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Abstract

I present a model of financing social enterprises to delineate the role of impact investors relative to "pure" philanthropists. I characterize the optimal scale and structure of a social enterprise when financed by grants, and when financed by investments. The analysis yields two heuristics to guide impact investors. First, investments allow a financier to discipline inefficient spending. Second, investments may enable a social enterprise to exploit new opportunities for profit, and may increase the enterprise's scale relative to when grant financed. I quantify these heuristics for the case of Husk Power, a social enterprise that has received impact investment.

Firms that create considerable social or environmental value without also generating positive discounted cash flows are often financed through government grants and philanthropic donations. However over the last several decades a growing class of *impact investors* has utilized the traditional tools of entrepreneurial finance to support some of these social enterprises. This paper is primarily concerned with private debt and equity impact investors, who subscribe to the view that by investing in companies with the dual mandate of profits and purpose, investors can achieve more of each than would be possible by pursuing these goals separately. Recent estimates indicate that the various types of private debt and equity investors with this dual objective manage upwards of $500 billion.[1] By comparison, total charitable giving in the United States was just over $410 billion in 2017.[2] Moreover, about 41% of large foundations in the United States now report using investments as a partial

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[1] See Mudaliar et al. (2019).

[2] See Giving USA (2017).

alternative to traditional grant-making activities to achieve their programatic goals.[3]

This paper addresses two fundamental questions in impact investing. When does an investment in a social enterprise generate more impact than a grant? And how do the optimal size and structure of a social enterprise depend on whether it receives an investment or grant? In considering these questions impact investors often suggest that they help firms scale and reach financial sustainability.[4] I argue that an analysis from first principles should recognize that scale and financial sustainability are choices about a firm's business model, which can be determined independently from choices about a firm's financing; even for-profit businesses sometimes receive grant financing. A second common assertion is that by making investments rather than donations, impact investors can expand the capital supply available to social enterprises.[5] While certainly correct, relative to donations investments impose a cost on social enterprises, who must one day relinquish even more capital than they initially raised. An explicit consideration of this cost is necessary in the analysis of when investment is superior to donation in supporting social enterprises.

To address these questions I present a simple model that isolates the relationship between a single investor and entrepreneur and allows for the separate determination of business model and source of financing. In particular I study a social entrepreneur who is distinguished from a "traditional" entrepreneur by the fact that he values not only profits but also the consumer welfare and aggregate externalities deriving from his production. I refer to these latter two objectives as the entrepreneur's mission. The entrepreneur chooses two factors: invested capital and the price charged for the good. In general, "price" is a stand-in for any business model decision that directly influences both the profitability and social impact of the organization. A financier must determine how much capital to funnel to, or extract from, the enterprise in each period.

At baseline I abstract from any incomplete information or incentive misalignment; rather, the financier and entrepreneur jointly maximize social welfare. I consider the following thought experiment. First, I characterize the optimal scale and structure of the social enterprise when the financier is constrained to only offer grants – i.e. the financier can continuously subsidize the firm's operation but cannot extract profits. Second, as a function of the enterprise's solution under this *grant financing regime*, I ask when allowing the financier to make investments, and freely extract profits from the firm, leads to a strictly better outcome. Because at baseline there is perfect incentive alignment, this latter *impact investing regime* can be understood as the first-best arrangement. The primary question of interest is then when does grant financing achieve the first-best outcome, and when not, how does investment financing differ in terms of its consequences on the social enterprise?

[3] See Buchanan et al. (2015).
[4] See eg. Cohen (2014), Hopkins and Olvera (2013), and Huppé and Silva (2013).
[5] See eg. Abt (2018), Keeler (2017), Ogden et al. (2018), Etzel (2015) and Faella and Gifford (2017).

The first contribution of the model is a definition and characterization of sustainable organizations. An organization has reached sustainability when it is a net distributor to its financiers rather than a net receiver. The model highlights that the decision to be organizationally sustainable depends on the balance of two concerns. First, does the financier prefer to retain her capital at the cost charging a fee to the firm's customers? A financier committed to social impact may find that she has no opportunities as impactful as allowing the firm's customers to retain their money, in which case she may prefer to continually finance operations. The second concern is the tradeoff between the firm's societal impact and its profits. In many cases the decision to maximize profits entails a compromise on social impact. For example, a firm may increase its profits by raising its prices, but doing so may exclude high value, low ability to pay customers.

For organizations supported by donations, I identify two kinds of sustainability. *Satiated firms* can finance their socially efficient scale through revenue generated from the sale of their goods and services. In contrast, *hungry firms* are constrained by their customers' willingness to pay for their goods and services, and operate below their socially efficient scale. Note that in both cases, the socially efficient scale of a sustainable firm is judged according to the value of customers' capital, rather than the financier's, as by definition the firm is sustained by payments from its customers.

Next, I demonstrate there is indeed a link between organizational sustainability and impact investing. In contrast to the claim that impact investing leads to organizational sustainability, I demonstrate that firms for which organizational sustainability is optimal when they are financed by grants are prime candidates for impact investments. When financed by grants alone, sustainable firms are not disciplined by the financier's cost of capital as they no longer rely on it. In contrast, because impact investors can extract profits from their portfolio firms, all such firms are disciplined by the cost of their financier's capital. In general, impact investors can either increase or decrease the scale of their portfolio firms relative to if they were financed by grants alone, and their consequence on the firm's scale depends on its kind of sustainability.

The shift from donations to investments creates two additional considerations for the firm's scale. First, a financier who makes an investment can withdraw profits from her portfolio firm if and when the firm reaches profitability. This is appealing in situations where the financier anticipates that the social entrepreneur will build a financially sustainable business but will exhaust his most impactful opportunities for investment or will extract a large fraction of his firm's profit through the distribution of retained earnings or overhead.[6] This result resembles the classic result of Jensen (1986), that investors can discipline their portfolio firms' inefficient usage of free cash flow. On its own this consideration enables the financier to reduce the long-run scale of her portfolio firm's operations.

The second consideration pushes in the opposite direction. Because she is a partial owner of the

[6]For ease of distinction, I arbitrarily refer to the entrepreneur as "he" and the financier as "she."

firm and its cash flows, an investor places higher value on the firm's profits than would a donor. Therefore relative to firms financed through donations alone, firms financed by impact investors have stronger incentive to exploit opportunities to expand their scale and profitability. This consideration is only relevant, however, when a firm has opportunities to generate profit that are somewhat, but not extremely misaligned with the firm's primary mission; even under grant financing a firm should exploit all opportunities that increase both profit and mission.

I demonstrate that relative to grant financing, impact investors may expand the scale of satiated firms precisely when they have unexploited opportunities for profit that are not severely misaligned with their social impact. In contrast, even though hungry firms operate below their socially efficient scale, judged according to their customers' cost of capital, impact investors always reduce their scale relative to if they were grant financed. By definition, hungry firms have exhausted all of their opportunities to generate profit by increasing their scale, and hence an investor reduces their scale until the marginal social value of their output is equated with the financier's, rather than the customers' cost of capital.

In both cases, the impact investor employs her comparative advantage in extracting and utilizing the profits of the firms she supports. Firms that cannot attract commercial investors have negative net present value. But once a socially oriented investor has subsidized their establishment, profitable opportunities to either expand or reduce their steady-state scale may arise. While exploiting these opportunities may inhibit the firm's narrowly defined social mission (i.e. the sum of the aggregate externality and consumer surplus), impact investors increase the total social impact of the firm, accounting for the value of additional profits.

Next I analyze the importance of the alignment between the firm's mission and its profits. Impact investors often refer to this alignment as "colinearity" or "mission lock." I discuss three distinct notions of this alignment that may offer some guidance as to when a firm is suitable for impact investment.

After characterizing the baseline model I analyze an extension in which there is a misalignment of incentives between the entrepreneur and financier. In particular the two parties may disagree about the relative importance of profits and social mission. I demonstrate that when the entrepreneur places less weight on profits than does the financier, the intuitions outlined above are virtually unchanged. In contrast, when the entrepreneur values profits more highly than does the financier, there is no longer a channel by which impact investment can induce an expansion of the firm's scale relative to grant financing. Instead, the primary way by which investments deliver additional value over grants is by reallocating the firm's marginal profits away from the private consumption of the entrepreneur.

This setting introduces a distinction between debt and equity. Debt allows the financier to place

a lower bound on how much profit the firm must produce (Jensen, 1986), and equity diminishes the weight that the entrepreneur places on profits (eg. Ross, 1973, Holmstrom, 1979). While this is typically viewed as a drawback of equity, my analysis highlights that when the entrepreneur overvalues profit relative to the firm's social mission, muting the firm's desire for profits may be an advantage.[7]

The *non-distribution* constraint on nonprofits may be the most common mechanism by which social entrepreneurs are induced to prioritize mission over profit. However, social enterprises seeking impact investment may sometimes be unable to incorporate as nonprofits, especially in settings where (for reasons outside of the model) equity is the most attractive form of financing. My analysis highlights that in such settings equity may nevertheless be an effective substitute in inducing social entrepreneurs to prioritize their mission over profits. This merit of equity is closely related to the analysis of Glaeser and Shleifer (2001) and Chowdhry et al. (2019), discussed further in the following section.

Finally as a demonstration of how to operationalize the ideas discussed above, I examine the case of Husk Power, a rural electrification company operating mainly in Bihar, India. Utilizing data on its costs of operation as well as estimates of elasticity of demand that it faces from its customers, drawn from Burgess et al. (2017) and Lassiter et al. (2018), I quantify the tradeoff that Husk Power faces between its profit and its social mission. I discuss how this informs whether Husk Power should be optimally supported through grants or investments.

This paper relates to several literatures. A large literature within economics explores the consequences of intrinsic motivation in principal-agent problems. Specifically, these papers posit that employees are motivated not only by the possibility of profit but also by the intrinsic value of their output (eg. Bénabou and Tirole, 2003, 2006, Besley and Ghatak, 2005, Prendergast, 2007). Rather than focusing on optimal incentive provision, I (primarily) abstract from any misalignment of incentives. Instead, under the premise of intrinsic motivation, I study a classic question in finance – what is the optimal mode of financing? While that literature primarily focuses on the distinction between various claims on a firm's assets (eg. Tirole, 2010), that financiers place intrinsic value on a firm's output introduces the possibility that grants are the optimal form of financing. Therefore at baseline I abstract from the distinction between claims on a firm's assets and instead ask when it is optimal to take any claim on the firm's assets versus offering a simple grant.

This paper also relates to the literature on corporate social responsibility, which explores when and why private companies engage in prosocial behavior (eg. Bénabou and Tirole, 2010). Rather than exploring when engaging in corporate social responsibility improves a firm's profitability (eg. Shleifer

[7]Models with non-pecuniary benefits in corporate finance typically posit that the manager overvalues them relative to the financier. The merit of equity in this setting arises from the possibility that the financier weighs non-pecuniary social benefit more highly than the firm's management.

and Summers, 1988, Besley and Ghatak, 2007, Margolis et al., 2007), I posit that the firm directly values social goals in addition to profits, and provide a tractable framework to study its operational decisions. In this sense my paper is related to Nilsson and Robinson (2017).

There is also a growing literature on impact investing, discussion of which is deferred to the following section.

1 A Brief Primer on Impact Investing

The term *impact investing* has evolved to encompass a wide range of investing activities, both in public markets and in privately held companies.[8] Impact investing sometimes refers to efforts to align public equity holdings with an investor's values, such as when investors prioritize holding shares of firms that meet certain environmental, social, and governance standards. In contrast, this paper is concerned with the segment of investors who allocate their capital with the explicit intention of creating social value in addition to private financial returns. The 2019 Annual Impact Investor Survey orchestrated by the Global Impact Investing Network estimates this market to comprise upwards of $500 billion in assets under management, about 69% of which are targeted toward privately held companies.[9]

Though there are no universal criteria for what constitutes an impact investment, the sectors representing the largest share of impact investing dollars are affordable housing, microfinance, energy, financial services, food and agriculture, water and sanitation, and healthcare. Together these seven sectors receive more than 65% of impact investments.

A variety of asset managers engage in impact investing, including venture capital and private equity firms, development finance institutions, high net worth individuals, family offices, and foundations. Foundations often utilize impact investments – in the form of program related investments (PRIs) – alongside their traditional grant making efforts, with the explicit intention, and requirement, of redeploying any proceeds generated from the investments. PRIs count towards the five percent of the endowment that must be spent on distribution to charitable causes and operating costs, required of foundations to maintain their tax-exempt status. About 41% of large foundations report making investments explicitly to achieve their programatic goals.[10] Since 2009 The Bill and Melinda Gates Foundation, in particular, has allocated more than $1.5 billion to a variety of for-profit and nonprofit firms through PRIs.[11]

Given the range of asset managers involved, it is unsurprising that impact investors also vary based

[8] For more comprehensive reviews of the field and its history, see Cole et al. (2018) and Trelstad (2016).
[9] Unless otherwise stated, all statistics about the impact investing field are drawn from Mudaliar et al. (2019).
[10] See Buchanan et al. (2015).
[11] See Brest (2016).

on whether they explicitly accept investments with below-market risk-adjusted expected returns. The Global Impact Investing Network estimates that 34% of impact investors target below-market returns while the remainder aim to generate market-rate returns while also pursuing social or environmental impact. Regardless of their anticipated financial returns, many impact investors primarily support firms that they hope will one day be able to attract commercial investments.

The body of academic research on impact investing is small but growing. Several papers investigate whether, as a matter of practice, impact investors earn lower returns than their commercial counterparts. Examining financial data from 53 private equity impact funds, Gray et al. (2015) argues that impact investors may achieve competitive financial returns. In contrast, using Preqin data, Barber et al. (2017) finds that self proclaimed impact funds earn on average 4.7% lower internal rate of return than their commercial counterparts, and that the limited partners who support impact funds are often willing to accept below market returns on their investments. Kovner and Lerner (2015) finds that firms supported by community development venture capital firms (CDVCs) are less likely to reach a successful exit than those supported by traditional venture capitalists, though it also concludes that CDVC firms in a region tend to attract traditional venture capitalists. Complicating the debate is that none of these studies have a measure of the impact achieved by each investment.

Taking an approach based on first principles, Brest et al. (2016) argues that for impact investors to achieve real impact they must provide money or support to social enterprises that would not have been provided in the investors' absence. On these grounds, they argue that many impact investments *must* yield below market return, as those that offer market rate returns are likely to have been financed even in the absence of impact investors. Oehmke and Opp (2019) formalize the idea that impact investors must focus on opportunities that would go unfinanced by commercial investors in order to maximize their impact, and study how the efforts of impact and commercial investors can complement one another. Alternatively, impact investors can have impact through different or superior non-monetary support than that of commercial investors. Geczy et al. (2019) provides a theoretical and empirical characterization of the various contractual terms employed by impact investors to monitor and facilitate impact.

In contrast to the aforementioned work, this paper investigates when it is better to support social entrepreneurship through an investment rather than a grant. The answer turns out to be related to the degree to which a socially oriented organization is positioned to generate a profit. In this sense my work is related to Nilsson and Robinson (2017), which investigates when profit oriented firms should engage in corporate social responsibility, and when socially oriented firms should generate profits.

Perhaps most closely related is Chowdhry et al. (2019), which also analyzes a social financier's decision of whether to make an investment or a donation. The authors of that paper focus on an agency problem between commercial and socially oriented investors and note that by diminishing

the commercial investor's equity stake in a firm, the impact investor can discipline the commercial investor to pursue socially desirable objectives. In contrast, I primarily abstract from contracting failures. Instead this paper can be thought of as an analysis of the first-best relationship between a socially oriented financier and a social entrepreneur. In doing so I identify situations in which the financier wishes to make an investment, thereby only partially subsidizing the firm's operations, rather than fully subsidizing the firm's operations through a donation.[12]

2 A Baseline Model of Social Entrepreneurship

Time is discrete and players discount the future at rate β. A social entrepreneur (alternatively referred to as a firm) and financier aim to serve a continuum of customers with mass 1. The social entrepreneur has a project which requires a startup investment α at time 0. If the startup investment is made, at all times 1 onwards the entrepreneur's project transforms capital into output such that he can serve y customers at cost $c(y)$ with $c'(y) > 0$ and $c''(y) > 0$. Each customer has a value v_i for the firm's product, with $v_i \sim U[0,1]$.[13]

In addition to output y, the entrepreneur chooses a price p. Each customer's best alternative use of funds yields a marginal utility of r_c. Thus, at a price of p, each customer i would like to purchase the firm's goods if $v_i \geq pr_c$. Therefore, the social entrepreneur faces a downward sloping demand curve $D(p) = \max\{1 - pr_c, 0\}$.

In addition to the private value v_i that each consumer enjoys, I assume there is a positive externality from the firm's output, represented in aggregate by $E(y) > 0$, with $E'' \leq 0$. That is, when y customers are served, the total positive externality is $E(y)$.

To abstract from dynamic considerations, I study a repeated static environment. Though events within a period unfold simultaneously, they can be understood as happening in sequence. First the firm sets a price for its goods. Then the firm's customers prepay for its goods – the firm need not meet the full demand for its goods at its chosen price. The firm then uses that revenue, potentially in conjunction with additional capital from the financier, to produce and deliver the promised goods to its customers. I assume that the firm cannot save between periods.

[12] The terms "donation" and "investment" are also used differently in Chowdhry et al. (2019) than in the present paper. In Chowdhry et al. (2019), a socially oriented investor and commercial investor can co-invest to support a social enterprise. Donations in Chowdhry et al. (2019) occur when the optimal outcome can be supported by a partial donation from the social investor, with the residual being supported by the commercial investor. In contrast, in my setting that arrangement would map to an investment. I study a single financier and ask when she would fully subsidize a firm's operations, which I refer to as a grant or donation, and when she would only partially subsidize a firm's operations, which I refer to as an investment.

[13] The analysis would be largely unchanged with an arbitrary distribution of v_i. The major exceptions are Propositions 4, which would no longer hold, and Proposition 7 which would instead reference the possibility of local improvements resulting from impact investing, rather than referencing the optimal arrangement under impact investing.

Specifically, a deep pocketed financier has cost of capital $r_f \equiv 1$ (representing the financier's next best use of capital, and contrasting the marginal utility of customers' consumption, r_c). At baseline I abstract from any misalignment of incentives and assume that the financier and entrepreneur jointly choose output y and price $p \geq 0$ to maximize social welfare. Mathematically, they solve

$$\max_{y,p} \left(\int_{1-y}^{1} v dv - pyr_c + E(y) \right) + (py - c(y)) \tag{1}$$

such that

$$y \leq D(p)$$

where $\int_{1-y}^{1} v dv = y - \frac{1}{2}y^2$ is the aggregate consumer welfare.[14] Let $\pi(y,p) \equiv py - c(y)$ represent the firm's profits and $M(y,p) \equiv \left(\int_{1-y}^{1} v dv - pyr_c + E(y) \right)$ represent the aggregate consumer surplus plus the positive externality. I refer to $M(y,p)$ as the firm's *mission*. The above objective function can be rewritten as $M(y,p) + \pi(y,p)$.

At baseline I assume that the entrepreneur cannot extract profit from the firm. He is paid a salary, already accounted for in $c(y)$ and the financier is the sole claimant on the firm's profits. This allows for analytic tractability, and emphasizes that the results to follow do not hinge on a misalignment of incentives. In Section 4 this assumption is relaxed so that the entrepreneur can extract profit from the firm and I demonstrate the qualitative insights are robust.

To identify the circumstances when an investment is superior to a grant, I separately consider two financing regimes. In the ***grant financing regime*** the financier is restricted to only give grants, and therefore she cannot extract positive profits from the firm. Because at baseline the financier is the sole claimant on the firm's profits, this translates to a constraint that the firm cannot generate positive profits. Therefore, in the grant financing regime the entrepreneur and financier jointly solve

$$\max_{y,p} M(y,p) + \pi(y,p) \tag{2}$$

such that

$$y \leq D(p)$$

and

$$\pi(y,p) \leq 0$$

I denote by p^{GF} and y^{GF} the optimal price and output under the grant financing regime. In Section 4 the grant financing regime corresponds to the restriction that the financier cannot extract profit from

[14] For simplicity I assume that when the firm produces y output it is allocated to the y highest value consumers.

the firm, while the entrepreneur is free to do so.

In the ***impact investing regime*****,** the financier can demand a claim on the firm's cash flows in exchange for up-front financing at time 0. At baseline I abstract from the distinction between debt, equity, and other claims on a firm's assets. Therefore the financier and entrepreneur solve the maximization problem outlined in (1), and are not constrained by $\pi(y,p) \leq 0$. I denote by p^{II} and y^{II} the optimal price and output under the impact investing regime.

The distinction between the two regimes is that in the impact investing regime the financier has the option to "recycle" the firm's profits by utilizing them at the same value as the rest of her capital. Though unmodeled, I assume that the financier can either reallocate the firm's profits to a new opportunity, or consume them. By contrast, in the grant financing regime I assume that the firm cannot generate positive profits, and is therefore restricted to invest all revenue into its production. In both regimes the firm is permitted to earn negative profits if it receives continued subsidy from the financier.

Clearly the impact investing regime always performs weakly better than the grant financing regime, as one can always replicate the grant outcome within the impact financing regime. The central question of this paper is when the impact investing regime performs strictly better than the grant financing regime, and how the optimal organizational structure differs under each regime. This analysis can therefore be understood as illuminating the potential benefits of impact investing, which would need to be compared to the unmodeled costs of establishing the relevant capabilities to assess and contract on such deals.

Under both regimes, the financier must decide at time $t = 0$ whether or not to invest α in order to create the firm. As inter-temporal saving is not possible, in all future periods $t \geq 1$ the players maximize the static objective functions above. Therefore at time 0 the financier invests in the firm if and only if the present discounted value of the future stream of value exceeds α.

2.1 A Running Example: Husk Power

I will periodically refer to Husk Power to ground the ideas in this paper. Husk Power is a for-profit social enterprise that has received financing from several impact investors. It provides affordable electricity in rural areas, operating primarily in Bihar, India. Husk produces electricity using solar power and by burning rice husk, both of which are cleaner than the most common alternatives – coal and diesel.

Husk charges a fee for its electricity but because it serves a poor population it has not yet succeeded in attracting commercial financing. However, that Husk generates consumer surplus for an impoverished population and that it produces a positive environmental externality are both reasons

that an impact investor might like to subsidize its operation. These are the two components of the firm's mission outlined in the previous section. Whether the optimal subsidy takes the form of a grant (full subsidy) or investment (partial subsidy) is the central question of this paper.

2.2 Further Discussion of Modeling Assumptions

A few of the model's features warrant further discussion. First, I assume the firm's output produces a positive externality. I assume that the aggregate externality is weakly concave and deliberately allow for the possibility that the aggregate positive externality is *decreasing* in the firm's output y. In the context of Husk Power this assumption might be satisfied if increasing its scale requires it to increase its reliance on rice husk and reduce its reliance on solar power, as the former is relatively more abundant but produces more carbon emissions than the latter. Alternatively Husk's increased scale might reduce the managers' ability to monitor product quality.

If instead the aggregate externality was restricted to be strictly increasing, all of the analysis to follow would be unchanged except for that of Proposition 7. In that proposition I demonstrate that for an impact investor to increase the scale of a firm relative to its optimal scale under grant financing, there must be a non-trivial tradeoff between the firm's scale and its mission.

Second, I assume that customers pay for the service prior to receiving it. Because I do not allow for the possibility of saving between periods, this assumption is necessary for the firm to use its revenue as a substitute for outside capital from the financier. It could alternatively be replaced by granting the firm access to short-term receivables financing to meet its demand each period.

Third, at baseline I have abstracted from any incentive misalignment between the financier and entrepreneur. The impact investing regime can therefore be understood as the first-best financial arrangement. And the comparison between the grant financing regime and the impact investing regime can be understood as investigating when a grant achieves the first-best arrangement, and when not, how the first-best arrangement differs both in terms of how the firm is financed and in terms of the firm's choices of price and scale. I explore the consequences of incentive misalignment in Section 4.

2.3 Comparison to a Neoclassical Profit-Maximizing Firm

This paper analyzes a social entrepreneur, who not only values the profit he generates, $\pi(y,p)$, but also internalizes his impact on aggregate consumer welfare and his positive externality, represented by $M(y,p)$. In contrast, a standard profit-maximizing firm only values its profits. To elucidate the model, in this section we analyze the traditional profit-maximizing firm.

We begin by considering the profit-maximizing firm in the impact investing regime. This corresponds to the standard monopolist's problem. Let $p(y)$ be the inverse demand curve – the maximum

price the firm can charge while still having demand $D(p) = y$. Because the firm is profit-maximizing, it always charges the maximum feasible price for any given quantity produced. The firm therefore solves

$$\max_y p(y)\, y - c(y)$$

In the grant financing regime, the firm is subjected to the constraint that its profits must be non-positive: $\pi(y, p) \equiv py - c(y) \leq 0$. Noting once again that the profit-maximizing firm always charges a price on the inverse demand curve, the optimization problem can be written as

$$\max_y p(y)\, y - c(y)$$

such that

$$p(y)\, y - c(y) \leq 0$$

Therefore the firm's solution is to produce nothing.

3 An Analysis of the Baseline Model

We analyze the model in two stages. First we study the optimal choices of price and output when the financier is constrained to offer grants, paying special attention to when the firm is sustainable under the grant financing regime. We subsequently relax this constraint to identify the circumstances in which a financier would strictly prefer to offer an investment, and the resulting consequences on the firm's structure relative to the grant financing regime.

3.1 Sustainability Under the Grant financing Regime

Our first point of analysis is to identify when a firm is optimally sustainable under the grant financing regime. We define a firm to be sustainable if after time 0, it is fully financed by its own revenue.

Definition 1. A firm is *sustainable under the grant financing regime* if $p^{GF} y^{GF} \geq c\left(y^{GF}\right)$. A firm is *sustainable under the impact investing regime* if $p^{II} y^{II} \geq c\left(y^{II}\right)$.

Our first proposition states that a firm is never sustainable under the grant financing regime if the customers' capital is more valuable than the financier's.

Proposition 1. *If $r_c > 1$, the firm is not sustainable under the grant financing regime.*

Charging a fee for the firm's goods reduces demand. So when money held by the firm's customers is more valuable than money held by the financier (because the financier has no investment opportunities as valuable as directly transferring wealth to the firm's customers), the firm optimally charges a price of 0 and is fully supported by the financier at all times t. From here out, we assume that the

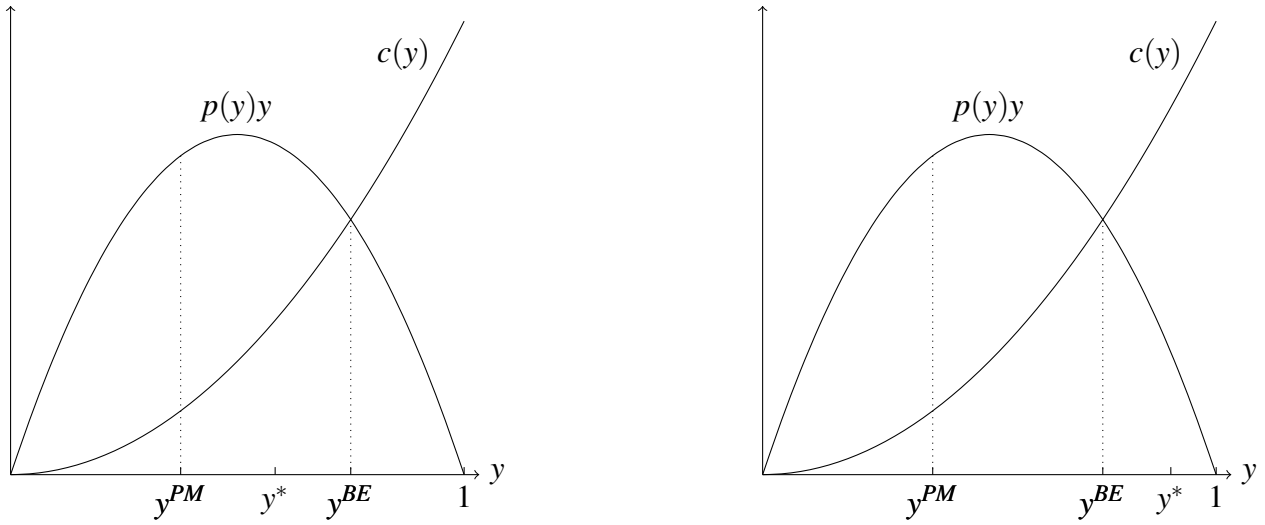

Figure 1

financier's capital is more valuable than the customers': $r_c < 1$.

Next, we aim to develop a graphical understanding of the firm's optimization problem. Recall that $p(y)$ is the inverse demand curve – the maximum price the firm can charge while still having demand y. Define the firm's maximal revenue curve to be $R(y) \equiv p(y)y$ – the revenue that the firm can generate by producing y and charging its customers their maximum willingness to pay.

Figures 1a and 1b plot the firm's maximal revenue curve and cost curve.

There are several points of interest in Figure 1. First, define the "break even" level of output y^{BE} to be the non-zero point at which $p\left(y^{BE}\right)y^{BE} = c\left(y^{BE}\right)$. That is, when the firm produces y^{BE} and charges the maximal price $p\left(y^{BE}\right)$, it generates just enough revenue to cover its costs, $c\left(y^{BE}\right)$. When no such point exists, we define $y^{BE} \equiv 0$. In the appendix we establish the following useful lemma.

Lemma 1. *There is at most one point $y > 0$ such that $p(y)y = c(y)$.*

Next, define y^* and p^* to be the solution to the relaxed variant of the grant financier's optimization problem (2)

$$\max_{y,p} M(y,p) + \pi(y,p)$$

such that

$$\pi(y,p) \leq 0$$

That is, y^* is the level of output the firm would provide if it was unconstrained by its customers' willingness to pay. Because $r_c < 1$, the firm covers its costs with its revenue rather than with its

financier's capital, so it charges its average cost $p = \frac{c(y)}{y}$. Therefore the above problem reduces to

$$\max_{y} M\left(y, \frac{c(y)}{y}\right)$$

which is a concave maximization problem, and y^* solves

$$1 - y^* + E'(y^*) = c'(y^*) r_c$$

Our next result characterizes the firm's choice of p and y when it is sustainable under the grant financing regime.

Proposition 2. *If the firm is sustainable under the grant financing regime it operates at*

$$y^{Sustainable} \equiv \min\left\{y^*, y^{BE}\right\}.$$

The corresponding price is

$$p^{Sustainable} \equiv \begin{cases} \frac{c(y)}{y} & \text{if } y^{Sustainable} = y^* \\ p\left(y^{BE}\right) & \text{if } y^{Sustainable} = y^{BE} \end{cases}$$

The break even level of output y^{BE} is the maximum scale at which the firm can operate without outside capital and y^* is the firm's unconstrained optimal scale when financed exclusively through the firm's own revenue. If the firm is sustainable, it operates at the lesser of these two points.

Definition 2. When a sustainable firm operates at y^* we say the firm is *satiated.* When a sustainable firm operates at y^{BE} we say the firm is *hungry.*

A satiated firm achieves its socially efficient scale, judged according to its customers' cost of capital. A hungry firm operates below its socially efficient scale judged according to its customers' cost of capital, but is constrained by its customers willingness to pay for its goods.

Now that we know where the firm operates when it is optimally sustainable, the remaining question is when the firm chooses to be sustainable versus when it relies on continued subsidy. If $y^* < y^{BE}$ the firm is optimally sustainable under the grant financing regime, as y^* is the firm's optimal scale under the grant financing regime, when unconstrained by customer demand, and when $y^* < y^{BE}$, it is also feasible.

Proposition 3. *If $y^{Sustainable} = y^*$, the firm is sustainable under the grant financing regime. The firm produces y^* and charges $p^* = \frac{c(y^*)}{y^*}$ to cover its costs.*

If $y^* > y^{BE}$ then the firm cannot self-finance at y^*. The following result characterizes when the

firm's optimum is y^{BE} and it is financially sustainable, versus when it relies on continued subsidy.

Proposition 4. *If $y^{Sustainable} = y^{BE}$, the firm is sustainable under the grant financing regime if and only if*

$$\frac{d}{dy}M\left(y^{BE}, p\left(y^{BE}\right)\right) + \frac{d}{dy}\pi\left(y^{BE}, p\left(y^{BE}\right)\right) < 0$$

The decision of whether to operate at y^{BE} or beyond it is a determination of whether the benefit to the firm's mission from expanding its scale outweighs the cost of the subsidy. The cost of the subsidy has two components. First, the financier must pay for the additional output and second, as increasing the supply results in moving down the customers' demand curve, the financier must also cover the resulting loss in revenue. When $y^{BE} < y^*$ and the cost of this subsidy outweighs the benefit, the firm is sustainable and operates at y^{BE}.

3.2 A Graphical Analysis of the Grant Financing Problem

The preceding discussion can be summarized and enriched via the following graphical analysis. Let

$$V^{GF}(y) \equiv \max_{p} M(y,p) + \pi(y,p)$$

such that

$$y \leq D(p)$$

and

$$\pi(y,p) \leq 0$$

be the financier's (and entrepreneur's) value as a function of output under the grant financing problem. Holding fixed $y \leq y^{BE}$, the firm optimally charges the average cost of production, $p = \frac{c(y)}{y}$, is fully financed by revenues, earns 0 profits, and rations its goods. Holding fixed $y > y^{BE}$, by definition the firm cannot finance production through revenue alone. In this case it charges $p(y)$ and the financier covers the shortfall $-\pi(y,p) = c(y) - p(y)$.

Further, note that for any p, the objective function $M(y,p) + \pi(y,p)$ can be rewritten as $M\left(y, \frac{c(y)}{y}\right) + (1 - r_c)\pi(y,p)$, where the first term reflects the firm's mission charging the full cost of production to its customers, and the second term reflects the value of the financier's subsidy (or, in the next section, the value of her profits).

Putting these two insights together we can write the grant financing value function as

$$V^{GF}(y) = M\left(y, \frac{c(y)}{y}\right) + \mathbb{I}\left(y \geq y^{BE}\right)(1 - r_c)\pi\left(y, p(y)\right)$$

where $\mathbb{I}(\cdot)$ is an indicator function. V^{GF} is plotted in Figure 2, for the case where $y^* < y^{BE}$ (2a), and

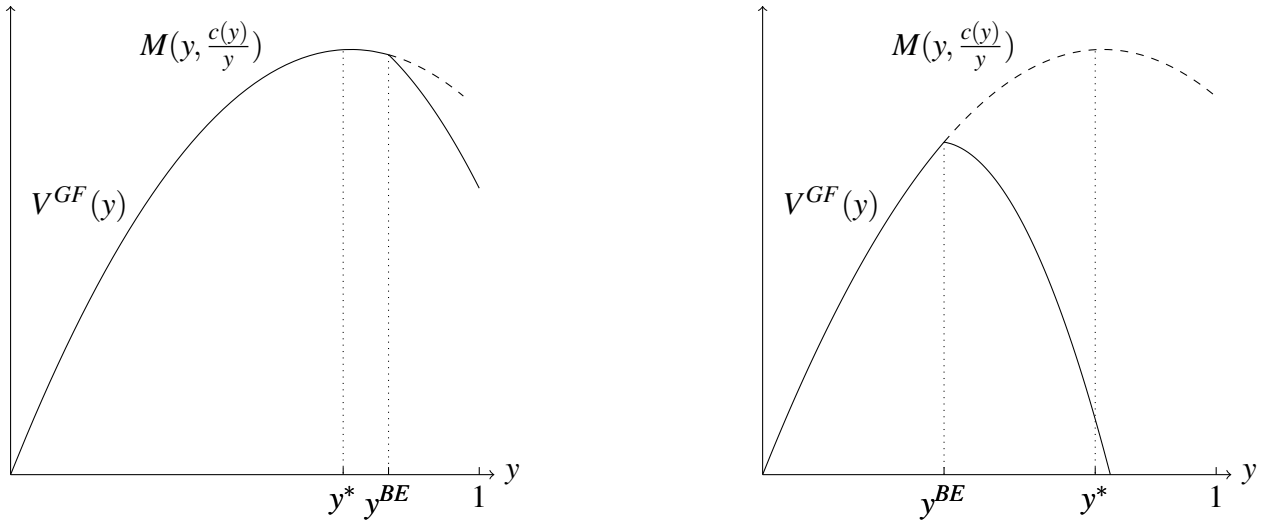

Figure 2

for the case where $y^{BE} < y^*$ (2b).

A few features of Figure 2 are of note. First, V^{GF} is kinked at y^{BE}. Below y^{BE} the firm is sustained through revenue alone and hence is fully supported by the relatively cheap cost of capital r_c belonging to its customers. Beyond y^{BE} the firm requires subsidy from the financier even when charging customers' full willingness to pay, $p(y)$, and hence its marginal cost of capital jumps discontinuously to $r_f \equiv 1$.

Next, as y^* is the peak of $M\left(y, \frac{c(y)}{y}\right)$, when $y^* < y^{BE}$ it coincides with the peak of $V^{GF}(y)$, as is the case in Figure 2a. Therefore, when feasible, y^* is the firm's optimal point of operation in the grant financing regime. Proposition 3 dictates that in this case the firm is sustainable and satiated, as $\frac{d}{dy}V^{GF}(y^*) = \frac{d}{dy}M\left(y^*, \frac{c(y^*)}{y^*}\right) = 0$.

Finally, consider the case where $y^* < y^{BE}$, as in Figure 2b. The firm operates at y^{BE} when

$$\lim_{y \to y^{BE}-} \frac{d}{dy}V^{GF}(y) \equiv \frac{d}{dy}M\left(y^{BE}, c\frac{\left(y^{BE}\right)}{y^{BE}}\right) > 0$$

and

$$\lim_{y \to y^{BE}+} \frac{d}{dy}V^{GF}(y) \equiv \frac{d}{dy}M\left(y^{BE}, \frac{c\left(y^{BE}\right)}{y^{BE}}\right) + \frac{d}{dy}(1-r_c)\,\pi\left(y^{BE}, p\left(y^{BE}\right)\right) < 0.$$

The first condition dictates that at its customers' cost of capital r_c, the firm finds it worthwhile to marginally expand production. The second condition dictates that at the financier's cost of capital $r_f \equiv 1$ it is not worth the necessary subsidy to expand production. The first condition is always met

when $y^{BE} < y^*$, as $M\left(y, \frac{c(y)}{y}\right)$ is concave in y. Proposition 4 therefore dictates that when the second condition is also met, the firm is sustainable and hungry.

Figure 2b depicts the case where the firm is sustainable and hungry (i.e. it operates at y^{BE}). If instead the slope of V^{GF} were positive directly to the right of y^{BE}, the figure would depict a firm whose optimal choice of operation would be beyond y^{BE}. In that case, the financier would find that the marginal value of additional output exceeded the necessary subsidy and the firm would not be sustainable under the grant financing regime.

Appendix Figure 5a provides an alternative graphical representation of the firm's grant financing solution, comparing the social benefit of the firm's output to its social cost.

3.3 Suitability for Impact Investing

In this section we characterize the firms for which the impact investing regime is strictly superior to the grant financing regime. This can be understood as studying the first-best problem, asking when its solution coincides with grant financing, and when not, how the solution differs. Our first result is that impact investing is superior for all organizations that are sustainable under the grant financing regime. In this sense, sustainability is a precondition for impact investing, rather than the other way around. In all cases in which the impact investor operates the firm at a different scale than a grant financier would have, the impact investor expands the total social impact of the firm, measured according to the firm's objective function. However, this comes from increasing the firm's profitability $\pi(y,p)$ and *reducing* its social mission $M(y,p)$.

Proposition 5. *The impact investing regime is (generically) strictly superior to the grant financing regime if and only if the firm is sustainable under the grant financing regime.*

Moreover, in all such cases, $\pi\left(y^{II}, p^{II}\right) > \pi\left(y^{GF}, p^{GF}\right)$ *and* $M\left(y^{II}, p^{II}\right) < M\left(y^{GF}, p^{GF}\right)$.

Under the grant financing regime all sustainable firms operate without the financier's capital, so their optimal scale is not disciplined by the financier's cost of capital. Because impact investors can extract profits from their portfolio firms, the size of every firm in the impact investing regime is governed by the financier's cost of capital. Therefore, impact investors change the size of all firms that are sustainable under the grant financing regime.

We next aim to understand this result graphically. Define $V^{II}(y)$ to be the financier's (and entrepreneur's) value under the impact investing regime. That is,

$$V^{II}(y) \equiv \max_p M(y,p) + \pi(y,p)$$

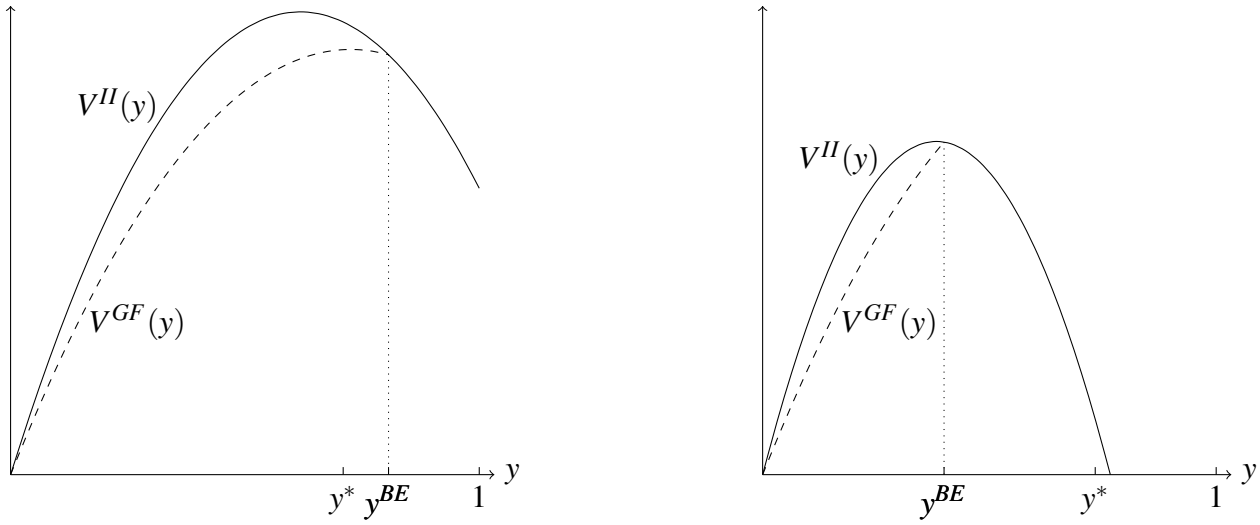

Figure 3

such that

$$y \leq D(p)$$

Now that the financier can pull profit out of the firm, and because $r_c < r_f \equiv 1$, the firm always charges $p(y)$. Therefore, following Section 3.2 we can rewrite the value function as

$$V^{II}(y) = M\left(y, \frac{c(y)}{y}\right) + (1 - r_c)\,\pi(y, p(y)).$$

This is depicted in Figure 3 , for the case where $y^* < y^{BE}$ (3a), and for the case where $y^{BE} < y^*$ (3b). Appendix Figure 5b presents an alternative graphical analysis of the firm's impact investing solution.

Compared to Figure 2, we can see that $V^{GF}(y)$ and $V^{II}(y)$ coincide to the right of y^{BE}. This is the region in which the firm receives ongoing subsidy from the financier and hence earns strictly negative profit. Firms that optimally operate in this region under the grant financing regime continue to do so in the impact investing regime, as the constraint that $\pi(y, p) \leq 0$ is non-binding. In contrast, to the left of y^{BE}, $V^{II}(y)$ is everywhere greater than $V^{GF}(y)$. In this region the financier in the impact investing regime withdraws profits from the firm.

In some cases the impact investor reduces the firm's scale relative to that in the grant financing regime and in other cases the impact investor increases the scale and profitability of her portfolio firm. As we will see in Propositions 6 and 7, whether the impact investor increases or reduces the scale of the firm depends on the nature of its sustainability.

Proposition 6. *Relative to the grant financing regime, impact investors always reduce the scale of*

hungry firms.

Recall that under the grant financing regime, hungry firms – those that operate at y^{BE} – are constrained by their customers' willingness to pay and therefore operate below their socially efficient scale, judged according to their customers' cost of capital. Nevertheless, the impact investor always reduces the scale of these firms. As discussed in Section 3.2, the fact that these firms are sustainable under the grant financing regime implies that the marginal social benefit of expanding the firm's scale is less than the financier's cost of capital, else the financier would have perpetually subsidized the firm under the grant financing regime. Therefore, as the impact investor is able to reduce the size of the firm and withdraw the resulting profit, this is always worthwhile on the margin. This resembles the classic result of Jensen (1986), that investors can discipline their portfolio firms' inefficient use of free cashflow.

Referring back to figure 3b, to the right of y^{BE}, the derivative of both value functions $V^{GF}(y)$ and $V^{II}(y)$ are equal. In contrast to $V^{GF}(y)$, $V^{II}(y)$ has a continuous derivative as the financier can always pull profits from the firm, and hence the fact that the firm was not worth expanding beyond y^{BE} under the grant financing regime implies that the impact investor optimally reduces its scale.

In effect, hungry firms create enough social value to justify their upfront investment α but will have exhausted their most impactful opportunities for investment before they have exhausted their opportunities to generate profit. Therefore impact investors optimally tax and reallocate the profit of these firms rather than allowing them to invest it into their operations (or, as described in Section 4, distribute it to their owners).

Proposition 7. *Relative to the grant financing regime, impact investors increase (decrease) the scale of a satiated firm if and only if* $\frac{d}{dy}\pi(y^*, p(y^*)) > 0$ *(*$\frac{d}{dy}\pi(y^*, p(y^*)) < 0$*).*

Moreover if $\frac{d}{dy}\pi(y^*, p(y^*)) > 0$ *then* $\frac{d}{dy}M(y^*, p(y^*)) < 0$.

Figure 3a depicts a firm that is satiated under the grant financing regime. Though in the grant financing regime the firm has already maximized its mission $M\left(y^*, \frac{c(y^*)}{y^*}\right)$, in the example depicted in the figure, the firm can further increase its profitability by reducing its scale. Proposition 7 dictates that the firm's scale always moves in the direction of increased profitability in the impact investing regime. In the general case this may result in increasing a satiated firm's scale in the impact investing regime.

At first this result may seem counterintuitive. In the grant financing regime, hungry firms would like to increase their mission by increasing their scale were they not constrained by their customers' willingness to pay, whereas satiated firms have maximized their mission. Yet impact investors always contract the scale of hungry firms, and may expand the scale of satiated firms.

This result derives from the fact that the impact investor's comparative advantage is helping firms increase their impact through utilizing profits, not elevating their mission. Because the impact investor can pull profit out of the firm, marginal profits are judged according to the impact investor's cost of capital, rather than at the marginal value of additional output. Firms financed by investors rather than donors therefore place higher value on profits. This is the mechanism that drives satiated firms to increase their scale when doing so increases profits. In contrast, by definition hungry firms have already exploited all of their opportunities to generate profit through increasing their scale, and so there is no scope for impact investors to increase their scale and profitability.

Two additional notes may further elucidate Proposition 7. First, even under the grant financing regime, firms already exploit each opportunity that increases profits when doing so also enhances the firm's mission. Therefore relative to grant financing, impact investors can only increase a satiated firm's scale when doing so increases the firm's profits but *inhibits* its mission. In this model, that occurs when the aggregate externality $E(y)$ is decreasing at y^*. This is certainly plausible in some cases, discussed in Section 2.2, but it is worth highlighting that when there is no tradeoff between a firm's mission and its scale, there is no channel for impact investors to increase the firm's scale.

Further, note that Proposition 7 dictates that impact investors increase the firm's scale when doing so is profitable irrespective of the corresponding harm to the firm's mission. In general, the derivative of the firm's objective function under the impact investing regime is $\frac{d}{dy}\pi(y, p(y)) + \frac{d}{dy}M(y, p(y))$, and hence the additional profits generated by increasing the firm's scale must be judged against the resulting harm to the firm's mission. However, because satiated firms have already maximized the firm's mission $M\left(y, \frac{c(y)}{y}\right)$, an envelope condition style argument implies that at y^* the value of increasing the firm's profits necessarily outweighs the harm to its social mission.[15]

Underlying Proposition 7 is that impact investing unlocks new opportunities to create value and profit. Essentially, impact investors facilitate the exploitation of profitable opportunities whose feasibility arises from the operation of a social enterprise. The opportunity may not be sufficiently lucrative that a commercial investor would finance the entire capital outlay required to start the enterprise (α, in the language of this paper), but once the capital outlay has been justified by its social value the new business opportunity becomes commercially viable. To take advantage of the new opportunity, the financier must be able to extract profit from the firm – therefore these opportunities may go unexploited when social enterprises are financed by donations alone.

3.4 Two Heuristics to Guide Impact Investors

The preceding analysis identifies two heuristics for judging when impact investing may be superior to grant financing. The first heuristic is an interpretation Proposition 6 in plain English: impact

[15] I thank Ray Fisman for pointing out this interpretation.

investors can discipline the spending of their portfolio firms. All firms that can attract support under the grant financing regime generate enough social value to justify their fixed cost α, judged according to the financier's cost of capital. But firms that generate sufficient revenue to be sustainable in steady state may allocate the marginal dollar they generate toward unproductive use. By making an investment rather than a grant, impact investors can extract and reallocate that dollar.

The second heuristic is a plain English interpretation of Proposition 7: by taking an ownership stake in the firm, a financier can empower the entrepreneur to exploit a profitable opportunity that arises as a result of investments made on the basis of their social value. Impact investors increase the value of a firm's profit by providing an additional avenue through which it can be used. Hence marginal opportunities to generate profit which also harm the firm's mission may only be worthwhile when the financier is a partial owner of the firm. In effect, impact investing enables profitable businesses to be built on the groundwork laid by social enterprises.

A Second Look at Husk Power

While the first heuristic is straightforward, it may be instructive to consider the second heuristic in the context Husk Power, introduced in Section 2.1.

Had Husk instead been financed exclusively by donations, its donors may have insisted it distribute its electricity for a very low fee, just to cover its variable cost of operation. Once it had incurred the cost of setting up the power plants and distribution networks (justified on the basis of their social value), Husk may have encountered an opportunity to generate a profit by on-boarding additional customers onto each of its electricity grids. If doing so would reduce the reliability of service for inframarginal customers, or would necessitate increasing Husk's carbon footprint at an outsized rate, and therefore would hamper Husk's mission of providing clean energy in rural areas, its donors may have discouraged it from exploiting this opportunity since it already covered its cost of operations.

Because Husk's financiers are investors, not donors, they have taken a claim on its future profits. They may therefore not only empower Husk to capitalize on this opportunity, but may also offer Husk more capital than donors would have, as they are positioned to benefit from the profits that capital would now generate. Notably, even if Husk could never generate enough revenue to attract commercial financing, impact investment may still improve upon donation by inducing Husk to generate additional profits on the margin.

3.5 Three Distinct Notions of the Alignment of Mission and Profit

Impact investors often refer to the degree to which a firm's mission and profits are in alignment using terms such as "colinearity" and "mission lock." In this section we articulate three distinct manners in which the alignment of a firm's mission and profits dictate its suitability for impact investment

relative to grant financing.

The first, termed *Profit-Mission Compromise*, refers to the tradeoff inherent in the business model between profits and social mission; *on the margin*, how much must a firm compromise on its social mission to generate additional profits. This is the tradeoff alluded to in the discussion of Proposition 7:

$$\frac{d\pi\left(y^{GF}, p\left(y^{GF}\right)\right)}{dy} > -\frac{dM\left(y^{GF}, p\left(y^{GF}\right)\right)}{dy}$$

By increasing the firm's scale, the impact investor may reduce the efficacy of the firm's social mission, narrowly defined. However, when the corresponding increase in profits is sufficiently large, it may offset the loss in the firm's mission and increase its broader social impact. In the model, the loss in the firm's social mission comes from expanding the firm's output beyond the point where its social value justifies the cost of the customers' capital. In the real world, the tradeoff between mission and profits may be broader, and may entail a firm branching out into new lines of operation. The point remains that if a firm is positioned to increase its profits without a substantial diminishment in its social mission, impact investors can facilitate that transition and increase the firm's total impact.

While impact investors and social entrepreneurs may prioritize social impact, many impact investors aim to one day sell their ownership stake in their portfolio firms to traditional commercial investors. The second tradeoff, termed *Vulnerability to Mission Drift,* reflects the danger inherent in relinquishing control of the firm to managers or subsequent investors who prioritize profits over social impact. Referring back to Figure 1, the profit-maximizing point is y^{PM} – the point at which the vertical distance between revenue and cost is greatest. In contrast to the firm's profit-mission compromise, which is an evaluation of the marginal tradeoff of impact and profit, vulnerability to mission drift reflects the difference in the firm's social impact at its most profitable point and at its most impactful point. Mathematically, vulnerability to mission drift is a comparison of the firm's impact at y^{PM} and y^{II}

$$M\left(y^{PM}, p\left(y^{PM}\right)\right) + \pi\left(y^{PM}, p\left(y^{PM}\right)\right) - \left(M\left(y^{II}, p\left(y^{II}\right)\right) + \pi\left(y^{II}, p\left(y^{II}\right)\right)\right)$$

Profit-mission compromise is low when on the margin, increasing the firm's profits results in only a small decrease in the firm's mission. Vulnerability to mission drift is low when the firm's most socially impactful point of operation is close to the firm's profit-maximizing point of operation.

Stepping slightly outside of the model, vulnerability to mission drift is low when the firm does not have access to opportunities that would result in commercial success without also expanding its social mission. Returning to the case of Husk Power is instructive in this regard. Husk's social mission is to expand power in rural India using solar power and burning rice husk. One justification of Husk's suitability for impact investment is that if Husk's management wants to increase the firm's profits, they must do so by expanding access to power in rural areas, as rice husk is only abundant in rural

areas. In contrast, Orb Energy – another renewable energy company in India – pursues a similar social mission by affixing solar panels to the roofs of rural customers. If Orb Energy wanted to one day pursue profit at the expense of its social mission, it could do so by shifting focus to urban areas, where customers may have higher willingness to pay for the same solar panels. But doing so might diminish its mission of expanding access to power in rural areas. In the language of this paper, Husk Power has low vulnerability to mission drift, whereas Orb Energy may not.[16]

Finally, stepping slightly farther from the confines of the model, a third dimension of the alignment between profits and social mission is *Profit-Mission Accountability.* Many socially oriented organizations are difficult for financiers to evaluate, as there are often few signals that a firm is delivering its services in an efficient manner to customers who value them highly. In settings where firms charge for their services, profits may be a reliable indicator for the social impact of the firm. On the other hand, a wide array of factors might distort the mapping between profits and social impact.

Returning once more to Husk Power, consider the challenge of evaluating its social impact if it were to distribute power for free. Even with detailed metrics of its costs, the number of power lines it set up, and the amount of power it distributed, a satisfying measure of social impact would be hard to derive without a notion of how much Husk's customers value its service. Because Husk does charge its customers for power, Husk's financiers can be assured it is delivering its power to high value users. But there are many reasons why profit might rise or fall even when its social impact does not. For instance, if Husk were to distribute its power at subsidized rates in a region stricken by drought, that would presumably lead to a reduction in its profits but an increase in its social impact. Or if during the same drought Husk could no longer afford to serve its poorest customers, its profits might rise but its social impact might fall. The extent to which profits track impact will vary from firm to firm, but when profits are a strong predictor of social impact, impact investors can help keep firms accountable by prioritizing profits.

4 Extension: The Entrepreneur Can Extract Profits

In this section we extend the model to allow for the possibility that the entrepreneur can extract profit from the firm. Therefore, even when grant financed, the firm can generate positive profits. The principal nuance is that the entrepreneur may place different value on profits (relative to the firm's mission) than does the financier. This divergence of interests between the financier and entrepreneur introduces an agency problem. So within the impact investing regime we now separately consider debt and equity and their consequences on the entrepreneur's behavior.

We analyze the problem in two stages. First, the financier offers the entrepreneur a contract that specifies a transfer of α capital from the financier to the entrepreneur at time 0, and a transfer from

[16] I thank Brian Trelstad for pointing me to this example.

the entrepreneur to the financier at all future periods. We separately consider debt contracts, which specify a transfer $\mathscr{R}$ in each period independent of the firm's profit, and equity contracts, which specify a transfer $\lambda\pi(y,p)$ in each period for $\lambda \in [0,1]$. The transfer $\mathscr{R}$ in debt contracts can be either positive or negative, reflecting the possibilities that the financier either continues to support the firm or extracts its revenues. We write $\mathscr{T}(\pi)$ to accommodate the possibility of either debt of equity (but we do not allow for transfers that are an arbitrary function of profits).

At all times $t \geq 1$, the entrepreneur then chooses p and y to solve

$$\max_{y,p} M(y,p) + r_e(\pi(y,p) - \mathscr{T}(\pi(y,p))) \tag{3}$$

s.t.

$$y \leq D(p)$$

$$\pi(p,y) \geq \mathscr{T}(\pi(y,p))$$

That is, given the transfer $\mathscr{T}(\pi)$, the entrepreneur chooses output and price to maximize the sum of his social mission $M(y,p)$, and the value of profits he extracts from the firm, measured by the firm's profits $\pi(y,p)$ minus the transfer the entrepreneur makes to the financier $\mathscr{T}(\pi)$. The entrepreneur values his own profits at rate r_e and we continue to normalize the value of the financier's profits to $r_f \equiv 1$. The entrepreneur is constrained by the demand curve $D(p)$ and the financing constraint which specifies that the firm's profit must exceed the transfer to the financier.

The financier's objective function is

$$V(y,p,\mathscr{T}(\pi)) \equiv M(y,p) + \mathscr{T}(\pi(y,p)).$$

Denoting the entrepreneur's solution to problem (3) by $(y(\mathscr{T}(\pi)), p(\mathscr{T}(\pi)))$, at time 0 the financier chooses $\mathscr{T}(\pi)$ to solve

$$\max_{\mathscr{T}(\pi)} \left[\sum_{t=1}^{\infty} \beta^t V(y(\mathscr{T}(\pi)), p(\mathscr{T}(\pi)), \mathscr{T}(\pi)) \right] - \alpha. \tag{4}$$

In the grant financing regime, the financier solves (4) subject to the constraint that $\lambda = 0$ in the case of equity and $\mathscr{R} \leq 0$ in the case of debt (i.e. the financier can continue to support the firm in periods 1 onwards but cannot extract profits from the firm), and in the impact investing regime the financier solves (4) without additional constraints. In both cases, the financier only proceeds in period 0 if her objective (4) is positive. When $r_e = 0$ this model reduces to that outlined in Section 2.

In the analysis to follow, we assume $r_e > r_c$ so that the entrepreneur values his own use of profits

more highly than his customers'. We separately analyze the cases where $r_e < 1$ so that the entrepreneur values his own profits less than does financier, and $r_e > 1$ so that he values his profits more highly than does the financier.

4.1 The Entrepreneur Values His Profits Less Than The Financier's: $r_e < 1$

We first analyze the case for which $r_e < 1$. Because the entrepreneur values his own profits less highly than does the financier the model and analysis are very similar to the preceding sections. In particular, Propositions 1, 5 and 6 continue to hold. Redefining satiated firms to be any firm that is optimally sustainable under the grant financing regime and that operates at $y < y^{BE}$ (rather than operating precisely at y^*), Propositions 2, 4, 7 continue to hold as well. Because the impact investor values profits more than does the entrepreneur, the firm will prioritize profits more highly under the impact investing regime. Therefore, the two primary advantages of impact investing highlighted in the previous section are qualitatively unchanged.

Further we have the following proposition.

Proposition 8. *When $r_e < 1$ debt is the optimal financial instrument.*

Debt allows the financier to ensure the first-best outcome. This is so because debt allows the financier to put a lower bound on the profits the firm must return each period (Jensen, 1986). Since when $r_e < 1$ the entrepreneur's inclination is to produce less profit than the financier would like, debt is always optimal.

4.2 The Entrepreneur Values His Profits More Than The Financier's: $r_e > 1$

We next analyze the case for which $r_e > 1$. We first consider an impact investor using debt financing. As above, after suitably redefining satiated firms Propositions 1, 2, and 4 continue to hold.

Proposition 5 can now be strengthened. In particular, we have the following result.

Proposition 9. *When $r_e > 1$, the impact investing regime is (generically) strictly superior to the grant financing regime if and only if the firm is sustainable under the grant financing regime. Under the optimal debt contract $\left(y^{GF}, p^{GF}\right) = \left(y^{II}, p^{II}\right)$.*

An impact investor using debt never changes the size of the portfolio firm relative to grant financing. This follows from the fact that, so long as the entrepreneur generates enough profit to repay $\mathscr{R}$ to the financier, his optimization is independent of $\mathscr{R}$. So if the financier sets $\mathscr{R} \leq \pi\left(y^{GF}, p^{GF}\right)$, the entrepreneur will choose the same scale of operation. The financier can change the scale of operation relative to the grant financing regime by demanding $\mathscr{R} > \pi\left(y^{GF}, p^{GF}\right)$, but as the entrepreneur already prioritizes profits more highly than the financier would like, this is never optimal. Therefore, in the impact investing regime, the financier sets $\mathscr{R} = \pi\left(y^{GF}, p^{GF}\right)$ and extracts the firm's full profits

without changing its scale of operation.

Recall, in Section 3 we highlighted two ways for an impact investor to create value relative to a grant financier. The first is that the impact investor can extract profits from her portfolio firm when they are they are no longer being put to good use on the margin, either because the firm has run out of impactful investment opportunities, or, in this case, because the entrepreneur consumes the firms marginal profits rather than reinvesting them. The second manner in which impact investors create value is by inducing the entrepreneur to exploit new opportunities for profit, as the impact investor can allocate those profits to other high value opportunities. When $r_e > 1$, the entrepreneur is already over-exploiting opportunities for profit relative to the impact investor's preferences, and therefore impact investors only create value by taxing and reallocating the profits the firm would have made even under the grant financing regime.

Further we have the following proposition

Proposition 10. *For r_e sufficiently large, equity is the optimal financial instrument.*

Unlike in the case where $r_e < 1$, the financier now faces a non-trivial tradeoff between debt and equity. While the debt contract allows the financier to extract all of the firm's profit, it does not change the entrepreneur's incentive to pursue profit on the margin. In contrast, equity depresses the entrepreneur's incentive to pursue profit and results in him placing a relatively higher weight on the firm's mission. While depressing the entrepreneur's incentive to pursue profit is typically viewed as a drawback of equity, in this context it is a merit as both the financier and the entrepreneur value a non-pecuniary benefit of the firm's output. This merit of equity is closely connected to the results of Glaeser and Shleifer (2001) and Chowdhry et al. (2019).

Stepping outside of the model, the non-distribution constraint on nonprofits may be the most common way that social entrepreneurs are encouraged to pursue their firm's mission over profits. Often, however, attracting financing from private investors requires the ability to sell equity and therefore precludes firms from incorporating as nonprofits. Proposition 10 suggests that in such settings, equity financing may impose a similar discipline on the entrepreneur.

5 A Case Study in Assessing the Suitability of Impact Investment: Husk Power

In this section we apply the heuristics developed formally in Section 3.3 and stated informally in Section 3.4 to the case of Husk Power. Husk Power, described in Section 2.1, is a rural electrification company that has received impact investment. We provide a quantitative analysis of their cost and revenue curves and use them to assess whether 1) an investor could productively tax the firms profits and reduce its scale or 2) justify an increase in the firm's scale and profitability.

In general, quantifying the forces in the above model requires three ingredients: An assessment of the social value of the firm's output, its cost of production, and the demand curve it faces. The first of these is often subjective and is required in the context of any philanthropic decision making. The latter two are straightforward to measure, and for the case of Husk Power, relevant estimates are provided by Burgess et al. (2017). The authors conducted an experiment in 2013 in collaboration with Husk Power in which they randomized the price of Husk's electricity and measured the resulting demand. In doing so they determined that reducing the price from 200 rupees to 100 rupees increased demand from 2 percent of eligible households up to 17 percent, where demand is measured by whether a household ever paid for a connection. The authors also note that the variable cost of production is 115 rupees per served household.

Extrapolating a linear curve through their estimates implies that demand is

$$D(p) = 32 - .15p$$

where demand is measured in percent of the population served. Company reports suggest that at the prevailing price of 160 rupees per connection, each power plant serves around 330 customers across several villages (Lassiter et al., 2018). Therefore we can equivalently write the demand curve on a per household basis as

$$D^{HH}(p) = 1320 - 6.1875p$$

Combining this estimated demand curve with the aforementioned variable cost of production, Figure 4 presents the empirical analogue of Figure 1: the revenue and cost curves of a single power plant operated by Husk Power.

The vertical line plots Husk Power's status quo point of operation. At the time of these estimates Husk operated very close to profit maximization (the profit maximizing point of operation is at a price of 164.2 rupees per household, reaching about 304 households per plant). This may not be a surprise, as Husk is supported primarily by impact investors rather than donors, who are thus positioned to benefit from the profits that Husk generates.

At the time that Husk Power was raising capital, its financiers could use figures like this to assess whether an investment or donation is more suitable. To assess the importance of the first heuristic, would-be donors could use Husk's business model projections to ask themselves if they are comfortable with the amount of money being disbursed to the firm's owners. If not, they may prefer to make an investment and reduce the profits available for the firms current owners.

The second heuristic – whether Husk is positioned to generate additional profits at sufficiently small cost to its social mission – is less relevant given its status quo plan of charging the profit-maximizing price. But suppose instead Husk planned to charge a price of, say, 125 rupees, reach

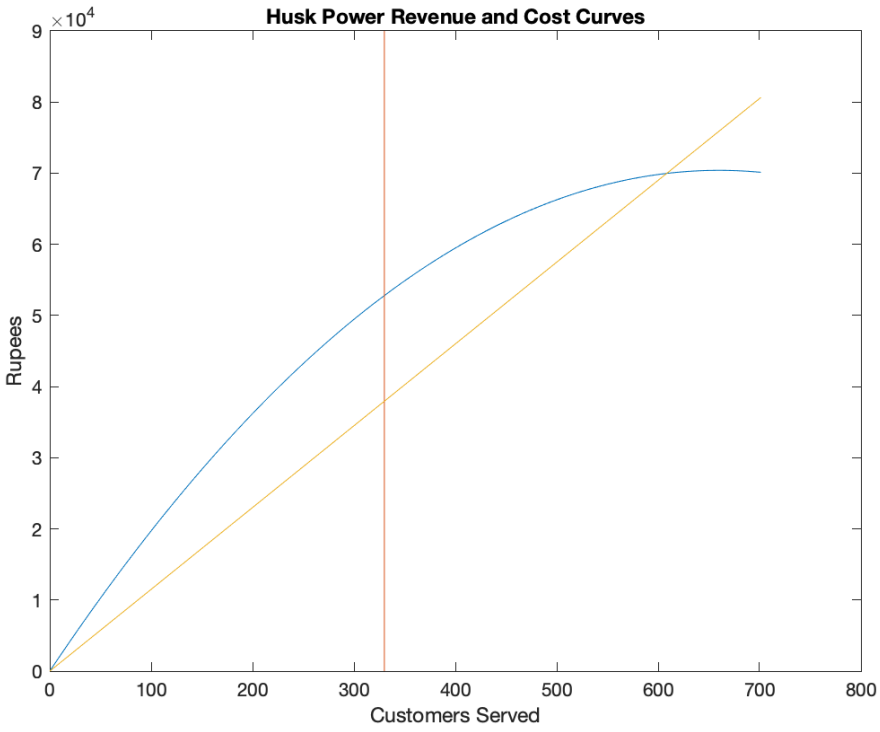

Figure 4

about 545 households per plant, and just barely break even. In that case, Husk's would-be donors could use Figure 4 to assess the tradeoff between generating additional profits by raising the price, and the resulting reduction in the number of households that received access to affordable electricity. If the reduction in Husk's social mission were deemed to be smaller than the value that Husk's financiers could generate using the additional profits, they may decide to invest in Husk rather than making a donation, and encourage it to raise its prices.

6 Discussion

This article has focused on the circumstances in which socially minded financiers can have more impact with an investment than with a grant. I critically examine two common explanations for the merits of impact investing. The first is that impact investments help firms reach scale and sustainability. The second is that philanthropic capital alone is insufficient to address societal problems and impact investments enable philanthropists to leverage their capital. I argue that neither of these can straightforwardly explain when an investment is superior to a grant. There is no direct link between the manner in which a firm is financed and whether or not it operates a sustainable business model; sustainable, even for-profit businesses are often the recipients of grant financing. And while impact investments do enable philanthropists to leverage their capital, they come at the cost of extracting resources from the organizations they aim to support. A theory that explains when an investment is superior to a grant must make this cost explicit.

Through the lens of a simple model I provide a formal definition of organizational sustainability and characterize the situations in which a social enterprise should be sustainable. I demonstrate that the cost of investments is justified when an organization would have been sustainable even if grant financed. In this sense, organizational sustainability leads to impact investing, rather than the other way around. I draw a distinction between sustainable firms that can self-finance their optimal scale, termed *satiated firms*, and sustainable firms whose scale is constrained by their customers' willingness or ability to pay for goods and services, termed *hungry firms*.

Impact investors may optimally increase or decrease the scale of a firm relative to if it were grant-financed. By providing firms an additional channel to productively allocate their profits, impact investors can help firms exploit new opportunities to generate profits and increase their scale. Doing so may be optimal when capitalizing on these opportunities is not in strong tension with a firm's mission. In contrast, when no such opportunities exist, impact investors may reduce the scale of sustainable organizations. These are the firms for which a financier has determined the marginal impact of an additional dollar invested does not justify its cost, either because the firm has run out of impactful opportunities or because marginal profits are channeled to the firm's owners rather than invested in the firm's mission. I demonstrate that whether impact investors expand or reduce the scale of the organization depends on the nature of its sustainability. By definition, hungry firms have exhausted their opportunities to generate profit through increased scale, and hence impact investors always reduce their scale. However, impact investors may increase the scale of satiated firms, which under grant financing have maximized their mission and so may not have exploited all opportunities for profit.

Put in other words, impact investors should hold two considerations when determining whether to offer a grant or investment to a particular organization. First, is this an organization that would be worth subsidizing into the indefinite future? If yes, a grant may be superior, as an investment will result in withdrawing capital from organizations that continue to have high impact. If no, an investment enables a financier to discipline relatively low-value spending of the organizations they support. Second, does this organization have opportunities to generate profits that would not come at significant cost to their primary mission? If yes, investing provides a channel to redirect those profits and increase their impact.

My analysis illuminates three distinct components of the alignment of profits and mission that influence when an investment delivers superior impact to a grant. I term these *Profit-Mission Compromise, Vulnerability to Mission Drift, and Profit-Mission Accountability.* The first represents the tradeoff that even socially oriented managers must incur between profits and mission, the second reflects the danger that a socially oriented organization might face if ever it ceded control to profit-maximizing managers, and the third reflects the ability to assess the firm's social impact by measuring its profits.

Next I consider the possibility that the financier and entrepreneur disagree on how much to prioritize profit relative to impact. I demonstrate that most insights from the earlier analysis are robust to this extension, and highlight a new distinction between debt and equity. Equity dampens the entrepreneur's marginal incentive to pursue profits. While this is typically viewed as a drawback of equity financing, in the context of social entrepreneurship it may be a merit, when the entrepreneur values profits too highly relative to the firm's social mission.

Finally, I provide a simple demonstration of how financiers might quantify the considerations in this paper in the context of Husk Power. Quantifying these considerations only requires basic cost and revenue projections, an estimate of the firm's demand curve, and an assessment of the social value of the firm's output.

I close this section by considering several other common justifications for impact investing. Impact investors enjoy control rights over their portfolio firms that traditional donors typically lack; Impact investors can help to professionalize their portfolio firms by ensuring they meet the accounting and transparency standards necessary for commercial investment; Impact investors can help signal a firm's ability to repay its debts, thereby enabling it to attract commercial capital. I argue each of these is fundamentally separable from distinction between a grant and an investment. In principle, grant financiers could contract on board seats and other control rights. Grants could be accompanied by consulting services to help professionalize grantees. And the signal of a firm's accountability could be derived from its balance sheet and income statement rather than by returning capital to its financiers. Therefore, I argue that the conditions outlined in this article should remain the primary justification for an investment rather than a grant.

6.1 An Alternative Interpretation of the Model

Economists typically understand the role of the nonprofit sector as serving societal goals that cannot be achieved by profit seeking firms, such as those that involve significant externalities or redistribution (eg. Bénabou and Tirole, 2010). Under this view of the world, it is taken for granted that firms that do not yield positive net present value return on invested assets, but serve some societal goal, should incorporate as nonprofits. Yet today, socially oriented firms such as the Chan Zuckerberg Initiative and the Omidyar Network, which claim not to have ambitions to earn outsized profits, are increasingly incorporating as for-profit limited liability corporations. The model presented in this article may cast some light on this organizational form. The comparison between the grant financing regime and the impact investing regime can be understood as a comparison between a firm that is prohibited from generating profits and one that is not. The analysis highlights that even if a firm will never fully recoup the cost of its invested assets, it may nevertheless be efficient to organize in such a way so as to be able to generate positive profits and distribute them to its owners.

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Additional Graphical Analysis of the Firm's Problem

In this section we provide an alternative graphical analysis of the firm's grant financing and impact investing problems, by comparing the social benefit of the firm's output to its social cost. It is best to read the analyses in Sections 3.1 - 3.3 prior to this one.

Define the social benefit of the firm's output to be

$$S(y) \equiv \int_{1-y}^{1} v dv + E(y).$$

In the grant financing problem, the firm relies exclusively on the customers' capital for production up until y^{BE}. Beyond that the firm charges customers' full willingness to pay, $p(y)$ and still relies on the financier's subsidy. Therefore we can write the social cost function in the grant financing problem as

$$c^{GF}(y) \equiv \begin{cases} r_c c(y) & \text{if } y \leq y^{BE} \\ c(y) - yp(y)(1-r_c) & \text{if } y > y^{BE} \end{cases}$$

Note that there is a kink in $c^{GF}(y)$ at y^{BE} where the financier must begin subsidizing production. We can write

$$V^{GF}(y) = S(y) - c^{GF}(y).$$

The firm's optimal scale when unconstrained by customer demand, y^*, is the point at which the slope of $r_c c(y)$ is equalized with the slope of $S(y)$. As before, if $y^* < y^{BE}$ then the firm's optimal point of operation is y^*. Else, if $y^{BE} < y^*$ then concavity of $S(y)$ and convexity of $c(y)$ imply that

$$\frac{d}{dy} S\left(y^{BE}\right) > \lim_{y \to y^{BE}-} \frac{d}{dy} c^{GF}(y) \equiv r_c c'(y).$$

If the financier does not find it worth subsidizing the firm to increase its scale beyond y^{BE}, that is

$$\frac{d}{dy} S\left(y^{BE}\right) < \lim_{y \to y^{BE}+} \frac{d}{dy} c^{GF}(y) \equiv c'(y) - \left(yp'(y) + p(y)\right)(1-r_c)$$

then the firm is optimally sustainable at y^{BE}. Else the firm receives continual subsidy and scales beyond y^{BE} in the grant financing regime.

Figure 5a plots the firm's grant financing problem for the case where $y^* > y^{BE}$.

Figure 5b presents a graphical analysis of the firm's impact investing problem. The social value of output $S(y)$ is the same as in the grant financing problem. However, because the impact investor can

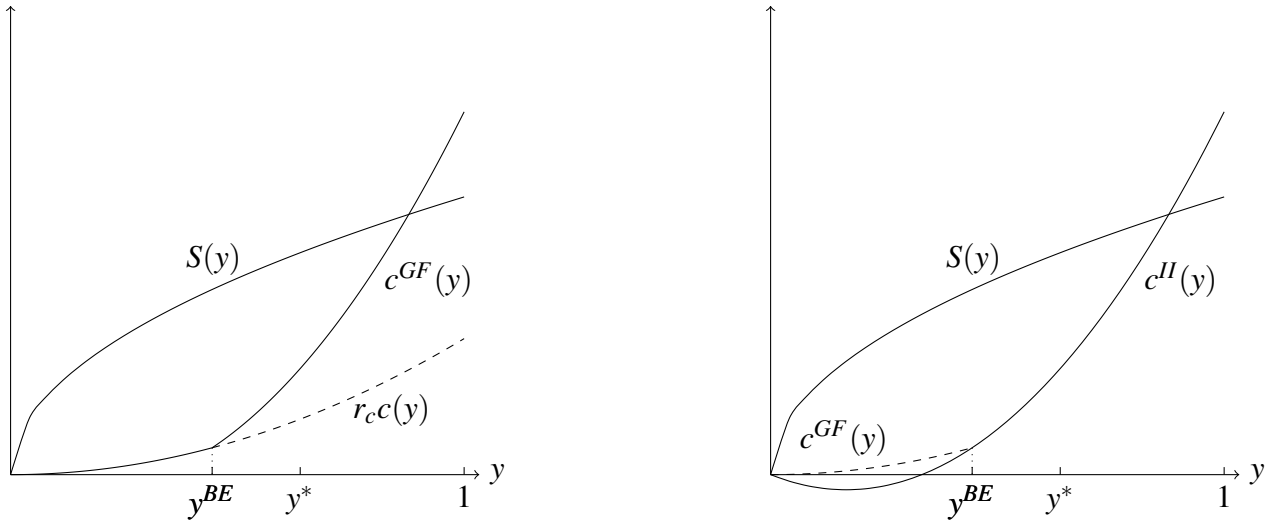

Figure 5

pull profits out of the firm, the social cost of production, $c^{II}(y)$, differs from that in the grant financing problem. In particular we have

$$c^{II}(y) \equiv c(y) - ypy(1 - r_c).$$

The impact investor always charges customers their full willingness to pay, and always withdraws any profits from production, or subsidizes any revenue shortfall. To the right of y^{BE}, $c^{II}(y)$ and $c^{GF}(y)$ coincide, as in this region the financier subsidizes the firm's output in both financing regimes and the negative profit constraint is non-binding.

To the left of y^{BE}, the social cost of production is always lower under the impact investing regime than it is under the grant financing regime. In the grant financing regime, the cost of producing $y < y^{BE}$ is $r_c c(y)$. In the impact investing regime, the financier receives the additional benefit of a transfer from the firm's customers, in the amount of $yp(y) - c(y)$, which she can put to use at higher social value than the customers' cost of capital.

Therefore, impact investing is strictly superior for all firms that operate to the left of y^{BE} under the grant financing regime – i.e. all firms that are sustainable under the grant financing regime. Whether the impact investor increases or decreases their scale once again depends on whether the sustainable firm is hungry or satiated, in line with the discussion in Section 3.3.

Omitted Proofs

Proof of Lemma 1

Suppose there are at least two points y_i such that $p(y_i)y_i = c(y_i)$. Because the revenue curve crosses the cost curve in at least two points, there is at least one point where the revenue curve crosses the cost curve from below. Therefore, there exists a point y_1 with $p(y_1)y_1 = c(y_1)$ and a point $y' < y_1$ with $p(y')y' < c(y')$. Because p is decreasing we have

$$\begin{gathered} p(y_1)y' < c(y') \\ \Longrightarrow \\ p(y_1)\frac{y'}{c(y')} < 1 = p(y_1)\frac{y_1}{c(y_1)} \\ \Longrightarrow \\ \frac{y'}{c(y')} < \frac{y_1}{c(y_1)} \end{gathered}$$

That is, the average output at y_1 is higher than the average output at $y' < y_1$ contradicting that c is convex. □

The economic intuition underlying this result is that when producing y', the firm can charge at least as high a price as it could charge when producing y_1, and if c is convex then it is strictly cheaper to produce the first y' units than it is to produce the next $y_1 - y'$ units. So if it is affordable to sell y_1 units at $p(y_1)$ then it is also affordable to sell y' units at $p(y') > p(y_1)$.

Proof of Proposition 2

y^* is the solution to the relaxed problem

$$\max_{y,p} M(y,p) + \pi(y,p)$$

such that

$$\pi(y,p) \leq 0.$$

Therefore when $y = y^*$ $p = \frac{y^*}{c(y^*)}$, is a feasible solution to the grant financed firm's problem it is the optimal solution. Lemma 1 guarantees that the firm's revenue curve and cost curve only cross in one spot (at y^{BE}). This guarantees that for all points $y < y^{BE}$, the revenue curve lies above the cost curve and therefore the firm can self finance and produce any such y. Therefore when $y^* < y^{BE}$, $y = y^*$ $p = \frac{c(y^*)}{y^*}$ is a feasible solution to the grant financed firm's problem.

When $y^* > y^{BE}$, Lemma 1 assures that it is not a feasible solution to the firm's problem. Solving for the firm's optimal scale under the additional constraint that the firm is self-financed amounts to

$$\max_y \int_{1-y}^{1} v dv + E(y) - c(y) r_c$$

such that

$$y \leq D\left(\frac{c(y)}{y}\right).$$

The social value of the firm's output $\left(\int_{(1-y)}^{1} v dv + E(y)\right)$ is concave, and therefore the optimal solution to the problem is the maximal feasible point of operation, y^{BE} (given that this is less than the optimum y^* of the same maximand unconstrained by the demand curve). □

Proof of Proposition 3

This is a consequence of the fact that $y = y^*$ $p = \frac{c(y^*)}{y^*}$ solves the firm's relaxed problem. □

Proof of Proposition 4

If $y^{BE} < y^*$ then concavity of $M\left(y, \frac{c(y)}{y}\right)$ dictates that $\frac{d}{dy} M\left(y, \frac{c(y^{BE})}{y^{BE}}\right) > 0$. This is the left derivative of the firm's objective function under the grant financing regime at y^{BE}. If the right derivative of the firm's objective function at y^{BE} is negative i.e. $\frac{d}{dy} M\left(y^{BE}, p\left(y^{BE}\right)\right) + \frac{d}{dy}\pi\left(y^{BE}, p\left(y^{BE}\right)\right) < 0$, then the optimal scale is y^{BE} and corresponding price is $p\left(y^{BE}\right)$.

Conversely, if the optimal scale $y^{GF} > y^{BE}$ then it must be that the right derivative at y^{BE} is positive. □

Proof of Proposition 5

First we show that when impact investing is superior to grant financing, the firm is sustainable under the grant financing regime. By concavity of the firm's objective function it must be that $\frac{d}{dy} M(y, p(y)) + \frac{d}{dy}\pi(y, p(y)) < 0$ for all $y > y^{II}$. If impact investing is superior to the grant financing regime then $\pi\left(y^{II}, p^{II}\right) > 0$. Therefore, by Lemma 1, $y^{II} < y^{BE}$. Hence by Proposition 4 the firm is sustainable under the grant financing regime.

If the firm is sustainable under the grant financing regime then $y^{GF} = \min\left\{y^{BE}, y^*\right\}$. If $y^{GF} = y^{BE}$ then by Proposition 4 we have that generically

$$\frac{d}{dy}\pi\left(y^{GF}, p\left(y^{GF}\right)\right) + \frac{d}{dy} M\left(y^{GF}, p\left(y^{GF}\right)\right) < 0.$$

The left hand side of the above inequality is the derivative to the unconstrained objective function

$$M\left(y,p\left(y\right)\right)+\pi\left(y,p\left(y\right)\right)$$

corresponding to the maximization problem under the impact investing regime. Thus the firm's objective function can be improved via a local decrease in its scale under the impact investing regime.

If $y^{GF}=y^{*}$ then, generically, $p^{GF}<p\left(y^{*}\right)$. Therefore the firm can make a local improvement by raising p^{GF} to $p\left(y^{*}\right)$ under the impact investing regime.

In all such cases, $\left(y^{II},p^{II}\right)\neq\left(y^{GF},p^{GF}\right)$. Therefore, by revealed preference, $\pi\left(y^{II},p^{II}\right)+M\left(y^{II},p^{II}\right)>\pi\left(y^{GF},p^{GF}\right)+M\left(y^{GF},p^{GF}\right)$. Further, as the only difference between the impact investing regime and the grant financing regime is the relaxation of the constraint that $\pi\left(y,p\right)\leq 0$, we have that $\pi\left(y^{II},p^{II}\right)>0\geq\pi\left(y^{GF},p^{GF}\right)$. Finally, because $\pi\left(y^{II},p^{II}\right)>0$, we know that $\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)$ is a feasible point of operation under the grant financing regime, with $\frac{y^{II}}{c\left(y^{II}\right)}<p\left(y^{II}\right)=p^{II}$. Moreover, $\pi\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)=0$. Therefore,

$$\begin{aligned}M\left(y^{GF},p^{GF}\right)&\geq M\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)\\&>M\left(y^{II},p\left(y^{II}\right)\right)\\&=M\left(y^{II},p^{II}\right)\end{aligned}$$

The first inequality follows because $\left(y^{II},\frac{y^{II}}{c\left(y^{II}\right)}\right)$ is an achievable point of operation under the grant financing regime that results in 0 profits. If the grant financing regime operates at a point with negative profits, it must achieve more of its mission. The second inequality follows from the fact that the firm's mission is declining in the price it charges, and the third equality follows from the fact that under the impact investing regime the firm always operates on the maximal revenue curve. □

Proof of Proposition 6

The difference between the impact investing regime and the grant financing regime is that the impact investing regime does not have the constraint $\pi\left(y,p\right)\leq 0$. Therefore, when $\left(y^{II},p^{II}\right)\neq\left(y^{GF},p^{GF}\right)$, it must be that $\pi\left(y^{II},p^{II}\right)>0$. By Lemma 1 we know that all y such that there exists a p with $\pi\left(y,p\right)>0$ satisfy $y<y^{BE}$. □

Proof of Proposition 7

Under the impact investing regime, the firm's objective function is

$$\begin{aligned} M(y,p(y))+\pi(y,p(y)) &= \left(\int_{1-y}^{1} v dv + E(y)\right) - c(y)r_c + (yp(y)-c(y))(1-r_c) \\ &= M\left(y, \frac{c(y)}{y}\right) + \pi(y,p(y))(1-r_c) \end{aligned}$$

By definition of y^*, $\frac{d}{dy}M\left(y, \frac{c(y)}{y}\right) = 0$.

Therefore the firm's objective function is increasing (decreasing) in y if and only if $\frac{d}{dy}\pi(y,p(y)) > 0$ ($\frac{d}{dy}\pi(y,p(y)) < 0$).

That $\frac{d}{dy}M(y^*,p(y^*)) < 0$ if and only if $\frac{d}{dy}\pi(y^*,p(y^*)) > 0$ follows from Proposition 5. □

Proof of Proposition 8

Let $\left(y^{II},p^{II}\right) = \underset{y,p}{\text{argmax}}\, M(y,p)+\pi(y,p)$. When $r_e < 1$, the debt contract that requires $\mathscr{R} = \pi\left(y^{II},p^{II}\right)$ induces the first-best outcome. (In contrast, any equity contract reduces the entrepreneur's (already too low) incentive to prioritize profit.) □

Proof of Proposition 9

Let $r_e > 1$. Suppose the firm is sustainable under the grant financing regime, and generates $\pi\left(y^{GF},p^{GF}\right)$ profit. Then under the grant financing regime the financier's per period utility is $M\left(y^{GF},p^{GF}\right)$. In contrast, the debt contract that requires payment $\mathscr{R} = \pi\left(y^{GF},p^{GF}\right)$ generates per period utility of $\pi\left(y^{GF},p^{GF}\right) + M\left(y^{GF},p^{GF}\right)$.

Now suppose that the firm is not sustainable under the grant financing regime. Then recalling that

$$\left(y^{GF},p^{GF}\right) \equiv \underset{y,p}{\text{argmax}}\, M(y,p)+r_e\pi(y,p)$$

we have that $\pi\left(y^{GF},p^{GF}\right) < 0$. Therefore the solution to $\max_{y,p} M(y,p)+\pi(y,p)$ also has negative profit, and impact investing will therefore induce the same outcome as grant financing. □

Proof of Proposition 10

Recall that $\left(y^{II},p^{II}\right) \equiv \underset{y,p}{\text{argmax}}\, M(y,p)+\pi(y,p)$, and that $\left(y^{PM},p^{PM}\right) \equiv \underset{y,p}{\text{argmax}}\, \pi(y,p)$. Fix any example where these two solutions do not coincide, and where $\pi\left(y^{II},p^{II}\right) \geq 0$. Now take r_e to be finite

but large. As r_e becomes arbitrarily large, the entrepreneur's choice of operation becomes arbitrarily close to $\left(y^{PM}, p^{PM}\right)$. The optimal debt contract extracts the full profits of the firm, but, for $r_e > 1$, it does not change the entrepreneur's point of operation, as the entrepreneur's marginal utility of profits is unchanged. In contrast, the equity contract which extracts a fraction $\frac{r_e-1}{r_e}$ of the profits induces the entrepreneur to choose $\left(y^{II}, p^{II}\right)$. As $r_e \to \infty$, the investor's utility from this contract converges to $M\left(y^{II}, p^{II}\right) + \pi\left(y^{II}, p^{II}\right)$. Therefore, for sufficiently large r_e, the optimal equity contract is superior to the optimal debt contract.

Now consider the case where $\pi\left(y^{II}, p^{II}\right) < 0$. It is no longer the case that the entrepreneur will choose $\left(y^{II}, p^{II}\right)$ under the equity contract that extracts a fraction $\frac{r_e-1}{r_e}$ of the firm's profits, as doing so would violate his budget constraint. However consider the optimal point of operation

$$\left(y^{II'}, p^{II'}\right) \equiv \underset{y,p}{\operatorname{argmax}}\, M(y,p) + \pi(y,p)$$

such that

$$\pi(y,p) \geq 0$$

There exists some equity share $s(r_e)$ such that if the financier takes $s(r_e)$ the entrepreneur will select $\left(y^{II'}, p^{II'}\right)$. As $r_e \to \infty$, $s(r_e) \to 1$, and the equity contract that takes $s(r_e)$ will dominate the debt contract which extracts the full profitability of the firm. □