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Turning Waste into By-Product

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Abstract

This paper studies how a firm can create and capture value by converting a waste stream into a useful and saleable by-product (i.e., implementing by-product synergy, BPS). We show that BPS creates an operational synergy between two products that are jointly produced. In essence, BPS is a process innovation that reduces the marginal cost of the original product and/or the by-product. The firm creates value through this process innovation and can capture this value by capturing newly created market opportunities, taking market share from competitors, or licensing the innovation to its competitors. We determine the optimal operating and licensing strategies for the firm and find market conditions under which the firm would benefit most from implementing BPS. We show that the optimal operating and licensing strategies are driven by the size of the cost reduction enabled by the BPS process innovation. We also show that leveraging the synergies between the original product and by-product can lead to counterintuitive profit-maximizing operating strategies such as increasing the amount of waste generated, and strategically increasing the quantity of original product above the business as usual production volume. We present a framework for assessing the environmental impact of BPS which incorporates the impact of the optimal operating and licensing strategies.

1 Introduction

Accompanying the production of every product in a manufacturing process is a stream of collateral output which is usually ascribed as waste. However, pressure to improve the bottom line and increased scrutiny on environmental impact have driven firms to innovate on ways to manage their waste streams. In this paper, we study how a firm can generate and capture value by converting its waste stream (through further processing) into a useful and saleable by-product, and how this practice can potentially benefit the environment. The conversion of a waste stream into by-product is commonly referred to as "by-product synergy" (BPS), a term we will use throughout this paper (U.S. Business Council for Sustainable Development 2011).

The BPS concept is not new. Many examples exist in the agriculture industry, e.g., manure from livestock is used to make fertilizer, animal hide is used to make leather. Often waste water or waste heat from a plant is used to cool or heat other facilities. Although BPS is prevalent in some industries (e.g., agriculture and petrochemical) this concept has not been universally embraced, leaving potentially valuable resources untapped. To promote and facilitate the identification of BPS opportunities, organizations around the world are organizing regional programs to bring companies together to share information about their waste streams. For example, the National Industrial Symbiosis Program was established to promote BPS in the UK, and the US Business Council for Sustainable Development has organized regional BPS programs in Chicago, Houston, Kansas City, New Jersey, and Puget Sound.

Although it is difficult to estimate the market potential of BPS opportunities, anecdotal evidence suggests that it could be significant. For example, the National Industrial Symbiosis Program helped over 400 firms save £26 million, preserving 173 jobs and creating 25 in the first 18 months of operation in the West Highlands (National Industrial Symbiosis Programme 2011). Working with the U.S. Business Council for Sustainable Development, Dow Chemical Company piloted the BPS concept within its own company. With six manufacturing plants from the Gulf Coast participating in the initial study, $15 million of annual cost savings were identified in addition to an annual reduction of 900 thousand MMBtu of fuel use and 108 million pounds of CO_2 emissions (U.S. Department of Energy, Energy Efficiency and Renewal Energy 2005).

This paper studies how a firm can create and capture value by implementing BPS. We explicitly consider the operational synergy between the original product and by-product, and jointly optimize their production given market characteristics. We show that BPS creates value by acting as a cost-reducing process innovation. A symbiotic relationship between the original product and by-product is created by the disposal cost and the virgin raw material cost. Producing by-product by "consuming" the original product's waste stream is a process innovation that avoids the disposal cost. Likewise, "feeding" its waste stream as input into by-product production is a process innovation that avoids the virgin raw material cost.

The firm can capture the value created by the BPS process innovation by capturing newly created market opportunities, taking market share from its competitors, or licensing the process innovation (if it is patentable). We show that the firm's optimal operating strategy depends critically on the size of the cost reduction enabled by the BPS process innovation, which is represented by the disposal cost or the virgin raw material cost. Since the waste stream is productively used in a BPS operation, more waste could actually be better. We derive conditions under which the firm should strategically "overproduce" or increase the quantity of the original product beyond the business as usual quantity, to leverage its cost advantage. Another interesting implication of valuable waste is that the firm could actually increase profit by generating more waste *per unit of original product*. Whereas the strategic increase in the production of the original product generates more waste proportionally, this result implies that it would be profitable for the firm to generate more waste *by being more wasteful*. This is opposite to the "cost minimization through waste reduction" prescription generally followed by manufacturing managers.

If the BPS process innovation can be patented, the firm can also capture value by licensing the innovation to its competitors. For example, Chaparral Steel/TXI developed a process to use steel slag, a waste stream from steel production, to produce Portland cement, and patented this process as the CemStar process (National Slag Association 2011). We focus our licensing analysis on the firm's competitor in the original market who can potentially implement BPS, and study two extreme forms of competition in the by-product market: perfect competition, and competition only between firms that implement BPS.

We find that if the by-product market is perfectly competitive, BPS acts purely as a cost reducing process innovation for the original product as it does not change the market dynamics in the by-product market. Thus, the market structure maps to that studied by Wang (1998), allowing us to apply the results therein. We show that the magnitude of the BPS-enabled cost reduction is determined by the disposal cost. If the disposal cost is high enough to enable a cost reduction that would completely squeeze out the competitor in the original market, licensing is not optimal. However, if the impact of the cost reduction is small enough to allow the competitor to remain in the original market, then licensing would be optimal using a per unit royalty fee. This fee allows the firm to capture the value BPS creates for the competitor.

If implementing BPS changes the market dynamics in the by-product market, then the licensing strategy becomes more complex. To focus on the effect of the BPS waste conversion process on the firm's licensing strategy, we examine the scenario where the by-product market is served only by firms in the original market that convert their waste streams into by-product. In this case, by licensing its BPS process, the firm introduces a competitor into the by-product market and increases competition in the original market (by allowing its existing competitor to reduce cost). In order to benefit from licensing, BPS must create enough value for the competitor, and the firm must be able to extract this value through license fees, to compensate for the increased competition. We find again that the disposal cost is critical in determining the optimal licensing strategy, however, its effect is non-monotonic. Licensing is not optimal for low and high disposal cost – either the value created for the competitor is too low, or the competitive advantage gained by keeping BPS proprietary is too high. We find that licensing is optimal for intermediate levels of disposal cost where enough value can be created and captured, but the opportunity cost of not leveraging the competitive advantage afforded by keeping BPS proprietary is not too high.

We also present a framework for assessing the environmental impact of implementing BPS that incorporates the production outcome of the optimal operating and licensing strategies. It is possible that the increase in total output and the changes in production technology could lead to a worse overall impact on the environment. There could also be a suboptimal environmental outcome if it were profit-maximizing for the firm to keep its process innovation proprietary, yet it is less polluting and thus would be better for the environment if everyone adopted it. This underscores the complexity of business decisions that involve environmental issues.

Contribution To the best of our knowledge, this paper is the first to introduce and analyze a BPS operation in the operations management literature. We formally model a BPS operation in a market setting and show how the firm can maximize value capture through a combination of operational optimization and licensing strategy, and how the two inter-relate.

Literature This paper takes an operational perspective of a subfield of industrial ecology called industrial symbiosis (Chertow 2000). We build on the existing literature in this field that describes specific examples of firms that exchange waste materials, energy, and water (cf. Ehrenfeld and Gertler 1997, Chertow and Lombardi 2005, Zhu et al. 2007, and van Beers et al. 2007). The industrial symbiosis research has primarily focused on the environmental impact of existing BPS operations, e.g., the reduction in toxic emissions or landfill use. In contrast, we take the perspective of the profit-maximizing firm that *optimizes* its operational and licensing decisions, and explicitly incorporate the joint production nature of the BPS process in a market setting.

This paper is also related to the literature on closed-loop supply chains that analyzes the flow of products from the consumer back to the manufacturer or another party in the supply chain (Fleischmann et al. 1997, Guide et al. 2003, Toffel 2003, Debo et al. 2005, Guide et al. 2006, Atasu et al. 2008). The focus of the closed-loop supply chain literature has been on the reverse logistics of getting post-consumer end-of-use products back from the consumer (Fleischmann et al. 1997, Jayaraman et al. 2003, Savaskan et al. 2004), and the resulting market dynamics of a remanufactured product competing with new products (Majumder and Groenevelt 2001, Ferguson and Toktay 2006, Ferrer and Swaminathan 2006, Atasu et al. 2008). Third party remanufacturers depend on the OEM for supply, but also are its competitor in the secondary market. This market structure can result in the OEM producing less new product and the remanufacturer helping to reduce the OEM's remanufacturing cost to induce higher new product quantity (Majumder and Groenevelt 2001). In contrast, BPS addresses the disposition of pre-consumer waste, i.e., the waste stream from the manufacturer, and competitors in the by-product market produce independently of the BPS firm's waste stream. Our results show that the BPS firm never decreases production of the original product and competitors in the by-product market never gain by helping the BPS firm. Moreover, BPS affects the operating cost structure. Therefore, it is essentially a process innovation which, if patentable, allows the firm to capture value through licensing, an option that does exist in remanufacturing. What is consistent across the remanufacturing and BPS settings is the profit tradeoff the firm makes between two markets: new product and secondary markets in the case of remanufacturing, and original and by-product markets in the case of BPS.

This paper complements the literature on coproduction systems that studies production systems that yield multiple products (cf. Bitran and Leong 1992, Bitran and Gilbert 1994, Hsu and Bassok 1999, Tomlin and Wang 2008). However, there is a fundamental difference between the quantity relationship in a coproduction system and in a BPS operation. In a coproduction system, the total quantity is fixed and the decision is how to divide the total quantity into various subclasses of product. For example, in a two-class system, to allocate one more unit to class i, one unit must be subtracted from class j. In a BPS operation, the quantity relationship is exactly the opposite. By increasing the quantity of original product, the (possible) quantity of by-product increases.

The market structure in our model is similar to multimarket oligopolies, which have been studied extensively in the economics literature. The research most closely related to ours are those papers that study firms which serve multiple markets from a single facility so that the cost structures of the products are related (cf. Bulow et al. 1985, Chen and Ross 2007). Whereas these papers investigate several cost structures (e.g., shared overhead), none capture the form used in our model which explicitly models the quantity relationship between two products that are jointly produced in a BPS operation. This allows us to derive operational insights such as strategically managing the proportion of waste generated, and understanding how the licensing strategy fits with the operating strategy.

We apply and build on the results from the licensing literature. Specifically, we use the licensing under Cournot competition results from Wang (1998) and show how the distinct BPS operating and market structures lead to different optimal licensing strategies.

The rest of the paper is organized as follows. Section 2 describes our model, Section 3 explores the operational implications of BPS in the monopoly case, Section 4 presents the licensing analysis, Section 5 examines the environmental impact of BPS, and Section 6 contains concluding remarks. All proofs are in the Appendix.

2 Model

We consider a manufacturing firm that produces product A, which we will call the original product. During the processing of the original product, waste is generated that can be further processed into a useful by-product, B. When the firm converts its waste stream into by-product B, we refer to this as implementing BPS. The waste from one unit of A can be used to generate $\gamma > 0$ units of B. Other raw materials may also be needed to produce B, but when the firm produces B using the waste stream of A, the proportion of the quantity of A to the quantity of B is 1:γ. We refer to the process that produces product A and its associated waste stream as Process X and the process for producing B as Process Y. Product B can be produced by using A's waste stream or by purchasing virgin raw material that substitutes for A's waste stream (this allows the firm to produce B independently of A). Although technically, B is a by-product only if it is produced using A's waste stream, we will use the term by-product to refer to B even if it is produced using virgin material. Figure 1 shows the process flow diagrams for the production of products A and B by the firm. Products A and B are completely different products that are sold into different markets.

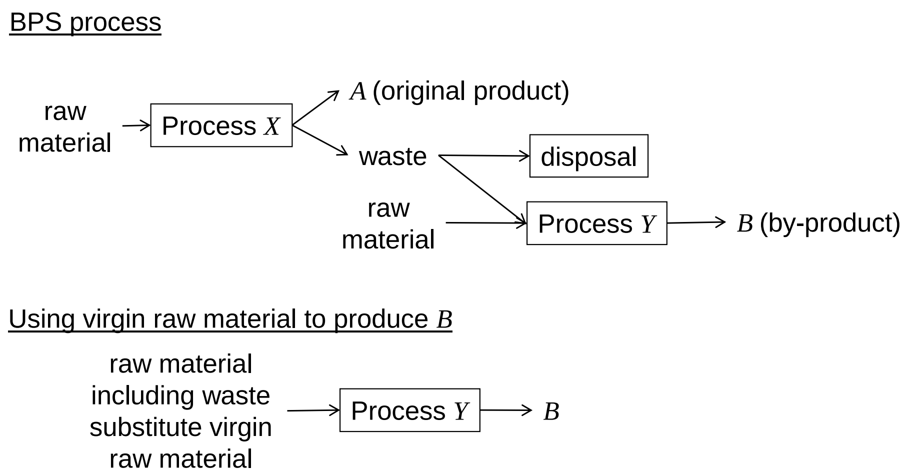


Figure 1: Process flow diagrams for the firm's production of products A and B.

For example, Cook Composites and Polymers is a gel coat manufacturer that developed a BPS process to convert its rinse styrene waste stream into a concrete coating by-product (Lee et al. 2009). In this case, product A is gel coat and product B is concrete coating. The firm can either produce concrete coating (product B) using the rinse styrene waste stream from the gel coat (product A) manufacturing process, or it can purchase virgin styrene to produce concrete coating. Gel coat and concrete coating are sold into two completely different markets .

The firm's cost of producing one unit of A (and its collaterally generated waste stream) using Process X is c_X. The disposal cost of the waste generated by the production of one unit of A is $c_w \geq 0$. The firm can produce B using the BPS process or independently from product A by purchasing virgin raw material. If the firm implements BPS, it incurs a processing cost of c_Y per unit of B, but it avoids $\frac{1}{\gamma}c_w$ in disposal cost. If the firm produces B using virgin raw material, it incurs the c_Y per unit processing cost and also a cost of c_r to purchase virgin raw material to substitute for A's waste stream.

We assume there is no quality difference between product B produced by the BPS process and using virgin raw material.

We assume general (inverse) demand functions $p_A(q)$ and $p_B(q)$ in markets A and B, respectively, that satisfy the following two commonly held assumptions.

Assumption 1 *Downward sloping demand functions:* $\frac{\partial p_j}{\partial q} < 0$, $j = A, B$.

Assumption 2 *The demand functions are such that the marginal revenue curve crosses the marginal cost curve from above:* $2\frac{\partial p_j}{\partial q} + q\frac{\partial^2 p_j}{\partial q^2} < 0$, $j = A, B$.

ndassumption For example, linear downward sloping demand curves $p_A = a - q$ and $p_B = b - q$ satisfy Assumptions 1 and 2. We assume that there are an infinite number of periods and at a certain point in time, the firm discovers that it has the opportunity to implement BPS. At this point, the firm chooses quantities q_A and q_B to maximize profit. If $q_B > 0$, then this means the firm has decided to implement BPS. For expositional simplicity, we perform the analysis for a single period. This simply avoids having to take the discounted total profit over an infinite horizon. We also make the following assumption which implies that $q_A > 0$.

Assumption 3 *The firm is profitable in market A without implementing BPS.*

What we are not explicitly considering in this model is the fixed cost of implementing BPS, e.g., for research and development or capital equipment. The firm could factor this into the analysis as it would for any strategic project. For example, it could calculate the net present value of the investment cost and future cash flow from implementing BPS. The important point from the perspective of this analysis is that however the investment decision is made, it should be based on the implementation that captures the maximum value from BPS.

3 Operational Implications: The Monopoly Case

To explore the operational implications of BPS, we use a simple benchmark case where the firm is a monopolist in both the original and by-product markets. We determine the drivers of the optimal operating strategy and use these to determine how the firm can capture value by implementing BPS. Given the market and operational characteristics of the BPS process, the firm's profit function is

$$\Pi \;=\; \begin{cases} \Pi^{(1)} \equiv p_A q_A + p_B q_B - (c_X + c_w) q_A - (c_Y - \frac{1}{\gamma} c_w) q_B, & \text{if } q_B < \gamma q_A \\ \Pi^{(2)} \equiv (p_A + \gamma p_B) q_A - (c_X + \gamma c_Y) q_A, & \text{if } q_B = \gamma q_A \\ \Pi^{(3)} \equiv p_A q_A + p_B q_B - (c_X - \gamma c_r) q_A - (c_Y + c_r) q_B, & \text{if } q_B > \gamma q_A \end{cases} . \qquad (1)$$

Additionally, we define $(q_A^{(1)}, q_B^{(1)}) \equiv \text{argmax}_{q_A,q_B} \Pi^{(1)}$, $q_A^{(2)} \equiv \text{argmax}_{q_A} \Pi^{(2)}$ (where $q_B^{(2)} = \gamma q_A^{(2)}$), and $(q_A^{(3)}, q_B^{(3)}) \equiv \text{argmax}_{q_A,q_B} \Pi^{(3)}$. Note that $(q_A^{(1)}, q_B^{(1)})$, $q_A^{(2)}$, and $(q_A^{(3)}, q_B^{(3)})$ are the unconstrained optimizers of $\Pi^{(1)}$, $\Pi^{(2)}$, and $\Pi^{(3)}$, respectively. Moreover, $q_A^{(1)}$ is the business as usual quantity of A.

Inspection of (1) shows that BPS acts as a process innovation that can reduce the marginal cost of producing A or B. When $q_B < \gamma q_A$, the firm primarily produces A and B "consumes" part of A's total waste stream. Therefore, $\frac{1}{\gamma} c_w$ is subtracted from B's cost because the production of one unit of B avoids $\frac{1}{\gamma} c_w$ in disposal cost for $\frac{1}{\gamma}$ inframarginal units of A. When $q_B > \gamma q_A$, the firm primarily produces B and A partially "feeds" the by-product process with its waste stream. Therefore, γc_r is subtracted from A's cost because the production of one unit of A avoids γc_r in raw material cost for γ inframarginal units of B. When $q_B = \gamma q_A$, neither the disposal cost nor the raw material cost appears in the profit function because the waste stream of A is matched exactly to the input stream of B. In this case, both the disposal cost and raw material cost are completely avoided.

The following proposition describes when implementing BPS is profitable and what the optimal operating strategy would be.

Proposition 1 *Implementing BPS increases the firm's profit if and only if* $q_B^{(1)} > 0$. *Assuming* $q_B^{(1)} > 0$, *the firm's optimal operating strategy is as follows. If* $q_B^{(1)} \leq \gamma q_A^{(1)}$, *then* $(q_A^*, q_B^*) = (q_A^{(1)}, q_B^{(1)})$ *and the firm's optimal operating strategy is "partial conversion" (i.e.,* $q_B^* < \gamma q_A^*$*), and its quantity of A is unaffected by BPS implementation. If* $q_B^{(1)} \geq \gamma q_A^{(1)}$ *and* $q_B^{(3)} \leq \gamma q_A^{(3)}$, *then*

$(q_A^*, q_B^*) = (q_A^{(2)}, q_B^{(2)})$, *and the firm's optimal operating strategy is "exactly-full conversion" (i.e.,* $q_B^* = \gamma q_A^*$*) and requires the firm to (weakly) increase production of* A*. If* $q_B^{(3)} \geq \gamma q_A^{(3)}$*, then* $(q_A^*, q_B^*) = (q_A^{(3)}, q_B^{(3)})$ *and the firm's optimal operating strategy is "full+ conversion" (i.e.,* $q_B^* > \gamma q_A^*$*) and requires the firm to source virgin raw material to produce* B *and increase production of* A*.*

It is possible that implementing BPS may not increase the firm's profit. However, we know that $q_A^{(1)} > 0$ from Assumption 3, therefore, BPS would increase the firm's profit if and only if the least costly unit of B sold to the highest willingness-to-pay consumer generates positive profit, i.e., $q_B^{(1)} > 0$. In this simple case where the firm is a monopolist in both markets, the firm creates value from implementing BPS because it introduces a cost-reducing process innovation in market A thereby creating new market opportunity, and creates a totally new market B. The firm captures part of the created value as profit, with the remainder going to consumers.

Once a BPS process is implemented, a natural inclination is to continue producing A business as usual and merely convert whatever waste stream that is collaterally generated into by-product, i.e., setting $q_B = \gamma q_A^{(1)}$. However, this would be optimal if and only if, coincidentally, the quantity of B that maximized profit in market B was *exactly* $\gamma q_A^{(1)}$. It is much more likely that the optimal operating strategy for the firm is to either increase its production of A beyond the business as usual quantity, or still dispose of some of its waste (partial conversion). Increasing q_A allows the firm to leverage its cost-reducing process innovation and *generate more waste* in order to capture value in market B. The managerial implication of Proposition 1 is that BPS should not be treated as an alternative to waste disposal. If exactly-full or full+ conversion is optimal, it requires a decision that simultaneously affects two product lines that must be managed at a strategic level. If partial conversion is optimal, the production of B should be managed strategically to capture the most value in market B, and the temptation to eliminate disposal cost entirely should be avoided.

The size of the cost reduction enabled by the BPS process innovation is represented by two parameters, the disposal cost c_w and the raw material cost c_r, which determine the optimal operating strategy of the firm. To present the results clearly, we use linear demand functions $p_A = a - q$ and

$p_B = b - q$ and we will continue to use these demand functions throughout the rest of the analysis. The following proposition shows how the firm's optimal operating strategy is parameterized by c_w or c_r.

Proposition 2 *If $b - c_Y \leq \gamma(a - c_X)$, then*

$$(q_A^*, q_B^*) \quad = \quad \begin{cases} (q_A^{(2)}, q_B^{(2)}) & \text{if } c_w \geq \hat{c}_w \\ (q_A^{(1)}, q_B^{(1)}) & \text{otherwise} \end{cases} \quad , \text{ where } \hat{c}_w = \frac{\gamma(a - c_X) - (b - c_Y)}{\gamma + \frac{1}{\gamma}}.$$

If $b - c_Y > \gamma(a - c_X)$, then

$$(q_A^*, q_B^*) \quad = \quad \begin{cases} (q_A^{(2)}, q_B^{(2)}) & \text{if } c_r \geq \hat{c}_r \\ (q_A^{(3)}, q_B^{(3)}) & \text{otherwise} \end{cases} \quad , \text{ where } \hat{c}_r = -\frac{\gamma(a - c_X) - (b - c_Y)}{1 + \gamma^2}.$$

If $\hat{c}_w = \hat{c}_r = 0$, this implies that conditions in markets A and B are such that, absent the disposal and virgin raw material costs, the optimal quantity of B matches exactly what can be produced using the entire waste stream from the optimal quantity of A. Whether c_w or c_r is critical in determining the optimal operating strategy depends on the balance of demand in markets A and B. The first part of Proposition 2 applies when, absent the disposal and virgin raw material costs, the demand for B would be less than what could be produced by the waste stream of A. In this case, c_w becomes more important because it represents the cost-reducing process innovation for the by-product. Increasing c_w lowers the marginal cost of B, thereby increasing demand for B. We find that when c_w is low (i.e., $c_w < \hat{c}_w$), partial conversion is optimal because the cost reduction for producing B is small (i.e., the value of "consuming" the waste stream of A is low) making it less attractive for the firm to produce B, and at the same time, the cost of producing A is low, making it attractive to produce A. However, as c_w increases, both these effects are mitigated thereby increasing the optimal quantity of B and decreasing the optimal quantity of A, eventually resulting in exactly-full conversion being optimal.

The second part of Proposition 2 applies when, absent the disposal and virgin raw material costs, the demand for B would be greater than what could be produced using the waste stream of A. The virgin raw material cost c_r represents the cost-reducing process innovation for the original product. If c_r is low (i.e., $c_r < \hat{c}_r$), full+ conversion is optimal because the cost reduction for producing A is small (i.e., the value of "feeding" the BPS process is low), making it less attractive for the firm to produce A, and at the same time, the cost of producing B is low, making it attractive to produce B. As c_r increases, both these effects are mitigated thereby increasing the optimal quantity of A and decreasing the optimal quantity of B, resulting in exactly-full conversion being optimal.

Notice that as c_w and c_r increase, the optimal operating strategy becomes exactly-full conversion, and hence, no matter how large c_w and c_r get after that point, they do not affect the profit of the firm. The magnitudes of c_w and c_r determine the impact of the BPS process innovation on the production of the original product and by-product. As these parameters increase, the cost reduction enabled by the process innovation for both products increase, thereby increasing the operational synergy between the two. The synergy is maximized when the waste stream of A matches exactly the input stream of B. An extreme form of this synergy is presented in the following proposition.

Proposition 3 *If $c_w > a - c_X > 0$ and $b - c_Y \geq 0$, producing A without BPS is not profitable, but producing A and B with BPS is profitable.*

If the disposal cost is high enough (i.e., $c_w > a - c_X$), producing A alone would be unprofitable. However, by jointly producing A and B, the disposal cost can be avoided thereby making both products profitable. This extreme situation highlights the fact that disposal cost is a key lever to inducing firms to think creatively about managing their waste streams and their operating strategies in general. Moreover, as regulation becomes more stringent for waste disposal (usually resulting in higher disposal cost), firms are more likely to pursue non-traditional solutions to waste management such as BPS. We see anecdotal evidence of this as the variety and number of firms participating in BPS forums across the U.S. and worldwide are increasing (National Industrial Symbiosis Programme 2011, U.S. Business Council for Sustainable Development 2011).

An interesting operational implication of BPS is that the proportion of "waste" generated by the production of A can be a measure of how efficiently the by-product is produced. Increasing the proportion parameter, γ, means that the quantity of B that can be produced per unit of A increases. The following proposition shows conditions under which the firm can capitalize on the increased efficiency of producing B as γ increases.

Proposition 4 *Assume the parameters a, b, c_X, c_Y, c_w, and c_r are such that the quantities $q_A^{(1)}$, $q_B^{(1)}$, $q_A^{(3)}$, and $q_B^{(3)}$ are all strictly positive. There exist $\gamma_2 > \gamma_1 > 0$ such that full+ conversion is optimal for $\gamma < \gamma_1$, exactly-full conversion is optimal for $\gamma \in [\gamma_1, \gamma_2]$, and partial conversion is optimal for $\gamma > \gamma_2$. Moreover, there exists $\hat{\gamma} \in (\gamma_1, \gamma_2)$ such that the firm's profit increases in γ for $\gamma < \hat{\gamma}$ and decreases in γ for $\gamma \geq \hat{\gamma}$.*

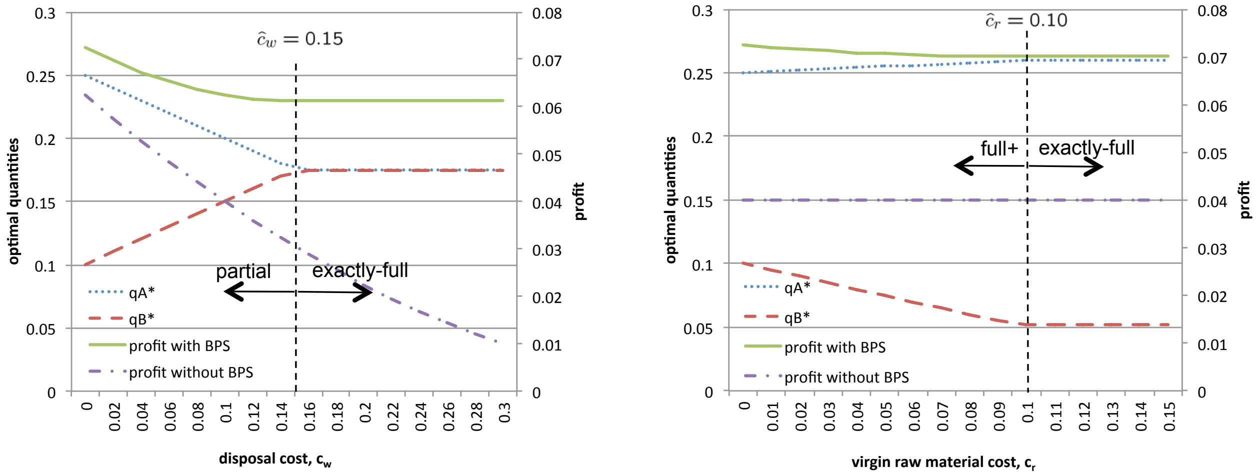
The proportionality parameter γ can either represent the efficiency at which A feeds the production of B, or the efficiency at which B consumes the waste stream of A. Which of these interpretations actually applies is determined by the firm's operating strategy. For example, when full+ conversion is optimal, the marginal unit of waste can be used productively to produce B, but the marginal unit of A would decrease the firm's profit in market A. Therefore, if it were possible to produce more waste without increasing the production of A (i.e., increase γ), overall profit would increase. Conversely, when partial conversion is optimal, reducing waste by one unit would avoid disposal cost, but decreasing the quantity of A would decrease the firm's profit in market A. Therefore, if it were possible to produce less waste without decreasing the production of A (i.e., decrease γ), then overall profit would increase. When exactly-full conversion is optimal, as γ increases, it transitions from representing the efficiency of the "feeding" process, to the inefficiency of the "consuming" process.

An interesting implication of Proposition 4 is that it may be optimal for the firm to *increase* the proportion of waste generated by the production of its original product. Unlike the result in Proposition 1, which shows that the firm can increase profit by generating more waste by proportionally increasing the quantity of original product, this result shows that the firm can increase profit *by being more wasteful.* This is opposite to the "cost minimization through waste reduction" prescription that is generally followed by manufacturing managers.

While it may not always be possible for a firm to control the proportion of waste it generates, when feasible, this result uncovers a potential operational synergy that the firm can exploit by implementing BPS. The gel coat manufacturer, Cook Composites and Polymers, uses an expensive, hazardous chemical to clean mixing vessels between production batches, which eventually becomes the rinse styrene waste stream. To minimize material and disposal costs, the firm strives to use as little styrene as possible. However, using too little would increase the probability of contamination between batches. By converting the waste rinse styrene into a saleable concrete coating by-product, the firm could use more styrene per batch of gel coat (i.e., increase γ), thereby decreasing the probability of contamination and also increasing the combined profitability of its gel coat and concrete coating products (Lee et al. 2009).

Numerical Example We illustrate the results of this section in a numerical example. We pick the following parameters to represent market A having a higher willingness to pay than market B, and consider the effects of c_w, c_r, and γ on the firm's optimal strategy. Let $a = 1$, $c_X = 0.5$, $b = 0.7$, and $c_Y = 0.5$.

The graphs in Figure 2 correspond to the results presented in Proposition 2. Figure 2a represents the case where the amount of B that can be produced using the waste stream of A exceeds the amount that would be demanded by market B, absent the disposal and virgin raw material costs. As disposal cost increases, the quantity of A decreases, and the quantity of B increases (to avoid disposal cost). For $c_w \geq \hat{c}_w = 0.15$, $q_A^* = \gamma q_B^*$ resulting in exactly-full conversion. Since the quantity of B demanded is inherently low, sourcing virgin raw material to produce B would never be optimal. Notice that profit decreases in c_w when partial conversion is optimal, even though B is consuming part of the waste stream of A. However, profit does not decrease as much as it would without BPS. For $c_w > \hat{c}_w$, profit with BPS is constant in c_w since the entire waste stream is consumed by production of B, however, profit without BPS continues to decrease in c_w. Thus, BPS can be used to mitigate the risk of the vagaries of waste disposal regulation.



(a) $b - c_y \leq \gamma(a - c_x)$, $\gamma = 1$ (b) $b - c_y > \gamma(a - c_x)$, $\gamma = 0.2$

Figure 2: Optimal quantities and profit as a function of disposal cost c_w and virgin raw material cost c_r, $a = 1$, $c_X = 0.5$, $b = 0.7$, and $c_Y = 0.5$.

Figure 2b represents the case where the amount of B that can be produced using the waste stream of A is less than the amount that would be demanded by market B, absent the disposal and virgin raw material costs. Since there is high demand in market B, it is optimal to convert the entire waste stream of A into B, therefore, disposal cost does not affect the optimal quantities. The only question is whether market B is attractive enough to warrant sourcing virgin raw material in addition to converting the waste stream of A. For low c_r, sourcing virgin raw material is optimal, however, q_B^* decreases in c_r. As virgin raw material cost increases, q_A^* increases to produce more waste to replace the virgin raw material. For $c_r \geq \hat{c}_r = 0.10$, $q_A^* = \gamma q_B^*$ resulting in exactly-full conversion, under which profit is constant in c_r.

Figure 3 corresponds to the results presented in Proposition 4. For low γ (i.e., $\gamma < \gamma_1 = 0.3$), full+ conversion is optimal, and q_A^* increases in γ. As γ increases, producing B using the waste stream of A essentially becomes cheaper. Therefore, instead of producing B using virgin raw material, the firm increases production of A to feed the BPS process. In this region, profit increases in γ because the feedstock for B is getting cheaper. At $\gamma = \gamma_1 = 0.3$, q_A^* has increased to the point where all B is produced using A's waste stream. For $\gamma \in [\gamma_1, \gamma_2]$, increasing γ has two opposing effects. On the one hand, feeding the BPS process becomes cheaper, but consuming the waste stream becomes more expensive. For $\gamma \in [\gamma_1, \hat{\gamma})$, the first effect dominates so profit continues to increase. However for $\gamma \in [\hat{\gamma}, \gamma_2]$, more waste is produced by A than is optimal for supplying market B, therefore, consuming waste becomes more expensive and profit starts to decrease. For $\gamma \in [\gamma_1, \gamma_2]$, exactly-full conversion is optimal, therefore, q_A^* decreases and q_B^* increases in γ, as more B is produced using less A. For $\gamma > \gamma_2$, producing B continues to be a less effective way of consuming the waste stream of A, therefore, q_B^* decreases as does the firm's profit.

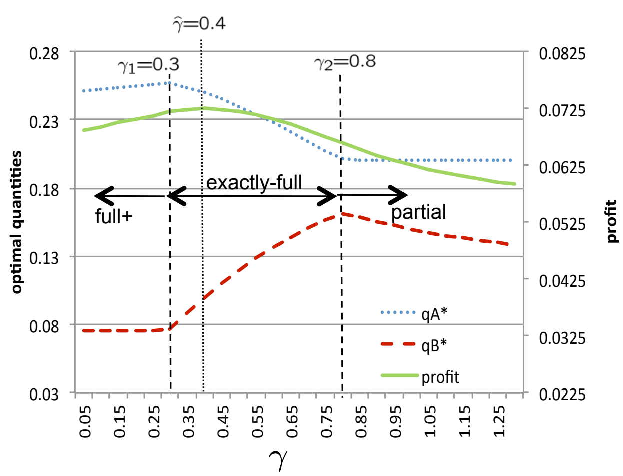


Figure 3: Optimal quantities and profit as a function of γ, $a = 1$, $c_X = 0.5$, $b = 0.7$, $c_Y = 0.5$, $c_w = 0.1$, and $c_r = 0.05$. Profit is maximized at $\hat{\gamma} = 0.33$.

4 Capturing Value under Competition: Licensing

We now explore how the firm can capture value from implementing BPS under different competitive scenarios. In particular, we consider the case where the firm can patent its BPS process and study how the firm can use licensing as a way to capture value. To understand how the BPS waste conversion process changes market dynamics and hence the firm's licensing strategy, we focus on the firm's interaction with a competitor in the original market who could potentially implement

BPS. We henceforth refer to the focal firm as firm 1, who can patent the BPS process, and the market A competitor as firm 2. We assume firms 1 and 2 compete in a Cournot duopoly in market A.

We consider two extreme market conditions in market B. In the first scenario, market B is perfectly competitive so that firms are price takers, no individual firm can make enough B to satisfy the market, and the quantity of B produced by any one firm can be sold at the market price. We assume that market B competitors will not vertically integrate to make A, and sourcing waste substitute virgin raw material is arbitrarily expensive (to focus the analysis on by-product production using the waste stream of A). This market structure is representative of commodity-type by-products such as cement (as in the Chaparral Steel example) or waste water.

In the second scenario, we consider the case where market B is served only by firms in market A that implement BPS. This extreme form of imperfect competition allows us to focus explicitly on the BPS-related competition between the two firms in market A. In essence, by licensing to firm 2, firm 1 introduces a competitor into market B that changes the market dynamics in B. This market structure is similar to that of the Cook Composites and Polymers example where the by-product concrete coating market is characterized by imperfect competition.

Note that firm 1 could exclusively license the BPS process to firm 2 so that only firm 2 implements BPS. Using a fixed fee license, this would result in exactly the same profit for firm 1 as not licensing to firm 2. We examine the more interesting case where firm 1 licenses to firm 2, but still implements BPS itself. We draw and build on results from Wang (1998) which compares royalty and fixed fee licensing for a cost-reducing innovation in a Cournot duopoly. The paper shows that licensing is optimal if and only if the cost reduction is *non-drastic*, i.e., the monopoly price with the reduced marginal cost enabled by the innovation is less than than the original marginal cost. Moreover, a per unit royalty license gives higher profit to the licensor than a fixed fee license because the licensor enjoys a cost advantage under a royalty fee, whereas both firms have the same cost under a fixed fee.

As in Section 3, we perform the analysis for a single period. If firm 1 licenses to firm 2, the two firms compete in both markets, otherwise, firm 2 sells only into market A and firm 1 sells into both markets. The focus of this section is on how firm 1 can use licensing to extract the value created from implementing BPS. For illustrative purposes, we examine the case where $\gamma = 1$, which fixes the value created at a particular level, and we show what licensing scheme firm 1 can use to capture as much of that value as possible. The licensing analysis for general γ gives the same qualitative insights presented in Propositions 5, 6, and 7, and is available from the author.

4.1 Perfect Competition in Market B

Since market B is perfectly competitive, neither firm 1 nor firm 2 can change the price, but both can sell into market B at the market price $p_B = c_b$. The following proposition describes firm 1's optimal licensing strategy when market B is perfectly competitive.

Proposition 5 *If $c_w \in (c_Y - c_b, \frac{a-c_X+c_Y-c_b}{2})$, it is optimal for firm 1 to license its BPS process to firm 2 using a royalty fee. If $c_w \geq \frac{a-c_X+c_Y-c_b}{2}$, it is optimal for firm 1 to not license to firm 2. If $c_w \leq c_Y - c_b$, it is not optimal for firm 1 to implement BPS.*

When market B is perfectly competitive, the impact of BPS on the competition between firms 1 and 2 is contained to market A. If $c_w > c_Y - c_b$, then BPS creates value by acting as a cost reducing innovation for producing A, thus we can directly apply the results from Wang (1998). Whether it is profitable for firm 1 to license to firm 2 depends on the size of the cost reduction, which in turn is determined by the disposal cost. If $c_w < \frac{a-c_X+c_Y-c_b}{2}$, the cost reduction is small and the innovation is non-drastic, therefore, it is optimal for firm 1 to license to firm 2 using a royalty license. However, if $c_w \geq \frac{a-c_X+c_Y-c_b}{2}$, the cost reduction is large and the innovation is drastic, therefore, firm 1 should keep its BPS process proprietary. The licensing decision boils down to the tradeoff between license fees and competitive advantage (by taking market share from firm 2 as a result of the BPS-enabled cost advantage). For high c_w, firm 1 is better off leveraging the competitive advantage afforded by keeping BPS proprietary. However, for low c_w, the cost reduction is not big enough to drive firm 2 out of the market, therefore, licensing is optimal.

4.2 Cournot Duopoly in Market B

Consider now the case where market B is served only by firm 1, and possibly firm 2, if firm 1 licenses its BPS process. Thus, firm 1 would be a monopolist in market B if it kept its BPS process proprietary, otherwise, if it licenses to firm 2, there would be a Cournot duopoly in market B. We now examine under which conditions it would be optimal for firm 1 to license its BPS process to firm 2, and what licensing scheme would be optimal. The following proposition shows that using only a fixed fee license is never optimal.

Proposition 6 *Firm 1 cannot increase its profit by licensing its BPS process to firm 2 using only a fixed fee license.*

If firm 1 licenses to firm 2, it gives up monopoly profit in market B. Moreover, the producer surplus in market B actually decreases when the market structure shifts from a monopoly to a duopoly. Therefore, even though firm 1 can extract the value created for firm 2 using the fixed licensing fee, its profit in market B decreases. Then, the only way the fixed fee licensing would increase firm 1's profit is if its profit in market A increased enough to compensate for the profit loss in market B. However, we know from Wang (1998) that royalty licensing is superior to fixed fee licensing in a Cournot duopoly. Therefore, this suggests that a licensing scheme that incorporates a per unit royalty component would be optimal. This intuition is formalized in the following proposition.

Proposition 7 *There exist $\underline{c}_w$ and $\bar{c}_w$ such that for $c_w < \underline{c}_w$ or $c_w > \bar{c}_w$, not licensing is optimal for firm 1. For $c_w \in [\underline{c}_w, \bar{c}_w]$, licensing using a combination royalty and fixed fee is optimal. If licensing is optimal, firm 1 implements exactly-full conversion, but firm 2 implements either partial or exactly-full conversion.*

Again we see that firm 1's optimal licensing strategy can be parameterized by the disposal cost c_w. Unlike the case where market B is perfectly competitive, here the disposal cost has a non-monotone effect on firm 1's licensing strategy. Not licensing is optimal for low and high disposal cost, and for intermediate disposal cost, licensing is optimal using a combination royalty and fixed fee. The difference in the effect of the disposal cost on firm 1's licensing strategy is driven by the fact that licensing now changes market dynamics in both markets. To understand the mechanisms that drive Proposition 7, it is instructive to examine the following numerical example.

Numerical Example We continue with the market parameters used in Section 3, i.e., $a = 1$, $c_X = 0.5$, $b = 0.7$, and $c_Y = 0.5$. Figure 4 shows the profits of firms 1 and 2 for various licensing schemes as a function of disposal cost c_w (notice that firm 1's profit is indicated on the left vertical axis and firm 2's profit is on the right). The curves that are generally increasing in c_w are firm 1's profit and those decreasing in c_w are firm 2's profit.

The key features of Figure 4 are that licensing is optimal for disposal cost between $\underline{c}_w = 0.11$ and $\bar{c}_w = 0.32$. For disposal cost less than or greater than this range, not licensing is optimal. The threshold disposal cost that determines when firm 1 shifts from partial conversion to exactly full conversion is $c'_w = 0.08$, whereas for firm 2, the threshold is higher at $c''_w = 0.27$.

For low disposal cost (i.e., $c_w < \underline{c}_w$), the value created by BPS for firm 2 is low, therefore, the value that can be extracted by firm 1 through licensing is too low to justify introducing firm 2 as a competitor in market B. However, as c_w increases to $c_w \geq \underline{c}_w = 0.11$, the value created increases and licensing becomes optimal. Firm 1 implements exactly-full conversion and firm 2 implements partial conversion because the royalty fee makes their costs asymmetric. As c_w increases, firm 2 increases its quantity of B to avoid disposal cost, and as a result, firm 1 can increasingly benefit from licensing BPS (as seen by the increasing difference between the "no licensing" and "royalty + fixed fee" profit curves for firm 1). At $c_w = c''_w$, the disposal cost is high enough so that firm 2 would convert its entire waste stream into by-product, even with the royalty fee. As c_w increases past this point, the royalty fee remains fixed because the optimal quantities are independent of

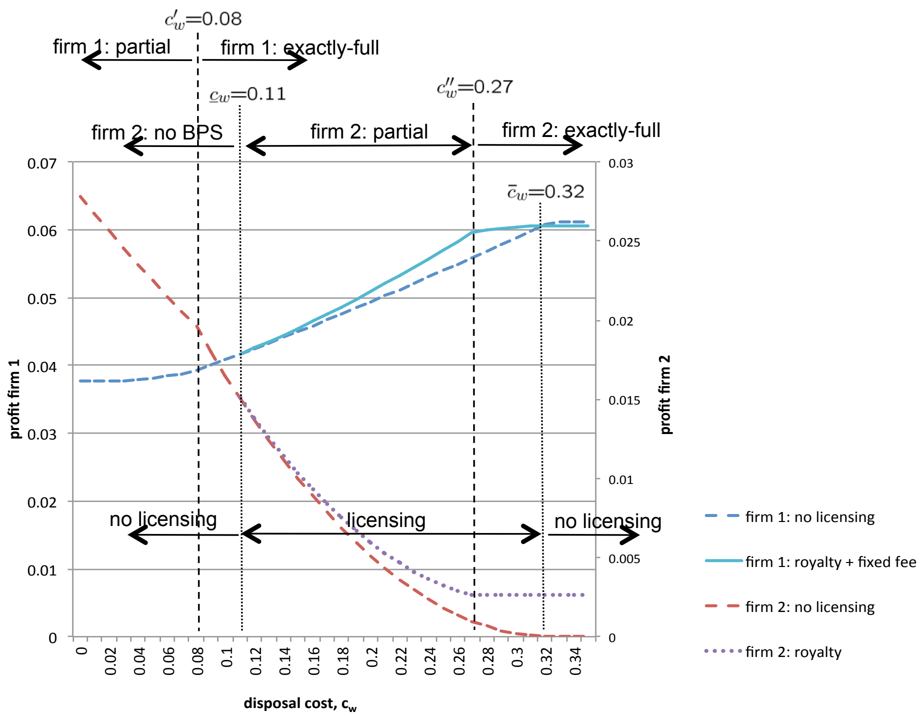


Figure 4: Firm 1 and firm 2's profits under no licensing and licensing as a function of disposal cost c_w, $a = 1$, $c_X = 0.5$, $b = 0.7$, and $c_Y = 0.5$.

disposal cost. However, the fixed fee portion of the license still increases because firm 2's profit under no licensing still decreases in c_w. Therefore, firm 1 can still extract the increasing difference between firm 2's licensing and no licensing profits using the fixed fee.

For $c_w \geq \bar{c}_w$, no licensing becomes optimal again. When the disposal cost is this high, the competitive advantage gained by the firm by keeping BPS proprietary outweighs the licensing profit, especially since the gain from avoiding disposal cost has already been exhausted. Note that firm 1 can increase profit using only a royalty fee, however, it would not capture all the benefit created for firm 2 because some of the value created for firm 2 is in the newly created market B opportunity. Therefore, using the combination royalty and fixed fee captures the entire benefit created for firm 2, and makes firm 2 indifferent between licensing and no licensing.

Again, the licensing decision boils down to the tradeoff between licensing profit and competitive advantage by keeping BPS proprietary. The licensing profit depends on how much value implementing BPS creates for firm 2. For low disposal cost c_w, value is created in market B, but ironically, it is low because producer surplus decreases as a result of the increased competition. Therefore, licensing is not optimal. For high disposal cost, the competitive advantage gained by firm 1 by keeping BPS proprietary is very high, especially since the value created by the waste conversion process is capped at the point where all the waste is converted into by-product. Moreover, under no licensing, even if firm 1 has converted its entire waste stream into by-product, it would still gain a competitive advantage over firm 2 who still has to pay for disposal. Therefore, licensing is optimal for intermediate levels of disposal cost where BPS creates enough value for firm 2 that can be extracted by firm 1, and at the same time, the opportunity cost of not leveraging the competitive advantage afforded by keeping BPS proprietary is not too high.

5 Environmental Impact

Although BPS is generally viewed as having a positive impact on the environment, careful analysis shows that this is not always the case. We focus here on the impact on emissions when a firm implements BPS (versus, for example, the environmental impact of waste disposal in landfill). Two environmental benefits of BPS that are often highlighted are: 1) less waste incineration, and 2) the by-product replaces competing products in market B that are produced in a manner that is worse for the environment, e.g., the displaced products use virgin raw material which creates emissions during the extraction process. We have adapted a framework from Baumgartner and Jost (2000) to determine the impact of BPS on emissions. Figure 5 shows the sources of potential emissions from firm 1 and the competitors in market B in the context of BPS.

Typically, it is implicitly assumed that the BPS process for producing B (Process Y) imposes less harmful impact on the environment than Process B used by the existing competitors in market B. For example, by using the BPS process developed by Chaparral Steel/TXI that uses steel slag to produce Portland cement, two energy-intensive steps were eliminated from the cement man-

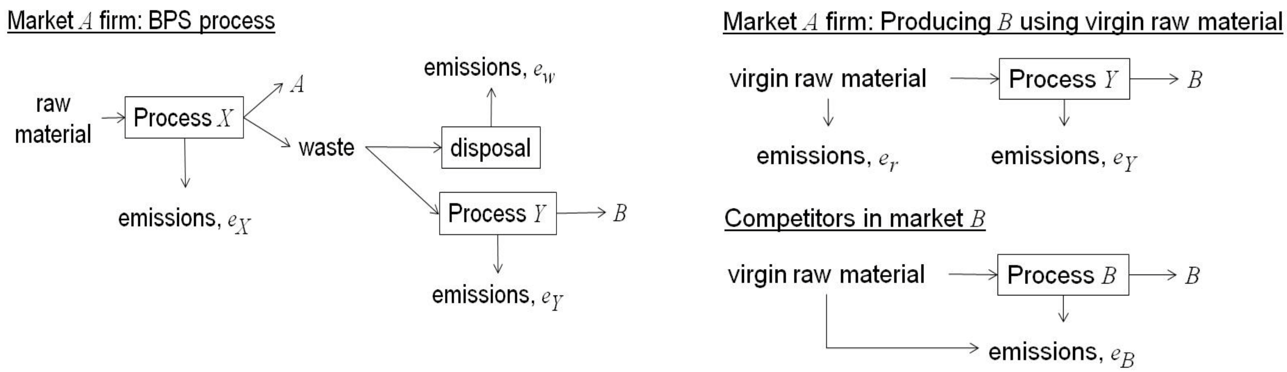


Figure 5: Emissions from the production of A and B when the market A firm implements BPS.

ufacturing process as the slag had already been partially processed in a useful manner for cement production. The steel mill also avoided the cost of incinerating the slag. These avoided steps resulted in 10% reduction in CO_2 emissions from the steel and cement plants and 10-15% energy savings (Forward and Mangan 1999). However, as shown in the previous analysis, by implementing BPS optimally, firm 1 may actually increase total production of A or B. Therefore, it matters what the relative polluting capabilities are for each process.

One general conclusion is that by implementing BPS and keeping it proprietary, the firm takes market share from its competitors who do not. Therefore, the competitors' production, and hence emissions, decrease. As a result, if the firm reduces its own emissions after implementing BPS, overall emissions will decrease. However, if the BPS process is less polluting, the firm could still face a conundrum of maximizing profit by keeping its process innovation proprietary, or sacrificing some profit and licensing the innovation so that other firms can reduce their environmental impact. As with many environmental issues, the implications of a seemingly straightforward and apparently beneficial concept such as utilizing a waste stream may in reality end up forcing a tradeoff between business and environmental objectives.

6 Conclusion

In a world where natural resources are becoming the binding constraint to growth, it can be advantageous for firms to consider efficient use of raw materials as well as efficient use of labor and capital as part of their operating strategy. Maximizing the use of raw material may not align with what firms have traditionally viewed as a corporate strategy that identifies the firm by the product it makes or the market it serves (Tripsas 2009). This traditional perspective has led to a natural distinction between product and waste, two output streams that have typically been managed differently. Taking the perspective of maximizing natural resource utilization challenges firms to think creatively about *all* outputs from their manufacturing process. To this end, the practice of BPS is one potential solution as it operationalizes the proverb, *"One man's trash is another man's treasure".*

In this paper, we derived the optimal operating and licensing strategies that allow a firm to capture the most value from implementing a BPS process. By converting a waste stream into a useful by-product, BPS creates value by acting as a cost reducing process innovation for either the original product or the by-product. Reducing the marginal cost of the original product or by-product allows the firm to create value, which it can then monetize by capturing newly created market opportunities, taking market share from its competitors, or licensing the BPS process to its competitors.

We found that the optimal operating and licensing strategies are driven by the disposal cost and virgin raw material cost, parameters that determine the size of the cost reduction enabled by the BPS process innovation. We showed that the joint production process must be managed strategically rather than simply used as an alternate form of waste disposal because the optimal operating strategy might entail generating more waste by overproducing the original product or operating in a more "wasteful" manner.

The licensing strategy for a BPS operation is complicated by the fact that licensing in the original market can change the market dynamics in both the original and by-product markets.

Therefore, the firm must tradeoff the gain from license fees with the competitive advantage afforded by keeping the process innovation proprietary. Again, we find that disposal cost is a key driver of the firm's strategy. When there is imperfect competition in the by-product market, low or high disposal cost favors not licensing. For low disposal cost, the value created for the competitor and hence the value that can be extracted by the firm through licensing is low. At the other end, the competitive advantage afforded by high disposal cost makes it optimal to keep BPS proprietary. At intermediate levels of disposal cost, enough value can be extracted by licensing and the opportunity cost of allowing competitors to implement BPS is low enough to make licensing optimal.

The general perception is that BPS is beneficial for the environment. However, since the total quantities produced in the two markets may change and how products are produced may change, the overall environmental impact will depend on the relative polluting ability of each process. Moreover, even if the BPS process is less polluting, the firm may still face a dilemma if the profit-maximizing strategy is to not license, but licensing to allow competitors to use its process innovation is better for the environment. This underscores the complexity of decision making in business situations that involve environmental issues.

Limitations and Extensions In our model, we assumed that a BPS opportunity has already been identified. However, the identification process is currently adhoc, making it difficult for firms to evaluate whether it is worthwhile to even investigate the feasibility of BPS, given the uncertainty of the outcome. Understanding the mechanisms for facilitating BPS opportunities and conditions under which they are likely to thrive is a fruitful avenue for future research

We also assumed a particular form of BPS implementation, i.e., the manufacturer generating the waste also converts it into by-product. Another feasible structure is one where the manufacturer pays a third party to do the conversion. This payment would obviously have to be less than the disposal cost, however, there may be economies of scale that allow the third party to more efficiently produce the by-product. There are a set of interesting issues surrounding the question of vertical integration, e.g., cost allocation and pricing, that would be an interesting avenue for future research.

In the realm of waste disposal always lurks the shadow of regulation. The Environmental Protection Agency regulates how hazardous wastes must be disposed of. It is unclear if a given BPS implementation involving hazardous waste would adhere to the existing regulations. If regulators want to promote productive use of waste streams, there will have to be a careful balance between encouraging BPS innovation that is good for business and the environment, and curtailing illegal waste disposal disguised as by-product sales. The interaction between market mechanisms and regulation in the BPS context is an important and interesting area for future research.

The Environmental Protection Agency conservatively estimates that the industrial solid waste generated annually in the U.S. is 7.6 billion tons (U.S. Environmental Protection Agency 2011). The industrial world has been given leeway to prolifically produce waste by not considering the impact of waste streams on human health, ecological diversity, and the welfare of future generations. We feel that the practice of productively using waste is an important concept that warrants further research as it can have both strategic benefits for the firm and beneficial impact on the environment.

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References

Atasu, A., V. D. R. Guide, and L. N. Van Wassenhove (2008, September-October). Product reuse economics in closed-loop supply chain research. *Production and Operations Management*.

Atasu, A., M. Sarvary, and L. N. Van Wassenhove (2008, October). Remanufacturing as a marketing strategy. *Mangement Science*.

Baumgartner, S. and F. Jost (2000, June). Joint production, externalities, and the regulation of production networks. *Environmental and Resource Economics 16*(2), 229–251.

Bitran, G. R. and S. M. Gilbert (1994, May/Jun). Co-production processes with random yields in the semiconductor industry. *Operations Research*.

Bitran, G. R. and T.-Y. Leong (1992, May). Deterministic approximatins to co-production problems with service constraints and random yields. *Management Science*.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy 93*(3), 488–511.

Chen, Z. and T. W. Ross (2007). Markets linked by rising marginal costs: Implications for multimarket contact, recoupment, and retailatory entry. *Review of Industrial Organization 31*(1), 1–21.

Chertow, M. R. (2000). Industrial symbiosis: Literature and taxononmy. *Annual Review of Energy and Environment 25*, 313–337.

Chertow, M. R. and D. R. Lombardi (2005). Quantifying economic and environmental benefits of co-located firms. *Environmental Science and Technology 39*(17), 65356541.

Debo, L. G., L. B. Toktay, and L. N. Van Wassenhove (2005, August). Market segmentation and product technology selection for remanufacturable products. *Management Science 51*(8), 1193–1205.

Ehrenfeld, J. and N. Gertler (1997). Industrial ecology in practice: The evolution of interdependence at kalundborg. *Journal of Industrial Ecology 1*(1), 67–79.

Ferguson, M. E. and L. B. Toktay (2006, Fall). The effect of competition on recovery strategies. *Production and Operations Management 15*(3), 351–368.

Ferrer, G. and J. M. Swaminathan (2006, January). Managing new and remanufactured products. *Management Science 52*(1), 15–26.

Fleischmann, M., J. M. Bloemhof-Ruwaard, R. Dekker, E. van der Laan, J. A. van Nunen, and L. N. Van Wassenhove (1997). Quantitative models for reverse logistics: A review. *European Journal of Operational Research 103*, 1–17.

Forward, G. and A. Mangan (1999, Spring). By-product synergy. *The Bridge 29*(1).

Guide, V. D. R., G. C. Souza, L. N. Van Wassenhove, and J. D. Blackburn (2006, August). Time value of commercial product returns. *Management Science 52*(8), 1200–1214.

Guide, V. D. R., R. H. Teunter, and L. N. Van Wassenhove (2003, Fall). Matching demand and supply to maximize profits from remanufacturing. *Manufacturing & Service Operations Management 5*(4), 303–316.

Hsu, A. and Y. Bassok (1999, Mar/Apr). Random yield and random demand in a production system with downward substitution. *Operations Research 47*(2), 277–290.

Jayaraman, V., R. A. Patterson, and R. Erik (2003). The design of reverse distribution networks: models and solution procedures. *European Journal of Operational Research 150*(2), 128–149.

Lee, D., M. W. Toffel, and R. Gordon (2009, January 14). Cook composites and polymers co. Case 9-608-055, Harvard Business School.

Majumder, P. and H. Groenevelt (2001, Summer). Competition in remanufacturing. *Production and Operations Management 10*(2), 125–141.

National Industrial Symbiosis Programme (2011). *National Industrial Symbiosis Programme*. National Industrial Symbiosis Programme. http://www.nisp.org.uk.

National Slag Association (2011). *CemStar Process: Slag Usage Raises Productivity, Operational*

Efficiency, Lowers Emissions. National Slag Association. www.nationalslag.org/archive/nsa_2011_cemstar_technical_bulletin.pdf.

Savaskan, R. C., S. Bhattacharya, and L. N. Van Wassenhove (2004, February). Closed-loop supply chain models with product remanufacturing. *Management Science 50*(2), 239–252.

Toffel, M. W. (2003, Spring). The growing strategic importance of end-of-life product management. *California Management Review 45*(3), 102–129.

Tomlin, B. and Y. Wang (2008). Pricing and operational recourse in coproduction systems. *Management Science 54*(3), 522–537.

Tripsas, M. (2009). Technology, identity, and inertia: Through the lens of 'the digital photography company'. *Organization Science 20*(2), 441–460.

U.S. Business Council for Sustainable Development (2011). *By-Product Synergy.* U.S. Business Council for Sustainable Development. http://usbcsd.org/projects.php.

U.S. Department of Energy, Energy Efficiency and Renewal Energy (2005, September). *Chemicals: Best Practices Plant-Wide Assessment Case Study.* U.S. Department of Energy, Energy Efficiency and Renewal Energy. http://www.usbcsd.org/pdf%5CDow_Case_Study.pdf.

U.S. Environmental Protection Agency (2011). *Guide for Industrial Waste Management.* U.S. Environmental Protection Agency. http://www.epa.gov/waste/nonhaz/industrial/guide/index.htm.

van Beers, D., G. Corder, A. Bossilkov, and R. van Berkel (2007). Industrial symbiosis in the australian mineral industry. *Journal of Industrial Ecology 11*(1), 55–72.

Wang, X. H. (1998, July). Fee versus royalty licensing in a cournot duopoly model. *Economics Letters 60*(1), 55–62.

Zhu, Q., E. A. Lowe, Y.-A. Wei, and D. Barnes (2007). A case study of the guitang group. *Journal of Industrial Ecology 11*(1), 31–42.

A Online supplement for "Turning Waste into By-Product": Proofs of main results

The following lemma is used to prove Proposition 1.

Lemma 1 *If $q_B^{(1)} \leq \gamma q_A^{(1)}$, then $q_B^{(3)} < \gamma q_A^{(3)}$. If $q_B^{(3)} \geq \gamma q_A^{(3)}$, then $q_B^{(1)} > \gamma q_A^{(1)}$. The optimal quantities that maximize the firm's profit are:*

$$(q_A^*, q_B^*) = \begin{cases} (q_A^{(1)}, q_B^{(1)}) & \text{if and only if } q_B^{(1)} \leq \gamma q_A^{(1)} \\ (q_A^{(2)}, q_B^{(2)}) & \text{if and only if } q_B^{(1)} \geq \gamma q_A^{(1)} \text{ and } q_B^{(3)} \leq \gamma q_A^{(3)} \\ (q_A^{(3)}, q_B^{(3)}) & \text{if and only if } q_B^{(3)} \geq \gamma q_A^{(3)} \end{cases}. \tag{1}$$

Proof of Lemma 1. We first show that $\Pi^{(1)}$, $\Pi^{(2)}$, and $\Pi^{(3)}$ are strictly concave in (q_A, q_B), therefore, each profit function has a single maximum. The following shows that $\Pi^{(1)}$ and $\Pi^{(3)}$ are strictly concave in (q_A, q_B):

$$\begin{aligned} \frac{\partial^2 \Pi^{(1)}}{\partial q_j^2} &= \frac{\partial^2 \Pi^{(3)}}{\partial q_j^2} = 2\frac{\partial p_j}{\partial q_j} + q_j \frac{\partial^2 p_j}{\partial q_j^2} < 0, j = A, B, \text{ by Assumption 2, and} \\ \frac{\partial^2 \Pi^{(1)}}{\partial q_A \partial q_B} &= \frac{\partial^2 \Pi^{(3)}}{\partial q_A \partial q_B} = 0, \text{ because } \Pi^{(1)} \text{ and } \Pi^{(3)} \text{ are additively separable in } q_A \text{ and } q_B. \end{aligned}$$

The Hessians for $\Pi^{(1)}$ and $\Pi^{(3)}$ have negative diagonals and zero off diagonals, which are clearly negative definite. Therefore, $\Pi^{(1)}$ and $\Pi^{(3)}$ are strictly concave in (q_A, q_B).

Since $q_B = \gamma q_A$, $\Pi^{(2)}$ is a function only of q_A. We show below that $\Pi^{(2)}$ is concave in q_A:

$$\begin{aligned} \Pi^{(2)} &= (p_A(q_A) + \gamma p_B(q_B(q_A)))q_A - (c_X + \gamma c_Y)q_A \\ \frac{\partial \Pi^{(2)}}{\partial q_A} &= \frac{\partial p_A}{\partial q_A} q_A + p_A + \gamma \frac{\partial p_B(q_B(q_A))}{\partial q_B} \frac{\partial q_B}{\partial q_A} q_A + \gamma p_B(q_B(q_A)) - (c_X + \gamma c_Y) \\ &= \frac{\partial p_A}{\partial q_A} q_A + p_A + \gamma^2 \frac{\partial p_A(q_B(q_A))}{\partial q_B} q_A + \gamma p_B(q_B(q_A)) - (c_X + \gamma c_Y) \\ \frac{\partial^2 \Pi^{(2)}}{\partial q_A^2} &= \frac{\partial^2 p_A}{\partial q_A^2} q_A + \frac{\partial p_A}{\partial q_A} + \frac{\partial p_A}{\partial q_A} + \gamma^2 \left(\frac{\partial \frac{\partial p_B(q_B(q_A))}{\partial q_B}}{\partial q_A} q_A + \frac{\partial p_B(q_B(q_A))}{\partial q_B} \right) + \gamma \frac{\partial p_B(q_B(q_A))}{\partial q_B} \frac{\partial q_B}{\partial q_A} \\ &= \frac{\partial^2 p_A}{\partial q_A^2} q_A + 2\frac{\partial p_A}{\partial q_A} + \gamma^2 \left(\frac{\partial \frac{\partial p_B(q_B(q_A))}{\partial q_B}}{\partial q_B} \frac{\partial q_B}{\partial q_A} q_A + \frac{\partial p_B(q_B(q_A))}{\partial q_B} \right) + \gamma^2 \frac{\partial p_B(q_B(q_A))}{\partial q_B} \\ &= \frac{\partial^2 p_A}{\partial q_A^2} q_A + 2\frac{\partial p_A}{\partial q_A} + \gamma^2 \left(\frac{\partial^2 p_B}{\partial q_B^2} q_B + 2\frac{\partial p_B}{\partial q_B} \right) < 0 \text{ by Assumption 2,} \end{aligned}$$

thus, $\Pi^{(2)}$ is concave in q_A.

Note that by subtracting $\Pi^{(1)}$ from $\Pi^{(3)}$, we know that

$$\Pi^{(1)} > \Pi^{(3)} \quad \text{if and only if} \quad q_B > \gamma q_A \tag{2}$$
$$\Pi^{(1)} = \Pi^{(3)} = \Pi^{(2)} \quad \text{if and only if} \quad q_B = \gamma q_A \tag{3}$$
$$\Pi^{(1)} < \Pi^{(3)} \quad \text{if and only if} \quad q_B < \gamma q_A. \tag{4}$$

We now show that if $q_B^{(1)} \leq \gamma q_A^{(1)}$, then $q_B^{(3)} < \gamma q_A^{(3)}$. Using (3), (4), and optimality, we have the following series of inequalities:

$$\Pi^{(3)}(q_A^{(3)}, q_B^{(3)}) > \Pi^{(3)}(q_A^{(1)}, q_B^{(1)}) \geq \Pi^{(1)}(q_A^{(1)}, q_B^{(1)}) > \Pi^{(1)}(q_A^{(3)}, q_B^{(3)}).$$

The first inequality is due to the optimality of ($q_A^{(3)}$,$q_B^{(3)}$). The second inequality is due to (3), (4) and the assumption that $q_B^{(1)} \leq \gamma q_A^{(1)}$. The third inequality is due to the optimality of ($q_A^{(1)}$,$q_B^{(1)}$). The inequality $\Pi^{(3)}(q_A^{(3)}, q_B^{(3)}) > \Pi^{(1)}(q_A^{(3)}, q_B^{(3)})$ (first and last terms) implies $q_B^{(3)} < \gamma q_A^{(3)}$ (from (4)).

Similarly, we now show that if $q_B^{(3)} \geq \gamma q_A^{(3)}$, then $q_B^{(1)} > \gamma q_A^{(1)}$. Using (2), (3), and optimality, we have the following series of inequalities:

$$\Pi^{(1)}(q_A^{(1)}, q_B^{(1)}) > \Pi^{(1)}(q_A^{(3)}, q_B^{(3)}) \geq \Pi^{(3)}(q_A^{(3)}, q_B^{(3)}) > \Pi^{(3)}(q_A^{(1)}, q_B^{(1)}).$$

The first inequality is due to the optimality of ($q_A^{(1)}$,$q_B^{(1)}$). The second inequality is due to (2), (3) and the assumption that $q_B^{(3)} \geq \gamma q_A^{(3)}$. The third inequality is due to the optimality of ($q_A^{(3)}$,$q_B^{(3)}$). The inequality $\Pi^{(1)}(q_A^{(1)}, q_B^{(1)}) > \Pi^{(3)}(q_A^{(1)}, q_B^{(1)})$ (first and last terms) implies $q_B^{(1)} > \gamma q_A^{(1)}$ (from (2)).

Consider the case where $q_B^{(1)} < \gamma q_A^{(1)}$ and $q_B^{(3)} \leq \gamma q_A^{(3)}$. Clearly, profit is maximized at $(q_A^{(1)}, q_B^{(1)})$ or some point $(q'_A, q'_B | q'_B \geq \gamma q'_A)$. Consider the point (q'_A, q'_B). For $q_B \geq \gamma q_A$, $\Pi = \Pi^{(3)}$ (and $\Pi^{(3)} = \Pi^{(2)}$ if $q_B = \gamma q_A$). But we know that $\Pi^{(1)}(q'_A, q'_B) \geq \Pi^{(3)}(q'_A, q'_B)$ because $\Pi^{(1)} \geq \Pi^{(3)}$ for all $q_B \geq \gamma q_A$. By the definition of optimality, $\Pi^{(1)}(q_A^{(1)}, q_B^{(1)}) > \Pi^{(1)}(q'_A, q'_B)$, so the optimal point is not (q'_A, q'_B). Since $\Pi = \Pi^{(1)}$ for $q_B < \gamma q_A$ and $q_B^{(1)} < \gamma q_A^{(1)}$, the firm's profit is maximized at $(q_A^*, q_B^*) = (q_A^{(1)}, q_B^{(1)})$.

Similarly, consider the case where $q_B^{(1)} \geq \gamma q_A^{(1)}$ and $q_B^{(3)} > \gamma q_A^{(3)}$. Clearly, profit is maximized at $(q_A^{(3)}, q_B^{(3)})$ or some point $(q'_A, q'_B | q'_B \leq \gamma q'_A)$. Consider the point (q'_A, q'_B). For $q_B \leq \gamma q_A$, $\Pi = \Pi^{(1)}$ (and $\Pi^{(1)} = \Pi^{(2)}$ if $q_B = \gamma q_A$). But we know that $\Pi^{(3)}(q'_A, q'_B) \geq \Pi^{(1)}(q'_A, q'_B)$ because $\Pi^{(3)} \geq \Pi^{(1)}$ for all $q_B \leq \gamma q_A$. By the definition of optimality, $\Pi^{(3)}(q_A^{(3)}, q_B^{(3)}) > \Pi^{(3)}(q'_A, q'_B)$, so the optimal point is not (q'_A, q'_B). Since $\Pi = \Pi^{(3)}$ for $q_B > \gamma q_A$ and $q_B^{(3)} > \gamma q_A^{(3)}$, the firm's profit is maximized at $(q_A^*, q_B^*) = (q_A^{(3)}, q_B^{(3)})$.

Consider the case where $q_B^{(1)} \geq \gamma q_A^{(1)}$ and $q_B^{(3)} \leq \gamma q_A^{(3)}$. The condition $q_B^{(1)} \geq \gamma q_A^{(1)}$ implies that the solution to

$$\begin{aligned} &\max_{q_A, q_B} p_A q_A + p_B q_B - (c_X + c_w) q_A - (c_Y - \frac{1}{\gamma} c_w) q_B \\ &\text{s.t. } q_B \leq \gamma q_A \end{aligned} \tag{5}$$

lies at the boundary $q_B = \gamma q_A$. Similarly, the condition $q_B^{(3)} \leq \gamma q_A^{(3)}$ implies that the solution to

$$\begin{aligned} &\max_{q_A, q_B} p_A q_A + p_B q_B - (c_X - \gamma c_r) q_A - (c_Y + c_r) q_B \\ &\text{s.t. } q_B \geq \gamma q_A \end{aligned} \tag{6}$$

lies at the boundary $q_B = \gamma q_A$. Therefore, $\Pi = \Pi^{(2)}$, which is maximized at $(q_A^*, q_B^*) = (q_A^{(2)}, q_B^{(2)})$. This concludes the proof. ■

Proof of Proposition 1. If the firm does not implement BPS, its profit function is $p_A q_A - (c_X + c_w) q_A$ leading to first-order condition

$$\frac{\partial p_A}{\partial q_A} q_A^m + p_A - (c_X + c_w) = 0, \tag{7}$$

where q_A^m is the optimal monopoly quantity without BPS. If $(q_A^*, q_B^*) = (q_A^{(1)}, q_B^{(1)})$, Lemma 1 shows that $q_B^{(1)} < \gamma q_A^{(1)}$, and $q_A^{(1)}$ is the solution to the first-order condition

$$\frac{\partial \Pi^{(1)}}{\partial q_A} = \frac{\partial p_A}{\partial q_A} q_A^{(1)} + p_A - (c_X + c_w) = 0, \tag{8}$$

which is identical to first-order condition (7). Therefore, $q_A^{(1)} = q_A^m$.

Consider the case where $(q_A^*, q_B^*) = (q_A^{(3)}, q_B^{(3)})$. Lemma 1 shows that $q_B^{(3)} > \gamma q_A^{(3)}$. The optimal quantity $q_A^{(3)}$ is derived from the following first-order condition:

$$\frac{\partial \Pi^{(3)}}{\partial q_A} = \frac{\partial p_A}{\partial q_A} q_A^{(3)} + p_A - (c_X - \gamma c_r) = 0. \tag{9}$$

Since the marginal cost of (9) is strictly less than the marginal cost of (8), i.e., $c_X - \gamma c_r < c_X + c_w$, and by Assumption 2, the marginal revenue curve crosses the marginal cost curve from above, we can conclude that $q_A^{(3)} > q_A^{(1)}$.

Consider the case where $(q_A^*, q_B^*) = (q_A^{(2)}, q_B^{(2)})$. Lemma 1 shows that $q_B^{(2)} = \gamma q_A^{(2)}$. From Lemma 1, we also know that if $(q_A^*, q_B^*) = (q_A^{(2)}, q_B^{(2)})$, then $q_B^{(1)} \geq \gamma q_A^{(1)}$ and $q_B^{(3)} \leq \gamma q_A^{(3)}$. If $q_B^{(1)} = \gamma q_A^{(1)}$, then $q_A^{(2)} = q_A^{(1)} = q_A^m$. However, if $q_B^{(1)} > \gamma q_A^{(1)}$, then the constraints in (5) and (6) are binding. We know from above that $q_A^{(3)} > q_A^{(1)}$. These two facts together imply that

$\frac{\partial \Pi(q_A^{(1)}, q_B^{(1)})}{\partial q_A} = \frac{\partial \Pi^{(3)}(q_A^{(1)}, q_B^{(1)})}{\partial q_A} > 0$. Since $\Pi^{(2)}(q_A, \gamma q_A) = \Pi^{(3)}(q_A, \gamma q_A)$, then $q_A^{(2)} > q_A^{(1)} = q_A^m$.

Let $\Pi^m = p_A q_A - (c_X + c_w) q_A$ be the firm's profit without BPS. By Assumption 3, the firm makes positive profit in market A without BPS, i.e., $q_A^{(1)} > 0$. Then if $q_B^{(1)} > 0$, BPS increases the firm's profit because the firm can always continue producing A business as usual and convert the collaterally generated waste stream into B, making positive profit in market B

If $q_B^{(1)} \leq 0$, that implies $q_B^{(1)} < \gamma q_A^{(1)}$, which in turn implies that partial conversion is optimal (from Proposition 1). Given that $\Pi^{(1)} = \Pi^m +$ (market B profit), if $q_B^{(1)} \leq 0$, market B profit would be zero. Therefore, BPS would not increase the firm's profit. ■

Proof of Proposition 2. Consider the effect of c_w on $q_A^{(1)}$ and $q_B^{(1)}$. The first-order condition that defines $q_A^{(1)}$ is given by (8), which shows that marginal cost of A increases in c_w. This together with Assumption 2 implies that $\frac{\partial q_A^{(1)}}{\partial c_w} < 0$. The first-order condition that defines $q_B^{(1)}$ is given by

$$\frac{\partial \Pi^{(1)}}{\partial q_B} = \frac{\partial p_B}{\partial q_B} q_B^{(1)} + p_B - (c_Y - \frac{1}{\gamma} c_w) = 0, \tag{10}$$

which shows that marginal cost of B decreases in c_w. This together with Assumption 2 implies that $\frac{\partial q_B^{(1)}}{\partial c_w} > 0$. By continuity, there exists $\hat{c}_w$ such that for $c_w \geq \hat{c}_w$, $q_B^{(1)} \geq \gamma q_A^{(1)}$. Note that (8) and (10) are independent of c_r.

Consider the effect of c_r on $q_A^{(3)}$ and $q_B^{(3)}$. The first-order condition that defines $q_A^{(3)}$ is given by (9), which shows that marginal cost of A decreases in c_r. This together with Assumption 2 implies that $\frac{\partial q_A^{(3)}}{\partial c_r} > 0$. The first-order condition that defines $q_B^{(1)}$ is given by

$$\frac{\partial \Pi^{(3)}}{\partial q_B} = \frac{\partial p_B}{\partial q_B} q_B^{(3)} + p_B - (c_Y + c_r) = 0, \tag{11}$$

which shows that marginal cost of B increases in c_r. This together with Assumption 2 implies that $\frac{\partial q_B^{(3)}}{\partial c_r} < 0$. By continuity, there exists $\hat{c}_r$ such that for $c_r \geq \hat{c}_r$, $q_B^{(3)} \leq \gamma q_A^{(3)}$. Note that (9) and (11) are independent of c_w.

Note that (8) and (10) cannot be satisfied with $c_w > 0$ if (9) and (11) are satisfied with $c_r > 0$ (and vice versa). Therefore, either (8) and (10) are satisfied and the optimal operating policy only depends on c_w as shown above, or (9) and (11) are satisfied and the optimal operating policy only depends on c_r as shown above, or $c_w = c_r = 0$ which leads to the degenerate solution $\Pi^{(1)} = \Pi^{(2)} = \Pi^{(3)}$.

Substituting $p_A = a - q_A$ and $p_B = b - q_B$ into (1), it is straightforward to show that the unconstrained optimizers for $\Pi^{(1)}$, $\Pi^{(2)}$, and $\Pi^{(3)}$ are:

$$q_A^{(1)} = \frac{a - c_X - c_w}{2} \quad \text{and} \quad q_B^{(1)} = \frac{b - c_Y + \frac{1}{\gamma}c_w}{2}, \tag{12}$$

$$q_A^{(2)} = \frac{a - c_X + \gamma(b - c_Y)}{2(1+\gamma^2)} \quad \text{and} \quad q_B^{(2)} = \gamma q_A^{(2)}, \tag{13}$$

$$q_A^{(3)} = \frac{a - c_X + \gamma c_r}{2} \quad \text{and} \quad q_B^{(3)} = \frac{b - c_Y - c_r}{2}. \tag{14}$$

Therefore, $q_B^{(1)} \geq \gamma q_A^{(1)}$ implies $c_w \geq \frac{\gamma(a-c_X)-(b-c_Y)}{\gamma+\frac{1}{\gamma}} = \hat{c}_w$, and $q_B^{(3)} \leq \gamma q_A^{(3)}$ implies $c_r \geq \frac{-\gamma(a-c_X)+(b-c_Y)}{1+\gamma^2} = \hat{c}_r$. This completes the proof. ■

Proof of Proposition 3. Suppose $c_w > a - c_X > 0$. Then $q_A^{(1)} < 0$ and the firm cannot make positive profit in market A without BPS. However, if $b - c_Y > 0$, then $q_A^{(2)} > 0$, which implies that the firm can implement BPS using exactly-full conversion and make positive profit producing A and B.

■

Proof of Proposition 4. From Lemma 1, we know that partial conversion is optimal if and only if $q_B^{(1)} \leq \gamma q_A^{(1)}$. Substituting from (12) gives the condition

$$\gamma^2(a - c_X - c_w) - \gamma(b - c_Y) - c_w \geq 0,$$

which has a negative real root and a positive real root because $a - c_X - c_w > 0$ and $c_w > 0$ by assumption (it does not matter what the sign of $b - c_Y$ is). When $\gamma = 0$, the expression is negative. Let γ_2 be the positive real root. Therefore, $\gamma^2(a-c_X-c_w)-\gamma(b-c_Y)-c_w$ is an increasing function that is negative for $0 < \gamma < \gamma_2$ and positive for $\gamma \geq \gamma_2$. Hence $q_B^{(1)} \leq \gamma q_A^{(1)}$ for $\gamma \geq \gamma_2$, implying partial conversion is optimal.

From Lemma 1, we also know that full+ conversion is optimal if and only if $q_B^{(3)} \geq \gamma q_A^{(3)}$. Substituting from (14) gives the condition

$$\gamma^2 c_r + \gamma(a - c_X) - (b - c_Y - c_r) \leq 0,$$

which has a negative real root and a positive real root because $c_r > 0$ and $b - c_Y - c_r > 0$ by assumption (it does not matter what the sign of $a - c_X$ is). When $\gamma = 0$, the expression is negative. Let γ_1 be the positive real root. Therefore, $\gamma^2 c_r + \gamma(a-c_X)-(b-c_Y-c_r)$ is an increasing function that is negative for $\gamma \in [0, \gamma_1]$ and positive for $\gamma > \gamma_1$. Hence $q_B^{(3)} \geq \gamma q_A^{(3)}$ for $\gamma \in [0, \gamma_1]$.

We also know from Lemma 1 that it is never the case that $q_B^{(1)} < \gamma q_A^{(1)}$ and $q_B^{(3)} > \gamma q_A^{(3)}$, therefore, it must be the case that $\gamma_1 < \gamma_2$ (we ignore the equality case as that ends up with the degenerate solution). Thus, we have shown that full+ conversion is optimal for $\gamma \in [0, \gamma_1)$, partial conversion is optimal for $\gamma > \gamma_2$, and $\gamma_1 < \gamma_2$. From Lemma 1, it follows that exactly-full conversion is optimal for $\gamma \in [\gamma_1, \gamma_2]$.

We now show that the firm's profit increases in γ when full+ conversion is optimal, decreases when partial conversion is optimal, and is contingent on γ when exactly-full conversion is optimal.

If full+ conversion is optimal, the profit function $\Pi^{(3)}$ is additively separable in q_A and q_B, and only the profit from q_A is affected by γ. The first-order condition with respect to q_A is given by

$$\frac{\partial \Pi^{(3)}}{\partial q_A} = \frac{\partial p_A}{\partial q_A} q_A^{(3)} + p_A - (c_X - \gamma c_r) = 0$$

Increasing γ uniformly decreases the marginal cost of a unit of A. Therefore, keeping q_A constant and increasing γ will increase $\Pi^{(3)}$. Re-optimizing q_A will increase $\Pi^{(3)}$ even further.

If partial conversion is optimal, the profit function $\Pi^{(1)}$ is additively separable in q_A and q_B, and only the profit from q_B is affected by γ. The first-order condition with respect to q_B is given by (10). Increasing γ uniformly increases the marginal cost of a unit of B. Therefore, keeping q_B constant and increasing γ will decrease $\Pi^{(1)}$. Re-optimizing q_B will decrease $\Pi^{(1)}$ even further.

Consider the case when exactly-full conversion is optimal and assume that the demand curves are linear, i.e., $p_A = a - q_A$ and $p_B = b - q_B$, where $a, b > 0$. The profit function $\Pi^{(2)}$ is:

$$\Pi^{(2)} = -(1+\gamma^2) q_A^2 + (a + b\gamma - c_X - \gamma c_Y) q_A.$$

Differentiating with respect to q_A and setting the first-order condition to zero gives

$$q_A^{(2)} = \frac{a + b\gamma - c_X - \gamma c_Y}{2(1+\gamma^2)}.$$

To show that $\Pi^{(2)}(q_A^{(2)}, \gamma)$ increases in γ, we use the Envelope theorem:

$$\frac{\Pi^{(2)}(q_A^{(2)}, \gamma)}{\partial \gamma} = \frac{\Pi^{(2)}(q_A, \gamma)}{\partial \gamma}\bigg|_{q_A = q_A^{(2)}} = -2\gamma (q_A^{(2)})^2 + (b - c_Y) q_A^{(2)} > 0 \text{ if and only if}$$

$$\frac{b - c_Y}{2} > \gamma q_A^{(2)} = \frac{\gamma(a - c_X + \gamma(b - c_Y))}{2(1+\gamma^2)} \Leftrightarrow \gamma < \frac{b - c_Y}{a - c_X} \equiv \hat{\gamma}.$$

The conditions from (1) imply that if $(q_A^*, q_B^*) = (q_A^{(2)}, q_B^{(2)})$, then

$$\gamma \leq \frac{q_B^{(1)}}{q_A^{(1)}} = \frac{b - c_Y + \frac{1}{\gamma} c_w}{a - c_X - c_w} \text{ and } \gamma \geq \frac{q_B^{(3)}}{q_A^{(3)}} = \frac{b - c_Y - c_r}{a - c_X + \gamma c_r}.$$

Clearly, $\hat{\gamma}$ lies between these two values. This concludes the proof. ■

Proof of Proposition 5. Market B is perfectly competitive and $p_B = c_b$. Firm 1 maximizes the following profit function:

$$\begin{aligned} \max_{q_{A1},q_{B1}} \Pi_1 &= p_A q_{A1} - (c_X + c_w) q_{A1} + c_b q_{B1} - (c_Y - c_w) q_{B1} \\ \text{s.t.} \quad & q_{B1} \le q_{A1} \end{aligned}$$

Note that for $c_b > c_Y - c_w$, firm 1 makes positive profit on every unit of B produced. Therefore, If $c_b > c_Y - c_w$, $q_{B1} = q_{A1}$ is optimal. For $c_b \le c_Y - c_w$, firm 1 makes zero or negative profit on every unit of B produced, therefore, $q_{B1} = 0$ is optimal. Therefore, for $c_b > c_Y - c_w$, firm 1's profit maximization reduces to

$$\max_{q_{A1}} \Pi_1 = p_A q_{A1} - (c_X + c_Y - c_b) q_{A1} \tag{15}$$

The profit equation in (15) shows that BPS is acting as a cost reducing innovation for producing A, with the reduced marginal cost equal to $c_X + c_Y - c_b$.

If firm 1 does not license to firm 2, then firm 2's profit maximization problem is

$$\max_{q_{A2}} \Pi_2 = p_A q_{A2} - (c_X + c_w) q_{A2}. \tag{16}$$

If firm 1 licenses its BPS process to firm 2, then firm 2 also reduces its marginal cost from $c_X + c_w$ to $c_X + c_Y - c_b$. Thus, we can apply the results from **?**.

Let $\varepsilon = c_X + c_w - (c_X + c_Y - c_b) = c_w + c_b - c_Y$ be the cost reduction enabled by the BPS innovation. **?** shows that in a Cournot duopoly, it is optimal for firm 1 to license to firm 2 using a per unit royalty fee if and only if $\varepsilon < a - c_X - c_w$. Moreover, licensing using a royalty fee is more profitable than using a fixed fee. Therefore, it is optimal for firm 1 to license to firm 2 using a per unit royalty fee if and only if $c_b > c_Y - c_w$ and $c_w + c_b - c_Y < a - c_X - c_w$. ■

Proof of Proposition 6. We first derive the equilibrium quantities and profits under no licensing (NL). Recall that we set $\gamma = 1$ and assume that c_r is arbitrarily high so that sourcing virgin waste substitute material is infeasible. The firms maximize their respective profit functions:

$$\begin{aligned} \max_{q_{A1},q_{B1}} \Pi_1 &= (a - c_X - c_w - q_{A1} - q_{A2}) q_{A1} + (b - c_Y + c_w - q_{B1}) q_{B1} \qquad (17) \\ & \text{s.t. } q_{B1} \le q_{A1} \\ \max_{q_{A2}} \Pi_2 &= (a - c_X - c_w - q_{A1} - q_{A2}) q_{A2} \qquad (18) \end{aligned}$$

If the BPS constraint is non-binding, market A is a duopoly and market B is a monopoly. The equilibrium quantities are

$$q_{A1}^{(1)} = q_{A2}^{(1)} = \frac{a - c_X - c_w}{3} \text{ and } q_{B1}^{(1)} = \frac{b - c_Y + c_w}{2}. \tag{19}$$

These quantities are optimal and partial conversion is optimal for firm 1 if and only if

$$q_{B1}^{(1)} < q_{A1}^{(1)} \Leftrightarrow c_w < \frac{2(a-c_X)-3(b-c_Y)}{5}. \tag{20}$$

The resulting profits for the two firms are thus

$$\Pi_1^{(1)NL} = (q_{A1}^{(1)})^2 + (q_{B1}^{(1)})^2 = \left(\frac{a-c_X-c_w}{3}\right)^2 + \left(\frac{b-c_Y+c_w}{2}\right)^2 \tag{21}$$

$$\Pi_2^{(1)NL} = (q_{A2}^{(1)})^2 = \left(\frac{a-c_X-c_w}{3}\right)^2. \tag{22}$$

If the BPS constraint is binding, i.e., $q_{A1} = q_{B1}$, then (17) becomes

$$\max_{q_{A1}} \Pi_1 = (a-c_X+b-c_Y-2q_{A1}-q_{A2})q_{A1}, \tag{23}$$

where $q_{B1} = q_{A1}$. Exactly-full conversion is optimal for firm 1, and solving (23) and (18) gives equilibrium quantities

$$q_{A1}^{(2)} = q_{B1}^{(2)} = \frac{a-c_X+c_w+2(b-c_Y)}{7} \text{ and } q_{A2}^{(2)} = \frac{3(a-c_X)-4c_w-(b-c_Y)}{7}. \tag{24}$$

The resulting profits for the two firms are thus

$$\Pi_1^{(2)NL} = 2(q_{A1}^{(2)})^2 = 2\left(\frac{a-c_X+c_w+2(b-c_Y)}{7}\right)^2 \tag{25}$$

$$\Pi_2^{(2)NL} = (q_{A2}^{(2)})^2 = \left(\frac{3(a-c_X)-4c_w-(b-c_Y)}{7}\right)^2. \tag{26}$$

If firm 1 licenses its BPS process to firm 2 for a fixed fee, the two firms become symmetric competitors in markets A and B. The firms maximize their profit functions:

$$\max_{q_{A1},q_{B1}} \Pi_1 = (a-c_X-c_w-q_{A1}-q_{A2})q_{A1} + (b-c_Y+c_w-q_{B1}-q_{B2})q_{B1} \quad \text{s.t. } q_{B1} \le q_{A1} \tag{27}$$

$$\max_{q_{A2},q_{B2}} \Pi_2 = (a-c_X-c_w-q_{A1}-q_{A2})q_{A2} + (b-c_Y+c_w-q_{B1}-q_{B2})q_{B2} \quad \text{s.t. } q_{B2} \le q_{A2} \tag{28}$$

Since the firms are identical, there will be a symmetric equilibrium where $q_{A1} = q_{A2}$ and $q_{B1} = q_{B2}$. We drop the firm level subscripts to simplify exposition, i.e., $q_A = q_{A1} = q_{A2}$ and $q_B = q_{B1} = q_{B2}$.

If the BPS constraints are not binding, i.e., $c_w < \frac{a-c_X-(b-c_Y)}{2}$, then the optimal quantities are

$$q_A = \frac{a-c_X-c_w}{3} \text{ and } q_B = \frac{b-c_Y+c_w}{3}, \tag{29}$$

resulting in profits

$$\Pi^{(1)L} = \left(\frac{a - c_X - c_w}{3}\right)^2 + \left(\frac{b - c_Y + c_w}{3}\right)^2. \tag{30}$$

If the BPS constraints are binding, i.e., $c_w \geq \frac{a - c_X - (b - c_Y)}{2}$, substituting $q_{A1} = q_{A2} = q_{B1} = q_{B2}$ into (27) and (28) gives us optimal quantities

$$q_A = q_B = \frac{a - c_X + b - c_Y}{6}, \tag{31}$$

resulting in profits

$$\begin{aligned} \Pi^{(2)L} &= \left(a - c_X + b - c_Y - 4\left(\frac{a - c_X + b - c_Y}{6}\right)\right)\left(\frac{a - c_X + b - c_Y}{6}\right) \\ &= \frac{1}{18}(a - c_X + b - c_Y)^2. \end{aligned} \tag{32}$$

In the following three cases, we compare firm 1's profit under licensing with a fixed fee with its profit under no licensing.

Case 1: $c_w < \frac{2(a-c_X)-3(b-c_Y)}{5}$, partial conversion is optimal under no licensing and licensing.

The fixed fee is equal to the difference between firm 2's profit under licensing (30) and under no licensing (22):

$$\begin{aligned} F &= \Pi^{(1)L} - \Pi_2^{(1)NL} = \left(\frac{a - c_X - c_w}{3}\right)^2 + \left(\frac{b - c_Y + c_w}{3}\right)^2 - \left(\frac{a - c_X - c_w}{3}\right)^2 \\ &= \left(\frac{b - c_Y + c_w}{3}\right)^2 \end{aligned}$$

Firm 1's total profit under licensing is its profit in (30) plus the fixed fee:

$$\Pi_1^L = \Pi^{(1)L} + F = \left(\frac{a - c_X - c_w}{3}\right)^2 + 2\left(\frac{b - c_Y + c_w}{3}\right)^2 \tag{33}$$

We show below that firm 1's total profit under licensing (33) is less than its profit under no licensing (21):

$$\begin{aligned} \Pi_1^L - \Pi_1^{(1)NL} &= \left(\frac{a - c_X - c_w}{3}\right)^2 + 2\left(\frac{b - c_Y + c_w}{3}\right)^2 - \left(\frac{a - c_X - c_w}{3}\right)^2 - \left(\frac{b - c_Y + c_w}{2}\right)^2 \\ &= -\frac{1}{36}\left(\frac{b - c_Y + c_w}{3}\right)^2 < 0. \end{aligned}$$

Case 2: $c_w \in [\frac{2(a-c_X)-3(b-c_Y)}{5}, \frac{a-c_X-(b-c_Y)}{2})$, exactly-full conversion is optimal under no licensing, but partial conversion is optimal under licensing.

The fixed fee is equal to the difference between firm 2's profit under licensing (30) and under no licensing (26):

$$F = \Pi^{(1)L} - \Pi_2^{(2)NL} = \left(\frac{a-c_X-c_w}{3}\right)^2 + \left(\frac{b-c_Y+c_w}{3}\right)^2 - \left(\frac{3(a-c_X)-(b-c_Y)-4c_w}{7}\right)^2$$

Firm 1's total profit under licensing is its profit in (30) plus the fixed fee:

$$\begin{aligned} \Pi_1^L &= \Pi^{(1)L} + F = \frac{2(a-c_X-c_w)^2 + 2(b-c_Y+c_w)^2}{9} \\ &\quad - \left(\frac{9(a-c_X-c_w)^2 - 6(a-c_X-c_w)(b-c_Y+c_w) + (b-c_Y+c_w)^2}{49}\right) \end{aligned} \tag{34}$$

We show below that firm 1's total profit under licensing (34) is less than its profit under no licensing (25):

$$\begin{aligned} \Pi_1^L - \Pi_1^{(2)NL} &= \frac{-(a-c_X-c_w)^2 + 17(b-c_Y+c_w)^2 - 18(a-c_X-c_w)(b-c_Y+c_w)}{441} \\ &< 0 \text{ if } (b-c_Y+c_w) < \frac{18}{17}(a-c_X-c_w), \end{aligned}$$

which is true by the assumption $c_w < \frac{a-c_X-(b-c_Y)}{2}$.

Case 3: $c_w \geq \frac{a-c_X-(b-c_Y)}{2}$, exactly-full conversion is optimal under no licensing and licensing.

The fixed fee is equal to the difference between firm 2's profit under licensing (32) and under no licensing (26):

$$F = \Pi^{(2)L} - \Pi_2^{(2)NL} = \frac{1}{18}(a-c_X+b-c_Y)^2 - \left(\frac{3(a-c_X)-(b-c_Y)-4c_w}{7}\right)^2$$

Firm 1's total profit under licensing is its profit in (32) plus the fixed fee:

$$\Pi_1^L = \Pi^{(2)L} + F = \frac{1}{9}(a-c_X+b-c_Y)^2 - \left(\frac{3(a-c_X)-(b-c_Y)-4c_w}{7}\right)^2 \tag{35}$$

We show below that firm 1's total profit under licensing (35) is less than its profit under no licensing (25):

$$
\begin{aligned}
\Pi_1^L - \Pi_1^{(2)NL} &= \frac{1}{9}(a - c_X + b - c_Y)^2 - \left(\frac{3(a-c_X) - (b-c_Y) - 4c_w}{7}\right)^2 \\
&\quad -2\left(\frac{a - c_X - c_w + 2(b - c_Y + c_w)}{7}\right)^2 \\
&= \frac{-50\alpha^2 + 80\alpha\beta - 32\beta^2}{441} \text{ (letting } \alpha = a - c_X - c_w, \beta = b - c_Y + c_w) \\
441(\Pi_1^L - \Pi_1^{(2)NL}) &< -50\alpha^2 + 80\left(\frac{113\alpha^2 - 31\beta^2}{206}\right) - 32\beta^2 \text{ (using the fact that } F > 0) \\
&= -6.12\alpha^2 - 44.04\beta^2 < 0.
\end{aligned}
$$

This completes the proof.

■

Proof of Proposition 7.

In Proposition 6, we showed that using only a fixed fee license was not optimal for firm 1. In the following three cases, we compare firm 1's profit under royalty + fixed fee licensing with its profit under no licensing. The equilibrium quantities and profits under no licensing are derived in the proof to Proposition 6.

Case 1: $c_w < \frac{2(a-c_X)-3(b-c_Y)}{5}$

For this range of c_w, under no licensing, partial conversion would be optimal for firm 1 (condition given in (20)). If firm 1 licenses to firm 2 for a per unit royalty of ρ, the profit functions would be

$$
\begin{aligned}
\max_{q_{A1},q_{B1}} \Pi_1 &= (a - c_X - c_w - q_{A1} - q_{A2})q_{A1} + (b - c_Y + c_w - q_{B1} - q_{B2})q_{B1} + \rho q_{B2} \quad (36) \\
&\quad \text{s.t. } q_{A1} \leq q_{B1} \\
\max_{q_{A2}} \Pi_2 &= (a - c_X - c_w - q_{A1} - q_{A2})q_{A2} + (b - c_Y + c_w - q_{B1} - q_{B2})q_{B2} - \rho q_{B2} \quad (37) \\
&\quad \text{s.t. } q_{A2} \leq q_{B2}
\end{aligned}
$$

If the BPS constraints are not binding, the optimal quantities are

$$
\begin{aligned}
q_{A1}^{(1)} &= q_{A2}^{(1)} = \frac{a - c_X - c_w}{3} \quad (38) \\
q_{B1}^{(1)} &= \frac{b - c_Y + c_w + \rho}{3} \text{ and } q_{B2}^{(1)} = \frac{b - c_Y + c_w - 2\rho}{3} \quad (39)
\end{aligned}
$$

Substituting (38) and (39) into (36) and differentiating with respect to ρ gives optimal license fee of $\rho^* = \frac{b-c_Y+c_w}{2}$. Substituting ρ^* back into (39) gives $q_{B1}^{(1)} = \frac{b-c_Y-c_w}{2}$ and $q_{B2}^{(1)} = 0$. Therefore, we end up back to the monopoly quantity for firm 1 and no waste conversion for firm 2. Thus, royalty licensing is not feasible for $c_w < \frac{2(a-c_X)-3(b-c_Y)}{5}$. Since we showed in Proposition 6 that using only a fixed fee licensing was not optimal, we conclude that licensing is not optimal for $c_w < \frac{2(a-c_X)-3(b-c_Y)}{5}$.

Case 2: $\frac{2(a-c_X)-3(b-c_Y)}{5} \leq c_w < \frac{39(a-c_X)-19(b-c_Y)}{58}$

The previous case showed that there is not a feasible royalty license when both firms implement partial conversion. We explore now whether there is a royalty licensing agreement where firm 1 would implement exactly-full conversion. The profit functions would be

$$\begin{aligned} \max_{q_{A1}} \Pi_1 &= (a - c_X + b - c_Y - 2q_{A1} - q_{A2} - q_{B2})q_{A1} + \rho q_{B2} \quad (40) \\ \max_{q_{A2},q_{B2}} \Pi_2 &= (a - c_X - c_w - q_{A1} - q_{A2})q_{A2} + (b - c_Y + c_w - q_{A1} - q_{B2})q_{B2} - \rho q_{B2} \quad (41) \\ & \text{s.t. } q_{A2} \leq q_{B2} \end{aligned}$$

If the BPS constraint for firm 2 is not binding, then the optimal quantities are

$$\begin{aligned} q_{A1}^{(2)} &= \frac{a - c_X + b - c_Y + \rho}{6} \quad (42) \\ q_{A2}^{(1)} &= \frac{5(a - c_X) - (b - c_Y) - 6c_w - \rho}{12} \text{ and } q_{B2}^{(1)} = \frac{-(a - c_X) + 5(b - c_Y) + 6c_w - 7\rho}{12} \quad (43) \end{aligned}$$

Substituting (42) and (43) into (40), we get:

$$\Pi_1^* = 2\left(\frac{a - c_X + b - c_Y + \rho}{6}\right)^2 + \rho\left(\frac{5(b - c_Y) - (a - c_X) - 7\rho + 6c_w}{12}\right). \quad (44)$$

It is straightforward to show $\frac{\partial^2 \Pi_1}{\partial \rho^2} < 0$, therefore, Π_1 is concave. Differentiating (44) with respect to ρ and setting the derivative to zero gives a license fee of

$$\rho^* = \frac{a - c_X + 19(b - c_Y) + 18c_w}{38}. \quad (45)$$

To ensure that (45) is optimal, we need to check that ρ^*, is positive, which by inspection, we see that it is, thus licensing increases firm 1's profit. Also, to check for feasibility for firm 2, we must have $q_{B2}^{(1)} > 0$. For $q_{B2}^{(1)} > 0$, substituting (45) into (43), we have

$$\begin{aligned} q_{B2}^{(1)} &= \frac{-(a - c_X) + 5(b - c_Y) + 6c_w - 7\rho^*}{12} > 0 \\ &= \frac{-45(a - c_X) + 57(b - c_Y) + 102c_w}{456} > 0 \\ \Leftrightarrow & \quad c_w > \frac{45(a - c_X) - 57(b - c_Y)}{102} \equiv \underline{c}_w. \quad (46) \end{aligned}$$

The above condition (46) implies that for $c_w \leq \underline{c}_w$, the optimal royalty fee is $\rho = \frac{-(a-c_X)+5(b-c_Y)+6c_w}{7} < \rho^*$, which gives $q_{B2}^{(1)} = 0$, which in turn implies that licensing is not optimal.

We also need to check that the partial conversion assumption for firm 2 holds. To check that $q_{B2}^{(1)} < q_{A2}^{(1)}$, we substitute (45) into (43):

$$q_{B2}^{(1)} = \frac{-45(a-c_X)+57(b-c_Y)+102c_w}{456} < \frac{189(a-c_X)-57(b-c_Y)+246c_w}{456} = q_{A2}^{(1)}$$

$$\Leftrightarrow c_w < \frac{39(a-c_X)-19(b-c_Y)}{58} \equiv \hat{c}_w \qquad (47)$$

It is straightforward to show that $\hat{c}_w > \underline{c}_w$. Therefore, for $c_w \in (\underline{c}_w, \hat{c}_w)$, firm 1 increases its profit by licensing its BPS process to firm 2 for a per unit royalty fee of $\rho^* = \frac{a-c_X+19(b-c_Y)+18c_w}{38}$. Let Π_2^R be firm 2's optimal profits under royalty licensing, i.e., substituting (42), (43), and (45) into (41). Since $q_{B2}^{(1)} > 0$, this means that firm 2's profit under royalty licensing, Π_2^R, is greater than its profit under no licensing in (26). Therefore, in addition to charging a royalty fee of (45), firm 1 can charge a fixed fee equal to Π_2^R-(26). This then makes firm 2 indifferent between licensing and no licensing, and firm 1 strictly better off by licensing. Firm 1 implements exactly-full conversion and firm 2 implements partial conversion.

<u>Case 3</u>: $c_w \geq \frac{39(a-c_X)-19(b-c_Y)}{58}$

Note that substituting $c_w = \frac{39(a-c_X)-19(b-c_Y)}{58}$ into (43) gives $q_{A2}^{(1)} = q_{B2}^{(1)} = \frac{3(a-c_X+b-c_Y)}{58}$. Thus for $c_w \geq \frac{39(a-c_X)-19(b-c_Y)}{58}$, the optimal quantities are independent of c_w because all waste from both firms is converted to by-product. However, the fixed fee portion of the license increases in c_w because firm 2's profit under no licensing (26) decreases in c_w. Therefore, firm 1's total profit under licensing still increases in c_w.

Notice that firm 1's profit under no licensing (25) also increases in c_w. However, we know that if $c_w = \frac{3(a-c_X)-(b-c_Y)}{4}$, then firm 2's quantity under no licensing is zero (from Equation (24)). For $c_w \geq \frac{3(a-c_X)-(b-c_Y)}{4}$, firm 1 makes monopoly profit in both markets, therefore, its profit under no licensing is strictly greater than its profit under licensing. Since profit under licensing is strictly greater than profit under no licensing when $c_w = \frac{39(a-c_X)-19(b-c_Y)}{58}$, and profit under no licensing is strictly greater than profit under licensing for $c_w \geq \frac{3(a-c_X)-(b-c_Y)}{4}$, by continuity, there exists $\bar{c}_w \in (\frac{39(a-c_X)-19(b-c_Y)}{58}, \frac{3(a-c_X)-(b-c_Y)}{4})$ such that for $c_w \leq \bar{c}_w$, licensing is optimal, and for $c_w > \bar{c}_w$, licensing is not optimal. This completes the proof. ■