Frenemies in Platform Markets: Heterogeneous Profit Foci as Drivers of Compatibility Decisions*

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Abstract

We study compatibility decisions of two competing platform owners that generate profits through both hardware sales and royalties from content sales. We consider a game-theoretic model in which two platforms offer different standalone utilities to users. We find that incentives to establish one-way compatibility—the platform owner with smaller standalone value grants access to its proprietary content application to users of the competing platform—can arise from the difference in their profit foci. As the difference in the standalone utilities increases, royalties from content sales become less important to the platform owner with greater standalone value, but more important to the other platform owner. One-way compatibility can thus increase asymmetry between the platform owners' profit foci and, given a sufficiently large difference in the standalone utilities, yields greater profits for both platform owners. We further show that social welfare is greater under one-way compatibility than under incompatibility. We also investigate how factors such as exclusive content and hardware-only adopters affect compatibility incentives.

Keywords: frenemies; compatibility; platform competition; profit foci

1 Introduction

An increasing number of markets today are organized around platforms that enable consumers to access complementary goods and services. These platforms are two-sided because both sides—consumers and complementors—need access to the same platform to interact or conduct transactions. A video game console like Xbox, PlayStation, or Wii, for example, serves as a platform connecting game players with independent game publishers who need access to the console's programming interface to develop games that can be sold to players. Other examples of platforms include smartphones, e-readers, credit cards, shopping malls, and social networking sites.

The literature on platform-based markets has examined strategies a platform owner can use to grow its business, such as two-sided pricing (e.g., Rochet and Tirole 2003; Parker and Van Alstyne 2005; Armstrong 2006; Hagiu 2006; Seamans and Zhu 2014; Cennamo and Panico 2015), quality investment (e.g., Zhu and Iansiti 2012; Casadesus-Masanell and Llanes forthcoming), adopting innovative business models (e.g., Economides and Katsamakas 2006; Casadesus-Masanell and Zhu 2010), enveloping adjacent platform markets (e.g., Eisenmann et al. 2011), and managing relationships with complementors (e.g., Carrillo and Tan 2008; Adner 2013; Hagiu and Spulber 2013; Huang et al. 2013; Kapoor 2013).

Our study complements the prior literature by examining competing platform owners' compatibility decisions. We are motivated by empirical observations that platform owners may become frenemies (friends and enemies)—they compete but at the same time cooperate with each other. Of particular interest are settings characterized by asymmetric compatibility where one platform owner hosts a rival's application but not vice versa. For example, in the e-reader market, two major platforms, Apple's iPad and Amazon's Kindle, compete against each other (e.g., Johnson 2013, 2014; De los Santos and Wildenbeest 2014; Dou 2014). These devices enable consumers to read e-books through their respective proprietary e-book apps, Apple's iBooks and Amazon's Kindle Reader. The Kindle device was introduced in 2007, the iPad in 2010. After Apple's entry into the market, Amazon cut Kindle's price by $70 as a competitive response. But shortly after, it decided to make its Kindle Reader app available on the iPad, thereby enabling consumers to read e-books purchased from Amazon on the iPad. Apple, well known for rejecting third-party applications that compete directly with its own offerings,[1] nevertheless approved Amazon's Kindle Reader app for the iPad, effectively rendering the two platform owners frenemies. Apple has not, however, made its iBooks app available for the Kindle.

As another example, in the automotive industry, cars' dashboards are becoming platforms to connect passengers to content and services. General Motors (GM) has developed a software system, OnStar, to provide value-added services such as vehicle diagnostics, navigation, internet access and app stores to its vehicles. At the same time, Google, a competitor to car manufacturers because it is building self-driving cars,[2] offers a smartphone integration system, Android Auto, that allows compatible vehicles to run many Android apps on the dashboard. Although GM's own OnStar app offers many features overlapping those of Android Auto, GM decided to make its cars compatible with Android Auto.[3]

What motivates competing platform owners to become frenemies by choosing this asymmetric equilibrium of one-way compatibility? How does such compatibility affect their profits? How do factors such as exclusive content affect compatibility incentives? To answer these questions, we develop a game-theoretic model in which two competing platform owners generate profits from both hardware sales and royalties from content sales. The hardware generates different standalone utilities to users. Both platform owners make compatibility decisions first and then set their hardware prices, and finally consumers purchase hardware and content. Compatibility is achieved when one decides to make its proprietary content application available on the competing platform and the competitor agrees.

We find that incentives to establish one-way compatibility—where one platform owner allows users of a competing platform to access its content without reciprocal access for its own users—can arise from the difference in the platforms' profit foci. As the difference in the standalone utilities of hardware increases, royalties from content sales become less important to the platform owner with greater standalone value but more important for the other platform owner. We further show that social welfare is greater under one-way compatibility than under incompatibility.

Our findings help explain the compatibility incentives that underline the examples discussed

[1] See, for example, http://almerica.blogspot.com/2008/09/podcaster-rejeceted-because-it.html, accessed June 2017.

[2] Source: http://www.dw.com/en/bmw-sees-apple-google-as-serious-competitors/a-18291512, accessed April 2017.

[3] Source: http://tinyurl.com/jfuoq29, accessed April 2017.

above. In the e-reader market, Apple's iPad provides many features beyond reading e-books, while Amazon's Kindle has been almost exclusively an e-reader device. As a result, in equilibrium, compared to Amazon, Apple's hardware profits are more important to its total profits. In contrast, for Amazon, royalties from e-book sales are more important to its total profits relative to Apple.[4] When this difference in profit foci is large enough, having the Kindle Reader app available on iPad is agreeable to both Apple and Amazon: Amazon's e-book sales increase because iPad users can now purchase e-books from Amazon and read them via the Kindle Reader app, while Apple's hardware sales increase because greater value accrues to the iPad with access to the Kindle Reader app than in the case of incompatibility. The additional profits Apple generates from hardware sales more than compensate for its loss in royalties from e-book sales through its iBooks app. Similarly, the additional profits Amazon generates from e-book sales are greater than its loss in Kindle device sales. Our model suggests a logic for incompatibility in the reverse direction: Neither Apple nor Amazon have incentives to make iBooks app available on Kindle. If iBooks app were available on Kindle, Apple would have lost to Amazon some hardware buyers who prefer the combination of the Kindle device and iBooks app (perhaps because of its better integration with Apple's iTunes store), and similarly, Amazon would have lost to Apple some books sales. Such losses would be significant for both firms given their profit foci.

In a similar vein, GM and Google choose to make GM cars compatible with Google's Android Auto because the car business remains the profit focus of GM, while ad-sponsored content provides the major source of profits to Google.[5] We are likely to have many buyers who prefer GM cars because of its reputation as a car manufacturer and Android Auto because they value its integration with their Android phones. Compatibility in this direction makes sense, since having Google's Android Auto on GM cars increases car sales for GM as well as ad revenue for Google because the bundle attracts these buyers. This logic also explains incompatibility in the reverse direction.

[4]The result is consistent with reports that Apple profits from every iPad sale, but Amazon earns no profits on Kindle sales. Source: http://www.forbes.com/sites/kellyclay/2012/10/12/amazon-confirms-it-makes-no-profit-on-kindles/, accessed March 2018.

[5]Although the exact business model for Google Car is not known because Google Car is still in development, with driverless technology, analysts have speculated that Google has the potential to capture value from an average of 50 min of a U.S. commuter's time in car through content delivered to car dashboards such as YouTube video, location-based search advertising, and so on (source: https://www.washingtonpost.com/news/wonk/wp/2016/02/25/how-much-of-your-life-youre-wasting-on-your-commute/). As a benchmark, Facebook users spent on average about 50 min on Facebook a day in 2016 (source: https://www.nytimes.com/2016/05/06/business/facebook-bends-the-rules-of-audience-engagement-to-its-advantage.html).

If GM's profit comes mostly from hardware sales and Google's profit comes mostly from content delivered through software, then it does not make sense for GM to have its OnStar system on Google Cars for two reasons. First, GM would lose to Google some car buyers who prefer the combination of the Google car and the GM system, who otherwise may have bought GM cars with the GM system. Second, Google also suffers as these car buyers do not use Android systems after buying Google Cars. By not having GM's OnStar system on Google Car, GM and Google can deter these buyers from adopting this bundle.

We extend our baseline model to examine how factors such as exclusive content and hardware-only adopters (i.e., consumers who do not purchase content after adopting a platform) influence the platform firms' compatibility incentives. We find that factors that reduce (increase) asymmetry in profit foci tend to reduce (increase) incentives to become one-way compatible. On the one hand, exclusive content on a platform with smaller standalone value increases the owner's reliance on content sales and thus heterogeneity in profit foci and the likelihood of compatibility. On the other hand, exclusive content increases the platform's value to users, thereby reducing the difference in utilities for the two firms. This reduction in heterogeneity reduces the likelihood of one-way compatibility. In the end, whether exclusive content increases the likelihood of one-way compatibility depends on its relative impact on utility difference and extra profits from additional content sales to the platform owner. This result is in contrast to studies in system competition (see, e.g., Katz and Shapiro 1985), where firms with larger networks tend to prefer incompatibility because, with compatibility, they would lose their market share advantage. We also find that the presence of hardware-only adopters induces both firms to focus more on their hardware sales. As a result, the two firms' profit foci become more similar, thereby reducing their incentives to have one-way compatibility.

The rest of the paper is organized as follows. We discuss the related literature in Section 2. In Section 3, we present the setup for our baseline model. Equilibrium results under incompatibility, two-way compatibility, and one-way compatibility are reported in Section 4. In Section 5, we derive the conditions under which one-way compatibility becomes the equilibrium outcome. In Section 6, we provide two extensions to our model. Finally, we discuss the implications of our results and conclude in Section 7. All technical proofs are included in Appendix A.

2 Literature Review

Our model builds on the theoretical literature on platform markets (e.g., Caillaud and Jullien 2003; Rochet and Tirole 2003; Bhargava and Choudhary 2004; Hao et al. 2015). Many of the extant theoretical models focus on competition between symmetric platform firms. The few papers examining the competition between asymmetric platforms tend to focus on platform firms with very different business models. Casadesus-Masanell and Ghemawat (2006) and Economides and Katsamakas (2006), for example, investigate the competition between proprietary and open-source platform firms; Casadesus-Masanell and Zhu (2010) investigate the competition between a platform firm that is both subscription-based and ad-sponsored, and one that is entirely ad-sponsored. Niculescu and Wu (2014) study different business models for selling software products, such as freemium and uniform seeding models. Our baseline model, in contrast, examines two platform firms with similar business models distinguished only by the amount of standalone value they create for users. We show that this difference alone yields opportunities to become frenemies.

A subset of this literature addresses compatibility. Doganoglu and Wright (2006), examining the difference between multi-homing and compatibility, find the latter to reduce incentives to pursue the former. Maruyama and Zennyo (2013) find compatibility to depend on product life-cycles: once most users have purchased hardware, platform firms' profits accrue largely from content purchases, whereupon competing platform firms have incentives to become compatible. The few studies that examine asymmetric platform firms typically find that weak firms seek compatibility in order to steal market share from stronger firms and that stronger firms have no incentive to establish compatibility. Casadesus-Masanell and Ruiz-Aliseda (2009), for example, explain large platform firms' preference for incompatibility in terms of the quest for market dominance, and Viecens (2011) shows that compatibility will always be preferred by a platform firm with smaller standalone value and never by its competitor.

The prior literature is thus unable to explain the mutual incentives for one-way compatibility among rivals. Our model, however, shows that as the difference in standalone utilities of two competing platform firms increases, both firms could become more willing to be compatible. Dou (2014) finds, in a model with vertically differentiated platforms and content, that when an inferior platform firm owns premium content, it is optimal for the inferior platform firm to offer such content to a superior platform firm. Dou's paper assumes that one-way compatibility can be established without the rival's permission. In our model, however, content quality does not have to differ across the two platform firms for compatibility incentives to emerge. More importantly, in our model one way compatibility arises as a consensus decision by both firms.

Our work is also related to the literature on system competition (e.g., Farrell and Saloner 1985, 1986; Katz and Shapiro 1985, 1994). This literature focuses on competing products that exhibit network effects. With compatibility, consumers of one product gain access to consumers of the other product (in the case of direct network effects) or to complementary applications designed for the other product (in the case of indirect network effects). These studies find that without heterogeneity between firms, firms always have incentives to be compatible because compatibility reduces competitive intensity. The same incentive also emerges in our model: we find that the two platform firms compete away content profit in the incompatible case but not in the case of compatibility. On the other hand, because in our model compatibility reduces differentiation between the two firms, the per-user content profit needs to be sufficiently high for two symmetric firms to prefer compatibility. More importantly, our paper identifies and focuses on heterogeneous profit foci between the two firms as an important driver for compatibility. A number of studies in this literature examine compatibility incentives where one firm has a larger installed base (e.g., Crémer et al. 2000; Malueg and Schwartz 2006; Farrell and Klemperer 2007; Chen et al. 2009), and find that it is less willing to be compatible because with compatibility, it has to share its network, while with incompatibility, it can maintain its market dominance. In contrast, our model shows that making a platform firm larger by allowing it to have exclusive content has an ambiguous effect on its compatibility incentive.

Finally, our work is related to the mix-and-match literature (e.g., Matutes and Regibeau 1988; Economides 1989; Matutes and Regibeau 1992; Kim and Choi 2015), which does not consider multi-sided market structures, but assumes that each system is made up of components and that a consumer needs to buy all of the components to use the system. Compatibility allows consumers to mix and match components from different system providers. Like the system competition literature and like our model, these studies find that system providers prefer compatibility because it reduces each firm's incentive to cut the price for a given component: without compatibility, a price cut leads to an increase in purchases of the whole system sold by each firm; with compatibility, a price cut only increases demand for that component. The firms in these settings capture value from selling components that are assumed to be symmetric to each other, whereas in our setting, profits are earned not only from hardware sales, but also from transactions conducted on the platform. Thus, the business models of the firms in these two settings differ. As a result, in equilibrium, a firm in the mix-and-match literature charges the same price for all its components, whereas in our model, firms can subsidize hardware in order to generate more revenue from transactions, which creates new incentives for compatibility.

Studies in all these streams of literature tend to focus on two-way compatibility because they often examine symmetric firms. In the case of asymmetric firms with different installed bases, one firm often prefers incompatibility. The two firms will thus either have no compatibility, or one firm can establish one-way compatibility through the use of a converter without its rival's consent (e.g., Farrell and Saloner 1992; Manenti and Somma 2008; Liu et al. 2011). Our study focuses on one-way compatibility based on mutual consent and examines the impact of other asymmetries between two firms, including different standalone values and different installed bases of content. We show that only those asymmetries that lead to more (less) differentiation in profit foci increase (decrease) incentives for one-way compatibility.

3 Model

We consider two platform firms, 1 and 2, that provide hardware devices, H_1 and H_2, and software applications, S_1 and S_2, respectively. Consumers use the software applications to consume content provided by third-party content providers. For example, in the case of the e-reader market, the hardware devices are the iPad and Kindle devices, the software applications are iBooks and Kindle Reader, and the content is e-books provided by book publishers. The content providers multi-home so the content is the same on these two platforms. The hardware devices may differ in their sizes, colors, and texture, and the software applications may differ in their interfaces and design. Therefore, consumers may have different preferences for hardware and software. We model the competition between the two platform firms as horizontally differentiated products. To capture consumer preference over both the hardware and software dimensions, we use a two-dimensional location model and consider a 1×1 square with firm 1 at the bottom-left corner $(0, 0)$ and firm 2 at the top-right corner $(1,1)$. A continuum of consumers of measure 1 is distributed across the square. Each consumer is characterized by a two-dimensional type (x, y), where $x \in [0,1]$ and $y \in [0,1]$. The x dimension represents a consumer's preference for hardware and the y dimension represents preference for software.

Consumer utility for each firm is the value a consumer derives from the platform net the price and disutility from the misfit between the firm and the consumer's taste. The degree of misfit in hardware (software) is measured by the distance between the firm's and a consumer's locations in the x (y) dimension. We assume that the unit hardware (software) misfit cost is t_h (t_s). We denote the hardware price of platform i as p_i and the consumer utility derived from the platform as U_i, $i \in \{1,2\}$. Consumers compare the two platforms and choose the one that offers greater value. Consistent with the practice of many markets (e.g., the e-reader market), we assume that the software from either platform is free. We also assume that content providers multi-home and platform firms use an agency model under which content providers set the content price directly. Therefore, the content quality and price are the same on the two platforms and can be omitted from the model because it does not alter consumers' platform choices.

When users of one firm's hardware can only use software from that same firm, which we refer to as the *incompatible case*, the utility for a consumer located at (x, y) from each firm can be formulated as

$$U_1 = v_1 - t_h x - t_s y - p_1, \tag{1}$$

$$U_2 = v_2 - t_h(1-x) - t_s(1-y) - p_2, \tag{2}$$

where v_i captures the value a consumer derives from using the platform (known to both the firms and consumers) such as access to the content and using other platform features. To capture heterogeneity in utilities that firms provide to users, without loss of generality, we assume that firm 1 offers superior standalone value: $v_1 > v_2$. The extra utility may come from additional functionalities offered by firm 1. For example, in the e-reader industry, Apple's iPad offers many mobile applications (such as a map app, a flashlight app, and the iTunes store) in addition to iBooks, while Amazon's Kindle Reader is primarily an e-book reader. Hence, Apple offers greater standalone value than Amazon. We denote the difference in the standalone utilities as $v_d = v_1 - v_2$.

When firm i's software, S_i, is also available on firm j's hardware, H_j, we call this the *compatible case*. Consumers who purchase H_j then have a choice of the two software applications, S_1 and S_2, and will choose the one with a lower misfit cost. In this case, to take into account consumers who purchase H_j but prefer S_i over S_j because of the lower misfit cost, we reformulate the software misfit cost for a consumer located at (x, y) from using platform j as $t_s \min\{y, 1-y\}$. We may have one-way compatibility in which only one firm's software is available on its rival's hardware, but not the reverse, or two-way compatibility, in which both firms' software applications are available on their rivals' hardware platforms.

We assume that consumer preferences for hardware (x) are uniformly distributed over $[0, 1]$. Given a consumer's preference for hardware at $x = z$, where z is a given number between 0 and 1, with probability $\beta \in [0, 1]$ his preference for software is $y = z$, and with probability $1 - \beta$ his preference for software is uniformly distributed over the interval $[0, 1]$. Notice that β measures the correlation between consumers' preferences for hardware and for software. When $\beta = 1$, consumers' preferences for hardware are perfectly correlated with their preferences for software. When $\beta = 0$, consumers' preferences for hardware and software are independent. Firm strategies can sometimes influence β. For example, branding hardware and software together might increase this correlation. The formulation allows us to consider arbitrary positive correlations in consumer preferences for hardware and software.

As consumers single-home and content providers multi-home, our setup is a model of "competitive bottlenecks" in the two-sided market literature (e.g., Armstrong 2006; Armstrong and Wright 2007). We assume that the platform owners charge a royalty on each sale transacted through their platforms.[6] We denote the average royalty earned from selling content to a consumer as γ. For ease of exposition, we assume the devices' marginal costs to be zero. We thus formulate firm profits from consumers and content providers as follows:

$$\pi_i = p_i D_{ih} + \gamma D_{is}, \tag{3}$$

where D_{ih} denotes the number of consumers who purchase hardware devices from firm i, and D_{is} the number of consumers who use firm i's software to consume content. In the incompatible case,

[6]Depending on the context, royalties can also be referred to as commission or referral fees.

the number of consumers who purchase hardware from a firm equals the number of consumers who use software offered by the same firm; that is, $D_{ih} = D_{is}$. In the compatibility case, the number of consumers who purchase a firm's hardware is likely to be different from the number of consumers who use the same firm's software to consume content. For example, when S_2 is available on H_1, some buyers of H_1 may use S_2 instead of S_1, in which case, $D_{1h} \geq D_{1s}$ and $D_{2h} \leq D_{2s}$.

The time sequence of the game is as follows. In stage 1, each firm simultaneously proposes whether it is willing to make its software available on its rival's hardware and whether it is willing to accept its rival's software on its hardware. Only with mutual consent, can this firm's software become available on its rival's hardware. In stage 2, both firms simultaneously price their hardware. In stage 3, the consumers make their hardware purchase decisions, and choose software to consume content.

For ease of exposition, we consider that hardware preference plays a more important role than software preference in determining consumer purchase decisions, such that the consumer with the strongest preference for a firm's hardware purchases that hardware even if he has the lowest preference for that firm's software.[7]

4 Equilibrium Analysis

We first derive the subgame-perfect equilibrium using backward induction. Based on the two firms' proposals in stage 1, we may have three types of subgames in stage 2: the *incompatible* case in which neither firm's software is available on its rival's hardware, the *two-way compatible* case in which each firm's software is available on its rival's hardware, and the *one-way compatible* case in which only one firm's software is available on its rival's hardware but the rival's software is not on that firm's hardware. One-way compatibility can take place when S_1 is available on H_2 or when S_2 is available on H_1. We focus on the one-way compatible case with S_2 being on H_1 in the paper. As we discuss in Appendix B, the other one-way compatible case with S_1 being on H_2 occurs only under very restrictive conditions and has little empirical relevance.[8] We first derive the equilibrium for each subgame in stage 2 and then derive the conditions under which the two firms

[7] As we show in the proof of Lemma 1, technically, this assumption requires $v_d \leq 3(t_h - t_s)$.

[8] As shown in Appendix B, a necessary condition for this one-way compatibility to become the equilibrium outcome is that under this one-way compatibility, the profit foci of the two firms flips: relative to each other, content sales become more important to firm 1 and hardware sales become more important to firm 2.

have incentives to be incompatible, two-way compatible, or one-way compatible in stage 1.

4.1 The Incompatible Case

When neither firm's software is available on the rival's device, competition between the firms is similar to the standard Hotelling setup, but with the three distinct features: (1) the two firms offer different standalone utilities, v_1 and v_2, (2) the revenue for each comes from two sources, hardware sales and royalties from content sales, and (3) the consumer type is two-dimensional.

In general, consumers who are close to the bottom-left corner prefer platform 1, and those who are close to the top-right corner prefer platform 2. As in the standard setup, letting $U_1 = U_2$, we can derive an indifference curve $x(y)$, over which all the consumers are indifferent in purchasing H_1 and H_2, as:

$$x(y) = \frac{v_d - (p_1 - p_2) + t_s + t_h - 2t_s y}{2t_h}. \tag{4}$$

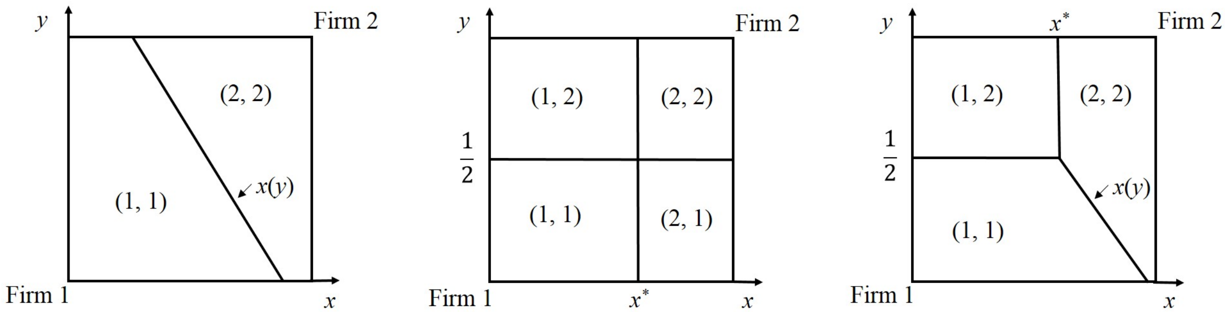
As illustrated in Figure 1(a), consumers located on the left-hand side of the curve purchase from firm 1, while those on the right-hand side purchase from firm 2. Under the assumption that hardware preference plays a dominant role, $x(0) \in [0, 1]$ in equilibrium. We can then formulate the demand function for firm 1 as:

$$\begin{aligned} D_{1h} = D_{1s} &= \beta \frac{v_d - (p_1 - p_2) + t_s + t_h}{2(t_s + t_h)} + (1-\beta) \int_0^1 \int_0^{x(y)} 1 dx dy && (5) \\ &= \frac{1}{2} + \frac{[t_h + (1-\beta)t_s](p_2 - p_1 + v_d)}{2t_h(t_s + t_h)}, && (6) \end{aligned}$$

where $x(y)$ is the indifference curve defined in Equation (4). The first term in Equation (5) measures the number of consumers who have the same degree of misfit for firm 1's hardware and software (located on the diagonal in Figure 1(a)), while the second term measures the other consumers who purchase from firm 1 (located on the left-hand side of the curve in Figure 1(a)). The demand for firm 2 is $D_{2h} = D_{2s} = 1 - D_{1h}$.

The profit functions of the two firms in Equation (3) can thus be specified as

$$\begin{aligned} \pi_1 &= p_1 D_{1h} + \gamma D_{1s}, && (7) \\ \pi_2 &= p_2(1 - D_{1h}) + \gamma(1 - D_{1s}). && (8) \end{aligned}$$



(a) Incompatible (b) Two-Way Compatible (c) One-Way Compatible

The tuples, $\{a, b\}$, identify regions where consumers will adopt hardware provided by firm a and the software application provided by firm b. Note that x^*s in Figures (b) and (c) have the same functional form, as defined by Equation (15). But because equilibrium prices are different in these two cases, the equilibrium x^*s are different. The same applies to the function $x(y)$ in Figures (a) and (c).

Figure 1: Consumers' purchase decisions

Solving the first-order conditions for the two profit-maximizing firms yields the equilibrium prices, profits, and demands, as summarized by the following lemma:

Lemma 1. *In the incompatible case, the equilibrium prices are:*

$$p_1 = \frac{1}{3}\left(\frac{3t_h(t_h+t_s)}{t_h+(1-\beta)t_s} + v_d - 3\gamma\right) \tag{9}$$

$$p_2 = \frac{1}{3}\left(\frac{3t_h(t_h+t_s)}{t_h+(1-\beta)t_s} - v_d - 3\gamma\right), \tag{10}$$

the equilibrium demands are:

$$D_{1h} = D_{1s} = \frac{1}{2} + \frac{[t_h+(1-\beta)t_s]v_d}{6t_h(t_h+t_s)} \tag{11}$$

$$D_{2h} = D_{2s} = \frac{1}{2} - \frac{[t_h+(1-\beta)t_s]v_d}{6t_h(t_h+t_s)}, \tag{12}$$

and the equilibrium profits are

$$\pi_1 = \frac{[3t_h(t_h+t_s)+t_hv_d+(1-\beta)t_sv_d]^2}{18t_h(t_h+t_s)[t_h+(1-\beta)t_s]} \tag{13}$$

$$\pi_2 = \frac{[3t_h(t_h+t_s)-t_hv_d-(1-\beta)t_sv_d]^2}{18t_h(t_h+t_s)[t_h+(1-\beta)t_s]}. \tag{14}$$

A number of observations related to the equilibrium are worth highlighting. First, note that $p_1 > p_2$, $D_{1h} > D_{2h}$, and $\pi_1 > \pi_2$. This result is expected because firm 1, being more attractive to users than firm 2 (captured by $v_d > 0$), can charge a higher price as well as garner a larger market share. Firm 1 consequently earns higher profits than firm 2.

Second, when the per-user content profit, γ, increases, both p_1 and p_2 decrease and can even become negative (i.e., below cost). In such cases, the platform firms have incentives to subsidize consumers' device purchases in return for profits from content sales. This pricing pattern and business model are similar to those for complementary products, such as printers and toners, or razors and blades.

Third, as the correlation in hardware and software preferences, β, increases, equilibrium prices increase. Because consumers have to buy hardware and software together in the incompatible case, greater correlation makes it easier for the two firms to target different consumer segments. As a result, the competition between the two firms is softened and the equilibrium hardware prices increase.

Finally, equilibrium profits are unrelated to γ, even though prices depend on it, because profits from content sales are competed away for the two firms. As long as a firm can attract a user, it earns an additional profit of γ. Firms are therefore willing to subsidize each user up to γ in a competitive setting.

4.2 The Two-Way Compatible Case

When each firm's software is available on its rival's hardware, regardless of which firm's hardware a consumer uses, the consumer can choose either software. A consumer's utility of purchasing either device can be expressed as $U_1 = v_1 - t_h x - t_s \min\{y, 1-y\} - p_1$ and $U_2 = v_2 - t_h(1-x) - t_s \min\{y, 1-y\} - p_2$. Because the software is free, consumers choose the software that has a better fit for their needs. As illustrated in Figure 1(b), half of the consumers choose the software offered by firm 1 and the other half choose the software offered by firm 2; that is, $D_{1s} = D_{2s} = 1/2$.

The hardware competition in this case is thus independent of software preferences, and the competition reduces to the standard Hotelling setup. Letting $U_1 = U_2$, we can derive the indifferent consumer's location as

$$x^* = \frac{v_d - (p_1 - p_2) + t_h}{2t_h}. \tag{15}$$

Consumers whose misfit with firm 1 is smaller than that of the indifferent consumer purchase H_1, and the rest purchase H_2. The profit functions of the two firms in Equation (3) can thus be specified as

$$\pi_1 = p_1 x^* + \frac{1}{2}\gamma, \tag{16}$$

$$\pi_2 = p_2(1 - x^*) + \frac{1}{2}\gamma. \tag{17}$$

It is worth highlighting that each firm's content revenue comes from some of the consumers who use this firm's device, as well as those who use its rival's device.

Solving the first-order conditions for the two profit-maximizing firms yields the equilibrium prices, profits, and the location of the indifferent consumer, as summarized by the following lemma:

Lemma 2. *In the two-way compatible case, the equilibrium prices are*

$$p_1 = \frac{1}{3}(3t_h + v_d) \tag{18}$$

$$p_2 = \frac{1}{3}(3t_h - v_d), \tag{19}$$

the equilibrium demands are

$$D_{1h} = \frac{1}{2} + \frac{v_d}{6t_h} \text{ and } D_{1s} = \frac{1}{2} \tag{20}$$

and $D_{2h} = 1 - D_{1h}$ *and* $D_{2s} = 1 - D_{1s}$, *and the equilibrium profits are*

$$\pi_1 = \frac{(3t_h + v_d)^2}{18t_h} + \frac{\gamma}{2} \tag{21}$$

$$\pi_2 = \frac{(3t_h - v_d)^2}{18t_h} + \frac{\gamma}{2}. \tag{22}$$

In equilibrium, as in the incompatible case, $p_1 > p_2$, $D_{1h} > D_{2h}$, and $\pi_1 > \pi_2$ because $v_d > 0$; that is, because its hardware has a valuation advantage than its rival, firm 1 charges a higher price for the hardware and has a greater market share than firm 2, and consequently earns greater profits. In addition, because consumers' hardware choices are no longer related to their software choices, equilibrium prices are independent of t_s, γ, and β. Although hardware prices are independent of the per-user content profit γ, the equilibrium profits increase with γ, which is different from the incompatible case.

4.3 The One-Way Compatible Case

When S_2 is available on H_1, consumers who purchase H_1 can choose whichever of the two software applications provides the better fit. The utility from firm 1 for a consumer located at (x, y) can be formulated as $U_1 = v_1 - t_h x - t_s \min\{y, 1-y\} - p_1$, and the utility from firm 2 takes the same form as in the incompatible case (i.e., Equation (2)).

In this case, for users who prefer firm 1's software application ($y \leq \frac{1}{2}$), if they buy hardware from firm 1, they use the software from firm 1. Therefore, the hardware choice for them between firms 1 and 2 stays the same as in the incompatible case, and the indifference curve has the same form as in Equation (4). For users who prefer firm 2's software application ($y > \frac{1}{2}$), if they buy hardware from firm 1, they will use the software from firm 2 instead; if they buy hardware from firm 2, they use the software from firm 2. Therefore, their software preference does not play a role in their hardware decision, the hardware choice stays the same as the two-way compatible case, and the indifference curve x^* stays the same as in Equation (15) by letting $U_1 = U_2$. For ease of exposition, we consider that $v_d - (p_1 - p_2) \geq 0$ such that $x^* \geq \frac{1}{2}$ in equilibrium.[9] The segmentation of the consumers is illustrated in Figure 1(c). All together, we can formulate the hardware demand function for firm 1 as:

$$D_{1h} = \beta \frac{v_d - (p_1 - p_2) + t_h}{2t_h} + (1-\beta) \int_0^1 \int_0^{\max\{x^*, x(y)\}} 1 dx dy = \frac{1}{2} + \frac{4(v_d - p_1 + p_2) + (1-\beta)t_s}{8t_h}, \tag{23}$$

where x^* and $x(y)$ are the indifference curves defined in Equations (15) and (4). Similar to the incompatible case, the first term in the formulation after the first equal sign measures the number of consumers who have the same degree of misfit for firm 1's hardware and software (located on the diagonal in Figure 1(c)), and the second term measures the other consumers who purchase from

[9] As implied by Lemma 3, technically, this assumption requires $t_s(1-\beta) \leq 2v_d$.

firm 1. The software demand function for firm 1 can be formulated as:

$$D_{1s} = \frac{\beta}{2} + (1-\beta)\int_0^{\frac{1}{2}}\int_0^{x(y)} 1 dx dy = \frac{\beta}{2} + \frac{(1-\beta)(2v_d - 2p_1 + 2p_2 + 2t_h + t_s)}{8t_h}. \tag{24}$$

The demand functions for firm 2 are $D_{2h} = 1 - D_{1h}$ and $D_{2s} = 1 - D_{1s}$. The profit functions take the same form as in Equations (7) and (8).

Solving the first-order conditions for the two profit-maximizing firms, we obtain the equilibrium prices, profits, and the location of the indifferent consumer, as summarized by the following lemma:

Lemma 3. *In the one-way compatible case with S_2 being available on H_1, the equilibrium prices are*

$$p_1 = \frac{1}{12}\left[12t_h + 4v_d + (1-\beta)t_s - 6(1-\beta)\gamma\right] \tag{25}$$

$$p_2 = \frac{1}{12}\left[12t_h - 4v_d - (1-\beta)t_s - 6(1-\beta)\gamma\right], \tag{26}$$

the equilibrium demands are

$$D_{1h} = \frac{1}{2} + \frac{(1-\beta)t_s + 4v_d}{24t_h} \tag{27}$$

$$D_{1s} = \frac{1}{2} - \frac{(1-\beta)[6t_h - 2v_d - (2+\beta)t_s]}{24t_h}, \tag{28}$$

and $D_{2h} = 1 - D_{1h}$ and $D_{2s} = 1 - D_{1s}$, and the equilibrium profits are

$$\pi_1 = \frac{[12t_h + 4v_d + (1-\beta)t_s]^2 + 18\gamma\left[8\beta t_h + (1-\beta^2)t_s\right]}{288t_h} \tag{29}$$

$$\pi_2 = \frac{[12t_h - 4v_d - (1-\beta)t_s]^2 + 18\gamma\left[8t_h - (1-\beta^2)t_s\right]}{288t_h}. \tag{30}$$

In equilibrium, as in the other two cases, firm 1 charges a higher price for H_1 than firm 2 charges for H_2 (i.e., $p_1 > p_2$). Notice that for the consumer segment with $y < \frac{1}{2}$, the competition is similar to that in the incompatible case, and for the consumer segment with $y > \frac{1}{2}$, the competition is similar to that in the two-way compatible case. In both the incompatible and two-way compatible cases, firm 1 charges a higher price than firm 2 because of its value advantage. In this one-way compatible case, which can be viewed as a hybrid of the other two cases, firm 1 naturally charges a higher price. Note that, in contrast to the two-way compatible case, the hardware prices are dependent of the per-user content profit γ; and in contrast to the incompatible case, the equilibrium profits increase in γ.

5 One-Way Compatibility as the Equilibrium Outcome

We next compare the equilibria in the three subgames discussed in Section 4, and examine the conditions under which one-way compatibility becomes the equilibrium outcome.

5.1 Comparison of Prices and Demands

We first investigate how the equilibrium prices and demands in the compatible cases differ from that in the incompatible case. Comparing equilibrium prices in the three subgames summarized in Lemmas 1, 2, and 3 yields the following result:

Proposition 1. *Compared to the incompatible case, (a) in the two-way compatible case, firms charge higher hardware prices if and only if* $\frac{\beta t_h t_s}{t_h+(1-\beta)t_s} < \gamma$*; (b) in the one-way compatible case, firm 1 charges a higher hardware price if and only if* $\frac{\beta t_h t_s}{t_h+(1-\beta)t_s} - \frac{1}{12}(1-\beta)t_s < \frac{1}{2}(1+\beta)\gamma$*, and firm 2 charges a higher hardware price if and only if* $\frac{\beta t_h t_s}{t_h+(1-\beta)t_s} + \frac{1}{12}(1-\beta)t_s < \frac{1}{2}(1+\beta)\gamma$*.*

Whether firms charge higher prices in the compatible case than in the incompatible case depends on the content royalty and the misfit costs. The intuition is as follows. Recall that firm revenue consists of hardware and content sales. In the incompatible case, firms rationally anticipate that bringing in one consumer can generate content revenue γ, in addition to the hardware revenue. As a result, in equilibrium both firms are willing to subsidize up to γ in hardware price (as shown in Equations (9) and (10)), in return for profits from content sales; in other words, content royalty drives hardware prices down in the incompatible case. In contrast, in the two-way compatible case, the firms always split the content demand evenly, and their hardware pricing does not factor in the content demand consideration. Therefore, compatibility reduces competition in this regard, and we call this the *content royalty effect*.

On the other hand, compatibility increases competition between the firms by reducing the strength of consumer preference. In the incompatible case, hardware is bundled with software, and the two devices differ in both hardware and software. In the two-way compatible case, the software is "unbundled" from the hardware, consumers can use either software regardless of the hardware they use, and the two devices differ only in hardware. Therefore, consumer preference is weaker in the two-way compatible case than in the incompatible case because of the decrease in the device differentiation, and consequently compatibility increases competition in this regard, which we call the *consumer preference effect*.

The left-hand side of the condition in Proposition 1(a) represents the consumer preference effect and the right-hand side is the content royalty effect. When the content royalty effect dominates the consumer preference effect, compatibility reduces the hardware competition and thus the firms charge higher hardware prices in the two-way compatible case.

The perfect-correlation case (i.e., $\beta = 1$) clearly demonstrates the balance between the two effects. When consumer preferences for hardware and software are perfectly correlated, the condition in Proposition 1(a) reduces to $t_s < \gamma$ and the consumer preference effect is presented by t_s. This is because in the incompatible case $U_1 = v_1 - (t_h + t_s)x - p_1$ and the strength of the consumer preference for the two devices is measured by $(t_h + t_s)$. In contrast, in the two-way compatible case, the strength of the consumer preference for the two devices is measured by t_h only, because consumers can choose either software. Therefore, compatibility reduces the strength of consumer preference from $(t_h + t_s)$ to t_h, and the difference t_s characterizes the consumer preference effect.

When the correlation β decreases, firms are more likely to charge higher prices in the two-way compatible case than in the incompatible case. In this case, the likelihood that a consumer who prefers the hardware from one firm also prefers the software from the same firm becomes smaller, and the consumer preference effect becomes weaker. In the case with independent preference for hardware and software (i.e., $\beta = 0$), whether software is bundled with hardware does not change the strength of consumer preference for the two devices, and the consumer preference effect becomes negligible. As a result, the content royalty effect always dominates, and the firms charge higher hardware prices in the two-way compatible case than in the incompatible case.

In the one-way compatible case, the competition for the consumer segment with $y < \frac{1}{2}$ is similar to that in the incompatible case, and the competition for the consumer segment with $y > \frac{1}{2}$ is similar to that in the two-way compatible case. Therefore, the one-way compatible case can be viewed as a hybrid of the other two cases, and hence the conditions in Proposition 1(b) continue to embody the consumer preference effect and the content loyalty effect. Meanwhile, the conditions for firms 1 and 2 to charge higher prices become asymmetric in this case (i.e., the term $\frac{1}{12}(1-\beta)t_s$ has opposite signs in the conditions). We have this asymmetry because although the one-way compatible case is a hybrid of the two other cases, the segmentation of the consumers into $y > \frac{1}{2}$ and $y < \frac{1}{2}$ gives firm 1 an extra advantage in the hardware competition. In particular, the competition for consumers with $y < \frac{1}{2}$ resembles the incompatible case, and in this segment consumers all prefer firm 1's software to firm 2's. Therefore, different from the incompatible case, firm 1 has an extra competitive advantage because of the favorable consumer preference for its software, in addition to the hardware valuation advantage. As a result, relative to the incompatible case, firm 1 becomes more likely to charge a higher price than firm 2 in the one-way compatible case.

We next look at the changes in the demand for each platform from the incompatible case to the compatible cases.

Proposition 2. *More consumers purchase H_1 and more consumers use S_2 in the compatible cases (in either the one-way or the two-way compatible case) than in the incompatible case.*

In the incompatible case, firm 1 has a valuation advantage in hardware, and in the competition with its rival, firm 1 leverages its advantage to obtain more market share for both hardware and software. In equilibrium, firm 2 has less than half market share for both hardware and software. Some consumers adopt firm 2's hardware because of their high misfit costs with firm 1's software. In the two-way compatible case, the hardware competition is independent of software preference. Hence, firm 1 leverages its valuation advantage solely for its hardware market share, and thus obtains greater hardware market share than in the incompatible case. In this case, half of the consumers choose S_2.

More consumers purchase the hardware offered by firm 1, H_1, in the one-way compatible case than the incompatible case, because compatibility allows consumers to use either software when adopting H_1 and thus makes H_1 more valuable. More consumers use the software offered by firm 2, S_2, in the one-way compatible case because some users who purchase H_1 will use S_2.

All together, we see that compatibility makes each firm experience increased demand for one component—either hardware or software—and decreased demand for the other. Conventional wisdom suggests that being compatible may make two firms more similar to consumers. In our case, compatibility "unbundles" the software component from the hardware component. This unbundling effect drives the differentiation of their profit models: one firm dominates the hardware market and the other the software market. Hence, in our model, compatibility enables increased differentiation.

5.2 Conditions for Compatibility

In this section, we study the equilibrium outcome in stage 1. In stage 1, each firm proposes whether it is willing to make its software available on its rival's hardware (which we denote as μ_i, $\mu_i \in$ {offer, not offer}) and whether it is willing to accept its rival's software (which we denote as ρ_i, $\rho_i \in$ {accept, decline}). In other words, firm i chooses (μ_i, ρ_i) in stage 1, where $i \in \{1, 2\}$, and $((\mu_1, \rho_1), (\mu_2, \rho_2))$ represents an action profile of the game in stage 1. Different action profiles correspond to different compatibility outcomes. For example, action profile ((offer, accept), (offer, accept)) corresponds to two-way compatible, and action profile ((not offer, accept), (offer, accept)) corresponds to one-way compatible.

To facilitate comparison, we use superscripts I, O, and T to distinguish outcome variables in the incompatible, one-way compatible, and two-way compatible cases. We define the profit difference for firm i under any two cases as $\Lambda_i^{j-k} \equiv \pi_i^j - \pi_i^k$, where $i \in \{1, 2\}$, and j, $k \in \{I, O, T\}$. For instance, Λ_1^{T-O} represents the profit difference for firm 1 between the two-way compatible and one-way compatible cases. The following proposition summarizes the conditions under which compatibility may occur as an equilibrium.

Proposition 3. *(a.1) Two-way compatibility supported by action profile ((offer, accept), (offer, accept)) can be sustained as an equilibrium when* $\Lambda_1^{T-O} \geq 0$, $\Lambda_1^{T-I} \geq 0$, $\Lambda_2^{T-O} \geq 0$, *and* $\Lambda_2^{T-I} \geq 0$; *(a.2) One-way compatibility supported by action profile ((not offer, accept), (offer, accept)) can be sustained as an equilibrium when* $\Lambda_1^{O-T} \geq 0$, $\Lambda_1^{O-I} \geq 0$, *and* $\Lambda_2^{O-I} \geq 0$.[10]

(b) When γ *increases, both firms are more willing to pursue two-way compatibility.*

(c) When v_d *increases, both firms are more willing to pursue one-way compatibility.*

(d) When β *increases, if* γ *is large (small), one-way compatibility becomes more (less) likely to become the equilibrium outcome.*

[10]Under action profile ((not offer, accept), (offer, decline)), one-way compatibility can also be sustained when $\Lambda_1^{O-I} \geq 0$ and $\Lambda_2^{O-I} \geq 0$, which are less restrictive and easier to satisfy than those presented in this proposition. The action profile ((offer, accept), (offer, decline)) cannot be sustained as an equilibrium, as we show in the proof.

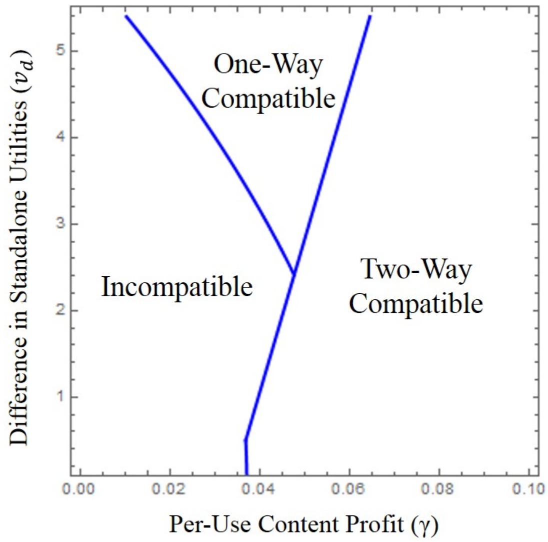


Figure 2: One-way and two-way compatibility ($t_h = 2$, $t_s = 0.2$, and $\beta = 0.2$)

The conditions for two-way compatibility equilibrium, $\Lambda_1^{T-O} \geq 0$ and $\Lambda_1^{T-I} \geq 0$, ensure that firm 1 has no incentive to deviate to one-way compatibility or incompatibility case; The conditions $\Lambda_2^{T-O} \geq 0$ and $\Lambda_2^{T-I} \geq 0$ ensure firm 2 has no incentive to deviate. Similar explanation applies to the conditions for one-way compatibility equilibrium. Figure 2 demonstrates the parameter space in which the conditions outlined in Proposition 3 can be satisfied such that one-way or two-way compatibility can arise as an equilibrium. As illustrated in Figure 2, the above proposition indicates that when the per-user content profit γ is large, both firms may have incentive to make their software available on their rival's hardware and two-way compatibility is likely to arise as an equilibrium. The intuition is that in the incompatible case, profits from content sales are competed away, while in the two-way compatible case, each firm earns $\frac{\gamma}{2}$ profits from content sales and the profits are increasing in the per-user content profit γ. As a result, when γ is larger, such that the content loyalty effect is more salient, firms are more likely to earn higher profits when they are two-way compatible compared to the incompatible case. Notice that the one-way compatible case can be viewed as the hybrid of the incompatible and two-way compatible cases. Firms benefit more from the two-way compatibility case than the one-way compatibility case.

The proposition also implies that when the difference in the standalone utilities v_d is large compared to content profit γ, one-way compatibility may arise as an equilibrium. As shown in Figure 2, as v_d increases, firm 1's competitive advantage in hardware becomes greater and H_1 sales become more important to its profitability. At the same time, royalties from content sales become more important to firm 2's profitability. Having S_2 available on H_1 increases H_1 sales and decreases content sales through S_1. It also increases content sales through S_2 and decreases H_2 sales. When v_d is sufficiently large, the two firms' profit foci are sufficiently different—the main revenue source for firm 1 is from hardware and the main revenue source for firm 2 is from content. As a result, firm 1 becomes willing to sacrifice content sales to increase H_1 sales, and firm 2 becomes willing to sacrifice hardware sales to increase its royalties from content sales. For the same reason, firm 1 has a disincentive to make its software available on its rival's hardware to avoid losing hardware sales.

The impact of β on firms' incentives to form one-way compatibility is more nuanced. Consider the situation where we have one-way compatibility. When consumers' preferences for hardware and software are more correlated, more consumers who adopt H_1 will adopt S_1. Firm 1 will thus have greater sales of its content as β increases. At the same time, as β increases, the demand for H_1 decreases because a higher correlation increases utility of consumers adopting H_2 and thus increases the demand of H_2. Therefore, whether firm 1 becomes more profitable as β increases depends on the per-user content profit, γ. When γ is large (small), the impact on content (hardware) sales dominates and hence firm 1 will find one-way compatibility more (less) profitable as β increases. For firm 2, as β increases, it improves the fit between consumers adopting H_2 and S_2. Hence, the demand for H_2 increases and the associated content sales from H_2 adopters also increase. Although firm 2 has less content sales from H_1 adopters, its overall profitability increases as β increases.

Two special cases are worth highlighting: the perfectly-correlated-preference case (i.e., $\beta = 1$) and the independent-preference case (i.e., $\beta = 0$). The following corollaries summarize the conditions under which compatibilities may emerge as an equilibrium in these two special cases.

Corollary 1. *In the perfectly-correlated-preference case (i.e., $\beta = 1$), (a) both firms are indifferent to having S_1 on H_2; (b) If and only if $9(\gamma - t_s) + v_d^2(\frac{1}{t_h} - \frac{1}{t_h+t_s}) \geq 0$, both firms have incentives to make S_2 available on H_1 and compatibility arises as an equilibrium; (c) both are more willing to pursue compatibility as the content royalty (γ) or the difference in standalone utilities (v_d) increases.*

Because of firm 2's hardware disadvantage, fewer than half of consumers buy H_2. When consumers' software preference is perfectly correlated with hardware preference, these consumers choose S_2 regardless of whether S_1 is available on H_2 because their misfit cost with S_2 is smaller. Having

S_1 available on H_2 devices therefore makes no difference. As a result, in this case, two-way compatibility and one-way compatibility become equivalent. Corollary 1 highlights that compatibility can arise in equilibrium when the content royalty (γ) or the difference in standalone utilities (v_d) is large for the same reason as explained in the general case above.

In this perfectly-correlated-preference case, by Proposition 1, the firms charge higher price in the compatible case than in the incompatible case if only if $\gamma \geq t_s$. When $\gamma \geq t_s$, by Corollary 1, the condition for the compatibility incentive is satisfied and the firms choose to be compatible, because higher prices can improve their profits. When $\gamma < t_s$, under which the firms charge lower prices in the compatible case, the firms might still have incentive to be compatible (when v_d is large, by Corollary 1). This is because when v_d is large, the firms have different profit foci, and after being compatible firm 1 sells more hardware (which can increase its hardware revenue, although the hardware price is lower) and firms 2 sells more content.

Corollary 2. *In the independent-preference case (i.e., $\beta = 0$), (a) two-way compatibility can be sustained as an equilibrium when $t_s(24t_h + t_s + 8\nu_d) < 18\gamma(8t_h - t_s)$; (b) The action profile ((not offer, accept), (offer, accept)) and one-way compatibility can be sustained as an equilibrium when $t_s(24t_h - t_s - 8\nu_d) < 18\gamma(8t_h - t_s) < t_s(24t_h + t_s + 8\nu_d)$.*

The above corollary highlights the condition under which one-way versus two-way compatibility may arise as an equilibrium: When standalone utility (v_d) is more salient than content royalty (γ) (i.e., the profit foci are sufficiently heterogenous), one-way compatibility emerges; otherwise, two-way compatibility may arise.

This exploration of the independent-preference case (i.e., $\beta = 0$) clarifies that the one-way compatibility in our model can arise even in the absence of demand-side preference correlation; that is, the heterogeneous profit foci across the two firms, which results from the difference in the standalone value of the hardware, are sufficient to drive the effect. When firm 1 makes more of its profit from hardware while firm 2 from content, one-way compatibility can be a win-win for both: making firm 2's software compatible with firm 1's hardware can be attractive to both firms because it increases firm 1's hardware sales (although it decreases firm 1's content sales, which is less important to firm 1) and increases firm 2's content sales (although it decreases firm 2's hardware sales, which is less important to firm 2). In this case, firm 1 has no incentives to put its software on firm 2 because, while it may generate extra profits from content sales for firm 1, it reduces firm 1's hardware sales, which are more important.

5.3 Impact of Compatibility on Social Welfare

We next examine the effect of compatibility on social welfare, defined as the sum of consumer utilities and firms' profits. In our setting, social welfare equals the total consumer value realized from the consumption of the products. For instance, the social welfare generated in the one-way compatible case can be formulated as follows:

$$\begin{aligned} W^O \;=\; & \beta\Big(\int_0^{x^*}(v_1 - t_h x - t_s \min\{x, 1-x\})dx + \int_{x^*}^1 \left[v_2 - (t_h + t_s)(1-x)\right] dx\Big) \\ & +(1-\beta)\Big(\int_0^1 \int_0^{\max\{x(y),x^*\}}(v_1 - t_h x - t_s \min\{y, 1-y\})dxdy + \\ & \int_0^1 \int_{\max\{x(y),x^*\}}^1 (v_2 - t_h(1-x) - t_s(1-y))dxdy\Big), \end{aligned} \tag{31}$$

where $x(y)$ and x^* are the indifference curves defined in Equations (4) and (15). The first term in the formulation represents the social welfare generated from the consumers who have the same degree of misfit for hardware and software, the second term represents the social welfare generated from the rest of consumers. Similarly, we can formulate the social welfare under the two-way compatible and incompatible cases.

The misfit cost associated with software applications plays an important role in driving the differences in social welfare. In the compatible cases, some consumers (in the one-way compatible case) or all consumers (in the two-way compatible case) have the option to choose the software that fits them better, whereas in the incompatible case consumers do not have this option. This option value, which leads to a better allocation of content buyers, is captured by the terms $\min\{x, 1-x\}$ and $\min\{y, 1-y\}$ in Equation (31), which increases social welfare in general. In addition, the social welfare is affected by consumer hardware choice in equilibrium. In the compatible cases, the software component is "unbundled" from the hardware component. The two firms will compete on hardware directly and consumers are more likely choose the hardware generating higher value for them. Therefore, being compatible also results in a better allocation of hardware. Indeed, comparing the social welfare under the compatible cases and the incompatible case, we can conclude the following:

Proposition 4. *The compatible case (either one-way or two-way compatible) generates greater social welfare than the incompatible case.*

6 Extensions

6.1 Exclusive Content

The baseline model assumes that all content publishers multi-home and all content is available on both platforms. We next examine the case where firm 2 has obtained some exclusive content. In the e-book reader market, Amazon operates an e-book self-publishing service so that some e-books are available only on Amazon's Kindle Reader.[11] In the smartphone market, more venues/advertisers provide content on Google Maps than on Apple's map app.[12]

We decompose the value v_j that consumers derive from platform j into two components: the value derived from consuming content (v_{jc}), and the value of using other functions (v_{jo}). In the baseline model, because content providers multi-home, consumer utility from content consumption is identical on both platforms; that is, v_{jc}, being the same for both platforms, is denoted as v_c. We can thus view $v_1 = v_c + v_{1o}$ and $v_2 = v_c + v_{2o}$, where $v_{1o} - v_{2o} = v_d$. In this extension, exclusive content for firm 2 affects consumer preferences between the two firms. We thus explicitly account for utility from consuming content.

We normalize the amount of multi-homing content on both platforms to be 1 and assume that firm 2 has k amount of exclusive content. We thus need to account for the extra utility consumers can derive, and the extra profits firm 2 can derive, from the exclusive content in each of the three cases. For example, in the incompatible case, the consumer utility from H_1 is the same as in the baseline model, $U_1 = v_{1o} + v_c - t_h x - t_s y - p_1$. The utility from H_2 changes to $U_2 = v_{2o} + (1+k)v_c - t_h(1-x) - t_s(1-y) - p_2$.[13] For ease of exposition, we normalize v_c to be 1.

[11]Conversations with Amazon and Apple in March 2014 revealed that both Amazon's Kindle Store and Apple's iBooks store had in excess of two million e-books and that Amazon had another approximately 500,000 exclusive titles (including self-published Kindle e-books) unavailable to readers anywhere else.

[12]See, for example, http://www.usatoday.com/story/tech/news/2016/05/24/google-maps-ads/84854240/.

[13]Note that while not required for our analysis, the value v_c can be interpreted in a more specific way. We assume that each content provider offers one unit of content. For a given unit of content, each consumer derives utility $\tilde{v}$, which is randomly drawn from a uniform distribution, $[0, \bar{v}_c]$. Each content provider is a monopoly for the content it provides, for which it charges price ρ. Consumers purchase all content from which they derive non-negative utility: $\tilde{v} - \rho \geq 0$. Given this setup, the optimal monopoly price that content providers set is $\rho^* = \bar{v}_c/2$, and each consumer purchases half of the content available on the platform. Hence, each consumer derives total utility $\int_{\rho^*}^{\bar{v}_c} (\tilde{v} - \rho^*) d\tilde{v} = \bar{v}_c^2/8$ from content on H_1 and $(1+k)\bar{v}_c^2/8$ on H_2. Therefore, under this setting, $v_c = \bar{v}_c^2/8$.

Because of the exclusive content, the value of H_2 is enhanced, and the value difference is smaller than in the baseline case. In addition, when both software applications are available on a hardware device, more consumers will consume content through firm 2's application because of its exclusive content.[14] We assume k to be small such that firm 1 continues to offer greater utility after taking exclusive content into account.

Our analysis focuses on how the amount of exclusive content, k, affects the equilibrium outcomes. The comparison of the outcomes from the incompatible, two-way compatible and one-way compatible cases yields the following result:

Proposition 5. *When content royalty, γ, is small, as the amount of exclusive content, k, increases, the incompatible case is more likely to become the equilibrium outcome. When γ is large, as k increases, the one-way compatible case is more likely to become the equilibrium outcome.*

The impact of k on compatibility choices depends on the level of γ. Exclusive content increases firm 2's reliance on content sales and, consequently, the heterogeneity between the firms' profit foci and thus the likelihood of compatibility. But exclusive content also increases the value of firm 2 to its users, thereby reducing the difference between the firms' utilities and thus the likelihood of compatibility. Whether exclusive content increases or decreases the likelihood of compatibility thus depends on its relative impact on the difference in utilities and extra profits from additional content sales. When γ is large, the exclusive content generates significant profits for firm 2 and the former effect is likely to dominate, in which case the exclusive content will increase the willingness to pursue one-way compatibility.[15]

6.2 Hardware-Only Adopters

As Dou (2014) points out, it is possible that some consumers adopt hardware because of standalone utilities, rather than the associated content. We consider this possibility by assuming that a small fraction of consumers, $\theta \in [0, 1]$, is not interested in content (referred to as *hardware-only adopters*),

[14] In our model exclusive content is accessible from a rival's platform under compatibility. If exclusive content cannot be accessed under compatibility, we can assume it to be part of the platform-specific utility (i.e., we will have a greater v_i).

[15] We have also examined the case where we let firm 1 have exclusive content. We find again that it is not always the case that the two firms are more willing to establish one-way compatibility. Because exclusive content increases the utility difference between the two firms but also increases the importance of content sales to firm 1, the two firms' profit foci may or may not become more different.

but the rest uses the software application to purchase associated content.

Software preferences and compatibility decisions do not affect the utility of these hardware-only adopters. Hence, the utility for a hardware-only adopter from adopting platform 1 is $U_1 = v_1 - t_h x - p_1$ and platform 2 is $U_2 = v_2 - t_h(1-x) - p_2$. The indifferent consumer is thus located at x^* as defined in Equation (15). The utility of the rest of consumers is the same as in our main model. Hence, under incompatibility, the profit functions of the two platforms are:

$$\begin{aligned}\pi_1 &= \theta p_1 x^* + (1-\theta)(p_1+\gamma)D_{1h} \\ \pi_2 &= \theta p_2(1-x^*) + (1-\theta)(p_2+\gamma)(1-D_{1h}),\end{aligned}$$

where D_{1h} is defined in Equation (6).

Similarly, under two-way compatibility, the profit functions are:

$$\begin{aligned}\pi_1 &= \theta p_1 x^* + (1-\theta)(p_1 x^* + \frac{1}{2}\gamma) \\ \pi_2 &= \theta p_2(1-x^*) + (1-\theta)[p_2(1-x^*) + \frac{1}{2}\gamma].\end{aligned}$$

Finally, under one-way compatibility, the profit functions are:

$$\begin{aligned}\pi_1 &= \theta p_1 x^* + (1-\theta)(p_1 D_{1h} + \gamma D_{1s}) \\ \pi_2 &= \theta p_2(1-x^*) + (1-\theta)[p_2(1-D_{1h}) + \gamma(1-D_{1s})],\end{aligned}$$

where D_{1h} and D_{1s} are defined in Equations (23) and (24).

We can similarly obtain the equilibrium prices, demands, and profits. Comparing the equilibrium profits, we have:

Proposition 6. *As the fraction of hardware-only adopters, θ, increases, the one-way compatible case is less likely to become an equilibrium.*

The intuition is that for hardware-only adopters, both firms make money from hardware only. As a result, as θ increases, their profit foci become more hardware-centric, which reduces the asymmetry in the profit foci. Firms are thus less likely to choose one-way compatibility as the equilibrium.

7 Discussion and Conclusions

The multi-sided nature of platform markets allows platform owners to generate profits from multiple groups of participants. It thus gives the owners flexibility to choose their profit foci and creates opportunities for competing platform owners with different profit foci to cooperate to capture more value for both. In this paper, we develop a model to show the general insight that competing platform owners can become frenemies when the difference in their profit foci is sufficiently large.

We have made a few simplifying assumptions in developing our model. For example, we study only the situation in which compatibility requires consensus by both platform owners. It is possible that in some contexts, compatibility can be achieved without mutual consent, such as by means of a converter (e.g., Farrell and Saloner 1992; Choi 1997). It is also possible that in some markets, it is illegal for a hardware platform not to accept an app made by its competitor. In such cases, compatibility can arise as an equilibrium more often than in our case with mutual consent, because the requirements to sustain an equilibrium in this case will be less restrictive.

Second, in our model, the two platform owners are horizontally differentiated. Future research could explore cases in which firms are vertically differentiated. In equilibrium, the platform owner with a higher quality will charge a greater hardware price and capture high-end users (e.g., users who are less price-sensitive), while the other owner will capture low-end users. If all users consume similar amounts of content, the two owners will continue to have asymmetric foci, with the owner of the higher-quality platform generating a greater portion of profits from hardware sales than the other owner does. We believe that these asymmetric profit foci will continue to provide incentives for the platform owner with lower quality to make its software application compatible to users of the rival platform.[16]

Third, we assume that the market is fully covered; that is, all users will adopt a platform. One would expect that if compatibility leads to market expansion, both firms might be more willing to establish compatibility. Similarly, our model does not allow tipping where one platform owner fully captures the market. Future work might relax these assumptions to conduct a more comprehensive analysis of compatibility decisions.

Finally, we analyze a scenario in which each firm offers both hardware and software. In practice,

[16]This intuition is also consistent with Cremer and Thisse (1991), in which they show that for every Hotelling model with horizontal differentiation, there is an equivalent vertical differentiation model.

it is possible that a firm may choose to offer only hardware or only software. For example, suppose Amazon makes a loss from selling its Kindle device and makes profits on e-book sales, could Amazon be better off by not offering the Kindle device? Two reasons may explain why Amazon may still prefer to offer both hardware and software. First, without the Kindle device, if Apple chooses not to have one-way compatibility, it will become monopoly in both the hardware and software markets (assuming all readers need either Kindle or iPad to read e-books). Amazon needs to reduce the risk by offering Kindle hardware. Second, even if Apple can commit to having the Kindle app on iPad, Amazon may still want to offer the Kindle device, because without the Kindle device, Apple would have monopoly power in the hardware market and have an incentive to charge a very high price for iPad. Amazon would then be forced to charge a small royalty so that some readers would still be able to afford e-books (taking readers' budget constraints into account). Therefore, having the Kindle device creates a competitive ceiling on Apple's ability to raise prices, which helps Amazon's ability to capture value from content sales. Future research can extend our model to endogenize firms' product portfolio decisions.

Our research offers important implications for both research and practice. Key to our analysis is the identification of asymmetric incentives. Recognizing the "room for maneuver" enabled by these asymmetries helps us understand the rise of collaboration in the face of competitive pressures (e.g., Adner 2013). Platform owners should seek opportunities to cooperate with rivals that take different approaches to value capture. Among platform owners that have begun to recognize such opportunities is Microsoft. Microsoft's Surface competes with Apple's iPad in the tablet market. The tablets are differentiated in that Surface, for example, comes with such Microsoft software applications as Microsoft Office, while iPad comes with many Apple-developed applications, such as Keynote. On March 27, 2014, Microsoft made Office available for purchase by iPad users. As in the case of iPad and Kindle, Microsoft's decision to achieve one-way compatibility is likely driven by a willingness to sacrifice some amount of Surface's share in the tablet market for additional profits from software sales to iPad users.[17] Our model likewise helps to explain Amazon's March 2015 opening of a store on Alibaba's Tmall.com, even though Amazon operates its own e-commerce site in China, Amazon.cn.[18] Amazon and Alibaba are competitors in the Chinese e-commerce market,

[17] In November 2014, Microsoft made the basic version of its Office app on iPad free, requiring users to pay only for premium features.

[18] Source: http://www.usatoday.com/story/tech/2015/03/08/amazon-on-alibaba-tmall/24610123/, accessed

but their profit foci differ. Amazon operates as a reseller, earning profits from consumers, while Alibaba is an intermediary that offers its service free to consumers and earns profits from merchants through store setup fees, advertising, and commissions. Amazon's market share in China being much smaller than Alibaba's, the collaboration enables Amazon to sell more products to Chinese consumers and Alibaba to earn more profits from service fees.

Our results also shed light on why many platform owners choose to remain incompatible. Casadesus-Masanell and Ruiz-Aliseda (2009) show that the market dominance incentive prevents many platforms from becoming compatible. Our study shows similarity in profit foci to be another reason that we do not see more instances of compatibility between competing platform firms. In the video game industry, for example, because Microsoft's Xbox and Sony's PlayStation offer similar sets of features and have closely matched pricing strategies, there is little interest on the part of either in pursuing platform compatibility. Overall, as platform firms' profit foci converge, they act less like friends and more like enemies.

Not all differences between platform owners create incentives to cooperate. As demonstrated by our extensions, the impacts of some differences can be ambiguous. We find, for example, that exclusive content on a platform does not necessarily increase or decrease its owner's compatibility incentive. Our analysis reveals that compatibility incentives result only from differences that generate greater asymmetry in platform owners' profit foci. Hence, it is important for platform owners to pay attention to how their competitors capture value in order to make compatibility decisions.

More generally, digitization is driving the decoupling of hardware and software in many traditional industries, enabling a proliferation of heterogenous business models that give rise to heterogenous profit foci. Firms in these industries will have to confront the frenemy dilemma. Our results provide guidelines that help to inform such strategic decisions.

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Appendix A: Proofs of Lemmas and Propositions

Proof of Lemma 1. By substituting D_{1h} and D_{1s} in Equation (6) into Equations (7) and (8) and using the first-order conditions, $\frac{d\pi_1}{dp_1}=0$ and $\frac{d\pi_2}{dp_2}=0$, we can derive the best response functions as follows:

$$\begin{aligned} p_1 &= \frac{1}{2}(\frac{t_h(t_h+t_s)}{t_h+(1-\beta)t_s}+p_2+v_d-\gamma) \\ p_2 &= \frac{1}{2}(\frac{t_h(t_h+t_s)}{t_h+(1-\beta)t_s}+p_1-v_d-\gamma). \end{aligned}$$

We can verify that the second-order derivatives of the profit functions are negative. Solving the two equations above yields the equilibrium prices of p_1 and p_2 in the lemma. Substituting the equilibrium prices into D_{1h} and D_{1s} in Equation (6) and into the profit functions in Equations (7) and (8) yields the equilibrium demands and equilibrium profits in the lemma.

Substituting the equilibrium prices into $x(0)$, we can verify that to ensure $x(0)\in[0,1]$ requires $v_d \leq 3(t_h-t_s)$. □

Proof of Lemma 2. Substituting x^* in Equation (15) into Equations (16) and (17) and using the first-order conditions, $\frac{d\pi_1}{dp_1}=0$ and $\frac{d\pi_2}{dp_2}=0$, we can derive the best response functions as follows:

$$\begin{aligned} p_1 &= \frac{1}{2}(p_2+t_h+v_d) \\ p_2 &= \frac{1}{2}(p_1+t_h-v_d). \end{aligned}$$

We can verify that the second-order derivatives of the profit functions are negative. Solving the two equations above yields the equilibrium prices of p_1 and p_2 in the lemma. Substituting the equilibrium prices into x^* in Equation (15) and into the profit functions in Equations (16) and (17) yields the equilibrium demands and equilibrium profits in the lemma. □

Proof of Lemma 3. By substituting D_{1h} and D_{1s} in Equations (23) and (24) into Equations (7) and (8) and using the first-order conditions, $\frac{d\pi_1}{dp_1}=0$ and $\frac{d\pi_2}{dp_2}=0$, we can derive the best response functions as follows:

$$\begin{aligned} p_1 &= \frac{1}{8}[4p_2+4t_h+4v_d-(1-\beta)(2\gamma-t_s)] \\ p_2 &= \frac{1}{8}[4p_1+4t_h+4v_d-(1-\beta)(2\gamma+t_s)]. \end{aligned}$$

We can verify that the second-order derivatives of the profit functions are negative. Solving the two equations above yields the equilibrium prices of p_1 and p_2 in the lemma. Substituting the equilibrium prices into D_{1h} and D_{1s} in Equations (23) and (24) and into the profit functions in Equations (7) and (8) yields the equilibrium demands and equilibrium profits in the lemma. □

To facilitate comparison, we use superscripts I , O, and T to distinguish outcome variables in the incompatible, one-way compatible, and two-way compatible cases.

Proof of Proposition 1. (a) According to Equations (9), (10), (18), and (19), we have

$$p_i^T - p_i^I = t_h - \frac{1}{3}\left(\frac{3t_h(t_h+t_s)}{t_h+(1-\beta)t_s} - 3\gamma\right) = \gamma - \frac{\beta t_h t_s}{t_h+(1-\beta)t_s},$$

where $i \in \{1,2\}$. Therefore, $p_i^T > p_i^I$ if and only if $\gamma > \frac{\beta t_h t_s}{t_h+(1-\beta)t_s}$.

(b) According to Equations (9) and (25), we have

$$\begin{aligned} p_1^O - p_1^I &= \frac{1}{12}\left[12t_h + (1-\beta)t_s - 6(1-\beta)\gamma\right] - \frac{1}{3}\left(\frac{3t_h(t_h+t_s)}{t_h+(1-\beta)t_s} - 3\gamma\right) \\ &= \frac{1}{2}(1+\beta)\gamma + \frac{1}{12}(1-\beta)t_s - \frac{\beta t_h t_s}{t_h+(1-\beta)t_s}. \end{aligned}$$

Therefore, $p_1^O > p_1^I$ if and only if $\frac{1}{2}(1+\beta)\gamma > \frac{\beta t_h t_s}{t_h+(1-\beta)t_s} - \frac{1}{12}(1-\beta)t_s$.

Similarly, according to Equations (10) and (26), we have

$$\begin{aligned} p_2^O - p_2^I &= \frac{1}{12}\left[12t_h - (1-\beta)t_s - 6(1-\beta)\gamma\right] - \frac{1}{3}\left(\frac{3t_h(t_h+t_s)}{t_h+(1-\beta)t_s} - 3\gamma\right) \\ &= \frac{1}{2}(1+\beta)\gamma - \frac{1}{12}(1-\beta)t_s - \frac{\beta t_h t_s}{t_h+(1-\beta)t_s}. \end{aligned}$$

Therefore, $p_2^O > p_2^I$ if and only if $\frac{1}{2}(1+\beta)\gamma > \frac{\beta t_h t_s}{t_h+(1-\beta)t_s} + \frac{1}{12}(1-\beta)t_s$.

□

Proof of Proposition 2. Because $0 \le \beta \le 1$ and $v_d > 0$, according to Equations (11) and (20),

$$D_{1h}^T - D_{1h}^I = \frac{v_d}{6t_h} - \frac{[t_h+(1-\beta)t_s]\nu_d}{6t_h(t_h+t_s)} = \frac{\beta t_s v_d}{6t_h(t_h+t_s)} > 0,$$

and, according to Equations (11) and (27),

$$D_{1h}^O - D_{1h}^I = \frac{(1-\beta)t_s + 4v_d}{24t_h} - \frac{[t_h+(1-\beta)t_s]\nu_d}{6t_h(t_h+t_s)} = \frac{(1-\beta)t_s(t_h+t_s) + 4\beta t_s v_d}{24t_h(t_h+t_s)} > 0.$$

Therefore, more consumers purchase H_1 in the compatible cases than in the incompatible case.

Similarly, according to Equations (12) and (20),

$$D_{2s}^T - D_{2s}^I = \frac{[t_h+(1-\beta)t_s]\nu_d}{6t_h(t_h+t_s)} > 0,$$

and, according to Equations (12) and (28),

$$\begin{aligned} D_{2s}^O - D_{2s}^I &= \frac{(1-\beta)[6t_h - 2v_d - (2+\beta)t_s]}{24t_h} + \frac{[t_h+(1-\beta)t_s]\nu_d}{6t_h(t_h+t_s)} \\ &= \frac{[4t_h + 2(1-\beta)(t_s-t_h)]v_d + (1-\beta)(t_h+t_s)[6t_h-(2+\beta)t_s]}{24t_h(t_h+t_s)} > 0, \end{aligned}$$

where the last inequality is because $t_s < t_h$. Therefore, more consumers use S_2 in the compatible cases than in the incompatible case. □

Proof of Proposition 3. (a.1) Two-way compatibility arises as an equilibrium only under action profile ((offer, accept), (offer, accept)). When $\Lambda_1^{T-O} \ge 0$, $\Lambda_1^{T-I} \ge 0$, $\Lambda_2^{T-O} \ge 0$, and $\Lambda_2^{T-I} \ge 0$, neither firm has an incentive to deviate, because deviating from (offer, accept) to (not offer, accept) or (offer, decline) only changes the two-way compatible case to the one-way compatible case, and deviating from (offer, accept) to (not offer, decline) only changes the two-way compatible case to the incompatible case. None of the deviations is profitable under the specified conditions.

(a.2) We consider action profile ((not offer, accept), (offer, accept)) to derive the one-way compatibility conditions. For firm 1, deviating from (not offer, accept) to (not offer, decline) changes the one-way compatible case to the incompatible case, which is nonprofitable when $\Lambda_1^{O-I} \geq 0$. Deviating from (not offer, accept) to (offer, accept) changes the one-way compatible case to the two-way compatible case, which is nonprofitable when $\Lambda_1^{O-T} \geq 0$. For firm 2, deviating from (offer, accept) to (not offer, accept) or (not offer, decline) changes the one-way compatible case to the incompatible case, which is nonprofitable when $\Lambda_2^{O-I} \geq 0$. Deviating from (offer, accept) to (offer, decline) does not make a difference.

The action profile ((offer, accept), (offer, decline)) cannot be sustained as a one-way compatible equilibrium, because $\frac{\partial \Lambda_2^{O-T}}{\partial \nu_d} = \frac{t_s(1-\beta)}{36t_h} \geq 0$, and thus

$$\Lambda_2^{O-T} \leq \Lambda_2^{O-T}|_{v_d=3(t_h-t_s)} = -\frac{(1-\beta)t_s[18\gamma(1+\beta)+(23+\beta)t_s]}{288t_h} \leq 0 \tag{32}$$

Therefore, firm 2 can profitably deviate from (offer, decline) to (offer, accept).

(b) Based on the equilibrium profits, we can derive the partial derivatives $\frac{\partial \Lambda_1^{T-I}}{\partial \gamma} = \frac{\partial \Lambda_2^{T-I}}{\partial \gamma} = \frac{1}{2}$, and

$$\begin{aligned}
\frac{\partial \Lambda_1^{T-O}}{\partial \gamma} &= \frac{(1-\beta)(8t_h - \beta t_s - t_s)}{16t_h} > 0 \\
\frac{\partial \Lambda_2^{T-O}}{\partial \gamma} &= \frac{t_s(1-\beta^2)}{16t_h} > 0.
\end{aligned}$$

Therefore, when γ increases, the conditions for two-way compatibility are more likely to be satisfied.

(c) We can derive the partial derivatives as follows:

$$\begin{aligned}
\frac{\partial \Lambda_1^{O-I}}{\partial \nu_d} &= \frac{\partial \Lambda_2^{O-I}}{\partial \nu_d} = \frac{t_s[(1-\beta)(t_h+t_s)+4\beta\nu_d]}{36t_h(t_h+t_s)} > 0 \\
\frac{\partial \Lambda_1^{O-T}}{\partial \nu_d} &= \frac{t_s(1-\beta)}{36t_h} > 0.
\end{aligned}$$

Therefore, when v_d increases, the conditions for one-way compatibility are more likely to be satisfied.

(d) We can derive the partial derivative as follows:

$$\frac{\partial \Lambda_1^{O-T}}{\partial \beta} = \frac{t_s(-12t_h - \ t_s - 4v_d + t_s\beta) + 18\gamma(4t_h - t_s\beta)}{144t_h}$$

Notice that $(4t_h - t_s\beta) > 0$ and $(-12t_h - \ t_s - 4v_d + t_s\beta) < 0$. Therefore, we can show that when γ is large, $\frac{\partial \Lambda_1^{O-T}}{\partial \beta} > 0$ and when γ is small, $\frac{\partial \Lambda_1^{O-T}}{\partial \beta} < 0$. We can similarly compute $\frac{\partial \Lambda_1^{O-I}}{\partial \beta}$ and $\frac{\partial \Lambda_2^{O-I}}{\partial \beta}$, and show that both expressions are greater (less) than 0 when γ is large (small). Combining these results, we find that as β increases, one-way compatibility is more (less) likely to be the equilibrium when γ is large (small). □

Proof of Corollary 1. (a) When $\beta = 1$, the equilibrium outcome in two-way compatibility case prescribed in Lemma 2 is identical to that in one-way compatibility case prescribed in Lemma 3. Therefore, both firms are indifferent to having S_1 on H_2. Intuitively, in these cases all users of the

H_2 device prefer S_2 to S_1, and, as a result, no users of the H_2 device use S_1 even if S_1 is available on H_2.

(b) When $\beta = 1$, $\Lambda_1^{O-I} \geq 0$ and $\Lambda_2^{O-I} \geq 0$ reduce to

$$\frac{1}{18}\left[9(\gamma - t_s) + v_d^2(\frac{1}{t_h} - \frac{1}{t_h + t_s})\right] \geq 0 \tag{33}$$

which is the condition prescribed in the corollary. Notice that if $\gamma > t_s$, this condition always holds; otherwise, the condition holds when $v_d^2 > \frac{9(t_s-\gamma)t_h(t_h+t_s)}{t_s}$.

(c) We can verify that $\frac{1}{18}\left[9(\gamma - t_s) + v_d^2(\frac{1}{t_h} - \frac{1}{t_h+t_s})\right]$ increases in v_d and γ. □

Proof of Corollary 2. (a) First, we notice that $\Lambda_1^{T-I} = \Lambda_2^{T-I} = \frac{\gamma}{2} > 0$. In addition, according to Equation (32), $\Lambda_2^{T-O} > 0$. Therefore, by Proposition 3(a), two-way compatibility can be sustained when $\Lambda_1^{T-O} > 0$. By Equations (21) and (29), we have

$$\Lambda_1^{T-O} = \frac{\gamma}{2} - \frac{t_s(24t_h + t_s + 8\nu_d + 18\gamma)}{288t_h}$$

The condition $\Lambda_1^{T-O} > 0$ is equivalent to $t_s(24t_h + t_s + 8\nu_d) < 18\gamma(8t_h - t_s)$.

(b) Because $\Lambda_1^{T-I} > 0$, when $\Lambda_1^{O-T} \geq 0$, $\Lambda_1^{O-I} \geq 0$ is guaranteed. Therefore, by Proposition 3(a), one-way compatibility can be sustained when $\Lambda_1^{O-T} \geq 0$ and $\Lambda_2^{O-I} \geq 0$. As in the proof of part (a), the condition $\Lambda_1^{O-T} > 0$ is equivalent to $18\gamma(8t_h - t_s) < t_s(24t_h + t_s + 8\nu_d)$. By Equations (13) and (29), we have

$$\Lambda_2^{O-I} = \frac{\gamma}{2} + \frac{t_s(-24t_h + t_s + 8\nu_d - 18\gamma)}{288t_h}$$

The condition $\Lambda_2^{O-I} \geq 0$ is equivalent to $t_s(24t_h - t_s - 8\nu_d) < 18\gamma(8t_h - t_s)$. □

Proof of Proposition 4. Substituting the equilibrium prices in Equations (25) and (26) into $x(y)$ and x^* in Equations (4) and (15), we can derive the equilibrium indifference curves. Substituting the equilibrium $x(y)$ and x^* into Equation (31), we have

$$W^O = \frac{18t_h[(4(v_d + 2v_2) - (3-\beta)t_s] + 10(1-\beta)v_d t_s + 20v_d^2 - 36t_h^2 + (1-\beta)(\beta+5)t_s^2}{144t_h}. \tag{34}$$

Similarly, we can derive W^T and W^I as follows.

$$\begin{aligned} W^T &= \beta\left(\int_0^{x^*}(v_1 - t_h x - t_s\min\{x, 1-x\})dx + \int_{x^*}^1[v_2 - (t_h + t_s)(1-x)]\,dx\right) \\ &\quad +(1-\beta)\Big(\int_0^1\int_0^{x^*}(v_1 - t_h x - t_s\min\{y, 1-y\})dxdy + \\ &\quad \int_0^1\int_{x^*}^1(v_2 - t_h(1-x) - t_s\min\{y, 1-y\})dxdy\Big) \\ &= v_2 - \tfrac{t_s}{4} - \tfrac{t_h}{4} + \tfrac{v_d}{2} + \tfrac{5v_d^2}{36t_h}, \end{aligned} \tag{35}$$

where the second equality is obtained by substituting the equilibrium prices in Equations (18) and

(19) into x^* in Equation (15) and by integration.

$$\begin{aligned}
W^I &= \beta\left(\int_0^{\frac{1}{2}+\frac{v_d}{6(t_h+t_s)}}(v_1-(t_h+t_s)x)dx+\int_{\frac{1}{2}+\frac{v_d}{6(t_h+t_s)}}^1\left[v_2-(t_h+t_s)(1-x)\right]dx\right)\\
&\quad +(1-\beta)\Big(\int_0^1\int_0^{x(y)}(v_1-t_hx-t_sy)dxdy+\\
&\quad \int_0^1\int_{x(y)}^1[v_2-t_h(1-x)-t_s(1-y)]dxdy\Big)\\
&= \frac{5v_d^2[t_h+(1-\beta)t_s]+18v_dt_h(t_h+t_s)-3(t_h+t_s)\left[3t_h((2-\beta)t_s-4v_2)+3t_h^2-(1-\beta)t_s^2\right]}{36t_h(t_h+t_s)},
\end{aligned} \tag{36}$$

where the second equality is obtained by substituting the equilibrium prices in Equations (9) and (10) into $x(y)$ in Equation (4) and by integration. The upper limit of the first integral in Equation (36), $\frac{1}{2}+\frac{v_d}{6(t_h+t_s)}$, is the intersection of equilibrium $x(y)$ and $y=x$.

Therefore, we have

$$W^T-W^I=\frac{t_s\left[5\beta v_d^2+3(1-\beta)\left(3t_h-t_s\right)\left(t_h+t_s\right)\right]}{36t_h\left(t_h+t_s\right)}>0 \tag{37}$$

$$W^O-W^I=\frac{t_s\left[10(1-\beta)v_d\left(t_h+t_s\right)+20\beta v_d^2+(1-\beta)\left(t_h+t_s\right)\left(18t_h-(7-\beta)t_s\right)\right]}{144t_h\left(t_h+t_s\right)}>0, \tag{38}$$

where the equality in each comparison is by simple algebra and the inequality is because of the assumption $t_h>t_s$. □

Proof of Proposition 5. The demand functions for the incompatibility case are:

$$\begin{aligned}
D_{1h}=D_{1s} &= \beta\frac{v_d-k-(p_1-p_2)+t_s+t_h}{2(t_s+t_h)}+\\
&\quad +(1-\beta)\left(\int_0^{\frac{v_d-k-(p_1-p_2)+t_h-t_s}{2t_h}}\int_0^1 1dydx+\int_{\frac{v_d-k-(p_1-p_2)+t_h-t_s}{2t_h}}^{\frac{v_d-k-(p_1-p_2)+t_h+t_s}{2t_h}}\int_0^{y(x)}1dydx\right).
\end{aligned}$$

For one-way compatibility, the demand functions are:

$$\begin{aligned}
D_{1h}^O &= \beta\frac{-p_1+p_2+t_h+v_d-k}{2t_h}+\\
&\quad (1-\beta)\left((\frac{-p_1+p_2+t_h+v_d}{2t_h})(1-\frac{t_s-k}{2t_s})+(\frac{-2p_1+2p_2+2t_h+t_s+2v_d-k}{4t_h})(\frac{t_s-k}{2t_s})\right)\\
D_{1s}^O &= \beta\frac{t_s-k}{2t_s}+(1-\beta)(\frac{-2p_1+2p_2+2t_h+t_s+2v_d-k}{4t_h})(\frac{t_s-k}{2t_s}).
\end{aligned}$$

For two-way compatibility, the demand functions are:

$$\begin{aligned}
D_{1h}^T &= \beta\frac{-p_1+p_2+t_h+v_d-k}{2t_h}+(1-\beta)(\frac{-p_1+p_2+t_h+v_d}{2t_h})\\
D_{1s}^T &= \frac{t_s-k}{2t_s}.
\end{aligned}$$

Taking the first-order conditions of the profit functions of all three cases, we can derive the equilibrium prices and profits.

To show that incompatibility is more likely to become the equilibrium outcome when γ is small, we only need to show that one of the two platforms is more likely to prefer incompatibility to one-way and two-way compatibility when γ is small. It is straightforward to show that indeed $\lim_{\gamma\to 0}\frac{\partial\Lambda_2^{I-O}}{dk} > 0$ and $\lim_{\gamma\to 0}\frac{\partial\Lambda_2^{I-T}}{dk} > 0$.

To show that one-way compatibility is more likely to become the equilibrium outcome, we can show that when γ is greater than a certain threshold, $\frac{\partial\Lambda_1^{O-I}}{dk} > 0$, $\frac{\partial\Lambda_1^{O-T}}{dk} > 0$, $\frac{\partial\Lambda_2^{O-I}}{dk} > 0$ and $\frac{\partial\Lambda_2^{O-T}}{dk} > 0$. □

Proof of Proposition 6. Since one-way compatibility requires the consensus of both platform owners, we will show that in this case as θ increases, Λ_1^{O-I} decreases. In other words, the profit difference between one-way compatibility and incompatibility shrinks as θ increases from 0 (our main model) to a larger fraction. Solving for first-order conditions, we can obtain:

$$\begin{aligned}
\pi_1^O &= [144t_h^2 + (4v_d + t_s(1-\beta)(1-\theta))^2 + 24t_h(4v_d + t_s(1-\beta)(1-\theta) + 6\gamma\beta(1-\theta)) \\
&\quad +18\gamma t_s(1-\beta)(1-\theta)(1+\beta+(1-\beta)\theta)]/(288t_h) \\
\pi_1^I &= [((t_s+t_s)(3t_h+v_d) - \beta t_s v_d)^2 + \beta t_s((t_h+t_s)(9\gamma t_h + 2v_d(3t_h+v_d)) - 2\beta t_s v_d^2)\theta \\
&\quad +\beta t_s(-9\gamma t_h(t_h+t_s) + \beta t_s v_d^2)\theta^2]/[18t_h(t_h+t_s)(t_h+t_s(1-\beta(1-\theta)))].
\end{aligned}$$

We can then verify that $\frac{\partial\Lambda_1^{O-I}}{d\theta} < 0$. □

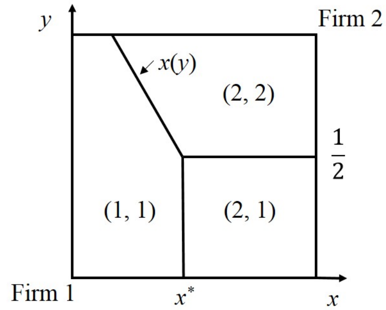

Figure 3: Consumers' purchase decisions under the other one-way compatibility

Appendix B: The One-Way Compatible Case with S_1 on H_2

We now consider the possibility of the other one-way compatibility as the equilibrium outcome where two platform owners agree to make S_1 compatible with H_2 but S_2 remains incompatible with H_1. Figure 3 illustrates consumers' purchasing decisions in this one-way compatible case. We can similarly derive the firms' profits in this case and compare them to the incompatible and two-way compatible cases to investigate when this one-way compatibility becomes the equilibrium outcome.

Proposition 7. *The one-way compatible case with S_1 on H_2 becomes the equilibrium outcome only when v_d is small and γ is intermediary.*

Proof. We use the superscript R to denote outcomes related to this one-way compatible case. We obtain the demand functions for the hardware and the software of the two platforms as:

$$\begin{aligned}
D_{1h}^R &= \beta\frac{-p_1+p_2+t_h+t_s+v_d}{2(t_h+t_s)} + \\
&\quad (1-\beta)\left(\int_0^{\frac{-p_1+p_2+t_h-t_s+v_d}{2t_h}}\int_0^1 dydx + \int_{\frac{-p_1+p_2+t_h-t_s+v_d}{2t_h}}^{\frac{-p_1+p_2+t_s+v_d}{2t_h}}\int_0^{\frac{-p_1+p_2+t_h+t_s+v_d-2t_hx}{2t_s}} 1dydx\right) \\
D_{1s}^R &= D_{1h}^T + (1-\beta)\frac{1}{2}\left(1-\frac{-p_1+p_2+t_h+v_d}{2t_h}\right)
\end{aligned}$$

and $D_{2h}^R = 1 - D_{1h}^R$ and $D_{2s}^R = 1 - D_{1s}^R$.

Solving the first-order conditions for the two profit-maximizing firms yields the equilibrium profits as follows:

$$\begin{aligned}
\pi_1^R &= ((\beta t_s(t_h+t_s-4v_d)+(t_h+t_s)(12t_h-t_s+4v_d))^2 - 18(\beta-1)r(t_h+t_s)(8t_h^2+(7-3\beta)t_ht_s \\
&\quad +(\beta-1)t_s^2))/(288t_h(t_h+t_s)(t_h-\beta t_s+t_s)) \\
\pi_2^R &= (t_h^2(18(-3\beta^2+2\beta+1)rt_s+(\beta^2-50\beta+193)t_s^2+8(13\beta-25)t_sv_d+16v_d^2) - \\
&\quad 2(\beta-1)t_ht_s(18(\beta+1)rt_s+(4v_d-t_s)((\beta-13)t_s+4v_d))+(\beta-1)^2t_s^2(18rt_s+ \\
&\quad (t_s-4v_d)^2)+144t_h^4-24t_h^3((\beta-13)t_s+4v_d))(288t_h(t_h+t_s)(t_h-\beta t_s+t_s)).
\end{aligned}$$

We next show that γ needs to be intermediary for this one-way compatibility to become an equilibrium. It is easy to see that when γ is sufficiently large (i.e., $\gamma > \gamma^{**}$), both firms will choose two-way compatibility because $\frac{\partial\Lambda_1^{T-R}}{d\gamma} > 0$ and $\frac{\partial\Lambda_2^{T-R}}{d\gamma} > 0$. We also note that $\frac{\partial\Lambda_1^{R-I}}{d\gamma} > 0$ and when $\gamma = 0$, firm 1 prefers incompatibility. Hence, there exists a threshold γ^* such that only when $\gamma > \gamma^*$, firm 1 prefers this one-way compatibility to incompatibility. These two results suggest that only when $\gamma^* < \gamma < \gamma^{**}$, this one-way compatibility can become the equilibrium outcome.

We then look at the threshold for v_d. Because $\frac{\partial \Lambda_2^{T-R}}{dv_d} > 0$, we could show that there exists a threshold v_d^* such that when $v_d > v_d^*$, firm 2 always prefers two-way compatibility to this one-way compatibility. Then we show that under the same condition, firm 1 prefers either incompatibility or two-way compatibility to this one-way compatibility. As a result, when $v_d > v_d^*$, this one-way compatibility cannot be the equilibrium outcome because it is dominated by either incompatibility or two-way compatibility. In other words, this one-way compatible case could become the equilibrium outcome only when $v_d < v_d^*$.

In sum, the analysis above shows that this one-way compatibility can become an equilibrium when $v_d < v_d^*$, and $\gamma^* < \gamma < \gamma^{**}$. □

This one-way compatible case shows up as the equilibrium under very narrow ranges of parameter spaces. The intuition is as follows. When v_d is not so big, the difference in the heterogeneity in profit foci of the two firms under incompatibility is small. With this one-way compatibility, the profit foci of the two firms flips: relative to each other, content sales become more important to firm 1 and hardware sales become more important to firm 2. When γ is not small, firm 1 finds that the gain from reduced competitive pressure and an increase in content sales exceed the loss from hardware sales. Similarly, firm 2 finds that the gain from reduced competitive pressure and an increase in hardware sales exceed the loss from content sales. Because hardware sales now become more important to firm 2, firm 2 does not want to make its software compatible with firm 1. Hence, this one-way compatible case becomes the equilibrium. When γ is large, both firms prefer two-way compatibility. When v_d increases, in contrast to the one-way compatible case with S_2 on H_1, this one-way compatible case is less likely to become the equilibrium because with a big v_d, profit foci are less likely to flip when firms switch from incompatibility to this one-way compatibility.

Overall, the proposition suggests that it is rare to observe one-way compatibility with S_1 on H_2 in practice as it requires a flip in firms' relative profit foci.