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# Wintel: Cooperation or Conflict

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**Abstract**

We study the incentives of complementors (producers of complementary products) to cooperate vs. compete and how these interact. In a system of complements, like the PC, the value of the final product depends on how well the different components work together. This, in turn, depends on the firms' investment in complementary R&D. We ask whether profit maximizing complementors will fully cooperate to make the final product as valuable as possible. Contrary to the popular view that two tight complements will generally have well aligned incentives, we demonstrate that natural conflicts emerge over pricing, the timing of investments, and who captures the greatest value at different phases of product generations.

Preliminary draft

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# 1 Introduction

Intel and Microsoft are perhaps the best known pair of complementary firms in the world. More than 80% of the personal computers worldwide ship with an Intel microprocessor running Microsoft's Windows operating system. Since 1980, when IBM chose both Intel and Microsoft as the core components of the first IBM PC, Intel and Microsoft have been joined at the hip. In this paper, we tap recently available data revealed by the U.S. Department of Justice to explore the Intel-Microsoft relationship and model the dynamics of competition between complementary players.[1] Contrary to the popular view that two tight complements will generally have well aligned incentives, we demonstrate that natural conflicts emerge over pricing, the timing of investments, and who captures the greatest value at different phases of product generations.

We study the incentives of complementors to cooperate vs. compete and how these interact. In a system of complements, like the PC, the value of the final product depends on how well the different components work together. This, in turn, depends on the firms' investment in complementary R&D. *We ask whether profit maximizing complementors will fully cooperate to make the final product as valuable as possible.* Our interest in the topic was sparked by claims from Intel and Microsoft executives that their "partner" was not doing enough to advance the PC platform.[2]

The PC industry is an ideal setting to study complementors and their relationships. Following IBM's decision to set up an open standard for its Personal Computer in 1980, the microcomputer industry became gradually more horizontal, which led to specialized players increasingly dominating each component layer. In effect, the horizontal industry structure produced a wide array of complementary firms that needed to work together. Undeniably, the two most important components in a PC are the microprocessor and the operating system, with Intel and Microsoft as the leading players in each of these domains. Most other components (disk drives, monitors, memory and other chipsets) are commodities. In fact, the combined profit of Intel and Microsoft during most years in the 1990s exceeded the total profit of the entire world PC industry. In 2004, for example, Intel and Microsoft earned over $15 billion in net profits while the three largest original equipment manufacturers (OEMs) in the world (Dell, HP, and IBM) made roughly $2.5 billion in profits from their PC operations.[3] Intel and Microsoft's decisions have a disproportionate effect on the structure of the PC industry.[4]

We approach the question by setting up a dynamic mixed-duopoly model. Intel and Mi-

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[1] In Section 2 we offer a brief account of the relationship between Intel and Microsoft. For more detail see Yoffie, Casadesus-Masanell and Mattu [17].

[2] For example, in an interview conducted by the authors on September 10, 2002, Frank Ehrig, former Microsoft relationship manager for Intel, asserted:

> Intel is always trying to innovate on hardware platform and thus, always needs software. When software lags, it creates a bottleneck for Intel. Microsoft, on the other hand, wants to serve the installed base of computers in addition to demand for new computers. Therefore, a natural conflict exists between both companies. In addition, the question always remains – who will get the bigger piece of the pie? The success of one is seen as ultimately taking money away from the other.

[3] IBM, alone, lost over $1 billion in PCs in 1998, and another $965 million between 2001 and 2004. Only Dell made material profits in the PC industry.

[4] For more on complements and the PC industry, see Gawer and Cusumano [8].

crosoft are both profit maximizers, but the shape of their objective functions differs because their product technologies are inherently different. As a software company, Microsoft derives profit from selling OSs and applications with each new PC, *and* from selling upgrades to the installed base. Intel's profits, however, come from the sale of microprocessors in new PCs.

The benchmark model is a two-stage game. The first stage is the value creation stage: Intel and Microsoft choose how much to invest in complementary research and development to make the microprocessor and the operating system work together as well as possible, and thus to raise the willingness to pay for the PC. In the second stage, value capture takes place. Both Intel and Microsoft set prices continuously over time to maximize the net present value of profit.

We find that Intel prefers to charge high prices early in the life of a generation, while Microsoft is better off charging low prices early to increase the installed base. Microsoft then raises prices gradually as the life of the generation approaches its end. Intel charges high prices early because it has no (direct) interest in the installed base and to take advantage of Microsoft's low prices. As a consequence of these pricing patterns, Intel's profits are larger than Microsoft's early in the life of a given PC generation. As time goes by, Microsoft's profits increase steadily because it can progressively take advantage of the growing installed base.

Contrary to intuition, we find that Intel and Microsoft's net present value of profits coincide. This is surprising because the objective functions are different and the equilibrium price paths are also different. We also find that the marginal effect on profits of stage one investments is the same for both Intel and Microsoft. As a result, initial investments in complementary R&D are equal. The symmetric effect of Intel and Microsoft prices on the PC demand function guarantees that every additional dollar of profit that Microsoft can earn by reducing price to increase current sales and grow the installed base can be exactly matched by Intel raising its price. This result suggests that there is no conflict regarding investments in complementary R&D. The only conflict that the benchmark model identifies (other than the usual free-rider problem) is that Intel sets prices too high and this forces Microsoft to sell its operating system at lower prices than desired.

Next, we extend the model to two consecutive PC generations. Toward this end, we allow Intel and Microsoft to decide (independently) when to invest and introduce a new generation. Intel decides when to switch to a new microprocessor and Microsoft when to introduce a new operating system. We find that Intel wants to invest and introduce the new generation earlier than Microsoft. Clearly, Intel would be better off if Microsoft's actions followed Intel's timing. Microsoft, however, is better off milking the first generation to the last drop and wait longer than Intel to introduce the new generation operating system. This result is a direct consequence of the fact that Microsoft drives profit from the installed base. Switching to a new generation implies that the first generation installed base will begin to fade. As a consequence, Microsoft has more of an incentive to stay longer with the first generation than Intel.

We conclude that there is a two-sided asymmetric conflict between Intel and Microsoft. From the point of view of Microsoft, Intel sets too high prices (especially early in the life of a given PC generation) and, from the point of view of Intel, Microsoft invests in and introduces its new generation operating systems too late.

The final part of the paper investigates possible solutions to both these conflicts. First, we look at the effects of Microsoft embracing AMD, an Intel competitor in microprocessors.

Second, we analyze Intel's support of Linux. As expected, Microsoft is able to force Intel to cut prices by making sure that AMD is a viable competitor. Intel, on the other hand, can also induce Microsoft to speed up its product introductions when Linux is present. At a more general level, both of these extensions show that one way to deal with complement conflict is by promoting competition. Competition, however, may not be "free" for the promoter. If the customers' willingness to pay for the new competitor's products is low, the promoter is forced to lower its price to ensure the viability of competition. Thus, there is a trade off between the benefits of complement competition and the costs to ensure that competition takes place.

While this model builds on a specific case, we believe the implications are more general. One can think of any two firms that produce complementary products, where one firm cares more about the installed base than the other (for example, Nokia, which makes money from new sales of phones, and service providers, who profit more from usage in the installed base.) This model also captures situations where there are two firms producing complementary products and one of them cares more about brand equity. Or both attach equal importance to brand equity, but one of them is more patient than the other.

The emergence of independent, complementary firms would seem to be expanding. An increasing number of industries have evolved away from vertical integrated to more horizontal structures. These industries range from computers and communications to consumer electronics, automobiles, and healthcare. In these new horizontal industries, products that used to be designed and manufactured by a single firm are now produced by different players that must coordinate activities. And of course, some of these players are complements to each other.

The result of these horizontal structures is that it is becoming more important for firms to create new skills for managing complementary relationships. In order to do this, one needs to understand the fundamental economics governing relations between complementors: What determines the relative bargaining power to appropriate value? How can one firm build a dominant position vis a vis complementors? How can a firm make sure that complementors invest enough in complementary R&D? etc...

To the best of our knowledge, there are no papers devoted to the formal study of relationships of cooperation and competition between complementors. Certainly, there is no previous dynamic mixed-duopoly model analyzing the relationships between complementors. Most of the literature on cooperation vs. competition focuses on relationships between competitors or between buyers and sellers. The analysis of complementors is not as well understood.

The paper is organized as follows. Section 2 summarizes the relationship between Intel and Microsoft. Section 3 presents the benchmark model. The main modeling assumptions are based on the specifics of the case. Section 4 extends the analysis to two product generations and studies the investment game played by Intel and Microsoft at the end of the first generation. Section 5 presents two extensions: competition in microprocessors and in operating systems. Section 6 concludes by discussing the managerial implications that can be drawn from the analysis.

# 2 Competitive Dynamics at Wintel

To a large degree, Intel and Microsoft have strong common goals to promote the "Wintel" standard. Historically, most versions of Microsoft Windows would only work on Intel architecture CPUs, and Intel CPUs required Microsoft's operating system to implement new features. Since both companies have the largest market share within their respective segments, growing the total market for personal computers grows both Intel and Microsoft's businesses. If Intel and Microsoft can also coordinate feature improvement and performance, the two companies can mutually increase barriers to imitation from competitive offerings in software and semiconductors.

At the same time, the profit functions for the two companies are very different. At the simplest level, Intel only makes money by selling new computers to new, or replacement purchasers. Microsoft, on the other hand, can make money by selling new machines as well as selling upgrades to the existing installed base. In addition, Intel and Microsoft have very different fixed and variable cost models. Intel is a capital intensive firm, which spends 10-20% of revenues on plant and equipment, and another 12-14% on R&D annually. With high fixed costs, Intel's profits depend on learning-by-doing, increasing yields in its factories, and driving capacity utilization. A typical fabrication facility in 2004 costs about $3 billion, fully equipped. Intel also worked on Moore's Law, driving to double CPU performance every 18 months, and introducing a major new architectural innovation for its CPUs every 4-5 years.

Microsoft, by comparison, has very little fixed investment: capital spending was limited to IT and office buildings. While Microsoft's R&D levels were comparable to Intel's, Microsoft had high selling and overhead (27% of revenue in 2003), driven by significant direct sales and ongoing support costs. Microsoft product cycles were generally longer than semiconductor product cycles. Microsoft generated a major new operating system roughly every 5-6 years, and minor upgrades every 2-3 years. As a general rule, software innovation lagged computing power.

While Intel and Microsoft were by far the leading providers in their product categories, they also faced somewhat different competitive environments. Intel's core microprocessor business was protected with a large number of patents and trade secrets, and its microcode – an embedded instruction set for the user – was copyrighted. Nonetheless, several competitors were cross-licensed to make compatible products. In 2004, Advanced Micro Devices and Via could legally design products that ran Intel's core microcode. As a result, Intel faced direct price competition in CPUs. Average CPU prices rose in the 1990s, and declined after the downturn in the market in 2000.

Since the mid-1990s, Microsoft had no compatible competitors in either its core operating system business or its productivity software business (Microsoft Office). When Microsoft transitioned from MS-DOS to Windows, the last compatible competitor (DR DOS, owned by Novell) could not make a directly compatible program without violating Microsoft trade secrets, patents and copyrights. While Intel faced little price competition from alternative architecture CPUs (such as IBM's PowerPC or Sun Microsystem's SPARC), Microsoft's primary price competition came from open source (free) software, such as Linux and Open Office. Programs written to take advantage of Windows were not directly compatible with Linux or Open Office, and users would have to use inefficient emulation programs or translators to read Windows or Office files. Since the early 1990s, average selling prices for Microsoft's operating system and office suite rose dramatically.

Intel and Microsoft not only depended on each other, but also a wide array of third party technologies to deliver their value to the end user. A new microprocessor, for example, might have improved processing speed, energy consumption, and security, but without concomitant improvements in the PC hardware and software platform (above and beyond the operating system), the R&D and added features would be wasted. As a result, Intel might invest in these new technologies themselves, which they did with new PC motherboards that had over 30 times the bandwidth of the original IBM PC bus. Intel would also recommend hardware or software specifications to third parties and subsidize their entry, which it did with multimedia software in the mid-1990s. And on occasion, Intel would directly invest in independent hardware and software vendors to accelerate the delivery of key technologies to the market, which it did through Intel Capital.[5] To facilitate the adoption of its wireless chips, Intel invested in dozens of companies to deliver wireless (802.11 or WiFi) hotspots around the world.

Microsoft had a similar set of complementary assets that were critical to their businesses. The most obvious was independent software vendors writing applications for Microsoft's operating system. An operating system, by itself, had very little value to an end-user. The role of an OS was to share scarce system resources, create file formats, and most important, provide a set of common services (such as input/output algorithms, graphics, and networking) to allow third parties to write applications. Without independent software vendors developing applications, the value of the operating system declines measurably – regardless of its technical merits. Microsoft spent heavily to court ISVs, providing them with tools, advanced information on new operating systems, and occasionally capital to underwrite new applications.

Microsoft also had a highly complementary relationship with computer hardware companies. New, faster hardware prompted people to upgrade to more powerful software and vice versa; Windows 95 was far more valuable on a Pentium-powered PC than on a 486 machine. Microsoft worked hard to evolve its operating system by sharing most of its interface specifications with complementors such as PC hardware and peripheral manufacturers while still maintaining its proprietary standards. This created a kind of "open but not open" platform interface that allowed Microsoft to keep significant influence on the design and future evolution of its products. In particular, Microsoft's control of its application program interfaces (APIs) was at the heart of its platform leadership. This gave Microsoft the ability to keep technical information from competitors such as Netscape and IBM, and yet, on other occasions provided enabling tools and technologies to facilitate the coordination of its products.

When IBM lost its reign of the PC platform, and the industry moved progressively towards a horizontal structure, Intel and Microsoft began to exert greater influence on the standards that defined the PC. Throughout the 1980s and 1990s, Microsoft and Intel found extensive opportunities to cooperate as well as to compete. As Microsoft progressed with software development and Intel created faster microprocessors, together they drove demand for PCs while also affecting the technological trajectory of the platform.

Yet despite the hype that ultimately developed around "Wintel," there was very limited interaction between the two companies until the development of the IBM PC 286 in the mid-

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[5] Intel Capital was Intel's venture capital arm. It was formed in the early 1990s with the aim to expand the market for Intel's products by investing in promising start-ups. Software companies in the portfolio optimized their applications for Intel processors and hardware companies incorporated Intel chips into their products. Besides financial support, companies in the portfolio (more than 400) benefited from technological assistance, insight into future trends, inclusion in marketing programs, access to distribution channels, strategic relationships with other sources of financing, networking with other portfolio companies, worldwide infrastructure, visibility, and corporate association.

1980s.[6] With the exception of the occasional software discussion and engineers exchanging information about DOS and Intel architecture microprocessors, the two companies did little to coordinate their respective businesses. By the mid-1980s, however, it become clear to both companies that greater coordination was not only desirable, but essential to their long-run business strategies. By the mid-1990s, a small number of full-time Intel engineers were stationed onsite at the Microsoft campus to facilitate knowledge sharing and to maintain trust at the engineering level.[7] Microsoft did not have any of its people located on the Intel campus.[8]

One of the biggest challenges for both companies was coordinating the release of new products. Microsoft compared any given release of its operating system to a train station.[9] The new Microsoft operating system could be seen as a train that had pulled into a station for a limited amount of time. During this small window, Intel and other passengers could board the train (e.g., recommend or provide software drivers and make other improvements). However, once the train had left the station, the opportunity to make additional enhancements was gone and "passengers" would have to wait for the next train—the next OS release—which generally took five years. Intel, on the other hand, managed its business according to Moore's law, which demanded significant new technologies be brought to the market every 18-24 months. Gates wanted Intel to be "hooked to the train," while Intel wanted Microsoft to accelerate OS releases and refinements to match Moore's Law.[10]

Despite obvious tensions, the two companies managed to collaborate on a wide range of activities that were mutually beneficial. When Microsoft was working with IBM to build the OS/2 to run on Intel's 286 processor in the late 1980s, it simultaneously supported Intel's push for the 386. Microsoft clearly recognized the added value of more processing power.[11] Furthermore, Microsoft also encouraged Intel's later introduction of the more powerful 486 processor since it ran Windows 3.0 much better than the 386.[12] As Gates once told Intel management, "We will fill the vessels you build with more software. . . ."[13]

The two companies worked well together on specific initiatives that benefited the entire PC community. For example, in the early 1990s, the ISA bus had become a bottleneck for the development of PC hardware components.[14] Intel had been working on a new bus (the PCI bus) that promised significant boost to PC performance while also maintaining an open specification available to everyone.[15] Intel realized that it was in the best interest of both OEMs and complementors to plug their products together in a uniform way making the development of complements easier and cheaper.[16] The challenge was to convince complementors that the new bus would become the next standard. Microsoft's strong support of PCI was critical in ensuring the industry's adoption of this new standard.

But the symbiotic relationship between Microsoft and Intel has frequently veered from win-win. Problems arose when one force wasn't pushing as hard as the other would like. For

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[6] Schlender [12].

[7] Casadesus-Masanell, Mattu, and Yoffie "Intel Corporation interview notes," Spring 2003.

[8] Ibid.

[9] Ibid.

[10] Ibid.

[11] Yu [18, p. 74].

[12] Ibid.

[13] Quote from Frank Ehrig, former Microsoft relationship manager for Intel.

[14] Gawer and Cusumano [8, p. 25] and Yu [18, p. 76].

[15] Yu [18, p. 76].

[16] Gawer and Cusumano [8, p. 29].

instance, Intel's 32-bit microprocessor (the 386) waited 10 years before Microsoft produced a 32-bit operating system (Windows 95).[17] Intel's growth was dependent on the demand for its newest processors. As the company strove to stay ahead of its competition in terms of microprocessor performance, Intel management became dependent on complementary software to stimulate demand for new products. Microsoft on the other hand, was motivated to not only meet the demand for new computers but also wanted to serve the installed base of computers.

Examples of conflict have been numerous. In 1991, Microsoft was a co-sponsor of the ACE consortium (Advanced Computing Environment), whose primary purpose was to develop and promote the RISC architecture in PCs which would replace Intel's CISC processor architecture.[18] The group consisted of 21 different companies including MIPS, the independent company that supplied RISC processors for workstation PCs, Compaq, Microsoft, and Dell. Microsoft was a particularly strong voice encouraging this architectural transition. To add to the persuasiveness of RISC, Microsoft announced its development of Windows NT that would take advantage of the higher-speed potential of the reduced instruction set architecture.[19] Although the ACE consortium ultimately failed, it pointed to the divergence of Intel and Microsoft's interest. ACE was aimed squarely at trying to build a non-Intel architecture as the center of the next generation of CPUs.

One of the biggest conflicts between the two companies emerged in 1995 over a technology called Native Signal Processing (NSP). At the time, multimedia applications were computationally consuming jobs carried out by sound and video processor add-ins. Intel wanted to promote multimedia applications because better quality audio, graphics, and video meant a better end-user experience that would ultimately broaden the PC market, thereby driving more hardware sales and the sale of more higher frequency CPUs.[20] Intel had developed a specialized software technology and application program interfaces that would help developers build applications with multimedia capabilities without the use of special chips.[21] NSP was a multimedia software solution that didn't tie into the Windows operating system since software developers could simply bypass Windows and give graphic handling instructions directly to the microprocessor.[22] Intel believed that by having more functions performed by the software, NSP gave PC buyers a greater incentive to buy faster microprocessors.[23]

Microsoft, however, was livid with Intel.[24] NSP not only displaced hardware components such as add-in chips and add-in circuit boards, but software as well. To further add salt to the wound, Intel had developed NSP as an extension to Windows 3.1, just as Microsoft was about to release Windows 95. Microsoft CEO Bill Gates stated in an email, "It sure seems like Intel is really dense. We told them clearly on the May 9 meeting that we had major objections about NSP and would recommend that people [OEMs] not install it."[25] Microsoft took the message one step further by publicly warning OEMs that it had no intention of supporting NSP currently or in any future release of Windows, forcing an "NSP

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[17] Casadesus-Masanell, Yoffie, and Mattu [5].

[18] Burgelman [2, p. 226].

[19] Jackson [10].

[20] Balkanski [1, p. 23].

[21] *Electronic Buyers' News* [7, p. 12] and Gawer and Cusumano [8, p. 153].

[22] Gawer and Cusumano [8, p. 153],

[23] Slater [13, p. 61].

[24] Jackson [10, p. 374].

[25] *United States v. Microsoft*: Government Exhibit 923, MS98 0168630 Confidential, carls email, May 31, 1995.

chill" among PC manufacturers. Ultimately, since Microsoft's cooperation was required to make NSP work, Intel was forced to capitulate.

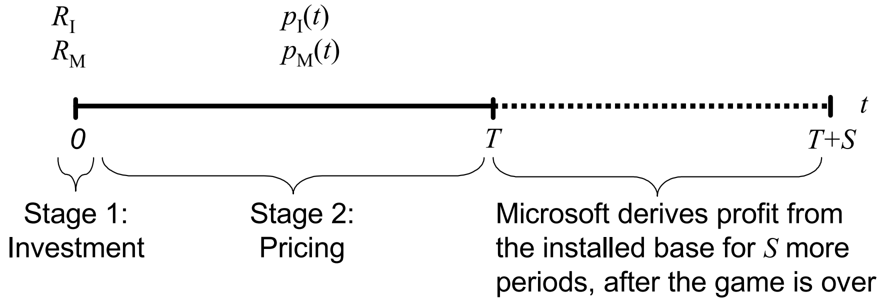
Similar conflicts later emerged over "MMX," a set of extensions in Intel's core processors that allowed the CPU to better handle multimedia, especially audio and video. PCs had not been designed to run graphic-intensive games or to play music or video clips. By adding 57 new instructions to the microprocessor, Intel wanted to increase the speed and quality of multimedia applications. To make MMX a success, Intel spent tens of millions of dollars in R&D resources and testing to develop the CPU extensions. In addition, it planned to spend another \$250 million to make it successful in the marketplace. Roughly \$100 million would be dedicated to underwrite the development of new software, which could take advantage of the new instruction set; and another \$150 million to market MMX as a brand new microprocessor that would drive consumers and business to buy new computers. Ultimately, however, these resources would be wasted if Microsoft did not support Intel. If the OS was not optimized to take advantage of MMX, (Microsoft had to add one switch), then most games or other applications would see few performance enhancements. Microsoft, however, was worried that a new set of Intel-specific instructions might fragment the PC platform. Intel's competitor, Advanced Micro Devices, was developing its own multimedia technologies (called 3DX), and AMD was also pushing hard for Microsoft's support. Fragmentation of multimedia standards would be painful for Microsoft: since Microsoft had to provide assistance to end-customers as well as independent software developers, multiple multimedia extensions would multiply Microsoft's support costs. In addition, to provide even basic support for MMX would require Microsoft to issue an interim upgrade for Windows '95. After protracted discussions, Microsoft demanded, and Intel acceded, to license AMD for free in exchange for Microsoft adding support for MMX.

In more recent years, Intel and Microsoft have gone through cycles of intense cooperation and conflict, ranging from highly coordinated efforts to launch the Universal Serial Bus in 1998 to well-publicized conflicts over support for Java and Linux (Microsoft strongly opposed any support for these programs, and Intel was worked to legitimize both. Intel even became one of the founders of the Open Source Development Laboratory, which provides ongoing funding and technical support for Linux creator, Linus Torvald.)

# 3 Benchmark Model

In this section we introduce a simple dynamic mixed-duopoly model to investigate the economics of cooperation and conflict between Intel and Microsoft. The case description in Section 2 suggests that the mixed nature of competition between Intel and Microsoft is due to technological reasons: Intel derives profit from sales of new PCs only whereas Microsoft also obtains profit from selling applications and upgrades to the installed base. The question we address in the model is whether this asymmetry alone is sufficient or not to generate conflict regarding investment in complementary R&D.

We model the relationship between Intel and Microsoft as a two-stage game. In the first stage, the "value creation" stage, the companies choose how much to invest in complementary R&D. In the second stage, the "value capture" stage, Intel and Microsoft set prices to capture as large a share of value as possible. We assume that the second stage has length $T \in (0, \infty]$ which, for now, is an exogenous parameter. Thus, Intel and Microsoft set prices $p_I(t)$ and $p_M(t)$ every instant $t$ from time 0 to $T$. After the life of the generation is over at time $T$, Microsoft may still realize some profit for $S \in [0, \infty]$ more periods. This additional profit originates from the sale of applications to the installed base. The following figure illustrates the game:



Let $q = \alpha\left(1 - \frac{p}{\beta}\right)$ be the demand function for PCs. Linearity is assumed for tractability. Parameter $\alpha$ (the horizontal intercept) is the size of the market and $\beta$ (the vertical intercept) indicates the willingness to pay for a PC of the individual who most values the PC.

Intel and Microsoft both invest in R&D to raise the value of the PC platform. In the first stage, Intel and Microsoft choose investments to affect the values of $\alpha$ and $\beta$. In particular, let $R_I$ and $R_M$ be Intel and Microsoft's period 0 investments in research and development. Then,

$$\alpha = \alpha\left(R_I, R_M\right) \text{ and } \beta = \beta\left(R_I, R_M\right). \tag{1}$$

These investments are complementary in the sense that the marginal effect of Intel's (Microsoft's) investment is larger when Microsoft (Intel) increases its investment. Formally, $\alpha$ and $\beta$ are supermodular functions: increasing $R_i$ raises the incremental returns associated with increases in $R_j$.

Because we are interested in isolating the effects of the asymmetry described above (that Microsoft cares more than Intel about the evolution of the installed base) on Intel and Microsoft's willingness to invest, we assume that $R_I$ and $R_M$ enter symmetrically both $\alpha$ and $\beta$. That is, $\alpha\left(R_I, R_M\right) = \alpha\left(R_M, R_I\right)$ and $\beta\left(R_I, R_M\right) = \beta\left(R_M, R_I\right)$ for all $R_I$ and $R_M$. Finally, the cost to invest for both Intel and Microsoft is the size of the investment itself.

The second stage is a continuous-time, finite-horizon game. In each period $t$, a new cohort of size $\alpha > 0$ enters the market. Given $\alpha$ and $\beta$, period $t$'s demand function is $q\left(t\right) = \alpha\left(1 - \frac{p(t)}{\beta}\right)$, where $p\left(t\right)$ is the sum of Intel and Microsoft's prices: $p\left(t\right) \equiv p_I\left(t\right) + p_M\left(t\right)$. At each time period, Intel and Microsoft compete by setting prices to capture the value of the PC. Both Intel and Microsoft's objectives are to maximize their net present value of profits. Yet Intel makes money primarily by selling microprocessors in new computers to new, or replacement purchasers, while Microsoft makes money by selling new machines as well as selling upgrades to the installed base. Also, marginal cost for Microsoft is negligible, but the marginal costs of a microprocessor are positive ($w \in [0, \beta]$).

Let $\delta \in [0, \infty)$ be the "rate of decay" or "death rate" of past sales. Then, the installed base at time $t$ is

$$y\left(t\right) = \left\{ \begin{array}{ll} y(0)e^{-\delta t} + \int_0^t e^{-\delta(t-\tau)} q\left(\tau\right) d\tau & \text{if} \quad t \in [0, T] \\ e^{-\delta(t-T)}\left(y(0)e^{-\delta T} + \int_0^T e^{-\delta(T-\tau)} q\left(\tau\right) d\tau\right) & \text{if} \quad t \in (T, T+S] \end{array} \right. . \tag{2}$$

To simplify notation, let

$$\pi_I\left(t\right) \equiv q\left(t\right)\left(p_I\left(t\right) - w\right) \tag{3}$$

and

$$\pi_M(t) \equiv q(t)\, p_M(t) + a y(t) \tag{4}$$

be Intel's and Microsoft's instant profits at time $t$, respectively. $a \geq 0$ is a parameter that captures Microsoft's profit from selling upgrades and applications to the installed base.[26]

Let $r \in [0, \infty)$ be the discount rate. Intel and Microsoft's objective functions are, respectively,

$$\Pi_I \equiv \int_0^T e^{-rt} \pi_I(t)\, dt \tag{5}$$

and

$$\Pi_M \equiv \int_0^T e^{-rt} \pi_M(t)\, dt + e^{-rT} y(T)\, V(S), \tag{6}$$

where $V(S)$ is the value to Microsoft of the installed base at time $T$, $y(T)$. Because every time period Microsoft gets $a$ from the installed base, we have that

$$V(S) = \int_0^S e^{-(r+\delta)t} a\, dt = \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)S}\right).$$

Differentiating the state variable $y$ (equation 2) with respect to $t$, we get the system dynamics:

$$\dot{y}(t) = \begin{cases} q(t) - \delta y(t) & \text{if} \quad t \in [0, T] \\ -\delta e^{-\delta(t-T)} y(T) & \text{if} \quad t \in (T, T+S] \end{cases}. \tag{7}$$

Finally, we have the initial condition $y(0) \equiv y_0 \geq 0$.

We now proceed to solving this dynamic game of complete but imperfect information. Given investment levels, we solve the pricing game and then given the outcome of the pricing game, investment levels are chosen.

## 3.1 Stage 2: The Pricing Game

In the second stage, Intel and Microsoft compete to capture the value of the PC platform by setting prices. We model this as a differential game with state variable $y$ and two decision variables: Intel chooses $p_I(t)$ to maximize (5) subject to (7) and $y(0) = y_0$ and Microsoft chooses $p_M(t)$ to maximize (6) subject to the same constraints.

This is a linear state differential game (see Dockner, Jørgensen, Van Long and Sorger [6, Chapter 7]). We solve for the Markov perfect equilibrium. Linear state games have the property that the open-loop equilibrium is Markov perfect.

**Proposition 1** *The equilibrium price paths are given by*

$$p_I^*(t) = \frac{1}{3}\left(\beta + 2w + \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) + V(S)\, e^{-(r+\delta)(T-t)}\right) \tag{8}$$

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[26] There are several interpretations for $a$. If Microsoft was a monopolist selling applications to the installed base and the demand for applications was given by $q = \varepsilon y\left(1 - \frac{p_M^A}{\gamma}\right)$, where $p_M^A$ is Microsoft's price for applications, then Microsoft's profit in period $t$ derived from the installed base would be $\frac{\varepsilon\gamma}{4} y(t)$. Thus, in this case, $a \equiv \frac{\varepsilon\gamma}{4}$. A second possibility is that the installed base has a unit elastic demand function. In this case Microsoft's revenue from sales to the installed base are $y(t)$ at each time period $t$.

*and*

$$p_M^*(t) = \frac{1}{3}\left(\beta - w - \frac{2a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - 2V(S)\,e^{-(r+\delta)(T-t)}\right). \tag{9}$$

*The equilibrium quantity sold in period t is*

$$q^*(t) = \frac{\alpha}{3\beta}\left(\beta - w + \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)}\right).$$

**Proof.** All proofs in the Appendix. ■

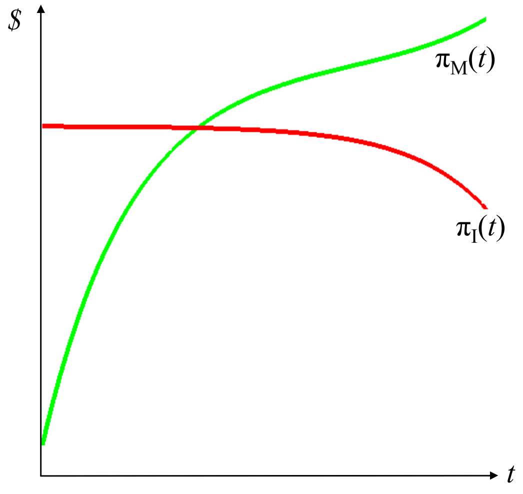
As long as Microsoft profits from selling to the installed base ($a > 0$), Intel's price cost margin is larger than Microsoft's. That is, $a > 0 \Rightarrow p_I^*(t) - w > p_M^*(t)$ and $a = 0 \Rightarrow p_I^*(t) - w = p_M^*(t)$. Intuitively, Microsoft sets lower prices to feed the installed base. Since Intel cannot profit from selling to the installed base, it takes advantage of Microsoft's price reductions by increasing the price of microprocessors.

Differentiating these expressions, we see that $\frac{dp_I^*}{dt} \leq 0$. Furthermore,

$$\operatorname{sgn}\left(\frac{dp_I^*}{dt}\right) = -\operatorname{sgn}\left(\frac{dp_M^*}{dt}\right) = \operatorname{sgn}\left(\frac{dq^*}{dt}\right).$$

Therefore, MS's price path is monotonically increasing throughout the life of an OS while that of Intel is monotonically decreasing. Prices are constant over time only if $T = \infty$, $S = \infty$, or $a = 0$.

Because $\frac{d\pi_I(t)}{dt} < 0$, Intel makes most of its profit at the beginning of the life a generation (microprocessor/OS). MS, instead, makes most of its profit in the later period of the life of the generation. The following figure illustrates the profit paths for Microsoft and Intel:



The more surprising result is the following:

**Proposition 2** *As long as $y_0 = 0$, Intel and Microsoft's net present value of profit is equal, despite the asymmetry in their objective functions, cost structures, and equilibrium price paths. In fact, Intel and Microsoft's net present value of profit can be expressed as:*

$$\Pi_I = \Theta_D(0) - \Theta_D(T) \tag{10}$$

*and*

$$\Pi_M = \left(\frac{a}{r+\delta}\left(1 - e^{-(r+\delta)T}\right) + V(S)\, e^{-(r+\delta)T}\right) y_0 + \Theta_D(0) - \Theta_D(T), \tag{11}$$

*where*

$$\Theta_D(t) \equiv \int -\frac{\alpha}{9\beta} e^{-rt}(\beta - w + \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) + V(S)\, e^{-(r+\delta)(T-t)})^2 dt.$$

The intuition for this result begins with the assumption that $y_0 = 0$, and both Intel and Microsoft are on a level playing field (zero profit) at $t = 0$. When $a = 0$, $\hat{p}_I(t)$ and $\hat{p}_M(t)$ are the prices Intel and MS would set, respectively. In this case, both objective functions are identical and $\hat{p}_I(t) - w = \hat{p}_M(t)$. As a consequence, both companies would earn the exact same amount, and the proposition is trivially satisfied.

Suppose now that $a > 0$. In the period immediately after $t = 0$, Microsoft will lower its price (compared to $\hat{p}_M(t)$) to build the installed base, foregoing some instant profit (sales of OSs in new PCs) but raising future profit (from selling to a larger installed base). Intel, however, will increase its price (compared to $\hat{p}_I(t)$) because it knows that Microsoft will maintain a lower price. Microsoft keeps dropping price up to the point in which the profit forgone from the lower price equals the increase in profit from selling upgrades to the installed base. *Because the demand function is symmetric in Intel and Microsoft's price*, every increase in profit that MS can get from selling at a lower price (building and selling to a larger installed base) is matched by Intel raising price and getting that additional profit immediately. Thus, all the extra profit that Microsoft gets from selling to the installed base is the same extra profit that Intel gets from higher prices. As a consequence, the net present value of profit of Intel and Microsoft coincide.

The symmetry on prices in the demand function acts as a "tube" connecting "two vessels:" Intel and Microsoft's profits. Just as in a system of connected vessels, both Intel and Microsoft profits are equated. A consequence of Proposition 2 is that Intel is better off when Microsoft's market power is greater in the upgrade and applications markets (larger $a$).

Finally, it is interesting to point out that the installed base $y(t)$ is not necessarily an increasing function of $t$. If the death rate $\delta$ is sufficiently large, $y$ increases first and then decreases as $t$ gets close to time horizon $T$.

### 3.1.1 Collusion

If Intel and Microsoft cooperate in choosing price $p(t)$, then the differential game becomes a straightforward dynamic optimization problem. The following proposition shows that consumer surplus and profit are larger in a monopolistic industry structure. The proposition shows that the standard result in a static model (see, for example, Cournot [4]) also holds in our dynamic setting.

**Proposition 3** *Let* $p^*(t)$ *and* $\Pi^*$ *be the optimal price path and NPV of profits in the case of collusion. Then* $p^*(t) < p_I^*(t) + p_M^*(t)$ *and* $\Pi^* > \Pi_I^* + \Pi_M^*$.

One implication from this proposition is that total welfare is larger under collusion. With competition, Intel does not internalize the effect of its high price on Microsoft's profits. Under collusion, this effect is taken into account and Intel lowers price. This allows for faster build up of the installed base. As a consequence, both total profit and consumer surplus increase.

## 3.2 Stage 1: The Investment Game

We now solve the first stage. We assume that Intel and Microsoft choose investments simultaneously to maximize overall profit. The Nash equilibrium investments $R_I^*$ and $R_M^*$ are found by solving

$$\left.\begin{array}{l} \max_{R_I} \Pi_I - R_I \\ \max_{R_M} \Pi_M - R_M \end{array}\right\}.$$

The following proposition shows that the asymmetry in objective functions does not directly imply the presence of conflict between Intel and Microsoft regarding investment in R&D.

**Proposition 4** $R_I^* = R_M^*$.

The proposition is stated in full generality: the result holds true independently of all parameter values, including $y_0$. We conclude that in the basic model there is *no* conflict regarding investments to advance the PC platform: Intel and Microsoft both invest the same amount. The asymmetry in the objective functions does not imply that Intel and Microsoft incentives to innovate differ.

Obviously, given the symmetry of $\alpha$ and $\beta$ on $R_I$ and $R_M$, if Intel and Microsoft cooperated in choosing investments at time zero, investments would still be equal. Thus, the result is independent of the degree of cooperation between Intel and Microsoft when choosing how much to invest.

Proposition 4 is somewhat disappointing. The ultimate objective of the modeling exercise is to better understand the real sources of conflict between Intel and Microsoft, as we described in the introduction. However, Proposition 2 suggests that the success of one *is* the success of the other and Proposition 4 shows that in the benchmark model there is no natural conflict regarding investment. Proposition 1, though, shows that there are conflict over price. Intel sets prices too high from the point of view of Microsoft. A profit maximizing Intel should have less interest in the installed base; in addition, it will take advantage of Microsoft's initially low prices, which are designed to build the installed base.

## 3.3 Comparison

We close the discussion of the benchmark model with a comparison of the profit levels that Intel and Microsoft would attain by cooperating instead of competing. We distinguish between four different cases, the combinations of Intel and Microsoft competing or cooperating in setting prices and in choosing investment levels. The following table sets up the notation:

| | | Price | |
|---|---|---|---|
| | | Competition | Cooperation |
| Investment | Competition | $\hat{\Pi}_i^{1,1}$ | $\hat{\Pi}_i^{1,0}$ |
| | Cooperation | $\hat{\Pi}_i^{0,1}$ | $\hat{\Pi}_i^{0,0}$ |

Of course, we obtain different investment levels depending on whether Intel and MS compete in setting prices in the second stage and whether they compete in choosing investments at time zero.

**Proposition 5** $\hat{\Pi}_i^{0,0} > \hat{\Pi}_i^{0,1} > \hat{\Pi}_i^{1,0} > \hat{\Pi}_i^{1,1}$.

The Proposition shows that the only conflict that Intel and Microsoft face regarding investment in the benchmark model (one generation) is the usual moral hazard in teams identified by Holmstrom [9] in his seminal 1982 paper on partnership games.

We conclude that Intel and Microsoft will want to cooperate in setting investment levels and prices and that such cooperation is welfare enhancing. Today, it would be illegal for Intel and Microsoft to cooperate on price. The model suggests, however, that antitrust authorities would enhance total welfare by allowing Intel and Microsoft to cooperate fully on R&D *and* pricing.

The ranking in Proposition 5 suggests that if Intel and Microsoft could cooperate in one dimension only, they would choose to do it in R&D (because $\hat{\Pi}_i^{0,1} > \hat{\Pi}_i^{1,0}$). However, Proposition 8 provides a rationale, in a setting with many generations, why such cooperation will be less than perfect.

# 4 Two Generations

We have shown that if there is one generation only, both Intel and Microsoft have the exact same willingness to invest in complementary R&D at time $t = 0$. We now investigate whether this "equal willingness" persists throughout the life of the first generation as Intel and Microsoft consider the profits that a second generation will eventually bring. In particular, we drop our assumption that the length of each generation is exogenous and fixed at $T$ and ask: who will want to release a new generation product first, Intel or Microsoft?

We change focus from studying the optimal size of the investments at a given time ($t = 0$) to studying the moment in time in which it is most desirable to invest and release a new microprocessor and/or a new operating system. To simplify the analysis, we fix the size of investments to some fixed value $R > 0$.

In what follows we assume that the effect of R&D investments (and, thus, knowledge) is cumulative. For example, suppose that Intel and Microsoft have each invested $R$ at time zero. The first generation demand parameters are then $\alpha(R,R)$ and $\beta(R,R)$. If Intel invests a second time but Microsoft does not, the new demand parameters (for the new generation) are $\alpha(2R,R)$ and $\beta(2R,R)$. For notational simplicity, we let $\alpha^{1,1} \equiv \alpha(R,R)$, $\alpha^{1,2} \equiv \alpha(R,2R)$, $\alpha^{2,1} \equiv \alpha(2R,R)$ and $\alpha^{2,2} \equiv \alpha(2R,2R)$. Similarly for $\beta$. Because of the symmetry of $\alpha$ and $\beta$ in $R_I$ and $R_M$, we have that $\alpha^{1,2} = \alpha^{2,1}$ and $\beta^{1,2} = \beta^{2,1}$.

## 4.1 Unilateral Incentives to Invest

We begin by studying Intel and Microsoft's *unilateral* willingness to invest. We assume that at time zero, both Intel and Microsoft invest $R$. These investments generate demand parameters $\alpha^{1,1}$ and $\beta^{1,1}$ for the first generation. Also at time zero we let *either* Intel *or* Microsoft decide how long will the a generation last. Let $t_I^*$ ($t_M^*$) be Intel's (Microsoft's) generation length choice. At the end of the first generation (at $t_I^*$ or $t_M^*$, depending on who chose) the firm that picked the length of the generation invests $R$. As a consequence, the demand parameters for the second generation are $\alpha^{2,1}$ and $\beta^{2,1}$ if Intel chooses the generation length and $\alpha^{1,2}$ and $\beta^{1,2}$ if Microsoft chooses.[27] Just as in the benchmark model, during the

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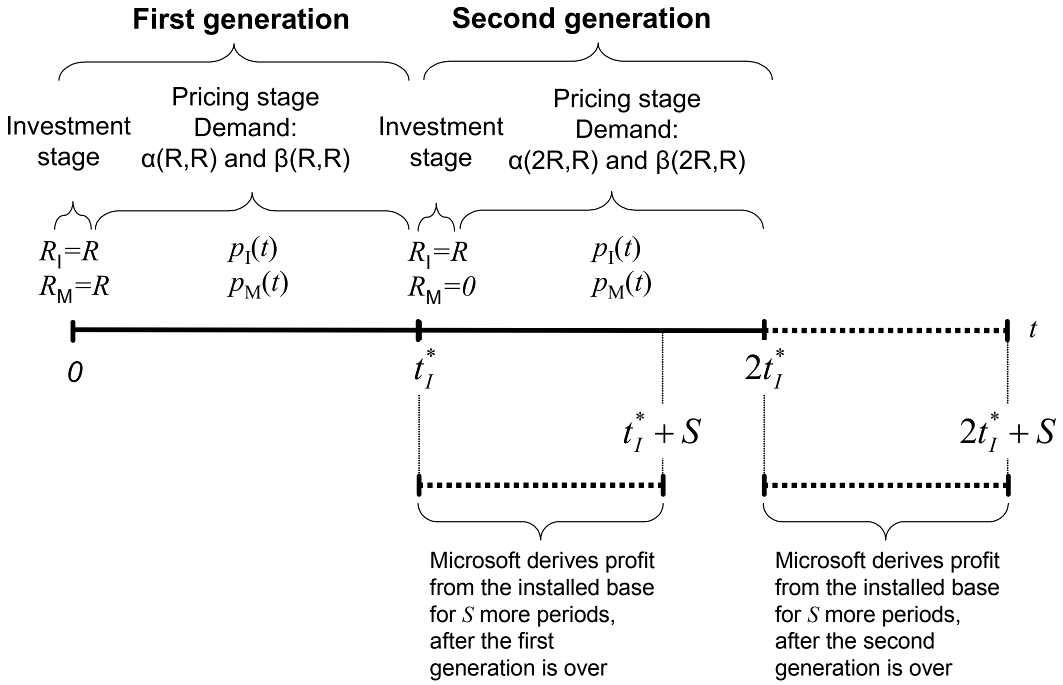
[27] As we pointed out above, the symmetry of $\alpha$ and $\beta$ implies that $\alpha^{2,1} = \alpha^{1,2}$ and $\beta^{2,1} = \beta^{1,2}$.

first and second generations Intel and Microsoft choose prices simultaneously to maximize the net present value of profit.

We perform two separate analyses, one with Intel choosing the length of the generation and another one with Microsoft choosing. The following figure illustrates the game when Intel is the firm choosing the length of the first generation $t_I^*$. Recall that $S$ can be any number from 0 to $\infty$. Of course the case where Microsoft chooses the length of the first generation, $t_M^*$, has an analogous figure.



Let

$$H_I(t_i) \equiv \int_0^{t_i} e^{-rt}\pi_I^{1,1}(t)\,dt + e^{-rt_i}\left(\int_0^{t_i} e^{-rt}\pi_I^{2,1}(t)\,dt - R\right)$$

and

$$\begin{aligned} H_M(t_i) \quad \equiv \quad & \int_0^{t_i} e^{-rt}\pi_M^{1,1}(t)\,dt + e^{-rt_i}V(S)\,y_1(t_i) \\ & + e^{-rt_i}\left(\int_0^{t_i} e^{-rt}\pi_M^{1,2}(t)\,dt - R + e^{-rt_i}V(S)\,y_2(t_i)\right), \end{aligned}$$

where $y_1(t)$ and $y_2(t)$ are the installed bases at time $t$ of the first and second generation products, respectively.[28]

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[28] Recall that $V(S)$ is the value to Microsoft of the installed base at the end of each generation. Because Microsoft gets $a$ out of $y^1(t_M)$ and $y^2(t_M)$ and the installed base fades out at rate $\delta$, we have:

$$V(S) = \int_0^S e^{-(r+\delta)t}a\,dt = \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)S}\right).$$

Intel and Microsoft choose $t_I^*$ and $t_M^*$ to maximize $H_I(t_i)$ and $H_M(t_i)$, the total net present value of both generations. Formally,

$$t_I^* = \arg\max_{t_i} H_I(t_i) \tag{12}$$

and

$$t_M^* = \arg\max_{t_i} H_M(t_i) \tag{13}$$

Because the equilibrium price paths for the first and second generations depend on the choice of $t_i^*$ (see equations (8) and (9)), as Intel (Microsoft) computes the optimal time to switch to the new generation it takes into account the effect on the price paths of a change in $t_i^*$. Intel (Microsoft) also takes into account the dependence of the installed base on $t_i^*$ (at the end of the first and second generations). The installed base at the end of the first and second generations affect Microsoft's profits directly and Intel's indirectly.

The following proposition shows that both Intel and Microsoft will want to invest (unilaterally) on a second generation product at exactly the same time.

**Proposition 6** $t_I^* = t_M^*$.

The result says that even if Intel makes most of its profit early in the life of a generation and Microsoft builds the installed base and makes most of its profit late, both complementors will want to (unilaterally) invest on a new generation at the same time.

Proposition 2 shows that regardless of the length of the first generation, Intel and Microsoft's profits are exactly the same for the first generation (provided that $y_0 = 0$, which we assume). In addition, the symmetry of $\alpha$ and $\beta$ together with Proposition 2 imply that Intel's profit from the second generation if Intel invests $R$ at time $t_I^*$ is exactly equal to Microsoft's profit from the second generation if Microsoft invests $R$ at time $t_M^*$. Thus, we have that Intel and Microsoft's total profit from the first and second generation are exactly equal. As a consequence the $t_I^*$ that maximizes Intel's total net present value of profit coincides with the $t_M^*$ that maximizes Microsoft's total net present value of profit.

## 4.2 The Investment Game

We have just showed that if asked at time zero when to invest and switch to a second generation, both Intel and Microsoft will agree in that the optimal time is $t^* \equiv t_I^* = t_M^*$. However, that both Intel and Microsoft want to unilaterally invest and release a second generation product at the same time does not necessarily imply that there is no conflict regarding investment. In fact, we now show that, unless the investments are very complementary, Intel and Microsoft will want to free-ride on each other's efforts.

Toward this end we analyze the normal form game that Intel and Microsoft play at time $t^*$:

| | | Microsoft | |
|---|---|---|---|
| | | Invest | Don't |
| Intel | Invest | A-R, A+V(S)$y_1$(t*)-R | B-R, B+V(S)$y_1$(t*) |
| | Don't | B, B+V(S)$y_1$(t*)-R | 0, V(S)$y_1$(t*) |

where

$$\begin{aligned} A &\equiv \int_0^{t^*} e^{-rt}\pi_I^{2,2}(t)\,dt = \int_0^{t^*} e^{-rt}\pi_M^{2,2}(t)\,dt + e^{-rt^*}V(S)\,y_2(t^*), \\ B &\equiv \int_0^{t^*} e^{-rt}\pi_I^{1,2}(t)\,dt = \int_0^{t^*} e^{-rt}\pi_M^{2,1}(t)\,dt + e^{-rt^*}V(S)\,y_2(t^*) \end{aligned}$$

and $y_1(t^*)$ is the installed base at the end of the first generation (time $t^*$).

To keep the analysis as simple as possible we begin by assuming that once time $t^*$as been reached if neither Intel nor Microsoft invest at that time, there will be no investment thereafter.[29]

If the investments are very complementary so that $A - R > B$, then the dominant strategy for both firms is to invest at $t^*$. In this case, there is no conflict. However, when the complementarity is not huge, such that $A - R < B$, there are two asymmetric pure strategy equilibria and one mixed-strategy equilibrium:

- <I, D>: Intel invests in a new microprocessor but Microsoft does not. Microsoft takes advantage of Intel's investment as its profit per period moves "for free" from $\pi_M^{1,1}$ to $\pi_M^{1,2}$.
- <D, I>: Intel does not invest in a new microprocessor and Microsoft invests in a new OS. Intel is now who takes advantage of Microsoft's investment.
- <$[\alpha]$ I + $[1-\alpha]$ D, $[\beta]$ I + $[1-\beta]$ D>, where $\alpha = \beta = \frac{B-R}{2B-A}$. In the mixed-strategy equilibrium, both Intel and Microsoft invest with the same probability.

We see that there is conflict regarding investment. In the two pure-strategy equilibria, only one firm invests. The firm that invests would be better off if the other firm invested as well. In the mixed strategy equilibrium both firms invest simultaneously with probability $\left(\frac{B-R}{2B-A}\right)^2$ only.

The conflict, however, is symmetric. Microsoft would like to make sure that the equilibrium played is <I, D> and Intel prefers the other pure strategy equilibrium. Unfortunately none of the standard theories for equilibrium selection apply here. For example, none of the two pure-strategy equilibria risk dominates the other. As a consequence we cannot apply Carlsson and van Damme's [3] "global game" approach to equilibrium selection. The case evidence in Section 2, however, suggests that the equilibrium played in the actual interactions of Intel and Microsoft is <I, D>.

The investment game has three equilibria because we have solved it as a simultaneous moves game. If, instead, the moves were sequential, the first mover could effectively pick the outcome: <I, D> or <D, I>. Thus, if Microsoft (Intel) could commit *not* to invest, it would then ensure that <I, D> (<D, I>) is the outcome of the game. Therefore, both Intel and Microsoft will have strong incentives to become committed first movers.

Microsoft, with its "train station" approach to the timing of new generations (see Section 2), is effectively committing not to invest too often so as to ensure that <D, I> is the equilibrium of the game. The question that we need to address now is: Why is Microsoft's commitment credible? Why couldn't Intel institute a similar policy and become the first mover? In the following subsection we argue that Microsoft (and not Intel) is the natural candidate to commit to such a policy.

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[29] In subsection 4.3, however, we analyze the incentives to invest and switch to the second generation once time $t^*$ has been reached.

## 4.3 Incentives to Invest at Time $s > 0$

In this subsection we allow Intel and Microsoft to "change their minds" regarding the optimal time to invest and we address the question: Will the equality $t_I^* = t_M^*$ persist throughout the life of the first generation? Or will Intel and/or Microsoft have an incentive to revise these choices as the first generation progresses? Further, if Intel and Microsoft have an incentive to revise their choices, will the equality $t_I^* = t_M^*$ persist?

To address these questions we analyze a natural generalization of the model above. Let

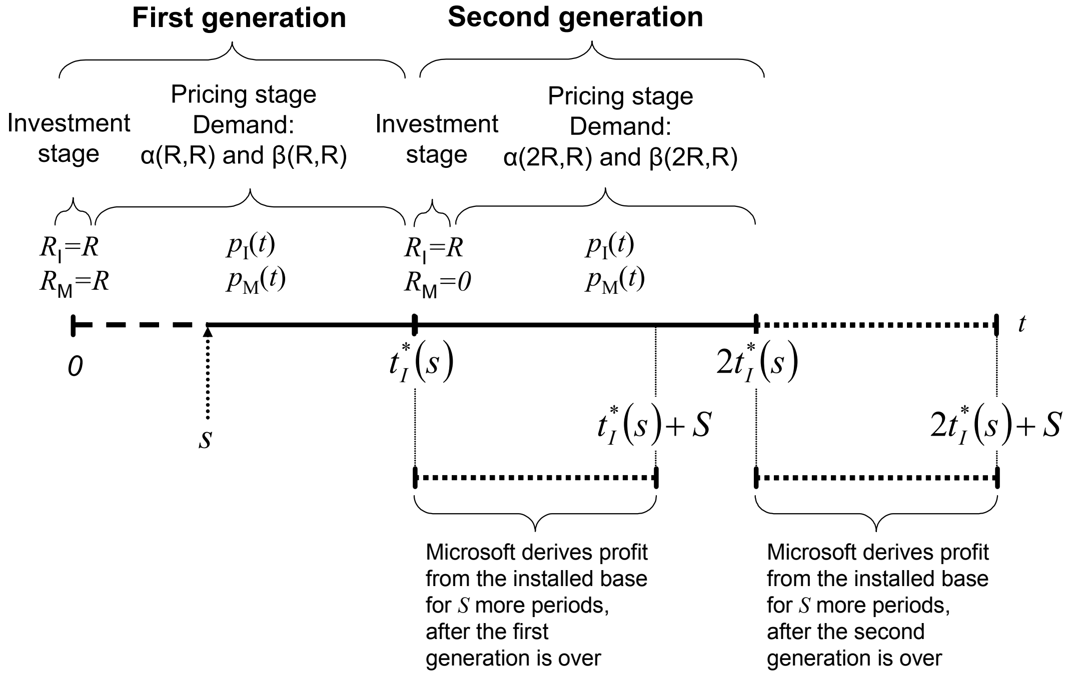
$$H_I(s, t_i) \equiv \int_s^{t_i} e^{-r(t-s)} \pi_I^{1,1}(t)\, dt + e^{-r(t_i - s)} \left( \int_0^{t_i} e^{-rt} \pi_I^{2,1}(t)\, dt - R \right)$$

and

$$\begin{aligned} H_M(s, t_i) \equiv & \int_s^{t_i} e^{-r(t-s)} \pi_M^{1,1}(t)\, dt + e^{-r(t_i - s)} V(S)\, y_1(t_i) \\ & + e^{-r(t_i - s)} \left( \int_0^{t_i} e^{-rt} \pi_M^{1,2}(t)\, dt - R + e^{-rt_i} V(S)\, y_2(t_i) \right). \end{aligned}$$

These are the total net present values (both generations) to Intel and Microsoft, respectively, looked at from moment $s > 0$. (Of course, equations $H_I(t_i)$ and $H_M(t_i)$ in the previous subsection coincide with $H_I(s, t_i)$ and $H_M(s, t_i)$ with $s = 0$, respectively.)

Again, we perform two separate analyses, one with Intel choosing the length of the generation and another one with Microsoft choosing. The following figure illustrates the game when Intel is the firm choosing the length of the first generation $t_I^*(s)$. Of course the case where Microsoft chooses the length of the first generation, $t_M^*(s)$, has an analogous figure.



Intel and Microsoft choose $t_I^*(s)$ and $t_M^*(s)$ to maximize $H_I(s,t_i)$ and $H_M(s,t_i)$, the total net present value of both generations. Formally,

$$t_I^*(s) = \arg\max_{t_i} H_I(s,t_i) \tag{14}$$

and

$$t_M^*(s) = \arg\max_{t_i} H_M(s,t_i) \tag{15}$$

We now allow $t_I^*$ and $t_M^*$ depend on the instant $s$ at which the evaluations are being made. Just as before, the equilibrium price paths for the first and second generations depend on the choice of $t_i^*(s)$.

The following proposition shows that $t_I^*(s) < t_M^*(s)$ for all $s > 0$ (unless $S = \infty$, in which case $t_I^*(0) = t_I^*(s) = t_M^*(s) = t_M^*(0)$).

**Proposition 7** *Suppose that $S < \infty$. Then,*

$$t_I^*(s) < t_M^*(s)$$

*for all $s > 0$. Further, $t_I^*(0) = t_M^*(0) < t_I^*(s)$ for all $s > 0$. Suppose that $S = \infty$. Then,*

$$t_I^*(s) = t_M^*(s)$$

*for all $s \geq 0$.*

The intuition for the result is simple. Contrary to the case in which $s = 0$, at $s > 0$ *there is* a positive installed base of Wintel machines. Microsoft sells applications and derives instantaneous profit $a$ from the installed base. At the moment of the switch to the second generation there are still $S$ more periods for Microsoft to profit from first-generation installed base. However, if $S < \infty$, the moment of the switch to the second generation is "the beginning of the end" of the profit-flow that Microsoft derives from the installed base. As a consequence, Microsoft has an incentive to delay this "beginning of the end." Since Intel does not derive profit (directly) from the installed base, it is willing to move to the second generation earlier. Clearly, if $S = \infty$, there is no end to the profit flow from the first generation installed base and, in this case, $t_I^*(s) = t_M^*(s)$ for all $s$. Also, we have that $t_I^*(0) = t_M^*(0)$ even if $S < \infty$ because at time $s = 0$ there isn't still an installed base that affects differentially Microsoft and Intel's incentives to switch to the new generation.

### 4.3.1 The Investment Game Revisited

Proposition 6 says that if asked at time $t = 0$, both Intel and Microsoft will agree that instant $t^* \equiv t_I^*(0) = t_M^*(0)$ is the optimal time to switch to the second generation. Proposition 7 implies that once $t^*$ is reached, Microsoft will want to delay the switch to the second generation further than Intel.

Microsoft is then the natural player to implement the "big trains" policy because as long as there is some installed base Microsoft prefers to switch to the new generation later than Intel. In other words, waiting to invest in a new generation microprocessor is less costly to Microsoft than to Intel.

## 4.4 The Investment Game Played Repeatedly

We have just shown that there are some structural features of the Intel/Microsoft relationship that tilt the balance towards <Invest, Don't> as the equilibrium of the investment game. However, the case evidence also reveals that from time to time, Microsoft invests in new generation operating systems. We can now extend the model to a sequence of generations and show that Microsoft also invests regularly in complementary R&D (although less frequently than Intel).

We divide the time line into equally spaced discrete portions of length $T$ and let Intel and Microsoft decide whether they want to invest a fixed quantity $R$ at the beginning of each one of these chunks.

For notational simplicity, for $k$ and $l$ positive integers, we let $\alpha^{k,l} \equiv \alpha(kR, lR)$ and $\beta^{k,l} \equiv \beta(kR, lR)$. For example, $\alpha^{3,1}$ and $\beta^{3,1}$ are the demand parameters if Intel has invested three times and Microsoft has invested once only. In other words, if firm $i$ has invested $k$ times, firm $i$'s total investment is $R_i = kR$. Of course, the symmetry of $\alpha$ and $\beta$ implies that $\alpha^{k,l} = \alpha^{l,k}$ and $\beta^{k,l} = \beta^{l,k}$.

To keep the model tractable, we assume that both Intel and MS are myopic ($r = \infty$); that is, they care about the short term only. Consistent with the rationale for equilibrium selection presented above, we will proceed by characterizing the equilibrium that is most favorable to Microsoft.

The stage game to be played after Intel has invested $k$ times and Microsoft $l$ times has the following form:

| | | Microsoft | | | |
|---|---|---|---|---|---|
| | | Invest | | Don't | |
| Intel | Invest | $\pi_I^{k+1,l+1} - R$ | $\pi_I^{k+1,l+1} - R$ | $\pi_I^{k+1,l} - R$ | $\pi_I^{k+1,l}$ |
| | Don't | $\pi_I^{k,l+1}$ | $\pi_I^{k,l+1} - R$ | 0 | 0 |

The term $V(S)\,y(T)$ is absent from the formulae above because $\lim_{r\to\infty} V(S) = 0$. Of course, this further simplifies the analysis. The strategies we propose as an equilibrium for the game are:

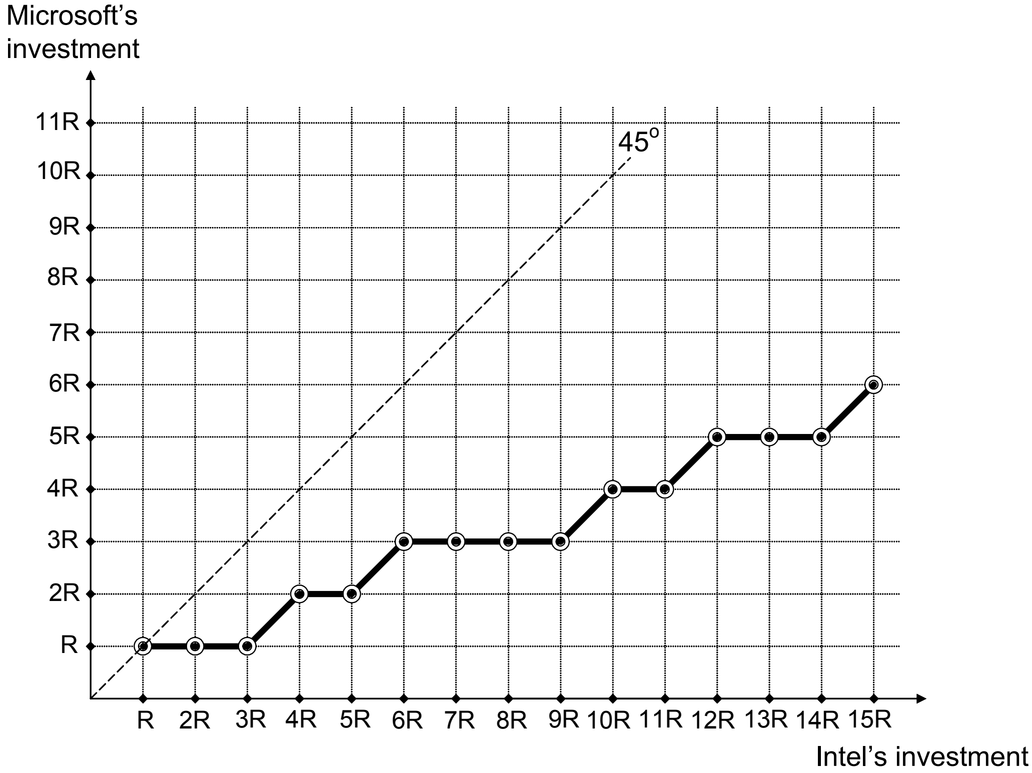
- Microsoft invests only in those periods in which it is a dominant strategy (of the stage game) for it to invest. As Intel keeps investing, the marginal effect on $\alpha$ and $\beta$ of Microsoft's investment keeps increasing because of supermodularity. Microsoft ends up investing after Intel has gone through some investment cycles.

- Intel invests in all those periods in which it is a dominant strategy (of the stage game) for it to invest. In addition, Intel also invests every period in which it is *not* a dominant strategy for Microsoft to invest.

**Proposition 8** *The proposed strategies constitute a Nash equilibrium.*

The symmetry of $\alpha$ and $\beta$ guarantees that with these strategies Intel will invest more often than Microsoft. Notice that Microsoft could *not* do better by coordinating its investment with Intel. This helps explain why there is not more cooperation between the two companies to invest in improving the platform. Intel, on the other hand, would very much like to see Microsoft invest more often.

The frequency of Microsoft's investment in equilibrium depends on the level of complementarity of the investments. At one extreme we have that $\alpha$ and $\beta$ could be highly supermodular. In this case, the investment path is exactly on the 45° line and there is no conflict between Intel and Microsoft. At the other extreme, when $\alpha$ and $\beta$ display no complementarities, only Intel invests and the path is a flat line. Finally, when complementarity is "average" we have a pattern as the one shown in the figure below (the exact shape, of course, depends on the specific functional forms of $\alpha$ and $\beta$):



## 4.5 Summary

We conclude that there is a two-sided problem in the Intel-Microsoft relationship. Intel, on the one hand, sets price too high, taking advantage of Microsoft's willingness to build the installed base. Intel's high price is undesirable to Microsoft because it slows down the expansion of the installed base. To compensate, Microsoft needs to lower its price further, which has a negative impact on its profit.

Microsoft, on the other hand, prefers to move to a new generation OS later than Intel's ideal time to move to a new generation microprocessor. Intel ends up investing more often than Microsoft. Intel dislikes the slow pace of investment by Microsoft because if Microsoft invested at Intel's pace, Intel would end up making substantially more profit.

In the following section we analyze two mechanisms used by which Microsoft and Intel attempt to neutralize the investment and pricing problems described above.

# 5 Extensions

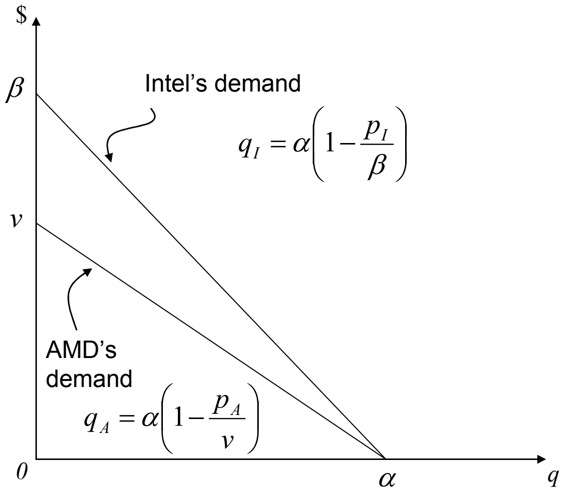
We present two extensions to the baseline model. In the first extension, Microsoft induces Intel to set lower prices for its microprocessors by backing Advanced Micro Devices (AMD), Intel's main competitor. We show that as competition between AMD and Intel intensifies, Intel sets lower prices and this helps Microsoft build the installed base and increase its profits. In the second extension, Intel induces Microsoft to invest in complementary R&D by encouraging the development of Linux, an operating system that also runs on Intel's architecture.

## 5.1 AMD

In this subsection we study the effect of competition in the microprocessor space on the equilibrium price paths and profits of both Intel and Microsoft. AMD was founded in 1971 with the primary mission of being a second source to Intel's innovations. At the time, most customers required innovators to license their products to second sources as a condition for winning designs. By 1976, AMD and Intel signed a broad cross-license: the deal required Intel license its newest technology in exchange for AMD delivering technology of "comparable value." In 1985, however, Intel believed that AMD could not offer comparable value, and it abrigated the cross-license. After a decade of litigation, Intel and AMD settled, with AMD gaining rights to Intel's 1995 microcode in exchange for monetary compensation.

Throughout the history of their competition, AMD and Intel have gone through cycles. Intel was usually the performance leader, but periodically AMD would introduce a new generation of products where it would gain a short-term advantage over Intel. For example, it took AMD almost five years to match Intel's 386, but when it did, it offered a product with a higher clock rate and lower power. Intel responded by aggressively expanding the 486 generation, and it took AMD almost two years to deliver a comparable microprocessor. In a more recent cycle, AMD started gaining ground on Intel in mid-1999. Independent testers showed that AMD's Athlon processors edged out comparable Intel's Pentium processors on certain benchmark tests. Moreover, in March 2000, in a move that took Intel by surprise, AMD introduced the 1 GHz Athlon microprocessor ahead of Intel. Intel immediately announced that it was ready to launch its own 1 GHz Pentium III processor priced 23% below the Athlon – the first time an Intel processor was priced below an AMD processor of the same clock-speed. One industry analysts observed: "Intel is not used to anyone being able to compete with them on performance. ... Intel has never had a competitor like this." Intel executives responded: "They are our most accomplished competitor; you've got to give them credit for that. ... The worst thing Intel can do is to take AMD lightly, and we don't." Intel had subsequently maintained a comfortable lead ahead of AMD with the introduction of its new Pentium IV generation microprocessors. While the competition over microprocessors began to shift away from speed alone in 2004, (moving to system performance and system architectures, such as 32 bits vs. 64 bits) Intel continued to offer products with higher clock rates than comparable AMD processors.

We start by assuming that Intel and AMD's products are vertically differentiated. That is, in steady state, customers generally view Intel's microprocessors as higher performance. Moreover, this vertical differentiation comes from Intel's history as a more reliable supplier than AMD, its deeper balance sheet which offers customers a greater sense of safety in buying from a secure supplier, and Intel's ability to supply complementary technologies, such as chips sets, which are necessary to deliver a complete system. Yet if the price differential is large, some customers will buy AMD. Because AMD is perceived as an inferior supplier to Intel, AMD needs to set prices below Intel's in order to generate demand for its microprocessors. Let $p_A(t)$ be AMD's price for its microprocessor at time $t$. Let $v$ be the vertical axis intercept for AMD's demand function. That the products are vertically differentiated means that $\beta > v$. The demand functions look as follows:



For given prices of Intel, AMD, and Microsoft, let $\hat{q}$ be the indifferent customer. That is,

$$\beta\left(1 - \frac{\hat{q}}{\alpha}\right) - p_M - p_I = \nu\left(1 - \frac{\hat{q}}{\alpha}\right) - p_A - p_M$$

or

$$\hat{q} = \alpha\left(1 - \frac{p_I - p_A}{\beta - \nu}\right).$$

Time $t$ profits to Intel, AMD, and Microsoft are, respectively:[30]

$$\pi_I = \alpha\left(1 - \frac{p_I - p_A}{\beta - \nu}\right) p_I,$$

$$\pi_A = \alpha\left(\left(1 - \frac{p_M + p_A}{\nu}\right) - \left(1 - \frac{p_I - p_A}{\beta - \nu}\right)\right) p_A,$$

and

$$\pi_M = \alpha\left(1 - \frac{p_M + p_A}{\nu}\right) p_M + ay.$$

With $p_A < p_I$, the total quantity of PCs sold is $\alpha\left(1 - \frac{p_M + p_A}{\nu}\right)$ and the quantity of microprocessors sold by AMD is

$$q_A = \underbrace{\alpha\left(1 - \frac{p_M + p_A}{\nu}\right)}_{a} - \underbrace{\alpha\left(1 - \frac{p_I - p_A}{\beta - \nu}\right)}_{b}$$

---

[30] To simplify the analysis, we assume zero marginal cost for both Intel and AMD.

where $a$ is the amount microprocessors that AMD would sell if Intel were not in the market, and $b$ are all those customers who prefer Intel to AMD.

The system dynamics are just as in the benchmark model:

$$\dot{y} = q - \delta y.$$

We assume that the parameters are such that all equilibrium prices are positive.[31]

The following proposition shows that competition in the microprocessor space is not necessarily beneficial to Microsoft.

**Proposition 9** *If AMD and Intel are close substitutes, the presence of AMD is beneficial to Microsoft.*

If the extent of vertical differentiation is large ($\nu$ far from $\beta$), Microsoft may be better off without AMD. Microsoft likes the presence of AMD because AMD and Intel compete and, all other things equal, when AMD is present, Intel needs to lower prices to avoid losing market share. The problem is that when the willingness to pay for AMD's microprocessors is low, if Microsoft sets a price larger than $\nu$ then AMD gets no demand and this neutralizes the effect of AMD's presence on Intel's pricing decisions. Microsoft, however, can always guarantee that AMD is a viable competitor (even if $\nu$ is very low) by setting the OS's price low enough.

Therefore, we see a trade off: on the one hand, Microsoft want to see AMD compete against Intel because it forces Intel to lower its prices. However, when $\nu$ is low, Microsoft has to lower its price to make sure that AMD is viable, which is costly. In fact, when $\nu$ is very low, Microsoft is better off not supporting AMD and competing against Intel only.

If the willingness to pay for AMD processors is very low, Microsoft will look for alternatives to lowering Windows' price to ensure that AMD stays alive in its competition against Intel. For example, Microsoft may choose to make periodic lump sum payments to AMD to subsidize its R&D expense to help reduce the vertical differentiation between Intel and AMD's products.

In fact, if the only instrument available to Microsoft to guarantee the viability of AMD is Windows' price, there is one paradoxical implication:

**Remark 10** *If Microsoft is committed to support AMD by means of low OS prices and the willingness to pay for AMD's products is sufficiently low, Intel is better off with competition from AMD than without it.*

Intuitively, if Microsoft is committed to support AMD and $\nu$ is low, the effect of Microsoft's lower prices to ensure the viability of AMD spills over to Intel who is now able to charge larger prices and earn a larger profit.

In summary, Microsoft benefits from the presence of AMD because, everything else equal, having an alternative microprocessor forces Intel to lower prices. We showed that as long as

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[31] It is easy to see that as long as

$$\nu \geq \frac{4\beta a\left(1 - e^{-(r+\delta)T}\right)}{3\beta\left(r+\delta\right) - a\left(1 - e^{-(r+\delta)T}\right)}$$

in equilibrium all prices are positive.

AMD and Intel are perceived as close substitutes ($\nu$ close to $\beta$), microprocessor competition is unambiguously beneficial to Microsoft. However, when the two microprocessors are highly vertically differentiated, Microsoft needs to take action if it wants to ensure that AMD remains a viable competitor. If the willingness to pay for AMD's products is very low, the cost to Microsoft of keeping AMD alive by means of low OS prices is larger than the benefits it obtains from the increased competition in the microprocessor space. In this case, Microsoft will be better off by just making lump sum payments to AMD than by lowering OS prices.

Notice finally that competition in the microprocessor space induces Intel to invest to ensure that its microprocessors stay ahead of AMD's. When AMD is present, the investments of Intel and Microsoft do not have symmetric effects on Intel's profitability. While Intel's investment benefits Intel and Microsoft, an investment by Microsoft benefits Microsoft, Intel *and* AMD. In other words, an investment by Microsoft does not help Intel differentiate its microprocessors from those of AMD to the same extent as a comparable investment by Intel.

## 5.2 Linux

In the same way AMD constrains Intel's pricing, Linux provides a similar constraint on Microsoft's pricing, especially in the higher end, server segment of the market. Linux, an operating system based on the UNIX OS originally developed at Bell Laboratories, is an open source software program. Unlike Microsoft which hires its own programmers and salespeople to build and sell software products, Linux encourages software developers all over the world to contribute code for free, which is then incorporated into the operating system. A key feature of Linux is that it is available to customers for free: anyone can download a copy of Linux at no cost from the Internet. Any company can also distribute Linux: while they cannot charge for the OS, companies such as Red Hat and SuSe Linux sell service and complementary software as part of a bundle. In addition, because Linux is built on the foundation of UNIX and not burdened by Microsoft's legacy with DOS and early versions of Windows, Linux is generally viewed in the technical community has more reliable, less prone to security flaws, and better performing that competing versions of Windows.

In this subsection we study the effect of the presence of Linux on Microsoft's incentives to invest. To simplify the analysis we center our attention on a variation of the investment game of Section 4.2. Intel and Microsoft decide simultaneously whether or not to invest in a new product generation that will last a time segment of length $t^*$.

Let $a$ be Intel's profit from sales of Lintel [Linux OS and Intel processor] PCs when both Intel and Microsoft have invested in a new generation.[32] Let $b$ be Intel's profit from Lintel machines when only Intel or Microsoft (but not both) has invested on a new generation. Finally, let $c$ be Intel's profit from sales of Lintel machines when there has not been investment on a new Wintel generation. To capture the idea that the Lintel standard is more desirable the lower is the performance of Wintel PCs we assume that:

$$a \leq b \leq c.$$

The normal form representation of the game is:

---

[32] A Lintel PC is one with an Intel microprocessor and a Linux operating system.

| | | Microsoft | |
|---|---|---|---|
| | | Invest | Don't |
| Intel | Invest | $A+a-R$, $A+V(S)y_1(t^*)-R$ | $B+b-R$, $B+V(S)y_1(t^*)$ |
| | Don't | $B+b$, $B+V(S)y_1(t^*)-R$ | $c$, $V(S)y_1(t^*)$ |

Recall that $A$ represents Intel and Microsoft's profits from the sale of Wintel machines when both Intel and Microsoft have invested in a new generation microprocessor and $B$ represents Intel and Microsoft's profits from the sale of Wintel machines when only Intel or Microsoft (but not both) has invested in a new generation.

The presence of Linux has a direct effect on Intel's payoffs as captured by the addition of $a$, $b$, or $c$ to Intel's payoffs. In addition, the mere existence of a substitute to Windows will affect the value of $A$ and $B$. Of course, even with the presence of Linux, we have that $A > B$.

The equilibria of this game depend on the relative values of the parameters.

**Proposition 11** *There is a range of values of $B$ such that the unique equilibrium is <Don't, Invest>.*

The effect of operating system competition on Microsoft's payoffs is captured by the value of $B$ (Microsoft's NPV of profit when only Microsoft or Linux invest). The effect of the lower $B$ (tougher OS competition) is tantamount to an increase in the complementarity of microprocessor and operating system. In other words, *the effect of an increase in competition is formally equivalent to an increase of complementarity.* It is this competition-led increase in complementarity that induces Microsoft to invest when Linux is present. However, the relative effect of $B$ on Intel and Microsoft's payoffs is asymmetric because Intel (but not Microsoft) benefits from the sale of Lintel PCs (parameter $b$).

When competition in the OS space is tough ($B$ close to zero), the game has three equilibria: either both companies invest, or none invests, or they invest with probability strictly between 0 and 1. In this mixed-strategy equilibrium the probability of Microsoft investing is larger than that of Intel. Finally, when competition is very mild ($B$ close to $A$), the presence of Linux does not modify the equilibria of the game in Section 4.2. In this mixed-strategy equilibrium Intel invests with larger probability than Microsoft.

We conclude that for most parameter values the presence of Linux breaks the "bad" equilibrium (from Intel's perspective) where Intel invests and Microsoft does not. It should then not be surprising that Intel supports the development of the Lintel platform in a variety of ways. Not only did Intel provide, along with IBM and Hewlett Packard, the funding for the Open Source Development Lab that coordinates Linux development, it also supports independent software developers who wish to develop applications for Linux and computer OEMs who want to incorporate Linux into their products. For any OS to work effectively on an Intel microprocessor, it also requires software drivers and optimizing the software to take advantage of specific features in the hardware. Intel employs large numbers of software engineers that do this work (for Linux as well as Microsoft) and usually give the software away.

# 6 Managerial Implications

We conclude with a summary of the main results and managerial implications of the analysis.

1. Even though complements need each other for value creation, they also compete to appropriate value. In the case of Wintel, the more Intel charges for the microprocessor, the more Microsoft is pressed to lower its price (and *vice versa*). This is true of all systems of complements: after investments have been made and the customer has some fixed valuation for the system, the more a complementor charges for its products, the less the scope for others to do so.

2. The symmetry of the demand with respect to the prices of complements implies some profit equalization between complementors, even if the objective functions are different: every additional dollar of profit that any one complementor can make by adjusting its price up or down, is also available to all other complementors. In the case of Wintel, we see that although Microsoft alone derives profits from the installed base, there is equalization of profits. This result is robust. In fact, even as the potential revenue that Microsoft derives from the installed base varies, the equality of profits persists. (Proposition 2.)

3. A direct managerial implication of the previous two points is that Intel is better off the more competitive Microsoft is in selling applications to the installed base. In terms of the model, Intel would like parameter $a$ to be as large as possible. Notice, though, that $a$ enters Microsoft's – but not Intel's – profit function. Parameter $a$ is large, for instance, if Microsoft has large market power in applications. When $a$ is large, Microsoft is willing to lower prices further because the installed base is more valuable. As a consequence, Intel can raise its prices and increase its profit.

4. We identified three instances of free riding in the relationship between Intel and Microsoft.

    (a) Free-riding by both Intel and Microsoft in the size of investment. (Proposition 5.)
    (b) Free-riding by Intel in pricing. (Proposition 1.)
    (c) Free-riding by Microsoft in the timing of investment. (Proposition 8.)

    If Intel and Microsoft could write enforceable contracts, free-riding would be neutralized.

5. The dynamics of competition at the value-capture stage have important implications on complementors' incentives to invest at the value-creation stage. In the case of Wintel, we see that the prospect of loosing the profit stream from the current installed base impels Microsoft to wait longer than Intel to switch to the next generation. (Proposition 7.)

6. Unless complementarity is very large (in which case it is a dominant strategy for complementors to invest), the investment-in-complementary-R&D game has multiple equilibria, none of which is Pareto superior. The game is one of coordination. Consequently, complementors will look for ways to ensure that their most-preferred equilibrium is selected. Commitment not to invest, competition in the microprocessor space,

and superior financial resources (milking the installed base) help Microsoft ensure that Intel invests often in new microprocessor generations. Microsoft effectively free-rides on Intel's efforts to push the platform forward. (Section 4.2.)

7. To appropriate as much value as possible from the system, complementors will encourage the development of competition in each others' spaces. But ensuring the presence of competition can also be costly. (Proposition 9.) As a consequence, the costs and benefits of encouraging competition need be weighted before committing to such a course of action. In the case of Wintel, we see that Microsoft promotes the development of Intel's imitators and substitutes and similarly, Intel supports Windows' substitutes, such as Linux.

# 7 Appendix

The proofs of all Propositions follow.

**Proof of Proposition 1.** See the derivation of equations (23) and (24) in the proof of Proposition 2. ■

**Proof of Proposition 2.** Let Intel and Microsoft maximize net present values of profit (eqns. 5 and 6) by choosing prices, given $\alpha$ and $\beta$.

The decision variables must satisfy the following Hamilton-Jacobi-Bellman equations:

$$rV_I - \frac{\partial V_I}{\partial t} = \max_{p_I} \left\{ \alpha \left( 1 - \frac{p_I + p_M}{\beta} \right) (p_I - w) + \frac{\partial V_I}{\partial y} \left( \alpha \left( 1 - \frac{p_I + p_M}{\beta} \right) - \delta y \right) \right\}, \quad (16)$$

$$rV_M - \frac{\partial V_M}{\partial t} = \max_{p_M} \left\{ \alpha \left( 1 - \frac{p_I + p_M}{\beta} \right) p_M + ay + \frac{\partial V_M}{\partial y} \left( \alpha \left( 1 - \frac{p_I + p_M}{\beta} \right) - \delta y \right) \right\}. \quad (17)$$

The prices that maximize the right hand of these expressions are:

$$p_I = \frac{1}{3} \left( \beta - 2\frac{\partial V_I}{\partial y} + \frac{\partial V_M}{\partial y} + 2w \right), \quad (18)$$

$$p_M = \frac{1}{3} \left( \beta + \frac{\partial V_I}{\partial y} - 2\frac{\partial V_M}{\partial y} - w \right). \quad (19)$$

We solve the system of two partial differential equations assuming the following linear value functions and showing that the value functions have indeed this form:[33]

$$V_I(y,t) = c_I(t)\, y + d_I(t), \quad (20)$$

$$V_M(y,t) = c_M(t)\, y + d_M(t). \quad (21)$$

which implies that $\frac{\partial V_I}{\partial y} = c_I(t)$ and $\frac{\partial V_M}{\partial y} = c_M(t)$.

Solving the system, we get:

$$c_I(t) = C_1 e^{(r+\delta)t},$$

---

[33] Because this is a linear state game, the value functions are linear. See Dockner, Jørgensen, Van Long, and Sorger [6, Chapter 7].

$$c_M(t) = \frac{a}{r+\delta} + C_2 e^{(r+\delta)t}$$

$$d_I(t) = (\int -\frac{\alpha}{9\beta} e^{-rt} (\beta + c_I(t) + c_M(t) - w)^2 dt + C_3) e^{rt},$$

$$d_M(t) = (\int -\frac{\alpha}{9\beta} e^{-rt} (\beta + c_I(t) + c_M(t) - w)^2 dt + C_4) e^{rt}.$$

Now, we solve for $C_1$ and $C_2$ by use of the boundary conditions (the values of $C_3$ and $C_4$ are unimportant at this point because as equations (18) and (19) reveal, the price paths depend on $c_I(t)$ and $c_M(t)$ only):

- Because Intel derives no profit from the installed base, we must have that

$$c_I(T) = C_1 e^{(r+\delta)T} = 0 \qquad \Rightarrow C_1 = 0.$$

- Because at time $T$ Microsoft derives $V(S)$ from the installed base, we have that

$$c_M(T) = \frac{a}{r+\delta} + C_2 e^{(r+\delta)T} = V(S) \qquad \Rightarrow C_2 = e^{-(r+\delta)T} \left( V(S) - \frac{a}{r+\delta} \right).$$

Thus,

$$\frac{\partial V_I(y,t)}{\partial y} = c_I(t) = 0$$

and

$$\frac{\partial V_M(y,t)}{\partial y} = c_M(t) = \frac{a}{r+\delta} + e^{-(r+\delta)(T-t)} \left( V(S) - \frac{a}{r+\delta} \right). \tag{22}$$

We obtain the explicit pricing strategies by substituting $c_I(t)$ and $c_M(t)$ into (18) and (19):

$$p_I^*(t) = \frac{1}{3} \left( \beta + 2w + \frac{a}{r+\delta} \left( 1 - e^{-(r+\delta)(T-t)} \right) + V(S) e^{-(r+\delta)(T-t)} \right), \tag{23}$$

$$p_M^*(t) = \frac{1}{3} \left( \beta - w - \frac{2a}{r+\delta} \left( 1 - e^{-(r+\delta)(T-t)} \right) - 2V(S) e^{-(r+\delta)(T-t)} \right). \tag{24}$$

We now show that despite the asymmetries in the objective functions and the asymmetry in the equilibrium price paths, the net present value of profit for both companies coincides. Let

$$\Theta_D(t) \equiv \int -\frac{\alpha}{9\beta} e^{-rt} (c_M(t) + c_I(t) + \beta - w)^2 dt.$$

Then, $d_I(t)$ and $d_M(t)$ can be simplified as follows:

$$d_I(t) = (\Theta_D(t) + C_3) e^{rt},$$

$$d_M(t) = (\Theta_D(t) + C_4) e^{rt}.$$

Using the boundary conditions ($V_I(T,y) = 0$ and $V_M(T,y) = V(S) y$) and the fact that $c_I(T) y = 0$ and $c_M(T) = V(S) y$, the value functions at $t = T$ can be expressed as:

$$(\Theta_D(T) + C_3) e^{rT} = 0,$$

$$\left(\Theta_D\left(T\right)+C_4\right)e^{rT}=0.$$

Therefore

$$C_3=-\Theta_D\left(T\right),$$

and

$$C_4=-\Theta_D\left(T\right).$$

As a consequence, we have that

$$d_I(t)=\left(\Theta_D\left(t\right)-\Theta_D\left(T\right)\right)e^{rt},$$

and

$$d_M\left(t\right)=\left(\Theta_D\left(t\right)-\Theta_D\left(T\right)\right)e^{rt}.$$

Finally, the value functions can be expressed as:

$$V_I\left(t,y\right)=c_I\left(t\right)y+d_I\left(t\right)=\left(\Theta_D\left(t\right)-\Theta_D\left(T\right)\right)e^{rt},$$

and

$$\begin{aligned} V_M\left(t,y\right) &= c_M\left(t\right)y+d_M\left(t\right) \\ &= \left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right)+V\left(S\right)e^{-(r+\delta)(T-t)}\right)y+\left(\Theta_D\left(t\right)-\Theta_D\left(T\right)\right)e^{rt}. \end{aligned}$$

At $t=0$:

$$\Pi_I\equiv V_I\left(0,y_0\right)=\Delta\Theta_D, \tag{25}$$

$$\Pi_M\equiv V_M\left(0,y_0\right)=\left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)T}\right)+V\left(S\right)e^{-(r+\delta)T}\right)y_0+\Delta\Theta_D. \tag{26}$$

where $\Delta\Theta_D\equiv\Theta_D\left(0\right)-\Theta_D\left(T\right)$. Therefore, $\Pi_I=\Pi_M$ as long as $y_0=0$. In other words, when $y_0=0$, the net present value of profit for both Intel and Microsoft is equal, regardless of the values of all other parameters. ■

**Proof of Proposition 3.**

The HJB is now:

$$rV-\frac{\partial V}{\partial t}=\max_p\left\{\alpha\left(1-\frac{p}{\beta}\right)(p-w)+ay+\frac{\partial V}{\partial y}\left(\alpha\left(1-\frac{p}{\beta}\right)-\delta y\right)\right\}. \tag{27}$$

The optimal price path is

$$p=\frac{1}{2}\left(\beta-\frac{\partial V}{\partial y}+w\right). \tag{28}$$

Substituting $p$ into equation (27) yields

$$rV-\frac{\partial V}{\partial t}=\frac{\alpha}{4}\left(1+\frac{1}{\beta}\frac{\partial V}{\partial y}-\frac{w}{\beta}\right)\left(\beta-\frac{\partial V}{\partial y}-w\right)+ay+\frac{\partial V}{\partial y}\left(\frac{\alpha}{2}\left(1+\frac{1}{\beta}\frac{\partial V}{\partial y}-\frac{w}{\beta}\right)-\delta y\right). \tag{29}$$

We solve this partial differential equation assuming that $V\left(y,t\right)$ is linear:

$$V\left(y,t\right)=c\left(t\right)y+d\left(t\right). \tag{30}$$

which implies that $\dfrac{\partial V}{\partial y} = c\,(t)$.

Solving the system, we get:

$$c\,(t) = \frac{a}{r+\delta} + C_1 e^{(r+\delta)t}$$

and

$$d(t) = (\int -\frac{\alpha}{4\beta} e^{-rt} (\beta + c(t) - w)^2 dt + C_2) e^{rt}.$$

Now, we solve for $C_1$ by use of the boundary condition. Because at time $T$ Microsoft derives $V\,(S)$ from the installed base, we have that

$$c\,(T) = \frac{a}{r+\delta} + C_1 e^{(r+\delta)T} = V\,(S) \qquad \Rightarrow C_1 = e^{-(r+\delta)T}\left(V\,(S) - \frac{a}{r+\delta}\right).$$

Therefore,

$$\frac{\partial V\,(y,t)}{\partial y} = c\,(t) = \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) + V\,(S)\, e^{-(r+\delta)(T-t)}. \tag{31}$$

Notice that $c\,(t)$ is the exact same expression as equation (22) in the Proof of Proposition 2.

We can now obtain the explicit joint pricing strategy by substituting $c\,(t)$ into (28):

$$p^*\,(t) = \frac{1}{2}\left(\beta + w - \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - V\,(S)\, e^{-(r+\delta)(T-t)}\right),$$

It is easy to check that the price under competition is higher than under collusion. As a consequence, consumer surplus is larger under collusion.

Let

$$\Theta\,(t) \equiv \int -\frac{\alpha}{4\beta} e^{-rt} (\beta + c(t) - w)^2 dt.$$

Then, $d\,(t)$ can be expresses as:

$$d\,(t) = (\Theta\,(t) + C_2)\, e^{rt}.$$

Using the boundary condition ($V\,(T,y) = V\,(S)\,y$), we have that

$$(\Theta\,(T) + C_2)\, e^{rT} = 0$$

or

$$C_2 = -\Theta\,(T)\,.$$

We can express the value function as:

$$V\,(t,y) = \left(\frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) + V\,(S)\left(e^{-(r+\delta)(T-t)}\right)\right) y + (\Theta\,(t) - \Theta\,(T))\, e^{rt}.$$

Let

$$\Delta\Theta = \Theta\,(0) - \Theta\,(T)\,.$$

Thus, at $t = 0$

$$\Pi = V(0, y_0) = \left( \frac{a}{r+\delta} \left(1 - e^{-(r+\delta)T}\right) + V(S)\, e^{-(r+\delta)T} \right) y_0 + \Delta\Theta,$$

In a duopolistic industry structure, the sum of profits is

$$\Pi_I + \Pi_M = \left( \frac{a}{r+\delta} \left(1 - e^{-(r+\delta)T}\right) + V(S)\, e^{-(r+\delta)T} \right) y_0 + 2\Delta\Theta_D$$

where (because equations (22) and (22) are equal):

$$\Theta_D(t) = \int -\frac{\alpha}{9\beta} e^{-rt} \left(\beta - w + c(t)\right)^2 dt.$$

Note that both $\Theta(t)$ and $\Theta_D(t)$ are negative. Furthermore, $2\Theta_D(t) > \Theta(t)$ (because $-\frac{2}{9} > -\frac{1}{4}$).

To prove that NPV of profit is higher in the monopolistic case we check that

$$\Pi - \Pi_D = \Delta\Theta - 2\Delta\Theta_D = \left(\Theta(0) - 2\Theta_D(0)\right) - \left(\Theta(T) - 2\Theta_D(T)\right) > 0.$$

To see this, notice that $\Theta(0) - 2\Theta_D(0)$ is negative because $2\Theta_D(t) > \Theta(t)$ for all $t$. Similarly, $\Theta(T) - 2\Theta_D(T) < 0$. But because $\Theta(t) - 2\Theta_D(t)$ decreases with $t$, we have that $\Theta(T) - 2\Theta_D(T) < \Theta(0) - 2\Theta_D(0) < 0$.

We conclude that total welfare increases with collusion. ■

**Proof of Proposition 4.** Looking at equations (25) and (26) we see that the marginal effect of $\alpha$ and $\beta$ on Intel and Microsoft's profits is the same. That is, $\frac{\partial \Pi_I}{\partial \alpha} = \frac{\partial \Pi_M}{\partial \alpha}$ and $\frac{\partial \Pi_I}{\partial \beta} = \frac{\partial \Pi_M}{\partial \beta}$. Because $\alpha$ and $\beta$ are assumed to be symmetric functions of $R_I$ and $R_M$, the equilibrium investments $R_I^*$ and $R_M^*$ coincide. ■

**Proof of Proposition 5.** Let

$$\Phi_0 \equiv \left( \frac{a}{r+\delta} \left(1 - e^{-(r+\delta)T}\right) + V(S)\, e^{-(r+\delta)T} \right) y_0,$$

$$\Phi_1 \equiv \int_0^T \frac{1}{4} e^{-rt} dt,$$

$$\Phi_2 \equiv \int_0^T \frac{1}{4} e^{-rt} \left( \frac{a}{r+\delta} \left(1 - e^{-(r+\delta)(T-t)}\right) + V(S)\, e^{-(r+\delta)(T-t)} - w \right)^2 dt$$

and

$$\Phi_3 \equiv \int_0^T \frac{2}{4} e^{-rt} \left( \frac{a}{r+\delta} \left(1 - e^{-(r+\delta)(T-t)}\right) + V(S)\, e^{-(r+\delta)(T-t)} - w \right) dt.$$

For $i \in \{I, M\}$ define

$$\Omega\left(s_i^0, s_i^1\right) \equiv s_i^0 \Phi_0 + s_i^1 \left( \alpha\beta\Phi_1 + \frac{\alpha}{\beta}\Phi_2 + \alpha\Phi_3 \right).$$

Using this notation, we can express second stage profits as

$$\Pi_C = \Omega\left(1,1\right), \, \Pi_I = \Omega\left(0, \tfrac{4}{9}\right) \text{ and } \Pi_M = \Omega\left(1, \tfrac{4}{9}\right).$$

In the first stage, Intel and Microsoft choose investments $R_I$ and $R_M$ to maximize profit. Thus, the investment game played in the first stage has Intel maximizing $\hat{\Pi}_I \equiv \Omega\left(s_I^0, s_I^1\right) - R_I$ by choice of $R_I$ and Microsoft maximizing $\hat{\Pi}_M \equiv \Omega\left(s_M^0, s_M^1\right) - R_M$ by choice of $R_M$.

We distinguish four cases.

1. Competition in stage 1 and in stage 2:

$$\left.\begin{array}{r} \max_{R_I} \Omega\left(0, \tfrac{4}{9}\right) - R_I \\ \max_{R_M} \Omega\left(1, \tfrac{4}{9}\right) - R_M \end{array}\right\}.$$

2. Competition in stage 1 cooperation in stage 2:

$$\left.\begin{array}{r} \max_{R_I} \Omega\left(\tfrac{1}{2}, \tfrac{1}{2}\right) - R_I \\ \max_{R_M} \Omega\left(\tfrac{1}{2}, \tfrac{1}{2}\right) - R_M \end{array}\right\}.$$

3. Cooperation in stage 1 and competition in stage 2:

$$\max_{R_I, R_M} \Omega\left(1, \tfrac{8}{9}\right) - R_I - R_M$$

   which is equivalent to

$$\left.\begin{array}{r} \max_{R_I} \Omega\left(1, \tfrac{8}{9}\right) - R_I \\ \max_{R_M} \Omega\left(1, \tfrac{8}{9}\right) - R_M \end{array}\right\}.$$

4. Cooperation in both stages:

$$\max_{R_I, R_M} \Omega\left(1,1\right) - R_I - R_M$$

   which is equivalent to

$$\left.\begin{array}{r} \max_{R_I} \Omega\left(1,1\right) - R_I \\ \max_{R_M} \Omega\left(1,1\right) - R_M \end{array}\right\}.$$

The four cases can be expressed in compact notation as

$$\left.\begin{array}{r} \max_{R_I} \Omega\left(s_I^0, s_I^1\right) - R_I \\ \max_{R_M} \Omega\left(s_M^0, s_M^1\right) - R_M \end{array}\right\}. \tag{32}$$

with values $s_I^0 = 0$, $s_M^0 = 1$ and $s_I^1 = s_M^1 = \frac{4}{9}$ for case 1, $s_I^0 = s_M^0 = s_I^1 = s_M^1 = \frac{1}{2}$ for case 2, $s_I^0 = s_M^0 = 1$ and $s_I^1 = s_M^1 = \frac{8}{9}$ for case 3, and $s_I^0 = s_M^0 = s_I^1 = s_M^1 = 1$ for case 4.

Because

$$\Omega\left(s_I^0, s_I^1\right) = s_i^0 \Phi_0 + s_i^1 \left(\alpha\beta\Phi_1 + \frac{\alpha}{\beta}\Phi_2 + \alpha\Phi_3\right)$$

and

$$\Phi_0 \equiv \left(\frac{a}{r+\delta}\left(1 - e^{-(r+\delta)T}\right) + V\left(S\right) e^{-(r+\delta)T}\right) y_0,$$

maximizing $\Omega\left(s_I^0, s_I^1\right)$ is equivalent to maximizing $\Omega\left(s_I^0, s_I^1\right) - s_i^0 \Phi_0$. In other words, in what follows it does not matter the value of $s_n^0$ because the solution to the maximization programs is independent of it. ■

**Lemma 12** *$R_n\left(s_n^0, s_n^1\right)$ is nondecreasing in $s_n^1$ (and independent of $s_n^0$).*

**Proof.** To prove this result we apply standard monotone comparative static results. Recall that $\alpha\left(R_I, R_M\right)$ and $\beta\left(R_I, R_M\right)$ are supermodular. We will now show that the game defined by the pair of objective functions in (32) is one with strategic complementarities. (See Milgrom and Shanon [11, p. 175].) Showing that this is a game with strategic complementarities implies that the equilibrium investments are nondecreasing in $s_n^1$ and $s_n^0$. (See Milgrom and Shanon [11, Thm. 13].)

1. To see that this is indeed a game with strategic complementarities, we have to show that:

    - $S_n$ (each player's strategy set) is a compact lattice. $R_I$ and $R_M$ take values in $\{R, 2R\}$. Therefore, the strategy sets are compact lattices.
    - $\Omega\left(R_I, R_M; s_n^0, s_n^1\right) - R_n$ is upper semi-continuous in $R_n$ for $R_{-n}$ fixed, and continuous in $R_{-n}$ for fixed $R_n$. Recall that

      $$\Omega\left(R_I, R_M; s_n^0, s_n^1\right) = s_n^0\Phi_0 + s_n^1\left(\alpha\beta\Phi_1 + \frac{\alpha}{\beta}\Phi_2 + \alpha\Phi_3\right)$$

      where $\Phi_0$, $\Phi_1$, $\Phi_2$ and $\Phi_3$ do not depend on $R_I$ or $R_M$. Because $\alpha\left(R_I, R_M\right)$ and $\beta\left(R_I, R_M\right)$ are continuous in $R_n$, $\Omega\left(R_I, R_M; s_n^0, s_n^1\right) - R_n$ is continuous in $R_n$.
    - $\Omega\left(R_I, R_M; s_n^0, s_n^1\right) - R_n$ is quasisupermodular in $R_n$ and satisfies the single crossing property in $(R_I; R_M)$.

      Let's begin by showing that $\Omega\left(R_I, R_M; s_n^0, s_n^1\right) \equiv s^0\Phi_0 + s^1\left(\alpha\beta\Phi_1 + \frac{\alpha}{\beta}\Phi_2 + \alpha\Phi_3\right)$ is supermodular in $R_I$ and $R_M$. Let

      $$g\left(z_1, z_2\right) = z_1 z_2\Phi_1 + \frac{z_1}{z_2}\Phi_2 + z_1\Phi_3.$$

      Notice that

      $$\frac{\partial^2 g\left(z_1, z_2\right)}{\partial z_1 \partial z_2} = \Phi_1 - \frac{1}{z_2^2}\Phi_2.$$

      By Milgrom and Shanon [11, Thm. 6], $g\left(z_1, z_2\right)$ is supermodular as long as $\Phi_1 \geq \frac{1}{z_2^2}\Phi_2$. Let $z_1 = \alpha\left(R_I, R_M\right)$ and $z_2 = \beta\left(R_I, R_M\right)$, then $\frac{\partial^2 g(\alpha,\beta)}{\partial\alpha\partial\beta} \geq 0$ when Microsoft prices are constrained to be positive. To see this, notice that

      $$p_M^*(t) = \frac{1}{3}\left(\beta - w - \frac{2a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - 2V(S)\,e^{-(r+\delta)(T-t)}\right)$$

      $$\begin{aligned} .p_M^*(t) > 0 \Rightarrow 0 &< \beta - \frac{2a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - 2V(S)\,e^{-(r+\delta)(T-t)} - w < \\ &< \beta - \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - V(S)\,e^{-(r+\delta)(T-t)} - w < \\ &< \beta - \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - V(S)\,e^{-(r+\delta)(T-t)} + w \Rightarrow \\ 0 &< \beta - \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) - V(S)\,e^{-(r+\delta)(T-t)} + w \Rightarrow \\ \beta &> \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w. \end{aligned}$$

So

$$p_M^*(t) > 0 \text{ if } \beta > \frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w \tag{33}$$

We know that

$$\frac{\partial^2 g(\alpha,\beta)}{\partial\alpha\partial\beta} = \Phi_1 - \frac{1}{\beta^2(R_I,R_M)}\Phi_2 =$$

$$= \int_0^T \frac{1}{4}e^{-rt}dt - \frac{1}{\beta^2}\int_0^T \frac{1}{4}e^{-rt}\left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w\right)^2 dt =$$

$$= \int_0^T \frac{1}{4}e^{-rt}\left(1-\frac{1}{\beta^2}\left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w\right)^2\right)dt.$$

So, this expression will be $\geq 0$ if

$$\left(1-\frac{1}{\beta^2}\left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w\right)^2\right) \geq 0 \Rightarrow$$

$$\frac{1}{\beta^2}\left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w\right)^2 \leq 1 \Rightarrow$$

$$\beta^2 \geq \left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)(T-t)}\right) + V(S)\,e^{-(r+\delta)(T-t)} - w\right)^2.$$

By (33) we know that this last expression is satisfied. We have shown that $\frac{\partial^2 g(\alpha,\beta)}{\partial\alpha\partial\beta} \geq 0$ when Microsoft prices are positive. Now by Topkis [14, Lemma 2.6.4], $g(\alpha,\beta)$ is supermodular on $R_I$ and $R_M$. We conclude that

$$\Omega\left(R_I, R_M; s_n^0, s_n^1\right) = s^0\Phi_0 + s^1 g\left(\alpha\left(R_I,R_M\right), \beta\left(R_I,R_M\right)\right)$$

is supermodular.

We are now ready to show that $\Omega\left(R_I, R_M; s_n^0, s_n^1\right) - R_n$ is quasisupermodular in $R_I$ and $R_M$ and satisfies the single-crossing property. This follows directly from Milgrom and Shanon [11, Thm. 10]. See also Topkis [14, p. 50].

We have just showed that the game defined by the pair of objective functions in (32) is one with strategic complementarities. We now use Milgrom and Shanon [11, Thm. 12] that (at least) one equilibrium exists. Moreover, by Milgrom and Shanon [11, Thm. 13] we conclude that the equilibrium investments $R_n\left(s_n^0, s_n^1\right)$ are nondecreasing in $s_n^1$. That $R_n\left(s_n^0, s_n^1\right)$ is independent of $s_n^0$ follows from the observation that because $\Omega\left(s_I^0, s_I^1\right) = s_i^0\Phi_0 + s_i^1\left(\alpha\beta\Phi_1 + \frac{\alpha}{\beta}\Phi_2 + \alpha\Phi_3\right)$ and $\Phi_0 \equiv \left(\frac{a}{r+\delta}\left(1-e^{-(r+\delta)T}\right) + V(S)\,e^{-(r+\delta)T}\right)y_0$, maximizing $\Omega\left(s_I^0, s_I^1\right)$ is equivalent to maximizing $\Omega\left(s_I^0, s_I^1\right) - s_i^0\Phi_0$. This proves the lemma.■

That $\hat{\Pi}_i^{1,1} > \hat{\Pi}_i^{0,1} > \hat{\Pi}_i^{1,0} > \hat{\Pi}_i^{0,0}$ follows immediately from the fact that $\Omega\left(R_I, R_M; s_n^0, s_n^1\right) - R_n$ are quasisupermodular functions. ■

**Proof of Proposition 6.** When Intel decides its (unilaterally) optimal time to invest, it solves:

$$\frac{dH_I(t_i)}{dt_i} = 0.$$

When Microsoft decides its (unilaterally) optimal time to invest, it solves:

$$\frac{dH_M(t_i)}{dt_i} = 0.$$

If we show that $H_I(t_i) = H_M(t_i)$ for all $t_i$ (because this implies that the derivatives are also equal at all points), then the proposition will be proven.

Proposition 2 together with the symmetry of $\alpha$ and $\beta$ imply that

$$\int_0^{t_i} e^{-rt}\pi_I^{1,1}(t)\,dt = \int_0^{t_i} e^{-rt}\pi_M^{1,1}(t)\,dt + e^{-r(t_i-s)}V(S)\,y_1(t_i)$$

and

$$\int_0^{t_i} e^{-rt}\pi_I^{2,1}(t)\,dt = \int_0^{t_i} e^{-rt}\pi_M^{1,2}(t)\,dt + e^{-rt_i}V(S)\,y_2(t_i).$$

Therefore $H_I(t_i) = H_M(t_i)$ for all $t_i$, and the proposition is proven. ■

**Proof of Proposition 7.** To simplify notation, we let $A_I(t_i)$ $(A_M(t_i))$ be Intel's (Microsoft's) profit from the second generation. That is,

$$A_I(t_i) \equiv \int_0^{t_i} e^{-rt}\pi_I^{2,1}(t)\,dt \qquad \text{and} \qquad A_M(t_i) \equiv \int_0^{t_i} e^{-rt}\pi_M^{1,2}(t)\,dt + e^{-rt_M}V(S)\,y^2(t_i).$$

Then we can write

$$H_I(s,t_i) \equiv \int_s^{t_i} e^{-r(t-s)}\pi_I^{1,1}(t)\,dt + e^{-r(t_i-s)}\left(A_I(t_i) - R\right)$$

and

$$H_M(s,t_i) \equiv \int_0^{t_i} e^{-r(t-s)}\pi_M^{1,1}(t)\,dt + e^{-r(t_i-s)}V(S)\,y_1(t_i) + e^{-r(t_i-s)}\left(A_M(t_i) - R\right).$$

Proposition 2 and the symmetry of $\alpha$ and $\beta$ on $R_I$ and $R_M$ imply that $A_M(t_i) = A_I(t_i)$, for all $t_i$ and all $S$. As a consequence,

$$\frac{dA_I(t_i)}{dt_i} = \frac{dA_M(t_i)}{dt_i}.$$

Thus, if we show that

$$\frac{d}{dt_i}\left(\int_0^{t_i} e^{-r(t-s)}\pi_M^{1,1}(t)\,dt + e^{-r(t_i-s)}V(S)\,y_1(t_i)\right) > \frac{d}{dt_i}\left(\int_s^{t_i} e^{-r(t-s)}\pi_I^{1,1}(t)\,dt\right)$$

so that

$$\frac{dH_M(s,t_i)}{dt_i} > \frac{dH_I(s,t_i)}{dt_i} \qquad \text{for all } t_i,$$

the result will be proven (because at $t_i$ that solves $\frac{dH_I(t_i)}{dt_i} = 0$ we will have that $\frac{dH_M(t_i)}{dt_i} > 0$ and Microsoft will be better off waiting some more time to switch).

The net present value of profit to Intel and Microsoft for the first generation beginning at time $s$ can be expressed as (the the proof of Proposition 2):

$$\text{Intel: } V_I(s,y;t_i) = \int_s^{t_i} e^{-r(t-s)} \pi_I^{1,1}(t)\,dt = c_I(s)\,y + d_I(s) = (\Theta_D(s) - \Theta_D(t_i))\,e^{rs}$$

and

$$\begin{aligned} \text{Microsoft} \quad : \quad V_M(s,y;t_i) &= \int_0^{t_i} e^{-r(t-s)} \pi_M^{1,1}(t)\,dt + e^{-r(t_i-s)} V(S)\,y_1(t_i) = c_M(s)\,y + d_M(s) \\ &= \left(\frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(t_i-s)}\right) + V(S)\,e^{-(r+\delta)(t_i-s)}\right) y(s) + (\Theta_D(s) - \Theta_D(t_i))\,e^{rs}, \end{aligned}$$

where $y(s)$ is the installed base at time $s$. Notice that $y(s)$ acts as the "initial installed base" for the time-$s$ problem of choosing $t_i$. It is important to see that $y(s) > 0$ for all $s > 0$.

We are interested in comparing

$$\frac{dV_M(s,y;t_i)}{dt_i} \quad \text{and} \quad \frac{dV_I(s,y;t_i)}{dt_i}.$$

Because

$$V_M(s,t_i) = \left(\frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(t_i-s)}\right) + V(S)\,e^{-(r+\delta)(t_i-s)}\right) y(s) + V_I(s,y;t_i),$$

we have that

$$\frac{dV_M(s,y;t_i)}{dt_i} = (a - (r+\delta)\,V(S))\,e^{-(r+\delta)(t_i-s)} + \frac{dV_I(s,y;t_i)}{dt_i}.$$

Now, because for all $S < \infty$

$$\frac{a}{r+\delta} > V(S) = \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)S}\right)$$

we conclude that

$$\frac{dV_M(s,y;t_i)}{dt_i} > \frac{dV_I(s,y;t_i)}{dt_i}$$

Therefore, when $S < \infty$, $t_I^*(s) < t_M^*(s)$ for all $s > 0$.

Finally, when $S = \infty$

$$V(S) = \frac{a}{r+\delta}$$

and

$$\frac{dV_M(s,y;t_i)}{dt_i} = \frac{dV_I(s,y;t_i)}{dt_i}.$$

Therefore, when $S = \infty$, $t_I^*(s) = t_M^*(s)$ for all $s$. ■

**Proof of Proposition 8.** The proof is immediate. Just notice that the proposed strategies constitute a sequence of Nash equilibria of each stage game. $r = \infty$ means that the future is unimportant to both Intel and Microsoft. ■

**Proof of Proposition 9.** To prove this result we follow the method used in the proofs of Propositions 2 and 3. The HJB equations are:

$$rV_I - \frac{\partial V_I}{\partial t} = \max_{p_I} \left\{ \left(1 - \frac{p_I - p_A}{\beta - \nu}\right) p_I + \frac{\partial V_I}{\partial y} \left(1 - \frac{p_M + p_A}{\nu} - \delta y\right) \right\},$$

$$rV_A - \frac{\partial V_A}{\partial t} = \max_{p_A} \left\{ \left(\frac{p_I - p_A}{\beta - \nu} - \frac{p_M + p_A}{\nu}\right) p_A + \frac{\partial V_A}{\partial y} \left(1 - \frac{p_M + p_A}{\nu} - \delta y\right) \right\},$$

and

$$rV_M - \frac{\partial V_M}{\partial t} = \max_{p_M} \left\{ \left(1 - \frac{p_M + p_A}{\nu}\right) p_M + ay + \frac{\partial V_M}{\partial y} \left(1 - \frac{p_M + p_A}{\nu} - \delta y\right) \right\}.$$

The prices that maximize the right hand of these equations are given by:

$$p_I = \frac{1}{6}\left(1 - \frac{\nu}{\beta}\right)\left(3\beta + \frac{\partial V_M}{\partial y} - 2\frac{\partial V_A}{\partial y}\right),$$

$$p_A = \frac{1}{3}\left(1 - \frac{\nu}{\beta}\right)\left(\frac{\partial V_M}{\partial y} - 2\frac{\partial V_A}{\partial y}\right),$$

and

$$p_M = \frac{\nu}{2} - \left(\frac{2}{3} - \frac{\nu}{6\beta}\right)\frac{\partial V_M}{\partial y} + \frac{1}{3}\left(1 - \frac{\nu}{\beta}\right)\frac{\partial V_A}{\partial y}.$$

Because this is a linear state game, the value function are linear:

$$V_I = c_I(t)\, y + d_I(t),$$

$$V_A = c_A(t)\, y + d_A(t),$$

and

$$V_M = c_M(t)\, y + d_M(t).$$

By substituting the prices and value functions into the HJB equations we obtain a system of differential equations that can be easily solved (just as we did in the proof of Proposition 2) to obtain:

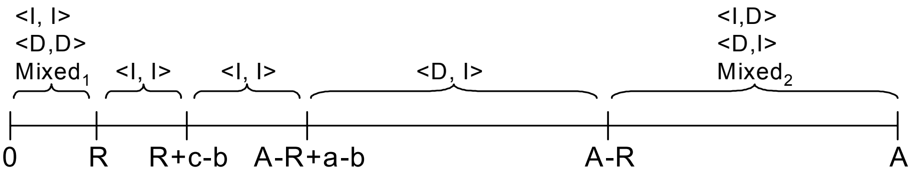
$$p_I^* = \frac{1}{6}\left(1 - \frac{\nu}{\beta}\right)\left(3\beta + \frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right)\right),$$

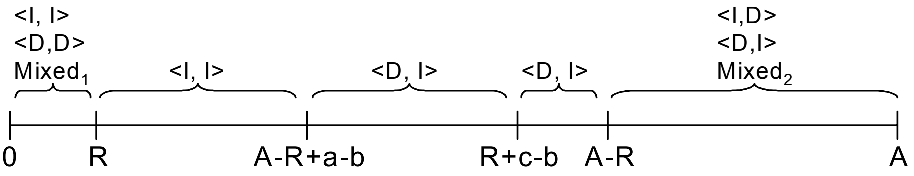
$$p_A^* = \frac{1}{3}\left(1 - \frac{\nu}{\beta}\right)\left(\frac{a}{r+\delta}\left(1 - e^{-(r+\delta)(T-t)}\right)\right),$$

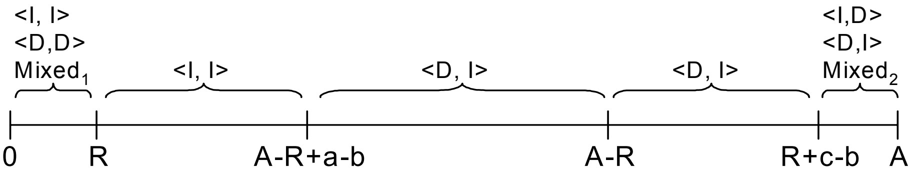
$$p_M^* = \frac{1}{6}\left(3\nu - \left(4 - \frac{\nu}{\beta}\right)\frac{a}{(r+\delta)}\left(1 - e^{-(r+\delta)(T-t)}\right)\right).$$

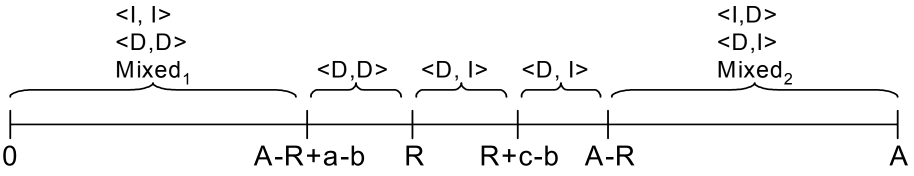
Notice that $p_I^* \to 0$ and $p_A^* \to 0$ as $\nu \to \beta$. In fact, $V_M$ increases as $\nu$ approaches $\beta$. ■

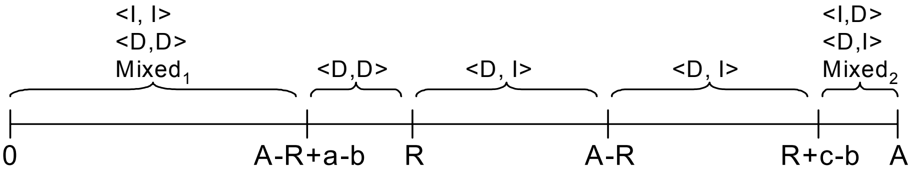
**Proof of Proposition 11.** The following figure summarizes (for all possible parameter configurations) the equilibria that obtains as the value of $B$ (which captures the intensity of competition between Linux and Windows) changes.

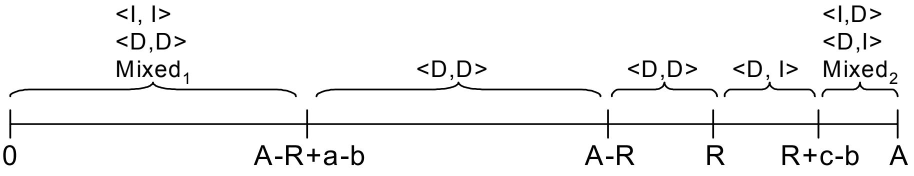












■

# References

[1] Balkanski, Alexander, September 1, 1995, "NSP—Friend or Foe?" *The Red Herring.*

[2] Burgelman, Robert A., 2002, *Strategy is Destiny*, New York: Free Press.

[3] Carlsson, Hans and Eric van Damme, 1993, "Global Games and Equilibrium Selection," *Econometrica* 61, 989-1018.

[4] Cournot, Augustine,1838, *Researches into the Mathematical Principles of the Theory of Wealth*, originally published in French, translated by Nathaniel Bacon, New York: Macmillan, 1927.

[5] Casadesus-Masanell, Ramon, David Yoffie, and Sasha Mattu, "Intel Corporation: 1968-2003," *Harvard Business School Case* 9-703-427.

[6] Dockner, Engelbert, Steffen Jørgensen, Ngo Van Long and Gerhard Sorger, 2000, *Differential Games in Economics and Management Science*, Cambridge University Press.

[7] *Electronic Buyers' News* No. 941, February 6, 1995, "Intel Puts Multimedia Inside – NSP Software May Alter Add-In Card Business."

[8] Gawer, Annabelle and Michael Cusumano, 2002, *Platform Leadership: How Intel, Microsoft, and Cisco Drive Industry Innovation*, Boston, MA: Harvard Business School Press.

[9] Holmstrom, Bengt, 1982, "Moral Hazard in Teams," *The Bell Journal of Economics* 13(2), 324-340.

[10] Jackson, Tim, 1997, *Inside Intel. Andy Grove and the Rise of the World's Most Powerful Chip Company*, New York: Penguin Group.

[11] Milgrom, Paul, and Christina Shannon, 1994, "Monotone Comparative Statics," *Econometrica* 62, 157-80.

[12] Schlender, Brent, July 8, 1996, "A Conversation with the Lords of Wintel," *FORTUNE* 134(1).

[13] Slater, Michael, November 1, 1995, "Signal Processing—Native and Otherwise...," *Computer Shopper* 15(11).

[14] Topkis, Donald M., 1998, *Supermodularity and Complementarity*, Princeton, NJ: Princeton University Press.

[15] *United States v. Microsoft*: Government Exhibit 923, MS98 0168630 Confidential, carls email, May 31, 1995.

[16] *United States v. Microsoft*: Government Exhibit 277. MS98 0169352 Confidential, billg email, Thursday May 25, 1995.

[17] Yoffie, David B., Ramon Casadesus-Masanell and Sasha Mattu, 2004, "Wintel (A): Cooperation or Conflict" *Harvard Business School Case* 9-704-419.

[18] Yu, Albert, 1998, *Creating the Digital Future*, New York: Free Press.