Inventory Pooling with Strategic Consumers: Operational and Behavioral Benefits

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Abstract

The practice of inventory pooling–serving two or more separate markets using a common inventory stock–is extensively studied in operations management. The operational benefits of this strategy are well known: when demand is stochastic, combining multiple markets reduces aggregate uncertainty and improves the firm's ability to efficiently match supply and demand, increasing profit as a result. In this paper, we explore a different aspect of pooling: its consequences for consumer purchasing behavior. We analyze a model in which consumers are forward-looking and anticipate end-of-season clearance sales, and may choose to strategically forgo purchasing items at a high price in order to obtain them at a discount. The firm may choose between a separated selling strategy (e.g., many physical stores to serve distinct geographic regions) or a pooled selling strategy (e.g., a single internet channel to serve the entire country). We demonstrate that in addition to the operational benefits of pooling, in this setting a behavioral dimension to pooling exists: by adopting a pooling strategy, the firm influences the amount of inventory available during the clearance sale and hence induces a change in consumer purchase timing. This behavioral dimension of pooling may benefit the firm (when margins are high and demands are negatively correlated) or may hurt the firm (when margins are low and demand is positively correlated). We also consider whether pooling benefits consumers, and find that in contrast to the claims of some retailers, inventory pooling may decrease consumer welfare, particularly if consumers are strategic. This happens because, despite the fact that inventory pooling increases product availability during high price sales, it may increase competition for scarce inventory and decrease product availability during clearance sales.

1 Introduction

Inventory pooling refers to a firm's ability to serve multiple markets–each with their own uncertain demand–from a single stock of inventory. The practice is often analyzed in the context of two distinct, but closely related, cases: location pooling and product pooling. Location pooling refers to the practice of pooling demands from separate geographic markets (e.g., combining the inventory from stores in two different physical locations). As information systems have improved and e-commerce has surged in popularity during the last decade, location pooling strategies have become the operational norm as large geographic regions are

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increasingly served by (either literally or virtually) centralized stocks (Cachon & Terwiesch 2009). For example, Clifford (2010) describes the efforts of the high-end U.S. department store chain Nordstrom to pool brick-and-mortar and internet inventories across the entire company, primarily using a combination of interconnected IT systems and extra employees to process transshipment requests at both retail stores and distribution centers; other retailers, such as the Jones Apparel Group, and e-commerce and logistics providers have made similar efforts (Fowler & Dodes 2010).

Product pooling, on the other hand, refers to the practice of meeting demand for multiple distinct products with a single, "universal" product capable of satisfying the needs of all customers. In this instance, the component markets need not be geographically separated but may be separated by customer requirements in terms of features, durability, performance, or aesthetic preferences (e.g., color). The pooling benefits of serving the same aggregate demand with less product variety are a key force behind the "SKU rationalization" movement, a retail approach to increase customer service (e.g., inventory availability) with reduced variety (Alfaro & Corbett 2003). Particularly in consumer products, these practices have gained an increasing amount of traction in recent years as cost concerns spur SKU rationalization even at large retailers such as Wal-Mart (Hamstra 2011) and companies with limited product lines, such as Apple, enjoy success both in managing inventory and in providing consumers a simpler menu of purchasing options (Burns 2009; Nosowitz 2010).

Both types of pooling–which we collectively refer to as *inventory pooling*–have been extensively studied in the operations management literature (e.g., Eppen 1979, Federgruen & Zipkin 1984, Corbett & Rajaram 2006). As an operational strategy, inventory pooling is frequently cited as an effective tool to mitigate demand uncertainty: combining inventory in this manner allows the firm to reduce demand variability, reduce operational costs, and increase profit, particularly if the component market demands are negatively correlated. However, despite the pervasiveness of pooling strategies in both the academic operations literature and in practice, little is known concerning how consumers themselves are impacted by and respond to inventory pooling techniques. These are precisely the issues that we explore in this paper, focusing on two key aspects of the inventory pooling problem: the impact of strategic consumer behavior on the value of pooling, and the impact of pooling on consumer welfare.

We analyze a simple model, following in spirit the seminal paper of Eppen (1979), in which a firm sells a product in multiple segregated markets and must choose between pooled and non-pooled operational systems. In the latter system, inventory is committed to each individual market well in advance of the resolution of demand uncertainty, and once committed to a specific market inventory cannot be transferred to any other market. In the former system, all demand is served from a single stock of inventory.

The difference between the previous literature and our paper is that we posit rational consumers that

strategically choose whether to purchase the product at the full price, or to wait until an end-of-season clearance sale in which the firm drastically discounts all remaining inventory. Specifically, while the existing literature assumes that consumers are myopic, purchasing the product at the full price without consideration paid to future price changes and inventory availability, the forward-looking consumers in our model make their purchasing decision (when and whether to buy the product) based on the selling price and their rational expectations of the chance of obtaining the product during the clearance sale. Such "strategic" consumer behavior is particularly problematic in precisely the sorts of retail industries that frequently employ pooling, such as fashion apparel, e.g., Nordstrom and the Jones Apparel Group (O'Donnell 2006), and short-lifecycle products such as consumer electronics, e.g., Best Buy, whose website lists inventory availability both online and at nearby stores.

In this setting, we first explore the impact of strategic consumer purchasing behavior on the value of pooling as an operational strategy. We demonstrate that a unique equilibrium to the game between the firm and consumers (in which the firm chooses inventory and consumers choose a purchase time) exists, then derive the equilibrium firm profit in both pooled and non-pooled systems, analyzing the behavior of the incremental value of pooling as a function of a variety of problem parameters. With myopic consumers, it is well known that pooling allows the firm to maintain a service target (e.g., an in-stock probability or critical ratio) while decreasing operational costs, thereby increasing profit (Eppen 1979). We find that under strategic consumer behavior, pooling generates value along two dimensions: the familiar operational dimension and a behavioral dimension which is new to our model. We show that the magnitude of the operational value of pooling is decreased by strategic consumer behavior, but otherwise it behaves in accordance with the intuition one might expect (e.g., it is always positive, and decreasing in the correlation of market demands).

In contrast, the behavioral dimension of pooling exhibits fundamentally different qualitative performance from the operational dimension. It may increase or decrease firm profit, depending on whether pooling decreases or increases clearance sale inventory availability. When pooling decreases clearance sale inventory availability, forward-looking consumers are less likely to strategically delay a purchase, hence there is a new source of value in pooling created by mitigating strategic consumer purchasing behavior. This case is most likely to hold if the product margins are high and the underlying markets are negatively correlated. When pooling increases clearance sale inventory availability, the behavioral effect encourages more consumers to strategically delay a purchase, potentially decreasing firm profit as a result. This case typically occurs when product margins are low and the underlying markets are positively correlated. Thus, our model demonstrates when pooling is likely to possess positive behavioral value to the firm (high margin products with negatively correlated demand) and when it is likely to possess negative behavioral value (low margin products with positively correlated demand), providing guidance to managers on when the behavioral benefits of pooling

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are greatest. In addition, we show that the behavioral value of pooling may increase in the correlation of market demands, precisely the opposite behavior of the operational value of pooling.

We also analyze the impact of pooling on consumer welfare. This is a particularly important aspect of pooling to consider, as many retailers publicize the customer service benefits of pooling; Nordstrom, for instance, emphasizes that consumers can instantly learn accurate, company-wide inventory availability and easily obtain any item that any location has in-stock, implying that consumers will benefit from pooling. As Nordstrom Direct president Jamie Nordstrom puts it when describing their pooling initiatives, "all the changes...were about satisfying customers" (Clifford 2010). But are customers *truly* better off when a firm pools inventory? We demonstrate that while availability at higher prices is typically increased by pooling, increasing welfare amongst those consumers willing to pay full price, if inventory is optimally chosen availability at lower prices can be reduced by pooling, which decreases welfare amongst the lowest value consumers. Thus, it is possible for pooling to decrease total consumer welfare once the firm optimally adjusts inventory, and whether this occurs depends on precisely which forces dominate. In a large scale numerical study, we demonstrate that under reasonable parameter values, pooling generally leads to an increase in consumer welfare (in over 78% of our sample), and we investigate conditions that dictate when pooling is a losing proposition for consumers. Most notably, we demonstrate that pooling is most likely to benefit consumers precisely when it least valuable to the firm.

Taken in sum, our results help to illuminate some of the behavioral consequences of a venerable operational strategy: inventory pooling. There are both behavioral benefits and costs to pooling, and by illustrating the driving forces behind each, we demonstrate precisely when consumer behavior may help (or hurt) a pooling initiative. Lastly, our model provides a word of caution to consumers that, while pooling does sometimes increase consumer welfare, very often this occurs when pooling least benefits the firm, implying that publicized pooling initiatives may lead to an overall reduction in consumer welfare.

2   Related Literature

Our model considers the practice of inventory pooling, which comprises a substantial stream of research within the operations literature. The seminal paper on this topic is Eppen (1979), who demonstrates that consolidating many individual newsvendor-type markets into a single market serving the aggregate demand is valuable to the firm, and the value is generally decreasing in the correlation of individual market demands. In an assumption that would become standard in the inventory pooling literature, Eppen (1979) employs multivariate normal component demands, which enables parsimonious analysis of pooled demand (since the sum of normal random variables is itself normal). Federgruen & Zipkin (1984) consider inventory pooling

in a supply chain setting with a centralized depot supplying multiple markets, and demonstrate the pooling effect with an expanded range of demand distributions (normal, exponential, and gamma).

More recently, Corbett & Rajaram (2006) extend the results of Eppen (1979) to general dependent demand distributions. Benjaafar *et al.* (2005) consider the value of pooling in production-inventory systems with production time variability. Also related to the inventory pooling literature are the literatures on resource or capacity pooling, including Fine & Freund (1990), Van Mieghem (1998), and Van Mieghem (2003), and product pooling and postponement, including Lee & Tang (1997) and Feitzinger & Lee (1997). Anupindi & Bassok (1999) are among the first to analyze an interaction of consumer behavior and pooling, demonstrating that if a large enough fraction of consumers are willing to search for available inventory, pooling may hurt a manufacturer selling to multiple retailers. The common factor in all pooling models is that combining multiple sources of uncertainty generally leads to reduced variability and lower costs. This need not always be the case, however; Alfaro & Corbett (2003) analyze the impact of pooling when the inventory policy in use is suboptimal, demonstrating that pooling can have negative value when inventory is not optimized properly. Another intuitive result is that pooling should lead to inventory levels closer to the mean demand; however, Gerchak & Mossman (1992) and Yang & Schrage (2009) demonstrate that this may not occur, depending on the distribution of demand.

Because we consider the combination of inventory pooling and strategic consumer behavior, our paper is also related to the recent stream of research on the topic of how customer purchasing behavior impacts firm operational decisions. The phrase "strategic consumers" has generally come to mean customers who anticipate future firm actions–such as price reductions–and take these anticipated actions into account when making their own purchasing decisions. There is increasing empirical evidence that consumers exhibit such behavior; recent work by Chevalier & Goolsbee (2009) (in the college textbook industry), Osadchiy & Bendoly (2010) (in a laboratory setting), and Li *et al.* (2011) (using data from the airline industry) all show that a small but substantial fraction of consumers behave in this manner and form rational expectations of future firm actions, on the order or 10-25% of the populations examined.

On the theoretical side, following early work in the economics literature focused primarily on pricing (Coase 1972; Stokey 1981; Bulow 1982), a large amount of recent attention has been focused on how strategic or forward-looking customer behavior influences the operational practices of a firm. Examples include supply chain contracting (Su & Zhang 2008), availability guarantees (Su & Zhang 2009), consumer return policies (Su 2009), multiperiod pricing (Aviv & Pazgal 2008), in-store display formats (Yin *et al.* 2009), price matching policies (Lai *et al.* 2010), opaque selling strategies (Jerath *et al.* 2010), dynamic pricing (Cachon & Feldman 2010), quick response inventory systems (Cachon & Swinney 2009; Swinney 2011), fast fashion production (Cachon & Swinney 2011), and product quality decisions (Kim & Swinney 2011). However, our

paper is the first, to our knowledge, that considers the impact of strategic customer behavior on the practice of inventory pooling.

3 Model

3.1 The Firm

A firm sells a single product in two distinct markets, labeled 1 and 2.[1] In each market, there are two populations of consumers: a "main population" of consumers who may purchase at a high price, and an infinite population of low-valuation "bargain hunting" consumers who only purchase at deeply discounted prices. Both of these populations, and the decisions they make, are described in greater detail in §3.2, below.

The size of the main consumer population in each market (i.e., the number of consumers) is stochastic and denoted by the random variable D_i, $i = 1, 2$. We denote the pooled demand (combined demand from both markets) by $D_P = D_1 + D_2$. Following the convention in much of the pooling literature, we assume that individual market demands (D_1 and D_2) are normally distributed, and we assume the component markets have identical mean μ and standard deviation σ. The correlation between market demands is given by ρ. The multivariate normal assumption implies that D_P is also normally distributed, with mean 2μ and standard deviation $\sigma\sqrt{2(1 + \rho)}$. Associated with the demand distribution are several related functions to which we will refer: $\Phi(\cdot)$ and $\phi(\cdot)$, which are the standard normal distribution and density functions, respectively, and $L(\cdot)$, which is the standard normal loss function, $L(z) = \int_z^\infty (t - z)\phi(t)\,dt$.

The firm may operate its supply chain in one of two systems. The first is referred to as the *non-pooled* system. In this system, inventory is committed to an individual market prior to the resolution of demand uncertainty, and once inventory has been committed to one market it cannot be used to satisfy demand in the other market (i.e., there is no transshipment, consumer search, or product substitution). In the non-pooled system, the amount of inventory destined for market i is given by q_i and expected firm profit in market i is $\pi_i(q_i)$, $i = 1, 2$. The second possible system is the *pooled* system, in which demand streams from both markets pull from a centralized stock of inventory. The subscript P will be used to refer to the pooled system, e.g., the total amount of inventory is denoted q_P and expected firm profit in the pooled system is $\pi_P(q_P)$. We frequently use generic inventory and profit expressions, $\pi_i(q_i)$, which may allude to markets 1 and 2 in the non-pooled system ($i = 1, 2$), or the "pooled market" ($i = P$) in the pooled system, as necessary.

In both systems, the dynamics of the selling season follow the classical newsvendor model with salvaging

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[1]We are agnostic as to whether the markets represent geographic regions or product markets (e.g., whether the type of pooling under consideration is location pooling or product pooling); for the majority of the analysis we simply refer to inventory pooling in a generic sense meant to capture both scenarios, and in §8, we discuss some potential differences between the product pooling and location pooling cases, and their possible implications.

(as in, e.g., Eppen 1979). There is a single selling season in which the product is sold at a constant price p, followed by a clearance sale at price s to dispose of remaining inventory. We refer to the full price selling season as "period 1" and the clearance sale as "period 2." Inventories are established before the start of period 1, and any period 1 demand in excess of supply is lost, while supply in excess of period 1 demand is sold in period 2. Each unit is produced or procured at constant marginal cost c, and to maintain consistency with and a fair comparison to the inventory pooling literature, we assume that all costs and prices are exogenously specified.

The economic characteristics of the products and the markets are identical: that is, the marginal production cost (c), full price (p), and clearance price (s) are the same, regardless of which market the items are sold in or which system (pooled or non-pooled) the firm employs. We ignore all secondary costs (fixed or variable) that may be associated with either a pooled or non-pooled system, in order to focus exclusively on the pooling effect on firm profit. To maintain finite solutions we require $s < c < p$. The firm chooses inventory in each market (in the non-pooled case) or in the pooled market (in the pooled system) to maximize expected profit, though it is easy to extend the model cover the case of arbitrary service targets (i.e., in-stock probabilities) without affecting any results.

3.2   Consumers

Recall there are two populations of consumers: the main population which arrives in period 1, and an infinite number of low-valuation bargain hunting consumers that arrive in period 2.[2] The consumers in the main population have heterogeneous valuations distributed according to the continuous distribution function $G(\cdot)$, where $\bar{G}(x) = 1 - G(x)$, with support on the interval (v_l, v_h). To maintain interesting solutions, we assume $v_l \geq s$ (any consumers with lower valuations would never purchase) and $v_h > p$ (otherwise, no consumers would ever purchase at the full price), and for technical purposes we assume the consumer valuation distribution has no mass on the endpoints ($\bar{G}(v_h) = G(v_l) = 0$).

Consumers in the main population are further subdivided into two segments: myopic or strategic. Myopic consumers purchase the product in period 1 (at the full price) if their valuation weakly exceeds the selling price, i.e., without consideration to future purchasing opportunities. Strategic consumers anticipate the period 2 clearance sale at price s, and take this lower price–in addition to the availability risk associated with the clearance sale–into account when deciding whether to purchase in period 1. We assume that a fraction α of the main population is strategic while a complementary fraction $(1 - \alpha)$ is myopic, and that

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[2]The existence of this "bargain hunting" segment simplifies our analysis by creating an infinite salvage market consistent with many newsvendor models and leading to closed form expressions for optimal inventory and expected firm profit, but is not necessary to generate any of our results. Strictly speaking, this market need be neither infinite nor deterministic: as long as the salvage market is sufficiently large so as to ensure all inventory is cleared in period 2 with probability one, our results continue to hold.
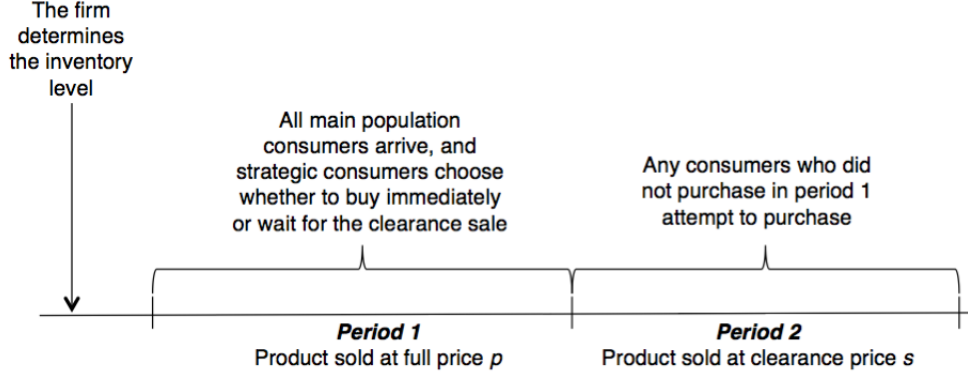
Figure 1. The sequence of events in the model.

the behavioral types are independent of the underlying consumer valuation structure.[3] Consequently, the period 1 demand from myopic consumers is $\bar{G}(p)(1-\alpha)D_i$. We assume that all myopic consumers who do not purchase in period 1 (i.e., those with valuations lower than p) return to the firm in period 2 and attempt to purchase, if the product is available. Hence, period 2 demand from myopic consumers is $G(p)(1-\alpha)D_i$.

The strategic consumers in the main population are forward-looking, risk-neutral utility maximizers who recognize that the product will be marked down to s during the clearance sale in period 2. Upon arrival in period 1, one of two cases holds for each individual consumer: either the firm is in-stock, or the firm is out-of-stock. If the latter, the game ends (i.e., there is no reason for the consumer to strategically delay). If the firm is in-stock, then the consumer chooses between a certain purchase at a high price and a delayed, but uncertain, purchase at the lower clearance price. Thus, a consumer with valuation v individually chooses whether to purchase the product at the high initial price and obtain surplus $v - p$, or wait for the markdown to obtain surplus $v - s$, taking into account the anticipated probability of obtaining a unit in the second period, $\tilde{\xi}_i$ in market i ($i = 1, 2, P$), where the $(\tilde{\cdot})$ symbol denotes a belief. The precise nature of these beliefs are discussed in the following section. For now, we merely state that all consumers possess common beliefs. If a consumer does not obtain a unit of the product in period 2, she receives zero surplus. We assume that a strategic consumer will purchase the product in period 1 if she is indifferent between the two periods, meaning a consumer in market i purchases in period 1 if $v - p \geq \tilde{\xi}_i(v - s)$.

The sequence of events is summarized in Figure 1, and the following proposition provides our first result concerning the purchasing behavior of the strategic consumer segment:

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[3]The latter assumption allows us to focus on purely behavioral differences in the underlying consumer population, rather than confounding behavioral differences with differences in valuations; see Su (2007) for a discussion of the alternative approach in which behavior and valuations are correlated.

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Segment	Number in Each Market	Valuations	Period 1 Demand	Period 2 Demand
Myopic Consumers	$(1-\alpha)D_i$	$\sim G(\cdot)$	$\bar{G}(p)(1-\alpha)D_i$	$G(p)(1-\alpha)D_i$
Strategic Consumers	αD_i	$\sim G(\cdot)$	$\bar{G}(v_i^*)\alpha D_i$	$G(v_i^*)\alpha D_i$
Bargain Hunters	∞	s	0	∞

Table 1. Consumer segments, population size, valuation structure, and resulting demand in each period.

Proposition 1. *There exists a critical consumer valuation in market i*

$$v_i^* = \min\left(v_h, \frac{p - \tilde{\xi}_i s}{1 - \tilde{\xi}_i}\right), \tag{1}$$

$i = 1, 2, P$, where all strategic consumers with $v \geq v_i^*$ purchase in period 1 and all strategic consumers with $v < v_i^*$ delay purchasing until period 2.

Proof. All proofs appear in the appendix. □

Based on this result, total market i demand from strategic consumers in period 1 is $\bar{G}(v_i^*)\alpha D_i$, and $G(v_i^*)\alpha D_i$ consumers delay until period 2; intuitively, high valuation consumers do not wish to risk waiting for a markdown and hence purchase early at a high price, while low valuation consumers (with less potential loss if there is a stock-out) are more willing to strategically delay a purchase. A summary of the characteristics of the consumer population is provided in Table 1.

Consumers are not the only entities possessing beliefs in our model; the firm possesses a belief concerning the critical consumer valuation in market i (and hence the total period 1 demand in each market), which we label $\tilde{v}_i$. Given these consumer characteristics and firm beliefs, we may now write the expression for expected firm profit in market i as a function of inventory given a particular belief about consumer purchasing behavior:

$$\pi_i(q_i) = \mathbb{E}_{D_i}\left[(p-s)\min(q_i, \beta(\tilde{v}_i)D_i) - (c-s)q_i\right], \tag{2}$$

where $\beta(\tilde{v}_i) = \bar{G}(\tilde{v}_i)\alpha + \bar{G}(p)(1-\alpha)$ is the fraction of the main population (both myopic and strategic consumers) that purchases in period 1. Lastly, we define a useful quantity

$$\Pi(\mu, \sigma) = (p-c)\mu - (p-s)\sigma L(\hat{z}) - (c-s)\hat{z}\sigma,$$

where $\hat{z}$ is the standard normal z-statistic corresponding to an in-stock probability of $\frac{p-c}{p-s}$. $\Pi(\mu, \sigma)$ is the optimal newsvendor expected profit when demand is normally distributed with mean μ and standard

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deviation σ; this will represent an upper bound on firm profit in our model.

4   Equilibrium to the Inventory-Purchase Timing Game

Having defined the underlying characteristics of both the firm and the consumer population, we may now define the equilibrium to the game and between the firm and consumers. We begin by discussing beliefs in the game, beginning with consumer beliefs about inventory availability. Consumers make their purchasing decisions based on the anticipated probability of obtaining a unit during the clearance sale, $\tilde{\xi}_i$. We assume that consumers do not directly observe and react to the inventory level of the firm when making their purchasing decisions. In other words, the firm does not act as a sequential leader in an inventory game; rather, we assume that consumers possess a common, fixed belief $\tilde{\xi}_i$ of the second period inventory availability, a belief which we will require to be correct in equilibrium (i.e., a rational expectations assumption, see Su & Zhang 2008; Cachon & Swinney 2009).[4] We make this assumption due to the fact that, in practice, it is difficult for individual consumers to accurately observe the inventory level of a firm (e.g., because inventory is held in many locations such as retail shelves, back rooms, warehouses and distribution centers, and throughout the supply chain) and moreover a firm does not necessarily have incentives to reveal this information to consumers (Yin *et al.* 2009), implying that even if consumers could directly observe inventory they may not believe that this information is credible.

Clearly, the probability that a consumer obtains a unit in the clearance sale will depend on a number of factors: the anticipated inventory in the clearance period, the anticipated demand from the myopic and strategic consumers, and the anticipated demand from the bargain hunting segment (which, as we have assumed, is infinite), and precisely when a particular consumer expects to arrive during the clearance sale, relative to all other consumers. A number of methods to model this probability have arisen in the literature. One method is to assume efficient rationing during the clearance sale (i.e., that higher valuation consumers are allocated inventory ahead of lower valuation consumers), as in Su & Zhang (2008). This is typically justified by claiming that higher valuation consumers are "more eager" to obtain a unit and hence are more likely to closely monitor the firm for a clearance sale and purchase immediately after a price reduction; however, efficient allocation is clearly a restrictive assumption. Another approach is to assume a completely random apportioning of inventory, resulting in the probability equaling the second period fill rate (fraction of fulfilled demand), as in Liu & van Ryzin (2008). However, because we have assumed an infinite population of bargain hunting consumers in period 2, this quantity is, strictly speaking, zero in our model.

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[4]Consumers might form such expectations via repeated interaction with the firm in many single shot games, learning about the average clearance sale availability on similar products over time. Su & Zhang (2009) demonstrate how consumer beliefs about availability can converge to the one-shot fixed expectation equilibrium in such a setting.

We thus follow a hybrid approach: we assume that, within the main population of consumers purchasing in period 2, allocation is completely random (all consumers have an equal probability of obtaining a unit, independent of their valuations), while the infinite population of bargain hunting consumers purchases after the main population. This might the case if, e.g., consumers who visited the firm in period 1 and intentionally delayed a purchase are already aware of the product and hence are more eager to buy in a clearance sale, but there is no priority within this class of customers based on their valuations; bargain hunters, on the other hand, may trickle into the firm more slowly over time, eventually exhausting inventory after the main population consumers have had a chance to purchase. As a result, the anticipated probability of obtaining a unit in the clearance sale is the *fill rate* of second period demand from the main population, i.e., the fraction of second period demand that is fulfilled. The fill rate is defined as the ratio between expected second period sales and expected second period demand (Deneckere & Peck 1995; Porteus 2002), both restricted to the main population of consumers (i.e., excluding bargain hunters). For brevity, we refer to this quantity as the *second period fill rate*.

Consequently, consumers and the firm play a simultaneous game: the firm chooses an inventory level subject to some belief about consumer purchasing behavior (i.e., what fraction of consumers will purchase at the full price) while consumers choose whether to purchase in period 1 or wait until period 2, subject to some belief about the second period fill rate. We call this game the **inventory-purchase timing game**, and an equilibrium to this game is a set of actions (inventory level and critical consumer valuation) in which both consumers and the firm choose optimal actions in response to their beliefs, which are consistent with the equilibrium outcome. Thus, we define the equilibrium as follows:

Definition 1. An equilibrium to the inventory-purchase timing game satisfies the following conditions, for $i = 1, 2$ in the non-pooled system and $i = P$ in the pooled system:

1. The firm maximizes expected profit subject to beliefs about consumer behavior: $q_i^* = \arg\max_{q_i} \pi_i(q_i, \tilde{v}_i)$.

2. Strategic consumers purchase in the period that maximizes their expected surplus subject to beliefs about product availability in the clearance period: $v_i^* = \min\left(v_h, \frac{p - \tilde{\xi}_i s}{1 - \tilde{\xi}_i}\right)$.

3. Firm beliefs are rational: $\tilde{v}_i = v_i^*$.

4. Consumer beliefs are rational: $\tilde{\xi}_i = \xi_i(q_i^*, v_i^*)$.

In the above definition, $\xi_i(q_i^*, v_i^*)$ represents the actual second period fill rate given an inventory level q_i^* and critical consumer valuation v_i^*. Second period demand from the main population equals $(1 - \beta(v_i^*)) D_i$,

while second period inventory equals $(q_i^* - \beta(v_i^*)D_i)^+$, where $(x)^+ = \max(x, 0)$. Thus, the fill rate is

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\begin{aligned}
\xi_i(q_i^*, v_i^*) &= \frac{\text{Expected Period 2 Sales}}{\text{Expected Period 2 Demand}} \\
&= \frac{\mathbb{E}_{D_i}\left[\min\left((1 - \beta(v_i^*))D_i, (q_i^* - \beta(v_i^*)D_i)^+\right)\right]}{\mathbb{E}_{D_i}\left[(1 - \beta(v_i^*))D_i\right]}.
\end{aligned}
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Next, we derive the firm's best response function (i.e., the optimal inventory decision given a particular belief about consumer purchasing behavior) in Proposition 2, for both the non-pooled and pooled systems.

Proposition 2. *(i) In the non-pooled system, the firm's best response inventory level in each market is $q_i^*(\tilde{v}_i) = \beta(\tilde{v}_i)(\mu + \hat{z}\sigma)$, and the expected profit of the firm in market i is $\pi_i^*(\tilde{v}_i) = \beta(\tilde{v}_i)\Pi(\mu, \sigma)$.*

(ii) In the pooled system, the firm's best response inventory level is $q_P^*(\tilde{v}_P) = \beta(\tilde{v}_P)\left(2\mu + \hat{z}\sigma\sqrt{2(1 + \rho)}\right)$, and the expected profit of the firm is $\pi_P^*(\tilde{v}_P) = \beta(\tilde{v}_P)\Pi\left(2\mu, \sqrt{2(1 + \rho)}\sigma\right)$.

With the firm's best response functions in-hand, we may now demonstrate the existence and uniqueness of an equilibrium to the inventory-purchase timing game, which the following proposition accomplishes:

Proposition 3. *In both the non-pooled and pooled systems, an equilibrium to the inventory-purchase timing game exists and is unique. In the non-pooled system, the equilibrium is symmetric across markets ($v_1^* = v_2^* \equiv v_{NP}^*$ and $q_1^* = q_2^* \equiv q_{NP}^*$). Moreover, the equilibrium critical consumer valuation in either system is always greater than the selling price ($v_{NP}^*, v_P^* \geq p$).*

The existence and uniqueness of an equilibrium is ensured in both systems because the firm's best reply is decreasing in the critical consumer valuation; a higher critical consumer valuation equates to less demand in period 1, which in turn leads to a lower optimal quantity. Conversely, the critical consumer valuation is increasing in the firm's inventory level: higher inventory means greater availability during the clearance sale, encouraging more consumers to strategically delay a purchase. Because one best reply (the firm's) is decreasing in the opponent's action and the other (consumer's) is increasing in the opponent's action, and both functions are continuous, a unique equilibrium must exist.

5   The Value of Inventory Pooling

Having established that a unique equilibrium to the inventory-purchase timing game exists, we may now proceed to analyze how that equilibrium is influenced by inventory pooling. We first characterize the equilibrium incremental value of pooling, which we define to be the difference between total expected firm profit in the pooled and non-pooled systems:

Proposition 4. *The incremental value of pooling is*

$$\pi_P^* - (\pi_1^* + \pi_2^*) = \beta(v_{NP}^*)\Delta + \alpha\left(G(v_{NP}^*) - G(v_P^*)\right)\Pi\left(2\mu, \sqrt{2(1+\rho)}\sigma\right),\tag{3}$$

where $\Delta \equiv \left((p-s)L(\hat{z}) + (c-s)\hat{z}\right)\left(2 - \sqrt{2(1+\rho)}\right)\sigma.$

As the proposition demonstrates, the incremental value of pooling can be represented as the sum of two distinct terms. We refer to the first term of equation (3) as the *operational value* of pooling, because this value is non-zero even when no strategic consumers are present in the market (i.e., when $\alpha = 0$). We label the second term in equation (3) the *behavioral value* of pooling due to the fact that it is non-zero only if some strategic consumers are present in the market ($\alpha > 0$) and if $v_{NP}^* \neq v_P^*$–i.e., only if pooling results in a change in equilibrium behavior amongst the strategic consumer segment.

The operational value of pooling is the product of two factors: the total fraction of demand that buys in the first period in the non-pooled system, $\beta(v_{NP}^*)$, and Δ, an expression that captures the change in the optimal expected newsvendor costs resulting from pooling demand. The operational value of pooling (in particular, the Δ term) is the source of value that has been explored by the vast majority of the operations literature on pooling, starting with Eppen (1979). Recalling that $\beta(v_{NP}^*) = \bar{G}(v_{NP}^*)\alpha + \bar{G}(p)(1-\alpha)$, observe that when there are no strategic consumers in the market ($\alpha = 0$), the operational value of pooling is $\bar{G}(p)\Delta$. If there are strategic consumers in the market ($\alpha > 0$), the operational value is $\left(\bar{G}(v_{NP}^*)\alpha + \bar{G}(p)(1-\alpha)\right)\Delta$; from Proposition 3, $v_{NP}^* \geq p$, so it follows that $\bar{G}(v_{NP}^*)\alpha + \bar{G}(p)(1-\alpha) \leq \bar{G}(p)$. In other words, the operational value of pooling is a smaller multiple of Δ if consumers are strategic, and hence strategic consumer behavior *lowers* the operational value of pooling by reducing full price demand–because there is less demand, there is less value in combining the demand streams from the individual markets.

The following proposition formally summarizes these observations, in addition to confirming a key result from the existing literature on pooling:

Proposition 5. *The operational value of pooling positive, lower if some consumers are strategic than if all consumers are myopic, and is decreasing in ρ.*

The last part of the proposition demonstrates that even when consumers behave strategically, a crucial finding of the pooling literature–that the operational value of pooling is decreasing in the correlation of market demands–continues to hold. Thus, even when strategic consumers exist in the market, the operational value of pooling behaves in accordance with our intuition: it is greatest when markets are negatively correlated.

We next move to the behavioral value of pooling. This benefit is proportional to two key factors: the incremental change in period 1 demand resulting from pooling ($\alpha\left(G(v_{NP}^*) - G(v_P^*)\right)$) and the newsvendor

optimal expected profit in the pooled system, $\Pi\left(2\mu, \sqrt{2(1+\rho)}\sigma\right)$. In other words, the behavioral value represents the incremental change in profit solely resulting from changing consumer behavior by influencing the critical consumer valuation. If $v_{NP}^* > v_P^*$, the behavioral value of pooling is positive, while if $v_{NP}^* < v_P^*$ it is negative; consequently, a key determinant of the behavioral impact of pooling is whether pooling increases or decreases the equilibrium critical consumer valuation.

Combining the equilibrium conditions from Definition 1, it follows that the critical consumer valuation is determined by the solution to

$$v_i^* = \min\left(v_h, \frac{p - \xi_i(q_i^*, v_i^*)s}{1 - \xi_i(q_i^*, v_i^*)}\right), \tag{4}$$

where $\xi_i(q_i^*, v_i^*)$ is the fill rate of second period demand from the main population. Thus, it is apparent that the critical issue is whether pooling increases or decreases the second period fill rate for a particular critical consumer valuation; if pooling decreases the second period fill rate, then the probability that a consumer obtains a unit during the clearance sale is lower under pooling, implying that pooling discourages consumers from strategically waiting for the clearance sale. This has the effect of decreasing the right hand side of (4) and hence decreasing the equilibrium critical consumer valuation.

At first glance, one might imagine that inventory pooling would unambiguously reduce the second period fill rate–after all, pooling reduces demand variability and hence should minimize supply-demand mismatch, decreasing the amount inventory remaining for the clearance sale and lowering product availability. This intuition, however, is only partially correct, and indeed pooling may result in a decrease *or increase* in the second period fill rate. This reason for this derives from the fact that pooling influences the second period fill rate via two competing mechanisms.

The first mechanism, which we label the *inventory effect*, results from the fact that, all else being equal, pooling decreases the total amount of inventory remaining after period 1, as argued above. This can be seen quantitatively by examining the expression for expected leftover inventory after the fist period, $\beta(v_i^*)\sigma\left(\hat{z} + L(\hat{z})\right)$. Holding v_i^* constant, total leftover inventory in the non-pooled system is $2\beta(v_i^*)\sigma\left(\hat{z} + L(\hat{z})\right)$, while total leftover inventory in the pooled system is $\sqrt{2(1+\rho)}\beta(v_i^*)\sigma\left(\hat{z} + L(\hat{z})\right)$. In other words, for a particular critical consumer valuation, pooling reduces the total amount of inventory available to sell during the clearance period, a result that holds even when $\hat{z}$ is negative (that is, the critical ratio is less than one half and safety stock is negative), because $\hat{z} + L(\hat{z}) \geq 0$; despite the fact that pooling results in an increase in inventory in these cases, the associated reduction in period 1 demand variability (and hence lost sales) more than makes up for the inventory increase. Consequently, pooling **reduces** the second period fill rate by reducing inventory in the second period.

If the inventory effect were the only mechanism by which pooling influenced product availability during
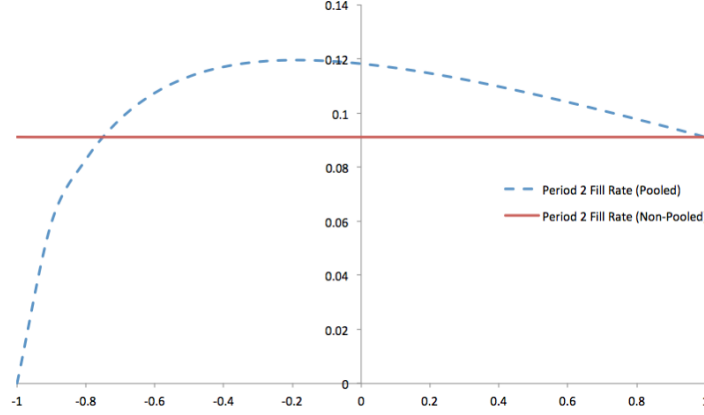
Figure 2. Example of the impact of pooling on equilibrium fill rates.

the clearance sale, then pooling would always result in a reduction in the second period fill rate. There is, however, another mechanism at work, which we call the *demand variability effect*, deriving from the fact that, all else being equal, pooling reduces the variability of second period demand from the main population of consumers. The demand variability effect occurs because second period demand is a multiple of the total market size, and pooling decreases the variability of the aggregate market size. Holding the amount of inventory during the clearance sale constant, less variable demand results in a greater second period fill rate; hence, pooling **increases** the average fill rate via by reducing demand variability in the second period. The inventory effect and the demand variability effect push the second period fill rate in opposite directions when pooling is adopted. If the inventory effect dominates, the fill rate is reduced by pooling, while if the demand variability effect dominates the fill rate is increased by pooling.

To illustrate how both cases can occur, consider the following simple example in which all consumers are myopic ($\alpha = 0$). The product economics are $p = 10$, $c = 9$, $s = 8$, meaning the critical ratio is $1/2$ and the corresponding $\hat{z}$ value is 0. Demand in each market is normally distributed with mean 150 and standard deviation 100. Consumers have valuations uniformly distributed in $[9, 11]$, which implies (because all consumers are myopic) that half of the consumers attempt to purchase in period 1 while the remaining half (with valuations less than 10, the selling price) wait until period 2. The parameter values in this example eliminate any behavioral effects, and in addition, as the critical ratio of $1/2$ implies no safety stock is held by the firm, aggregate inventory in the non-pooled and pooled systems is the same. Figure 2 plots the second period fill rate in this example as a function of the demand correlation in both the non-pooled and pooled systems. In the non-pooled system, the fill rate for second period demand is 9.1% in each market. The second period fill rate in the pooled system, however, may be higher or lower than this value, depending on the demand correlation. In particular, if the correlation is less than approximately -0.75, the fill rate is lower under pooling, while if the correlation is greater than this value, the fill rate is higher under pooling.

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The reason for this is that for highly negatively correlated markets, the inventory effect dominates; in this case, pooling results in such a substantial decrease in second period inventory that this effect overwhelms the demand variability effect. When demand is more positively correlated, the inventory effect is weaker, and the reduction in second period demand variability begins to dominate leading to a net increase in the second period fill rate.

This example demonstrates that pooling influences the second period fill rate via opposing mechanisms–decreasing both inventory and demand variability in the second period–and as a result, this implies that pooling may influence the critical consumer valuation in either direction, leading to positive or negative behavioral value to the firm. Consequently, it is important to characterize conditions that lead to one case over the other, as the following proposition accomplishes:

Proposition 6. *There exists some $\bar{\rho}$ such that, for all $\rho < \bar{\rho}$, pooling decreases the equilibrium critical consumer valuation and results in positive behavioral value.*

Proposition 6 shows that if the underlying markets are sufficiently negatively correlated, pooling possesses positive behavioral value. This aligns with the intuition discussed in the above example: for highly negatively correlated markets, pooling results in a substantial reduction in second period inventory, leading the inventory effect to dominate the demand variability effect. In these cases pooling leads to a reduction in the second period fill rate, which in turn reduces strategic consumer incentives to delay a purchase, encouraging more consumers to buy at the full price. Thus, for sufficiently negatively correlated markets pooling unambiguously benefits the firm: the operational and behavioral values are both positive, meaning the total value of pooling is positive.

In a similar manner to Proposition 6, the following proposition characterizes the impact of pooling on the critical consumer valuation as a function of the z statistic:

Proposition 7. *There exists some $\bar{z}$ such that, for all $\hat{z} > \bar{z}$, pooling decreases the equilibrium critical consumer valuation and results in positive behavioral value.*

Recalling that $\hat{z}$ is determined by the critical ratio, Proposition 7 is equivalent to saying that if the critical ratio is sufficiently high (implying high product margins or a high clearance price), pooling possesses positive behavioral value. When the critical ratio is high, then the firm carries substantial safety stock–in these cases, the inventory effect on the fill rate is pronounced (i.e., because the safety stock is substantial, pooling results in a dramatic decrease in safety stock and hence period 2 availability) and dominates the demand variability effect.

Taken together, Propositions 5-7 help to provide a complete picture of the impact of strategic consumer behavior on the practice of inventory pooling. When consumers behave strategically, there are two dimensions
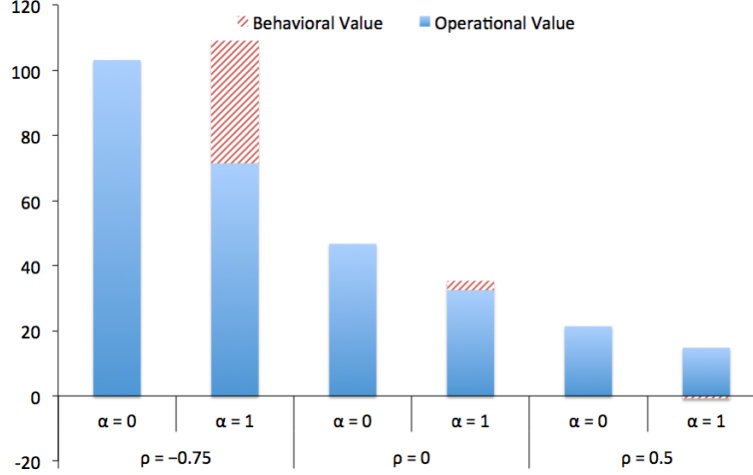
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Figure 3. Three possible ways that strategic consumer behavior can impact the total value of pooling. In the example, $p = 10$, $c = 6$, $s = 2$, demand is normally distributed with mean 100 and standard deviation 50 in each market, and consumer valuations are uniformly distributed between 8 and 12.

to the value of inventory pooling: operational and behavioral. The operational dimension, which exists even when all consumers are myopic, is reduced in magnitude by strategic consumer behavior but otherwise behaves in accordance with our intuition (e.g., it is always positive and decreasing in demand correlation).

The behavioral dimension, on the other hand, exists only when consumers are strategic, and may increase or decrease firm profit, depending on the product and market characteristics. The behavioral value is most likely to be positive if the markets are negatively correlated or if the product margins and critical ratio are high. When markets are sufficiently positively correlated or the product margins and critical ratio are sufficiently low, pooling can increase the second period fill rate, leading to an increase in the number of consumers who strategically wait for the clearance sale. Thus, in these cases, unlike the well-known operational value of pooling, the behavioral dimension can lead to a reduction in firm profit even, when the firm employs an optimal inventory policy and pooling is "free" (i.e., has no additional fixed or marginal costs).

Because one component of the value of pooling (operational) is reduced by strategic behavior, but another source of potential value (behavioral) is introduced, it is thus possible for strategic consumer behavior to lead to an increase or decrease in the total value a firm assigns to inventory pooling. We differentiate between three cases, depicted graphically in Figure 3:

1. *The total value of pooling is increased by strategic consumer behavior.* If the markets are negatively correlated or the critical ratio is high, then the behavioral benefit of pooling is large and more than makes up for the reduction in the operational benefit, leading the firm to value pooling more if consumers are strategic than if they are myopic. This scenario seems to align with recent examples of

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pooling implementations in the retail apparel industry (Fowler & Dodes 2010), where strategic consumer behavior is a well-known phenomenon (O'Donnell 2006) and gross margins are typically high. This case is illustrated in the leftmost scenario in Figure 3.

2. *The total value of pooling is reduced by strategic consumer behavior, but both the components are positive.* Provided the markets are not too positively correlated or the critical ratio is not too low, then the behavioral benefit is still positive, but is not large enough to make up for the reduction in the operational value of pooling. Consequently, the net value of pooling is reduced by strategic consumer behavior, even though both dimensions of pooling increase the profit of the firm. This case is illustrated in the middle scenario in Figure 3.

3. *The total value of pooling is reduced by strategic consumer behavior, and the behavioral value of pooling is negative.* If the markets are sufficiently positively correlated or the critical ratio is sufficiently low, then the behavioral value of pooling is negative, meaning that by pooling inventory the firm encourages more strategic consumers to wait for the clearance sale. In these scenarios, a purely operational analysis that ignores the behavioral impact of pooling is likely to overestimate the value of a pooling system. This case is illustrated in the rightmost scenario in Figure 3.

In Figure 3, observe that in the rightmost scenario, when pooling has negative behavioral value, the magnitude of the behavioral value is quite small relative to the operational value. This is no coincidence and indeed is representative of many examples that we have observed. The reason for this is that the behavioral value of pooling is proportional to firm profit in the pooled system. Recalling that pooling is likely to have negative value when demand is positively correlated and margins are low, this is precisely when pooled profit is smallest; as a result, we typically a observe that when the behavioral value of pooling is negative it is small in comparison to the operational value, whereas when the behavioral value is positive it is of comparable magnitude to the operational value, a feature we explore further in §7, in addition to analyzing how frequently the behavioral value is negative over a wide range of parameter combinations.

Lastly, we note that the behavioral value of pooling is not necessarily monotonic in the correlation of market demands. Figure 4 provides an example of this effect, which derives from the similarly non-monotonic impact of demand correlation on the second period fill rate depicted in Figure 2. Thus, while it is true that the behavioral value of pooling is greatest when demands are very negatively correlated (i.e., the behavioral value is decreasing in ρ for sufficiently negative ρ), it is also true that the behavioral value can be increasing in the demand correlation for greater ρ (though this depends on the problem parameters, and need not always be the case). Consequently, the qualitative behavior of this component of pooling can be precisely the opposite of the operational value of pooling.
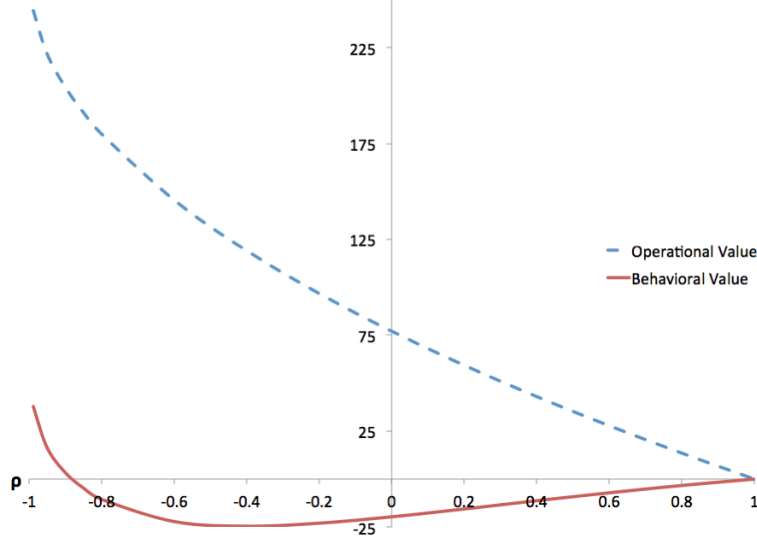
Figure 4. Example of non-monotonicity of the behavioral value of pooling as a function of market correlation. In the example, $p = 10$, $c = 7$, $s = 2$, demand is normally distributed with mean 150 and standard deviation 100 in each market, consumer valuations are uniformly distributed between 8 and 12, and all consumers are strategic.

6 Consumer Welfare

Thus far, we have focused on understanding the impact of consumer behavior on the value of inventory pooling to the firm. In this section, we reverse our stance, considering the impact of inventory pooling on consumer welfare. Because inventory pooling directly impacts service levels, inventory availability, and consumer purchase timing, it will clearly have an effect on the welfare of the consumer population. Indeed, many retailers (e.g., Nordstrom and Jones Apparel Group, Fowler & Dodes 2010) and fulfillment providers emphasize the customer service aspect of inventory pooling, claiming that increased inventory availability enabled by pooling will help customers avoid stock-outs and find products that they desire. This argument, however, fails to address the fact that the firm's optimal inventory level differs under a pooled system from the optimal inventory in a non-pooled system. Once the firm optimally adjusts its inventory in response to a pooling system, it is unclear how and to what extent consumer welfare will be impacted. Thus, a natural question to ask is: does pooling benefit consumers, and if so, in what ways?

To answer this question, we analyze equilibrium consumer welfare in the pooled and non-pooled systems, where "welfare" is defined to be the average consumer surplus on a successful purchase times the total number of sales. We assume, for simplicity, that consumers have valuations for the product uniformly distributed on the interval $[v_l, v_h]$, rationing is random (i.e., all consumers have equal probability of obtaining a unit), and that bargain hunting consumers receive zero surplus. To build an intuition for the impact of pooling on welfare, we focus on two extreme cases: either all consumers are myopic ($\alpha = 0$) or all consumers are strategic ($\alpha = 1$), starting with the former.

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Proposition 8. *If all consumers are myopic ($\alpha = 0$), pooling increases total consumer welfare in period 1, and if ρ is sufficiently negative, decreases total consumer welfare in period 2.*

As the proposition shows, with myopic consumers, expected sales in the first period (at the full price) always go up due to pooling, a natural consequence of the reduction in demand variability; this implies that consumer welfare in period 1 always increases. Thus, retailers that claim pooling benefits consumers are correct, at least in part: myopic consumers who buy at high prices are better off under pooling. However, expected sales in the second period (at the clearance price) may go up or down due to pooling. Once again, the two opposing forces that impact the fill rate–the inventory effect and the demand variability effect–make the impact of pooling on second period welfare ambiguous, i.e., it is possible (if demand is sufficiently positively correlated) for pooling to increase the second period fill rate and hence increase welfare in the second period. Nevertheless, the opposite case is also possible, meaning that, depending on which effect dominates (the increase in period 1 welfare or the decrease in period 2 welfare), pooling can decrease total consumer welfare. Because the proposition covers the case of purely myopic consumers, pooling does not impact consumer purchase timing in this case, and this effect is driven entirely by changes in expected sales.

If consumers are strategic, the same basic consequences of pooling on consumer welfare persist, except with the added complication that pooling may induce more strategic consumers to purchase at the high price. If this is the case, there are lower valuation consumers in period 1 in the pooled system than in the non-pooled system. Thus, while the expected period 1 sales increase under pooling, the average surplus of a consumer in period 1 on a successful purchase decreases. If all consumers who wished to purchase in period 1 were able to do so, this would not reduce welfare; however, there may be rationing even in period 1, and as a result low valuation consumers may "block" high valuation consumers from purchasing at the high price. Consequently, it's possible for pooling to decrease consumer welfare even in period 1 if consumers are strategic. A welfare reduction in period 2 continues to be possible, just as in the myopic case, and as a result it's possible for pooling to unambiguously reduce consumer welfare if consumers are strategic.

These observations illustrate an important consequence of retailer initiatives to pool inventory: despite claims to the contrary, pooling need *not* benefit consumers, even those consumers wishing to purchase at a high price. The negative impact of pooling on consumers is driven by two factors: the fact that pooling reduces inventory availability during clearance sales, and the resultant shift in consumer purchase timing once strategic consumers begin to purchase at the full price rather than delaying until the clearance sale, leading to increased competition for scarce inventory at the high price. We investigate both of these effects in more detail in our numerical study in §7.

Parameter	Values
p	10
c	$\{5, 7, 9\}$
s	$\{2, 4\}$
μ	150
σ	$\{50, 75, 100\}$
ρ	$\{-0.75, -0.5, -0.25, 0, 0.25, 0.5, 0.75\}$
α	$\{0, 0.25, 0.5, 0.75, 1\}$
v_h	$\{12, 14, 16\}$
v_l	$\{6, 8, 10\}$

Table 2. Parameters used in numerical study.

7 Numerical Study

7.1 The Value of Pooling to the Firm

Thus far, we have shown that pooling possesses both operational and behavioral sources of value when consumers are strategic. We've demonstrated that the operational value of pooling is reduced by strategic consumer behavior, while the behavioral value may be positive (if demands are sufficiently negatively correlated or if the critical ratio is sufficiently high) or negative. However, a number of interesting questions remain regarding the magnitude and frequency of the effects we have identified. To investigate these, we employ an extensive numerical study, focusing on the following three questions:

1. How likely is it that the behavioral value is negative given reasonable parameter values, and under what conditions does negative behavioral value occur?

2. What is the magnitude of the behavioral value, relative to the operational value?

3. Is it ever the case that the total value of pooling (operational plus behavioral) is negative?

The study consists of 5,670 problem instances comprised of every combination of the parameter values in Table 2. The parameters were chosen to represent a wide range of realistic scenarios, e.g., narrow valuation distributions (uniform on $[10, 12]$) to wide valuation distributions (uniform on $[6, 16]$). Coefficients of variation of market size were chosen to be less than 1, as to ensure a low probability of negative demand with the underlying normal distribution. Critical ratios range from 0.125 to 0.833. For each parameter combination in the sample, we calculated the equilibrium expected firm profit in both the non-pooled and pooled systems, and thus were able to derive the value of pooling.

The behavioral value of pooling was negative in 42.9% of the sample (2,435 cases). Thus, cases where pooling has behavioral effects that are detrimental to the firm seem quite common. Comparing instances where pooling has negative behavioral value to instances where pooling has positive behavioral value, negative

| | Average Value When Pooling... | |
Parameter	...Has Positive Behavioral Value	...Has Negative Behavioral Value
c	6.5	7.6
s	3.1	2.9
σ	68	85
ρ	-0.055	0.073

Table 3. Summary of average parameter values and their impact on the behavioral value of pooling.

value occurs when, on average, costs are higher, salvage values are lower, demand is more variable, and demand is positively correlated (see Table 3). This aligns with the intuition provided in Proposition 6, i.e., that the behavioral value of pooling is likely to be positive on high critical ratio products with negatively correlated market demands, and demonstrates that the behavioral value is likely to be negative in the opposite conditions.

The average operational value of pooling in our sample was 64, while the average behavioral value of pooling, when positive, was 14. When the behavioral value was negative, the average was -7. The range of behavioral values was -66 to 252. Over the entire sample, the (absolute) magnitude of the behavioral value was, on average, 25% of the operational value. Thus, we conclude that the behavioral value of pooling is significant relative to the operational value, implying that behavioral considerations play an important role in the managerial decision of whether to invest in inventory pooling.

Despite the sometimes substantial negative behavioral value of pooling observed in our sample, in no instances did we observe total negative value of pooling (i.e., operational plus behavioral value). Hence, we conclude that while we cannot rule out negative total value with strategic consumers, due to the forces that we discussed in §5 (i.e., that the behavioral value is most likely to be small in magnitude when it is negative) it is unlikely given reasonable parameters; however, it is important to note that this analysis was performed ignoring any pooling costs (fixed or variable), and hence the reduced total value of pooling due to negative behavioral value may in fact lead to a decrease in profit from pooling once, e.g., fixed costs such as information systems or transshipment capabilities have been incorporated.

7.2 The Impact of Pooling on Consumer Welfare

Next, we perform a numerical study on the impact of pooling on consumer welfare, focusing on the following key question: how likely is it that the consumer welfare is reduced by pooling given reasonable parameter values, and under what conditions does this occur? For the consumer welfare study, we use the parameter values in Table 2, restricted to the $\alpha = 0$ and $\alpha = 1$ cases.

In our numerical analysis, we found that when consumers are entirely myopic ($\alpha = 0$), pooling results in an increase in consumer welfare in 84% of cases. In Proposition 8 and the subsequent discussion, we

| | Average Value When Pooling... | |
Parameter	...Increases Welfare	...Decreases Welfare
c	7.3	5.5
s	2.9	3.2
σ	77	60
ρ	0.025	-0.13

Table 4. Summary of average parameter values and their impact on consumer welfare with myopic consumers.

| | Average Value When Pooling... | |
Parameter	...Increases Welfare	...Decreases Welfare
c	7.6	5.3
s	2.9	3.3
σ	79	62
ρ	0.036	-0.12

Table 5. Summary of average parameter values and their impact on consumer welfare with strategic consumers.

argued that consumer welfare with myopic consumers is likely to decrease because of pooling when demand correlation is sufficiently negative and when critical ratios are high. This intuition is verified by examining our numerical sample. Instances in which pooling decreased consumer welfare were characterized by, on average, lower cost and higher salvage values (i.e., higher critical ratios), lower demand variability, and negatively correlated demands; see Table 4. The combination of these factors indicates that myopic consumers stand to lose the most in a pooled system when safety stocks in the non-pooled system are high (critical ratios are high) and a move to a pooled system leads to a large reduction in safety stock (e.g., because demand is negatively correlated).

As a function of the problem parameters, the same intuition derived in the myopic consumer case applies in the strategic consumer ($\alpha = 1$) case: as demonstrated in Table 5, pooling is more likely to be detrimental to consumers if the critical ratio is higher (cost is lower and salvage value higher), demand variability is lower, and demand is negatively correlated. Overall, we observed that when all $\alpha = 1$, pooling increases consumer welfare in 77% of cases analyzed. Thus, pooling is more likely to decrease consumer welfare if consumers are strategic than if they are myopic. As discussed in §6, this happens because, in addition to the forces that exist in the myopic consumer case, pooling may also induce strategic consumers to buy earlier, increasing competition among consumers for scarce inventory in period 1 and lowering the average surplus of a successful period 1 purchase.

Tables 4 and 5, combined with Table 3, lead to an interesting conclusion: pooling is most likely to benefit consumers precisely when the firm values pooling the least. Indeed, in every instance in our sample in which pooling had negative behavioral value to the firm (and hence the least overall value), pooling also increased consumer welfare; when pooling had positive behavioral value it increased consumer welfare only 63% of the time. In total, pooling was a "win-win" solution, resulting in both positive behavioral value to the firm

(because of inducing strategic consumers to pay a higher price) and increased welfare to consumers (because of greater product availability) in just 42% of the sample, highlighting that while consumers and the firm may mutually benefit from pooling, it is equally likely that pooling is detrimental to at least one party.

8   Discussion

Although pooling has been widely studied in the operations literature as a strategy for reducing demand uncertainty, making matching supply and demand easier for firms serving multiple markets, the consequences of pooling in the presence of rational, forward-looking, utility maximizing consumers have not been analyzed. In this paper, we have sought to fill that gap. We showed that strategic consumer behavior decreases the well-known operational value of pooling but introduces a new value along a behavioral dimension. This behavioral value of pooling results from a change in product availability during the clearance sale, and may be positive or negative depending on the circumstances. Moreover, we also examined how pooling impacts consumer welfare, demonstrating that it is possible for pooling to decrease consumer welfare because of a reduction in availability during the clearance period and because of increased competition for scarce inventory during the full price period.

These results are of managerial importance for at least four reasons. First, they show that strategic consumers do not impact the value of pooling in a trivial manner; rather, they interact with the underlying product characteristics (the critical ratio) and market characteristics (correlation of demands), affecting the value of pooling in different ways depending on the situation. Our results thus provide managerial insight about how strategic consumer behavior influences the value of inventory pooling: the presence of strategic consumers enhances the value of pooling on high margin, high critical ratio products (or, alternatively, products for which a target service level is set at a high value) with negatively correlated markets. This observation can help managers to target specific products or markets for pooling initiatives, and also to justify pooling initiatives in scenarios when they were previously believed to be too costly solely based on operational benefits (i.e., because of the potentially large behavioral benefits). The latter observation helps to explain (at least in part) the recent adoption of pooling practices at retailers in industries particularly prone to strategic consumer behavior, such as Nordstrom and the Jones Apparel Group. In contrast, on low margin products or markets that are highly positively correlated, pooling possesses negative behavioral value, leading firms to be less likely to invest in pooling when consumers are strategic under these market conditions.

Second, our results offer some insight regarding which type of pooling (product or location) is benefitted (or harmed) most by strategic consumer behavior. Broadly speaking, product pooling is most likely to be

characterized by negative demand correlations (if demand within a product category is relatively stable) while location pooling is most likely to be characterized by positive demand correlations (if the popularity of a product between geographic regions is based on common causes such as economic conditions, international fashion trends, weather, etc.). Our model demonstrates that strategic consumer behavior is more likely to increase the value of pooling if demands are negatively correlated–that is, in the product pooling case. When demands are positively correlated (which is more symptomatic of location pooling), it is more likely that strategic consumer behavior decreases the value of pooling. Generally speaking, the type of costs associated with each type of pooling may differ as well: product pooling is more likely to increase marginal costs (e.g., because of a more complex universal product) while location pooling is more likely to increase fixed costs (e.g., because of infrastructure like fulfillment capabilities and information systems). An increase in marginal costs due to product pooling will decrease the corresponding critical ratio, decreasing clearance period availability and thus enhancing the (already likely to be positive) behavioral benefit; hence, it seems even more likely that product pooling has greater behavioral benefits than location pooling, once the marginal costs of product pooling are considered.

Third, the results show that an operational practice–inventory pooling–has behavioral consequences that are potentially significant in comparison to the known, operational benefits, with qualitatively different behavior. This underscores the importance of considering wider ranging implications of operational and supply chain strategies when consumers may, themselves, be impacted by and react to those practices: ignoring the behavioral aspect of pooling could cause a firm to both over- and under-value the impact of a pooling initiative.

Lastly, the results demonstrate that pooling may, in some cases, be detrimental to consumer welfare. While this happens in a minority of cases (16% with myopic consumers, 23% with strategic consumers), it generally happens on particularly profitable, high salvage value products with negative demand correlations–precisely when the firm values pooling the most. Thus, consumers should be wary of firm claims that pooling is all about customer service–pooling provides a number of profit benefits (operational and behavioral) to the firm, and indeed may harm consumers as a result.

A   Proofs

Proof of Proposition 1. Because $\tilde{\xi}_i \in [0, 1]$, first period surplus is increasing in the consumer valuation faster (in the weak sense) than expected second period surplus. Hence, either $v_h - p \geq \tilde{\xi}_i(v_h - s)$, implying there exists some v (possibly less than v_l) such that $v - p = \tilde{\xi}_i(v - s)$, or $v_h - p < \tilde{\xi}_i(v_h - s)$, implying all strategic consumers delay purchasing until the second period. Because there is no mass at the endpoints of

the consumer valuation distribution by assumption, the result follows. □

Proof of Proposition 2. From equation (2), following the newsvendor solution, the optimal inventory level in market $i = 1, 2$ is $q_i^*(\tilde{v}_i) = \beta(\tilde{v}_i)F_i^{-1}\left(\frac{p-c}{p-s}\right)$, where $F_i^{-1}(x)$ is the inverse cumulative distribution function in market i. Due to the normal demand assumption, this may be written $q_i^*(\tilde{v}_i) = \beta(\tilde{v}_i)(\mu + \hat{z}\sigma)$ in the non-pooled system and $q_P^*(\tilde{v}_P) = \beta(\tilde{v}_P)\left(2\mu + \hat{z}\sigma\sqrt{2(1+\rho)}\right)$ in the pooled system, where $\hat{z}$ is the standard normal z-statistic corresponding to an in-stock probability of $\frac{p-c}{p-s}$. The expressions for expected profit follow from the newsvendor profit function evaluated at the quantities above. □

Proof of Proposition 3. (i) *The Non-pooled System.* From Proposition 2 and Definition 1, the equilibrium in the non-pooled system must satisfy the following conditions in each market: (1) the firm chooses the optimal inventory level, $q_i^* = \beta(\tilde{v}_i)(\mu + \hat{z}\sigma)$, (2) consumers purchase in the period that maximizes their utility, $v_i^* = \min\left(v_h, \frac{p - \tilde{\xi}_i s}{1 - \tilde{\xi}_i}\right)$, and expectations are rational, (3) $\tilde{v}_i = v_i^*$, and (4) $\tilde{\xi}_i = \xi_i(q_i^*, v_i^*)$. Combining conditions (1) with (3) and (2) with (4) yields $q_i^* = \beta(v_i^*)(\mu + \hat{z}\sigma)$ and $v_i^* = \min\left(v_h, \frac{p - \xi_i(q_i^*, v_i^*)s}{1 - \xi_i(q_i^*, v_i^*)}\right)$. Thus, a simultaneous solution to these two equations will provide the equilibrium. To derive this equilibrium, we must provide a functional form of $\xi_i(q_i^*, v_i^*)$, the actual probability that (in equilibrium) a consumer will obtain a unit if she delays until the clearance sale, i.e., the period 2 fill rate excluding bargain hunting consumers. Inserting the expression for the optimal inventory level of the firm and rearranging terms, the second period fill rate as a function solely of v_i^* is

$$\xi_i(v_i^*) = \frac{\mathbb{E}_{D_i}\left[\min\left(D_i, \frac{\beta(v_i^*)}{1-\beta(v_i^*)}\left((\mu + \hat{z}\sigma) - D_i\right)^+\right)\right]}{\mu}.$$

The term $\frac{\beta(v_i^*)}{1-\beta(v_i^*)}$, the ratio between first and second period demand, is decreasing in v_i^*. As a result, it follows that $\xi_i(v_i^*)$ is decreasing in v_i^*. Moreover, because the individual market demands are identically distributed, any equilibria must be identical in the two non-pooled markets and we replace i with NP. Define $\Omega_{NP}(v) \equiv \min\left(v_h, \frac{p - \xi_{NP}(v)s}{1 - \xi_{NP}(v)}\right)$. To see that a unique fixed point to $v_{NP}^* = \Omega_{NP}(v_{NP}^*)$ (which determines the equilibrium) exists, observe that $\xi_{NP}(v) \geq 0$, hence for any v, $\Omega_{NP}(v) = \frac{p - \xi_{NP}(v)s}{1 - \xi_{NP}(v)} \geq \frac{p - 0 \times s}{1 - 0} = p$. Thus, $\Omega_{NP}(v)$ is continuous and always lies in the compact interval $[p, v_h]$, implying Brouwer's Fixed Point Theorem applies (and, moreover, any fixed point must satisfy $v_{NP}^* \geq p$). Finally, $\Omega_{NP}(v)$ is decreasing in v since $(p - xs)/(1 - x)$ is increasing in x and $\xi_{NP}(v)$ is decreasing in v. Consequently, there must be exactly one point where $v_{NP}^* = \Omega_{NP}(v_{NP}^*)$ on the interval $[p, v_h]$, as demonstrated graphically in Figure 5. Because a unique v_{NP}^* exists, clearly a unique q_{NP}^* also exists, thus proving the proposition.

(ii) *The Pooled System.* The proof follows analogously to part (i), replacing the individual market demand mean and standard deviation (μ and σ, respectively) with the pooled market mean and standard deviation (2μ and $\sigma\sqrt{2(1+\rho)}$). □
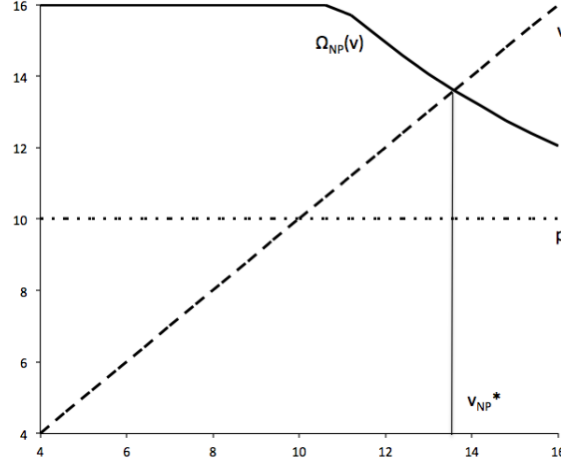
Figure 5. An example of equilibrium existence and uniqueness.

Proof of Proposition 4. From Propositions 2 and 3, equilibrium firm profit in the non-pooled system is

$$\pi_1^* + \pi_2^* = 2 \left(\bar{G}(v_{NP}^*)\alpha + \bar{G}(p)(1-\alpha) \right) \Pi(\mu, \sigma).$$

Also from Propositions 2 and 3, equilibrium profit in the pooled system is

$$\pi_P^* = \left(\bar{G}(v_P^*)\alpha + \bar{G}(p)(1-\alpha) \right) \Pi \left(2\mu, \sqrt{2(1+\rho)}\sigma \right).$$

Rewriting this expression,

$$\pi_P^* = \left(\bar{G}(v_P^*)\alpha - \bar{G}(v_{NP}^*)\alpha \right) \Pi \left(2\mu, \sqrt{2(1+\rho)}\sigma \right) + \left(\bar{G}(v_{NP}^*)\alpha + \bar{G}(p)(1-\alpha) \right) \Pi \left(2\mu, \sqrt{2(1+\rho)}\sigma \right).$$

Thus, the value of pooling is given by (3). □

Proof of Proposition 5. The fact that the operational value is positive and lower under strategic behavior is demonstrated in the discussion preceding the proposition. Δ is decreasing in ρ, since $\frac{d\Delta}{d\rho} = -\frac{((p-s)L(\hat{z})+(c-s)\hat{z})\sigma}{\sqrt{2(1+\rho)}}$, and, since $L(\hat{z}) = \phi(\hat{z}) - \hat{z}(1 - \Phi(\hat{z}))$ (see Porteus 2002), $L(\hat{z}) + \hat{z} = \phi(\hat{z}) + \hat{z}\Phi(\hat{z}) \geq 0$ implies $(p - s)L(\hat{z}) + (c - s)\hat{z} \geq 0$. The coefficient $\bar{G}(v_{NP}^*)\alpha + \bar{G}(p)(1-\alpha)$ is independent of ρ, hence the total operational value is decreasing in ρ. □

Proof of Proposition 6. Suppose $\rho = -1$. Then, pooled demand is deterministic, and the firm's optimal inventory in the pooled system is precisely enough to cover period 1 demand. Consequently, the period 2 fill rate is zero, and thus the equilibrium critical consumer valuation is $v_P^* = p \leq v_{NP}^*$. Because firm profit is continuous in ρ, the result follows. □

Proof of Proposition 7. To show the result, we will show that, for fixed consumer critical valuation

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v_i^*, pooling decreases the second period fill rate if $\hat{z}$ is sufficiently large; in turn, because the equilibrium critical consumer valuation is increasing in the second period fill rate (see the proof of Proposition 3) this will imply that the critical consumer valuation is decreased by pooling for large $\hat{z}$. The identity Total Sales = Expected 1st Period Sales + Expected 2nd Period sales implies that the second period fill rate may be written: 2nd Period Fill Rate = (Total Sales - Expected 1st Period Sales)/(Expected 2nd Period Demand). Thus,

$$\xi_i(q_i^*, v_i^*) = \frac{\mu_i - \sigma_i L\left(\frac{q_i^* - \mu_i}{\sigma_i}\right)}{(1 - \beta(v_i^*))\mu_i} - \frac{\beta(v_i^*)\left(\mu_i - \sigma_i L\left(\frac{q_i^* - \beta(v_i^*)\mu_i}{\beta(v_i^*)\sigma_i}\right)\right)}{(1 - \beta(v_i^*))\mu_i}.$$

Substituting the optimal inventory of the firm, $q_i^* = \beta(v_i^*)(\mu_i + \hat{z}\sigma_i)$, and setting $\lambda = \mu_i/\sigma_i$ leads to

$$\xi_i(v_i^*) = \frac{1 - \frac{1}{\lambda}L\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)}{1 - \beta(v_i^*)} - \frac{\beta(v_i^*)\left(1 - \frac{1}{\lambda}L(\hat{z})\right)}{1 - \beta(v_i^*)}.$$

We observe here that, for fixed v_i^*, pooling (which increases λ) unambiguously increases expected 1st period sales (the second term in the above expression). Hence, a sufficient condition for pooling to decrease the second period fill rate for fixed v_i^* is for pooling to decrease expected total sales divided by expected second period demand, i.e., for pooling to increase $\omega \equiv \frac{1}{\lambda}L\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)$. Because pooling equates to an increase in λ, this is equivalent to requiring that

$$\frac{d\omega}{d\lambda} = -\frac{1}{\lambda^2}L\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right) + \frac{1 - \beta(v_i^*)}{\lambda}\bar{\Phi}\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)$$

be positive (where, from the properties of the unit normal loss function, $L'(x) = -(1 - \Phi(x))$). Observe that $\frac{d\omega}{d\lambda}$ is positive if and only if

$$\frac{d\omega/d\lambda}{\bar{\Phi}\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)} = -\frac{1}{\lambda^2}\frac{L\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)}{\bar{\Phi}\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)} + \frac{1 - \beta(v_i^*)}{\lambda}$$

is positive. In the limit as $\hat{z}$ grows to infinity, by L'Hôpital's rule,

$$\lim_{\hat{z}\to\infty} -\frac{1}{\lambda^2}\frac{L\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)}{\bar{\Phi}\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)} + \frac{1 - \beta(v_i^*)}{\lambda} = \lim_{\hat{z}\to\infty} -\frac{1}{\lambda^2}\frac{\bar{\Phi}\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)}{\phi\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)} + \frac{1 - \beta(v_i^*)}{\lambda}$$

The normal distribution has increasing failure rate that converges to infinity, meaning $\frac{\bar{\Phi}(\beta(v_i^*)(\lambda+\hat{z})-\lambda)}{\phi(\beta(v_i^*)(\lambda+\hat{z})-\lambda)}$ is decreasing in $\hat{z}$ and $\lim_{\hat{z}\to\infty}\frac{\bar{\Phi}(\beta(v_i^*)(\lambda+\hat{z})-\lambda)}{\phi(\beta(v_i^*)(\lambda+\hat{z})-\lambda)} = 0$. Hence,

$$\lim_{\hat{z}\to\infty} -\frac{1}{\lambda^2}\frac{L\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)}{\bar{\Phi}\left(\beta(v_i^*)(\lambda + \hat{z}) - \lambda\right)} + \frac{1 - \beta(v_i^*)}{\lambda} = \frac{1 - \beta(v_i^*)}{\lambda} \geq 0,$$

meaning that, for sufficiently large $\hat{z}$, pooling results in a decrease in the second period fill rate, proving the result. $\square$

Proof of Proposition 8. *(i) The Non-Pooled System.* Let $S_{t,i}^*$ and $W_{t,i}^*$ be the equilibrium sales and welfare in market i in period t, respectively. If all consumers are myopic ($\alpha = 0$), the average surplus of a consumer purchasing in period 1 is $(v_h + p)/2 - p$, hence the consumer welfare in period 1 in market i is $W_{1,i}^* = \left(\frac{v_h + p}{2} - p\right) S_{1,i}^*$. Given the optimal firm inventory level, $S_{1,i}^* = \bar{G}(p)(\mu - \sigma L(\hat{z}))$. This implies total welfare in period 1 (across both markets) is $W_{1,1}^* + W_{1,2}^* = 2\left(\frac{v_h + p}{2} - p\right) \bar{G}(p)(\mu - \sigma L(\hat{z}))$.

In the second period, for any particular realization of demand D_i, the sales to the main population of consumers equals $\min(D_i, q_i^*) - \min(\bar{G}(p)D_i, q_i^*)$, i.e., total sales to the main population minus sales in the first period. Thus, expected sales in period 2 equals $S_{2,i}^* = \mathbb{E}\min(D_i, q_i^*) - S_{1,i}^*$. The average surplus of a consumer purchasing in the second period is $(p + v_l)/2 - s$, hence consumer welfare in period 2 in market i is

$$W_{2,i}^* = \left(\frac{p + v_l}{2} - s\right)\left(\mathbb{E}\min(D_i, q_i^*) - S_{1,i}^*\right).$$

$S_{1,i}^* = \bar{G}(p)(\mu - \sigma L(\hat{z}))$ and $\mathbb{E}\min(D_i, q_i^*) = \mu - \sigma L(\bar{G}(p)\hat{z} - G(p)\frac{\mu}{\sigma}))$. Hence,

$$W_{2,i}^* = \left(\frac{p + v_l}{2} - s\right)\left(G(p)\mu - \sigma\left(L(\bar{G}(p)\hat{z} - G(p)\frac{\mu}{\sigma}) - \bar{G}(p)L(\hat{z})\right)\right),$$

and the total welfare in period 2 (across both markets) is

$$W_{2,1}^* + W_{2,2}^* = 2\left(\frac{p + v_l}{2} - s\right)\left(G(p)\mu - \sigma\left(L(\bar{G}(p)\hat{z} - G(p)\frac{\mu}{\sigma}) - \bar{G}(p)L(\hat{z})\right)\right).$$

(ii) The Pooled System. Following in an analogous manner, first period welfare under pooling is

$$W_{1,P}^* = \left(\frac{v_h + p}{2} - p\right)\bar{G}(p)\left(2\mu - \sqrt{2(1+\rho)}\sigma L(\hat{z})\right) \geq W_{1,1}^* + W_{1,2}^*.$$

Second period welfare under pooling is

$$W_{2,P}^* = \left(\frac{p + v_l}{2} - s\right)\left(G(p)2\mu - \sqrt{2(1+\rho)}\sigma\left(L\left(\bar{G}(p)\hat{z} - G(p)\frac{2\mu}{\sqrt{2(1+\rho)}\sigma}\right) - \bar{G}(p)L(\hat{z})\right)\right).$$

Observe that $\bar{G}(p)\hat{z} - G(p)\frac{2\mu}{\sqrt{2(1+\rho)}\sigma} \leq \hat{z}$, hence $L\left(\bar{G}(p)\hat{z} - G(p)\frac{2\mu}{\sqrt{2(1+\rho)}\sigma}\right) \geq L(\hat{z}) \geq \bar{G}(p)L(\hat{z})$. This expression is non-monotonic in ρ, but in the limit as $\rho \to -1$, $W_{2,P}^* \to 0$ (i.e., there are no second period sales), leading to the result. $\square$

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