UNCERTAINTY IN LEARNING CURVES:

IMPLICATIONS FOR FIRST MOVER ADVANTAGE

Michael J. Leiblein
Fisher College of Business
The Ohio State University
leiblein.1@osu.edu

John S. Chen
Warrington College of Business Administration
University of Florida
john.chen@warrington.ufl.edu

Hart E. Posen
School of Business
University of Wisconsin-Madison
hposen@wisc.edu

March 2021

Acknowledgements: The authors contributed equally to the study and are listed in the order determined by a random draw. The authors have benefitted from conversations with seminar participants at the University of Pennsylvania, University of Michigan, the University of Southern California, Rensselaer Polytechnic Institute, the University of Colorado, and the 2019 Academy of Management Review theory development workshop. In addition, we are grateful for specific comments and suggestions provided by Paul Adler, Jay Barney, Janet Bercovitz, Ashton Hawk, Michael Jensen, Steven Postrel, Jeff Reuer, Richard Saouma, Jordan Seigel, Tony Tong, and Maggie Zhou.

UNCERTAINTY IN LEARNING CURVES:
IMPLICATIONS FOR FIRST MOVER ADVANTAGE

ABSTRACT: The existence of a learning curve, in which costs decline with cumulative experience, suggests that early entry (and production) provides learning opportunities that create advantage by reducing future production costs relative to later entrants. We argue that this proposition is subject to an under-appreciated limitation — that progress down the learning curve may be uncertain. If there is uncertainty in the learning curve, then the taken-for-granted wisdom regarding the benefits of learning curves may over- or under-emphasize the value of early entry. We consider two forms of uncertainty — prospective (future production costs) and contemporaneous (current production costs). We demonstrate computationally that while prospective uncertainty in the learning curve enhances the learning benefits of early entry and production, contemporaneous uncertainty reduces these benefits. Further, we examine the implication of these findings for competition and learning curve spillovers between leader and laggard firms. We conclude with implications for future research regarding learning curves and the pursuit of early mover advantage.

1. INTRODUCTION

The learning curve is often used to explain the presence and persistence of early mover advantage (e.g., Lieberman & Montgomery, 1988, 1998). A significant body of literature has described the learning curve — cost reduction or quality improvement that results from cumulative production experience — in a wide variety of industries (e.g., Arrow, 1962; Lieberman, 1984, 1987; Argote & Epple, 1990). Spence (1981) was the first to identify and systematically study the strategic implications of the learning curve. When learning curves are significant, firms may enter a market early, set prices below their marginal cost, and enjoy a competitive advantage as their costs subsequently fall below those of later-entering rivals. This claim has been widely adopted not only in the theoretical literature on early mover advantage, but also in management practice (e.g., Pisano, 1994; Reeves, Haanaes, & Sinha, 2015; Kiechel, 2010).

Implicit in the logic linking learning curves to early entry and advantage is the assumption that the rate of learning is deterministic and known to firms (e.g., Spence, 1981; Ghemawat & Spence, 1985). Firms, however, often face considerable uncertainty about the rate at which they will learn. Classic research on learning curves, such as Yelle (1979) and Argote and Epple (1990), document wide variation in the rate of learning across organizations and even between facilities within the same organization. Firms do not know with certainty, *ex ante*, the rate at which they will learn because the learning curve is the result of a complex oft repeated process that requires defining myriad of micro problems, determining which are worth addressing, intuiting possible solutions, choosing solutions, and implementing these solutions (Mishina, 1999). This has substantial performance implications. For example, Douglas Aircraft mis-estimated the learning curve in the production of the DC-9 — an error that led to their eventual (forced) merger with McDonnell Aircraft (Abernathy & Wayne, 1974).

The objective of this paper is to explore the strategic implications of uncertainty in the learning curve. It has been suggested that committed investment in the face of uncertainty is a defining feature of the strategic management field (e.g., Ghemawat, 1991; see also Leiblein, Reuer, and Zenger, 2018, and Van den Steen, 2018). Uncertainty may be the most salient challenge faced by firms as they seek to leverage learning curves to capture early mover advantage. In the absence of uncertainty, the strategic implications of learning curve logic are straightforward. In particular, increases in the learning curve lead to earlier entry with its associated benefits, and increases in learning curve spillovers from earlier to later entrants offset these benefits, potentially yielding late mover advantages (Spence, 1981; Ghemawat & Spence, 1985). However, when learning is uncertain, the managerial implications of early entry are ambiguous and the model of Spence (1981) may over- or under-emphasize the value of early entry.

We consider the implications of a firm's uncertainty about the *rate* at which it will progress down the learning curve for entry timing and early mover advantage.[1] More specifically, we consider both *prospective uncertainty* and *contemporaneous uncertainty* (Leiblein, Chen, & Posen, 2017; Posen, Leiblein, & Chen, 2018) in the learning curve. Prospective uncertainty results because firms may have limited knowledge about the extent to which production in the current period will lead to learning that reduces production costs in future periods. Recognition of prospective uncertainty in the learning curve allows us to conceptualize investments in early entry as a temporal series of real options. Early investments in production lead to learning that

[1]We use the term "uncertainty" for consistency with prior literature based on Black and Scholes (1973), and in particular, work on real options upon which we build (e.g., Dixit & Pindyck, 1994). In this extensive literature, the term "uncertainty" is used in place of the term "risk," even though the assumption is that the distribution is known.

creates options to engage or disengage in future production upon receipt of new information about the extent to which production costs have declined. Contemporaneous uncertainty results because firms may have limited knowledge about current period production costs. Given the imprecision of data, the complexity of the learning process, and the limits of organizational information processing routines, the firm may be unable to precisely estimate how much it has learned. Recognition of contemporaneous uncertainty in the learning curve allows us to conceptualize investments in early entry as a process of learning from noisy feedback (Knudsen and Levinthal, 2007), which raises the possibility of decision-making errors surrounding earlier versus later entry.

Industry contexts are characterized by varying degrees of both prospective and contemporaneous uncertainty, and as such, their joint consideration is critical to understanding the implications of learning curves for early mover advantage. The structure of our analysis follows Spence (1981). In our first experiment, we examine the single firm case. The key result is that learning curve uncertainty may increase or decrease what the literature describes as early or first mover advantages. In particular, when prospective uncertainty in the learning rate increases, the merits of early entry increase. In contrast, when contemporaneous uncertainty increases, the merits of early entry decrease. In a second experiment, we examine the two-firm case, first with competition alone, then with learning curve spillovers from a leader that enters first to a laggard firm that enters second. We find that, in general, combinations of the two forms of uncertainty tend to increase early mover advantage (due to preemption) at lower levels of competition, but decrease it at higher levels of competition for at all but the highest levels of prospective uncertainty. With regard to learning curve spillovers, we find that the results depend on the type of spillover. For example, when the spillover reduces the laggard's initial cost, higher prospective uncertainty reduces the damages to the leader, while the opposite is true from spillovers that increase the laggard's learning rate. Finally, spillovers from the leader to the laggard can be beneficial to the leader when the knowledge that spills over reduces the uncertainty in the laggard's learning curve.

In sum, the established claim that the existence of a learning curve provides a rationale for earlier entry is subject to an under-appreciated limitation — that progress down the learning curve may be uncertain. Our model provides predictions regarding how learning curve uncertainty impacts whether and when firms should invest in early entry and production to gain an early mover advantage. We identify conditions under which uncertainty in the learning curve may, counterintuitively, increase, rather than decrease, the rewards to earlier entry. More generally, when learning curve uncertainty is large relative to the expected learning rate, it is uncertainty, rather than expectations about the rate, that determines the extent of early mover advantage.

2. REVIEW

The literature on early mover advantage describes the "benefit enjoyed by a firm as the consequence of its early entry into a new market" (Lieberman, 2016: 1). This literature highlights the entry timing advantages associated with a diverse array of conceptual mechanisms including experience, preemption of valuable and scarce assets, the formation of buyer switching costs, and the creation of network effects. Disadvantages associated with free rider effects, adverse markets, and technical change are also described (Lieberman & Montgomery, 1988; Suarez & Lanzolla, 2007).

We focus on one of the core mechanisms associated with early mover advantage — the learning curve (Lieberman & Montgomery, 1988).[2] The literature on learning curves is long and

[2] Related literature on forward pricing, limit pricing, and umbrella pricing notes how reductions in pricing may deter entry. These studies recognize how learning curve-related cost reductions facilitate the ability to set a limit price. There is also a literature on

distinguished, with seminal contributions by Wright (1936), Hirsch (1952), Arrow (1962), and Alchian (1963). The literature also provides ample empirical support for learning curve effects in experience- or scale-driven industries such as aircraft production (Wright, 1936), chemical processing (Lieberman, 1984), aircraft and trucks (Argote, 1990), semiconductor production (Gruber, 1992), and even pizza production (Darr, Argote, & Epple, 1995).

Learning curves are not simply about cost reduction in the domain of classic production contexts. As highlighted in Argote (1999) and Thompson (2012), learning curves have been observed across a variety of dimensions of performance (e.g., costs, survival, recalls, customer satisfaction) and levels of analysis (e.g., groups, plants, firms). Stiglitz and Greenwald (2015: 36) argue that the learning-by-doing (Arrow, 1962) that underlies learning curves is particularly relevant today. They note that "even advances in leading-edge technology are typically the result of small improvements...They are the result of... learning from doing."

The basic logic linking learning curves and early mover advantage is that production in the current period has implications for both current and future performance. The logic first put forward by Spence (1981) states that "when learning occurs, part of the firm's short-run marginal cost can be regarded as an investment, which reduces the cost of production in future periods" (Lieberman, 1984: 214). The implication for current performance is a function of the profit (or loss) achieved through current production. The implication for future performance is a function of future cost reductions from progress down the learning curve.

Other foundational studies on the implications of learning curves examine the consequences of learning curve spillovers across rival firms. For instance, Ghemawat and Spence (1985: 839) state that it is "widely believed... that learning often cannot be kept entirely proprietary." While

---

strategic behavior (e.g., Fudenberg & Tirole, 1983; Dasgupta & Stiglitz, 1988). For instance, work has considered preemption in a deterministic setting (e.g., Fudenberg & Tirole, 1985), which has been extended to incorporate external shocks that take the form of Brownian motion (Dixit & Pindyck, 1994)).

proprietary learning curve effects may provide an early mover advantage, spillovers reduce the magnitude of subsequent cost differentials and attenuate that advantage. In sum, the implications of the basic model of learning curves for early movers are potentially offset by the presence of spillovers across firms (Lieberman, 1987).

A central, often implicit, assumption in the extant literature is that the rate of learning is deterministic and known by firms. This assumption belies the substantial empirical evidence of uncertainty about the rate of progress down a learning curve. Thompson (2012: 221), in a relatively recent review of the learning curve literature, concludes that "in the standard formulation of organizational learning, cost reductions are obtained as a *predictable* by-product of accumulated production volume … (yet) not only are variations in the rate of learning difficult to predict, they are difficult to understand after the fact" (italics added). Yelle (1979) notes differences across plants in learning rates, even those employing the same production technology. Argote and Epple (1990: 924) conclude that "there is great variation in the rate at which organizations learn, ranging from production programs with little or no learning to those with impressive productivity growth." These authors further state that there "is often more variation across organizations or organizational units producing the same product than within organizations producing different products" (p. 921). Indeed, Thompson (2012), using well-known data from WWII Liberty ship building, shows that different firms producing near identical ships exhibited very different learning rates. In sum, it seems that the assumption that learning curves are deterministic and known to managers is often inappropriate.

3. UNCERTAIN LEARNING CURVES

We examine how uncertainty in the learning curve impacts early mover advantage. We recognize that learning curves may differ in several dimensions, each of which may be subject to uncertainty. For instance, learning curves may differ in initial cost (i.e., starting point of learning), rate of progress (i.e., slope of learning), and long-run potential for cost reductions (i.e., asymptote of learning). We focus on the rate of progress in the learning curve as it is arguably the most salient dimension highlighted in the literature, but we return to a brief analysis of uncertainty in the asymptote of learning later in our analysis of the model.

We further recognize that prominent theories of decision-making under uncertainty formalize different forms of uncertainty. For instance, uncertainty about the future state of the world, which we call prospective uncertainty, is highlighted by the real options model. In contrast, uncertainty about the current state of the world, which we call contemporaneous uncertainty, results from noisy feedback. In the strategy literature it is often considered via the bandit model with belief updating taking the form of fractional updating or a Kalman (Bayes) filter.

As an example of these two forms of uncertainty, consider a semiconductor firm deciding, at the start of 2018, whether or not to begin production using a new 10nm process technology.[3] The unit production cost of such a fabrication process is, in part, dependent on “yield,” which is the fraction of dies on a silicon wafer that provides a usable output from the manufacturing process.. At any point in time, the firm faces two types of uncertainty in its decision-making regarding whether or not to produce with the 10nm process. The firm faces prospective uncertainty due to its limited knowledge about the extent to which production experience with the 10nm process in 2018 will reduce production cost in 2019 and beyond. The firm also faces contemporaneous uncertainty. It has limited knowledge about its current cost of producing using the 10nm process. Contemporaneous uncertainty may manifest at the start of 2018, as the firm may be uncertain of

[3] This industry has been subject to several well-received studies of experience curve effects (e.g., Gruber, 1992, Irwin & Klenow, 1994).

its true initial cost at the outset of production, even before it starts learning with the 10nm technology. Contemporaneous uncertainty may also manifest at the end of 2018 (start of 2019) about its costs at that time. That is, the firm may know that it learned from its 2018 production using the 10nm process, but remain unsure about precisely how much it has learned. As a consequence, at the end of 2018 the firm must commit to producing (or not) in 2019 without certainty about its current cost. That is, data constraints, organizational information processing, behavioral biases, and influence costs lead to noisy estimates of learning curve effects.

We illustrate the learning curve with and without these two forms of uncertainty in Figure 1. Panel A depicts the case in which the learning curve is deterministic and fully known by the firm. Here, the "Known Cost Learning Curve" is equivalent to traditional conceptions of the learning curve (as in the seminal Spence (1981) paper). Panel B depicts the early mover decision when there is prospective uncertainty in the rate of learning, from the viewpoint of a firm at $t = 1$ (illustrated by the stick figure in the diagram). The "Known Cost Learning Curve" is now an "Expected Cost Learning Curve" and the shaded area surrounding the Expected Cost Learning Curve depicts, at $t = 1$, the range of potential learning curves that a firm may anticipate realizing through $t = 2$. Panel C depicts the early mover decision when there is contemporaneous uncertainty in the rate of learning, from the viewpoint of a firm at $t = 2$. In this panel, the firm knows it has learned (between $t = 1$ and 2), as represented by the Realized Cost Learning Curve line, but there is still uncertainty about its current production costs, represented in Panel C by a cloud that distorts one's perception of the true cost of production at $t = 2$ (the firm can also face contemporaneous uncertainty at $t = 1$).

<< Insert Figure 1 about here >>

3.1 Prospective Uncertainty

In this subsection, we consider how prospective uncertainty in the rate of learning affects early mover advantage. In our setting, prospective uncertainty reflects the extent to which production in the current period will result in learning that reduces future production costs in subsequent periods. Uncertainty of this kind is discussed in the real options literature (e.g., Dixit & Pindyck, 1994). This form of uncertainty, sometimes termed volatility, focuses on the future, and thus we refer to it as *prospective uncertainty* (Posen, Leiblein, & Chen, 2018).

We begin by noting a common model of the learning curve. This model employs an exponential formulation (Argote, 1999; Heathcote, Brown, & Mewhort, 2000) and can be written in recursive form, first in levels (uppercase) and then in logs (lowercase):

$$K_t = K_{t-1} e^{-q_{t-1}\gamma} \tag{1}$$

$$k_t = k_{t-1} - q_{t-1}\gamma \tag{2}$$

where k_t is the cost of producing in period *t*, $k_t = ln(K_t)$, γ is the learning rate, and q_t is the firm's (known) quantity of production in period *t*.[4] This familiar formulation of the learning curve assumes that there are diminishing returns to experience in the sense that each additional unit of experience reduces costs by a decreasing amount. Moreover, this formulation assumes costs decline at a known (deterministic) rate.

To account for uncertainty in the rate of learning, Equation 2 can be modified as:

$$k_t = k_{t-1} - q_{t-1}(\gamma + \phi) \tag{3}$$

where ϕ is a stochastic process representing prospective uncertainty in the learning rate. In this formulation, the extent of cost reduction due to an additional unit of production is not deterministic in the sense that it may be smaller or larger than the mean rate of cost reduction, γ.

[4] In the non-recursive formulation, $K_t=C[exp(-q_{t-1}\gamma)]$, where C is a constant. K_{t-1} in Equation 1 is $K_{t-1}=C[exp(\gamma\Sigma_{j=1}^{i-2}q_j)]$. The basic assumption in this formulation is that all production-relevant costs are variable costs.

When there is uncertainty in the rate of learning, as in Equation 3, a portion of an investment in production can be conceptualized as providing a real option to produce in the future at a (potentially) lower cost. We are not the first to recognize the optionality associated with learning curves. Majd and Pindyck (1989), for instance, apply a real options model to learning curves in a setting where market prices are stochastic, but the rate of learning is deterministic. In contrast, we provide a real option model that recognizes that the rate of learning may itself be uncertain. This is an important distinction. In Majd and Pindyck (1989), the uncertainty is exogenous to the firm's decision since it unfolds whether or not the firm produces, whereas in our learning curve setting, uncertainty is endogenous in the sense that it only unfolds when a firm produces, and as such one cannot wait for uncertainty to evolve before making a decision.

In order to clarify the symmetry between the uncertain learning curve and real options models, we explicitly describe how the Black and Scholes (1973) equation models asset movement over time, after which we draw a parallel to uncertain learning curves. Consider a European call option on an asset with value in logs, s, where t - 1 is the time at which the value of the asset is assessed and t is the (maturation) time at which the firm must make its option exercise/termination decision. In this formulation, the Black and Scholes equation describes the distribution of the expected future value of the underlying asset at t from the perspective of a firm at $t-1$ based on a geometric Brownian motion process. This distribution is defined by the initial value of the underlying asset in logs, s_{t-1}, the expected drift in the value of the underlying asset, μ, and the extent of prospective uncertainty, σ_p.[5] A realization of the asset's value at time t may be written in log form such that:

[5] An alternative solution process for discrete time models is provided by the binomial option pricing model introduced by Cox, Ross, and Rubinstein (1979). As management scholars are more familiar with the intuition behind the Black and Scholes solution, we emphasize this process in our discussion.

$$s_t = s_{t-1} + \mu - \sigma_p^2/2 + \sigma_p \nu_p, \qquad (4)$$

where ν_p is a draw from the unit normal distribution and the $\sigma_p^2/2$ term is an adjustment such that uncertainty is mean preserving when the asset value is converted to levels from logs.

This mapping between real options and uncertain learning curves allows us to conceive of uncertainty in the learning curve in real option terms. The formulation of asset movement in Equation 4 is structurally equivalent to the uncertain learning curve in Equation 3. More precisely, the $\mu - \sigma_p^2/2 + \sigma_p \nu_p$ term, which defines potential changes in the value of the underlying asset over time in Equation 4, corresponds to the $-q_{t-1}(\gamma + \phi)$ term, which defines the potential cost reduction due to learning in Equation 3. In particular, the μ term, in equation 4, is the expected knowledge gain from learning-by-doing, which is analogous to the γ in the learning curve model of equation 3, and the $-\sigma_p^2/2 + \sigma_p \nu_p$ term represents the level of uncertainty in this knowledge gain. This implies that we can write the uncertain learning curve as Equation 5 below, where uncertainty follows geometric Brownian motion, as:

$$k_t = k_{t-1} - q_{t-1}(\gamma + \sigma_p^2/2 - \sigma_p \nu_p) \ . \qquad (5)$$

The option that exists when the learning curve is uncertain reflects the right, but not the obligation, to produce in *t+1* at a lower production cost than would otherwise be possible (i.e., if it had not produced in *t*). Notice that *uncertainty only evolves when the production quantity is non-zero*. This distinguishes uncertain learning curves from other forms of uncertainty evolution in which the uncertainty evolves independently from a firm's production decisions.

The existence of prospective uncertainty complicates the early entry decision. In the absence of prospective uncertainty in the learning curve, the firm will be willing to produce in $t = 1$ if its production cost is less than a threshold, $k_1^{thresh} = m + D$, where m is the market price, and D is the loss one is willing to bear in the deterministic learning curve setting because learning is known to reduce future costs (e.g., Spence, 1981). Recognition of prospective uncertainty in the rate of learning indicates that early production also yields an option value, *G*. The option value results because, if the realized rate of learning is faster than expected the firm has even lower costs when it produces in the next period (i.e., exercise the option), but if the rate is slower than expected the firm can halt future production (i.e., terminate the option). Thus, a profit maximizing firm will be willing to produce in $t = 1$ at cost less than $k_1^{thresh} = m + D + G$.

3.2 Contemporaneous Uncertainty

Learning curves may also be subject to an additional form of uncertainty — a firm may be uncertain about precisely how much it has learned in the prior period of production. Because this uncertainty focuses on a firm's knowledge about cost in the present time period, we refer to it as *contemporaneous uncertainty*. This uncertainty may be due to measurement costs (Barzel, 1982), limitations in the ability of accounting systems to fully assess labor and equipment costs, lumpiness and variability in measuring output in learning-intensive contexts, and the potential for errors introduced by organizational challenges such as influence costs (Holmstrom & Milgrom, 1991; Milgrom & Roberts, 1990).

A case study within the Boeing B-17 heavy bomber program provides an illustration of the challenges that lead to contemporaneous uncertainty in the learning curve. Mishina (1999) details the processes through which cost reduction occurred in Boeing's Plant No. 2 and the challenges ascribing these reductions to learning. The description of learning in the B-17 bomber program highlights that underlying the learning curve are continuous investments in production that represent, at least in part, a stream of small experiments that may facilitate subsequent cost reductions. These experiments led to changes in production scale, access to external experiences, product alterations, changes to the production system (including labor), and learning by doing.

Mishina (1999: 148-149) states, “It is generally difficult, if not impossible, to properly decompose whatever variations there are in the unit cost data into the part explained by the rate of output, that is, the effect of scale, and the part explained by cumulative output, that is, the effect of learning, so along as the effort to do so relies exclusively in numerical data analysis.”

Following prior research, we add *contemporaneous uncertainty* to Equation 5 at the time of the option exercise/termination decision:

$$\hat{k}_t = k_{t-1} - q_{t-1}(\gamma + \sigma_p^2/2 + \sigma_c^2/2 - \sigma_p \nu_p - \sigma_c \varepsilon_c) \quad (6)$$

where prospective uncertainty is σ_p, contemporaneous uncertainty is σ_c, and ν_p and ε_c are realizations from independent unit normal distributions (Posen, Leiblein, & Chen, 2018). While the outcome of ν_p becomes a part of the firm’s true production cost k_t, the outcome of ε_c is not part of this cost—it is a transient noise component in the decision maker’s observation of its production cost. Thus, the contemporaneous uncertainty term, $\sigma_c \varepsilon_c$, indicates that the firm may be uncertain about precisely how much it has learned in the prior period.

Posen, Leiblein, and Chen (2018) show that contemporaneous uncertainty can be incorporated into the real option to calculate an adjusted option value. The foundational assumption is that given a current cost signal that is noisy due to contemporaneous uncertainty, firms exercise or terminate the option based on a belief, b_t, that is formed via Bayesian updating such that:

$$b_t = \Omega \hat{k}_t + (1 - \Omega)(k_{t-1} - \gamma - \sigma_p^2/2 - \sigma_c^2/2) + \sigma_c^2 \Omega/2 + \sigma_c^2/2 \quad (7)$$

where $\Omega = \sigma_p^2/(\sigma_p^2 + \sigma_c^2)$. Equation 7 indicates that at the time when the firm decides whether to produce in period *t*, it knows (i.e., observes) the prior period cost, k_{t-1}, but does not know with certainty the current period cost, k_t. Rather, it observes a noisy representation of the current period cost, $\hat{k}_t$. Thus, the decision to exercise the option to produce in the next period is based on the firm's subjective belief about that cost, b_t, and is subject to decision-making errors.

The existence of contemporaneous uncertainty in the learning curve further complicates and changes the firm's assessment of the merits of entering early. Recognizing the existence of contemporaneous uncertainty leads us to observe that a profit maximizing firm will be willing to produce in $t = 1$ at cost less than $k_1^{thresh} = m + D + G - Y$, where additional parameter, *Y*, represents the loss due to the decision-making errors induced by contemporaneous uncertainty. Thus, in our model, the firm will be willing to produce in $t = 1$ at cost $k_1^{thresh} = m + D + G - Y$ where *D* is increasing in the expected learning rate, γ, *G* is increasing in prospective uncertainty, σ_p, and *Y* is increasing in contemporaneous uncertainty, σ_c.[6]

3.3 Learning Curve Spillovers

In examining the implications of learning curves, knowledge spillovers are of substantial importance. Spence (1981: 66) observes that it "is possible, and in fact likely, that the learning effects are not entirely firm-specific. Rather, they spill over from one firm to the next through the hiring of rivals' employees and other channels. When this occurs, some of the benefits of a firm's accumulated volume pass to other firms." In prior studies considering the effect of learning curve spillovers, it is often assumed that spillovers change the laggards *rate* of learning (e.g., Ghemawat & Spence, 1985).

If we accept that there is uncertainty in learning curves, then it may be useful to redefine the concepts and theory we use to assess learning curve spillovers. Consider a leader that makes its entry decision at time *t* and a laggard that makes its entry decision at *t* + 1 in the presence of learning curves that are uncertain. In this setting, we highlight three types of learning curve

---

[6] This equation is additive for illustrative purposes only. There may also be interactions between the terms.

spillovers — spillovers that affect the laggard's initial costs, its rate of learning, and its learning curve uncertainty.

First, a standard caricature of learning curve spillovers emphasizes knowledge flows from leader to laggard firms that reduce the laggards' initial production costs. This type of spillover allows laggard firms to anticipate some fraction of the problems that have been corrected by the leader in the initial period and initiate production at a lower initial cost than would otherwise be possible.

A second form of learning curve spillover affects the downward slope of the learning curve experienced by the laggard. This type of spillover accelerates the laggard's rate of learning-by-doing by providing it with a deeper understanding of the physical principles, equipment, or processes required for efficient production. Learning-by-doing, which underlies the learning curve, "occurs when, in the course of engaging in productive activity, problems are identified, experiments are performed as solutions are sought, and solutions are implemented" (Posen & Chen, 2013: 1701). The deeper understanding provided by this type of spillover allows the laggard to avoid some of the costs associated with a leader's failed experiments in the course of learning, making possible faster learning.

Consideration of uncertainty in the learning curve suggests the existence of a third form of spillover that has not previously been recognized in the literature — a spillover that reduces the uncertainty faced by the laggard because the laggard's learning is correlated with that of the leader. This type of spillover is specific to the context of uncertain learning curves. In a setting where the learning of leader and laggard firms is related, knowledge regarding the leader's realized rate of learning-by-doing provides information regarding the laggard's future learning rate. The cost reductions of the leader and laggard firms are correlated in the sense that if the leader's realized learning was faster than γ, the laggard expects that its learning rate is likely to be faster than γ. By observing the leader's realized learning, the laggard faces lower effective levels of both prospective and contemporaneous uncertainty than does the leader.

4. COMPUTATIONAL MODEL & RESULTS

4.1 Model Setup

We develop a computational model to consider the strategic implications of learning curve uncertainty for early mover advantage. In the model, we simulate a firm choosing whether or not to produce in each period of a three-period game.[7] The firm's objective is to maximize cumulative profit over the three periods. We assume an inverse demand curve in which price is $m = 1 + \beta(1\text{-}Quantity)/2$ where β, which we refer to as competition, is the slope of the inverse demand curve. In the two-firm case it represents the degree of competitive interactions between firms. To keep the model simple, we assume binary production in each period (either one or zero units), with capacity fixed at one unit (we relax this assumption in later analysis). We assume a firm knows the critical production parameters, which are constant over model time: prospective uncertainty, σ_p, contemporaneous uncertainty, σ_c, and the expected learning rate, γ. The firm also knows its initial production cost, k_1, in expectation, but this knowledge may be clouded by contemporaneous uncertainty.

The firm's production cost evolves over time following the geometric Brownian motion process in Equation 5. At the end of each period in which it chooses to produce, the firm receives noisy feedback about its realized production costs according to Equation 6 and forms beliefs based on its costs following the Bayesian logic in Equation 7.[8] The production decision can be

[7] This implies an implicit discount rate of zero in the first three periods, and full discounting in any subsequent periods.
[8] The firm is assumed to face contemporaneous uncertainty at $t = 1$, modeled as an initial belief that is a draw from a normal distribution centered around the initial cost, $k_1 - (\sigma_c)^2/2 + \sigma_c\epsilon_c$.

viewed as a real option because choosing to produce allows the firm to learn whether the future production cost will be low enough so as to induce further production, or else the firm will exit. Costs only evolve when a firm makes investments in learning (i.e., when it produces). Therefore, there is an asymmetry in that the firm's future profit will grow with good realizations of prospective uncertainty, but will not diminish much with bad realizations because the firm has the option to not produce.

To simulate the model, we use Monte Carlo and backward induction for a given σ_p, σ_c, and γ. Pseudo code that more fully describes the single-firm model is available in Appendix 1. The basic intuition is as follows. In each period, we calculate a cost threshold, k_t^{thresh}, which reflects the production cost in period t at which losses in that period are perfectly offset by learning curve benefits in terms of lower costs in future periods. We start by calculating k_3^{thresh}, which is simply the price m since there are no future periods in which to benefit from moving down the learning curve. We then calculate k_2^{thresh}. To do so, we conduct a grid search over k_2 using Monte Carlo with one million iterations of σ_p and σ_c at each k_2 in the grid to find the cost at which the firm is indifferent to producing at $t = 2$. This cost is potentially higher than the price, *m,* because of the future period benefits of current period learning. We then repeat this process to calculate k_1^{thresh}. Having calculated the three cost thresholds, we then proceed to run the model forward to calculate, for any given k_1 the expected profit over the three periods, conditional on σ_p, σ_c, and γ. For instance, at $t = 1$, the firm will produce if $k_1 < k_1^{thresh}$. If the firm does not produce, it “exits” and does not produce during the length of the simulation.[9] If the firm produces, we use Monte Carlo over one million iterations of the cost evolution in $t = 1$, and

[9] If a firm does not produce in $t = 1$, expected cumulative (three-period) profit from producing in $t = 2$ is always less than that in $t = 1$, because there is one less period in which to reap future rewards from early learning.

then proceed to do the same in the subsequent periods. We note that it is also possible, with some important limitations, to analytically model this learning process (see Appendix 2).[10]

The two-firm model introduces competition. We follow Spence (1981: 53) which employs exogenous entry times, with one firm going first and the other entering later, and define the leader firm as making its entry decision starting at $t = 1$, while the laggard makes its entry decision starting at $t = 2$. While the laggard knows the existence of the leader at $t = 2$ by virtue of the leader's production in the market, the leader does not know of the existence of the laggard at $t = 2$. We assume that the laggard plays a stochastic game (Shapley, 1953) in that it factors both the leader's and its own distributions of future cost reductions in making its entry decision. At $t = 3$, leader and laggard engage in a standard Cournot game to determine output.

Table 1 provides summary definitions of each of the key parameters and outcome variables in the model. Table 2 provides a summary of the key predictions from our model as well as a set of further implications of these results amenable to empirical testing. We relax the assumptions stated above in later analysis in which we consider factors such as prospective uncertainty in the learning asymptote rather than rate and the prospect of capacity expansion.

<< Insert Tables 1 and 2 here >>

4.2 Single-Firm Case — Prospective & Contemporaneous Uncertainty in Learning Curves

In our first experiment, we consider the single-firm case to examine implications of learning curves for entry timing across levels of prospective and contemporaneous uncertainty. We compare the impact of uncertain learning curves to the deterministic learning curve described

[10] An analytical model using a binary tree is feasible, if one forgoes diminishing returns to experience. This model is presented and solved in Appendix 2. However, diminishing returns to experience is a critical feature of most learning curve models. In results available from the authors, we demonstrate that in the absence of diminishing returns to experience, optimal production and profits is convex in both prospective uncertainty and the expected learning rate, which would imply a type of risk-loving behavior.

in Spence (1981). We calculate the *early learning premium*, which we define as the maximum initial loss a firm is willing to incur in entering earlier in order to capture future benefits from progress down the learning curve, $exp(k_1^{thresh}) - exp(m)$. To make this concept concrete, we consider a firm making its entry decision at time *t*. The premium for entering at *t* = 1 relative to deferring entry to *t* + 2 results from the future period cost reductions and thus, increased profits from learning in period *t* = 1.

We begin by examining how prospective uncertainty regarding the rate of learning affects the early learning premium. We demonstrate that early mover advantage associated with an uncertain learning curve is increasing in the level of prospective uncertainty. We examine four levels of prospective uncertainty, σ_p = 0, 0.5, 1, or 2, while setting contemporaneous uncertainty, σ_c, to zero. Thus, at time *t*, a firm knows its current production costs, k_t, but does not know its future production costs, k_{t+1} with certainty. We conduct this analysis across a range of expected learning rates, γ, between 0 and 1. When γ = 1, expected costs in the subsequent period, k_{t+1}, are reduced by a factor of 1/e from the cost in the current period.

<< Insert Figure 2 here >>

Figure 2 reports the results of this first experiment. The x-axis indicates the expected learning rate – the rate at which the decision-maker assumes that costs will decline with cumulative experience. The y-axis indicates the early learning premium. The dashed line represents the cost threshold for $\sigma_p = 0$, which is the deterministic learning curve setting examined by Spence (1981). We refer to this as the Spence (deterministic) component of the early learning premium. The upward sloping lines in the body of the figure indicate that the early learning premium increases with the expected rate of learning across all levels of prospective uncertainty. We refer to the difference between the $\sigma_p = 0$ line and the line for any given level of prospective uncertainty as the option component of the early learning premium.

The results show that prospective uncertainty affects the early learning premium in several ways. First, prospective uncertainty confers an early learning premium even when the expected learning curve is zero. This can be observed in the left side of the figure at $\gamma = 0$ for cases when $\sigma_p > 0$. This result stands in contrast to Spence's (1981) model, which suggests that, in the absence of learning curve effects, the firm should follow the well-worn profit maximizing rule and invest up to the point where marginal cost equals marginal revenue.[11] The intuition is that even with zero expected learning, early investments provide the right to produce at a lower cost in future periods should the actual learning rate prove to be positive.

Second, the early learning premium shifts upward with prospective uncertainty. Prospective uncertainty increases the merits of entering early at any given expected learning rate and indicates that firms facing prospective uncertainty should "overproduce" in the first period relative to the level of production that would occur in the absence of prospective uncertainty (i.e., as in the Spence, 1981 model). In contrast to traditional financial discounting models such as NPV which suggest that uncertainty mitigates the desire to enter earlier and produce more, we show that the option value associated with prospective uncertainty in the learning curve may encourage earlier entry. The claim provided by early entry is valuable because future production decisions are contingent on the realized learning rate, which can be faster or slower than the expected learning rate, γ, and firms that have entered earlier have the discretion to choose whether to produce in subsequent periods at the lower cost (if the realized learning rate is faster than expected) or to halt production (if the realized learning rate is slower than expected).

[11] The point where marginal revenue equals marginal cost is represented in Figure 2 at the point where the expected learning rate is $\gamma = 0$ and there is no prospective uncertainty, in which case the early learning premium is zero.

Third, the level of early learning premium in the presence of prospective uncertainty converges with the Spence curve as the expected learning rate increases. This indicates that the option value associated with prospective uncertainty in the learning curve and the value of the expected rate of learning are substitutes in the sense that greater early learning premium can come from either increasing the expected learning rate or increasing prospective uncertainty. Indeed, at low levels of prospective uncertainty (e.g., $\sigma_p = 0.5$) and very high learning rates (e.g., $\gamma = 1$), option value is almost entirely displaced by learning effects. This substitution occurs because the option to produce in the future at a lower cost, as conferred by current production, is more likely to be "in the money" when learning is intense. Thus, the Spence (deterministic learning curve) component of the laggard's early learning premium is increasing in γ, but the option component of the early learning premium is decreasing in γ .

We now turn to the implications of contemporaneous uncertainty. The addition of contemporaneous uncertainty complicates the firm's analysis as it is now uncertain about both future and current production costs. More specifically, contemporaneous uncertainty recognizes that the firm may be uncertain about how much it has learned in the prior period. In contrast to prospective uncertainty, we demonstrate that early mover advantage associated with an uncertain learning curve is decreasing in the level of contemporaneous uncertainty. We examine four levels of contemporaneous uncertainty, σ_c = 0, 0.5, 1, or 2, while setting prospective uncertainty to $\sigma_p = 1$. The results of these simulations are reported in Figure 3.

<< Insert Figure 3 here >>

The main result is that contemporaneous uncertainty reduces the early learning premium. For instance, at a learning rate of 0.6, the early learning premium decreases in magnitude from 0.88 to 0.80 when contemporaneous uncertainty increases from 0.5 to 1. This result arises because contemporaneous uncertainty induces errors in a firm's subsequent production decisions. In the presence of contemporaneous uncertainty, a firm does not know (with certainty) its current production costs because it may mistakenly over- or under-estimate the amount of learning that has occurred and make errors of commission or omission in its entry timing decisions. A firm is likely to make an error of commission, erroneously producing in the next period, if it believes its cost reduction due to learning in the prior period was greater than actually realized. That is, the firm produces in the next period when it should "shelve" or terminate the option created by early entry. Likewise, a firm is likely to make an error of omission, erroneously deciding not to produce in the next period, if it believes its cost reduction due to learning in the prior period was smaller than was actually realized. Of course, in any given situation, the magnitude of errors of omission or commission need not be the same.[12]

An interesting question is the point at which the negative impact of contemporaneous uncertainty offsets the benefits of prospective uncertainty. Figure 3 allows us to consider the proportion of gains from learning and/or prospective uncertainty (relative to the Spence line) that are lost to contemporaneous uncertainty for a given level of prospective uncertainty ($\sigma_p = 1$). If the learning rate is relatively shallow ($\gamma = 0.2$), the early learning premium gain above the Spence line due to prospective uncertainty erodes by approximately 31% when contemporaneous uncertainty increases from $\sigma_c = 0$ to 1. At a steeper learning rate $(\gamma = 1)$, this erosion increases to approximately 53%. Thus, not only do contemporaneous and prospective uncertainty have opposing influences on the early learning premium, their relative importance varies at different levels of learning.

[12] It should be noted that errors in production decisions reduce profits in expectations, not every time. A commission error can be "redeemed" by unexpectedly large cost reductions. An omission error might be superior to the unrealized counterfactual: that is, if the firm had correctly produced, this might nevertheless have led to an unexpectedly small cost reduction that would have made the firm worse off.

In unreported results, we find that the Figure 3 findings are broadly consistent across a wide range of levels of prospective and contemporaneous uncertainty. At extremely high levels of prospective and contemporaneous uncertainty, the two forms of uncertainty are not fully counteracting. Essentially, the option effect associated with early entry that is due to prospective uncertainty eclipses the detrimental effects of decision-making errors due to a similarly high level of contemporaneous uncertainty.

In sum, distinguishing between prospective and contemporaneous uncertainty in learning curves is critical for early mover decisions since the former increases the rewards to entering early — early learning premium — while the latter decreases it.

4.3 Two-Firm Case — Competition and Spillovers

In the second and third experiments, we consider the two-firm case to examine implications of learning curves for entry timing across levels of prospective and contemporaneous uncertainty when a leader firm competes with a laggard. We organize this section into two groups of experiments. In the first group of experiments we vary the level of product market competition. In the second group of experiments, we add learning curve spillovers from the leader to the laggard.

Figure 4 reports our findings regarding the association between competition and early mover advantage with and without uncertainty. The horizontal axis in the figure reports the degree of competition, β. The vertical axis reports the early mover advantage defined as the cumulative profit (in levels, across all periods) a leader earns from making its entry decision at $t = 1$, less the profit a laggard earns by making its entry decision at $t = 2$. The body of the figure reports the association between competition and early mover advantage with no uncertainty (Spence line) as well as at low and high levels of prospective ($\sigma_p = 0.5$ *or* 2) and contemporaneous uncertainty ($\sigma_c = 0.5$ *or* 2).

As expected, the no uncertainty condition indicates that the early mover advantage is increasing in competition, β, due to preemption effects and exhibits a discontinuous jump as competition increases above $\beta = 0.5$. The uncertainty lines demonstrate that, relative to the no uncertainty condition, early mover advantage is increasing in the level of prospective uncertainty (σ_p) and decreasing in contemporaneous uncertainty (σ_c) across levels of competition (β). It is notable that there are conditions under which uncertainty increases early mover advantage at low levels of competition (β) but decreases early mover advantage at high levels of competition. That is, uncertainty in the learning curve and competition in the environment have an interactive effect on early mover advantage and this interactive effect runs in opposite directions for prospective and contemporaneous uncertainty.

<< Insert Figures 4 and 5 here >>

To better understand the mechanisms underlying these results, we decompose early mover advantage in Figure 5. Specifically, the Figure reports associations between competition, prospective and contemporaneous uncertainty, cumulative leader and laggard profits, and leader and laggard aggregate production quantities. We begin by assessing the no-uncertainty case across all four panels. We observe that increasing competition, β, initially reduces leader and laggard profits until $\beta > 0.5$, at which point the laggard's entry is preempted (zero quantity) and leader profits return to their baseline (no competition) level. This is expected given the specified inverse demand curve with binary production. We also observe the effect that competition has on production quantity. In the no uncertainty case the leader enters the market and the laggard enters only if competition is modest (e.g., $\beta < 0.5$). Again, this is expected given model parameters.

Figure 5 indicates that both leader and laggard profits are decreasing in the level of competition. The Figure further indicates that the positive association between competition and early mover advantage in Figure 4 results because the leader's profits decrease slower than the laggards. In addition, Figure 5 indicates that prospective uncertainty tends to more dramatically increase the leader's profits than it does the laggards, but contemporaneous uncertainty tends to more dramatically decrease the leader's profits than it does the laggards. We see that the leader always produces less when there is uncertainty, even though it achieves greater profits. Finally, we observe the central role of laggard preemption. Uncertainty reduces laggard production (relative to the no-uncertainty case) at low levels of competition, but increases it at higher levels of competition. Since, at high levels of competition, the leader's reduced production is more than offset by the laggard's increased production, total industry production increases when learning curves are uncertain.

In the second group of two-firm experiments we add learning curve spillovers from the leader to the laggard. We assess the extent to which learning curve uncertainty magnifies or attenuates the effect of learning curve spillovers on the advantage a leader holds in being the first mover. Our model and parameter settings are identical to those used previously except we now specify an initial cost for the leader and laggard, k_1^{ldr} and k_2^{lag}, to calculate the value of spillovers to the laggard and the leader's early mover advantage. In our simulations, we set both k_1^{ldr} and k_2^{lag} to 75% of the laggard's k_2^{thresh}, the cost at which the laggard has zero expected profit for entry (less a nominal amount to induce laggard entry). In doing so, we are able to induce sufficient entry to see preemptive effects and isolate changes in the laggard's profits due to spillovers by holding laggard profits due to entry constant across the other parameter settings.

The model considers three types of learning curve spillovers. In the first spillover scenario, initial cost reducing spillovers decrease the laggard's costs at the time of its entry decision (at t = 2). This initial cost reduction is defined as the fraction, ω, of the leader's first period cost reduction. That is, if the leader's cost after a period of learning is k_2^{ldr}, then the laggard's initial cost when it makes its entry decision is $k_2^{lag,s} = k_2^{lag,ns} - \omega \cdot max(k_2^{ldr} - k_1^{ldr}, 0)$ where $k_2^{lag,s}$ and $k_2^{lag,ns}$ are the laggard's initial production cost with and without spillovers, respectively.[13] To model the laggard with this spillover, we simulate the leader's actual cost-reduction between its first and second periods of production, calculate the laggard's initial production cost $k_2^{lag,s}$, and use Monte Carlo to compute the laggard's behavior and profits over the remaining period.

In our second spillover scenario, learning rate increasing spillovers enhance the rate at which the laggard expects to learn and reduce its costs over time. If the leader's expected learning rate is γ^{ldr} then the laggard's learning rate will be $\gamma^{lag} = (1 + \psi)\gamma^{ldr}$, where ψ represents the fractional increase in the laggard's expected learning rate due to information obtained from the leader's observed learning. To model the laggard with this spillover, we simulate the laggard's behavior and profits over its two periods of production using an expected learning rate of γ^{lag}.

In the third spillover scenario, uncertainty reducing spillovers diminish the effective levels of the laggard's prospective and contemporaneous uncertainty in its learning curve. In the presence of uncertainty reducing spillovers, the laggard knows that its realized and observed cost reductions are correlated with those of the leader with a correlation coefficient of ϱ. When $\varrho = 0$, the leader's realized cost reductions provide no additional information to the laggard about its future cost reductions. When $\varrho = 1$, the leader's realized cost reductions fully inform

[13] The max function in this expression captures the idea that a laggard will ignore spillovers from a leader whose costs increase rather than decrease over the first period of learning.

the laggard of its future cost reductions such that the laggard experiences a deterministic learning curve. When $0 < \varrho < 1$, the laggard knows that its costs are correlated with the leader's. Because of this knowledge, the laggard may construct its notional learning curve parameters γ', σ_p' and σ_c' from the leader's observed cost reduction, $k_2^{ldr} - k_1^{ldr}$, and ϱ. That is, the laggard observes a leader's cost reduction, and can use its knowledge of stochastic cost reduction (e.g., Equation 5) to infer that the leader's draw of prospective uncertainty is $v_p^{ldr} = x$. Using Bayesian inference, the laggard deduces the conditional distribution of its prospective uncertainty draw. Analytically, this is given by $v_p^{lag}|[v_p^{ldr} = x] \sim N[\varrho x, 1 - \varrho^2]$, which implies $\gamma' = \gamma + (\sigma_p^2\varrho^2/2) - \sigma_p\varrho x$, $\sigma_p' = \sigma_p\sqrt{1 - \varrho^2}$, and $\sigma_c' = \sigma_c\sqrt{1 - \varrho^2}$. To model the laggard with an uncertainty reducing spillover, we simulate the laggard's behavior and profits over its two periods of production using these notional parameters.[14]

<< Insert Table 3 here >>

We report our findings regarding the extent to which learning curve uncertainty magnifies or attenuates the effect of spillovers in Table 3. The columns in Table 3 indicate percentage changes in early mover advantage, leader and laggard profit, and leader and laggard quantity relative to the no spillover condition. The rows indicate the different types of spillovers and different levels of prospective and contemporaneous uncertainty. The body of the table reports the percentage change in outcomes (e.g., early mover advantage) for representative levels of initial cost reducing, learning rate increasing, and uncertainty reducing spillovers (in particular, we set spillover levels of $\omega = 0.8$, $\psi = 400$, and $\varrho = 0.8$, respectively) as compared to a base case

[14] We use the term lower "effective" levels of uncertainty because increasing the correlation between the leader and laggard is not identical to a direct reduction in the laggard's level of prospective and contemporaneous uncertainty. A direct reduction in the laggard's uncertainty to zero would imply that the laggard realizes and observes a learning rate of γ. Correlated realizations function differently. When the laggard's learning is fully correlated with the leader's, the laggard faces no uncertainty after it observes the leader's realized learning outcome, but the laggard will, nevertheless, have a realized learning rate that is either higher or lower than γ (i.e., equal to the leader's realized γ).

where there are no spillovers. In all instances, we hold competition at a moderate level (β = 0.25). As expected, in the no uncertainty case, the introduction of spillovers increases laggard profit and decreases the leader's early mover advantage for both the initial cost reducing and learning rate increasing spillovers. Sensibly, in this no-uncertainty setting, there is no effect due to uncertainty reducing spillovers.

The overarching result from Table 3 is that the introduction of uncertainty in the learning curve has significant implications for the effect of spillovers on the laggard's profitability outcome and the leader's early mover advantage. We highlight two specific effects. First, the effect of uncertainty on early mover advantage in the presence of spillovers is contingent on both the form of uncertainty and the type of spillover. For example, while higher prospective uncertainty magnifies the negative effect of cost reducing spillovers on a leader's advantage, the effects are the polar opposite for learning rate increasing spillovers. This difference between the two types of spillovers occurs because the former type of spillover directly impacts the likelihood of laggard entry, while the latter type of spillover only impacts outcomes conditional on laggard entry. In contrast, increasing contemporaneous uncertainty magnifies the negative effect of cost reducing spillovers and learning rate increasing spillovers on a leader's early mover advantage, particularly at high levels of prospective uncertainty.

A second effect highlights conditions under which spillovers from the leader to the laggard actually increase the leader's early mover advantage. Recognition of uncertainty in the learning curve implies the potential existence of spillovers that reduce the uncertainty a laggard faces regarding its learning curve. This uncertainty reduction enables the laggard to make more informed entry decisions than would otherwise be possible. In this case, the laggard improves its profitability by reducing production quantity and in so doing improves industry structure. Thus, such spillovers from the leader to the laggard may improve the leader's early mover advantage, as well as both leader and laggard profits.

4.4 Model Extensions: Further Consideration of Uncertain Learning Curves

Our models are amenable to a variety of extensions that enable consideration of the manner in which uncertain learning curves affect strategically interesting phenomena. We consider four such phenomena below.

First, there are several plausible ways to characterize learning curves. While our model emphasizes prospective and contemporaneous uncertainty in the *rate* of learning, it is possible to conceive of learning curves that differ in their: (a) *initial cost* in terms of the starting point of learning, (b) *rate of learning* in the sense that the slope of the curve may be more or less steep when problems are solved faster or slower, and (c) *asymptote of learning* in terms of the long-run potential for cost reductions due to, perhaps, scientific fundamentals that limit the potential advancement within a given technology (see Rockart and Dutt, 2015). Our model is amenable to all of these generalizations.

Of particular interest is prospective uncertainty in the asymptote and how it differs from prospective uncertainty in the rate of learning. We extend the model to facilitate this analysis. The evolution of cost (in levels) when a firm produces under prospective uncertainty in the asymptote is given by $K_t = K_{t-1}exp[\ -q_{t-1}\gamma] + K_A exp[-(\sigma_p^a)^2/2 + (\sigma_p^a \nu_p^a)]$, where K_A is the asymptote (which is set to zero in our main models) as the 'a' superscript on σ_p indicates our focus on prospective uncertainty regarding the asymptote rather than the rate.

In Figure 6, we show how prospective uncertainty in the learning rate and asymptote differ in the impact they have on the early learning premium. The results here are for the single-firm model. In both panels, the expected asymptote is set at 50% of price and $\sigma_c = 0$. In the left panel, we examine prospective uncertainty in the rate of learning, σ_p^r, per the main models in the paper, holding $\sigma_p^a = 0$. In the right panel we examine prospective uncertainty in the asymptote of learning, σ_p^a, holding $\sigma_p^r = 0$. Panel A, depicts the implications of prospective uncertainty in the rate of learning and is similar to Figure 2 in the paper (but differs quantitatively due to the different asymptote used here). Results in Panel B, depicting the implications of prospective uncertainty in the asymptote of learning, are similar to that of Panel A in that increasing prospective uncertainty increases the early learning premium.

The key difference from our main results on learning rate uncertainty occurs at the y-intercept. We describe the intuition underlying the rate uncertainty case (panel a) in section 4.2. Specifically, we note how prospective uncertainty in the rate of learning generates option value that results in an early learning premium, even when the expected rate of learning is zero. In the asymptotic uncertainty case (panel b), the lines converge at the intercept. This convergence occurs when the rate of learning, γ, is sufficiently slow that uncertainty in the asymptote never binds. That is, the option value due to prospective uncertainty is small because at a very slow learning rate, early entry is unlikely (the deterministic component of the learning curve is nowhere near sufficient to induce early entry), and the uncertainty in the asymptote has little influence on the entry (production) decision. While the specific nature of the effect of uncertainty in the learning curve on the early learning premium differs across learning curves that highlight the rate of learning and those that highlight the learning asymptote, the general result, that prospective and contemporaneous uncertainty affect the association between the expected learning and the early learning premium, remains unchanged.

<< Insert Figure 6 here >>

Second, one might wonder whether or how allowing the leader to adjust production quantity in response to learning affects our results. While our main models assume fixed production capacity, so as to focus attention on learning curve uncertainty, we extend our model to allow for capacity expansion. In particular, the firm chooses to produce 1 unit or 0 units in its first period (i.e., the choice set is {0,1}). If the firm produces, it increases its capacity to 1.5 units in the subsequent period (i.e., the production choice set is expanded to {0, 1, 1.5}), and likewise, if the firm produces in two prior periods, it increases its capacity to 2 units in the third period (i.e., the production choice set is expanded to {0, 1, 1.5, 2}). This model structure allows us to evaluate the payoffs to the leader's choices, to assess to what extent payoffs are due to preemption, and to explore how these effects are influenced by uncertainty.

Figures 7 and 8 report results from simulations where capacity adjustments are possible. These figures indicate that allowing the leader to add more capacity in later periods increases the range over which the laggard is preempted, and this is magnified by greater prospective uncertainty and reduced by greater contemporaneous uncertainty. The added complexity in Figure 7 (as compared to Figure 4) occurs because capacity expansion affects the extent to which demand constraints bind. A comparison of the "quantity" panels in Figures 5 and 8 make this clear. When capacity expansion is feasible, uncertainty interacts with competition to affect early mover advantage in at least two interesting ways. First, the potential for early mover advantage at high prospective uncertainty is much greater when capacity expansion is possible. This occurs because early investment provides the leader with a preferential ability to add capacity after learning about (resolving uncertainty about) the learning curve. This ability is more valuable with greater levels of prospective uncertainty since greater uncertainty implies a wider range of costs. Thus, the leader can increase capacity upon receiving a positive signal or shut down production upon receiving a negative signal. The value of this option also declines with the level of competition as competition mitigates the leaders discretion (Smit and Trigeorgis, 2012). It is also shifted lower by contemporaneous uncertainty as this form of uncertainty engenders decision-making errors. Second, the general impact of prospective and contemporaneous uncertainty does not change when capacity expansion is possible. As above, there are conditions under which uncertainty increases early mover advantage (at low levels of competition) and where uncertainty decreases early mover advantage (at high levels of competition).

<< Insert Figures 7 and 8 here >>

Third, while the model presented in this paper treats prospective and contemporaneous uncertainty as separable and orthogonal, it is possible that these two forms of uncertainty in learning curves may be correlated. In our model, contemporaneous and prospective uncertainty are defined by $\sigma_c \varepsilon_c$ and $\sigma_p \nu_p$, where the ε_c and ν_p terms refer to the underlying unit normal distributions and the σ terms refer to the variances in these distributions. The simplest form of correlation exists when high levels of prospective uncertainty, σ_p, are accompanied by high levels of contemporaneous uncertainty, σ_c. An alternative, and potentially more interesting, form of correlation exists in the realizations of uncertainty that occur within a given model. That is, we may not observe independent draws from the two unit uncertainty distributions, ε_c, ν_p, but understand that these draws are correlated. Intuitively, the positive correlation between ν_p and ε_c increases the likelihood that a good realization of prospective uncertainty (faster learning than expected) tends to be accompanied by a realization of contemporaneous uncertainty in the same direction (the signal is contaminated by noise that suggests even faster learning). We report findings from this model extension in Appendix 3, Figure A3.1. In that figure, we show that a positive correlation in the realizations of prospective and contemporaneous uncertainty tends to increase the early learning premium.

A fourth extension to our model introduces differences in technologies across leader and laggard firms when there are differences in uncertainty in the rate of learning across the new and existing technology. For instance, one may consider a laggard firm that is confronted with choosing whether to invest in a new technology employed by the leader, which is known to have a lower asymptotic (long-run) cost, or stick with an existing, tried-and-true, technology. We consider how: (a) the existing technology's initial cost at the start of the learning curve, and (b) uncertainty reducing spillovers from the leader in the new technology impact the laggard's technology choice. We explore the trade-offs facing the laggard in situations characterized by different levels of prospective and contemporaneous uncertainty. A key result, reported in Appendix 3, Figure A3.2, is that increasing prospective uncertainty induces the laggard to move to the new technology while contemporaneous uncertainty may do the opposite.

In addition, we considered a number of robustness checks. For instance, in our main models, contemporaneous uncertainty is constant across time. We have examined contexts in which contemporaneous uncertainty is increasing or decreasing over time. The qualitative pattern of results is robust to these alternative model specifications.

5. DISCUSSION AND CONCLUSION

This paper develops a theory that describes how uncertainty about the rate of progress along the learning curve affects early mover advantage. Consistent with prior work, our theory recognizes that early entry provides potential profit from both production in the current period and from potential cost reductions due to progress down the learning curve in future periods. In addition, our theory is consistent with prior work recognizing that the magnitude of learning curve effects due to earlier entry may be mitigated by competition and knowledge spillovers from earlier to later entrants. Our theory extends the literature by demonstrating how uncertainty in the learning curve influences the benefits of earlier and later entry as well as the extent to which knowledge spillovers to later entrants undermines early mover advantage.

Our research builds on the claim that decisions related to future learning are complicated by a firm's uncertainty regarding the precise nature of the learning curve. While applications of learning curve logic often *assume* that managers know the future learning rate, there are reasons to believe that substantial uncertainty exists in estimates of learning curves (Thompson, 2012). Our simulation allows us to weigh the benefits of learning due to early entry against the benefits and costs of early entry under uncertainty. Specifically, we show that the value associated with early entry is contingent upon both the magnitude and form of uncertainty. Prospective uncertainty increases, while contemporaneous uncertainty decreases, the benefits of early learning. While the ideas of prospective and contemporaneous uncertainty are intimately linked to well-established theories (involving optionality and feedback learning), we lack measures that readily isolate these two concepts. Thus, research that develops and validates constructs or proxies consistent with these ideas would be valued. While ideally, empirical research would leverage valid and reliable constructs for prospective and contemporaneous uncertainty, it may also be possible to develop useful tests that leverage proxies for these concepts (e.g., the Boston Consulting Group's use of Compustat data to generate proxies for volatility and malleability). Such research would also facilitate empirical testing of our claims that learning curve uncertainty affects entry timing and early mover advantage.

Recognizing the importance of learning curve uncertainty further reveals a boundary condition to prior arguments that the existence of a learning curve provides an unqualified rationale for early entry. Spence (1981: 49) claims that, "when additions to output lower future costs, it is appropriate for the firm to go beyond the short-run maximizing level of output." We note that prospective and contemporaneous uncertainty refine this result, by demonstrating that uncertainty in the learning curve and competition in the environment jointly and interactively affect the benefits of learning. This joint and interactive effect suggests a linkage to prior efforts aimed at developing methods to link corporate strategy and corporate finance (Smit and Trigeogis, 2004). For instance, our theory suggests the effect of uncertainty on the early mover advantage is contingent on the level of competition — increasing it at low levels of competition and decreasing it at high levels of competition.

Our theorizing also contributes to discussions regarding the effect of learning curve spillovers in the presence of uncertainty. If learning "spills over" from one firm to another (e.g., Irwin & Klenow, 1994), then later movers may utilize the experience gained by the leader to reduce their costs or improve product quality (e.g., Ghemawat & Spence, 1985). Our paper contributes to the existing learning curve spillover discussion in three ways. First, we reveal an additional mechanism through which learning curve spillovers may affect competition. In addition to affecting the laggard's expectations regarding its learning, learning curve spillovers may also affect the level of uncertainty faced by the laggard. Second, we illustrate the unique consequences of three spillover mechanisms. These spillovers are differentially sensitive to both forms of uncertainty and affect leader and laggard behavior in different ways. Finally, we demonstrate conditions under which uncertainty reducing spillovers from leader to laggard may, counterintuitively, increase the leader's early mover advantage by improving industry structure. While spillovers always benefit the laggard, we show that uncertainty interacts with the different types of spillovers, in some cases magnifying spillover's deleterious effect for the leader, but in other cases, actually mitigating its deleterious effects. While the relative importance of the three different types of spillovers highlighted in our model is ultimately an empirical question, our findings suggest a connection to extant work suggesting alternative reasons why spillovers may benefit an originating firm (e.g., Polidoro & Toh, 2011; Yang, Phelps, & Steensma 2010; Pacheco-de-Almeida & Zemsky 2012).

In recognizing the importance of both initial cost reducing, learning rate increasing and uncertainty reducing spillovers, this paper also suggests a way to consider contingencies between managerial actions and the firm's ability to benefit from spillovers. Whereas Spence (1981) views spillovers as phenomena that affect all firms equally, work on absorptive capacity notes that firm's may invest in research and development to enhance their ability to absorb external knowledge (e.g., Cohen and Levinthal, 1990). However, if there are different types of spillovers then it is possible that firms vary in their pursuit of these different spillovers. For instance, one might imagine that different investments or actions are required to develop absorptive capacity relevant to initial cost reducing (e.g., imitation), learning rate increasing (e.g., understanding of scientific principles), or uncertainty reducing spillovers (e.g., competitive intelligence). If so, we would expect that firms that correctly diagnose the benefits of particular forms of spillovers (based on observed levels of prospective and contemporaneous uncertainty) and invest in the form of spillover appropriate for that environment would enjoy an advantage over others that did not make these investments. This contingency further implies that laggard firms that act to benefit from particular types of spillovers are more likely to achieve higher performance in specific situations.

Our paper also casts new light on debates regarding the effect of learning on early and late mover advantage. We provide two main contributions. First, we show how the magnitude of entry timing advantages due to learning are affected by two forms of uncertainty. Whereas prior studies develop verbal arguments regarding the effect of market and technical evolution on first mover advantage (e.g., Suarez & Lanzolla, 2007), our model shows how uncertainty in the learning curve may affect early mover advantage. We trace more precisely how earlier or later investment decisions are affected by prospective and contemporaneous uncertainty (and various forms of learning spillovers). In doing so, we provide a means to link discussions from the marketing, strategy, and technology literature on early mover advantages to prominent perspectives on sequential decision-making under uncertainty. Second, our approach allows us to address a challenge in the empirical early mover literature associated with the ability to measure timing advantages against a clear counterfactual (Lieberman & Montgomery, 2013: 313; see also Suarez & Lanzolla, 2007: 377). By measuring the magnitude of learning effects defined by different levels and types of uncertainty and spillovers, we compare estimates of advantages due to learning across several scenarios and suggest empirically testable propositions regarding the benefits of early or late entry in similar situations.

The insights offered by this study suggest that it may also be worthwhile to explore how uncertainty affects managerial decision-making in other phenomena of interest. For instance, the differing effects of prospective and contemporaneous uncertainty on the benefits of learning curves suggest it may be worthwhile to explore whether and how these forms of uncertainty affect the firm’s ability to generate competitive advantage through bidding in uncertain strategic factor markets (e.g., Barney, 1986). It may be possible to devise experiments that vary the level and type of risk or uncertainty (or even unknown unknown forms of uncertainty as discussed in Ehrig and Foss [2022]) and assess their impact on investment behavior. Because learning curve uncertainty affects the presumed benefits of learning curve strategies, it may also be useful to generate propositions regarding how prospective and contemporaneous uncertainty affect the evaluation of multiple technologies with different initial technical performance characteristics, expected rates of performance improvement, or asymptotic performance limits (i.e., S-curves). Our examination of the types of spillovers identified in this paper also allows us to think about new ways in which knowledge might spill over and affect the evolution of more or less “related” technologies.

Our paper is not without limitations. Future research may refine our model to develop alternative notions of competition between early and late movers. In addition to the directions suggested by the model extensions outlined in Section 4.4, one might also consider adding additional types of competition (e.g., Bertrand or Stackelberg) or adding uncertainty to the demand side of our model. Of course, our model results are dependent on our assumptions. Thus, efforts to test the assumptions in our theoretical model would also be valuable.

Scholars in the field of strategic management have shown considerable interest in learning curves and early mover advantage. We have explored the implications of uncertainty in learning curves by modeling the learning curve via a series of behavioral real options subject to both prospective and contemporaneous uncertainty (Posen et al., 2018). Consequently, investments in learning to generate subsequent advantage may or may not pay off as expected. Results obtained from this model show that these two forms of uncertainty have countervailing effects on the value of early investments in learning. The implication is that, depending on the nature of the uncertainty facing the firm, early investments in learning may or may not generate early mover advantage.

REFERENCES

Abernathy, WJ, Wayne, K. 1974. The limits of the learning curve. *Harvard Business Review*.

Alchian A. 1963. Reliability of progress curves in airframe production. *Econometrica,* 31(4): 679–693.

Argote L, Epple D. 1990. Learning curves in manufacturing. *Science,* 247(4945): 920–24.

Argote L. 1999. *Organizational Learning: Creating, Retaining, and Transferring Knowledge,* Kluwer Academic Publishers, MA.

Arrow KJ. 1962. The economic implications of learning by doing. *Review of Economic Studies,* 29(3): 155–73.

Barney, JB. 1986. Strategic factor markets: Expectations, luck, and business strategy. *Management Science*, 32(10): 1231–1241.

Barzel, Y. (1982). Measurement cost and the organization of markets. *The Journal of Law and Economics*, 25(1), 27-48.

Black, F, Scholes, M. 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81(3): 637-654.

Cohen, W, Levinthal, DA. 1990. Absorptive capacity: A new perspective on learning and innovation. *Administrative Science Quarterly* 35(1): 128-152.

Darr, E, Argote, L, Epple, D. 1995. The acquisition, transfer, and depreciation of knowledge. *Management Science* 41(11): 1750-1762.

Dasgupta, P, Stiglitz, J. 1988. Learning-by-Doing, Market Structure and Industrial and Trade Policies. *Oxford Economic Papers,* 40(2): 246-268.

Dixit AK, Pindyck RS. 1994. *Investment under Uncertainty*, Princeton, NJ: Princeton University Press.

Ehrig, T, Foss, N. Forthcoming. Unknown unknowns and the treatment of firm-level adaptation in strategic management research. *Strategic Management Review.*

Fudenberg, D, Tirole, J. 1983. Learning-by-Doing and Market Performance. *The Bell Journal of Economics* 14(2): 522-530.

Fudenberg, D, Tirole, J. 1985. Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies,* 52(3): 383-401.

Ghemawat, P. 1991. *Commitment: The Dynamic of Strategy*. Simon and Schuster. New York, NY

Ghemawat, P., & Spence, A. M. 1985. Learning curve spillovers and market performance. The *Quarterly Journal of Economics*, 100(Supplement), 839-852.

Gruber, H. 1992. The Learning Curve in the Production of Semiconductor Memory Chips. *Applied Economic*s 24(8): 885-94.

Heathcote A, Brown S, Mewhort DJ. 2000. The power law repealed: the case for an exponential law of practice. *Psychonomic Bulletin & Review,* 7(2): 185–207.

Hirsch WZ. 1952. Manufacturing Progress Functions. *The Review of Economics and Statistics*, 34(2): 143–155.

Holmstrom, B., & Milgrom, P. (1991). Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics, and Organization*, 7(special), 24-52.

Irwin, Douglas and P. Klenow. 1994. "Learning by doing spillovers in the semiconductor industry." Journal of Political Economy 102(6): 1200-1227.

Kiechel W. 2010. The Lords of Strategy: The secret intellectual history of the new corporate world. Harvard Business School Press.

Knudsen, T and Levinthal, D. 2007. Two faces of search: Alternative generation and alternative evaluation. *Organization Science*, 18(1):39-54.

Leiblein, MJ, Chen, JS, Posen, HE. 2017. Resource Allocation in Strategic Factor Markets: A Realistic Real Options Approach to Generating Competitive Advantage. *Journal of Management* 43(8): 2588-2608.

Leiblein, MJ, Reuer, JJ, Zenger, T. 2018. What Makes a Decision Strategic. *Strategy Science* 3(4): 558-573.

Lieberman MB. 1984. The learning curve and pricing in the chemical processing industries. *The RAND Journal of Economics*, 15(2): 213-228.

Lieberman MB 1987. The learning curve, diffusion, and competitive strategy. *Strategic Management Journal,* 8(5): 441-452.

Lieberman, M. 2016. First-Mover Advantage. In The Palgrave Encyclopedia of Strategic Management. Palgrave Macmillan, London

Lieberman MB, Montgomery DB. 1988. First−mover advantages. *Strategic Management Journal,* 9: 41-58. doi:10.1002/smj.4250090706

Lieberman MB, Montgomery DB. 1998. First−mover (dis)advantages: retrospective and link with the resource−based view. *Strategic Management Journal,* 19: 1111-1125.

Lieberman MB, Montgomery DB. 2013. Conundra and Progress: Research on Entry Order and Performance. *Long Range Planning,* 46(4): 312–324.

Majd S, Pindyck RS. 1989. The learning curve and optimal production under uncertainty. *The RAND Journal of Economics*. 20(3): 331.

Milgrom, P., Roberts, J. 1990. Bargaining costs, influence costs, and the organization of economic activity. In J. E. Alt & K. A. Shepsle (Eds.), *Perspectives on Positive Political Economy*: 57–89. Cambridge: Cambridge University Press.

Mishina, K. 1999. Learning by new experiences: Revisiting the flying fortress learning curve. In NBER Chapters, National Bureau of Economic Research, Inc.

Pisano GP. 1994. Knowledge, integration, and the locus of learning: An empirical analysis of process development. *Strategic Management Journal,* 15: 85-100.

Posen, HE, Chen, JS. 2013. An advantage of newness: Vicarious learning despite limited absorptive capacity. *Organization Science* 24(6): 1701-1716.

Posen HE, Leiblein MJ, Chen JS. Chen. 2018. Toward a behavioral theory of real options: Noisy signals, bias, and learning. *Strategic Management Journal,* **39**(4) 1112–1138.

Reeves M, Haanæs K, Sinha J. 2015. *Your Strategy Needs a Strategy: How to Select and Execute the Right Approach to Strategy*. Boston, MA: Harvard Business Review Press.

Rockart SF, Dutt N. 2015. The rate and potential of capability development trajectories. *Strategic Management Journal,*36(1): 53-75.

Shapley, L. S. 1953. Stochastic games. *Proceedings of the National Academy of Sciences of the United States of America*, 39(10): 1095–1100.

Smit H, Trigeorgis L. 2004. *Strategic investment: Real options and games*. Princeton University Press.

Spence AM. 1981. The learning curve and competition. *Bell Journal of Economics,* 12(1): 49–70.

Stiglitz, J. E., Greenwald, B. C. 2015. *Creating a Learning Society: A New Approach to Growth, Development, and Social Progress*, Columbia University Press, NY.

Suarez FF, Lanzolla G. 2007. The role of environmental dynamics in building a first mover advantage theory. *Academy of Management Review,* 32(2): 377-392.

Thompson P. 2012. The relationship between unit cost and cumulative quantity and evidence for organizational learning-by-doing. *Journal of Economic Perspectives. 26(3): 203-224.*

Van den Steen E. 2018. A formal theory of strategy. *Management Science*. 63(8): 2616-2636.

Wright TP. 1936. Factors affecting the cost of airplanes. *Journal of the Aeronautical Sciences*, 3(4): 122–128.

Yelle LE. 1979. The learning curve: Historical review and comprehensive survey." *Decision Sciences,* 10(2): 302–28.

TABLES

Table 1. Summary definitions of the key variables in our model

Variable	Definition
Learning rate (γ)	The expected rate at which future production costs are reduced by additional current production, typically modeled as a constant in exponential form. Spence (1981) assumes this rate is deterministic and known by the firm, we think of this as the expected learning rate.
Prospective uncertainty (σ_p)	Uncertainty about *future* production cost. This results from uncertainty about the rate at which future production costs are reduced due to current period production.
Contemporaneous uncertainty (σ_c)	Uncertainty about *current* production cost. This results from noise that contaminates a firm's ability to estimate the rate at which production costs declined due to production by the end of the prior period.
Early learning premium	Maximum initial loss a firm is willing to incur (via early entry) in order to capture future benefits (cost reductions) from further progress down the learning curve (i.e., a comparison of a focal firm's expected profit from entering at t = 1 versus t = 2).
Early mover advantage	A leader's profit advantage over a laggard when a leader makes its entry decision one period earlier than the laggard (i.e., a comparison between a focal firm's expected profit from its entry decision at t = 1 relative to a rival's entry decision at t = 2).
Competition (β)	The downward slope in the inverse demand curve, which, in the two firm case, represents the extent of competitive interactions between firms.
Initial cost reducing spillovers (ω)	Knowledge spillovers from a leader to a laggard that reduce the laggard's initial cost.
Learning rate increasing spillovers (ψ)	Knowledge spillovers from a leader to a laggard that increases the laggard's learning rate.
Uncertainty reducing spillovers (ϱ)	Knowledge spillovers from a leader to laggard that reduce the laggard's uncertainty about its learning rate.

Table 2. Summary of key predictions and further implications from our model

DETERMINISTIC LEARNING CURVES (e.g., Spence, 1981)	
Key Predictions	**Further Implications / Empirical Predictions**
A faster expected **learning rate** leads to a greater **early learning premium** and **early mover advantage**.	In the presence of a learning curve, a firm will enter earlier (i.e., produce more) than would be appropriate under short-term profit maximization. Such entry (production), even at a short-term loss, reduces future production costs.
Knowledge spillovers reduce a leader's **early mover advantage**.	In the presence of learning curve spillovers from a leader to a laggard, a leader will enter later (i.e., produce less) than would be appropriate in absence of spillovers.
UNCERTAIN LEARNING CURVES	
Key Predictions	**Further Implications / Empirical Predictions**
Higher **prospective uncertainty** about the learning rate leads to a greater **early learning premium**.	Prospective uncertainty confers option value—if the realized learning rate is greater than expected, then the firm captures the benefit, but halts production if it is lower. The early learning premium due to prospective uncertainty may be positive even when the expected learning rate is zero. This implies a firm will enter earlier than predicted by a deterministic learning curve. As the expected learning rate increases, the contribution of prospective uncertainty to the early learning premium decreases (i.e., negative moderation).
Higher **contemporaneous uncertainty** about the learning rate leads to a lower **early learning premium**.	Contemporaneous uncertainty, which results from noisy estimates of current production cost (i.e., uncertainty about how much has been learned previously), engenders errors in the entry decision. This implies that a firm will enter later than predicted by a deterministic learning curve.
Prospective and **contemporaneous uncertainty** interact with **competition** to influence **early mover advantage**.	Uncertainty increases the early mover advantage at low levels of competition and decreases it at high levels of competition. While prospective and contemporaneous uncertainty have opposing effects, combinations of these uncertainties tend to reduce the laggard's production quantity at low levels of competition and increase its production quantity at high levels of competition as compared to the no uncertainty condition.
The leader's **early mover advantag**e is contingent on spillover type as well as **prospective** and **contemporaneous uncertainty**.	The interaction between learning curve uncertainty and spillovers affects the early mover advantage. The nature of this effect differs across the type of spillover. Increasing prospective uncertainty magnifies the negative (positive) effect of cost reducing (learning rate increasing) spillovers on a leader's advantage. Increasing contemporaneous uncertainty magnifies the negative effect of cost reducing and learning rate increasing spillovers on a leader's early mover advantage, particularly at high levels of prospective uncertainty. Cost reducing and learning rate increasing spillovers always undermine the leader's advantage, but uncertainty reducing spillovers may increase the leader's early mover advantage.

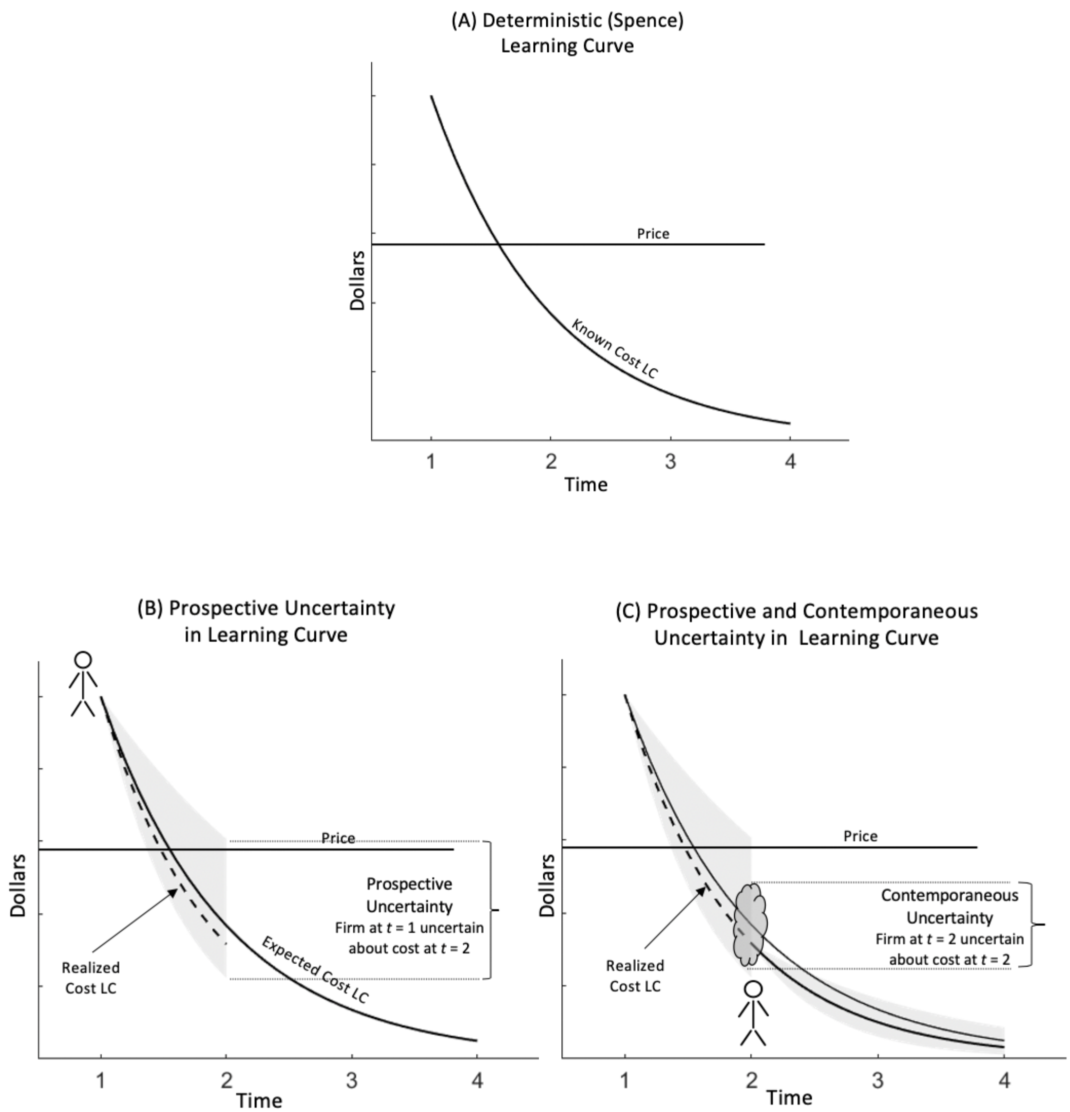
Table 3. Change in EMA, profits, and quantities *due to an increase in spillovers*

Spillover Type		% change in EMA	% change in leader profit	% change in laggard profit	% change in leader quantity	% change in laggard quantity
cost reducing	Spence (no unc)	-49.6	0.0	49.6	0.0	0.0
	low σ_p, low σ_c	-72.3	-7.1	65.1	-0.2	56.9
	high σ_p, low σ_c	-86.7	-3.1	83.6	0.0	25.7
	low σ_p, high σ_c	-71.9	-1.5	70.4	0.4	14.6
	high σ_p, high σ_c	-105.1	-4.2	100.9	-0.2	33.5
rate increasing	Spence (no unc)	-83.2	0.0	83.2	0.0	0.0
	low σ_p, low σ_c	-68.5	-7.9	60.6	-1.8	67.0
	high σ_p, low σ_c	-12.8	-1.4	11.4	-0.1	11.2
	low σ_p, high σ_c	-71.8	-1.9	69.9	-0.3	14.1
	high σ_p, high σ_c	-18.9	-1.6	17.3	-0.3	15.1
uncertainty reducing	Spence (no unc)	0.0	0.0	0.0	0.0	0.0
	low σ_p, low σ_c	-10.5	-2.7	7.8	0.6	15.1
	high σ_p, low σ_c	-6.8	-2.0	4.8	0.1	11.4
	low σ_p, high σ_c	3.5	6.1	2.5	6.1	-60.9
	high σ_p, high σ_c	-6.9	-0.6	6.3	0.1	-4.9

Table Notes: Percent change in EMA and other outputs when going from no-spillover to spillovers, with $\beta = 0.25$, high/low uncertainty is 0.5 and 2. Baseline is relative to the no-spillover case. The spillover levels for cost-reducing, rate-increasing, and uncertainty-reducing spillovers are $\omega = 0.8$, $\psi = 400$, and $\varrho = 0.8$, respectively. Results are qualitatively similar across a range of spillover levels.

FIGURES

Figure 1. Depiction of uncertainty in learning curves



Note: We demarcate the x-axis in units of time, which is equivalent to cumulative production under the assumption that production is binary (one unit per period).

Figure 2. Early learning premium at varying levels of prospective uncertainty and no contemporaneous uncertainty

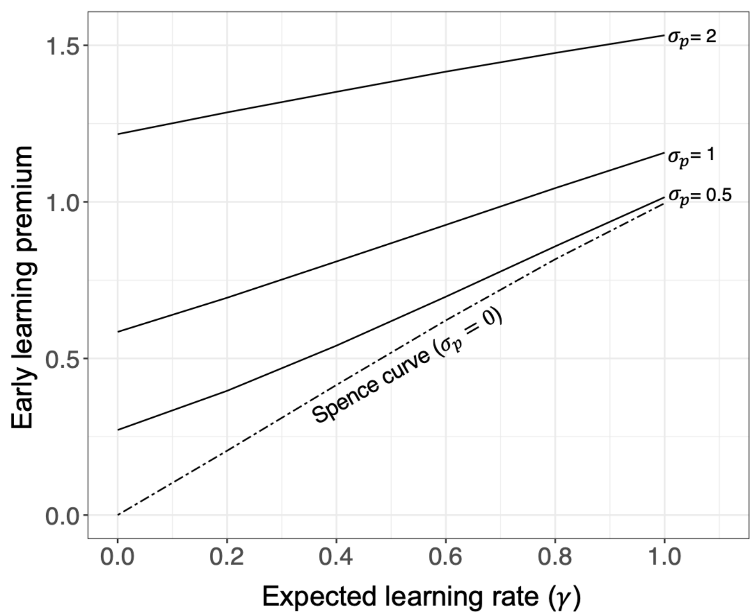


Figure 3. Early learning premium at varying levels of contemporaneous uncertainty

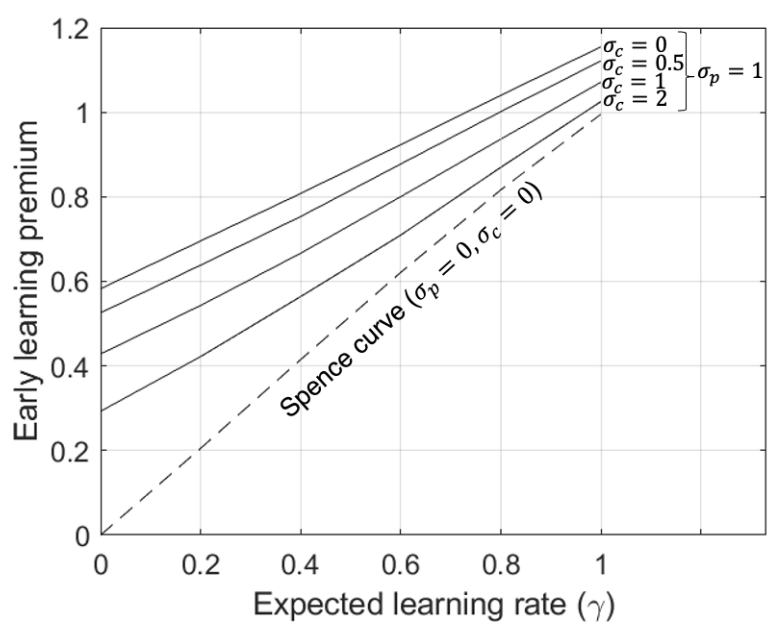


Figure 4. Early mover advantage and Competition

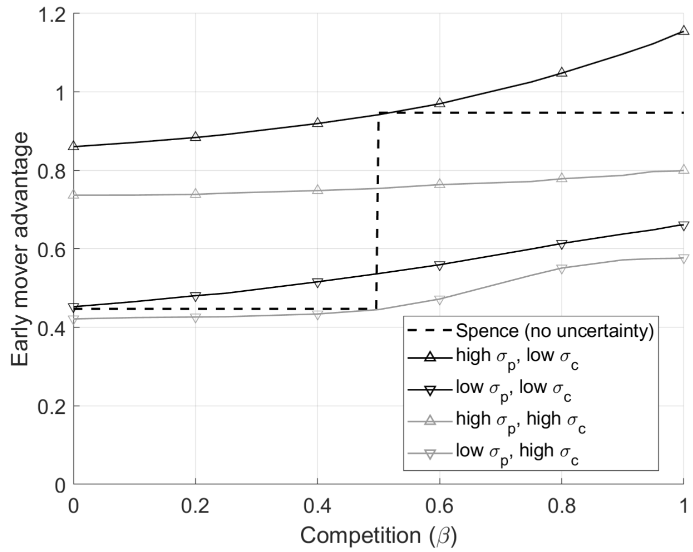


Figure 5. Decomposition of leader/laggard profit and quantity

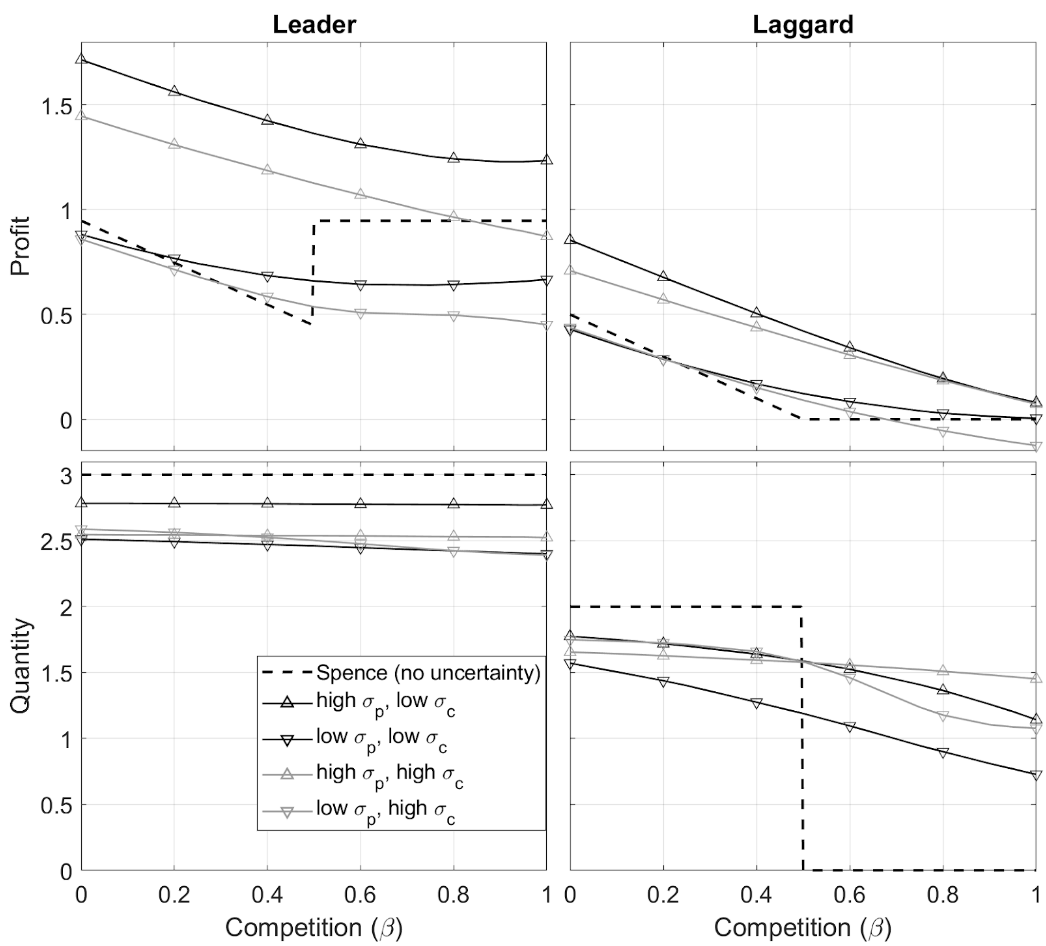


Figure 6. Early learning premium — learning rate versus asymptotic uncertainty

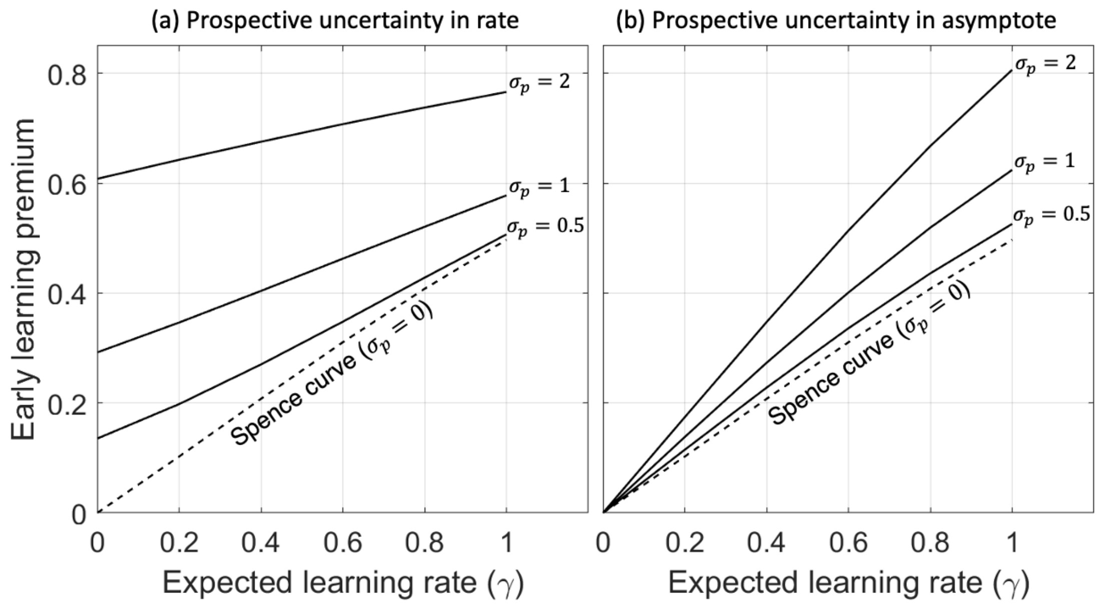


Figure 7. Capacity expansion: Early mover advantage and Competition

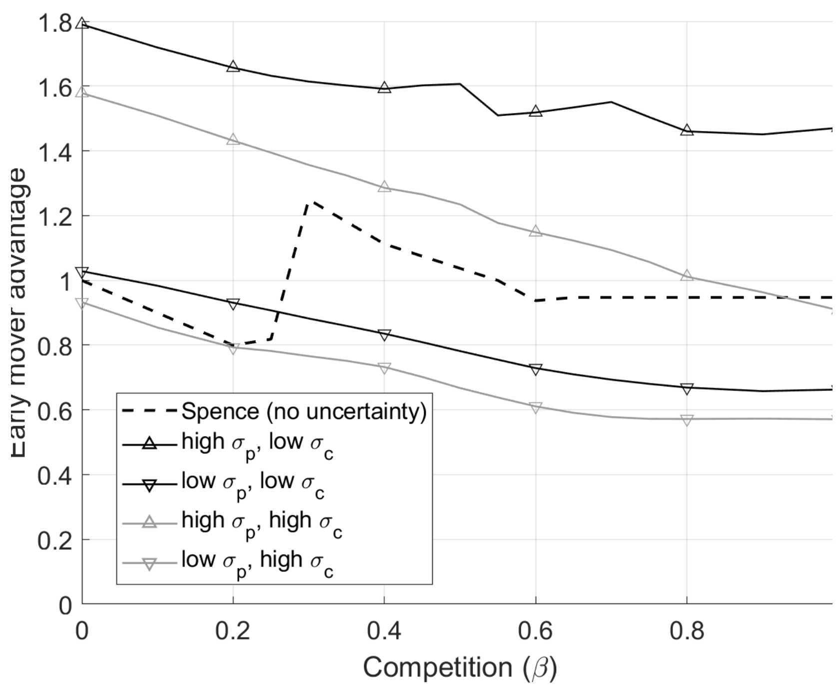
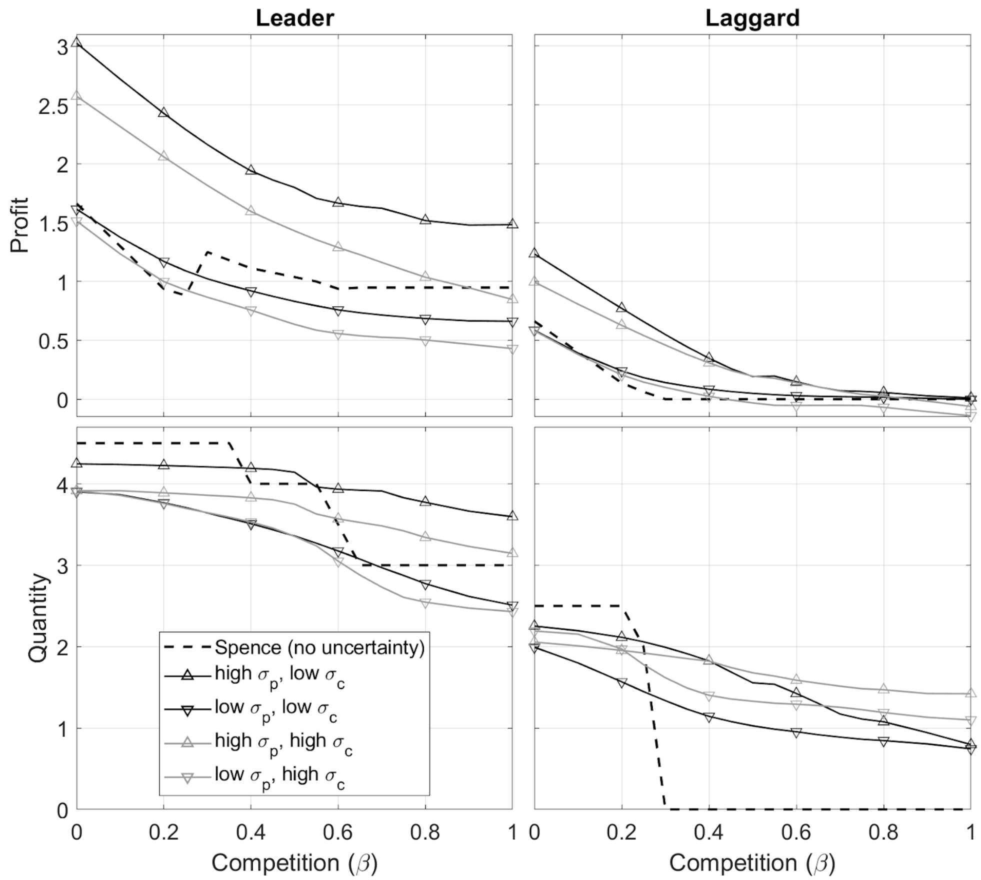


Figure 8. Capacity Expansion: Decomposition of leader/laggard profit and quantity



APPENDIX 1

COMPUTATIONAL MODEL PSEUDO CODE — LEARNING CURVE UNCERTAINTY

The computational model uses backward induction and a Monte Carlo approach to calculate the expected cumulative (three-period) profit from entry at $t = 1$ for a firm with initial cost, k_1, when learning curves are subject to prospective and contemporaneous uncertainty. The model parameters, in addition to initial cost, are: price m, prospective uncertainty σ_p, contemporaneous uncertainty σ_c, and expected learning rate γ. Equation numbers below refer to equations in the paper.

Before proceeding, we define as a function the mechanism underlying the evolution of a firm's production cost from period t - 1 to t under prospective and contemporaneous uncertainty, per our theoretical model given in the paper. The ***Cost Evolution Function*** takes as inputs prior period cost, current period production threshold, and prior period binary production decision $(k_{t-1}, k_t^{thresh}, decision_{t-1})$, and generates outputs in the form of the current period cost and production decision, $[k_t, decision_t]$. The pseudo code for this function is at the end of this document.

1. **Calculate k_3^{thresh}, the production decision threshold in the final period (t = 3).**
    a. Since there is no option value from production at $t = 3$, the firm will produce if its belief on cost at $t = 3$, b_3 (per equation 7), is less than the price.
    b. Thus, the cost threshold for the production decision is simply $k_3^{thresh} = m$.

2. **Calculate k_2^{thresh}, the production decision threshold in the next-to-last period (t = 2).**
    a. Expected profit over $t = 2$ and 3 is $E(\pi_2 + \pi_3) = E[exp(m) - exp(k_2) + exp(m) - exp(k_3)]$
    b. At $t = 2$, conduct a grid search to find the cost k_2^{thresh} at which the cumulative expected profit in periods $t = 2$ and 3 is zero, $E(\pi_2 + \pi_3) = 0$, conditional on production at $t = 2$.
        i. Define a search grid over costs at $t = 2$, $X = \{k_2^{Low}, k_2^{Low} + \Delta k, k_2^{Low} + 2\Delta k, ..., k_2^{High}\}$, where expected profit, $E(\pi_2 + \pi_3)$, is positive at cost k_2^{Low} and negative at k_2^{High}.
        ii. For each $k_2 \in X$ use a Monte Carlo to simulate stochastic cost movement between t = 2 and 3 by invoking, one million times, ***Cost_evolution***$(k_2, k_3^{thresh}, 1)$ function to generate the cost and production decisions at $t = 3$, $[k_3, decision_3]$.
            - Calculate expected profit $E(\pi_2 + \pi_3)$ by averaging realized profits over the million iterations that were simulated at cost k_2.
            - $E(\pi_2 + \pi_3) = \frac{1}{10^6} \sum_{i=1}^{10^6} \{exp(m) - exp(k_2) + [exp(m) - exp(k_3^i)] \cdot decision_3^i\}$, where k_3^i and $decision_3^i$ are the period 3 cost and production decision of the i*th* iteration.
        iii. The output is the cumulative expected profit at period $t = 2$ for each cost $k_2 \in X$.

c. Identify the $k_2^* \in X$ such that $E(\pi_2 + \pi_3) = 0$, then $k_2^{thresh} = k_2^*$.

3. **Calculate k_1^{thresh}, the production decision threshold in the first period (t = 1).**

a. Expected profit over t = 1 through 3 is $E(\pi_1 + \pi_2 + \pi_3) = \sum_{i=1}^{3} E(exp(m) - exp(k_i))$.

b. At $t = 1$, conduct a grid search to find the cost k_1^{thresh} at which the cumulative expected profit in periods t = 1, 2 and 3 is zero, $E(\pi_1 + \pi_2 + \pi_3) = 0$, conditional on production at t = 1.

i. Define a grid over the set of cost in t = 1, $X = \{k_1^{Low}, k_1^{Low} + \Delta k, k_1^{Low} + 2\Delta k, ..., k_1^{High}\}$, where expected profit, $E(\pi_1 + \pi_2 + \pi_3)$, is positive at cost k_1^{Low} and negative at k_1^{High}.

ii. For each $k_1 \in X$ use a Monte Carlo approach to simulate stochastic cost movement between t = 1 and 2 and then t = 2 and 3 by invoking, one million times, ***cost_evolution***(k_1, k_2^{thresh}, 1) function to generate [k_2, *decision$_2$*] followed by ***cost_evolution***(k_2, k_3^{thresh}, decision$_2$**)** to generate [k_3, *decision$_3$*]**.**

- Calculate expected profit $E(\pi_1 + \pi_2 + \pi_3)$ by averaging realized profits over the million iterations that were simulated at cost k_1
- $E(\pi_1 + \pi_2 + \pi_3) = \frac{1}{10^6} \sum_{i=1}^{10^6} \{exp(m) - exp(k_1)\} + [exp(m) - exp(k_2^i)] \cdot decision_2^i$ $+ [exp(m) - exp(k_3^i)] \cdot decision_3^i$, where k_t^i and $decision_t^i$ are the period t cost and production decision of the i*th* iteration within step ii.

iii. The output is the cumulative expected profit at period t = 3 for each cost $k_1 \in X$.

c. Identify the $k_1^* \in X$ such that $E(\pi_1 + \pi_2 + \pi_3) = 0$, then $k_1^{thresh} = k_1^*$. The ***early learning premium*** is then $exp(k_1^{thresh}) - exp(m)$ in Experiment 1.

4. **Calculate cumulative profit at any given k_1 over the three-periods**

a. For any given initial cost, k_1, calculate the expected cumulative (three-period) profit from entry at t = 1, $E(\pi) = \sum_{i=1}^{3} E(\pi_i) = \sum_{i=1}^{3} E(exp(m) - exp(k_i))$, taking into consideration contemporaneous uncertainty in the initial period.

b. At the chosen initial cost, k_1, use a Monte Carlo approach with one million iterations:

i. Draw ε_c from the standard normal distribution.

ii. Produce in t = 1 at a cost of k_1 if $\hat{k}_1 = k_1 - \sigma_c^2/2 + \sigma_c \varepsilon_c > k_1^{thresh}$ and set $decision_1 = 1$.

iii. Simulate stochastic cost movement between t = 1 and 2 by invoking ***cost_evolution***(k_1, k_2^{thresh}, $decision_1$) function to generate [k_2, *decision$_2$*].

iv. Simulate stochastic cost movement between t = 1 and 2 by invoking ***cost_evolution***(k_2, k_3^{thresh}, decision$_2$**)** to generate [k_3, *decision$_3$*].

c. Calculate expected cumulative profit over the three periods at k_1 by averaging realized profits over the million iterations that were simulated in step b. This is given by
$\frac{1}{10^6}\sum_{i=1}^{10^6}[exp(m) - exp(k_1)] \cdot decision_1^i + [exp(m) - exp(k_2^i)] \cdot decision_2^i$
$+ [exp(m) - exp(k_3^i)] \cdot decision_3^i$, where k_t^i and $decision_t^i$ are the period t cost and production decision of the i*th* iteration within step a. k_1 is fixed, so there is no need of an index to denote the iteration.

Note on calculating early mover advantage in the leader-laggard spillover model. The leader makes its entry decision at $t = 1$ and thus has three periods as described in the pseudo code above. The laggard makes its entry decision at $t = 2$ and thus has only two periods, with its learning parameters adjusted for the spillover as described in the paper. The necessary modifications to the pseudo code are mostly self-evident. The leader's **early mover advantage** is $E(\pi_{Leader}) - E(\pi_{Laggard})$.

Cost Evolution Function *(Input)* ⇒ [Output]:

Cost_evolution ($k_{t-1}, k_t^{thresh}, decision_{t-1}$) ⇒ [k_t, $decision_t$]

1. This function simulates one period of stochastic cost movement. (k_t, k_{t-1}) are the true costs at time t and t - 1. k_t^{thresh} is a cost threshold above which the firm will not produce at time t, and $(decision_t, decision_{t-1})$ are the binary production decisions at time t and t - 1.
2. If $decision_{t-1} = 0$, i.e. no production at t-1, set $k_t = k_{t-1}$ and $decision_t = 0$.
   a. This is because, if the firm did not produce in t - 1 because its cost was too high, then it does not move down the learning curve, and its cost will again be too high in t.
3. If $decision_{t-1} = 1$, i.e. production at t - 1, then:
   a. Draw ν_p and ε_c from the standard normal distribution
   b. Cost evolves per equation 5 as: $k_t = k_{t-1} - (\gamma + \sigma_p^2/2 - \sigma_p\nu_p)$ due to prospective uncertainty.
   c. Observed cost, which is contaminated by noise, per equation 6, such that $\hat{k}_t = k_{t-1} - q_{t-1}(\gamma + \sigma_p^2/2 + \sigma_c^2/2 - \sigma_p\nu_p - \sigma_c\varepsilon_c) = k_{t-1} - \gamma - \sigma_p^2/2 - \sigma_c^2/2 + \sigma_p\nu_p + \sigma_c\varepsilon_c$, the latter equality results because $q_{t-1} = 1$ when production occurs at t - 1.
   d. Form an updated belief about the profits from producing at t per equation 7. Here, b_t is the firm's updated belief about cost when the prior period's cost is k_{t-1}:
   $b_t = \Omega\hat{k}_t + (1 - \Omega)(k_{t-1} - \gamma - \sigma_p^2/2 - \sigma_c^2/2) + (1 + \Omega)\sigma_c^2/2$, where $\Omega = \sigma_p^2/(\sigma_p^2 + \sigma_c^2)$
   e. If $b_t < k_t^{thresh}$. set $decision_t = 1$ (i.e., produce at t), and otherwise set $decision_t = 0$.

APPENDIX 2

ALTERNATIVE APPROACH - ANALYTICAL MODEL

The analytical model below presents an alternative approach that may be used to examine the strategic implications of prospective and contemporaneous uncertainty in learning curves. The binomial tree formulation is easily tractable if one forgoes the standard learning curve formulation of diminishing returns to experience (by implications, however, production and profits are convex in prospective uncertainty and expected learning rate). We first present the two-period model then extend it to three periods.

Two-Period Model

Consider a two-period production model with $t = 1, 2$, and with a per-period discount factor of $\delta \equiv (1+r)^{-1}$, where $r > 0$ is the discount rate. Let the inverse demand curve in each period be $m(q_t) = \alpha - q_t$. The learning curve is $k(Q_t) = k_1 - \tilde{\gamma} Q_t$, where k_1 is the initial cost and $Q_t \equiv \sum_{s=1}^{t-1} q_t$ is cumulative output (note $Q_1 = 0$).[1] $\tilde{\gamma}$ is a random variable that reflects *prospective* uncertainty in the learning rate. For simplicity, we assume $\tilde{\gamma}$ takes on the two values $\gamma_H > \gamma_L$ with equal probability (i.e., high and low learning rates are equiprobable). Thus, learning rate has a mean $\gamma \equiv (\gamma_H + \gamma_L)/2$ and standard deviation $\sigma_p \equiv (\gamma_H - \gamma_L)/2$, which is prospective uncertainty. As in the paper, the firm knows the realization of $\tilde{\gamma}$ perfectly after period one production (assuming contemporaneous uncertainty is zero).

We introduce *contemporaneous* uncertainty as mistakes in the firm's assessment of whether a high or low learning rate outcome has occurred. The probability the firm incorrectly perceives the learning draw in each period is represented by a Bernoulli random variable $0 \leq p_c \leq 1/2$, such that greater p_c represents greater contemporaneous uncertainty. That is, the firm will, with probability p_c, erroneously believe that $\tilde{\gamma} = \gamma_L$ when $\tilde{\gamma}$ is actually γ_H (and vice-versa). In the paper, contemporaneous uncertainty is expressed as the variance in the noisy signal received by the firm, σ_c. The variance of the Bernoulli distribution is $p_c(1-p_c)$, which implies that $\sigma_c = \sqrt{p_c(1-p_c)}$ and therefore $0 \leq \sigma_c \leq 1/2$.

Let $\hat{\gamma}$ be the observed learning rate at $t = 2$. Using Bayes' Rule, or by considering the diagram below, the probability that the firm has the correct belief on learning rate given that it observes γ_H is $1 - p_c$, and the probability that the firm has the correct belief on learning rate given that it observes γ_L is also $1 - p_c$.

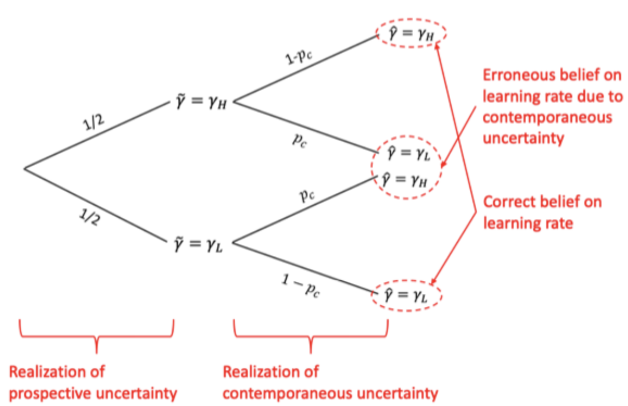


Figure 1: Simple production model for one period of decision-making

[1] This linear learning curve facilitates a closed form solution for optimal production, at the expense of the diminishing returns to experience that is typically seen in the learning curve literature.

As a Bayesian decision-maker, the firm at $t = 2$ will maximize profit based on what it observes, taking into consideration the probability of error as noted above. Thus, the expected learning rate given an observation of $\hat{\gamma} = \gamma_H$ is $[(1 - p_c)\gamma_H + p_c\gamma_L]q_1$, leading to an expected profit when $\hat{\gamma} = \gamma_H$ at $t = 2$ to be:

$$\pi_2(q_2)|_{\hat{\gamma}=\gamma_H} = \{\alpha - q_2 - k_1 + [(1 - p_c)\gamma_H + p_c\gamma_L]\, q_1\}\, q_2 \tag{1}$$

Similarly, the $t = 2$ profit when $\hat{\gamma} = \gamma_L$ is:

$$\pi_2(q_2)|_{\hat{\gamma}=\gamma_L} = \{\alpha - q_2 - k_1 + [(1 - p_c)\gamma_L + p_c\gamma_H]\, q_1\}\, q_2 \tag{2}$$

Taking first order conditions and solving for q_2 gives:

$$\begin{aligned} q_2^*|_{\hat{\gamma}=\gamma_H} &= \frac{\alpha - k_1 + [(1 - p_c)\gamma_H + p_c\gamma_L]\, q_1}{2} \\ q_2^*|_{\hat{\gamma}=\gamma_L} &= \frac{\alpha - k_1 + [(1 - p_c)\gamma_L + p_c\gamma_H]\, q_1}{2} \end{aligned} \tag{3}$$

Plugging these optimal values back into the profit equations give:

$$\begin{aligned} \pi_2^*(q_1)|_{\hat{\gamma}=\gamma_H} &= \left[\frac{\alpha - k_1 + [(1 - p_c)\gamma_H + p_c\gamma_L]\, q_1}{2}\right]^2 \\ \pi_2^*(q_1)|_{\hat{\gamma}=\gamma_L} &= \left[\frac{\alpha - k_1 + [(1 - p_c)\gamma_L + p_c\gamma_H]\, q_1}{2}\right]^2 \end{aligned} \tag{4}$$

Using backward induction and the fact that low and high learning rates are equally observed (but not without error) at $t = 2$, the expected profit at $t = 1$ is given by:

$$\begin{aligned} \pi_1(q_1) = (\alpha - q_1 - k_1)\, q_1 &+ \frac{\delta}{2}\left[\frac{\alpha - k_1 + [(1 - p_c)\gamma_H + p_c\gamma_L]\, q_1}{2}\right]^2 \\ &+ \frac{\delta}{2}\left[\frac{\alpha - k_1 + [(1 - p_c)\gamma_L + p_c\gamma_H]\, q_1}{2}\right]^2 \\ &= (\alpha - q_1 - k_1)\, q_1 + \frac{\delta}{4}\left[(\alpha - k_1 + \gamma q_1)^2 + (1 - 2p_c)^2\sigma_p^2 q_1^2\right] \end{aligned} \tag{5}$$

The first term is just the price/quantity tradeoff. The second is the expected profit from learning in the absence of uncertainty (per Spence, 1981). The third term reflects prospective and contemporaneous uncertainty. Notice that the early mover premium increases (decreases) with prospective (contemporaneous) uncertainty, as our paper shows. Also notice that at full contemporaneous uncertainty — that is, when $p_c = 1/2$ and learning signals become pure noise — the prospective uncertainty gain (the third term) is zero, as we would expect.

This formulation allows for the calculation of optimal production quantity and corresponding profit at $t = 1$, and how this is impacted by various factors (learning rate, prospective and contemporaneous uncertainty, etc.). Solving the first-order conditions of Equation 5 and substituting σ_c for $\sqrt{p_c(1 - p_c)}$, we obtain:

$$\boxed{\begin{aligned} q_1^* &= \frac{(2 + \delta\gamma)(\alpha - k_1)}{4 - \delta\left[\gamma^2 + \sigma_p^2(1 - 4\sigma_c^2)\right]} \\ \pi_1(q_1^*) &= \frac{1}{4}(\alpha - k_1)^2\left\{\delta + \frac{(2 + \delta\gamma)^2}{4 - \delta\left[\gamma^2 + \sigma_p^2(1 - 4\sigma_c^2)\right]}\right\} \end{aligned}} \tag{6}$$

Three-Period Model

The key to solving the three-period model is to recognize that after second period production, we are faced with exactly the same scenario as at the beginning of the first period of a two-period production model, except that our starting cost is lower. Recall our assumption in the paper that all contemporaneous uncertainty is resolved after units are actually produced, which is quite reasonable as the true costs are paid at that point.

In particular, the decision tree of Figure 1 will be applicable at that time, and will result in a functional form for profit analogous to Equation 5 (but shifted in time):

$$\pi_2(q_2, k) = (\alpha - q_2 - k)q_2 + \frac{\delta}{4}[(\alpha - k + \gamma q_2)^2 + (1 - 2p_c)^2\sigma_p^2 q_2^2] \tag{7}$$

Using backwards induction, we can now apply the same logic as in Equations 1 and 2. That is, conditional on seeing a high learning signal at $t = 1$, the firm knows that the learning draw was actually high with probability $1 - p_c$. Thus, the firm will rationally conclude that its expected profit is:

$$\pi_2(q_2)|_{\hat{\gamma}=\gamma_H} = p_c\pi_2(q_2, k_1 - \gamma_L q_1) + (1 - p_c)\pi_2(q_2, k_1 - \gamma_H q_1) \tag{8}$$

Likewise, the expected profit conditional on seeing a low learning draw will be:

$$\pi_2(q_2)|_{\hat{\gamma}=\gamma_L} = p_c\pi_2(q_2, k_1 - \gamma_H q_1) + (1 - p_c)\pi_2(q_2, k_1 - \gamma_L q_1) \tag{9}$$

Symmetry in our setup implies that low and high learning draws are observed equally, even with contemporaneous uncertainty, so the total expected profit at the initial time period (before any production decision has been made) is:

$$\pi_1(q_1) = (\alpha - q_1 - k_1)q_1 + \frac{\delta}{2}\pi_2(q_2)|_{\hat{\gamma}=\gamma_H} + \frac{\delta}{2}\pi_2(q_2)|_{\hat{\gamma}=\gamma_L} \tag{10}$$

Solving first order conditions in Equation 10 gives $\pi_1(q_1)$, the profit as a function of q_1:

$$\pi_1(q_1) = \frac{1}{4}\left[2\delta^2\gamma\alpha' q_1 + \delta^2\alpha'^2 + \delta^2 q_1^2\sigma_p^2 - 4\delta\gamma q_1^2 + 4\alpha' q_1 - 8q_1^2 + \frac{(\delta\gamma + 2)^2\left(\delta\alpha'(2\gamma q_1 + \alpha') + 4q_1^2\right)}{4 - \delta\left(\gamma^2 + (1 - 4\sigma_c^2)\sigma_p^2\right)}\right] \tag{11}$$

where we set $\alpha' = \alpha - k_1$ for notational brevity. Solving first order conditions for Equation 11 gives q_1^*, the optimal initial production quantity:

$$\boxed{q_1^* = \frac{\alpha' + \frac{1}{2}\delta^2\gamma\alpha' + \frac{\delta\gamma\alpha'(\delta\gamma+2)^2}{2\left(4-\delta\left(\gamma^2+(1-4\sigma_c^2)\sigma_p^2\right)\right)}}{2\delta\gamma + 4 - \frac{\delta^2\sigma_p^2}{2} - \frac{2(\delta\gamma+2)^2}{4-\delta\left(\gamma^2+(1-4\sigma_c^2)\sigma_p^2\right)}}} \tag{12}$$

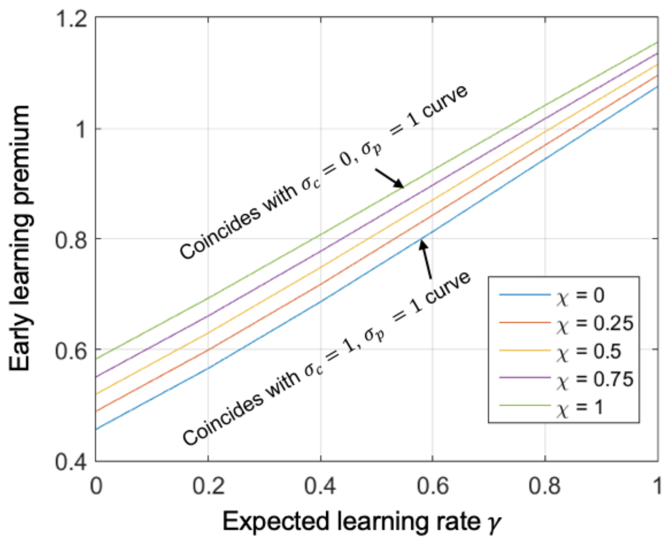
Optimal profit is $\pi_1(q_1^*)$.

APPENDIX 3

MODEL EXTENSIONS

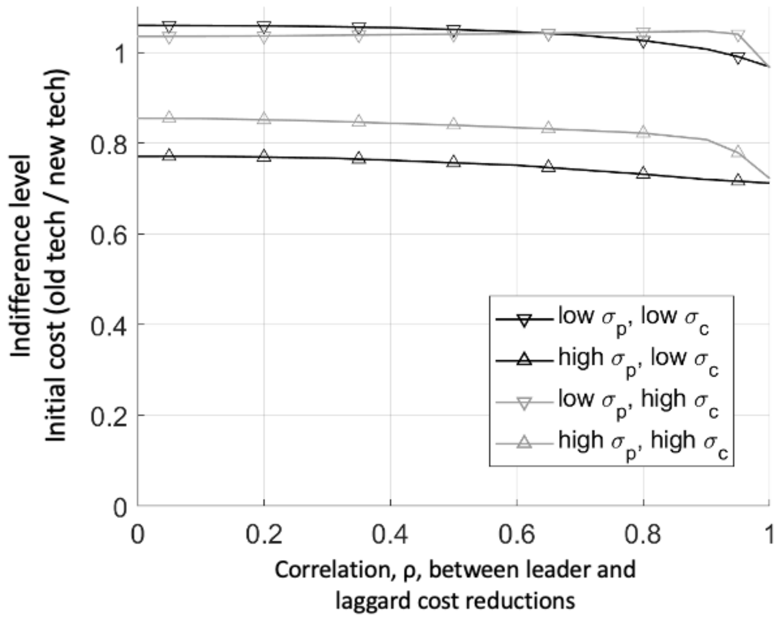
This document covers results for model extensions in Section 4.4 of the paper, but not included in the main body. In these model extensions, parameters are the same as in the paper unless otherwise noted.

Figure A3.1. Early learning premium when prospective and contemporaneous uncertainty realizations are correlated.



Note: This is the single-firm model with analysis conducted at $\sigma_c = \sigma_p = 1$. This model addresses the possibility of an association between prospective and contemporaneous uncertainty affects the early learning premium. A firm may not experience independent draws from the two unit uncertainty distributions, ε_c, ν_p, but instead may experience draws that are correlated at a level of χ (which is known to the firm). The key result is that, as the correlation increases, so too does the early learning premium. When $\chi = 1$, the firm knows that draws from ν_p and ε_c are identical, and can therefore infer the true cost reduction. More generally, as the correlation in realizations of the two forms of uncertainty increases, the effective level of contemporaneous uncertainty decreases. Thus, as the correlation increases, so too does the early mover premium. Ultimately, the $\chi = 0$ line in the Figure is equivalent to the $\sigma_c = \sigma_p = 1$ in Figure 3 of the paper and the $\chi = 1$ in the Figure is equivalent to the $\sigma_p = 1$ with $\sigma_c = 0$ line in Figure 3 of the paper.

Figure A3.2. Leader using a new technology versus a laggard choosing between old and new technology - Indifference curve



Note: Two-firm model. The leader employs the new technology and enters at $t = 1$. The laggard, which enters at $t = 2$, can choose the existing or new technology. The expected learning rate is $\gamma = 0.2$ for both technologies. The new technology is superior to the existing technology in that the former has a lower asymptotic cost [i.e., asymptote (old_tech) = asymptote (new_tech) + 0.1]. The existing technology differs from the new technology in three other ways because the old technology is "tried-and-true." First, there is no prospective or contemporaneous uncertainty in the existing tech as its learning curve is well understood. Second, in using the existing technology, the laggard will not benefit from spillovers from the leader using the new technology, since the technologies are different. Third, the existing technology may have a lower initial cost (at the start of the learning curve) than the new technology because it is so well understood.

We vary (a) the initial cost at the start of the learning curve for the existing technology relative to the new technology, represented on the y-axis and (b) the extent of uncertainty reducing spillovers from the leader to the laggard if the laggard uses the new technology on the x-axis. The lines are indifference curves that indicate regions where the laggard is indifferent between the two technologies. The lows and high values for both σ_p and σ_c are 0.5 and 2, respectively. The competition level $\beta = 0.25$.

The key result is that, if the initial cost of the existing technology is sufficiently low, and the spillovers from the leader using the new technology are also sufficiently low, then the laggard will choose to use the existing technology even though it is inferior in the long run. Increasing prospective uncertainty induces the laggard to move to the new technology. The existing technology has to be at a substantially lower initial cost than the new to induce adoption by the laggard when there is substantial prospective uncertainty. When σ_p is low, the laggard will choose the existing technology as long as its initial cost is lower than that of the new technology, regardless of the level of uncertainty reducing spillovers. When σ_p is high, the laggard will choose the existing technology only if its initial cost at the start of the learning curve is less than roughly 80 percent of that of the new technology. As uncertainty reducing spillovers in the new technology, ρ, increase, the laggard needs an even lower initial cost if it is to choose the existing technology. Finally, note that, at high levels of prospective uncertainty, increasing contemporaneous uncertainty induces the laggard to move to the existing technology (in contrast to increasing prospective uncertainty).