what happens if prices are sticky in the short run. In particular, we take the short-run price level P_1 to be fixed, while allowing the long-run price level P_2 to remain flexible.

The cause for the price stickiness will not be modeled here, and the reason for the deviation of prices from equilibrium prices will not enter our analysis. It seems natural to imagine that prices were set in advance based on economic conditions that were expected to prevail and that conditions turned out differently than expected. Equation (19) shows what determines the price level consistent with full employment. If any of the exogenous variables in this equation are other than what was anticipated, and the price level is unable to change, the economy will be forced to deviate from the classical flexible-price equilibrium. One notable possibility, for instance, is fluctuations in A_2, which we have interpreted as reflecting confidence about future economic growth.

With a fixed price level, there are two cases to consider: the price level can be stuck too low, or it can be stuck too high. If the price level is too low, the goods market will experience excess demand. Such a situation is sometimes called "repressed inflation." If the price level is too high, the goods market will experience excess supply. In this case, which might be called the "Keynesian regime," firms will be unable to sell all they want at the going price and so some productive resources will be left idle. Because our goal is to understand optimal policy during recessions, our analysis will focus on this latter case.[1]

Formally, the equations describing the sticky-price equilibrium closely resemble equations (9) through (13) from the flexible-price model. One difference is that because nominal rigidity prevents full employment of capital in period one, equation (9), $Y_t = A_t K_t$, may not hold for $t = 1$. Moreover, AK_1 needs to be replaced with Y_1 in equation (12), which now becomes

$$C_1 = Y_1 - K_2 - G_1.$$

Of course, the presence of a sticky price level in period one breaks the monetary neutrality of the flexible price model. Here, monetary policy affects the real economy's equilibrium quantities.

The equilibrium of this model is described by the following equations:

$$C_1 = \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{(1+i_1)\, P_1}, \tag{31}$$

[1] As an aside, we note that much of the new Keynesian literature makes this case canonical, and precludes the case of repressed inflation, by assuming monopolistic competition. Firms in such industries charge prices above marginal cost and, as long as prices are not too far from equilibrium, are always eager to sell more at the going price.

$$C_2 = A_2 \frac{M_2}{(1+i_1)\, P_1}, \tag{32}$$

$$I_1 = \frac{1}{(1-g_2)} \frac{M_2}{(1+i_1)\, P_1}, \tag{33}$$

$$Y_1 = \frac{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 (1-g_2)}{(1-g_2)} \frac{M_2}{(1+i_1)\, P_1} + G_1, \tag{34}$$

$$Y_2 = A_2 \frac{1}{(1-g_2)} \frac{M_2}{(1+i_1)\, P_1}, \tag{35}$$

$$P_2 = \frac{(1+i_1)}{A_2} P_1. \tag{36}$$

Equation (34) can be viewed as an aggregate demand curve. It yields a negative relationship between the output Y_1 and the price level P_1.

This set of equations also yields another famous Keynesian result: the paradox of thrift. If β rises, households want to consume less and save more. In equilibrium, however, saving and investment are unchanged, because output falls. That is, because aggregate demand influences output, more thriftiness does not increase equilibrium saving.

Note that all the real equilibrium quantities above depend on the ratio

$$\frac{M_2}{(1+i_1)\, P_1}. \tag{37}$$

Expression (37) succinctly captures the policy position of the central bank. It also hints at our findings detailed below, where we show that the various tools available to the central bank can act as substitutes.

In this setting, the monetary policy that generates full employment can be read directly from (34) by equating Y_1 with $A_1 K_1$:

$$\frac{M_2}{(1+i_1)\, P_1} = \frac{(1-g_2)}{1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 (1-g_2)} (A_1 K_1 - G_1)\,. \tag{38}$$

To maintain full employment, monetary policy needs to respond to present and future technology, present and future fiscal policy, and household preferences.

To illustrate the implications of this solution, consider the impact of a negative shock to future technology A_2. (We maintain the assumption that $\sigma < 1$). In the absence of a policy response, the effect on the economy's short-run equilibrium can be seen immediately from equations (31) through (36). Consumption falls in both periods. Output falls in period two, even though the economy is at full employment, as worse technology reduces potential output in that period. Most important for our purposes, output falls in the first period due to weak aggregate demand. Potential output in period one is unchanged because A_1 and K_1 are fixed. Thus, a decline in "confidence" as reflected in A_2 causes resources in period one to become idle.

5 Optimal Policy when Monetary Policy is Sufficient to Restore the Flexible-Price Equilibrium

In this section, we begin to examine optimal policy responses to a drop in aggregate demand. For concreteness, we focus on a negative shock to future technology A_2. Formally, let a hat over a variable denote the value of that variable anticipated when prices were set. We assume that the price level was set to achieve full employment based on an expected value $\hat{A}_2$, but once prices are set, the actual realized value is A_2, where $A_2 < \hat{A}_2$. We begin with conventional monetary policy, where the central bank adjusts the short-term nominal interest rate, and derive the threshold value for A_2 above which conventional monetary policy is sufficient to replicate the flexible-price equilibrium. We also characterize optimal fiscal policy in this scenario. Then, we examine the options for monetary policy when A_2 falls further and the economy hits the zero lower bound.

Whenever monetary policy is sufficient to restore the flexible-price equilibrium, optimal fiscal policy follows classical principles, satisfying expression (23) from the flexible-price equilibrium. Therefore, the post-shock equilibrium with optimal fiscal policy can be summarized with the following set of equations:

$$C_1 = \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{(1+i_1)\,P_1}, \tag{39}$$

$$C_2 = A_2 \frac{M_2}{(1+i_1)\,P_1}, \tag{40}$$

$$I_1 = (1+\theta)\,\frac{M_2}{(1+i_1)\,P_1}, \tag{41}$$

$$Y_1 = (1+\theta)\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]\frac{M_2}{(1+i_1)\,P_1}, \tag{42}$$

$$Y_2 = A_2\,(1+\theta)\,\frac{M_2}{(1+i_1)\,P_1}, \tag{43}$$

$$G_1 = \theta \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \frac{M_2}{(1+i_1)\,P_1}, \tag{44}$$

$$G_2 = \theta A_2 \frac{M_2}{(1+i_1)\,P_1}. \tag{45}$$

Optimal monetary policy is implied by (42) and the full-employment condition $Y_1 = A_1 K_1$:

$$\frac{M_2}{(1+i_1)\,P_1} = \frac{1}{(1+\theta)\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1. \tag{46}$$

In our canonical case in which $\sigma < 1$, a fall in A_2 raises the right-hand side of this expression. Thus, a decline in confidence about the future causes optimal monetary to be more expansionary, as reflected in either a fall in short-term interest rate i_1 or an increase in the future money supply M_2.

5.1 Conventional Monetary Policy

The conventional monetary policy response to weak aggregate demand is to lower the short-term nominal interest rate i_1. For now, assume that this conventional response is the central bank's only response, so that the long-term money supply remains at its pre-shock level (i.e., $M_2 = \hat{M}_2$). Fiscal policy is at its classical optimum derived above. With these assumptions, we can rearrange result (46) and substitute it along with i_1 into result (42) and solve for first-period output after the shock:

$$Y_1 = \left[\frac{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1+\left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2}\right] \frac{(1+\hat{\imath}_1)}{(1+i_1)} A_1 K_1. \tag{47}$$

Manipulating expression (47) yields a threshold value for A_2 above which conventional policy is sufficient to restore the flexible-price equilibrium. We denote this threshold $A_2|_{conventional}$, and it is:

$$A_2|_{conventional} = \left[\frac{\left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2 - \hat{\imath}_1}{\left(\frac{1}{\beta}\right)^{\sigma}(1+\hat{\imath}_1)}\right]^{\frac{1}{1-\sigma}}. \tag{48}$$

Note that a higher initial value of $\hat{\imath}_1$ implies a lower threshold $A_2|_{conventional}$. This result parallels much recent discussion suggesting that higher normal levels of nominal interest rates would increase the scope for conventional monetary responses to adverse demand shocks. (See, e.g., Blanchard, Dell'Ariccia, and Mauro 2010.) To show this clearly, note that if $\hat{\imath}_1 = 0$ this expression reduces to:

$$A_2|_{conventional} = \hat{A}_2.$$

That is, if the nominal interest rate is normally zero, then conventional monetary policy has no power in response to an adverse shock.

The value of the short-term interest rate i_1 that generates full employment satisfies:

$$(1+i_1) = \left[\frac{1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{1+\left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2}\right](1+\hat{\imath}_1). \tag{49}$$

At this value of the interest rate, consumption, investment, and output all equal their values in the flexible-price equilibrium.

The limiting case in which σ approaches zero may be instructive. In this case, the interest rate needed to restore full employment simplifies to

$$(1+i_1) = \left(\frac{1+A_2}{1+\hat{A}_2}\right)(1+\hat{\imath}_1). \tag{50}$$

Thus, when our measure of confidence A_2 falls below what was anticipated when prices were set, the gross nominal interest rate must move in the same direction. How far A_2 can fall before the central bank hits the zero lower bound depends solely on the normal interest rate $\hat{\imath}_1$.

5.2 Long-term Monetary Expansion

If A_2 falls below $A_2|_{conventional}$, the central bank will be unable to obtain the flexible-price equilibrium with conventional monetary policy. As recent events have shown, monetary authorities may look beyond conventional policy in this situation. One much-discussed option is to try to affect the long-term nominal interest rate. We consider that option in a later section, where we specify a variation on this baseline model in which the economy has three periods, not two.

In this baseline model, the central bank has one tool other than the short-term interest rate: the long-term level of money supply M_2. Condition (42) implies that any shock to future technology can be fully offset by changes to M_2. Formally, the M_2 required to restore the flexible-price equilibrium after the shock $A_2 < A_2|_{conventional}$ when $i_1 = 0$ satisfies:

$$\frac{M_2}{P_1} = \frac{1}{(1+\theta)\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1, \tag{51}$$

Note that the right hand side of (51) is decreasing in A_2, so that (as expected) a large negative shock to future technology calls for a long-term nominal expansion.[2]

[2] The role of future monetary policy in influencing the short-run equilibrium has, of course, been widely discussed. See, for example, Krugman (1998) and Eggertsson and Woodford (2003).

5.3 Summary when Monetary Policy is Unrestricted

A sufficiently flexible and credible monetary policy is always sufficient to stabilize output following an adverse demand shock, even if the zero lower bound on the short-term interest rate binds. Once monetary policy has restored the flexible-price equilibrium, the role of fiscal policy is entirely passive and is determined by classical principles that equate the marginal utility of government purchases to the marginal utility of private consumption.

One noteworthy, and perhaps surprising, result concerns the influence of these expansionary moves in monetary policy on inflation. In this model, the current price level P_1 is fixed, but equation (36) shows how monetary policy influences the future price level P_2. A cut in the interest rate i_1 reduces the future price level. The explanation is that the lower interest rate stimulates investment and increases future potential output; for any given future money supply M_2, higher potential output means a lower price level. Similarly, an increase in future money M_2 does not raise the future price level because it stimulates current output and investment; the increase in future potential output offsets the inflationary pressure of a greater money supply. Thus, while the various tools of monetary policy can increase aggregate demand and output in this economy, they do not increase future inflation until the economy reaches full employment.

Of course, as has been made clear in recent debates over U.S. monetary policy, the ability of the central bank to fulfill its potential is vulnerable to real-world constraints on policymaking. The central bank may not be willing or able to commit to the expansionary long-term money supply M_2 that is required for stabilization. As a consequence, monetary policy may be insufficient to restore the flexible-price equilibrium, raising the question of whether and how fiscal policy might supplement it. We turn to that question in the next section.

6 Optimal Fiscal Policy when Monetary Policy is Restricted

Imagine an economy that had been hit by an adverse shock to A_2. The central bank has set $i_1 = 0$, but that policy move has been insufficient to restore output to full employment. In addition, the central bank is, for some reason, unable to commit to an expansion in the future money supply M_2. (In the notation of the previous section, this implies $A_2 < A_2|_{conventional}$, $i_1 = 0$, and $M_2 = \hat{M}_2$.) How might fiscal policy respond to such a scenario?

We consider two fiscal stimulus policies in this section, each intended to raise one of the components of aggregate demand. First, we consider an increase in G_1, government purchases in the first period. Second, we examine a subsidy s aimed at boosting investment I_1. Both of these policies are financed by increased lump-sum taxes. The timing of these taxes is immaterial because we have assumed all households are forward-looking. In a later section, we relax the assumption of completely forward-looking households. As the households in that example choose consumption in part based on a rule of thumb tied to current disposable income, adjusting the timing of taxes has the potential to raise consumption C_1.[3]

6.1 The Government's Fiscal Policy Problem

In this scenario, the government faces the following optimization problem:

$$\max_{\left\{\{G_t,T_t\}_{t=1}^{2},s\right\}} \left\{u\left(C_1\right)+v\left(G_1\right)+\beta\left[u\left(C_2\right)+v\left(G_2\right)\right]\right\},$$

where s is an investment subsidy so that the cost of one unit of investment to a firm in period one is $(1-s)$, and the values for $\{C_1, C_2, K_2\}$ as a function of government policies are chosen optimally by households and firms. The government is constrained by the following balanced budget condition:

$$P_1\left(T_1-G_1-sI_1\right)+\frac{P_2\left(T_2-G_2\right)}{\left(1+i_1\right)}=0. \tag{52}$$

Some of the equations that determined equilibrium in the model of Section 1 must be altered to take into account the investment subsidy. Equation (10), which result from the firm's profit maximization, becomes

$$\left(1-s\right)\left(1+i_1\right)=A_2\frac{P_2}{P_1}, \tag{53}$$

while the government budget constraint (6) becomes (52).

We begin with the simplest fiscal stimulus: an increase in current government purchases G_1. For now, we set the investment subsidy s to zero. But we will return to it shortly.

6.2 Government Purchases under Flexible Prices

As a benchmark, recall the condition (23) on fiscal policy in the flexible-price allocation:

$$u'\left(C_t\right)=v'\left(G_t\right) \text{ for all } t.$$

[3] One can imagine other fiscal instruments as well. In particular, a retail sales tax (or subsidy) naturally comes to mind. The effects of such an instrument in this model depends on what price is assumed to be sticky. If the before-tax price is sticky, then a sales tax gives policymakers the ability to control directly the after-tax price, which is the price relevant for demand. This in turn allows policymakers to overcome all the inefficiencies that arise from sticky prices. After a decline in aggregate demand, a cut in the sales tax can reduce prices to the level consistent with full employment. On that other hand, if the after-tax price is assumed to be sticky, then a sales tax has no use as a short-run stabilization tool.

The most important implication of this relationship is that public and private consumption move together. Intuitively, if a shock induces households to save more and spend less, it raises the marginal utility of consumption. The optimal response of fiscal policy under flexible prices is to follow the private sector's lead by lowering government expenditure. As a result of the decline in G_1, consumption falls less in all periods than it would have if fiscal policy were to remain fixed at its pre-shock levels.

For future reference, the optimal level of government spending under flexible prices is:

$$G_1^{flex} = \frac{\theta\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{(1+\theta)\left[1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1.$$

Under our maintained assumption that $\sigma < 1$, optimal government spending falls in response to the negative shock to future technology A_2.

6.3 Government Purchases under Short-run Sticky Prices

Let's now return to a setting with sticky prices. As shown in expressions (31) and (32), if the economy is operating below full employment, the equilibrium levels of consumption do not depend on the choice of G_1. That is, as long as some productive resources are idle, an increase in public consumption has an opportunity cost of zero. Therefore, as long as the marginal utility of government services is positive, the government should increase spending until the economy reaches full employment.

The government-spending multiplier here is precisely one. This result is akin to the balanced-budget multiplier in the traditional Keynesian income-expenditure model. Here, as in that model, an increase in government spending puts idle resources to work and raises income. Consumers, meanwhile, see their income rise but recognize that their taxes will rise by the same amount to finance that new, higher level of government spending. As a result, consumption and investment are unchanged and the increase in income precisely equals the increase in government spending.[4]

Formally, one can show that the following first-order conditions characterize the government's optimum:[5]

$$v'(G_1) = \beta A_2 v'(G_2),$$

$$u'(C_t) > v'(G_t) \text{ for all } t.$$

[4] Woodford (2010) discusses how new Keynesian models tend to produce government-spending multipliers that equal unity if the real interest rate is held constant. In a later section, we present an extension of our model that yields a multiplier greater than one.

[5] Readers interested in seeing a more explicit (if laborious) demonstration of these and other results should consult the online appendix.

Because government spending puts idle resources to use, optimal spending on public consumption rises above the point that equates its marginal utility to that of private consumption.[6]

The optimal level of government spending in the first period is:

$$G_1^{sticky} = \frac{\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} \left\{ 1 - \frac{(1+\hat{\imath}_1)\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]}{(1+\theta)\left[1 + \left(\frac{1}{\beta \hat{A}_2}\right)^{\sigma} \hat{A}_2\right]} \right\} A_1 K_1.$$

One can show that optimal government spending exceeds the level that would be set at the flexible-price equilibrium. That is, $G_1^{sticky} > G_1^{flex}$. Whether the optimal G_1^{sticky} is a stimulus relative to pre-shock G_1 is a bit more complicated. For a shock that just barely pushes into the zero lower bound region (that is, A_2 equal to or slightly worse than the threshold in expression 48), the optimal G_1^{sticky} falls below pre-shock G_1, indicating the optimality of fiscal contraction. In this case, the central bank has the capacity to offset most of the shock with conventional monetary easing, and government spending can fall below its pre-shock level toward its new, lower flexible-price level. For larger shocks, however, optimal G_1^{sticky} will be greater than pre-shock G_1, indicating the optimality of fiscal expansion. In this case, there is a lot of idle capacity for fiscal policy to use up.

One can derive a full set of equations comparing the equilibrium with optimal fiscal policy as described here to the flexible-price equilibrium. They establish that:

$$C_1^{sticky} < C_1^{flex},$$

$$C_2^{sticky} < C_2^{flex},$$

$$I_1^{sticky} = I_1^{flex},$$

$$G_1^{sticky} > G_1^{flex},$$

$$G_2^{sticky} > G_2^{flex},$$

$$Y_1^{sticky} = Y_1^{flex},$$

$$Y_2^{sticky} = Y_2^{flex}.$$

[6] One surprising implication is that government consumption in period two also expands beyond the classical benchmark. The reason is that, according to equation (33), increased second-period government consumption stimulates first-period investment. Why? Intuitively, higher g_2 tends to reduce second-period consumption for a given output, which in turn tends to increase the second-period price level (recall $M_2 = P_2 C_2$). Higher expected inflation would tend to reduce the real interest rate, stimulating investment. In the final equilibrium, however, investment and potential output expand sufficiently to leave C_2 and P_2 unaffected.

The bottom line is that when monetary policy fails to achieve full employment, it is optimal for the government to put those idle resources to work by increasing its spending. This fiscal policy is second-best, however, because it fails to produce the same allocation of resources achieved under flexible prices. Public consumption will be higher in both periods, but private consumption will be lower. As a result, households will end up with a lower level of welfare.

6.4 Investment Subsidy

Next, we expand the government's fiscal tools by allowing it to subsidize investment by choosing $s > 0$.

As with government purchases, the investment subsidy can cause idle capital to be brought into production. When output is below its full-employment level in period one, a positive investment subsidy is welfare-improving. That is true if government spending is unchanged or if it is set to its new flexible-price optimum. (See the online appendix for details). In general, the subsidy that generates full employment is a complicated function of the economy's parameters.

One special case, however, clarifies the intuition for the role of the investment subsidy. In much traditional Keynesian analysis, the real interest rate does not much affect private consumption. We might interpret this as suggesting that the elasticity of intertemporal substitution is very small. If we take the limit as $\sigma \to 0$, then the optimal investment subsidy is:

$$s = -i_1, \tag{54}$$

where i_1 is the interest rate chosen in (50) that reproduces the flexible-price equilibrium. Government spending in this equilibrium is once again set on classical principles. Result (54) shows that the government sets the investment subsidy rate equal to the opposite of the optimal negative nominal interest rate. Intuitively, the investment subsidy allows the government to provide the same incentives for investment as the negative interest rate would have, if the latter were possible, thereby reproducing the flexible-price equilibrium.[7]

For the more general case of positive σ, we rely on numerical simulations to judge the welfare consequences of policy change. We offer such calculations in the next section.

6.5 Comparing Welfare Gains to Output Gains from Fiscal Tools

It is common for policy debates to focus on the output stimulus achievable by various policy options. Using our results above, we now turn to a numerical evaluation of whether this focus on "bang for the buck" is a good guide to policymaking. As an alternative, we also calculate a welfare-based measure of policy

[7] The use of tax instruments as a substitute for monetary policy is also examined in recent work by Correia, Farhi, Nicolini, and Teles (2010).

effectiveness.

Suppose the economy begins at full employment and the zero lower bound. If it is then hit by a negative shock to A_2, conventional monetary policy is ineffective, and we assume that future monetary expansion is impossible. We want to compare several alternative fiscal policies, all aimed at achieving full employment:

- An increase in current government spending G_1, holding future government spending G_2 constant.
- An increase in both current and future government spending, maintaining the government's intertemporal Euler equation.
- An investment subsidy, holding constant government spending.
- An investment subsidy, allowing government spending to optimally adjust.

These four policies are all compared to a benchmark in which fiscal policy is held fixed at its pre-shock level. For each policy, we calculate a version of what is usually called the "multiplier" or "bang for the buck." This statistic is the increase in current output (Y_1) divided by the increase in the current government budget deficit.[8] We also calculate a welfare-based measure of the returns to each fiscal policy option. In particular, we calculate the percent increase in current consumption (C_1) in the benchmark economy that would raise welfare in the benchmark economy to that under each fiscal policy option.

Table 1 shows the results of these calculations for a variety of parameter values. Three parameters are important to the model. First, our baseline value for the intertemporal elasticity of substitution is $\sigma = 0.5$, well within standard ranges for macroeconomic models, and we consider both higher and lower values for σ. Second, our baseline value for the household's relative taste for government consumption is $\theta = 0.24$. As we showed above, θ equals the ratio G/C at the flexible-price equilibrium, and this ratio is 0.24 in the U.S. national income accounts for 2009. We consider higher and lower values for θ as well. Finally, our baseline value for the size of the shock to future technology is 25%, but we also consider shocks of 10% and 40%.

[8] The increase in the deficit is calculated as the increase in G_1 plus any loss in revenue from the investment subsidy s. Implicitly, this holds current lump-sum tax revenue T_1 fixed. Recall that the timing of tax payments is irrelevant to the equilibrium of the model economy because all households are forward-looking.

Table 1: Alternative Metrics to Evaluate Fiscal Policies						
		Fiscal Policy Option				Monetary Policy
Model variant	Measure	(1) Current purchases, G_1	(2) Optimal mix of G_1, G_2	(3) Investment subsidy, s	(4) Optimal mix of G_1, G_2, s	Unrestricted; Yields flexible-price equilibrium
Baseline (σ=0.5; θ=0.24; shock=25%)	Welfare gain	4.1%	4.1%	6.7%	8.5%	8.6%
	Bang for the buck	1.0	1.2	1.1	0.6	na
Low EIS (σ=0.33)	Welfare gain	4.3%	4.9%	12.9%	15.8%	15.9%
	Bang for the buck	1.0	1.7	1.2	0.8	na
High EIS (σ=0.67)	Welfare gain	1.3%	1.5%	1.6%	3.0%	3.0%
	Bang for the buck	1.0	0.4	1.0	0.2	na
Low taste for G (θ=0.18)	Welfare gain	4.8%	5.1%	9.1%	10.4%	10.6%
	Bang for the buck	1.0	1.5	1.2	0.8	na
High taste for G (θ=0.30)	Welfare gain	2.9%	2.9%	4.3%	6.6%	6.7%
	Bang for the buck	1.0	0.9	1.0	0.4	na
Small shock (shock=10%)	Welfare gain	2.2%	2.3%	2.7%	2.9%	2.9%
	Bang for the buck	1.0	1.3	1.2	0.7	na
Large shock (shock=40%)	Welfare gain	4.4%	4.4%	9.5%	16.6%	17.0%
	Bang for the buck	1.0	1.0	1.0	0.4	na

The results shown in Table 1 suggest that the conventional emphasis on the output multiplier may be substantially misleading as a guide to optimal policy. In none of the variants considered does the policy with the largest multiplier also generate the greatest welfare gain.

One pattern is particularly striking: Across all parameter values that we consider, the policy that is best for welfare (option 4) is the worst according to the bang-for-the-buck metric. The reason is that this policy recommends a large investment subsidy in the first period, generating a deficit nearly twice as large as the next largest deficit among the other three policies. While generating much less bang for the buck, this investment subsidy allows policymakers to stabilize output with lower public consumption. This raises private consumption in both the first and second periods, relative to the other policy options, and moves the economy closer to the flexible-price equilibrium. The final column of Table 1 shows that this policy generates nearly as large a welfare gain as would fully flexible monetary policy.

7 Unconventional Monetary Policy in a Model with Three Periods

In this section we add a third period to the baseline model. As the main features of the model are unchanged, our purpose in adding a third period is specific: to expand the set of tools available to the central bank.

The U.S. Federal Reserve has recently pursued policies aimed at lowering long-term nominal interest rates. Adding a third period to the model allows us to clarify the role of such a policy in stabilizing aggregate demand.

Three periods implies two nominal interest rates, which we denote i_1 and i_2. The variable i_2 is a future short-term interest rate; hence, by standard term-structure relationships, a change in i_2 will move long-term rates in the first period in the same direction. The long-term money supply is now denoted M_3. We focus on the case when the price level in period one is fixed; prices are flexible in periods two and three. To keep things simple, we omit all fiscal policy in this section (that is, $\theta = 0$ for all t, so $G_t = 0$ as well).

The expression for equilibrium output when output is demand-constrained is the following:

$$Y_1 = \left[1 + \left(\frac{1}{\beta A_3}\right)^{\sigma} A_3 + \left(\frac{1}{\beta^2 A_2 A_3}\right)^{\sigma} A_2 A_3\right] \frac{M_3}{P_1 (1+i_1)(1+i_2)}.$$

This expression shows the monetary policy tools that can offset a shock to aggregate demand. If $\sigma < 1$, a fall in future productivity (A_2 or A_3) reduces output for a given monetary policy. The central bank has three tools to offset such a shock. It can lower the current short-term interest rate i_1, it can reduce long-term interest rates by reducing the future short-term rate i_2, or it can raise the long-term nominal anchor M_3.

Two conclusions about the efficacy of monetary policy are apparent. First, if the long-term nominal anchor M_3 is held fixed, the ability to influence long-term interest rates expands the central bank's scope for restoring the optimal allocation of resources. Formally, one can derive thresholds for A_2 above which conventional and unconventional policies are sufficient to restore the flexible-price equilibrium. One can show that:

$$A_2|_{long-term-interest} < A_2|_{conventional}.$$

Second, as before, if the central bank can control the long-term nominal anchor M_3, there is no limit to its ability to restore the flexible-price equilibrium.

8 Government Investment

So far, all government spending in this model has been for public consumption. We now consider one way in which public investment spending might be incorporated into the model. We return to our baseline model with two periods, with one addition. In addition to private investment, we also have investment by the government, denoted G^I. Government consumption is now denoted G^C.

The production function is:

$$Y_t \leq A_t^F K_t^F + A_t^G \kappa \left(K_t^G\right),$$

where K_t^F and K_t^G are the private and public capital stocks, A_t^F and A_t^G are exogenous technology parameters specific to private (firm) and public (government) capital. The function $\kappa(\cdot)$ reflects that the two forms of capital are not perfect substitutes in production. To ensure a sensible interior solution, we assume $\kappa'(\cdot) > 0$ and $\kappa''(\cdot) < 0$.

Under flexible prices, the solution to the government's optimal policy problem satisfies the following conditions

$$u'(C_1) = v'\left(G_1^C\right),$$

$$u'(C_2) = v'\left(G_2^C\right),$$

$$v'\left(G_1^C\right) = A_2^F \beta v'\left(G_2^C\right) \tag{55}$$

$$\kappa'\left(K_2^G\right) = \frac{A_2^F}{A_2^G}. \tag{56}$$

The first three of these should be familiar by now, as they are the same classical conditions as in the baseline model. The last of these is a new condition showing that optimal fiscal policy sets the marginal product of public capital equal to that of private capital. It implies that the optimal amount of public capital depends on the relative productivities of private and public capital. For example, a fall in the productivity of private capital $\left(A_2^F\right)$, holding constant the productivity of public capital (A_2^G), increases optimal investment in public capital.

If prices are sticky, the following equations describe the economy's equilibrium:

$$C_1 = \left(\frac{1}{\beta A_2^F}\right)^{\frac{1}{\gamma}} A_2^F \frac{M_2}{(1+i_1) P_1},$$

$$C_2 = A_2^F \frac{M_2}{(1+i_1) P_1}.$$

$$I_1 = \frac{1}{\left(1-g_2^C\right)} \frac{M_2}{(1+i_1) P_1} - \frac{A_2^G}{A_2^F} \kappa\left(G_1^I\right)$$

$$P_2 = \frac{(1+i_1)}{A_2^F} P_1$$

$$Y_1 = \frac{1 + \left(\frac{1}{\beta A_2^F}\right)^{\frac{1}{\gamma}} A_2^F \left(1-g_2^C\right)}{\left(1-g_2^C\right)} \frac{M_2}{(1+i_1) P_1} + G_1^C - \frac{A_2^G}{A_2^F} \kappa\left(G_1^I\right) + G_1^I$$

$$Y_2 = \frac{A_2^F}{\left(1-g_2^C\right)} \frac{M_2}{(1+i_1) P_1}$$

These are close analogues to expressions (31) through (34), modified to include government investment. If monetary policy is unrestricted, the central bank can use this solution to derive optimal policy and achieve the first-best flexible-price equilibrium. We focus on the case, however, in which monetary policy is limited, in order to examine the possible role of fiscal policy.

Optimal fiscal policy changes surprisingly little with the introduction of government investment. In particular, it remains true, as in our previous analysis under sticky prices, that

$$u'(C_1) > v'\left(G_1^C\right),$$

$$u'(C_2) > v'\left(G_2^C\right),$$

that is, the government increases public consumption beyond where a classical criterion would indicate. However, conditions (55) and (56) continue to hold. Investment in public capital is still determined by equating the marginal products of the two types of capital.

One might ask, why doesn't public investment rise even further to help soak up some of the idle capacity? It turns out that, in this model, public investment crowds out private investment. In particular, private investment at the zero lower bound is determined by

$$I_1 = \frac{1}{\left(1-g_2^C\right)}\frac{M_2}{P_1} - \frac{A_2^G}{A_2^F}\kappa\left(G_1^I\right).$$

At the optimum, as determined by equation (56), $\partial I_1/\partial G_1^I = -1$. The intuition behind this result is the following. When the government increases public investment, other things equal, it tends to increase second-period output and consumption. An increase in second-period consumption for a given money supply tends to push down second-period prices, raising the first-period real interest rate. Private investment falls, leaving the effective capital stock $\left(K_2^F + \frac{A_2^G}{A_2^F}\kappa\left(K_2^G\right)\right)$ unchanged. As a result, public investment is an ineffective stabilization tool and therefore continues to be set on classical principles.[9]

As with the baseline model, an investment subsidy can implement the flexible-price optimum in this model in the limit as $\sigma \to 0$. The optimal subsidy matches the size of the negative nominal interest rate that would implement the flexible-price equilibrium if negative rates were available, as in expression (54).

9 Tax policy in a Non-Ricardian Setting

Throughout the analysis so far, households have been assumed to be forward-looking utility maximizers, and thus their behavior accords with Ricardian equivalence. Changes in tax policy have important effects in the

[9] The mechanism here resembles Eggertsson's (2010) "paradox of toil," according to which positive supply-side incentives reduce expected inflation, raise real interest rates, and depress aggregate demand and short-run output.

model if they influence incentives (as in the case of investment subsidies) but not to the extent that they merely alter the timing of tax liabilities.

Many economists, however, are skeptical about Ricardian equivalence. Moreover, much evidence suggests that consumption tracks current income more closely than can be explained by the standard model of intertemporal optimization. (See, e.g., Campbell and Mankiw 1989). In this subsection, we build non-Ricardian behavior into our model by assuming that households choose consumption in the first period in part as maximizers and in part as followers of a simple rule of thumb. Such behavior can cause the timing of taxes to affect the economy's equilibrium through consumption demand, and it opens new possibilities for optimal fiscal policy.

Formally, a share $(1-\lambda)$ of each household's consumption in a given period is determined by what a maximizing household above would choose, while a share λ is set equal to a fraction ρ of current disposable income. We denote these two components of consumption C_t^M for the maximizing share and C_t^R for the rule-of-thumb share, where

$$C_t^R = \rho\left(Y_t - T_t\right),$$

and a household's total consumption is

$$C_t = \left(1-\lambda\right) C_t^M + \lambda C_t^R.$$

We choose a value for ρ that sets $C_t^M = C_t^R$ prior to any shocks. That is, the proportionality coefficient in the rule of thumb is assumed to have adjusted so that the level of consumption was initially optimal. But in response to a shock, households will continue to follow this rule of thumb, potentially causing consumption to deviate from the utility-maximizing level.

Adding rule-of-thumb behavior has minor implications for the conditions determining equilibrium. The one equation directly affected by it is the household's intertemporal Euler condition, where now only the maximizing component of consumption satisfies this condition. As with the analysis of the Ricardian baseline model, we characterize optimal monetary and fiscal policy in a variety of settings after the economy has suffered an unexpected shock to future technology A_2. We assume the budget was balanced $(G_1 = T_1)$ prior to the shock.

The first result to note is that optimal fiscal policy is the same in the flexible-price scenario and in the fixed-price scenario with fully effective monetary policy. In both cases, output remains at the full-employment level. This is similar to what we have seen previously. However, in this non-Ricardian model, the optimal timing of optimal tax policy responds to the shock to A_2. To the extent that households follow the rule of thumb for consumption, they fail to reduce their first-period consumption appropriately in response to their lower wealth. To set first-period consumption equal to its optimal value, the government should raise taxes T_1.

Formally, the optimal fiscal policy is described by these equations:

$$T_1 = \frac{(1+\theta) + \theta \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2 \left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]}{(1+\theta)\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]^2} A_1 K_1,$$

$$G_1 = \frac{\theta \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2}{(1+\theta)\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]} A_1 K_1.$$

The optimal budget balance would be:

$$T_1 - G_1 = \frac{1}{\left[1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right]^2} A_1 K_1.$$

In words, a decline in the economy's wealth due to a reduction in future productivity should induce a budget surplus. Just as the forward-looking consumers start saving more in response to new circumstances, the government also tightens its own belt by reducing spending to its flexible-price equilibrium level and setting taxes above that level, thereby increasing public saving as well.

Now consider the case in which prices are sticky and monetary policy is restricted to a conventional policy of reducing the short-term interest rate. If the shock to aggregate demand is sufficiently large, this monetary policy may be insufficient to restore the economy to full employment and the optimal allocation of resources. In this case, fiscal policy may play a valuable role in increasing aggregate demand.

The reduced-form solution for output as a function of policy is the following

$$Y_1 = \frac{1}{1-\lambda\rho} G_1 - \frac{\lambda\rho}{1-\lambda\rho} T_1 + \frac{1 - \lambda\rho \left(\frac{1}{A_2\beta}\right)^{\sigma} A_2}{A_2(1-\lambda\rho)} G_2 + \frac{\lambda\rho \left(\frac{1}{A_2\beta}\right)^{\sigma} A_2}{A_2(1-\lambda\rho)} T_2 + \frac{1 + (1-\lambda\rho)\left(\frac{1}{A_2\beta}\right)^{\sigma} A_2}{1-\lambda\rho} \frac{M_2}{(1+i_2) P_1}$$

Notice that if $\lambda > 0$ the timing of taxes influences equilibrium output. Moreover, the government purchases multiplier now exceeds unity. Particularly noteworthy is that the government-spending and tax multipliers in this model (the coefficients on the first two terms) resemble those in the traditional Keynesian income-expenditure model, where $\lambda\rho$ takes the place of the marginal propensity to consume. However, it is not possible to vary G_1 or T_1 without also changing some other fiscal variable to satisfy the government budget constraint.

One can show that optimal fiscal policy in this setting satisfies the following conditions:

$$u'(C_1) = v'(G_1),$$

$$v'(G_1) = \beta A_2 v'(G_2).$$

These conditions are two of the same classical principles that characterize fiscal policy in the baseline flexible-price equilibrium. There is, however, an important exception: the intertemporal Euler equation for private consumption is no longer included in the conditions for the optimum. The reason is that when the economy has idle resources, the real interest rate fails to appropriately reflect the price of current relative to future consumption. Thus, optimal policy in this non-Ricardian setting induces household to consume more than they would on their own if they were intertemporally maximizing.

To get a better sense for these results, it is useful to compare the optimal allocation in this non-Ricardian sticky-price model with that in the corresponding Ricardian sticky-price model examined earlier. We denote the current section's model with the superscript "*nonR*." One can show the following:

$$C_1^{nonR} > C_1^{sticky},$$

$$C_2^{nonR} = C_2^{sticky},$$

$$I_1^{nonR} < I_1^{sticky},$$

$$G_1^{nonR} < G_1^{sticky},$$

$$G_2^{nonR} < G_2^{sticky},$$

$$Y_1^{nonR} = Y_1^{sticky},$$

$$Y_2^{nonR} < Y_2^{sticky}.$$

These inequalities show that optimal fiscal policy in this non-Ricardian model can move the equilibrium allocations of consumption and government purchases closer to the baseline flexible-price optimum. In particular, because the government here can use taxes to stimulate consumption demand, it relies less on increases in government purchases.

If we reintroduce the investment subsidy into the model, the results change even more dramatically. In particular, fiscal policymakers now have sufficient tools to fully restore the flexible-price equilibrium. The online appendix shows the proof, but the intuition is straightforward. Because fiscal policy can influence consumption through the lump-sum tax, investment through the investment subsidy, and government purchases directly, it has complete control over the allocation of resources. When A_2 falls, optimal policy in the first period calls for a decrease in government spending (because society is poorer), an increase in taxes (so the non-Ricardian component of consumption responds appropriately to the lower permanent income), and an investment subsidy (to stimulate investment spending and aggregate demand). Thus, sufficiently flexible fiscal policy can yield the first-best outcome even when monetary policy cannot.

10 Some Tentative Policy Conclusions

The goal of this paper has been to explore optimal monetary and fiscal policy for an economy experiencing a shortfall in aggregate demand. The model we have used is in many ways conventional. It includes short-run sticky prices, long-run flexible prices, and intertemporal optimization and forward-looking behavior on the part of firms and households. It is simple enough to be tractable yet rich enough to offer some useful guidelines for policymakers. These guidelines are tentative because, after all, our model is only a model. Yet with this caveat in mind, it will be useful to state the model's conclusions as clearly and starkly as possible.

One unambiguous implication of the analysis is that how any policy instrument is used depends on which other instruments are available. To summarize the results, it is fair to say that there is a hierarchy of instruments for policymakers to take off the shelf when the economy has insufficient aggregate demand to maintain full employment of its productive resources.

The first level of the hierarchy applies when the zero lower bound on the short-term interest rate is not binding. In this case, conventional monetary policy is sufficient to restore the economy to full employment. That is, all that is needed is for the central bank to cut the short-term interest rate. Fiscal policy should be set based on classical principles of cost-benefit analysis, rather than Keynesian principles of demand management. Government consumption should be set to equate its marginal utility with the marginal utility of private consumption. Government investment should be set to equate its marginal product with the marginal product of private investment.

The second level of the hierarchy applies when the short-term interest rate hits against the zero lower bound. In this case, unconventional monetary policy becomes the next policy instrument to be used to restore full employment. A reduction in long-term interest rates may be sufficient when a cut in the short-term interest rate is not. And an increase the long-term nominal anchor is, in this model, always sufficient to put the economy back on track. This policy might be interpreted, for example, as the central bank targeting a higher level of nominal GDP growth. With this monetary policy in place, fiscal policy remains classically determined.

The third level of the hierarchy is reached when monetary policy is severely constrained. In particular, the short-term interest rate has hit the zero bound, and the central bank is unable to commit to future monetary policy actions. In this case, fiscal policy may play a role. The model, however, does not point toward conventional fiscal policy, such as cuts in taxes and increases in government spending, to prop up aggregate demand. Rather, fiscal policy should aim at incentivizing interest-sensitive components of spending, such as investment. In essence, optimal fiscal policy tries to do what monetary policy would if it could.

The fourth and final level of the hierarchy is reached when monetary policy is severely constrained and fiscal policymakers rely on only a limited set of fiscal tools. If targeted tax policy is for some reason unavailable, then policymakers may want to expand aggregate demand by increasing government spending, as well as cutting the overall level of taxation to encourage consumption. In a sense, conventional fiscal policy is the demand management tool of last resort.

11 A Methodological Epilogue

Economists rely on simple models to develop and hone their intuition about how the economy works. When considering the role of fiscal policy for dealing with an economy in recession, the first thought of many economists is the famous income-expenditure model, also known as the "Keynesian cross," which they learned as undergraduates. With a minimum of algebra and geometry, the model shows how fiscal policy can increase aggregate demand and thereby close the gap between output and its potential level. Indeed, some of the more sophisticated econometric models used for macroeconomic policy analysis are founded on the logic of this simple model.

The conventional application of these macroeconomic models for normative purposes, however, is hard to reconcile with more basic economic principles. Ultimately, all policy should aim to improve some measure of welfare, such as the utility of the typical individual in society. The output gap matters not in itself but rather because it must in some way be an input into welfare.

A common aphorism (often attributed to James Tobin) is that "it takes a heap of Harberger triangles to fill an Okun's gap."[10] The saying is invoked to suggest that when the economy is suffering from the effects of recession, microeconomic inefficiencies should become a lower priority than bringing the economy back to potential. This conclusion, however, may be too glib. Policymakers have various tools at their disposal with which they can influence aggregate demand. Which tools they use to bring the economy back to full

[10] For economists under the age of 40, who may be less familiar with these archaic terms, maybe we should explain. A Harberger triangle is the area in a supply and demand diagram that measures the deadweight loss of a tax or other market distortion. An Okun's gap is the loss in output and employment when the economy falls below potential because of insufficient aggregate demand.

employment can profoundly influence the level of welfare achieved. That is, welfare is a function of both Okun's gaps and Harberger triangles, and policymakers need to be mindful of this fact when they conduct demand management.

The policy guidelines we have derived in this paper are based on the standard tools of welfare economics. Much debate over fiscal policy compares the policy alternatives using metrics that are quite different from a welfare measure. In particular, the commonly used "bang for the buck," which measures how much GDP rises for each dollar added to the budget deficit, is a potentially misleading guide to evaluating alternative policies. The reason is that welfare depends not only on the level of output, but also on its composition among the various components of spending. In other words, policymakers should aim to close the Okun's gap not at the smallest budgetary cost but, instead, while creating the smallest Harberger triangles.

Finally, we finish by reminding the reader that our specific policy conclusions are based a particularly simple model. We have chosen this approach because simple models can clarify thinking more powerfully than complex ones. Our model includes those elements that we believe are most crucial for the topic at hand. But there is no doubt that many features of the real economy have been left out. To be specific: We have not included financial frictions and problems of financial intermediation. We have not incorporated any open-economy features. We have lump-sum rather than distortionary taxes. We have no uncertainty. We incorporate sticky prices, but we do not take into account that firms' price setting is staggered or that different sectors may have different degrees of price rigidity. We have not formally modeled the political process that allows some policy tools to be used more easily than others. Future work may well modify our framework and, in doing so, call some of our tentative conclusions into question. We hope that the simple and transparent model presented here provides a useful starting point for those future investigations.

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