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# Contractual Restrictions and Debt Traps\*

Ernest Liu<sup>†</sup> and Benjamin N. Roth<sup>‡</sup>

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## Abstract

Microcredit and other forms of small-scale finance have failed to catalyze entrepreneurship in developing countries. In these credit markets, borrowers and lenders often bargain over not only the interest rate but also implicit restrictions on types of investment. We build a dynamic model of informal lending and show this may lead to endogenous debt traps. Lenders constrain business growth for poor borrowers yet richer borrowers may grow their businesses faster than they could have without credit. The theory offers nuanced comparative statics and rationalizes the low average impact and low demand of microfinance despite its high impact on larger businesses.

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Capital constraints pose a substantial obstacle to small-scale entrepreneurship in the developing world. Experimental evidence from “cash drop” studies paints a remarkably consistent picture: across a broad range of contexts including Mexico, Sri Lanka, Ghana, and India, small-scale entrepreneurs enjoy a *monthly* return to capital in the range of 5%–10%.<sup>1</sup> Surprisingly, however, many experimental evaluations of microfinance find that it has only modest or even no impact on entrepreneurship.<sup>2</sup> This may be especially puzzling in light of the fact that the interest rates charged for microloans are well below the estimates of marginal return to capital. If credit is available, and interest rates are below entrepreneurs’ marginal return to capital, what prevents them from using it to pursue their profitable investment opportunities?

We address this puzzle through a theory that predicts that increasing access to credit can actually constrain entrepreneurship, and that these constraining effects may be strongest precisely when entrepreneurs have access to the most productive investment opportunities. We rely on two special features of informal credit markets for small-scale borrowers in the developing world.

First, we highlight that a borrower who successfully grows his business and builds a stock of pledgeable collateral may eventually gain access to a more active credit market and graduate from his informal lender. We further assume that long-term contracting is infeasible, so that a borrower and his informal lender suffer from a hold-up problem; while the borrower can commit to repay his current loan, he cannot commit to share any benefits he derives once he has stopped borrowing from his informal lender.

Second we assume that in the informal sector, the borrower and lender bargain not only over the interest rate, but also over a contractual restriction that determines whether the borrower can make investments in fixed capital or only in working capital. We assume that fixed capital investments help the borrower build his stock of pledgeable assets while working capital investments generate a consumption good but are not useful for growing the borrower’s business.

While microfinance institutions rarely contract over specific investments that a borrower must make, it is common for microfinance institutions to impose rigid requirements that loan repayments begin immediately after disbursement and in frequent installments thereafter. A growing body of experimental research finds that by relaxing this requirement and allowing borrowers to match their repayments to the timing of their cashflows, borrowers exhibit higher demand for credit, make longer-term investments, and see substantial and persistent increases in their sales and profits (e.g. [Field et al. \(2013\)](#), [Takahashi et al. \(2017\)](#), [Barboni and Agarwal \(2018\)](#), and [Battaglia et al. \(2019\)](#)).<sup>3</sup> Following

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<sup>1</sup>See e.g., [De Mel et al. \(2008\)](#), [Fafchamps et al. \(2014\)](#), [Hussam et al. \(2017\)](#), and [McKenzie and Woodruff \(2008\)](#).

<sup>2</sup>See [Banerjee et al. \(2015\)](#) and [Meager \(2017\)](#) for overviews of the experimental evaluations of microfinance.

<sup>3</sup>The insistence on early and frequent repayment is sometimes attributed to deterring default but there is little empirical support for this claim. Of the four studies cited above, only [Field et al. \(2013\)](#) finds an increase in default from allowing borrowers flexibility in the timing of repayment, and the additional default is quite small. That study reports that on

this empirical evidence, we assume that the fixed capital project takes longer to materialize output than does the working capital project, and that when a lender insists on initial payments early in the loan's tenure the borrower must choose his working capital project. We refer to contracts that insist on early repayment as *restrictive contracts* and those that allow for flexible repayment as *unrestrictive contracts*.

That the borrower may graduate from his informal lender generates an incentive for the informal lender to constrain the borrower's business growth. And that the lender can impose contractual restrictions on the borrower gives her the means to do so. We build a dynamic theory of informal lending on top of these two modeling ingredients to understand when a lender may hold her borrower captive.

We focus on the relationship between a single informal lender and her borrower. The borrower's outside option is to invest in either of his two projects without the help of the lender. The lender wishes to prolong the period over which she can extract rents from her borrower. By offering a restrictive contract, the lender can keep the borrower captive, but in doing so the lender must offer her borrower a relatively low interest rate to compensate him for forgoing the fixed capital project. In contrast, the borrower readily accepts a high interest rate for unrestrictive contracts, which help him to grow his business faster than he could have in the lender's absence.

Our first main result characterizes when the lender offers a restrictive contract in equilibrium and keeps her borrower captive. Contractual restrictions are inefficient; the lender imposes restrictions in order to inhibit borrower growth and retain future rents from lending, at the expense of joint surplus creation. Whether restrictions arise in equilibrium depends on the confluence of two factors: the borrower's bargaining position and the hold-up problem. The borrower's bargaining position reflects the borrower's option to reject the lending contract and instead operate his business under autarky. When the borrower's bargaining position is strong, the lender must offer low interest rates for the borrower to accept a restrictive loan. In contrast, unrestrictive loans always allow the borrower to grow his business faster than he could have alone and hence may exceed the borrower's outside option. The lender's payoff from unrestrictive contracts is thus insensitive to the borrower's bargaining position; consequently, borrowers with high bargaining positions tend receive unrestrictive contracts.

The hold-up problem pushes in the opposite direction. Because the borrower cannot commit to long-term contracts, the lender's value upon letting the borrower grow his business is determined by the speed at which the borrower can grow his business and the borrower's bargaining position at higher business sizes. Therefore the lender can extract less rent through unrestrictive contracts the higher is the borrower's bargaining position in the future and the faster he can grow his business through fixed capital investment.

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average, borrowers who received a flexible repayment contract defaulted on an extra Rs. 150 per loan. However three years later, these same borrowers earned on average an additional Rs. 450 to Rs. 900 *every week*.

We provide an analytic characterization of the probability the lender offers a restrictive loan at every borrower wealth level and show that the lender may offer restrictive contracts and stop the borrower from growing his business even when business growth is socially efficient and even when the borrower would have grown his business in autarky. This is the sense in which increasing access to credit may constrain entrepreneurship. Restrictive contracts are likely when the borrower can only grow his business very slowly on his own, so that his bargaining position is weak. Restrictive contracts are also likely when the borrower has access to very productive fixed capital investment opportunities that require the lender's capital, so that the hold-up problem limits how much rent the lender can extract through unrestrictive contracts. This may explain why microfinance has had so little impact on entrepreneurship despite, or perhaps because of, the high return to capital found amongst microentrepreneurs in the research cited above.

Our theory predicts that borrowers who are close to entering the formal sector will receive unrestrictive contracts. These are borrowers with strong bargaining positions, so lenders find it too costly to offer them restrictive loans they will accept. This helps reconcile a second pattern from the experimental literature on the impacts of microfinance, which finds that while on average microfinance has had little impact on entrepreneurship, relatively wealthier borrowers do enjoy business growth from microcredit.<sup>4</sup>

The model further yields nuanced comparative statics that shed light on the dynamic interlinkages of wealth accumulation. We show that improving the attractiveness of the formal sector unambiguously improves the welfare of relatively rich borrowers who are close to the formal sector. Increasing the attractiveness of the formal sector improves the bargaining position of these borrowers, who are resultantly less likely to receive restrictive contracts, and when they do the contracts are more generous. Yet, exactly the same phenomenon may reduce the welfare of poorer borrowers. Because of the hold-up problem, the very fact that borrowers have stronger bargaining positions if they grow larger reduces the rent the lender can extract from unrestrictive contracts to poorer borrowers, who may therefore become more likely to receive restrictive contracts and remain in captivity. Thus there is a “trickle-down” nature of the comparative statics in our model, highlighting that policies that seem to improve the welfare of some borrowers may backfire on the poorest borrowers they aimed to help.

Interpreting borrower captivity as a poverty trap, our model also offers a counterpoint to the standard intuition that poverty traps are driven by impatience. We show that increasing borrower patience relaxes the poverty trap for rich borrowers, yet higher patience may *amplify* the poverty trap for poorer borrowers, causing them to get trapped at even lower levels of wealth. This is again due to the “trickle down” effect whereby lenders react to richer borrowers becoming more demanding by tightening

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<sup>4</sup>See [Angelucci et al. \(2015\)](#), [Augsburg et al. \(2015\)](#), [Banerjee et al. \(2015\)](#), [Crepon et al. \(2015\)](#), and [Banerjee et al. \(2017\)](#). This is also consistent with abundant anecdotal evidence that MFIs relax contractual restrictions such as rigid repayment schedules for richer borrowers, although this fact may be explained by a number of other theories.

contractual restrictions on poorer borrowers and preventing their growth.

At this point it is useful to contrast our theory to the seminal work of [Petersen and Rajan \(1995\)](#) (PR henceforth), who also study financing with limited commitment and show that improving borrower's future outside option—modeled through intensified competition from other lenders—can make the borrower worse off. Taken at face value, the prediction that improvements in the formal sector may worsen the poverty trap for poor borrowers seems similar to the message of PR. However, the economic forces differ fundamentally. The result of PR follows from the fact that early-stage financiers may not be able to recoup the initial costs of lending if they must operate in a competitive market once their borrowers' businesses have grown. In our model, the lender is motivated by prolonging the period over which she can extract rents from her borrower. This difference manifests itself in two key ways.

First, in our model, borrowers always reach the formal sector under autarky; yet, in repeated lending relationships, lenders hold borrowers captive in order to prolong rent extraction. In contrast, borrowers in PR are only “trapped”—and their projects unfinanced—because the lender may refuse to lend. In other words, borrowers are trapped because of the *presence of lending* in our model and are trapped because of the *lack of lending* in PR.

A second conceptual distinction is that the severity of the poverty trap in our model depends crucially on borrower's bargaining position. In PR, intensifying future competition always worsens the initial financial friction. In contrast, improvements in the formal sector in our model may actually alleviate the trap for borrowers with strong bargaining positions. Intuitively, when borrowers have strong bargaining positions, the lender cannot extract substantial rents by keeping them in captivity. Thus improving access to the formal sector always reduces the likelihood of captivity for these borrowers. We expand on these points in Section [3.1](#).

Our theory offers a novel explanation for why credit may have low impact on entrepreneurship. While no one theory is likely to be solely responsible, we argue that our model is distinct from existing theories for several reasons.

First, as documented above, the marginal return to capital microentrepreneurs typically demonstrate is substantially above the interest rates charged by microfinance institutions. Because many entrepreneurs have ready access to microcredit, a theory that explains why microcredit has not catalyzed entrepreneurship must appeal to some feature of the loans beyond the transfer of capital and the interest rate. Our theory highlights that microcredit contracts are multidimensional and that constraints imposed by the lender may limit the usefulness of these loans for making long-term investments.

Theories that rationalize contractual restrictions based on deterring default arising from adverse selection and moral hazard can explain these constraints if they serve to distinguish amongst different

types of borrowers or discipline their behavior (eg., [Ghatak \(1999\)](#) and [Banerjee et al. \(1994\)](#)). But as we argued above, we do not think the empirical evidence supports these stories.

Our paper contributes to the literature on debt traps resulting from limited pledgeability. The topic has a long tradition in the literature on development finance (e.g., [Bhaduri \(1973\)](#), [Ray \(1998\)](#)). The inefficient contractual restrictions in our model arise due to lender’s desire and ability to capture future rents; hence, so long as market power is present, the inefficiency we identify and analyze could be relevant in developed financial environments as well ([Drechsler et al. \(2017\)](#), [Scharfstein and Sunderam \(2017\)](#)). The dynamic inefficiency in our model stands in contrast to other papers that study lending inefficiencies arising from information frictions (e.g., [Stiglitz and Weiss \(1981\)](#), [Dell’Ariccia and Marquez \(2006\)](#), [Fishman and Parker \(2015\)](#)), common agency problems (e.g., [Bizer and DeMarzo \(1992\)](#), [Parlour and Rajan \(2001\)](#), [Brunnermeier and Oehmke \(2013\)](#), [Green and Liu \(2017\)](#)), and strategic default (e.g., [Breza \(2012\)](#)). Our work also relates to [He and Xiong \(2013\)](#), who analyze restrictive investment mandates in the context of asset management. Finally, [Donaldson et al. \(2019\)](#) study how different varieties of intermediaries emerge due to differences in funding costs and lack of commitment. Their paper also features borrower captivity due to inefficient project choice. The project choice is made by the entrepreneur in [Donaldson et al. \(2019\)](#); by contrast, the lender in our model effectively makes the choice through contractual restrictions. Because the lender must respect the borrower’s individual rationality constraint reflecting the borrower’s ability to choose an inefficient version of either project, our analysis yields a different set of predictions about when a lender may hold her borrower captive.

The rest of the paper proceeds as follows. In [Section 1](#) we describe the baseline model in which borrowers have only two business sizes—one in which they interact with their informal lender and one in which they are in the formal sector. [Section 2](#) characterizes the equilibrium of this game. [Section 3](#) extends the model so that the borrower has several business sizes in which he only has access to the informal lender, and discusses how the bargaining position of relatively richer borrowers influences the welfare of poorer borrowers when subjected to a forward-looking lender. [Section 4](#) summarizes the relationship of our results with existing empirical evidence and concludes. The appendix contains several model extensions including a variant of the model with continuous states, a discussion of institutional features of microfinance that aren’t captured by our model, and all of the proofs.

## 1 The Baseline Two-State Model

**Players, Production Technologies, and Contracts** We study a dynamic game of complete information and perfectly observable actions. There are two players, a borrower (he) and a lender (she), both of whom are risk neutral. Each period lasts length  $dt$  and players discount the future at rate  $\rho$ . For analytical convenience we study the continuous time limit as  $dt$  converges to 0; a period is

therefore an instant. Each period is subdivided into an early and late portion of the period, though no discounting happens within a period. The game has two states  $w \in \{1, 2\}$ . State 1 is the only active state. If the game reaches state 2 it ends, with both parties receiving fixed payoffs described below.

Within each period the borrower has access to two production technologies: a working capital project and a fixed capital project. Due to limited attention he must choose only one project within each period.

His working capital project transforms resource inputs into consumption goods, and has two scales. Without any outside capital (i.e. with only the borrower's labor), the borrower can produce  $y^{aut}$  consumption goods under autarky. With  $\kappa$  units of outside capital the borrower can produce  $y$  goods, with  $y - \kappa > y^{aut}$ . We assume that the working capital project produces output in the early portion of the period.

The fixed capital project transforms resources into consumption goods, once again with two scales at rates  $y^{aut}$  and  $y$ , but with two differences from the working capital project. First, we assume that the fixed capital project takes more time—hence output is only realized in the late portion of the period. Second, the fixed capital project offers the entrepreneur the possibility to grow his business. Without outside capital, the borrower's business grows to state 2 at the end of the period with probability  $g^{aut} dt$ . With  $\kappa$  units of outside capital, his business grows to state 2 with probability  $g dt$ , where  $g > g^{aut}$ . With the complementary probability the borrower's business does not grow and the state remains at  $w = 1$ .

Over each instant, the lender makes a take-it-or-leave-it offer of a loan contract  $c = \langle R, a \rangle \in \mathcal{C} \equiv \mathbb{R}^+ \times \{0, 1\}$ , which specifies an upfront transfer of  $\kappa$  to the borrower,  $R$  is the (contractable) repayment from the borrower to the lender, and  $a$  is the contractual restriction.

If the borrower rejects the contract, the borrower chooses one of the two projects to perform without any outside capital.

If the borrower accepts the contract, the lender transfers  $\kappa$  to the borrower, and the borrower must repay  $R$  to the lender. If the contract specifies  $a = 1$ , the lender insists on being repaid in the early portion of the period. As such the borrower must choose the working capital project. If instead the contract specifies  $a = 0$ , then the borrower is free to repay the lender in the late portion of the period, and as such can choose either of his projects. We refer to contracts which specify  $a = 1$  as *restrictive* loans and those that specify  $a = 0$  as *unrestrictive* loans.

Regardless of whether the contract is accepted or rejected, the two players meet again in the next instant and a new contract is proposed unless the terminal state  $w = 2$  is reached.

If the game ever reaches the terminal state, both players cease acting. The borrower receives a

continuation payoff  $U$  (understood as the payoff resulting from reaching the formal sector), and the lender receives a payoff of 0 (having lost her customer). Though the borrower always repays his loans, he cannot commit to share his proceeds in state 2.

We assume that the borrower cannot pledge his continuation payoff or his growth prospects to the lender; that is, the maximal repayment to the lender is the entirety of the consumption good generated by his investments,  $R \leq y$ .

### Timing and Summary of Setup

1. The lender makes a take it or leave it offer  $c = \langle R, a \rangle$  to the borrower.
2. The borrower decides whether to accept the contract.
  - (a) If he rejects the contract, he chooses one of his two projects.
    - i. The lender's flow payoff is 0 and the borrower's flow payoff is  $y^{aut}$ .
    - ii. If the borrower invested in fixed capital then his business grows from state 1 to 2 with probability  $g^{aut} dt$ , and remains constant otherwise.
  - (b) If he accepts the contract, the lender transfers  $\kappa$  to the borrower. If the contractual restriction is  $a = 1$  then he must invest in the working capital project. Else he can choose either project. The borrower then repays  $R$  to the lender.
    - i. The lender's flow payoff is  $R - \kappa$ . The borrower's flow payoff is the output of the consumption good, net of repayment,  $y - R$ .
    - ii. If the borrower invested in fixed capital then his business grows from state 1 to 2 with probability  $g dt$ , and remains constant otherwise.
3. If the state is  $w = 1$ , the period concludes and after discounting the next one begins.<sup>5</sup>

**Equilibrium** Our solution concept is the standard notion of *Stationary Markov Perfect Equilibrium* (henceforth *equilibrium*)—the subset of the subgame perfect equilibria in which strategies are only conditioned on the payoff relevant state variables. An equilibrium is therefore characterized by the lender's state contingent contractual offers  $(R_w, a_w) \in \mathcal{C}$ , the borrower's state and contract-contingent accept/reject decision  $d_w : \mathcal{C} \rightarrow \{\text{accept, reject}\}$ , and the borrower's state, contract and acceptance-contingent investment decision  $i_w : \mathcal{C} \times \{\text{accept, reject}\} \rightarrow \{\text{work, fixed}\}$ . At all states all strategies

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<sup>5</sup>Note that for tractability we have implicitly assumed that the borrower cannot save her consumption good between periods. The primary vehicle of savings in this model is investment in the fixed capital project.

must be mutual best responses. We defer discussion of the value functions that pin down these equilibrium response functions to Section 2.

By studying Stationary Markov Perfect Equilibria, we impose that the lender uses an impersonal strategy: borrowers with the same business size must be offered the same (potentially mixed set of) contracts. This may be an especially plausible restriction in the context of large lenders, such as microfinance institutions whose policy makers may be far removed from their loan recipients, thereby rendering it infeasible to offer personalized contracts that condition on the borrowers' investment histories.

## Model Discussion

The model has three features that merit further discussion.

### State $w = 2$ : The formal sector

We assume that if the borrower reaches state  $w = 2$  then the game ends. We imagine that this regime change represents the borrower gaining access to formal lenders. Though unmodeled, we imagine these formal lenders can only lend to borrowers with sufficiently large business (e.g. because large businesses have significant pledgeable assets, as in Rampini and Viswanathan (2013)). In contrast, we assume that the informal lender operating in state  $w = 1$  has a superior enforcement technology, and thus can make loans to borrowers with smaller businesses without fear that they will default.<sup>6</sup>

While the informal lender can perfectly enforce short-term contracts, we assume that long-term contracts are infeasible. This reflects that enforcing long-term contracts—especially those that specify transfers after the borrower's business has grown, long after the initial loan—may require strong legal institutions and outside enforcement. Therefore, the borrower always repays his outstanding short-term debt  $R$  but cannot pledge the benefits of reaching the formal sector to his informal lender.

Thus, when the borrower's business is of size  $w = 1$  his informal lender is a monopolist. When he reaches the formal sector at  $w = 2$  he enters a more competitive lending market, receiving a reduced form payoff of  $U$  while his lender loses her customer and receives a payoff of 0.<sup>7</sup>

### Fixed and working capital projects

The borrower has access to two projects, a working capital project and a fixed capital project. The working capital project corresponds to buying a liquid asset such as inventory, which the borrower can sell quickly. The fixed capital project takes longer to realize a return, but also gives the borrower

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<sup>6</sup>Indeed, through tactics such as joint liability, enforcing repayment from small borrowers is often understood to be the primary innovation of microfinance (see e.g. Banerjee (2013)).

<sup>7</sup>The model would work in exactly the same way if the informal lender received a low payoff rather than 0, reflecting the possibility that the borrower stays with his current lender but captures more of the surplus due to increased competition.

a chance to build up a stock of pledgeable assets and expand his business (e.g. the borrower could buy a durable asset, or expand his store front, and hence take longer to recoup his costs). Following [Donaldson et al. \(2019\)](#) we assume that the borrower can only choose one of these two projects within a given period, due to limited attention or energy. But the borrower can switch projects between periods. In [Appendix A.3](#) we provide an extension in which the borrower can utilize both projects within the same period.

For simplicity, while the fixed capital project returns a consumption good only in the late portion of the period, we assume that it produces consumption goods at the same rate as the working capital project. Therefore, because there is no discounting within a period, when the borrower is in autarky the fixed capital project strictly dominates the working capital project. However the model can easily accommodate working capital and fixed capital projects that return consumption goods at different rates.

### **The contractual restriction $a = 1$**

We assume that when the lender offers contract that specifies  $a = 1$  the borrower must repay his debt  $R$  in the early portion of the period, and therefore must invest in the working capital project. In contrast, when the contract specifies  $a = 0$  the borrower is free to repay his debt in the late portion of the period, and so can invest in either of his two projects. This formulation is consistent with a growing body of empirical evidence about microfinance. A standard microfinance contract requires that borrowers begin repaying their loans immediately after receiving them, and continue to do so in frequent installments throughout the loan cycle. [Field et al. \(2013\)](#), [Takahashi et al. \(2017\)](#), [Barboni and Agarwal \(2018\)](#), and [Battaglia et al. \(2019\)](#) find that by relaxing this feature of the loan contract and allowing borrowers to match the timing of their repayments to the cashflow of their businesses, borrowers exhibit higher demand for credit, make larger and longer-term investments, and their business grow faster as a result.

Our theory will illuminate when and why the lender might design contracts to discourage her borrowers from making long-term investments even when such investments do not cause the lender any short-run loss.

## **2 Analysis of the Two-State Model**

In this section we characterize the equilibrium in the two-state model. In doing so we highlight two competing factors; the strength of the borrower's bargaining position and the hold-up problem induced by the fact that the borrower cannot commit to share the benefits of the formal sector with his informal lender. The borrower's bargaining position determines the level of rent that the lender can extract through restrictive contracts. In contrast, the hold-up problem limits the rents that the lender

can extract through unrestrictive contracts. After providing an analytic formula to characterize the interplay of these two forces we study comparative statics that shed new light on several empirical phenomena in the microfinance literature. Then, in Section 3.2 we extend the model to have many states and explore the dynamic nature of comparative statics within the richer model.

## 2.1 Autarky and Relationship Value Functions

**The Borrower's Autarky Problem** First consider the borrower's autarky problem, without having access to a lender. That is, the economic environment is as in Section 1, but the borrower is forced to reject the lender's contract at all times and can always choose flexibly between the two projects.

The borrower's value of being in state 1 under autarky,  $B^{aut}$ , solves the following HJB equation:

$$\rho B^{aut} = y^{aut} + \max \{0, g^{aut} (U - B^{aut})\}. \quad (1)$$

The first term on the right-hand-side represents the flow payoff from consumption good, produced by either the working or fixed capital projects. The second term captures the potential payoff from implementing the fixed capital investment, which, with Poisson rate  $g^{aut}$ , could result in a successful expansion of his business size, raising his value function by  $(U - B^{aut})$ . When the formal sector is unattractive ( $U$  is low), the borrower always chooses the working capital project and does not grow beyond state 1 under autarky. To make the analysis interesting, we assume throughout the paper that *the borrower does have the desire to grow to the formal sector under autarky*.

**Assumption 1.**  $U > y^{aut} / \rho$ .

Under the assumption, borrower's autarky value is

$$\rho B^{aut} = (1 - \alpha) y^{aut} + \alpha \rho U, \quad \text{where } \alpha \equiv \frac{g^{aut}}{\rho + g^{aut}} < 1. \quad (2)$$

To understand this, note that the flow payoff in state 1 under autarky is  $y^{aut}$ , and the continuation value under state 2 is  $U$ . Borrower's value function in state 1 is therefore a weighted average between  $y^{aut} / \rho$  and  $U$ . Intuitively, the borrower spends a fraction  $(1 - \alpha)$  of his expected discounted life in state 1 and a fraction  $\alpha$  in state 2, enjoying flow payoffs of  $y^{aut}$  and  $\rho U$ , respectively; the weights,  $(1 - \alpha)$  and  $\alpha$ , depend on the rate of growth  $g^{aut}$  and his discount rate  $\rho$ .

**Relationship Value Functions** We now outline the borrower and lender's relationship maximization problems and describe their value functions.

Let  $B^*$  and  $V^*$  denote the borrower and lender's equilibrium value in state 1. If the borrower accepts an unrestrictive contract ( $a = 0$ ) for an instant, he solves the investment problem and, under

Assumption 1, always prefers the fixed capital project, obtaining an instantaneous payoff

$$v(\langle R, 0 \rangle) \equiv y - R + g(U - B^*). \quad (3)$$

On the other hand, if the borrower accepts a restrictive contract ( $a = 1$ ), the borrower must invest all resources into the working capital project, realizing an instantaneous payoff

$$v(\langle R, 1 \rangle) \equiv y - R.$$

Finally, the borrower's instantaneous payoff from rejecting a contract is

$$\underline{v} \equiv y^{aut} + g^{aut}(U - B^*).$$

Intuitively, the borrower always has the option of rejecting the contract, ensuring a minimum flow value  $\underline{v}$  of operating the fixed capital project under autarky for the current instant while maintaining the option to interact with the lender again over the next instant, with continuation value  $B^*$ .

Given equilibrium contract  $\langle R^*, a^* \rangle$ , the borrower's value function  $B^*$  satisfies the HJB equation:

$$\rho B^* = \max \{ \underline{v}, v(\langle R^*, a^* \rangle) \}.$$

The first term is the borrower's instantaneous payoff from rejecting the contract and invest in the fixed capital project while being in autarky for the instant. If a contract is accepted, the borrower optimally chooses his project subject to contractual restrictions  $a^*$ , and his flow payoff depends on two terms: the residual consumption value (net of repayment),  $y - R^*$ , and the potential change in continuation payoff due to successful investment in fixed capital.

The lender chooses a contract that maximizes her flow payoff, subject to the borrower's individual rationality constraint:

$$\rho L^* = \max_{\langle R, a \rangle} R - \kappa - I(\langle R, a \rangle) \cdot g \times L^* \quad \text{s.t. } v(\langle R, a \rangle) \geq \underline{v}.$$

$I(\cdot)$  is an indicator for whether the borrower invests in fixed capital given the contract offered.

## 2.2 Analysis of the Equilibrium

We now analyze the equilibrium structure of lending contracts. In what follows, we first separately characterize the optimal restrictive and unrestrictive contracts, assuming that the lender is forced to offer the respective type of contracts and not the other. We then study which type of contract the lender offers. We use  $\tilde{\cdot}$  to denote variables ( $\tilde{R}$ ,  $\tilde{L}$ , and  $\tilde{B}$ ) under the restrictive regime and  $\hat{\cdot}$  to denote variables

$(\hat{R}, \hat{L}, \text{ and } \hat{B})$  under the unrestrictive regime. We continue to denote variables in equilibrium—when the lender can choose which type of contracts to offer—with  $\cdot^*$ ; specifically,  $L^*$  and  $B^*$  are the value equilibrium value functions for the lender and the borrower, respectively.

### 2.2.1 Restrictive Contracts Are Maximally Extractive

First suppose the lender must always offer restrictive contracts, which, once accepted, dictate that all resources must go into the working capital project. Because the lender's flow payoff is increasing in repayment she always chooses the highest  $R$  possible subject to the borrower's individual rationality constraint, making the borrower indifferent between accepting the restrictive contract versus rejecting it and investing in the fixed capital project on his own.

**Lemma 1.** *If contracts are always restrictive, then the borrower's value is the same as under autarky  $\tilde{B} = B^{aut}$ . The equilibrium repayment is  $\tilde{R} = y - \rho B^{aut}$ . Lender's value function is  $\tilde{L} = (\tilde{R} - \kappa) / \rho$ .*

Lemma 1 pins down the repayment amount the lender must offer if the borrower is to accept a restrictive contract. The lender's decision between restrictive and unrestrictive contracts is a tradeoff between offering the borrower a lower repayment amount and suspending his growth (restrictive) instead of offering a loan with a higher repayment amount that allows him to invest as he pleases (unrestrictive).

### 2.2.2 Unrestrictive Contracts May Leave Rents to the Borrower

Now suppose the lender must offer unrestrictive contracts. The lender's value function solves

$$\rho \hat{L} = \max_{R \leq y} R - \kappa - g \times \hat{L} \quad \text{s.t. } y - R + g \times (U - \hat{B}) \geq y^{aut} + g^{aut} \times (U - \hat{B}).$$

Under an unrestrictive contract, the borrower always chooses the fixed capital project. Thus the lender's payoff consists of not only the flow value  $(R - \kappa)$  of consumption but also, with Poisson rate  $g$ , a decline in value  $(-\hat{L})$  for losing a client when the borrower successfully grows into state 2. The inequality constraint reflects that the instantaneous payoff to the borrower must be weakly higher than his payoff obtained by rejecting the contract.

To solve the lender's problem, note that under unrestrictive contracts, both the borrower and lender's value functions are weighted averages of the payoffs in state 1 and 2. Specifically, let  $\beta \equiv g / (\rho + g)$ ; then the lender's value function is  $\rho \hat{L} = (1 - \beta) (\hat{R} - \kappa) + \beta \times 0$  and the borrower's is  $\rho \hat{B} = (1 - \beta) (y - \hat{R}) + \beta \rho U$ . That is, if the borrower grows into state 2 at rate  $g$ , then both players expect to spend  $(1 - \beta)$  fraction of their discounted lifetime in state 1 and  $\beta$  fraction in state 2.

The lender's problem can be rewritten as

$$\max_{R \leq y} (1 - \beta)(R - \kappa) \quad \text{s.t.} \quad (1 - \beta)[y - R] + \beta \rho U \geq y^{aut} + g^{aut}(U - \hat{B}). \quad (4)$$

Because of limited pledgeability  $R \leq y$ , the borrower's individual rationality constraint may not bind. That is, the optimal unrestrictive contract may entail the lender capturing all flow output—setting repayment  $\hat{R} = y$ —and that borrower's entire value is derived from the growth prospect,  $\hat{B} = \beta U$ .

**Lemma 2.** *If equilibrium contracts are always unrestrictive, then the players' value functions are*

$$\hat{B} = \max \{ \beta U, B^{aut} \}, \quad \hat{L} = \left[ \beta U + (1 - \beta) \left( \frac{y - \kappa}{\rho} \right) \right] - \hat{B}.$$

Lemma 2 illuminates an important force in our model. The borrower's value is always weakly higher under an unrestrictive contract than under a restrictive one, and, in general, the inequality may be strict. When the borrower invests in working capital, all of the value of the investment materializes in the same period. Hence when the lender offers a restrictive contract, she can perfectly tailor the repayment to push the borrower to his outside option; that is, the borrower's utility is exactly determined by his bargaining position. In contrast, when the borrower invests in fixed capital through an unrestrictive contract, part of the value he derives materializes after he has graduated to the formal sector, and hence the lender cannot extract the full value of these investments. In effect the non-pledgeability of value once the borrower reaches the formal sector induces a hold-up problem, whereby the borrower cannot commit to share the benefits of reaching the formal sector with his lender.

We term this difference between the borrower's value under unrestrictive and restrictive contracts,  $\hat{B} - \tilde{B}$ , the borrower's *expansion rent*, as it is the additional utility the borrower enjoys when he is allowed to expand his business. The expansion rent may be positive because of the hold-up problem. It will play an important role in the following analysis.

### 2.2.3 Characterizing the Equilibrium Contract

We are now ready to characterize the equilibrium. It is first useful to observe that this game admits a unique equilibrium.

**Proposition 1.** *An equilibrium exists and is generically unique.*

The lender may in general follow a mixed strategy between offering unrestrictive and restrictive contracts. Let  $p^*$  be the equilibrium probability that contracts are restrictive;  $p^*$  is a key object of our equilibrium and subsequent comparative static analysis.

**Proposition 2.** *Generically,*<sup>8</sup> *the lender offers restrictive contracts with probability*

$$p^* = \begin{cases} 1 & \text{if } (\hat{L} - \tilde{L}) \leq 0, \\ 0 & \text{if } \rho (\hat{L} - \tilde{L}) \geq g^{aut} (\hat{B} - \tilde{B}), \\ \frac{(\rho + g) \left(1 - \frac{\rho(\hat{L} - \tilde{L})}{g^{aut}(\hat{B} - \tilde{B})}\right)}{\rho + g - \frac{\rho(\hat{L} - \tilde{L})}{g^{aut}(\hat{B} - \tilde{B})} (g - g^{aut})} & \text{otherwise.} \end{cases}$$

The borrower's equilibrium value function is

$$B^* = s\tilde{B} + (1 - s)\hat{B},$$

with  $s \equiv \frac{p^*(\rho + g^{aut})}{\rho + p^*g^{aut} + (1 - p^*)g} \in [0, 1]$ . The lender's value function is  $L^* = \max\{\hat{L}, \tilde{L}\}$ . The unrestrictive contract offered is  $\langle \hat{R}, 0 \rangle$  and the restrictive contract offered is  $\langle y - (y^{aut} + g^{aut}(U - B^*)), 1 \rangle$ .

The proposition completely characterizes the equilibrium contracts and value functions. When  $p^* = 1$ , all contracts are restrictive, and the informal lender endogenously keeps the borrower captive, even though the borrower would eventually reach the formal sector in the absence of credit. Even when  $p^* < 1$ , the borrower grows at a rate  $(1 - p^*)g$ , which may be slower than under autarky,  $g^{aut}$ .

The reason that  $p^*$  may be interior is that the lender lacks commitment. Consider the case  $\hat{L} > \tilde{L}$ , under which the lender prefers offering *unrestrictive contracts with probability one* over offering *restrictive contracts with probability one*. However, due to lack of commitment, the lender may have an incentive to deviate and offer restrictive contracts when the borrower expects the lender to only offer unrestrictive one. Specifically, the borrower's individual rationality constraint, reproduced below, always binds when the lender offers a restrictive contract over the instant:

$$y - R \geq y^{aut} + g^{aut}(U - B^*).$$

The amount that the borrower is willing to repay,  $R$ , depends on how much he values being in state 1 ( $B^*$ ). When staying in state 1 is relatively more attractive, the borrower is willing to accept a restrictive contract with a greater repayment (over rejecting the contract); hence, restrictive contracts become more attractive to the lender as she can extract more rents. Because of expansion rents  $(\hat{B} - \tilde{B}) \geq 0$  and that  $B^* = (1 - s)\hat{B} + s\tilde{B}$ , the borrower's value is high exactly when he expects to receive *unrestrictive contracts* more often (i.e.  $s$  is large), but, because of her higher willingness to accept restrictive contracts, this is also exactly when the lender finds *restrictive contracts* more attractive. This logic generates the scope for interior  $p^*$  in the unique equilibrium. Note that the

<sup>8</sup>The proposition holds except in the non-generic case where  $\hat{B} = \tilde{B}$  and  $\hat{L} = \tilde{L}$ , under which both players are indifferent between restrictive and unrestrictive contracts, and there is a continuum of equilibria corresponding to all  $p^* \in [0, 1]$ .

lender's equilibrium value  $L^*$  is always equal to  $\hat{L}$  whenever  $p^* < 1$  and is equal to  $\tilde{L}$  only when  $p^* = 1$ .

Proposition 2 admits the following corollary.

**Corollary 1.** *The lender offers a restrictive contract with probability  $p^* = 1$  if and only if*

$$\underbrace{\beta \left( U - \frac{y - \kappa}{\rho} \right)}_{\substack{\text{"efficiency gain"} \\ \text{from investing in fixed capital}}} \leq \underbrace{\hat{B} - \tilde{B}}_{\substack{\text{"expansion rent"} \\ \text{captured by the borrower}}} . \quad (5)$$

Inequality (5) is a rearrangement of the condition  $\hat{L} - \tilde{L} \leq 1$ . The left-hand side can be understood as the social efficiency gain of investing in fixed capital relative to working capital. The borrower and lender spend a discounted fraction  $\beta$  of their lives in the formal sector, where the joint payoff is  $U$ , and they forgo their joint production in state 1,  $\frac{y - \kappa}{\rho}$ . The right-hand side is the borrower's expansion rent—the additional value he derives from unrestrictive contracts relative to restrictive contracts because of the hold-up problem. The lender's benefit from unrestrictive contracts is the entire social efficiency gain of helping the borrower grow his business minus the borrower's expansion rent. Therefore when Inequality (5) holds, the lender offers only restrictive contracts and keeps the borrower captive.

**Corollary 2.** *If  $\rho U \leq y - \kappa$ , the lender offers restrictive contracts with probability  $p^* = 1$ .*

Corollary 2 is an immediate consequence of Corollary 1. When it is socially inefficient to grow to the formal sector, the lender always offers restrictive contracts. This is so because the left-hand side of Inequality (5) is negative, whereas the right-hand side is always weakly positive.

Understanding that the lender offers only restrictive contracts when it is socially efficient to stay in state 1, we now make the following assumption to guarantee that it is socially efficient to grow to state 2, creating the possibility that the borrower receives unrestrictive contracts in equilibrium.

**Assumption 2.**  $U > (y - \kappa) / \rho$ .<sup>9</sup>

This is a strengthening of Assumption 1.

**Interpretations** Proposition 2 highlights that expanding access to credit can *inhibit* entrepreneurship. This statement can be interpreted in three ways.

First, under Assumption 1, the borrower would invest in fixed capital and reach the formal sector under autarky, but he may fail to grow or grow at a slower rate in the presence of credit. Second, under Assumption 2 it is socially efficient for the borrower to invest in fixed capital and reach the

<sup>9</sup>Note that when Assumption 2 does not hold, we have fully characterized the equilibrium.

formal sector even when he has access to a lender in state 1. Third, if the borrower rejects a restrictive contract (off the equilibrium path), he invests in fixed capital and grows his business. Therefore, entrepreneurial growth is stymied only *because of* the presence of the very lender—it is the lender actively discourages the borrower from making fixed capital investments by offering him restrictive contracts and compensating him with a lower repayment amount.

That expanding access to credit may inhibit entrepreneurship may shed some light on the disappointing impacts of microfinance repeatedly found across studies around the world, cited in the introduction.

Finally, we note that while introducing a lender can constrain entrepreneurship relative to autarky, the lender can never reduce the borrower’s welfare relative to autarky. This is because of the voluntary nature of the loans—the lender must always satisfy the borrower’s individual rationality constraint.

## 2.3 Comparative Statics

In this section we study comparative statics of the equilibrium to understand the interplay between the the strength of the borrower’s bargaining position and the hold-up problem. In doing so we shed light on the circumstances in which restrictive contracts and borrower captivity are especially likely and argue that this can explain a number of empirical facts about microfinance.

Our first comparative static is with respect to  $g^{aut}$  (or equivalently  $\alpha \equiv \frac{g^{aut}}{\rho + g^{aut}}$ ), the rate at which the borrower can grow his business without outside credit. We hold all other values of the production functions fixed.

**Proposition 3.** *Increasing  $g^{aut}$  makes restrictive contracts less likely in equilibrium. Formally,  $dp^*/dg^{aut} \leq 0$  with strict inequality if  $p^* \in (0, 1)$ .*

Examining Inequality (5) makes the logic underlying this comparative static clear. Increasing  $g^{aut}$  improves the borrower’s bargaining position as investing in fixed capital on his own is now relatively more attractive, and therefore this reduces the maximum repayment he is willing to accept along with a restrictive contract. This reduces the borrower’s expansion rent. However, the efficiency gain from investing in fixed capital is unaffected. Hence unrestrictive contracts become relatively more attractive and therefore more likely in equilibrium.

Our second comparative static is with respect to  $g$  (or equivalently  $\beta \equiv \frac{g}{\rho + g}$ ), the rate at which the borrower expands when receiving unrestrictive contracts.

**Proposition 4.** *Increasing  $g$  makes restrictive contracts more likely in equilibrium. Formally,  $dp^*/dg \geq 0$  with strict inequality if  $p^* \in (0, 1)$ .*

In contrast to Proposition 3 where making the fixed capital project more productive *under autarky* makes restrictive contracts less likely, now in Proposition 4 making the fixed capital project more pro-

ductive *upon receiving investment*  $\kappa$  makes restrictive contracts more likely. Because of the hold-up problem, increasing  $g$  reduces the rent that the lender can extract from unrestrictive contracts. The lender can only extract rent from the borrower so long as he remains in state 1, and increasing  $g$  reduces the amount of time after which the borrower leaves the lender when receiving unrestrictive contracts. In contrast, because the borrower can only take advantage of the improved fixed capital project with the lender's help, the borrower's bargaining position is left unchanged, so the lender's utility from restrictive contracts is unchanged. Hence, increasing  $g$  increases the likelihood of restrictive contracts and borrower captivity.

Together Propositions 3 and 4 start to paint a picture about when introducing a lender is likely to inhibit entrepreneurship. In particular, borrowers with poor ability to grow without the lender's help (low  $g^{aut}$ ), and strong prospects for business growth with the lender's help (high  $g$ ) are likely to be offered restrictive contracts. This may be precisely the case for many microfinance borrowers around the world. In particular, while it may have at first seemed counterintuitive that microfinance has had low impact on entrepreneurship despite strong evidence that unconditional cash grants lead to substantial and persistent business growth, Propositions 3 and 4 suggest that it may be *because* of these attractive investment opportunities, and an inability for borrowers to grow their businesses in the absence of outside capital, that microfinance has had little impact.

Our next comparative static concerns the time it takes for the borrower to reach the formal sector. Consider simultaneously raising both  $g^{aut}$  and  $g$ , which increases the rate of growth from the fixed capital project under any investment level. This may be interpreted as reducing the borrower's distance from achieving the regime change and entering the formal sector.

**Proposition 5.** *There exists a  $\bar{g}$  such that for  $g > g^{aut} > \bar{g}$ , the borrower always receives unrestrictive contracts with positive probability.*

Proposition 5 implies that when it is socially efficient to grow to the formal sector, borrowers sufficiently near the formal sector always receive unrestrictive contracts with positive probability. This does not follow immediately from Propositions 3 and 4 because increasing both  $g^{aut}$  and  $g$  pushes in opposite directions. Once again examining Inequality (5), we can see that as  $\alpha$  and  $\beta$  both converge to 1 and the expansion rent converges to 0. Hence, the efficiency gain of regime change always outweighs the borrower's expansion rent. Effectively, for borrowers with sufficiently strong bargaining positions, the lender cannot extract substantial rents using restrictive contracts. This renders the hold-up problem irrelevant, as the lender can always extract positive rent through unrestrictive contracts.

So borrowers near the formal sector always receive unrestrictive contracts and the presence of the lender increases the rate at which they grow their business. This result is consistent with the large body of experimental evidence on the impact of microfinance. Many of these studies find that

while the average impact of microfinance on entrepreneurship is low, borrowers with relatively larger businesses who are closer to graduating out of microfinance do see substantial business growth.<sup>10</sup>

For completeness we note that comparative statics can also be done with respect to the output of the consumption good,  $y^{aut}$  and  $y$ . Increasing  $y^{aut}$  improves the borrower's bargaining position, and just as in Proposition 3 this pushes towards unrestrictive contracts. Increasing  $y$  makes state 1 more attractive. This increases the likelihood of restrictive contracts as it reduces the social efficiency gain of growing to the formal sector, and the lender is more willing to offer concessionary repayment amounts to keep the borrower captive.

Our final comparative static of this section concerns the attractiveness of the formal sector,  $U$ .

**Proposition 6.** *Increasing  $U$  makes restrictive contracts less likely. Formally,  $dp^*/dU \leq 0$  with strict inequality if  $p^* \in (0, 1)$ .*

Increasing  $U$  increases the value that the borrower places on business growth, and hence strengthens his bargaining position. So as  $U$  increases, borrowers become more demanding when receiving restrictive contracts. In contrast, borrowers are weakly more willing to accept any unrestrictive contract as  $U$  increases, since unrestrictive contracts allow them to grow their business and reach the formal sector. Therefore increasing the attractiveness of the formal sector reduces the likelihood of borrower captivity. We will see in the next section, when the borrower must grow his business more than once in order to reach the formal sector, that this conclusion may be reversed.

### 3 Contractual Restrictions and State Dependence

In this section we extend the 2-state model to have several active states before the borrower graduates to the formal sector. The goal is to develop new insights stemming from the forward-looking nature of the lending relationship. We demonstrate that comparative statics that improve the welfare of relatively rich borrowers may, but will not always, harm the welfare of poorer borrowers. In effect, because of the hold-up problem, improving the bargaining position of richer borrowers may reduce the rent the lender can extract from unrestrictive contracts to poorer borrowers and increase the likelihood that they are held captive. For instance, increasing the attractiveness of the formal sector may harm poorer borrowers by virtue of helping richer ones. This is similarly true when increasing the borrower's patience. We characterize when these forces can on net increase the likelihood of captivity for poorer borrowers.

Formally, there are  $n - 1$  business sizes through which borrowers must grow their businesses before they reach the formal sector in state  $w = n$ . Because there are several active states, we now index all

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<sup>10</sup>See Angelucci et al. (2015), Augsburg et al. (2015), Banerjee et al. (2015), Crepon et al. (2015), and Banerjee et al. (2017).

parameters and value functions by  $w$ . We assume that  $y_w^{aut} \leq y_{w+1}^{aut}$  and  $y_w - \kappa_w \leq y_{w+1} - \kappa_{w+1}$  for all  $w$  so that the borrower's output grows in the state, and that  $U > (y_{n-1} - \kappa_{n-1})/\rho$  so that it is socially efficient to invest in fixed capital when the borrower is one state away from the formal sector.

We begin by developing an intuition for comparative statics in a 3-state model and then extend our analysis to the more general case. In the appendix we characterize a variant of the model with deterministic growth and a continuous set of states prior to graduation to the formal sector.

### 3.1 A 3-State Model

We can directly apply the analysis from Section 2 to the last active state  $w = 2$ . Moreover, we can extend Proposition 2 to state  $w = 1$ . Specifically, in equilibrium the borrower receives a restrictive contract in state  $w = 1$  with probability 1 if and only if

$$\underbrace{\beta_1 \left( (B_2^* + L_2^*) - \frac{y_1 - \kappa_1}{\rho} \right)}_{\text{"efficiency gain" from investing in fixed capital}} \leq \underbrace{\hat{B}_1 - \tilde{B}_1}_{\text{"expansion rent" captured by the borrower}}. \quad (6)$$

There are two differences from Inequality (5). First, upon moving to state 2 the borrower and lender can enjoy a total continuation utility of  $B_2^* + L_2^*$ , rather than  $U + 0$  when they move to the formal sector. Second, the borrower's equilibrium continuation utilities depend on his state 2 value rather than directly on  $U$ :

$$\begin{aligned} \tilde{B}_1 &\equiv (1 - \alpha_1)y_1/\rho + \alpha_1 B_2^*, \\ \hat{B}_1 &\equiv \max \{ \beta_1 B_2^*, (1 - \alpha_1)y_1^{aut}/\rho + \alpha_1 B_2^* \}. \end{aligned}$$

where  $\alpha_w \equiv \frac{g_w^{aut}}{\rho + g_w^{aut}}$  and  $\beta_w \equiv \frac{g_w}{\rho + g_w}$ . Therefore the model can be solved via backward induction: the equilibrium can first be characterized in state  $w = 2$  just as it was in Section 2; then, as a function of continuation values in state  $w = 2$ , the equilibrium can be analytically characterized in state  $w = 1$ .

The 3-state model also features a richer manifestation of the hold-up problem. In the 2-state model the lender's value from unrestrictive contracts was limited by the speed  $g$  at which the borrower grows his business when offered unrestrictive contracts. In the 3-state model the lender's value from unrestrictive contracts is also determined by her value  $L_{w+1}^*$  in the next state. In the final state the lender's value is exogenously determined to be 0. However, in state 2 the lender's value is in part a reflection of the borrower's bargaining position. Thus because of the hold-up problem, increasing the borrower's state 2 bargaining position may reduce the lender's ability to extract rents through unrestrictive contracts in state 1. We will see in Proposition 6 that this is sometimes, but not always the case.

In general the borrower may receive restrictive contracts in either or both of the active states.

Propositions 2, 3, 4, and 5 continue to offer guidance as to when the borrower is likely to be captive in state  $w = 1$  or  $w = 2$  (or both, or neither).

However, while Proposition 6 continues to characterize the comparative statics of the equilibrium in state 2 with respect to the formal sector payoff  $U$ , the comparative statics of the equilibrium in state 1 with respect to  $U$  are markedly different. Let  $p_w^*$  be the equilibrium probability of a restrictive contract in state  $w$ .

**Proposition 7.** *When the lender offers restrictive contracts in state 2, increasing  $U$  makes restrictive contracts more likely in state 1. Otherwise, increasing  $U$  makes unrestrictive contracts more likely in state 1. Formally,*

$$dp_1^*/dU \begin{cases} \geq 0 & \text{if } p_2^* = 1, \\ \leq 0 & \text{if } p_2^* < 1. \end{cases}$$

*The inequalities are strict if  $p_1^* \in (0, 1)$ .*

We know from Proposition 6 that increasing  $U$  always makes unrestrictive contracts more likely in state 2. However, Proposition 7 highlights that the comparative statics in state 1 now hinge on the equilibrium contract offered in state 2.

If  $p_2^* = 1$ , then increasing  $U$  always makes restrictive contracts more likely in state 1. This can be understood with reference to Inequality (6). First, consider the right-hand side of the inequality—the borrower’s expansion rent. Because  $p_2^* = 1$ , the borrower’s state 2 value is  $B_2^* = \alpha_2 U$ , which grows at rate  $\alpha_2 \times dU$ . The borrower’s state 1 bargaining position is  $B_1^{aut} = \alpha_1 B_2^*$  which grows at rate  $\alpha_1 \alpha_2 \times dU$ , as his outside option of investing in fixed capital at the autarkic rate becomes more attractive. Hence the borrower’s continuation utility when offered restrictive contracts in state 1,  $\tilde{B}_1$ , grows at the same rate. In contrast, because of the hold-up problem, the borrower cannot commit to demand less than  $B_2^*$  value if ever he reaches state 2. So, the borrower’s value upon receiving an unrestrictive contract in state 1,  $\hat{B}_1 = \beta_1 B_2^*$ , grows at rate  $\beta_1 \alpha_2 \times dU > \alpha_1 \alpha_2 \times dU$ . On net the expansion rent in state 1 is therefore increasing in  $U$ .

Now consider the left-hand side of Inequality (6)—the social efficiency gain from unrestrictive contracts. Because  $p_2^* = 1$ , the borrower is captive in state 2 and will never reach the formal sector on the equilibrium path. Therefore, in state 1 the social efficiency gain of unrestrictive contracts is unaffected by increasing  $U$ , as state 2 joint surplus is fixed. On net, increasing  $U$  increases the expansion rent while not changing the social efficiency gain from unrestrictive contracts. So, while increasing  $U$  decreases the likelihood of borrower captivity in state 2, when  $p_2^* = 1$  the very same force increases the likelihood of captivity in state 1.

Another way to understand this phenomenon is that increasing  $U$  increases the borrower’s bargaining position in state 2. Because the lender offers the borrower a restrictive contract in state 2,

strengthening the borrower’s bargaining position results in a transfer from the lender to the borrower in the form of a lower repayment amount. From the perspective of state 1 this has two consequences. First, the borrower’s improved state 2 bargaining position improves the borrower’s state 1 bargaining position, by increasing the value of growing at the autarkic rate. This makes restrictive contracts less attractive to the lender. Second, because of the hold-up problem, the lender anticipates that unrestrictive contracts become less attractive, as the borrower cannot commit not to exercise his improved bargaining position in state 2. The preceding discussion implies that when  $p_2^* = 1$  the latter force always dominates and the lender shifts towards restrictive contracts in state 1.

Conversely, when  $p_2^* < 1$ —the borrower receives unrestrictive contracts with positive probability in state 2—increasing  $U$  always reduces the probability of contractual restrictions *in both states*. As in the above case, increasing  $U$  increases  $B_2^*$  and therefore strengthens the borrower’s bargaining position in state 1. This makes restrictive contracts less attractive to the lender in state 1. However the lender’s payoff from unrestrictive contracts in state 1 is unaffected. To see this, note that the lender has a weak preference to offer unrestrictive contracts in state 2, and her state 2 value is  $\hat{L}_2 = (1 - \beta_2) \left( \frac{y_2 - R_2}{\rho} \right)$ . This is invariant in  $U$ , as unrestrictive contracts in state 2 exceed the borrower’s bargaining position. Put in other words, increasing  $U$  increases the borrower’s state 2 value and bargaining position, but because the lender was already offering unrestrictive contracts, this does not induce a transfer from the lender to the borrower. Increasing  $U$  does not reduce the lender’s continuation value in state 2 and therefore does not reduce the value of unrestrictive contracts in state 1. Hence when  $p_2^* < 1$ , increasing  $U$  reduces the likelihood of borrower captivity in both states.

The preceding discussion implies that when  $p_2^* = 1$ , increasing the attractiveness of the formal sector can cause a Pareto disimprovement as the welfare of both parties in state 1 diminishes due to the hold-up problem. Therefore, policies that improve the benefits of graduating from microfinance and informal lending may backfire.

Before proceeding to the next result it is worth contrasting Proposition 7 with the result of **Petersen and Rajan (1995)** (henceforth PR) that improving late-stage finance can reduce access to early-stage finance. The result of PR is driven by the fact that early-stage financiers may not be able to recoup the initial costs of lending if they must operate in a competitive market once their borrowers’ businesses have grown. In contrast, in our model the lender is not motivated by recouping her upfront costs of lending but rather by prolonging the period over which she can extract rents from her borrower. This difference manifests itself in two ways. In PR, business growth slows down because lenders withdraw from the early-stage market when competition increases in the late-stage market. In contrast, in our model business growth slows down only when the lender actively enters the early-stage market and shifts towards offering restrictive contracts to increase rent extraction. Therefore our model highlights that the introduction of credit can slow business growth.

Second in our analysis the borrower’s bargaining position plays a key role in determining whether he remains captive to the lender. In contrast to PR, once the formal sector becomes sufficiently attractive (so much so that the lender offers unrestrictive contracts in state 2, as is dictated by Proposition 6), then further increases in formal sector attractiveness always reduce the likelihood of captivity in state 1 and speed growth. Intuitively, once the borrower’s state 2 bargaining position becomes sufficiently strong, the lender can no longer extract substantial rents by keeping the state 2 borrower in captivity. At this point, increasing the formal sector attractiveness improves the state 1 borrower’s bargaining position without influencing the amount of rent the lender can extract from unrestrictive contracts, and so only serves to help the state 1 borrower.

Our next result concerns the role of the borrower’s patience in determining her captivity. When the equilibrium is such that the borrower receives a restrictive contract in state 1 it can be understood as an endogenous poverty trap. Borrowers that start in state 1 never grow out of it, despite finding it worthwhile to grow their business if they reach a larger state. The standard intuition about poverty traps is that they are driven by impatience, and that sufficiently patient agents never succumb to them (e.g. Azariadis (1996)). However, this is a model in which increasing the borrower’s patience can make the poverty trap in state 1 more likely. Let  $\rho^B$  denote the borrower’s patience, and  $\rho^L$  denote the lender’s patience.

**Proposition 8.** *Increasing the borrower’s patience (decreasing  $\rho^B$ ) can increase the likelihood of restrictive contracts in state 1. Formally, when  $p_2^* = 1$ ,  $dp_1^*/d\rho^B$  may be negative.*

The intuition underlying Proposition 8 resembles that of Proposition 7. Increasing the borrower’s patience increases his value for investing in fixed capital and growing his business. This in turn increases his state 2 bargaining position and decreases the repayment amount he is willing to accept for restrictive contracts in state 2. Just as in Proposition 7 because of the hold-up problem this can increase the likelihood of captivity (and hence a poverty trap) in state 1.

Our final result of the section concerns the role of repetition in inducing borrower captivity. Borrower captivity arises not because of *lending* per se—having access to external funds always expands production sets—but because of the *repeated* nature of lending relationships. The lender has an incentive to trap the borrower only because of future rents, and the trap completely disappears if the relationship is short-lived. This result provides a counterpoint to the standard intuition that repeated interaction facilitates cooperation and efficient contracting (eg. Tirole, 2010). Our model highlights that under motives to maintain dynamic rents, repeated interaction strictly lowers welfare.

**Proposition 9.** *If the borrower and lender only interact one time, the lender always offers unrestrictive contracts.*

The borrower always accepts a higher repayment amount for unrestrictive contracts than for restrictive contracts. Therefore, a lender who anticipates no future relationship always myopically prefers

to offer unrestrictive contracts and help her borrower grow. It is only because of the possibility of rent extraction in the future that the lender might incur a short-term loss in order to keep her borrower captive. This result resembles, but also stands in contrast with results from the relationship banking literature, which dictate that lenders with long-standing relationships with their borrowers may have informational advantages and can use this to extract rents from their borrowers. While that literature highlights that repetition may lead to information rents, it is the information rent and not the repetition per se that leads to rent extraction. In contrast our model highlights that *repetition* may be the key factor in exacerbating the lender's rent extraction motive.

### 3.2 An $n$ State Model

In this section we extend the model above to have an arbitrary number of active states before the borrower graduates to the formal sector. In doing so we characterize the equilibrium of the full model and provide a partial characterization of the model's "trickle-down" comparative statics.

As in Section 3.1 we can generalize the condition for the borrower to receive restrictive contracts in state  $w$ :

$$\underbrace{\beta_w \left( (B_{w+1}^* + L_{w+1}^*) - \frac{y_w - \kappa_w}{\rho} \right)}_{\text{"efficiency gain" from investing in fixed capital}} \leq \underbrace{\hat{B}_w - \tilde{B}_w}_{\text{"expansion rent" captured by the borrower}} \quad (7)$$

As in Section 3.1, we can fully solve for the equilibrium contracts via backward induction on the state. The results in Section 2 still apply to the final state, and the results in Section 3.1 still apply to the final two states. Further, the results and intuitions generated in Section 2 could be readily extended to the  $n$  state model to form similar intuitions for when and why the lender may keep her borrowers captive.

An example equilibrium is depicted below, again with each circle representing a state and shaded circles representing states in which restrictive contracts are offered.



Our remaining goal in this section is to highlight that the comparative statics remain quite tractable in this more general case. Specifically, we present the consequence of increasing  $U$  on the equilibrium loan offers.

Note that without loss of generality we can identify  $m$  disjoint, contiguous sets of states  $\{\underline{w}_1, \dots, \bar{w}_1\}$ ,  $\dots$ ,  $\{\underline{w}_m, \dots, \bar{w}_m\}$  such that  $\bar{w}_m = \max \{w : p_w^* = 1\}$ ,  $\underline{w}_m = \max \{w : p_w^* = 1, p_{w-1}^* < 1\}$ , and in general for  $l \geq 1$ ,  $\bar{w}_l = \max \{w < \underline{w}_{l+1} : p_w^* = 1\}$   $\underline{w}_l = \max \{w \leq \bar{w}_l : p_w^* = 1, p_{w-1}^* < 1\}$ . An arbitrary

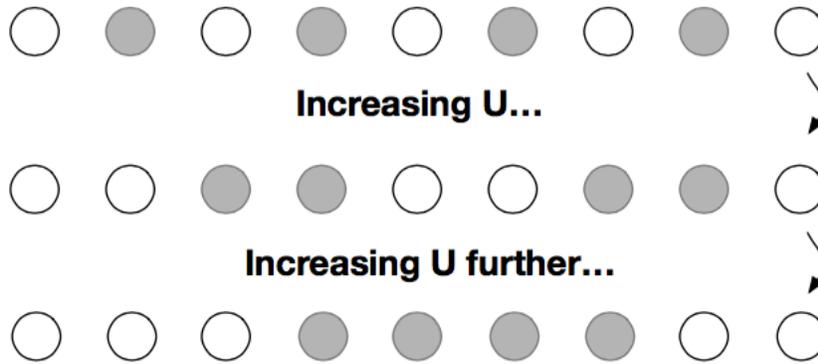
set  $\{\underline{w}_l, \dots, \bar{w}_l\}$  is a contiguous set of states where restrictive contracts are offered with probability 1, and each pure restrictive state is contained in one of these sets.

We consider an impatient borrower and establish the following result.

**Proposition 10.** *For impatient borrowers, the regions of contiguous restrictive states merge together as the formal sector becomes more attractive.*

That is, there exists a  $\bar{\rho}$  such that for  $\rho > \bar{\rho}$ ,  $\frac{dp_w^*}{dU} < 0$  for  $w \in \{\bar{w}_m + 1, \dots, n\}$ ,  $\frac{dp_w^*}{dU} > 0$  for  $w \in \{\bar{w}_{m-1} + 1, \dots, \underline{w}_m - 1\}$ ,  $\frac{dp_w^*}{dU} < 0$  for  $w \in \{\bar{w}_{m-2} + 1, \dots, \underline{w}_{m-1} - 1\}$ , and so on.

Proposition 10 states that the highest region of pure restrictive states moves leftward, the second highest region moves rightward, and so on. This is depicted in the following figure.



The intuition is as follows. The analysis to understand the shift in  $\{\underline{w}_m, \dots, \bar{w}_m\}$  follows that of Section 3.1. Borrowers with business size  $w > \bar{w}_m$  receive unrestrictive contracts with positive probability. As  $U$  increases, it becomes more expensive to offer restrictive contracts, but no more expensive to offer unrestrictive contracts. Hence the lender shifts towards unrestrictive contracts in this region. In the region  $\{\underline{w}_m, \dots, \bar{w}_m\}$ , the lender has a strict preference to offer restrictive contracts, and so continues to do so. But because  $U$  has increased, the borrower values business growth more highly—i.e. his bargaining position has improved—and so the lender must compensate him with a lower repayment amount. Directly to the left of  $\{\underline{w}_m, \dots, \bar{w}_m\}$ , the lender anticipates that the borrower will have a stronger bargaining position once he reaches  $\underline{w}_m$ , and so tightens the contractual restrictions in response to the hold-up problem. Hence the region  $\{\underline{w}_m, \dots, \bar{w}_m\}$  moves leftward.

Two forces change the borrower's continuation utility in state  $\underline{w}_m - 1$ . First the borrower has higher continuation utility in state  $\underline{w}_m$ , which raises his continuation utility in state  $\underline{w}_m - 1$ . Second the borrower receives more restrictive contracts in state  $\underline{w}_m - 1$  which lowers his continuation utility. When the borrower is impatient, the latter force dominates and his continuation utility in state  $\underline{w}_m - 1$  declines. Therefore the preceding analysis applies in reverse to states  $\{\bar{w}_{m-2} + 1, \dots, \underline{w}_{m-1}\}$ .

## 4 Concluding Remarks

We have presented a model to formalize the intuition that, because informal lenders may not be able to offset the costs of supporting borrower growth by extracting the benefits in the future, they may impose contractual restrictions that inhibit long-term, profitable investments.

Our simple theory is able to organize many of the established facts about microfinance. First, the model reconciles the seemingly inconsistent facts that small-scale entrepreneurs enjoy very high return to capital yet are unable to leverage microcredit and other forms of informal finance to realize those high returns, despite moderate interest rates charged by the lenders.<sup>11</sup> In our model, firms that borrow from the informal lender may see their growth stalled, and remain in the relationship indefinitely, even though they would have continued to grow in the absence of a lender. Put simply, in this model, having access to a lender can *reduce* business growth.

While the experimental studies cited above find, on average, low marginal returns to credit, a number of them find considerable heterogeneity in returns to credit across borrowers of different size. In particular, they consistently find that treatment effects are higher for businesses that are more established.<sup>12</sup> Our model sheds light on this heterogeneity, as the borrowers nearest to the formal sector receive unrestrictive contracts and grow faster than they could have in autarky.

One further puzzling fact in the microfinance literature is that, despite the fact that loan products carry low interest rates relative to the returns to capital, demand for microcredit contracts is low in a wide range of settings.<sup>13</sup> Our model offers a novel explanation. Despite low interest rates, contractual restrictions that impose constraints on business growth push borrowers exactly to their individual rationality constraints; these borrowers do not benefit at all from having access to informal lending. In Appendix A.4, we formalize low take-up rate of informal loans through a model extension in which the lender is incompletely informed about the borrower's outside option. While our argument is intuitive, it stands in sharp contrast to standard intuitions based on borrower-side financial constraints, which predict that credit constrained borrowers should have high demand for additional credit at the market interest rate.

Our theory also offers nuanced predictions about the lending relationship as the economic environment changes. Increasing formal sector attractiveness improves the bargaining position of rich borrowers, increases their welfare, and relaxes contractual restrictions, making borrower captivity less likely. However, because of a hold-up problem, the same improvement may harm the welfare

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<sup>11</sup>See Banerjee et al. (2015) and Meager (2017) for overviews of the experimental evaluations of microfinance, and see, for instance, De Mel et al. (2008), Fafchamps et al. (2014), Hussam et al. (2017), and McKenzie and Woodruff (2008) for evidence that microentrepreneurs have high return to capital.

<sup>12</sup>See Angelucci et al. (2015), Augsburg et al. (2015), Crepon et al. (2015), and Banerjee et al. (2017).

<sup>13</sup>See e.g. Banerjee et al. (2014), and Banerjee et al. (2015).

of poorer borrowers. Anticipating that rich borrowers have improved bargaining positions, the lender shifts towards restrictive loans for poor borrowers to prevent them from reaching higher levels of wealth and exploiting their improved positions. Thus improving the formal sector can make both the borrower and lender worse off in equilibrium.

In addition to the theories cited in the introduction, it is worth contrasting our theory with two other classes of theories prominent in development economics. The first might sensibly be labeled “blaming the borrower.” These theories allude to the argument that many borrowers are not natural entrepreneurs and are primarily self-employed due to a scarcity of steady wage work (see e.g., [Schoar \(2010\)](#)). While these theories have some empirical support, they are at best a partial explanation of the problem as they are inconsistent with the large impacts of cash grants, as cited in our introduction.

Second are the theories that “blame the lender” for not having worked out the right lending contract. These theories implicitly guide each of the experiments that evaluate local modifications to standard contracts.<sup>14</sup> While many of these papers contribute substantially to our understanding of how microfinance operates, none have so far generated a lasting impact on the models that MFIs employ. Therefore it seems unlikely that a lack of contract innovation is the sole constraining factor on the impact of microfinance.

Our theory, in contrast, assumes that borrowers have the competence to grow their business and that lenders are well aware of the constraints imposed on borrowers by the lending paradigm. Instead we focus on the rents that lenders enjoy from retaining customers and the fact that sufficiently wealthy customers are less reliant on their informal financiers. Therefore our theory may offer a useful, and very different lens with which to understand the disappointing impact of microfinance. For instance our theory suggests that policy makers might increase the impact of microfinance by regulating the types of contracts that lenders can offer, rather than by offering business training to borrowers, or consulting to lenders. Part of the value of this theory, therefore, may stem from the distance between its core logic and that of the primary theories maintained by empirical researchers and policy makers.

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<sup>14</sup>See e.g. [Gine and Karlan \(2014\)](#), [Attanasio et al. \(2015\)](#), and [Carpena et al. \(2013\)](#) on joint liability, [Field et al. \(2013\)](#) on repayment flexibility, and [Feigenberg et al. \(2013\)](#) on meeting frequency.

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## A Model Extensions

### A.1 A Continuous State Model

In this section we extend the model above to have a continuous set of active states before the borrower graduates to the formal sector. In particular, the business is indexed by  $w \in [0, n]$  through which borrowers must grow their businesses before they reach the formal sector after graduating from state  $n$ . As in Section 3.1 we now index all parameters and value functions by  $w$ . Beyond the continuous state-space the model differs from Section 3.1 in two ways:

1. Projects are now discrete and opportunities to perform a project arrive in each interval of length  $dt$  with probability  $\psi dt$ .
2. Projects are now deterministic. When a project opportunity arises,
  - (a) the working capital project continues to deliver  $f_{0,w}$  or  $f_{1,w}$  consumption goods in the early portion of the period depending on whether or not a loan is offered,
  - (b) the fixed capital project continues to deliver  $f_{0,w}$  or  $f_{1,w}$  consumption goods in the late portion of the period depending on whether or not a loan is offered. At the end of the period the state moves from  $w$  to  $w + g_0$  or  $w + g_1$  depending on whether or not a loan is offered.

All other modeling components are the same as in Section 3.1.

We can now strengthen Proposition 1.

**Proposition 11.** *Generically the game has a unique subgame perfect equilibrium (SPE).*

The argument for uniqueness of MPE follows that of Proposition 1. To see how to strengthen it to unique SPE, define  $\bar{w}_1$  such that  $\bar{w}_1 + g_{0,\bar{w}_1} = n$ . Note that when the borrower is in state  $w \in [\bar{w}_1, n]$  he can guarantee himself the formal sector payoff  $U$  plus consumption  $f_{0,w}$  upon the first project arrival. If  $U > \frac{\Psi}{\rho} \max \{f_{1,w} - \bar{k}_w\}$  (i.e. if it is socially efficient for the borrower to grow his business in state  $w$ ), then the lender must offer the borrower an unrestrictive contract in the unique SPE. Otherwise, if  $U < \frac{\Psi}{\rho} \max \{f_{1,w} - \bar{k}_w\}$  then it is socially efficient for the borrower to stay in state  $w$  and the unique equilibrium is for the lender to offer a restrictive contract that meets the borrower's individual rationality constraint. Therefore behavior and continuation values are uniquely pinned down in SPE for states  $w \in [\bar{w}_1, n]$ .

Next define  $\bar{w}_2$  such that  $\bar{w}_2 + g_{0,\bar{w}_2} = \bar{w}_1$ . For states  $w \in [\bar{w}_2, \bar{w}_1]$ , when a project arrives the borrower's outside option is to guarantee herself the payoff of  $B_{w+g_0,w}^* + f_{0,w}$ , which as argued above is uniquely pinned down in SPE. To improve upon the lender's MPE payoff in state  $w$ , she must offer

an unrestrictive contract. To see this note that conditional on offering a restrictive contract in MPE, total surplus is fixed. Therefore to increase total surplus she must switch to an unrestrictive contract. But we know that upon delivering an unrestrictive contract the borrower grows to state  $w' > w + g_{0,w}$ , and we just argued above that SPE payoffs are unique in state  $w'$  and therefore the payoff to any SPE, the payoff to any unrestrictive contract in state  $w$  must coincide with the payoff to that contract in the unique MPE. Therefore equilibrium behavior is unique in states  $w \in [\bar{w}_2, \bar{w}_1]$ . The rest of the argument proceeds by backward induction.

We have the following immediate corollary of Proposition 11.

**Corollary 3.** *When growing to the formal sector is efficient, borrowers in state  $w \in [\bar{w}_1, n]$  receive unrestrictive contracts.*

This corollary is an analogue to Proposition 5, implying that relatively wealthier borrowers receive unrestrictive contracts and reach the formal sector faster than they would in the lender's absence.

Further, we can extend condition 6 for the borrower to receive contracts with probability 1 in state  $w$ :

$$\underbrace{(B_{w+g_{1,w}} + L_{w+g_{1,w}}) - \frac{\psi}{\rho} (f_{1,w} - \bar{k}_w)}_{\text{"efficiency gain" from investing in fixed capital}} \leq \underbrace{\hat{B}_w - \tilde{B}_w}_{\text{"expansion rent" captured by the borrower}} \quad (8)$$

where

$$\tilde{B}_w \equiv f_{0,w} + B_{w+g_{0,w}}^*$$

and

$$\hat{B}_w \equiv \max \{ f_{0,w} + B_{w+g_{0,w}}, B_{w+g_{1,w}} \}$$

Inequality (8) has much the same interpretation as that of (6) in Section 3.1. The left-hand side of Inequality (8) is the efficiency gain of allowing business expansion—the first two terms are the borrower and lender's continuation utilities after the borrower grows, and the third term is the foregone consumption value were the borrower and lender to invest in the working capital project in state  $w$  whenever it arises. The right-hand side is the borrower's expansion rent, which as in Section 3.1 may be positive because the lender cannot fully extract the borrower's value of business growth due to the holdup problem.

Without further structure on the model we cannot characterize the arrangement of restrictive and unrestrictive states. However, the results and intuitions generated in Section 2 could be readily extended to the continuous state model to form similar intuitions for when and why the lender may keep

her borrowers captive. Further, Proposition 10 continues to hold.<sup>15</sup>

## A.2 The Borrower Never Graduates

In principle the model can be extended to the case where the borrower never graduates, so that the state space is  $[0, \infty)$ . Inequality (8) continues to characterize when the lender offers restrictive contracts. Underlying the possibility that the borrower receives a restrictive contract in state  $w$  is that the borrower's expansion rent is larger than the efficiency gain from growth. This occurs when the borrower's productivity in the absence of the lender grows faster than does their joint surplus, which would be the case if the borrower were to gradually reach self-sufficiency (e.g. if  $(f_{1,w} - \bar{k}_w) - f_{0,w}$  is decreasing in  $w$ ).

## A.3 The Borrower Can Operate Both Projects Within a Period

In this section we consider an extension to the two state model in which the borrower can operate both projects within the same period. Suppose the borrower is endowed with 1 unit of labor in each period, and can choose how to allocate his labor between his two projects. Let  $l$  be the labor allocated to the working capital project and  $(1-l)$  be the labor allocated to the fixed capital project. Suppose that when  $k \in \{0, \bar{k}\}$  capital is allocated to the working capital project it returns  $lf_0$  or  $lf_1$  output in the early portion of the period, and similarly when  $(1-l)$  energy and  $k$  capital are allocated to the fixed capital project it returns  $(1-l)f_0$  or  $(1-l)f_1$  output in the late portion of the period and  $(1-l)g_0dt$  or  $(1-l)g_1dt$  probability of growth.

Suppose further that when the lender offers a restrictive contract  $c = \langle R, 1 \rangle$ , the borrower must invest enough labor and capital to repay  $R$  in the early period but then is free to invest the remainder of both into the fixed capital project.

It is straightforward to see that Inequality (7) will still determine when the borrower receives a restrictive contract and remains captive so long as

$$\max_l \frac{lf_1 + (1-l)f_0 - R}{\rho} \left( 1 - \frac{(1-l)g_0}{\rho + (1-l)g_0} \right) + \frac{(1-l)g_0}{\rho + (1-l)g_0} B_{w+1}^*$$

such that

$$lf_1 \geq R$$

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<sup>15</sup>The definitions of contiguous states of borrower captivity must be suitably redefined to accommodate the continuous state space. For general  $l \geq 1$  we now define  $\bar{w}_l \equiv \sup \{w < \underline{w}_{l+1} : p_w^* = 1\}$  and  $\underline{w}_l \equiv \sup \{w \leq \bar{w}_l : p_w^* < 1\}$ . An arbitrary set  $[\underline{w}_l, \bar{w}_l]$  is a contiguous set of states where restrictive contracts are offered with probability 1, and each pure restrictive state is contained in one of these sets.

is less than

$$\frac{f_1 - R}{\rho}$$

Even if the borrower is free to allocate some fraction of his labor to the fixed capital project when he receives a restrictive contract, the fixed capital project is socially efficient, and in autarky he would invest in fixed capital, the borrower may still find it worthwhile to invest all of his labor into the working capital when he receives a restrictive contract in equilibrium. This occurs because capital and labor are complements, so when the repayment demands sufficiently high labor in the working capital project, the borrower may find he prefers to invest the residual of his labor into the working capital project and enjoy the consumption output rather than growing his business slowly.

When Inequality (7) is the valid condition for restrictive contracts in all states, it is straightforward to verify that the remainder of our results hold as well.

#### **A.4 The Borrower is Privately Informed About His Outside Option**

In this section we explore an extension in which the borrower maintains some private information about his outside option. In particular, we augment the model such that the borrower's growth rates  $g_0$  and  $g_1$  are privately known. In particular if he invests in fixed capital we assume he grows at rate  $g_{0,v} = g_0 + v$  or  $g_{1,v} = g_1 + v$ . Let  $v_t \stackrel{\text{iid}}{\sim} F$  be a random variable, privately known to the borrower and redrawn each period in an iid manner from some distribution  $F$ .

Now, if the lender offers the borrower a restrictive contract, she will face a standard screening problem. Because she would like to extract the maximum acceptable amount of income, borrowers with unusually good outside options will reject her offer. This is encoded in the following proposition.

**Proposition 12.** *The borrower may reject restrictive contracts with positive probability.*

This intuitive result offers an explanation for the low take-up of microcredit contracts referenced in the introduction. Lenders who offer restrictive contracts to borrowers aim to extract the additional income generated by the loan, but in doing so lenders are sometimes too demanding and therefore fail to attract the borrower. In contrast, because lenders who offer unrestrictive contracts leave the borrower with excess surplus, demand for these contracts is high.

## **B Additional Interpretation Issues**

In this section we provide additional discussions on how our modeling assumptions map to the institutional features of microfinance.

## Non-profit MFIs

The debt trap in our model arises when a profit-maximizing lender prolongs the period over which she can extract rents from her borrower. Yet the low returns from microfinance have been observed across a range of microfinance institutions spanning both for-profit and non-profit business models. We believe there are a number of ways in which the forces identified in our model might similarly apply to non-profit MFIs. First, because the two business models share many practices, features that are adaptive for profit-maximizing MFIs may have been adopted by non-profits. A second possibility operates through the incentives of loan officers who are in charge of originating and monitoring loans. Across for-profit and non-profit MFIs many loan officers are rewarded for the number of loans they manage, so losing clients through graduation may not be in their self-interest. Put another way, even in non-profit MFIs, the loan officers often have incentives that make them look like profit-maximizing lenders.

A final possibility is that, while non-profits may not aim to maximize rent extraction, they may still need to respect a break-even constraint. If microfinance institutions incur losses to serve their poorest borrowers, then they may need to inhibit their wealthier borrowers' ability to graduate in order to maintain profitability.

## Infinite stream of borrowers

One crucial feature of our debt trap is that when the borrower becomes wealthy enough to leave his lender, the lender loses money. In reality there are many unserved potential clients in the communities in which MFIs operate. Why, then, can't an MFI offer unrestrictive contracts and then replace borrowers who have graduated with entirely new clients? The proximate answer is that unserved clients are unserved primarily because they have no demand for loans (e.g. [Banerjee et al. \(2014\)](#)). This may be unsatisfactory, as demand for microfinance would presumably increase if the lender lifted contractual restrictions and allowed the borrowers to invest more productively. But even in this case, the pool of potential borrowers who would find this appealing is likely limited. For instance, [Banerjee et al. \(2017\)](#) and [Schoar \(2010\)](#) argue that only a small fraction of small-scale entrepreneurs are equipped to put capital to productive use. Thus, it is reasonable to assume that MFIs lose money when a borrower terminates the relationship.

## C Omitted Proofs

This section contains all omitted proofs. Recall that  $\alpha \equiv \frac{g^{aut}}{\rho + g^{aut}}$ ,  $\beta \equiv \frac{g}{\rho + g}$ , and we now define  $\delta(p) \equiv \frac{pg^{aut} + (1-p)g}{\rho + pg^{aut} + (1-p)g}$ .

## Proof of Lemma 1

Suppose in equilibrium, the lender offers the restrictive contract  $\langle \tilde{R}, 1 \rangle$  with probability 1. That  $\tilde{R} = y - \rho \tilde{B}$  follows from the fact that the lender sets the repayment at the highest level that surpasses the borrower's outside option.

It remains to show that  $\tilde{B} = B^{aut}$ . First, note that if (off the equilibrium path) the borrower were to reject the lender's contract, he would invest in fixed capital. This follows from the fact that if the borrower had a weak preference to invest in working capital upon rejecting the contract, the borrower would also accept a restrictive contract with repayment  $R = y - y^{aut}$ . In such an equilibrium, the borrower's value would be  $B = \frac{y^{aut}}{\rho}$ , which, by Assumption 1, would contradict the borrower's weak preference for investing in working capital.

Hence the borrower's outside option is to invest in the fixed capital project. Because the lender sets repayment  $\tilde{R}$  such that the borrower is indifferent between accepting the contract versus his outside option, the borrower's continuation utility is  $\tilde{B} = (1 - \alpha) \frac{y^{aut}}{\rho} + \alpha U \equiv B^{aut}$ .  $\square$

## Proof of Lemma 2

We first demonstrate that for an unrestrictive contract  $\langle \hat{R}, 0 \rangle$  offered in equilibrium, the borrower's equilibrium value is  $\hat{B} = \max \{ \beta U, B^{aut} \}$ . If  $\beta U \geq B^{aut}$ , then the unrestrictive contract that specifies  $\hat{R} = y$  and induces  $\hat{B} = \beta U$  satisfies the borrower's IR constraint. To see this note that if the borrower were to reject the contract and invest in fixed capital his flow payoff would be

$$\begin{aligned} y^{aut} + g^{aut} (U - \beta U) &\leq y^{aut} + g^{aut} (U - B^{aut}) \\ &\leq g(U - \beta U) \end{aligned}$$

where the first inequality follows by the assumption that  $\beta U \geq B^{aut}$  and the second follows by the fact that  $g(U - \beta U)$  is the borrower's equilibrium flow payoff, corresponding to value  $\beta U$ , and that  $y^{aut} + g^{aut} (U - B^{aut})$  is the borrower's autarkic flow payoff, corresponding to value  $B^{aut}$ .

If instead  $\beta U < B^{aut}$  then the inequalities above would be reversed and the unrestrictive contract  $R = y$  would not meet the borrower's IR constraint. Therefore the borrower's IR constraint would always bind and by the logic in Lemma 1 the borrower's equilibrium value would be  $\hat{B} = B^{aut}$ .

That the lender's equilibrium payoff is  $\hat{L} = \left[ \beta U + (1 - \beta) \left( \frac{y - \kappa}{\rho} \right) \right] - \hat{B}$  follows from the fact that the first term is the sum of the borrower and lender's values in the active state and the second term is the borrower's value in the active state.  $\square$

## Proof of Propositions 1 and 2

The existence of an equilibrium follows standard arguments (see Maskin and Tirole (2001)). In this section we prove that generically the equilibrium is unique.

The logic in the proof of Lemma 1 establishes that if the borrower receives a restrictive contract in equilibrium, he receives his outside option flow payoff

$$y^{aut} + g^{aut}(U - B^*).$$

In contrast, the logic in the proof of Lemma 2 establishes that if the borrower receives an unrestrictive contract in equilibrium, his flow payoff may strictly exceed his outside option, namely he receives

$$\max \{y^{aut} + g^{aut}(U - B^*), g(U - B^*)\}.$$

Now conjecture that in equilibrium the lender offers the borrower a restrictive contract with probability  $p^*$ . Consider the following two cases.

**Case 1:**  $B^{aut} \geq \beta U$ .

In this case, by the logic in the proof of Lemma 2, the borrower's outside option binds for both unrestrictive and restrictive contracts. Her equilibrium payoff is therefore  $B^* = B^{aut}$ , independent of the equilibrium probability of restrictive contracts. The lender's value from offering restrictive contracts is  $\tilde{L}$  and her value from unrestrictive contracts is  $\hat{L}$ . Generically one of the two will be strictly greater and in the unique equilibrium she therefore offers the corresponding contract with certainty.

**Case 2:**  $B^{aut} < \beta U$  In this case the logic in the proof of Lemma 2 dictates that upon receiving an unrestrictive contract the borrower's flow payoff is

$$g(U - B^*) > y^{aut} + g^{aut}(U - B^*),$$

and her IR constraint is strictly satisfied. To see this note that

$$g(U - B^*) \geq g(U - \hat{B}) = \rho \hat{B}$$

with strict inequality when  $p^* < 1$ , and

$$y^{aut} + g^{aut}(U - B^*) \leq \rho B^* \leq \rho \hat{B}$$

where the second inequality is strict when  $p^* > 0$ , as  $B^*$  is derived below.

Therefore the borrower's equilibrium flow payoff (unconditional on the contract she is offered) is

$$\rho B^* = p^* (y^{aut} + g^{aut} (U - B^*)) + (1 - p^*) g (U - B^*)$$

Hence

$$(\rho + g^{aut}) B^* = (\rho + g^{aut}) (p^* \tilde{B} + (1 - p^*) \hat{B}) + (1 - p^*) (g - g^{aut}) (\hat{B} - B^*)$$

and therefore

$$B^* = \frac{p^* (\rho + g^{aut}) \tilde{B} + (1 - p^*) (\rho + g) \hat{B}}{\rho + p^* g^{aut} + (1 - p^*) g}$$

Clearly  $B^*$  is decreasing in  $p$ .

When the lender offers a restrictive contract, it is accompanied by a repayment

$$\begin{aligned} R &= y - (y^{aut} + g^{aut} (U - B^*)) \\ R &= y - \left( y^{aut} + \frac{g^{aut}}{\rho + \delta(p^*)} (\rho U - p^* y^{aut}) \right) \end{aligned}$$

The repayment the borrower demands in return for accepting a restrictive contract is increasing in the probability she expects restrictive contracts in equilibrium, as her value of remaining in the active state is increasing in the probability of restrictive contracts.

By the assumption that  $\beta U > B^{aut}$ , when she offers an unrestrictive contract, the lender offers a repayment  $R = y$ .

If she offers a restrictive contract in equilibrium (and hence has at least a weak preference for restrictive contracts) her equilibrium value is

$$\begin{aligned} L^* &= \frac{R - \kappa}{\rho} \\ &= \frac{y - \kappa}{\rho} - \left( \frac{y^{aut}}{\rho} + \frac{g^{aut}}{\rho + \delta(p^*)} \left( U - p^* \frac{y^{aut}}{\rho} \right) \right) \end{aligned}$$

If she offers an unrestrictive contract in equilibrium (and hence has at least a weak preference for unrestrictive contracts) her equilibrium value is

$$L^* = (1 - \beta) \frac{y - \kappa}{\rho}.$$

That conditional on offering restrictive contracts the lender's value is decreasing in the probability of

restrictive contracts, and that conditional on offering unrestrictive contracts the lender's value is constant in the probability of restrictive contracts implies the two can be equal at at most one probability  $p^*$  of restrictive contracts. Hence the equilibrium is unique.

When the lender offers unrestrictive contracts with positive probability, her continuation value is

$$L^* = \hat{L} = (1 - \beta) \frac{y - \kappa}{\rho}.$$

When she offers restrictive contracts with certainty her continuation value is

$$\begin{aligned} L^* &= \tilde{L} \\ &= \frac{y - \kappa}{\rho} - \tilde{B} \end{aligned}$$

Finally, we derive the equilibrium probability  $p^*$  of a restrictive contract, when  $p^*$  is interior.

If the lender is indifferent between restrictive and unrestrictive contracts we have

$$\begin{aligned} (1 - \beta)(y - \kappa) &= (y - \kappa) - (y^{aut} + g^{aut}(U - B^*)) \\ &\implies \\ \beta(y - \kappa) &= y^{aut} + g^{aut}(U - B^*) \end{aligned}$$

Noting that  $\rho \hat{L} = (1 - \beta)(y - \kappa)$ ,  $\rho \tilde{L} = (y - \kappa) - \rho \tilde{B}$ , we have that

$$\begin{aligned} \rho(\tilde{L} - \hat{L}) + y^{aut} + g^{aut}(U - \tilde{B}) &= \beta(y - \kappa) \\ &= y^{aut} + g^{aut}(U - B^*) \\ &\implies \\ \rho(\hat{L} - \tilde{L}) &= g^{aut}(B^* - \tilde{B}) \\ &= g^{aut} \left( \frac{(1 - p^*)(\rho + g)(\hat{B} - \tilde{B})}{\rho + p^*g^{aut} + (1 - p^*)g} \right) \end{aligned}$$

The proof can trivially be extended to the many state model via backward induction.  $\square$

### Proof of Propositions 3, 4, and 6

When  $p^*$  is interior, it is implicitly defined by the following equation, which sets the lender's equilibrium value from unrestrictive contracts,  $\hat{L}$ , equal to her value from offering restrictive contracts

with probability  $p^*$ .

$$(1 - \beta) \frac{y - \kappa}{\rho} = \frac{y - \kappa}{\rho} - \left( \frac{y^{aut}}{\rho} + \frac{g^{aut}}{\rho + \delta(p^*)} \left( U - p^* \frac{y^{aut}}{\rho} \right) \right)$$

The left-hand side is invariant in  $g^{aut}$ . Holding fixed  $p^*$ , the right-hand side is decreasing in  $g^{aut}$ . Therefore, to maintain equality,  $\frac{d}{dg^{aut}} p^* < 0$ .

The left-hand side is decreasing in  $g$ . Holding fixed  $p^*$ , the right-hand side is increasing in  $g$ . Therefore, to maintain equality,  $\frac{d}{dg} p^* > 0$ .

The left-hand side is invariant in  $U$ . Holding fixed  $p^*$ , the right-hand side is decreasing in  $U$ . Therefore, to maintain equality,  $\frac{d}{dU} p^* < 0$ .  $\square$

## Proof of Proposition 5

Recall that when the lender offers restrictive contracts with probability 1, her value is

$$\tilde{L} = \frac{y - \kappa}{\rho} - \frac{g^{aut}}{\rho + g^{aut}} U.$$

For  $g^{aut}$  sufficiently large,  $\tilde{L} < 0$ . In contrast,  $\hat{L} > 0$  for all values of  $g^{aut}$  and  $g$ . Hence when both are large the lender never offers restrictive contracts with probability 1.  $\square$

## Proof of Proposition 7

When  $p_1^*$  is interior, it is implicitly defined by the following equation, which sets the lender's equilibrium value from unrestrictive contracts,  $\hat{L}_1$ , equal to her value from offering restrictive contracts with probability  $p_1^*$ .

$$(1 - \beta_1) \frac{y_1 - \kappa_1}{\rho} + \beta_1 L_2^* = \frac{y_1 - \kappa_1}{\rho} - \left( \frac{y_1^{aut}}{\rho} + \frac{g_1^{aut}}{\rho + \delta_1(p_1^*)} \left( B_2^* - p_1^* \frac{y_1^{aut}}{\rho} \right) \right) \quad (9)$$

First suppose  $p_2^* = 1$ , so that

$$\begin{aligned} L_2^* &= \tilde{L}_2 \\ &= \frac{y_2 - \kappa_2}{\rho} - \tilde{B} \\ &= \frac{y_2 - \kappa_2}{\rho} - \left( (1 - \alpha_2) \frac{y_2^{aut}}{\rho} + \alpha_2 U \right). \end{aligned}$$

Clearly  $\frac{d}{dU}L_2^* < 0$  and  $\frac{d}{dU}B_2^* = \frac{d}{dU}\tilde{B}_2 > 0$ . Moreover, because the lender offers a restrictive contract with probability 1 in state 2, increasing  $U$  does not change the sum of  $L_2^* + B_2^*$ . Hence,  $\frac{d}{dU}L_2^* = -\frac{d}{dU}B_2^*$ . Rearranging Equation (9) we have

$$\beta_1 L_2^* + \frac{g_1^{aut}}{\rho + \delta_1(p_1^*)} \left( B_2^* - p_1^* \frac{y_1^{aut}}{\rho} \right) = \frac{y_1 - \kappa_1}{\rho} - \frac{y_1^{aut}}{\rho} - (1 - \beta_1) \frac{y_1 - \kappa_1}{\rho}$$

The left-hand side is invariant in  $U$  and the right-hand side is decreasing in  $U$ . Hence  $\frac{dp_1^*}{dU} > 0$ .

Now consider the case where  $p_2^* < 1$ . Then  $L_2^* = \hat{L}_2$  is invariant in  $U$ . Therefore the left-hand side of Equation (9) is invariant in  $U$  while, holding  $p_1^*$  fixed, the right-hand side is decreasing in  $U$ . Therefore  $\frac{dp_1^*}{dU} < 0$ .  $\square$

## Proof of Proposition 8

Consider an example in which  $p_2^* = 1$  and the lender has a strict preference for restrictive contracts,  $p_1^* = 1$  and the lender is indifferent between the two contracts, and  $\rho^L = \rho^B$ .

It is straightforward to show that the lender's indifference condition in state 1 implies that

$$\beta_1^L L_2^* + \beta_1^B B_2^* - \beta_1^L \frac{y_1 - \kappa_1}{\rho^L} = \beta_1^B B_2^* - \left( \frac{y_1^{aut}}{\rho^L} (1 - \alpha_1^B) + \alpha_1^B \frac{\rho^B}{\rho^L} B_2^* \right) \quad (10)$$

where  $\beta_w^L = \frac{g_w}{\rho^L g_w}$ , and  $\beta_w^B$  and  $\alpha_w^B$  are defined similarly.

Consider first the thought experiment of raising the borrower's patience (decreasing  $\rho^B$ ) in state 2 but holding it fixed in state 1. This makes the borrower more demanding of restrictive contracts in equilibrium as he values growth more highly, and therefore has the same consequence as increasing  $U$ . So just as in Proposition 7, this pushes the lender to offer more restrictive contracts in state 1.

Now consider the thought experiment of raising the borrower's patience in state 1 but holding it fixed in state 2. This has the reverse consequence of raising the borrower's value of business growth in state 1 and increasing the lender's preference for unrestrictive contracts.

Examination of Equation (10) demonstrates that this latter force is second order when  $\rho^B$  and  $\rho^L$  are large. Hence decreasing  $\rho^B$  can on net increase the probability of restrictive contracts in equilibrium.  $\square$

## Proof of Proposition 9

This is a straightforward consequence of the fact that the lender's flow payoff from unrestrictive contracts is always weakly higher than her flow payoff from restrictive contracts.  $\square$

## Proof of Proposition 10

The proof is similar to that of Proposition 7.

First consider state  $n$ . If  $\bar{w}_m \neq n$  then  $p_n^* < 1$ . If  $p_n^* = 0$  then the theorem holds trivially. Else, if  $p_n^* \in (0, 1)$  then it is determined by

$$(1 - \beta_n) \frac{y_n - \kappa_n}{\rho} = \frac{y_n - \kappa_n}{\rho} - \left( \frac{y_n^{aut}}{\rho} + \frac{g_n^{aut}}{\rho + \delta_n(p_n^*)} \left( U - p_n^* \frac{y_n^{aut}}{\rho} \right) \right)$$

As in the prior analysis,  $\frac{d}{dU} p_n^* < 0$ ,  $\frac{d}{dU} B_n^* > 0$ , and  $\frac{d}{dU} L_u^* = 0$ .

The proof proceeds in the same manner for all states  $w > \bar{w}_m$ .

Next consider state  $\bar{w}_m$ . By definition  $p_{\bar{w}_m}^* = 1$  and generically the lender has a strict preference for restrictive contracts. Hence the lender's value  $\frac{d}{dU} L_{\bar{w}_m}^* = \frac{d}{dU} \tilde{L}_{\bar{w}_m} < 0$  and  $\frac{d}{dU} B_{\bar{w}_m}^* = \frac{d}{dU} \tilde{B}_{\bar{w}_m} > 0$ , as  $\frac{d}{dU} B_{\bar{w}_m+1}^* > 0$ . This remains true for all  $w \in \{\underline{w}_m, \dots, \bar{w}_m\}$ .

Now consider state  $\underline{w}_m - 1$ . As in the proof of Proposition 7, because  $p_{\underline{w}_m+1} = 1$ , we have that have established that  $\frac{d}{dU} L_{\underline{w}_m}^* = -\frac{d}{dU} B_{\underline{w}_m}^*$ . Therefore following the same steps as in the proof of Proposition 7 we can show that  $\frac{d}{dU} p_{\underline{w}_m-1}^* \leq 0$  with strict inequality if  $p_{\underline{w}_m-1}$  is interior. The lender's value  $\frac{d}{dU} L_{\underline{w}_m-1}^* = \frac{d}{dU} \hat{L}_{\underline{w}_m-1} < 0$ . The borrower's value

$$B_{\underline{w}_m-1}^* = \left( 1 - \frac{\delta_{\underline{w}_m-1}(p_{\underline{w}_m-1}^*)}{\rho + \delta_{\underline{w}_m-1}(p_{\underline{w}_m-1}^*)} \right) \frac{p_{\underline{w}_m-1}^* y^{aut}}{\rho} + \frac{\delta_{\underline{w}_m-1}(p_{\underline{w}_m-1}^*)}{\rho + \delta_{\underline{w}_m-1}(p_{\underline{w}_m-1}^*)} B_{\underline{w}_m}^*$$

Raising  $U$  imposes two forces on  $B_{\underline{w}_m-1}^*$ . First, it increases  $B_{\underline{w}_m}^*$  which increases  $B_{\underline{w}_m-1}^*$ . Second it reduces  $p_{\underline{w}_m-1}^*$  which reduces  $B_{\underline{w}_m-1}^*$ . As in the proof of Proposition 8, when the borrower is sufficiently impatient, the latter force dominates and  $B_{\underline{w}_m-1}^*$  decreases.

In this case the proof proceeds in reverse for states  $w \in \{\underline{w}_{m-1} - 1, \dots, \underline{w}_{m-1} - 2\}$ , reverses once again for states  $w \in \{\underline{w}_{m-2} - 1, \dots, \underline{w}_{m-2} - 2\}$ , and so on.  $\square$

## Proof of Proposition 11

The proof for uniqueness of MPE follows that of Proposition 1. The proof that the unique MPE is the unique SPE closely follows the verbal argument in the text and is thus omitted.  $\square$

## Proof of Proposition 12

Let the distribution  $F$  be such that  $v = 0$  with probability  $1 - \varepsilon$  and  $v = \frac{1}{\varepsilon}$  with probability  $\varepsilon$ . Fix all parameters such that when  $\varepsilon = 0$  the lender offers restrictive contracts with probability 1. Then fixing

all other parameters and for  $\varepsilon$  sufficiently low, if the lender continues to offer restrictive contracts with probability 1. Moreover, when Assumption 2 holds, for  $\varepsilon$  sufficiently low, the lender cannot afford to satisfy the borrower's IR constraint when  $v = \frac{1}{\varepsilon}$ . Hence the borrower rejects the restrictive contract with probability  $\varepsilon$  on the equilibrium path.  $\square$