The Comprehensive Effects of Sales Force Management:
A Dynamic Structural Analysis of Selection, Compensation, and Training

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Abstract
This study provides a comprehensive model of an agent’s behavior in response to multiple sales management instruments, including compensation, recruiting/termination, and training. The model on agents’ behavior takes into account many of the key elements that constitute a realistic sales force setting—allocation of effort; forward-looking behavior; present bias; effectiveness of training; and employee selection and attrition. By understanding how these elements jointly influence agents’ behavior, the study provides guidance on the optimal design of sales management policies. Counterfactual experiments show ways in which alternative compensation schemes, recruiting and termination policies, and sales training opportunities affect the selection and performance of the sales force. A field validation, by comparing counterfactual and actual outcomes under a new policy, attests the accuracy of the model. In addition, the study offers a key methodological contribution by providing formal identification conditions for hyperbolic time preferences. The key to identification is that, under a multi-period nonlinear incentive scheme, an agent’s proximity to a goal affects only future payoffs in non-pecuniary benefit periods, providing exclusion restrictions on the current payoff.

Keywords: sales compensation; training; selection; recruiting; termination; hyperbolic discounting; present-bias; dynamic structural models; exclusion restriction; identification.
1. Introduction

Effective management of the sales force is vital to the success of many sales-driven organizations. Approximately 15 million salespeople in the United States, representing about 10% of the entire U.S. labor force, serve as links between the customer and the firm (U.S. Department of Labor, 2018). Investments in these salespeople are approximated to be 10% of sales revenues and can reach up to 40% in certain industries (Heide, 1999). The U.S. economy spends more than $800 billion on sales forces each year, nearly five times the estimated $169.5 billion spent on media advertising and more than 20 times the estimated $39.5 billion spent on Internet advertising (Zoltners et al., 2013). As these significant figures suggest, personal selling represents one of the most important elements of the firm’s marketing mixes, highlighting the importance of managing and motivating salespeople to achieve the organization’s objectives.

Since the earliest days of personal selling, organizations have utilized three main sales (force) management instruments—compensation, recruiting/retention (of high-ability employees), and training—to better control and motivate the sales force. Figure 1 illustrates the relation between these instruments and the organization’s outcome—i.e., the sales performance. Performance is the result of salespeople’s behavior, and the sales management instruments are utilized to train and motivate proper behavior as well as to select the right type of people. The three key instruments (levers) not only differ in cost and effectiveness across different types of people, but they also are interconnected in their efficacy to change behavior and, thus, the desired performance outcome. This study aims to jointly examine the effectiveness of multiple sales management instruments in the selection and performance of various types of salespeople.

A key, if not the most important, instrument in sales management is compensation. Organizations frequently use compensation to motivate and control the behavior of salespeople. A sales force compensation plan typically consists of fixed- and variable-pay components, where each component plays a distinct role in managing sales force behavior. Fixed pay (base salary) compensates for risk and, thus, provides stability and security of income (Arrow, 1971; Basu et al., 1985; Harris & Raviv, 1978; Hölstrom, 1979; Lal & Srinivasan, 1993). Variable pay, on the other hand, provides a direct link between the sales outcome and financial rewards, thereby inducing
motivation to achieve superior performance. Examples of variable pay include commissions given as a portion of sales and lump-sum bonuses contingent on meeting a preset quota. According to Joseph and Kalwani (1998), 95% of firms in the United States utilize some form of variable pay to incentivize their salespeople, with the most frequently used forms being commissions and quota-based bonuses. Organizations also change the components of their incentive plans frequently; nearly 80% of U.S. firms revise their compensation programs every two years or less, to better motivate salespeople and to tailor their behavior to the goal of the sales organization (Zoltners et al., 2012).

However, simply providing the salespeople an optimal menu of compensation is insufficient to achieve the desired outcomes of the sales organization. To support the productivity of their sales force, firms frequently rely on sales force training. Firms use sales training as a means to increase productivity, stimulate communication inside and outside the organization, reduce inter- and intra-department misunderstandings, enrich sales force morale, and decrease selling costs (Stanton & Buskirk, 1987; Churchill et al., 1993; Dubinsky, 1996). Sales organizations in the U.S. invest $15 billion annually in sales training programs and devote more than 33 hours per year to training each salesperson (Lorge & Smith, 1998; Ingram et al., 2015). Thus, to effectively allocate resources among the firm’s sales management instruments, it is essential to properly assess and evaluate the outcomes of an organization’s sales training programs.

Whereas compensation and training serve to induce the right behavior, selection (recruitment/termination) affects firm performance through change of the sales force composition. Salespeople are known to exhibit a high rate of turnover: the estimated annual turnover rate of 27% is more than twice that of the average work force in the U.S. (Richardson, 1999). There are two types of employee selection: (i) firm-induced selection, which involves recruiting, retention, and termination; and (ii) employee-induced selection, or voluntary turnover. When properly managed, selection allows the firm to maintain a healthy sales force through retaining high-quality employees and terminating persistent low performers. However, selection—especially voluntary turnover—also involves substantial costs to the firm, including costs related to hiring and training, jeopardized customer relationships, and territory vacancies (Griffeth & Hom, 2001; Boles et al., 2012). Hence,
deriving a proper policy to control for sales force selection is vital to the success of a sales organization.

Despite the ubiquitous use of the above three sales management instruments, however, there is little insight into their joint effect on various behavioral outcomes. For instance, how should a firm design its compensation plan to select the right salespeople—retain the high performers while discouraging the low—over time? Which is more effective in motivating the salespeople to meet their goals—increasing the level of monetary compensation or providing sales training opportunities? Can recruiting/termination policy replace the role of compensation and, if so, at what cost?

Separately identifying each of the above issues turns out to be problematic because various behavioral outcomes are often interrelated and occur simultaneously. Heterogeneous salespeople exhibit differences in productivity, time preferences, and responsiveness to compensation components and training, which, in combination, determine individual performance. The performance outcome results in compensation that influences employee turnover, which naturally leads to the selection of heterogeneous types of salespeople. Hence, this interrelated nature of behavioral outcomes necessitates an integrated model of sales force management.

There are two key challenges in modeling and identifying salespeople’s response to various management instruments. First, data at the salesperson level on various management practices are difficult to obtain, as many organizations treat HR information as highly confidential. As a result, previous studies have narrowed their focus to a single sales management instrument, such as compensation (Steenburgh, 2008; Misra & Nair, 2011; Chung et al., 2014). Second, a researcher observes neither the agent’s effort nor his or her time preference—the degree to which immediate utility is favored over delayed utility—but only the attrition decision and performance outcome over a time period that is likely correlated with the agent’s forward-looking allocation of effort and outside opportunities. This requires a behavioral assumption about the link between a sales agent’s motives (e.g., how close the person is to achieving quota at the end of the period) and his or her allocation of effort over time.

To overcome these challenges, we collaborate with a major multinational firm and formulate a comprehensive model of sales force behavior in response to various sales management practices. The
model takes into account many of the key realistic elements in salespeople’s behavior, including allocation of effort, stay-or-leave decision, forward-looking behavior, present-bias, and learning from training opportunities. Overall, we seek to gain insights into ways in which employee training, outside employment opportunities, and various elements of compensation jointly affect the selection and performance of heterogeneous salespeople.

This study also provides a key methodological contribution to the economics and marketing literature. The study provides a formal proof on the identification conditions of a hyperbolic discounting model—a more general structure than an exponential discounting model. An agent’s distance-to-quota (DTQ), under a nonlinear incentive contract, affects only his or her future payoffs in non-pecuniary benefit periods, providing exclusion restrictions on current payoffs to identify the agent’s time preferences. However, identifying time preferences in a hyperbolic discounting model becomes challenging when confronted with an agent’s continuous choice (e.g., effort). Existing studies are largely built upon a discrete choice setting (Fang & Wang, 2015; Abbring & Daljord, 2019), and, thus, the identification results do not fully translate. This study offers a formal discussion on the associated limitations and provide proper regularity conditions for identifying a hyperbolic discounting model under continuous choice. Building on the theoretical identification results, the study’s empirical application shows support for agents’ quasi-hyperbolic discounting time preferences. A hyperbolic discounting model can potentially explain agents’ seemingly irrational behaviors (such as extreme procrastination) that are difficult to explain with a standard discounting model but are commonly observed in the real world (for a review, see Ainslie (1992), Kirby (1997), and Frederick et al. (2002)).

The estimation results reveal the existence of different types of salespeople that possess heterogeneous utility and time preferences. A series of counterfactual experiments shows ways in which an agent’s performance and selection changes with alternative compensation plans, recruiting/termination policies, and sales training opportunities. The counterfactual results demonstrate a trade-off relation between adjusting fixed- and variable-pay components towards performance; a potential drawback of hiring only high-type salespeople; the short- and long-term performance outcomes of hiring rookies vs. experienced salespeople; how a collective leave package
can lead to selective departure of the sales force; and how sales training can serve as an alternative to providing additional compensation.

A field validation— that compares the actual sales records (that accompany changes in sales management instruments) with the simulated counterfactual outcomes— demonstrates the accuracy and applicability of the model. Hence, this study’s framework and model can provide a practical application for sales organizations to foresee the effect of multiple sales management instruments on the behavior of the sales force.

The remainder of the paper’s structure is as follows. Section 2 summarizes the related literature. Section 3 describes the institutional settings and provides model-free evidence that facilitates the empirical analyses. Section 4 presents the modeling framework of sales management and an agent’s dynamic optimization problem. Section 5 illustrates the identification of dynamic models under different time preferences. Section 6 describes the estimation procedure. Section 7 discusses the estimation results, counterfactual simulations, and field validation. Section 8 concludes.

2. Related Literature

This study on multi-dimensional sales force management contributes to several streams of research. First and foremost, the study relates to the strand of literature on sales force compensation. The theoretical studies on this topic find controversial results regarding which component constitutes the optimal plan. Early work by Basu et al. (1985) and Rao (1990), under the principal-agent framework of Hölmstrom (1979) in a single-period setting with a risk-averse agent, finds that the optimal incentive pay combines salary and nonlinear commission. More granularly, Hölmstrom and Milgrom (1987) and Lal and Srinivasan (1993) show that, in a multi-period setting, only a linear contract can achieve the first-best outcome. In contrast, Oyer (2000) finds that quota-based bonus compensation, along with linear over-achievement commissions, is uniquely optimal when the participation constraints are unbinding. More recently, Schöttner (2016) derives the conditions under which commission dominates a bonus plan and vice versa, depending on the degree of the agent’s responsiveness to incentives.
The findings from empirical studies on compensation also reflect the discrepancy over differential effects on motivation and productivity. Oyer (1998), using aggregate sales data, finds that quota-bonus pay can induce salespeople to manipulate the timing of sales, thereby negatively affecting productivity. On the contrary, analyzing at the individual-level, Steenburgh (2008) concludes that quota-bonus pay induces additional effort that provides net improvement in sales. A similar disparity is reported using dynamic models; for example, Misra and Nair (2011) and Chung et al. (2014) report contrasting findings regarding the effect of quota-based bonuses on sales performance.

This study’s contribution to this stream of literature is twofold. First, the study expands the scope of compensation plans to discuss the dynamic selection of sales agents, and, thus, provide a better understanding of how a firm’s compensation plan could facilitate the restructuring of its existing portfolio of sales agents. In addition, the study examines agents’ training records with their performance. Both sales training and compensation serve as significant investments for firms, and, thus, this study allows the evaluation of the relative effectiveness of these different sales management instruments. To the best of our knowledge, this study is the first to jointly examine the effect of multiple sales management instruments on sales force selection and performance.

As selection, by definition, accompanies employee turnover, this study relates to the strand of literature on the antecedents of sales force turnover. In marketing, studies have put greater emphasis on the negative aspects of sales force departure. Richardson (1999) derives managerial measures for assessing the direct and indirect costs of turnovers, and Darmon (2008) proposes a cost/benefit analysis of turnover to provide management efficiency. Using empirical analyses, Shi et al. (2017) find that the negative effects vary, and Sunder et al. (2017) find that turnover risk is the greatest for salespeople with moderate performance.

The abovementioned studies, however, are limited to evaluating the short-term effect of territory absence and potentially overlook the selection process that simultaneously takes place. That is, if a firm can select the right salespeople, then despite the short-term loss, employee turnover may result in greater long-term profitability. Hence, this study contributes to the appropriate evaluation of turnover by investigating salespeople’s latent future potential. In addition, the structural approach
of the study allows for various counterfactual policy simulations, whereas descriptive studies limit this applicability.

This study also relates to the strand of literature on sales training effectiveness. Although various studies have emphasized the pivotal role of sales training on performance and have proposed conceptual frameworks (Walker et al., 1977; El-Ansary, 1993; Honeycutt et al., 1995; Attia et al., 2005), only a handful of empirical studies have followed this footprint, likely due to difficulties in collecting data. In addition, early empirical studies, mostly relying on survey measures, have generated strikingly mixed findings, ranging from enhancing performance up to 50% (Martin & Collins, 1991; Roman et al., 2003) to being largely uninfluential (Christiansen et al., 1996; Dubinsky, 1996).

More recently, Kumar et al. (2014), investigating the effect of voluntary training opportunity on the salesforce lifetime value (net present value of a salesperson’s output), show that sales training indeed has a positive effect in both the short term and the long term. However, they evaluate only the correlation between salespeople’s self-selected training and their outcomes, refraining from developing causal inference. To identify the causal effect of training, Atefi et al. (2018) conduct a controlled field experiment that varies training policies across retail stores. They find a positive relation between the proportion of salespeople who receive training and sales outcomes; however, they analyze it only at the aggregate (store) level due to the institutional and experimental settings.

This study provides several insights into the literature by measuring the effectiveness of sales force training. First, by analyzing the training records at the individual salesperson level, the study associates different types of salespeople to the effectiveness towards training—thereby investigating the differential effect across heterogeneous salespeople. Second, the dynamic model allows us to analyze the comprehensive effects of training, in which it affects not only intertemporal performance outcomes, but also subsequent selection of the sales force. Lastly, the structural formulation of the model allows the evaluation of the compensation-equivalent monetary value of training, providing guidance to firms on resource allocation for sales training.

Finally, this study relates to the economics and psychology literatures on time preferences and intertemporal decision-making. The discounted utility model, established by Samuelson (1937) and
further developed by Thaler (1981), posits that individuals discount future payoffs relative to immediate payoffs. In capturing the behavioral responses to immediate versus delayed outcomes, mainly, two models of time preferences exist: exponential and (quasi-) hyperbolic discounting. The exponential discounting model assumes that discounting occurs at a fixed rate per unit of time (Samuelson, 1937; Dhami, 2016), representing stationarity and time-invariance. Meanwhile, a hyperbolic discounting model posits that the longer the time horizon before a reward is received, the lower the per-period discount rate becomes (Ainslie, 1975; Ainslie & Herrnstein, 1981; Loewenstein & Prelec, 1992; Laibson, 1997; O’Donoghue & Rabin, 1999), implying present-bias and time-inconsistent behavior. That is, when the time delay is short, the per-period discount rate is high and individuals are more likely to become impatient by selecting smaller but earlier rewards; but when the time delay is long, the per-period discount rate declines and individuals exhibit greater patience for larger but later rewards.

In terms of identifying agents’ time preferences, Rust (1994) shows that the discount factor is generally not identified from naturally occurring data without imposing further restrictions. Magnac and Thesmar (2002) generalize this idea to provide conditions on exclusion restrictions, the existence of variables that do not affect an agent’s current payoff but only his or her future payoff. This allows for the identification of the discount factor. Empirical studies in economics and marketing apply the exclusion restrictions to identify time preferences (discount factor) across various contexts, including new and used durable goods (Chevalier & Goolsbee, 2009; Ishihara & Ching, 2019); cellular phone usage (Yao et al., 2012); hardware and software platforms (Lee, 2013); sales force compensation (Chung et al., 2014); and consumer learning and inventory (Ching et al. 2014; Akca & Otter, 2015; Ching & Osborne 2019).

In a recent debate, Fang and Wang (2015) and Abbring and Daljord (2019) discuss identification regarding time preferences in dynamic discrete choice models. Fang and Wang (2015), by extending the above exclusion restriction arguments (Magnac & Thesmar 2002), consider conditions to identify various discounting behaviors, including exponential, quasi-hyperbolic, and naïve time preferences. Abbring and Daljord (2019) consider exclusion restrictions on model primitives and argue that the
arguments presented in Fang and Wang (2015), under weaker conditions, may not allow for point identification of the discount factor.

Different from the above studies (in which choice is directly observed), this study’s identification allows the agent’s action to be unobserved (e.g., effort) and only indirectly inferred from the observable output (e.g., performance). Regarding identification of models involving unobserved choice variables1, Hu and Xin (2019) provide a general framework under which the conditional choice probability and the law of motion for state variables are separately identified. Their identification leverages exclusion restrictions that affect only the conditional choice probability, but not the state transition probability. Similar to Hu and Xin (2019), this study identifies the agent’s unobserved effort using variation in output (sales performance) in response to his or her state (DTQ). The agent’s DTQ, under a nonlinear incentive contract, provides exclusion restrictions by affecting only future payoff (through the evolution of state variables), but not current period payoff, in non-pecuniary benefit periods.

This study contributes to this stream of literature by expanding the scope of identification to a quasi-hyperbolic discounting model that incorporates continuous choice of the agent’s actions. The study shows the limitations in applying the results of identification in a discrete choice setting (Magnac & Thesmar, 2002; Fang & Wang, 2015; Abbring & Daljord, 2019) to a hyperbolic discounting structure that accommodates continuous choices and provide regularity conditions for identification—for both exponential and quasi-hyperbolic discounting time preferences. Building on the identification arguments, the study’s empirical application presents support for agents’ quasi-hyperbolic discounting time preferences that exhibit heterogeneity in both the present-bias and the long-term discount factors.

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1 Empirical studies that examine unobserved choice variables in a dynamic setting include Misra and Nair (2011) and Chung et al. (2014), which analyze sales force behavior with effort unobserved, and Cosguner et al. (2018), which estimate a dynamic oligopoly pricing model where retail prices are observed but wholesale prices are not.
3. Institutional Details and Descriptive Analysis

This section presents the focal institution’s sales environment, its compensation plan (used for the empirical analysis of this study), and model-free evidence on forward-looking behavior and allocation of effort, which justify the dynamic structural formulation of the agent’s model.

3.1. Sales Environment

The firm under study is a multinational generic pharmaceutical company, offering a portfolio of branded prescription products through its own direct sales force. The data come from the firm’s sales operations in Turkey. Some notable aspects of the Turkish pharmaceutical market are worth mentioning. First, the government heavily regulates prices. Second, the nation’s universal healthcare system induces a high level of competition among the generics companies. Third, the country’s regulations prohibit direct-to-consumer (DTC) advertising, as is the case in most other markets.²

As such, personal selling plays an important and only role in the firm’s marketing and go-to-market strategies. Thus, to recruit and maintain a sustainable pool of salespeople and to train and motivate them properly are critical factors for success.

The data consist of salespeople’s turnover, hours of training, and performance records during a three-year period (2015-2017). Table 1 shows the number of employees who joined and departed and the corresponding turnover rate for each year. The firm’s average (voluntary) turnover rate was 14.60% over the three years.³ We focus our attention to those individuals who have remained in the firm (stay), and who have voluntarily separated (quit). In addition, to minimize the effect of the initial learning curve, we discard individuals with observations of fewer than or equal to three months since being hired (i.e., who joined on or after October 2017). The data-cleaning process leaves us with 554 salespeople. Table 2 shows the corresponding descriptive statistics. Employees who have decided to stay with the firm tend to perform better with higher variable pay and to have longer tenure.

² As of 2018, only Brazil, New Zealand, and the United States allowed direct-to-consumer advertising, with varying restrictions on mode and content.
³ To focus on salespeople’s behavior towards selection (voluntary turnover), we treat layoffs as a separate strategic decision by the firm and do not consider involuntary departures. The involuntary turnover rate of the firm was 7% in 2017; the majority of those who left were salespeople in their probation period (less than one year since hire).
The firm offers three types of sales training programs: primary training session; year-end sales session; and new sales-employee orientation. The 12-hour primary training session took place twice during the data observation period: once in January 2015, targeted at the salespeople from the primary care division (representing half of the entire sales force), and the other in April 2016, targeted towards senior salespeople across all divisions. The three-hour year-end sales session, which took place in December 2016, was mandatory for all salespeople. In 2017, the firm introduced a new sales-employee orientation program (designed for salespeople with less than one year of tenure) that ran for three hours. The estimated hourly cost of sales training per salesperson was $37 in 2016.

The firm operates its sales activity by route call sales: each salesperson has a preplanned series of meetings with either physicians or pharmacists over his or her exclusive territory. On average, a salesperson makes 20 calls per day. During each meeting, the salesperson exerts effort to promote the firm’s products over a wide range of competitors’ products.

3.2. The Firm’s Compensation Plan

The firm’s compensation plan consists of three components: a base salary, a quota-based bonus, and an overachievement commission. Figure 2 illustrates an overview of the plan, and Table 3 describes the specifics of the quota-bonus payment schedule. The salespeople receive an average fixed monthly salary of $1,500. At the end of the first three quarters, a salesperson receives a $1,700 bonus if he or she attains the respective quotas. At the end of the year, the firm pays a $3,400 bonus if the salesperson has met the annual quota. In addition, salespeople receive an overachievement commission of approximately $170 (2% of the combined bonuses of $8,500) per any excess percentage points above the annual quota. The firm caps the overachievement

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4 Because the firm chose participants in the primary training sessions based on an entire division or seniority, there exists unique variation in training hours that is exogenous to individual performance. In addition, the salespeople that joined the firm during the observation period add to this variation, as they missed the training opportunity in the earlier periods.

5 All monetary figures in this study are in U.S. dollars, converted using the exchange rate at the beginning of the data (January 2015).

6 As shown in the Table 3, salespeople receive a diminishing fraction of the bonus when her performance is at 90-99% of the quota, starting from the second quarter. Hence, in strict terms, ‘to meet the quota’ would mean the sales/quota level is at or above 90%. However, the firm avoids using this definition to discourage underachievers from believing that they are performing adequately. Thus, we follow the firm’s terminology, indicating that a salesperson meets quota when his or her performance is at or above 100%.
commission at $8,500, attained when the salesperson’s performance (sales/quota realization) reaches 150%.

In setting and updating the quota for each territory, the firm uses a prominent market research company (that possesses a majority of the pharmaceutical sales data per region, including the firm’s competitors’) to incorporate information on market-level growth and potential (rather than the sales outcomes of previous years) to mitigate possible ratcheting concerns. Some features of the firm’s quota-based bonus system are noteworthy. First, the quotas are set to be cumulative from the beginning of the year. Second, the firm defers the unearned bonus amounts in each quarter to the subsequent quarter. That is, if a salesperson misses the quota in a given quarter, the respective bonus amount is added to the total amount attainable in the next quarter. For example, if a salesperson meets both Q1 and Q2 quotas, he or she would receive $1,700 in both March and June. However, if a salesperson met only the Q2 quota and not the Q1 quota, he or she would receive $3,400 only in June.

This payout structure creates unique dynamics in the forward-looking behavior of the salespeople. On the one hand, it motivates them to keep up the pace from the beginning of the year. When a salesperson performs adequately and achieves bonus in a given period, he or she remains motivated due to the attainable quarterly bonus in the next period. Even if the salesperson does not meet quota in a given quarter, the motivation to exert effort becomes greater in the subsequent periods, as the total bonus amount increases due to the deferred bonus amounts from the previous quarters.

On the other hand, however, the cumulative nature of the performance evaluation raises potential concerns that poor performers lose motivation and give up. Because the sales/quota realization accumulates from the beginning of the year, the effect of several negative sales shocks can have a lasting effect throughout the year. This could demotivate the salespeople with poor-performance outcomes during the early part of the year, whereas they would have received a fresh start under an independent quarterly-quota system.
3.3. Model-Free Evidence: Forward-Looking Behavior

If variables related to the proximity to bonuses (DTQ) affect salespeople’s performances (in non-bonus periods), it suggests forward-looking behavior (Chung et al., 2014). Specifically, the state of proximity to quota would affect the performance of those who have a reasonable chance of achieving it. Hence, to show evidence of forward-looking behavior, we divide salespeople by their cumulative quota achieved (%QA). When %QA>0.8, salespeople have a reasonable probability of achieving quota by the quarterly-bonus periods and by the end of year, whereas when %QA<0.8, there is little chance. In addition, the probability likely decreases as time passes within the year. Table 4 reports the results of a regression analysis, with each column showing monthly performance as the dependent variable and the %QA up to the corresponding month as the explanatory variable, for each group of reps who are %QA>0.8 and %QA<0.8.

Consistent with forward-looking behavior, the (state) variable %QA remains significant throughout the year for salespeople with %QA>0.8. However, for those with %QA<0.8, %QA is significant only during earlier periods of the year. This is the case because, despite some bad outcomes during the earlier months of the year, there still exists some probability of meeting the annual quota by achieving high performance for the remainder of the year. However, the chance of achieving the annual quota decreases as months with low performance accumulate, and, by mid-year, low-performing salespeople (%QA<0.8) start to give up.

For a graphical illustration of forward-looking behavior, Figure 3 displays the scatterplot and the best fitting nonparametric smoothed polynomial (and its 95% confidence interval) of the salespeople’s performance in March, June, September, and December, against %QA by the previous month. Three items stand out from Figure 3. First, from March through September, a considerable number of salespeople with states %QA<0.8 achieve monthly performance (sales/quota) greater than 100%. However, in December, very few in the lower group exhibit excess performance. Consistent with the results in Table 4, salespeople likely give up in December when they are far

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7 Hereafter, the term ‘performance’ denotes sales normalized by the agent’s corresponding monthly quota, which are used to construct the cumulative interim and annual quotas. As indicated in Section 3.2, quotas are set by a well-established outside consulting firm, taking into account territorial and seasonal fluctuations in demand.
from quota because there is no way of achieving the annual quota with simply one month of superior performance. Second, a salesperson’s effort increases as he or she is on track to meet quota but flattens once he or she has met quota. The proximity to achieving quota motivates the salesperson, but, once he or she surpasses quota (%QA>1), the motivation is no longer intact. Lastly, a salesperson’s marginal effort with regard to his or her state (%QA of the previous month) increases with time in a calendar year. That is, the slope of the fitted line is steeper in December than in March. There are likely two reasons: (i) the presence of the overachievement commission in December motivates salespeople to exert greater effort towards the end of the year, and (ii) the large end-of-the-year bonus (including the overachievement commission) is less discounted due to temporal proximity and, thus, motivates salespeople more towards the end of the year.

4. Model

This section presents a comprehensive model of a sales agent’s behavior based on the sales management framework, illustrated in Figure 1. The discussion proceeds in three parts: (i) the agent’s per-period utility and performance response functions; (ii) the dynamic allocation of effort and stay-or-leave decisions; and (iii) time preferences (discount factors of exponential and quasi-hyperbolic discounting models).

An agent derives utility and disutility from compensation and effort, respectively, and faces intertemporal employment (stay-or-leave) decisions. Compensation is nonlinear and dependent on history of performance (e.g., quarterly sales outcomes). Thus, an agent exhibits forward-looking behavior and dynamically allocates effort over a specific time horizon.

4.1. Per-Period Utility and Performance Response

Agent $i$ in period $t$ derives per-period utility from the pecuniary benefits conditional on his or her choice of actions—whether to stay with the firm ($d_i$) and, (if so), how much effort to exert ($e_i$)—such that

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8 The period in the empirical application is a month.
If the agent decides to stay with the firm \((d_a = 1)\), he or she receives positive pecuniary utility \(M(W_{it})\) as a function of compensation \(W_{it} = W(q_{it}, s_{it}; \psi_{it})\), the level of which is determined by the agent’s performance \(q_{it}\) and state \(s_{it}\), given the firm’s compensation plan \(\psi_{it}\). Concurrently, the agent incurs disutility \(C(e_{it})\) from exerting effort \(e_{it}\), which affects the performance outcomes in the contemporaneous period. If the agent decides to quit \((d_a = 0)\), he or she receives reservation value \(\rho_i\) in perpetuity. The reservation value represents the agent’s outside option. The decision to leave the firm is an absorbing state (i.e., permanent), and, thus, once an action is taken, the agent cannot return to the firm.\(^9\)

In addition to the deterministic elements, the per-period utility includes a structural error term \(\varepsilon_{dit}\), which represents the state unobserved by the researcher but observed by the agent in his or her stay/leave decision-making \(d_{it}\).\(^{10}\) The structural error follows a Type-I extreme value distribution with location parameter zero and scale parameter \(\sigma_x\) and is assumed to be independently and identically distributed across choices and agents over time.

The agent’s per-period performance, \(q_{it}\), is a function of his or her individual effect \(\alpha_i\), effort \(e_{it}\), and an idiosyncratic performance shock \(\xi_{it}\) such that

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q_{it} = \exp(\alpha_i + e_{it} + \xi_{it}),
\]

or in logarithmic terms, \(\ln(q_{it}) = \alpha_i + e_{it} + \xi_{it}\). The log-linear specification allows the agent’s sales performance to be always positive, consistent with the empirical setting.

\(^9\) No agent, in the data, returned after departing from the firm.

\(^{10}\) As is standard in the literature, the error term satisfies the conditional independence assumption (Rust, 1987) in that, in a given period, it is not a function of an agent’s effort allocation decision \((e_{it})\). That is, the error term (unobserved state) realizes ex-ante of the agent’s current-period effort decision.
The individual effect (heterogeneity) $\alpha_i$ represents the agent’s baseline ability—i.e., performance attained without any effort.\textsuperscript{11} The performance shifters $x_{it}$ affect individual heterogeneity such that $\alpha_i = \alpha_0 + \alpha_1 x_{it}$, where $x_{it}$ includes the agent’s tenure, training, tenure-training interaction, and the level of higher education. An agent’s cumulative hours of sales training forms the training variable to capture the long-run persistence effect of training. Based on this structure, each training hour accumulates to form the agent’s stock of expertise, which carries over to the subsequent periods and affects his or her performance in the long run. The distribution of the performance shock $\xi_i$ (common knowledge to the agent\textsuperscript{12}) follows $N(0, \sigma_i^2)$ and is independent of the agent’s state $(s_{it}, \alpha_i, \varepsilon_{it})$ and effort $e_{it}$ for any $s \leq t$.

By the performance response function in Equation (2), the agent’s unobserved effort $e_{it}$ is (stochastically) linked to his or her observed performance $q_{it}$, in a given period. The performance outcome $q_{it}$, in turn, has both (i) a direct effect on contemporaneous compensation $W_t = W(q_{it}, s_{it}; \psi_{it})$ in bonus/commission periods; and (ii) an indirect effect on future compensation through the evolution of the subsequent state variables $s_{i,t+1} = f(q_{it}, s_{it}; \psi_{it})$, where $f(\cdot)$ is the state transition function conditional on the firm’s compensation plan $\psi_{it}$.

Equation (1) represents the ex-post utility of the agent, as the performance shock $\xi_i$ in Equation (2), which affects $q_{it}$ and $W_t$ in a given period, has yet to be realized when making the stay/leave and effort decisions. To form the basis of decision making, the agent takes expectation over his or her compensation $W_t = W(q_{it}, s_{it}; \psi_{it})$, given effort $e_{it}$ (that determines performance outcome $q_{it}$ under current state $s_{it}$). In this manner, the ex-ante utility function of the agent is

\[
U(d_{it}, e_{it}, s_{it}, \varepsilon_{it}) = \begin{cases} 
\mathbb{E}[M(W_{it}) \mid e_{it}, s_{it}] - C(e_{it}) + \varepsilon_{it} & \text{if } d_{it} = 1, \\
\rho_t + \varepsilon_{it} & \text{otherwise},
\end{cases}
\]

\textsuperscript{11} Strictly speaking, individual heterogeneity can also interpret as the baseline level of effort. As we cannot directly observe effort, the two effects (baseline ability and baseline effort) are not distinguishable from each other. Thus, effort $e_{it}$ represents the agent’s additional contribution to performance from the baseline.

\textsuperscript{12} More formally, the agent has rational expectation on the law of motion: the agent knows the distribution of the performance shock $\xi_i$, which affects the transition probability of future states.
where the functions $M$ and $C$ take on a parametric functional form. The pecuniary utility $M$ of wealth $W_d$ takes the form of mean-variance utility such that

$$E[M(W_d) \mid e_{it}, s_{it}] = E[W_d \mid e_{it}, s_{it}] - \gamma_i \text{Var}[W_d \mid e_{it}, s_{it}],$$

where $\gamma_i > 0$ represents the agent’s risk preference. The disutility $C$ is specified to be convex in effort $e_{it}$, such that

$$C(e_{it}) = C(e_{it}; \theta_i) = \theta_i e_{it}^2,$$

where $\theta_i > 0$ denotes the agent’s ease and flexibility in exerting effort. An implicit benefit of the mean-variance-utility specification is that it provides, by construction, scale and location normalization of utility. This allows us to estimate, rather than to normalize, the agent’s reservation value $\rho_i$ and the scale parameter $\sigma_e$ of the structural error in the empirical application (see also Section 5.4 for a detailed discussion of identification).

Given these specifications, the ex-ante utility (hereafter, stated simply as utility) function can be represented as

$$U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) = \begin{cases} E[W_d \mid e_{it}, s_{it}] - \gamma_i \text{Var}[W_d \mid e_{it}, s_{it}] - \theta_i e_{it}^2 + \varepsilon_{dit} & \text{if } d_{it} = 1, \\ \rho_i + \varepsilon_{dit} & \text{otherwise.} \end{cases}$$ (3)

The reservation value shifters $z_i$ affect the agent’s reservation value $\rho_i$, such that $\rho_i = \rho_{0i} + \rho_i z_{it}$, where $\rho_{0i}$ represents agent $i$’s baseline reservation value. Reservation value shifters $z_{it}$ include tenure and the level of higher education.

### 4.2. Compensation and State Variables

The focus of this study is on a class of nonlinear compensation schemes of which the payout depends on aggregate performance outcome over a certain time horizon. These schemes typically include components such as quarterly and annual bonuses or end-of-year commissions, which are commonly administered in practice (Joseph & Kalwani, 1998). By providing a reward at the end of the quota evaluation cycle consisting of multiple periods, the compensation plan stimulates the sales

---

13 The mean-variance utility represents a second-order approximation to a general concave utility function with constant absolute risk-aversion (CARA).
agent’s forward-looking behavior, as the agent’s effort exerted today accumulates to influence his or her future payoff. This cumulative effort is captured by a subset of the state variables in $s_t$, whose subsequent-period values $s_{t+1}$ evolve as a function of current-period performance $q_t$ and state $s_t$.

The firm’s incentive scheme ($\psi_i$) includes the following components: (i) individual specific monthly base salary $w_{it}$; (ii) the maximum attainable quarterly-bonus amount $Q_t$ (including the deferred amount from previous periods), common across all agents but varying across years; (iii) quarterly-bonus payout rate $R_{qt}$; and (iv) the end-of-year overachievement commission rate $R_{yt}$.$^{14}$

Formally, the components $Q_t$, $R_{qt}$, and $R_{yt}$ are as follows:

$$Q_t = \begin{cases} 
1,700 & \text{if } s_{it} = 3, \\
3,400 & \text{if } s_{it} = 6, \\
5,100 & \text{if } s_{it} = 9, \\
8,500 & \text{if } s_{it} = 12, \\
0 & \text{otherwise}, 
\end{cases}$$

$$R_{qt} = \begin{cases} 
0.35 & \text{if } 0.90 \leq s_{2,t+1} < 0.91, \text{ and } s_{it} = 12 \\
0.41 & \text{if } 0.91 \leq s_{2,t+1} < 0.92, \text{ and } s_{it} = 12 \\
0.47 & \text{if } 0.92 \leq s_{2,t+1} < 0.93, \text{ and } s_{it} = 12 \\
0.53 & \text{if } 0.93 \leq s_{2,t+1} < 0.94, \text{ and } s_{it} = 12 \\
0.59 & \text{if } 0.94 \leq s_{2,t+1} < 0.95, \text{ and } s_{it} = 12 \\
0.65 & \text{if } 0.95 \leq s_{2,t+1} < 0.96, \text{ and } s_{it} \in \{6,9,12\} \\
0.72 & \text{if } 0.96 \leq s_{2,t+1} < 0.97, \text{ and } s_{it} \in \{6,9,12\} \\
0.79 & \text{if } 0.97 \leq s_{2,t+1} < 0.98, \text{ and } s_{it} \in \{6,9,12\} \\
0.86 & \text{if } 0.98 \leq s_{2,t+1} < 0.99, \text{ and } s_{it} \in \{6,9,12\} \\
0.93 & \text{if } 0.99 \leq s_{2,t+1} < 1.00, \text{ and } s_{it} \in \{6,9,12\} \\
1.00 & \text{if } 1.00 \leq s_{2,t+1}, \text{ and } s_{it} \in \{3,6,9,12\} \\
0 & \text{otherwise}, 
\end{cases}$$

$$R_{yt} = \begin{cases} 
1.00 & \text{if } 1.00 \leq s_{2,t+1} < 1.01, \text{ and } s_{it} = 12, \\
1.02 & \text{if } 1.01 \leq s_{2,t+1} < 1.02, \text{ and } s_{it} = 12, \\
\vdots \\
1.98 & \text{if } 1.49 \leq s_{2,t+1} < 1.50, \text{ and } s_{it} = 12, \\
2.00 & \text{if } 1.50 \leq s_{2,t+1}, \text{ and } s_{it} = 12, \\
0 & \text{otherwise}, 
\end{cases}$$

$^{14}$ Although illustrated based on the institutional setting, our model is applicable to a wide class of nonlinear compensation schemes.
where the state variable $s_{1t}$ denotes the month-type ($\{1, 2, ..., 12\}$) and $s_{2t}$ denotes and percentage of the quota achieved (%QA) by the end of the previous month. The above components are collected to represent the firm’s incentive scheme by the vector $\psi_{it} = \{w_{it}, Q_{it}, R_{qt}, R_{yt}\}$.

Given the incentive scheme $\psi_{it}$, an agent $i$ receives compensation $W_{it} = W(q_{it}, s_{it}; \psi_{it})$ at time $t$, conditional on performance $q_{it}$ and state $s_{it}$. Compensation $W_{it}$ is comprised of three components: (i) monthly base salary $w_{it}$; (ii) quarterly (and annual) bonus $QB_{it}$; and (iii) end-of-year overachievement commission $OC_{it}$, in the following form:

$$W_{it} = w_{it} + QB_{it} + OC_{it},$$

whose elements $QB_{it}$ and $OC_{it}$ are expressed as follows:

$$QB_{it} = \max\left\{ Q \cdot R_{qt} \left( \frac{(s_{1t} - 1) \cdot s_{2t} + q_{it}}{s_{3t}} \right) - s_{3t}, 0 \right\},$$

$$OC_{it} = Q \cdot R_{qt} \left( \frac{(s_{1t} - 1) \cdot s_{2t} + q_{it}}{s_{1t}} \right),$$

where $s_{3t}$ represents the amount of bonus accrued (%BA) in previous quarters (hence, limiting the maximum quarterly-bonus amount attainable for those agents who previously received the bonus). Note that in non-bonus periods, $QB_{it} = 0$, and, thus, $W_{it}$ depends solely on $w_{it}$; note also that $OC_{it}$ distributes only in December.

The state variables directly linked to compensation include: (i) the month-type within the year, $s_{1t}$; (ii) the percentage of cumulative quota achieved (%QA), $s_{2t}$; and (iii) the amount of annual bonus accrued (%BA), $s_{3t}$.

The state variables evolve as follows:

1. **Month-type**

   $$s_{1t} = \begin{cases} 
   1 & \text{if } t \text{ is the start of the year}, \\
   s_{1(t-1)} + 1 & \text{otherwise}.
   \end{cases}$$

2. **Percentage of cumulative quota achieved (%QA)**

   $$s_{2t} = \begin{cases} 
   0 & \text{if } t \text{ is the start of the year}, \\
   \frac{s_{1(t-2)} \cdot s_{2(t-1)} + q_{(t-1)}}{s_{1(t-1)}} & \text{otherwise}.
   \end{cases}$$
3. Percentage of annual bonus accrued (%BA)

\[
    s_{3t} = \begin{cases} 
        0 & \text{if } t \text{ is the start of the year,} \\
        \max \left\{ Q_{t(t-1)} \cdot R_{q(t-1)} \left( \frac{s_{t(t-2)} \cdot s_{2(t-1)} + q_{t(t-1)}}{s_{t(t-1)}} \right), s_{3(t-1)} \right\} & \text{otherwise.} 
    \end{cases}
\]

Whereas the month-type evolves in a purely self-contained manner, the latter two state variables evolve (stochastically) based on the agent’s performance. The percentage of cumulative quota achieved (%QA) evolves every month, conditional on the performance in the previous periods. The percentage of annual bonus accrued (%BA) evolves stepwise every quarter, conditional on receiving the quarterly-bonus. The state variables that directly affect compensation are represented by the vector \( s_t = \{ s_{1t}, s_{2t}, s_{3t} \} \).

4.3. Dynamic Allocation of Actions

The per-period utility function in Equation (3), when linked with the aforementioned course of actions, outcomes, and state transitions, naturally leads to a dynamic formulation of the model. An agent chooses an action that solves the dynamic optimization problem, maximizing the sum of current and future payoffs where he or she receives a stream of utility over discrete time periods \( (t=1,2,...,\infty) \). An agent’s value function is defined as the agent’s discounted present value of the expected utility stream such that

\[
    \bar{V}(s_t, e_{\delta t}) = \mathbb{E} \left[ \sum_{\tau=t}^{\infty} \phi(\tau - t) \left\{ \max_{d_\tau, e_\tau} U(d_\tau, e_\tau, s_t, e_{\delta t}) \right\} \right] 
\]

\[
    = \max_{d_t, e_t} \left\{ U(d_t, e_t, s_t, e_{\delta t}) + \mathbb{E} \left[ \sum_{\tau=t+1}^{\infty} \phi(\tau - t) \left\{ \max_{d_\tau, e_\tau} U(d_\tau, e_\tau, s_t, e_{\delta t}) \right\} \right] \right\},
\]

where \( \phi(j) \) denotes the agent’s discount function for the utility from future \( j \)-periods forward \( (j=0, 1, 2, 3, ...) \) and \( \phi(0)=1 \), and \( U(d_t, e_t, s_t, e_{\delta t}) \) represents the utility function in Equation (3). Hence, the present value is represented by the expected utility flow upon making an infinite sequence of optimal decisions \( (d_\tau, e_\tau : \tau=t, t+1,...) \), governed by the discount function \( \phi(\cdot) \). The expectation is taken with regard to both the idiosyncratic performance shock \( \xi \) and the structural error \( \varepsilon \) for each period \( \tau \geq t+1 \).
The choice-specific value with respect to each action pair \((d_{it}, e_{it})\), which represents the discounted present value when the agent chooses actions \(d_{it}\) and \(e_{it}\) given state variables \(s_{it}\) and \(\varepsilon_{dit}\), is defined as

\[
V(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) = U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) + \mathbb{E}\left[\sum_{\tau=t+1}^{\infty} \phi(\tau - t) \left\{ \max_{\substack{d_{\tau}, e_{\tau}, s_{\tau}, \varepsilon_{d\tau}}} \mathbb{E} U(d_{\tau}, e_{\tau}, s_{\tau}, \varepsilon_{d\tau}) \right\} | d_{it}, e_{it}, s_{it}, \varepsilon_{dit} \right],
\]

(4)

In each period, the agent incorporates the information contained in the current states \((s_{it}, e_{dit})\) to evaluate the future outcome of current-period actions: employment and effort.

The agent’s effort policy (the optimal level of effort), as a function of current states and stay-or-leave decision, is given by

\[
e_{it} = e(d_{it}, s_{it}) = \begin{cases} \arg\max_{e} \{V(1, e, s_{it}, \varepsilon_{it})\} & \text{if } d_{it} = 1, \\ 0 & \text{otherwise}. \end{cases}
\]

That is, the agent chooses the optimal level\(^{15}\) of effort \(e_{it}\), which maximizes the discounted stream of expected utility flow, conditional on the current states and on staying with the firm. The temporal trade-off of exerting effort \(e_{it}\) (in non-bonus/commission periods) arises between the per-period disutility \(C(e_{it})\) in Equation (1), and the state-transition \(s_{i,t+1}\) (updated through the performance outcome \(q_{it}\) in Equation (2)) towards a higher probability of future pecuniary benefit at the end of the compensation cycle.

The agent decides to continue with the firm if the choice-specific value \(V(1, e_{it}, s_{it}, \varepsilon_{it})\) of staying and exerting effort is greater than the value \(V(0, 0, s_{it}, \varepsilon_{0it})\) of leaving\(^{16}\). That is,

\[
d_{it} = \begin{cases} 1 & \text{if } V(1, e_{it}, s_{it}, \varepsilon_{it}) \geq V(0, 0, s_{it}, \varepsilon_{0it}), \\ 0 & \text{otherwise}. \end{cases}
\]

The summary of the model dynamics is as follows: After observing his or her current state (with regard to future pecuniary benefit), an agent exerts effort and incurs disutility. Exerted effort, in combination with an idiosyncratic shock, determines the agent’s current-period sales performance.

\(^{15}\) For brevity, we suppress the optimality (*) notation throughout the paper.

\(^{16}\) Note that once the agent leaves the firm \((d_{it} = 0)\), the absorbing state implies (i) effort \(e_{it} = 0\) in all subsequent periods, and (ii) the recursive formulation in Equation (4) degenerates to receiving the reservation value \(r_{i}\) in perpetuity.
This performance affects both the current-period payoff and the probability distribution of state variables in the subsequent period. Hence, the agent’s effort helps preserve the states in healthy conditions, increasing the chance of receiving a monetary payoff in later periods. However, if the current state shows a limited chance of receiving future payoffs (e.g., after several periods of low performance), the agent may give up exerting effort in order to reduce current-period disutility. Furthermore, if the value of staying becomes lower than the outside option, the agent will decide to leave the firm.

4.4. Time Preferences

The above forward-looking model naturally leads to a conceptual question: How does an agent discount the stream of future utility to derive the optimal policy? In other words, what is the agent’s time preference, the degree to which immediate utility is favorable over delayed utility? The question can be addressed through varying the structure of \( \phi(j) \), the discount function of the dynamic optimization problem in Equation (4). We consider two models of time preference: exponential discounting and quasi-hyperbolic discounting.

4.4.1. Exponential Discounting

The exponential discounting model (Samuelson, 1937) postulates that an agent’s discount function for the \( j \)th future period takes the form

\[
\phi(j) = \delta^j \text{ for } j = 0, 1, 2, ..., 
\]

where \( \delta \in (0, 1) \). The model implies time-consistent behavior by featuring stationary discounting where expected future utility geometrically decays. Because of its analytical convenience, exponential discounting is frequently assumed in the economics and marketing literatures.

The dynamic optimization problem can be decomposed into an infinite sequence of single-period decisions. Assuming exponential discounting, the infinite sum of the discounted future utility flow in Equation (4) can be replaced by the subsequent period value function such that
Henceforth, for brevity of exposition, subscripts $i$ and $t$ are suppressed, and the subsequent period $(t+1)$ is denoted by a prime (′) symbol when possible.

Let $v(d,e,s)$ denote the deterministic portion of the choice-specific value in Equation (4) (i.e., $v(\cdot) = V(\cdot) - \varepsilon_{dt}$) and define it as the choice-specific value function. Similarly, let $u(d,e,s)$ denote the deterministic portion of the utility function (i.e., $u(\cdot) = U(\cdot) - \varepsilon_{dt}$). Assuming additive separability and serial independence of the structural errors, the above equation simplifies to

$$v(d,e,s) = u(d,e,s) + \delta E\left[\max_{d',e'} \left\{ v(d',e',s') + \varepsilon_d' \right\} \bigg| d,e,s \right].$$

(5)

4.4.2. Quasi-Hyperbolic Discounting

The quasi-hyperbolic discounting model (Phelps & Pollak, 1968; Laibson, 1997) posits that an agent’s discount function for the $j$th future period takes the form

$$\phi(j) = \begin{cases} 1 & \text{if } j = 0, \\ \beta^j & \text{if } j = 1, 2, 3, \ldots, \end{cases}$$

where $\delta \in (0,1)$ is the standard discount factor, and $\beta \in (0,1]$ is the present-bias factor. Often referred to as the Beta-Delta preferences, the model parsimoniously captures present-bias and, thus, time-inconsistency. The standard discount factor $\delta$ captures long-term, time-consistent discounting, and the present-bias factor $\beta$ captures short-term impatience and the discontinuity between the present and the future (O’Donoghue & Rabin, 1999). Note that exponential discounting is a special case of quasi-hyperbolic discounting when $\beta=1$—i.e., the agent is not present-biased.

Given quasi-hyperbolic discounting, the choice-specific value in Equation (4) becomes

$$v(d,e,s) = u(d,e,s) + \beta \delta E\left[\max_{d',e'} \left\{ \tilde{v}(d',e',s') + \varepsilon_d' \right\} \bigg| d,e,s \right].$$

(6)
where
\[ \hat{v}(d, e, s) = u(d, e, s) + \delta E \left[ \max_{d', e', s'} \left\{ \hat{v}(d', e', s') + \varepsilon'_d \right\} d, e, s \right] . \] (7)

Unlike the case of exponential discounting, however, the quasi-hyperbolic discounting model does not allow a recursive representation of a single value function. The flow of future utility involves an additional value function \( \hat{v}(\cdot) \) due to the agent’s time-inconsistency. Hence, the optimal choice of effort \( e \) in the present becomes different from that of the future (i.e., present-biased).

The structure in Equations (6) and (7) requires solving two equations for two functions. This leads to a challenge in identification, which we discuss, in detail, in the following section.

5. Identification

This section presents the formal identification arguments and proceeds in the following order. First, we discuss the primitives of static utility—performance response, pecuniary utility of wealth, and disutility of effort—and then, the agents’ time preferences—both exponential and quasi-hyperbolic. Finally, we provide an intuitive discussion of identification. The formal arguments build upon those of Magnac and Thesmar (2002), who propose exclusion restrictions to identify the standard (exponential) discount factor. We expand identification of time preferences to consider the present-bias factor in a quasi-hyperbolic discounting model that accommodates continuous choice. Refer to the Appendix for proofs regarding formal arguments.

5.1. Static Utility

Suppose that the data \((d_i, s_i, q_i, \psi_i)\) for agents \(i = 1, 2, \ldots, N\) over time \(t = 1, 2, \ldots, T\) are observed\(^{17}\) and that these observations are independent and identically distributed across agents. We first consider the identification of the performance response function in Equation (2). The challenge in identifying the performance response of unobserved effort \(e_u\) is in controlling for individual heterogeneity \(\alpha_u\). Because one does not directly observe either construct, separately

\(^{17}\) Note that the agent’s state variables \((s_i)\) are computed given one’s performance history \((q_i)\) and firm’s compensation plan \((\psi_i)\).
identifying effort from individual heterogeneity typically is infeasible without further restrictions. The issue becomes further complicated because an agent’s effort policy is likely a function of individual heterogeneity $\alpha_i$. That is, an agent takes into account his or her own baseline productivity when making the effort decisions.

The agent’s behavior under nonlinear compensation schemes provides conditions for the effort policy to be separately identified from individual heterogeneity. The idea is to exploit observations in which the optimal effort is trivially a corner solution. Consider the following assumption:

**Assumption 1 (Corner Solution).** Suppose there exists a subset $S_a$ with a positive probability measure in the support of state variables $s$ such that if $s \in S_a$, the derivative of the value function with respect to $e$ is non-positive, i.e., $\frac{\partial v(s,e,s_t)}{\partial e} < 0$ for any $e \geq 0$.

That is, if $s$ takes a value in $S_a$, the agent exerts zero effort. The set $S_a$ exists when the agent is far above the quota (i.e., the bonus is already attained) or far below the quota (i.e., the bonus is not within reach). In either state, the agent’s additional performance provides limited gains and, thus, the agent is better off not incurring any effort (and avoiding the associated disutility).

**Proposition 1.** Under Assumption 1, (i) the agent’s effort policy $e_a$, (ii) individual heterogeneity $\alpha_a$, and (iii) the distribution of performance shock $\xi_a$ are identified.

**Proof.** See Appendix A. The proof is based on a nonparametric regression approach in a similar vein to Hu and Xin (2019).

Proposition 1 governs the relation among unobserved effort, individual heterogeneity, and observed performance, and stipulates the forward-looking behavior of the agents. From Proposition 1, the agent’s effort policy conditional on staying with the firm, $e_a = e(1,s_a)$, is identified. Even if an agent leaves the firm, the optimal effort is identified during the period in which the agent stayed with the firm.
Regarding the identification of the choice-specific value function in Equation (5), the following lemma holds:

**Lemma 1.** Under Assumption 1, the difference in choice-specific value functions, \( v(1, e_{it}, s_{it}) - v(0, 0, s_{it}) \), is nonparametrically identified up to location and scale at the optimal level of effort \( e_{it} \) (identified in Proposition 1).

**Proof.** See Appendix B. The proof uses the conditional choice probability approach in Magnac and Thesmar (2002).

The location and scale of the payoff function are generally not identified in binary choice models without further normalizations. However, via the model specification in Equation (3), the value of leaving, \( v(0, 0, s_{it}) \), no longer depends on the agent’s effort choice or state variables. Hence, Lemma 1 implies that the value of staying, \( v(1, e_{it}, s_{it}) \), is identified at the optimal level of effort.

Given nonparametric identification of the choice-specific value function \( v \), what remains for identification are the primitives of the utility function and the time preferences (discount factor).

### 5.2. Exponential Discounting Model

The identification of the exponential discounting model materializes from the exclusion restriction (Magnac & Thesmar, 2002) provided by a nonlinear compensation plan.

**Assumption 2 (Exclusion Restriction).** Suppose that the state variables \( s \) can be partitioned into two vectors, \( s_1 \) and \( s_2 \), where \( s_1 \) is a vector of variables that satisfies the following condition: there exists a subset \( S_1 \) in the support of \( s_1 \) such that if \( s_1 \in S_1 \), both

1. \( u(d, e, s_1, s_2) = u(d, e, s_1, \tilde{s}_2) \) for any \( s_1 \) and \( \tilde{s}_2 \), and
2. \( v(d, e, s_1, s_2) \neq v(d, e, s_1, \tilde{s}_2) \) for some \( s_1 \) and \( \tilde{s}_2 \) hold.

That is, if \( s_1 \) takes a value in \( S_1 \), \( s_2 \) does not affect the present utility. In this study’s empirical application, the variables month type and an agent’s DTQ play the role of \( s_1 \) and \( s_2 \), respectively. For example, there is no performance-based lump-sum payment in October or November, so the DTQ does not affect the per-period utility. In these months, the per-period utility depends only on...
the disutility of effort, which does not include \( s \). However, the future expected utilities would differ according to the DTQ.

**Proposition 2.** Suppose that the agent’s true time preferences follow the exponential discounting model. Under assumptions 1-2, the instantaneous utility function \( u \) at the optimal effort and the discount factor \( \delta \) are nonparametrically identified.

**Proof.** See Appendix C. The proof uses exclusion restrictions similar to Corollary 3 and Proposition 4 in Magnac and Thesmar (2002).

Once \( u \) is nonparametrically identified, the parametric identification of the mean-variance and reservation value within \( u \) is straightforward. As there is no variation in pecuniary utility during non-bonus periods, we have \( \theta \) identified in those months. Parameter \( \gamma \) is identified using \( u(d, e, s) + \theta e^2 \) during the bonus-paying months. The remaining \( \rho \) and \( \sigma_e \) are identified using the variation in agents’ stay-or-leave decisions (see Section 5.4 for an intuitive discussion).

### 5.3. Quasi-Hyperbolic Discounting Model

Under the quasi-hyperbolic discounting model in Equations (6) and (7), there exist two value functions, \( v \) and \( \hat{v} \), and two discount factors, \( \beta \) and \( \delta \). The quasi-hyperbolic discounting model is more general than the exponential discounting model, as the latter is a special case of the quasi-hyperbolic discounting model when \( \beta = 1 \).

As Lemma 1 equally applies to the quasi-hyperbolic discounting model, the choice-specific value function \( v \) is nonparametrically identified (up to location and scale). However, identification of \( \hat{v} \) is not as straightforward, as Equations (6) and (7) form a system of equations. Multiplying Equation (7) by \( \beta \) and subtracting it from Equation (6) becomes

\[
v(d, e, s) - \beta \hat{v}(d, e, s) = (1 - \beta) u(d, e, s).
\]

Because \( \beta > 0 \), the above equation simplifies to

\[
\hat{v}(d, e, s) = \frac{\beta - 1}{\beta} u(d, e, s) + \frac{1}{\beta} v(d, e, s).
\]
Finally, inserting the above equation into Equation (6) establishes that

\[ v(d, e, s) = u(d, e, s) + \delta E \left[ \max_{d', e'} \left\{ \left( \beta - 1 \right) u(d', e', s') + v(d', e', s') + \beta \varepsilon' \right\} \right] \]  

(8)

In Equation (8), the distribution of \( \varepsilon' \) and the value function \( v \) are known, whereas the per-period utility \( u \) and the discount factors, \( \delta \) and \( \beta \), are unknown. Because this equation summarizes the system of equations, parameters \( (u, \delta, \beta) \) are identified if there exists a unique solution to Equation (8).

In Mathematics, the structure of Equation (8) is known as a nonlinear Fredholm integral equation of the second kind (Arfken & Weber, 1999; Polyanin & Manzhirov, 1998; Vetterling et al., 1992). Solving the integral equation for the unknown utility function \( u(d, e, s) \) is an ill-posed inverse problem due to the maximum function and integration taken over the utility function. Because a lot of information is “integrated out” and naturally lost during the process, it is well known that the solution to this ill-posed inverse problem may not exist, or even if a solution exists, it may not be unique.\(^\text{18}\)

The essence of this problem arises due to the continuity of the choice variable. If the choice variable is discrete, the integral equation in Equation (8) can be replaced by a matrix algebra, and the problem is simplified to finding the inverse of the matrix. For example, Abbring and Daljord (2019) and Fang and Wang (2015), in a discrete choice setting, rely on matrix algebra to find the inverse for identification. This is not applicable to our setting—in which the choice variable is continuous—as solving for the inverse of an integral equation is ill-posed. Thus, without further restrictions, the utility function and the discount factors cannot be nonparametrically identified, even if the aforementioned exclusion restrictions hold. Intuitively, the ill-posed problem in our case is due to the fact that the continuous choice in the model requires the utility function to be an infinite dimensional object (absent parametric assumptions), whereas in a discrete choice model, the utility function is represented by a finite dimensional vector (as in Fang and Wang (2015)). Because

\[^{18}\text{Conceptually, obtaining the solution to this problem is equivalent to finding an inverse mapping of the nonlinear integral. Even if there exists a unique solution, it is known to be extremely difficult, if not impossible, to obtain.}\]
of this difference, a finite number of exclusion restrictions is insufficient to nonparametrically identify
the utility function of a continuous choice hyperbolic discounting model.

The exponential discounting model bypasses this issue because the utility function does not
enter the integral due to its recursive nature. That is, the value function for the future payoffs is
identified directly from the choice probabilities. In contrast, in the quasi-hyperbolic discounting
model, the utility function enters the integral as in Equation (8). This change in the value function
creates complications in solving for the equation, leading to uncertainty about the existence of the
solution and, if it does exist, its uniqueness.

A typical solution for an ill-posed inverse problem is “regularization.” In a broad sense, to
regularize is to provide additional assumptions that can aid the existence, uniqueness, and numerical
stability of a solution. Some common examples include discretization of variables (Magnac &
Thesmar, 2002; Fang & Wang, 2015; Abbring & Daljord, 2019); parameterization of functions; and
Lasso-type penalization methods.

The parametric assumption in Equation (3) on the per-period utility function \( u \) serves as a
regularization to identify quasi-hyperbolic time preferences in the presence of a continuous choice
variable. To illustrate, given the parameter vector \( \mu = (\gamma, \theta, \rho, \sigma) \), the agent’s optimal effort (in the
subsequent period) conditional on staying with the firm is

\[
e'(s | \mu) = \arg \max_{e} \left\{ (\beta - 1)u(1,e,s) + v(1,e,s) \right\}.
\]

Note that prior to parametrization, this optimal effort for the subsequent period was intractable.

Given the extreme value distribution assumption, the future payoff component within the
expectation in Equation (8), conditional on \( s' \), now becomes

\[
\max_{d', e'} \left\{ (\beta - 1)u(d',e',s' | \mu) + v(d',e',s') + \beta e' \right\} \\
= \beta \sigma \log \left\{ \exp \left( \frac{(\beta - 1)u(1,e',s' | \mu) + v(1,e',s')}{\beta \sigma} \right) + \exp \left( \frac{(\beta - 1)\rho_0 + v(0,0,s')}{\beta \sigma} \right) \right\} \\
= \Lambda(s' | \mu, \beta).
\]

The expectation of the above future payoff over \( s' \), given the current period state and choice variables,
becomes
\[ \mathbb{E} \max_{d',e'} \left\{ (\beta - 1)u(d',e',s' \mid \mu) + \nu(d',e',s') + \beta \varepsilon' \right\} \mid d,e,s = \int \Lambda(s' \mid \mu, \beta) f(s' \mid d,e,s) ds'. \]

Thus, the identification criteria in Equation (8) simplify to a function of the parameters \((\mu, \delta, \beta)\), where

\[ u(d,e,s \mid \mu) - v(d,e,s) + \delta \int \Lambda(s' \mid \mu, \beta) f(s' \mid d,e,s) ds' = \Pi(d,e,s \mid \mu, \delta, \beta). \]

The true parameter vector \((\mu, \delta, \beta)\) solves the above equation \(\Pi(d, e, s \mid \mu, \delta, \beta) = 0\) for all \((d, e, s)\).

Thus, for identification, the assumption of a full-rank condition is sufficient.

**Assumption 3 (Rank Condition).** Denote the agent’s decision and state variables by \(x = (d, e, s)\).

There exists a subset \(X = \{x_j : j = 1, 2, \ldots, J\}\) in support of \(x\) such that \(\left\{ \frac{\partial \Pi}{\partial \mu, \delta, \beta} : j = 1, 2, \ldots, J \right\}\) has a rank greater than or equal to the number of parameters.

The assumption rules out the case in which different values of parameters yield identical observations in the model. Mathematically, the assumption holds if no parameters are linearly dependent. The nonlinear nature of the model (the agent’s effort enters the mean-variance utility nonlinearly and the disutility function quadratically) readily satisfies the rank condition assumption.

The sufficient conditions for Assumption 3 are formally stated in Appendix D.

**Theorem 1.** Suppose that the agent’s true time preferences follow the quasi-hyperbolic discounting model. Under Assumptions 1-3, the standard discount factor \(\delta\), present-bias factor \(\beta\), and parameters of the utility function \(\mu\) are parametrically identified.

**Proof.** See Appendix E. The proof uses the local identification approach to find the unique solution to \(\Pi(d, e, s \mid \mu, \delta, \beta) = 0\).

**5.4. Intuitive Discussion of Identification**

In addition to the formal identification arguments described above, we discuss model identification in our empirical context. First, we provide intuition regarding the identification of static utility. Then, we discuss identification regarding the discount factor(s).
A key challenge is identifying unobserved effort and utility parameters with limited variations in the agent’s compensation contract. There exists some variation in the compensation contract—specifically in the quarterly and annual bonus amounts across years. Thus, variations in performance across the different compensation regimes enable identification. In addition, the relation between an agent’s sales performance and his or her state variables help identification. The agent likely exerts more effort when close to quota versus when far from quota. Thus, systematic differences in sales performance at different DTQs identify effort and, thus, facilitate identification of the disutility of effort (Misra & Nair, 2011; Chung et al., 2014). Suppose that there are two agents with the same states, but one has higher performance than the other does. Then, we can infer that the agent with higher performance has lower disutility of effort. Similarly, suppose there are two agents both of whom have no chance of meeting quota (DTQ is very low), but one has higher performance than the other does. Then, we can infer that the agent with higher performance has higher baseline ability. The extent to which an agent over- or under-performs on quota identifies the risk-aversion parameter. A risk-averse agent would constantly over- or under-achieve (especially in March, June, and September, when no overachievement incentives are offered), whereas a risk-neutral agent would just meet quota. The variation in sales in the same states within an agent identifies the distribution of the performance shocks. The variation in sales with variation in performance shifters identifies the performance response parameters.

As described in Section 4.1, the parametric functional form on the agent’s payoff (specifically, the mean-variance utility function) provides location and scale normalization to facilitate identification. The mean-variance-utility specification implicitly presumes that the constant term of utility is zero and the parameter associated with the mean of wealth is unity. Thus, the mean ($\mu$) and variance ($\sigma^2$) of the outside option is identified under this specification. Intuitively, if, given a level of income, salespeople are frequently leaving the firm, we can infer that the value of the outside option is high. Relatedly, the observed attrition behavior at different levels of income identifies the variance of the outside option. For example, if salespeople’s attrition behavior do not change much
with changes in income, we can infer high variance in the value of the outside option. Naturally, the variation in reservation shifters identifies the corresponding parameters.

As explained in Section 5.2, an agent’s DTQ in non-bonus periods act as an exclusion restriction to identify discount factor(s). Suppose that there are two agents with the same characteristics who display same behavior (and, thus, performance) at the end of the year (final period of a compensation cycle). However, suppose that, in non-bonus periods, one agent performs better than the other does, even though both are in the same state (DTQ). We can infer that the agent with high performance in non-bonus periods has a higher discount factor (or a lower discount rate). The hyperbolic discounting model, under the functional form specification of utility, is identified if there exist more than two periods with exclusion restrictions. The performance of an exponential discounter would be more consistent and smooth throughout the year as compared to that of a hyperbolic discounter.

6. Estimation

The estimation procedure follows the full-solution method (Rust, 1987), using maximum likelihood, rather than the conditional choice probability approach (Hotz & Miller, 1993; Bajari et al., 2007), as the two-step estimation procedure can generate biases if the state variables in the policy function are correlated with the first-stage errors. In addition, the maximum likelihood approach has the minimum variance achievable by a consistent and asymptotically normally distributed estimator.

6.1. Individual Likelihood

Given the value function (Equation (4)) and the empirical specification of the per-period utility function (Equation (3)), one can obtain the expected value function through the inner loop in the conventional nested fixed-point algorithm (NFXP) such that

$$EV(d, e, s) = E_{\xi, \xi'}[V(s', \xi' \mid d, e, s)] = \int_{\xi} \sigma_{\xi} \log \left[ \sum_{\xi' \in \{0,1\}} \exp \left[ \frac{\max_{\xi} \{u(d', e', s') + \phi(1)EV(d', e', s')\}}{\sigma_{\xi}} \right] \right] d\xi'.$$

Then, the choice probability of stay-or-leave, \(\pi_{\xi \in \{0,1\}}\), conditional on the agent’s state, is obtained by solving the agent’s dynamic optimization problem.
\[ \pi_{it} = \Pr(d_t \mid s_t) = \frac{\exp\left(\frac{(\max_{e \in \mathcal{E}}\{u(d, e, s) + \phi(1)\EV(d, e, s)\})}{\sigma_{e, i}}\right)}{\sum_{e \in \mathcal{E}} \exp\left(\frac{(\max_{e \in \mathcal{E}}\{u(d, e, s) + \phi(1)\EV(d, e, s)\})}{\sigma_{e, i}}\right)}. \] (9)

In the process, the optimal level of effort \( e_{it} \), given agent \( i \)'s state in period \( t \), is inferred by the level at which the expected value function is maximized. The attained effort enters the performance response function in Equation (2).

By combining Equation (2) and Equation (9), one can compute the likelihood of the agent’s observations. Given the history (data) of an agent with observations over \( T \)-periods, the agent’s likelihood is

\[
L_i(\Omega; q_i, d_i, s_i) = \prod_{t=1}^{T} \left( \phi_{e, i}(\ln(\hat{q}_{it}) - \ln(q_{it})) \cdot \pi_{it}^{d_i} \cdot \pi_{0it}^{(1-d_i)} \right),
\]

where the vector \( \Omega = \{ \delta_i, \theta_i, \gamma_i, \theta_f, \rho, \sigma_e, \alpha, \sigma_{\xi} \} \) is the set of parameters of the time preferences, utility and performance response functions; \( \hat{d}_{it} \) denotes the observed stay-or-leave decision, and \( \hat{q}_{it} \) is the observed per-period performance for agent \( i \) at time \( t \); and \( \phi_{e, i} \) denotes the probability density function of a normal distribution with mean zero and variance \( \sigma_{e, i}^2 \).

### 6.2. Unobserved Heterogeneity

Discrete segments accommodate unobserved heterogeneity (Kamakura & Russell, 1989). Assume that salesperson \( i \) belongs to one of \( K \) segments \( k \in \{1, \ldots, K\} \), with relative probabilities

\[ m_k = \frac{\exp(\lambda_k)}{\sum_k \exp(\lambda_k)}. \]

Let \( L_{ait} = L(\Omega_k \mid k; q_{it}, d_{it}, s_{it}) \) be the likelihood of parameters for individual \( i \) at time \( t \), conditional on unobservable segment \( k \), given the agent’s data. Then, the likelihood of the segment-level parameters upon observing an individual’s history is

\[ L_k(\Omega_k; q_i, d_i, s_i) = m_k \left( \prod_{t=1}^{T} L_{ait} \right). \]

By summing over all of the unobserved states \( k \in \{1, \ldots, K\} \), the overall likelihood of individual \( i \) becomes:
\[ L(\Omega; q_i, d_i, s_i) = \sum_{k=1}^{K} L_k(\Omega_k; q_i, d_i, s_i), \]

where \( \Omega = \{\Omega_1, \ldots, \Omega_k\} \) contains the segment-level parameters. Hence, the log-likelihood over the \( N \) sample of individuals becomes:

\[
\sum_{i=1}^{N} \log \left( L(\Omega; q_i, d_i, s_i) \right) = \sum_{i=1}^{N} \log \left( \sum_{k=1}^{K} m_k \left( \prod_{t=1}^{T} L_{ikt} \right) \right).
\]

7. Results

This section presents the results in the following order. First, we show the results of the exponential and quasi-hyperbolic discounting models and discuss their implications. Then, we show how the changes in compensation plan has led to sales force selection across heterogeneous agents. Next, we show the results of counterfactual simulations that addresses the substantive questions of this study—how sales management instruments (compensation, recruiting/termination, and training policies) affect the performance and selection of salespeople. Finally, we compare actual performance and turnover—with real changes in the firm’s sales management instruments—in the post-data-analysis period with simulated outcomes—accompanying the changes—to validate the accuracy of our model.

7.1. Parameter Estimates

Table 5 shows the parameter estimates of the exponential and quasi-hyperbolic discounting models. Based on the Bayesian information criterion (BIC), the three-segment model shows the best fit.\(^1\)

Regarding time preferences, in the exponential discounting model, the discount factor \( (\delta) \) is 0.895, 0.975, and 0.983, respectively, for segments 1, 2, and 3. The range of the standard discount factor is consistent with the behavioral and empirical studies on time preferences (Frederick et al., 2002; Yao et al., 2012; Chung et al., 2014). In the quasi-hyperbolic discounting model, the standard discount factor \( (\delta) \) is 0.996, 0.976, and 0.994, and the present-bias factor \( (\beta) \) is 0.477, 0.999, and

\(^1\) The BIC values for one- and two-segment quasi-hyperbolic discounting models are 11,793.79 and 11,169.31, respectively.
0.980, respectively, for segments 1, 2, and 3. **Figure 4** depicts a graphical illustration of time preference (and, thus, the amount of discounting towards the future) by segment. The solid lines represent exponential and the dotted lines represent quasi-hyperbolic discounting. Segment 1 shows myopic and present-biased behavior, while segments 2 and 3 show forward-looking and time-consistent discounting behavior.

The quasi-hyperbolic discounting model is a more general model than the exponential discounting model. Furthermore, the BIC value of the two models indicates that the quasi-hyperbolic discounting model fits the data better, implying that some sales agents are present-biased in their time preferences. Hence, we use the results of the quasi-hyperbolic discounting model for inference. For the structural parameters of the utility function, the disutility parameter ranges from 0.556 to 23.911. The disutility parameter is small for segment 3 (hereafter referred to as the high type), representing the agents' ease and flexibility in exerting effort. Conversely, the estimate is large for segments 1 and 2 (hereafter referred to as the low and moderate types). Hence, the following pattern appears in terms of segmentation. Segments 2 and 3 exhibit forward-looking behavior, which is expected of moderate-to-high-performing agents who seek the end-of-year bonus and commissions. The low-type agents, on the other hand, show myopic behavior.

The reservation value is low for segments 1 and 2, reflecting limited outside opportunities for these type of agents, and high for segment 3, implying the various potential opportunities outside the firm. For reservation value shifters, education and tenure are statistically insignificant, potentially reflecting the nature of personal selling, where interpersonal and relational skills are of greater importance than such observable characteristics.20

Regarding the parameters of the performance response function, tenure (both the mid-level and the senior dummy variables) has a positive effect on performance. In addition, training improves performance; however, the interaction effect of mid-level and senior dummies with training is

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20 In addition, tenure is within the focal firm, which may lead to underestimating its effect on the outside option (compared to industry tenure).
negative and significant. Hence, training does not benefit the senior salespeople as much as the junior salespeople.

7.2. Selection

The change in the firm’s compensation plan likely has led to the selection of its sales force. **Table 6** shows the share of the three segments and their descriptive characteristics, across the full three years of the data. Segment 1, the myopic low type, has the smallest share of 13.29%; segments 2 and 3 represent bigger shares of 64.24% and 22.47%, respectively. Consistent with the parameter estimates in **Table 5**, segment 3 achieves the highest performance, with the largest portion meeting the annual quota. The average base salary is higher for this segment, reflecting their tenure. Segment 2, the forward-looking moderate type, falls short on performance compared to segment 3 and has lower tenure. The myopic low type, segment 1, falls short in every performance dimension and exhibits a stark difference in annual variable pay compared to the other segments.

In terms of selection induced over time, **Table 7** shows the segment portfolio (in percentages) of the total sales force by each year-end—reflecting how the portfolio of salespeople has changed over time. The share of segment 2, the forward-looking moderate type, has constantly increased. In contrast, the shares of segments 1 and 3 have decreased over the years. This reflects the frequent quitting within these segments, likely due to insufficient compensation from lack of productivity for segment 1, and good outside opportunities for segment 3.

7.3. Counterfactual Simulations

This section shows the results of several counterfactual simulations that address the key substantive question of this study: how to manage, motivate, and sustain a healthy sales force using the sales management instruments outlined in **Figure 1**. The counterfactuals evaluate agents’ performance and selection according to changes in: (i) compensation structure, (ii) training hours, and (iii) recruiting and termination policies.

The basis of the counterfactuals suppose that the firm is undertaking its annual compensation plan design at the beginning of 2018 (i.e., following the data observation period) with its remaining portfolio of salespeople (N=400). The changes are from the 2017-plan components. For each new
regime, we simulate 200 paths per each individual-segment pair, using the parameter estimates of the quasi-hyperbolic discounting model. Then, we allocate individuals into each segment, based on segment probabilities. Finally, we aggregate performance and selection.

### 7.3.1. Alternative Compensation Structures

The challenge in designing a compensation plan is to determine the optimal ratio of fixed and variable pay. The theory predicts that when a firm increases the portion of fixed pay, employee turnover will likely decrease. But how would heterogeneous salespeople react differently to the change in terms of both performance and selection? Hence, the first counterfactual exercise examines a change in fixed versus variable pay, while keeping other components constant. Table 8 depicts the performance, turnover, and compensation amount outcomes of the counterfactual simulations under the new regimes.

First, we increase the base salary by 5, 10, and 15% and keep everything else constant. As anticipated, the turnover rate decreases across all segments. However, a notable aspect is that sales productivity also decreases. This is driven mainly by the retention effect: being granted higher rent, the low-performing agents, who otherwise would have left the firm, are now more likely to stay with the firm. The retention effect is greater for the low types, reflected by the more pronounced decrease in productivity. Next, we increase the bonus amount by 5, 10, and 15% and keep everything else constant. Again, employee turnover decreases; however, compared to the case of the increase in base salary, the reduction is smaller. Moreover, the effect on productivity is positive, especially with the high-type agents, as an increase in the bonus amount helps motivate these agents to a greater extent.

The experiment demonstrates the trade-off between adjusting fixed versus variable pay on performance and selection. While increasing the fixed salary could serve as a simple remedy to reduce employee turnover, this could, on average, hurt the overall performance of the sales force. In contrast, an increase in variable pay does not harm performance but has a smaller effect on employee turnover. The effect of policy changes applies heterogeneously across segments, which affects the resulting portfolio of the remaining salespeople.
7.3.2. Sales Training

The next counterfactual simulation involves changes in sales training. As discussed in Section 7.1, sales training positively affects productivity, but mostly for junior salespeople. The increase in performance affects not only current period utility but also future outcomes, which brings about changes in the dynamic optimization of effort over time. To evaluate the role of sales training in performance and selection, we provide 6, 12, and 24 hours of sales training (cost-wise equivalent to 1.25, 2.5, and 5% annual salary increase, respectively) in January for all agents. Table 9 shows the results.

As anticipated, sales training leads to increased performance across all segments. In addition, the employee turnover rate decreases across all segments. Overall, the general trend of providing sales training is similar to that of increasing the bonus amount—training helps agents obtain better performance, which, in turn, raises the probability of attaining the bonus (evidenced by the increase in compensation amount). Hence, the counterfactual results shown in the past two subsections provide a practical tool for firms to analyze the cost-benefit evaluation of compensation and training.

7.3.3. Recruiting and Termination Policy

A firm can induce selection of the sales force through its recruiting and termination policy. We consider two cases: (i) changes in the firm’s recruiting policy; and (ii) changes in its termination policy.

The recruiting policy relates to the type of salespeople that the firm should target during its hiring process. First, suppose the firm can observe the agents’ latent types. Should the firm focus on targeting the high types, who are more likely to be skilled but require greater compensation to keep? Alternatively, should the firm target the moderate or low types at lower costs? To evaluate the outcomes of the recruiting policy, we simulate the firm to hire 50 of each type (low, moderate, and high), and compare the differences in performance and turnover over a five-year horizon. Figure 5a depicts the annual performance (solid lines) and cumulative turnover rate (columns) by
As anticipated, the high types show better performance than the low types. However, the high types’ high performance comes at a cost: these agents are more apt to depart the firm due to better outside opportunities, leaving their territories vacant. Hence, in Figure 5b, we report the turnover-adjusted performance, which accounts for the territory vacancy (treated as zero outcome). Although in the short run, hiring high-type agents leads to greater performance, in the long run, territory vacancy can be detrimental to the firm’s objectives. Therefore, without any changes in effort to retain high-type salespeople, simply recruiting a large number of them can have limited positive effects on the firm’s productivity.22

The above experiment, however, is not directly applicable in practice—as firms cannot observe the agents’ hidden types. Hence, we examine a scenario where the firm possesses information about the candidates’ tenure, which provides ex-ante understanding of the agents’ experience and, thus, the underlying type. The dilemma is whether to poach the rival’s experienced salespeople, who are more likely to be pre-equipped with sales techniques but require greater compensation to hire, or to target unexperienced rookies, who require a lower base salary with the potential to be trained from the outset of their career.23 Hence, to answer the question, we simulate the firm to hire 50 salespeople—either experienced (tenure 3-7 years) or rookies (tenure 0-2 years).24 To capture the nurturing opportunity, the firm provides the rookie salespeople with a 24-hour sales training each year (cost-wise equivalent to the salary difference with experienced salespeople). Figure 6a shows the performance and turnover results. As expected, experienced salespeople perform better than rookies in the short run. However, the gap narrows as sales training accumulates for the rookies, and they eventually outperform the experienced. Further, this gain in productivity lowers the turnover rate of the rookies, and, as shown in Figure 6b, rookie salespeople exhibit better net

21 The performance figures tend to be lower than the segment characteristics in Table 6, as the respective new hires have zero tenure and no training.
22 This counterfactual exercise can also be viewed as the impact of not targeting any particular type of salespeople during the recruitment process. We thank the Associate Editor for providing this intuition.
23 Whether to recruit experienced versus rookie salespeople was one of the main concerns for the firm.
24 To compute the segment probabilities conditional on tenure, we apply the Bayes’ theorem. For example, the probability of an experienced salesperson to belong in segment 3 is given by Pr(Segment 3 | Mid-level) = Pr(Mid-level|Segment 3)Pr(Segment3)/Pr(Mid-level) = 0.284/0.226/0.244 = 0.264. An implicit assumption is that the tenure within the focal firm reflects the sales force characteristics at the industry-level.
productivity (i.e., turnover-adjusted performance) by the fifth year. Therefore, a firm should consider the outcome priority (e.g., short- vs. long-term performance) and the associated efforts (e.g., nurturing vs. retention) when setting its recruitment policy.

On the flip side of recruiting is the firm’s policy for terminating its salespeople. In most nations, including Turkey, firm-initiated employee termination (layoff) is limited due to labor force regulations. Hence, to terminate a salesperson by discretion, the firm must provide a leave package that the person will agree to. We evaluate the effect of a leave package by providing a lump sum of $4,500, $9,000, and $18,000 (equivalent to 3-, 6-, and 12-months base salary, respectively) that agents can opt into. **Table 10** shows the results that the leave package mostly affects the low- and moderate-type agents (segments 1 and 2), as the marginal value of the package is higher for these segments. The average firm productivity increases, as agents with less potential are the ones mainly to accept the package and leave. Hence, the termination policy counterfactual reveals that strategically providing leave packages can potentially lead to better outcomes and to the firm’s desired selection of salespeople.

### 7.4. Field Validation

In the beginning of 2018, the focal firm, based on the results of the above counterfactual analyses, decided to raise its bonus amount by 20% and to offer an additional 12 hours of training (6 hours each in the first and second halves of the year). To validate the accuracy of our model, we obtain the performance and turnover records under the new regime (January-June 2018), and compare the actual data and the counterfactual outcomes, projected based on changes in the sales management instruments.

**Figure 7** compares the actual (solid line) and projected (dotted black line) performance outcomes over the 6-month period. The model simulation projects the general trend, though a bit less cyclical. Overall, the projected monthly performance results fit the actual outcomes well with a mean absolute percentage error of 3.74%. In terms of employee turnover, the model predicts that 11 salespeople would leave the firm during the 6-month period. In reality, the actual departure occurred from 9 salespeople.
The comparison shows the competence of the model to predict and, thus, evaluate the outcomes under a new policy that includes multiple sales management instruments. We also simulate performance outcomes in case the firm had not made any changes (i.e., keep the 2017 plan) to its sales management instruments. The results (dotted gray line) show that the firm’s performance would likely have been lower throughout the period if not for the change.

Organizations should approach with caution when changing their sales management policy as it can be quite costly. The cost includes not only the direct cost of amending administrative functions but also opportunity costs and the cost of “getting it wrong.” For example, when a firm initially gives a bonus but takes it away later, salespeople’s performance can be lower than having not given the bonus in the first place, because of erosion in intrinsic motivation (Lepper et al., 1973; Chung and Narayandas, 2017). In addition, the firm’s management can lose credibility with its employees, when management policies repeatedly change. Hence, the framework and model of this study provide rigorous yet practical means for firms to foresee the result of a change in alternative sales management policies.

8. Conclusion

Managing a sales force is an intricate task with multidimensional outcomes. If properly managed, firms can induce greater performance from their sales force while keeping their top performers from leaving the firm. This study develops and estimates a dynamic structural model of a comprehensive response to multiple sales management instruments, including compensation, training and recruiting/termination policies. The agent’s model takes into consideration many elements that constitute a realistic working environment—allocation of continuous effort; forward-looking behavior, including present-bias; effectiveness of sales training; and employee attrition. Substantively, the study provides guidance to firms on (i) evaluating the differential outcomes of various compensation policies; (ii) assessing the selection of different types of employees regarding changes in the recruiting and termination policies; and (iii) addressing the value of sales force training.

A summary of the study’s results are as follows. An increase in fixed salary positively affects employee retention but may decrease aggregate sales because low-type agents, who otherwise would
have left the firm, are likely to stay. In contrast, an increase in variable pay enhances sales productivity but has limited effect on employee retention. Because of the focal firm’s selection process over time, high performers have steadily left the firm, while mid-to-low performers have remained. However, if the firm were to focus mainly on recruiting high-performing experienced salespeople, sales would increase in the short-term but would likely decrease in the long-term due to the territory vacancies created by salespeople’s attrition. Hence, firms should focus on retention efforts along with their recruiting efforts of high performers. In addition, providing adequate leave packages can lead to an appropriate selection of salespeople to maintain a healthy sales force. Furthermore, sales training, a novel management instrument that both academics and practitioners have often overlooked, is an effective long-term performance driver that aids salespeople in their early careers to improve their performance, which, in turn, helps the firm increase sales performance and employee retention. A field validation, comparing post-analysis actual and counterfactual outcomes, verifies the accuracy of the model in this study. The field validation supports the practical applicability of the model in the real-world—as means that can predict changes in behavior (and, thus, sales and employee attrition outcomes) under various sales management policies, using multiple instruments.

Methodologically, the study introduces a new insight to the marketing and economics literature by providing formal proof regarding the identification of discount factors in a hyperbolic discounting model, accompanying continuous and unobserved choices. The key to identification is the aggregation of performance outcomes over a specific time horizon when evaluating compensation: an agent’s distance-to-quota for obtaining a bonus payment (in non-bonus periods) serves as an exclusion restriction that affects only future utility and not current utility. The study provides conditions under which both an exponential and a hyperbolic discounting model are identified, and through the empirical application, find evidence of present-bias in salespeople’s behavior.

This study has some limitations that open up avenues for future research. First, it does not consider multidimensional effort regarding different products (Chung et al., 2019) or customer types (Kim et al., 2019), whereby agents could exhibit dynamic substitution across products and customers or both. For example, in the early periods of a quota-evaluation cycle, an agent might focus on high-
ticketed products that, if sold, could satisfy a large portion of his or her quota. However, as periods pass, an agent might gradually shift to low-ticketed and easy-to-sell products. Second, free goods as a sales promotion tool, which is common in the pharmaceutical industry, can induce additional dynamics in a sales agent’s behavior. While free goods reduce the agent’s short-term returns on performance, they can induce greater long-term outcomes by building a stronger relationship with a customer. Relatedly, an agent’s effort, in addition to the immediate short-term effect, can also have a long-term effect on sales through augmented customer relationships. Lastly, this study considers time-invariant unobserved heterogeneity; however, time-variant unobserved factors (Arcidiacono and Miller, 2011; Hu and Shum, 2012; Chou et al., 2019) may affect the agent’s effort decision. Although not addressed in this research due to data limitations and model parsimony, the abovementioned topics would provide exciting avenues for future research.

In summary, this research offers a comprehensive, practical, yet rigorous application to understand the roles of multiple sales management instruments—compensation, training, recruiting and termination—in the selection and performance of salespeople. We believe that the results will guide firms in their sales management practices to help recruit, compensate, train, and, thus, maintain a healthy sales force to achieve their desired outcomes.


Appendix

A. Proof of Proposition 1. Because the agent chooses effort after observing the state variables $(s_{it})$, individual heterogeneity ($\alpha_i$), and the utility shock ($\varepsilon_{dit}$), his or her optimal effort policy $e_{it}$ is a function of $s_{it}$, $\alpha_i$, and $\varepsilon_{dit}$ (i.e., $e_{it}=e(s_{it}, \alpha_i, \varepsilon_{dit})$). However, $\varepsilon_{dit}$ does not affect effort because it is invariant to the effort choice conditional on the stay-or-leave decision, $d_{it}$. Hence, the performance response function can be represented as
\[
\ln(q_{it}) = \alpha_i + e(s_{it}, \alpha_i) + \xi_{it},
\]
where $q_{it}$ and $s_{it}$ are observed, but $\alpha_i$ and $\xi_{it}$ are not.

By Assumption 1, when $s_{it} \in S_{\alpha}$, the value function is a decreasing function of effort. Hence, the optimal effort is zero (i.e., $e(s_{it}, \alpha_i)=0$ for $s_{it} \in S_{\alpha}$). Therefore, we have $\ln(q_{it}) = \alpha_i + \xi_{it}$. Independence between $\xi_{it}$ and $s_{it}$ implies
\[
E(\ln(q_{it}) | s_{it} \in S) = \alpha_i + E(\xi_{it} | s_{it} \in S) = \alpha_i,
\]
from which $\alpha_i$ is identified.

Once $\alpha_i$ is identified (from observations $s_{it} \in S_{\alpha}$), the performance response (when $s_{it} \not\in S_{\alpha}$) takes the form of a nonparametric regression with a known intercept: $e(s_{it}, \alpha_i)$ is a regression function of $\ln(q_{it}) - \alpha_i$ on $s_{it}$ and $\alpha_i$. Thus, the optimal effort $e_{it}$ is identified from $E(\ln(q_{it}) - \alpha_i | s_{it}, \alpha_i)$ using nonparametric regression methods. The distribution of the residuals is a consistent estimator for the distribution of $\xi_{it}$.

(Q.E.D.)

B. Proof of Lemma 1.\textsuperscript{25} Consider an agent with $(s_{it}, \varepsilon_{0it}, \varepsilon_{1it})$. The agent chooses to stay with the firm only if
\[
v(1, e, s_{it}) + \varepsilon_{1it} \geq v(0, 0, s_{it}) + \varepsilon_{0it}.
\]

\textsuperscript{25} The proof builds upon that of Lemma 1 in Magnac and Thesmar (2002).
Although $\varepsilon_{0it}$ and $\varepsilon_{1it}$ are unobserved, their joint distribution is assumed to be known. Thus, the probability of staying with the firm can be written as

$$
\Pr(d_i = 1 \mid s_i) = \Pr(v(1, e, s_i) - v(0, 0, s_i) \geq \varepsilon_{0it} - \varepsilon_{1it})
= F_{\varepsilon_{0it}}^{-1}(v(1, e, s_i) - v(0, 0, s_i)),
$$

where $F_{\varepsilon_{0it}}(.)$ is the cumulative distribution function of $\varepsilon_{0it} - \varepsilon_{1it}$. As the probability of staying with the firm can be computed from the observable data $(d_i)$, one can obtain the difference in choice-specific value function via

$$
v(1, e, s_i) - v(0, 0, s_i) = F_{\varepsilon_{0it}}^{-1}(\Pr(d_i = 1 \mid s_i)).
$$

(Q.E.D)

C. Proof of Proposition 2.\textsuperscript{26} The value functions at states $(s_i \in S_1, s_2)$ and $(s_i \in S_1, \tilde{s}_2)$, which satisfy Assumption 2, can be evaluated such that

$$
v(d, e, s_1, s_2) = u(d, e, s_1, s_2) + \delta E[\max_{d', e'} \{v(d', e', s') + \varepsilon_i'\} \mid d, e, s_1, s_2],
$$

(A.1)

$$
v(d, e, s_i, \tilde{s}_2) = u(d, e, s_i, \tilde{s}_2) + \delta E[\max_{d', e'} \{v(d', e', s') + \varepsilon_i'\} \mid d, e, s_i, \tilde{s}_2].
$$

(A.2)

From Lemma 1, $v(1, e_0, s_i)$ is identified up to location and scale. Therefore, the difference between Equations (A.1) and (A.2) is identified up to scale. By the exclusion restriction, subtracting Equation (A.2) from Equation (A.1) cancels out the per-period utility:

$$
v(d, e, s_1, s_2) - v(d, e, s_i, \tilde{s}_2) = \delta \left\{E[\max_{d', e'} \{v(d', e', s') + \varepsilon_i'\} \mid d, e, s_1, s_2] - E[\max_{d', e'} \{v(d', e', s') + \varepsilon_i'\} \mid d, e, s_i, \tilde{s}_2]\right\}.
$$

Because the unidentified scale parameter of the value function is cancelled out, the difference in the expected future utility can be computed utilizing the identified difference of value functions and the law of motion. Therefore, the discount factor $\delta$ is the only unknown in this equation, and is uniquely identified. The per-period utility $u$ is identified from either Equation (A.1) or (A.2), given the value function and the discount factor. The scale of the per-period utility function is normalized by the functional form of the mean-variance utility.

(Q.E.D.)

\textsuperscript{26} The proof uses a similar argument in the proofs for Corollary 3 and Proposition 4 in Magnac and Thesmar (2002).
D. Sufficient Conditions for Assumption 3 (Rank Condition). Observe that the derivatives of $\Pi$ with respect to the parameters are given by

$$\frac{\partial \Pi(x | \omega)}{\partial \theta} = -c^2 - \delta(\beta - 1) \int \left\{ (e')^2 \cdot \exp \left[ \frac{(\beta-1)\mu}{\beta} u(1, e', s' | \mu) + \frac{1}{\beta} v(1, e', s') \right] \right. \left\} f(s' | d, e, s) \right\} ds'$$

$$\frac{\partial \Pi(x | \omega)}{\partial \gamma_0} = 1 + \delta(\beta - 1) \int \left\{ \exp \left[ \frac{(\beta-1)\mu}{\beta} u(1, e', s' | \mu) + \frac{1}{\beta} v(1, e', s') \right] \right. \left\} f(s' | d, e, s) \right\} ds'$$

$$\frac{\partial \Pi(x | \omega)}{\partial \gamma_1} = \text{E}(W | d, e, s) + \delta(\beta - 1) \int \left\{ \text{E}(W' | d', e', s') \cdot \exp \left[ \frac{(\beta-1)\mu}{\beta} u(1, e', s' | \mu) + \frac{1}{\beta} v(1, e', s') \right] \right. \left\} f(s' | d, e, s) \right\} ds'$$

$$\frac{\partial \Pi(x | \omega)}{\partial \gamma_2} = -\text{Var}(W | d, e, s)$$

$$\frac{\partial \Pi(x | \omega)}{\partial \delta} = \int \left\{ \Lambda(s' | \mu, \beta) f(s' | d, e, s) \right\} ds'$$

$$\frac{\partial \Pi(x | \omega)}{\partial \beta} = \int \left\{ u(1, e', s' | \mu) \cdot \exp \left[ \frac{(\beta-1)\mu}{\beta} u(1, e', s' | \mu) + \frac{1}{\beta} v(1, e', s') \right] \right. \left\} f(s' | d, e, s) \right\} ds'.$$

The following conditions are sufficient for Assumption 3 (Rank Condition):

(i) For any $x$ and $\tilde{x}$ in $X$, there is a first-order stochastic dominance relationship between $f(s' | x)$ and $f(s' | \tilde{x})$.

(ii) $\text{E}(W | d, e, s)$ and $\text{Var}(W | d, e, s)$ are weakly monotone in $s$, but are not constant functions on $X$.

(iii) $e'$, $\text{E}(W | d, e, s)$, and $\text{Var}(W | d, e, s)$ are linearly independent on $X$.

(iv) $\Lambda(s | \mu, \beta)$ is strictly increasing in $s$ on $X$.

The above conditions are readily satisfied in the study’s empirical setting: condition (i) holds when $x$ only affects the mean of $s'$; conditions (ii) and (iii) are implied by the nonlinear structure of the compensation scheme; and condition (iv) follows from the extreme value distribution assumption.

If $f(s' | x)$ first-order stochastically dominates $f(s' | \tilde{x})$, we have that $\int (a(s')) f(s' | x) ds' \geq \int (a(s')) f(s' | \tilde{x}) ds'$ for any weakly increasing function $a$. Thus, conditions (i) and (ii) jointly imply
that the expected future payoffs vary across \( x \). The derivatives with respect to \( \theta, \gamma, \beta \) are linearly independent by condition (iii). By condition (iv), the derivative with respect to \( \delta \) is linearly independent of the other derivatives.

E. Proof of Theorem 1. Let \( \omega = (\mu, \delta, \beta) \) denote the vector of parameters, and suppose that \( \omega_0 \) is the true value of these parameters. The vector of true parameters \( \omega_0 \) is said to be locally identified if there exists a positive number \( \zeta \) such that no other parameter value \( \tilde{\omega} \neq \omega_0 \) satisfies \( \Pi(\tilde{\omega}) = 0 \) and \( \| \tilde{\omega} - \omega_0 \| < \zeta \). That is, \( \omega_0 \) is the unique solution to \( \Pi \) within a certain radius.

The true parameter \( \omega_0 \) solves \( \Pi(x \mid \omega_0) = 0 \) for all \( x \). Although \( x \) has infinite support, the information necessary for identification is up to the number of the parameters. Let \( \{x_1, x_2, \ldots, x_J\} \) be a subset of the support of \( x \) satisfying Assumption 3. Let us denote the equations evaluated at the subset by

\[
\pi(\omega) = \begin{pmatrix}
\Pi(x_1 \mid \omega) \\
\Pi(x_2 \mid \omega) \\
\vdots \\
\Pi(x_J \mid \omega)
\end{pmatrix}
\]

and \( \pi' = \frac{\partial \pi(\omega)}{\partial \omega} \). A sufficient condition for local identification of \( \alpha_0 \) is \( \text{rank}(\pi') = \dim(\omega) \) (Chen et al. 2014), which directly follows from Assumption 3.

(Q.E.D.)

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27 The local identification approach is the standard definition of identification in the economics literature (e.g., Chen et al., 2014). Global identification can be achieved by assuming that the second derivative is globally convex or concave.