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Abstract

This paper provides a comprehensive model of an agent's behavior in response to multiple sales management instruments, including compensation, recruiting/termination, and training. The model takes into account many of the key elements that constitute a realistic sales force setting: allocation of continuous effort; forward-looking behavior; present bias; effectiveness of training; and employee selection and attrition. The paper provides insights into the way that employee training, recruiting policies, and various compensation components affect both the selection and performance of heterogeneous sales agents. In addition, the paper provides two key methodological contributions. First, we provide identification results for dynamic models with continuous and unobserved choice variables, which arise in many empirical applications but are often ruled out due to theoretical intractability. Second, we consider a hyperbolic discounting model as well as an exponential discounting model and provide conditions under which each model is identified. The key to identification is that, under a multi-period nonlinear incentive scheme, an agent's proximity to a goal affects only future payoffs in non-pecuniary periods, providing exclusion restrictions on the current payoff.

Key words: sales compensation; training; selection; recruiting; termination; hyperbolic discounting; present-bias; dynamic structural models; exclusion restriction; identification.

1. Introduction

Effective management of the sales force is vital to the success of many sales-driven organizations. Approximately 20 million salespeople in the United States, representing 10% of the adult population, serve as links between the customer and the firm (Zoltners et al., 2008). Investments in these salespeople are approximated to be 10% of sales revenues and can reach up to 40% in certain industries (Heide, 1999). The U.S. economy spends more than $800 billion on sales forces each year, nearly five times the estimated $169.5 billion spent on media advertising and more than 20 times the estimated $39.5 billion spent on Internet advertising (Zoltners et al., 2013). As these significant figures suggest, personal selling represents one of the most important elements of the firm's marketing mixes, highlighting the importance of managing and motivating salespeople to achieve the organization's objectives.

Since the earliest days of personal selling, organizations have utilized three main sales (force) management instruments—compensation, recruiting/retention (of high-ability employees), and training—to better control and motivate the sales force. **Figure 1** illustrates the relation between these instruments and the organization's outcome—i.e., the sales performance. Performance is the result of salespeople's behavior, and the sales management instruments are utilized to train and motivate proper behavior as well as to select the right type of people. The three key instruments (levers) not only differ in cost and effectiveness across different types of people, but they also are interconnected in their ability to effect behavioral change and, thus, the desired performance outcome. This research aims to jointly examine the effectiveness of multiple sales management instruments in the selection and performance of various types of salespeople.

A key, if not the most important, instrument in sales management is compensation. Organizations frequently use compensation to motivate and control the behavior of salespeople. A sales force compensation plan typically consists of fixed- and variable-pay components, where each component plays a distinct role in managing sales force behavior. Fixed pay (base salary) compensates for risk and, thus, provides stability and security of income (Arrow, 1971; Basu et al., 1985; Harris & Raviv, 1978; Hölmstrom, 1979; Lal & Srinivasan, 1993). Variable pay, on the other hand, provides a direct link between the sales outcome and financial rewards, thereby inducing motivation to achieve superior performance. Examples of such variable pay include commissions given as a portion of sales and lump-sum bonuses contingent on meeting a preset quota. According to Joseph and Kalwani (1998), 95% of firms in the United States utilize some form of variable pay to incentivize their salespeople, with the most frequently used forms being commissions and quota-based bonuses. Organizations also change the components of their incentive plans frequently; nearly 80% of U.S. firms revise their compensation programs every two years or less, to better motivate salespeople and to tailor their behavior to the goal of the sales organization (Zoltners et al., 2012).

However, simply providing the sales agents an optimal menu of compensation is not sufficient to achieve the desired outcomes of the sales organization. To support the productivity of their sales force, firms frequently rely on sales force training. Sales training is suggested as a means to increase productivity, stimulate communication inside and outside the organization, reduce inter- and intra-department misunderstandings, enrich sales force morale, and decrease selling costs (Stanton & Buskirk, 1987; Churchill et al., 1993; Dubinsky, 1996). Sales organizations in the U.S. invest $15 billion annually in sales training programs and devote more than 33 hours per year to training each salesperson (Lorge & Smith, 1998; Ingram et al., 2015). Thus, to effectively allocate resources among the firm's sales management instruments, it is essential to properly assess and evaluate the outcomes of an organization's sales training programs.

Whereas compensation and training serve to induce the right behavior, selection (recruitment/termination) affects firm performance through change of the sales force composition. Salespeople are known to exhibit a high rate of turnover: the estimated annual turnover rate of 27% is more than twice that of the average work force in the U.S. (Richardson, 1999). Selection is driven largely by two factors: (i) firm-induced selection, which involves recruiting, retention, and termination; and (ii) agent-induced selection, or voluntary turnover. When properly managed, selection allows the firm to maintain a healthy sales force through retaining high-quality agents and terminating persistent low performers. However, selection—especially voluntary turnover—also involves substantial costs to the firm, including costs related to hiring and training, jeopardized customer relationships, and territory vacancies (Griffeth & Hom, 2001; Boles et al., 2012). Hence, deriving a proper policy to control for sales force selection is vital to the success of a sales organization.

Despite the ubiquitous use of the above three sales management instruments, however, there is little insight into their joint effect on various behavioral outcomes. For instance, how should a firm design its compensation plan to select the right salespeople—retain the high performers while discouraging the low—over time? Which is more effective in motivating the agents to meet their goals—increasing the level of monetary compensation or providing sales training opportunities? Can recruiting/termination policy replace the role of compensation and, if so, at what cost?

Separately identifying each of the above issues turns out to be problematic because various behavioral outcomes are often interrelated and occur simultaneously. Heterogeneous salespeople exhibit differences in productivity, time preferences, and responsiveness to compensation components and training, which, in combination, determine individual performance. The performance outcome results in compensation that influences agents' turnover, which naturally leads to the selection of heterogeneous types of salespeople. Hence, this interrelated nature of behavioral outcomes necessitates an integrated model of sales force management.

There are two key challenges in modeling and identifying salespeople's response to various management instruments. First, data at the salesperson level on various management practices are difficult to obtain, as many organizations treat HR information as highly confidential. This is one of the reasons why previous studies have narrowed their focus to a single sales management instrument, such as compensation (Steenburgh, 2008; Misra & Nair, 2011; Chung et al., 2014). Second, a researcher directly observes neither the agent's effort exerted nor his or her time preferences—the degree to which immediate utility is favored over delayed utility—but observes only the attrition decision and performance outcome over a time period that is likely correlated with agent's forward-looking allocation of effort and outside opportunities. This requires a behavioral assumption about the link between a sales agent's motives (e.g., how close the person is to achieving quota at the end of the period) and his or her allocation of effort over time.

To overcome these challenges, we collaborate with a major multinational firm and formulate a comprehensive model of sales force behavior in response to various sales management practices. The model takes into account many of the key realistic elements in salespeople's behavior, including allocation of effort, stay-or-leave decision, forward-looking behavior, present-bias, and learning from training opportunities. Overall, we seek to gain insights into the way that employee training, outside employment opportunities, and various elements of compensation jointly affect the selection and performance of heterogeneous salespeople.

This paper also provides several key methodological contributions to the economics and marketing literature. We provide a formal proof that contemporaneous utility and time preferences can be non-parametrically identified in a model with the following two features. First, we allow for the choice variable to be *continuous*. Identifying time preferences in a dynamic choice model becomes challenging when confronted with continuous choice (e.g., effort) by the agent. Existing studies are largely built upon the discrete choice framework (Magnac & Thesmar, 2002; Fang & Wang, 2015; Abbring & Daljord, 2019), and, thus, the identification results do not fully translate into many real-world phenomena of continuous choice. We discuss the associated limitations and provide proper regularity conditions for identifying a model of continuous choice. Second, our model accompanies *unobserved* choice (effort), an essential feature in models representing the principal-agent problem. As is standard in the literature, we assume that the outcome (sales) is observed instead of the unobserved choice (effort). The key to identification is the presence of a non-linear incentive contract, in which an agent's distance-to-quota (DTQ) affects only his or her future payoffs in non-pecuniary periods and provides exclusion restrictions on current payoffs.

In addition, we provide conditions under which a hyperbolic discounting model—a more general structure than an exponential discounting model—is identified. Building upon the identification results, we estimate both exponential and quasi-hyperbolic discounting time preferences in our empirical application. A hyperbolic discounting model can potentially explain agents' seemingly irrational behaviors (such as extreme procrastination) that are difficult to explain with a standard discounting model but are commonly observed in the real world (for reviews, see Ainslie (1992), Kirby (1997), and Frederick et al. (2002)).

Our analysis reveals the existence of different types of salespeople and a selection across different types occurring during the firm's shift towards a more aggressive compensation plan. In addition, we identify the existence of heterogeneous time preferences (in both the present-bias and the long-term discount factor) that determine an agent's optimal level of effort. Subsequently, we run a series of counterfactual experiments to determine the changes in sales force performance and selection induced by alternative compensation plans, recruiting/termination policies, and sales training opportunities. Our findings reveal the trade-off relationship between adjusting fixed- and variable-pay components towards performance and selection. We also show the potential drawback of hiring only high-type agents; how a collective leave package can lead to selective departure of agents; and how sales training can serve as an alternative to providing additional compensation.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature. Section 3 presents a general framework of sales management and an agent's dynamic optimization problem. Section 4 discusses the identification of dynamic models under different time preference assumptions. Section 5 describes the institutional settings and provides model-free evidence that facilitates the empirical analyses. Section 6 presents the empirical specification and estimation. Section 7 discusses the estimation and counterfactual simulation results. Section 8 concludes.

2. Related Literature

This study on multi-dimensional sales force management contributes to several streams of research. First and foremost, the paper relates to the strand of literature on sales force compensation. The theoretical studies on this topic find controversial results regarding which component constitutes the optimal plan. Early work by Basu et al. (1985) and Rao (1990), under the principal-agent framework of

Hölmstrom (1979) in a single-period setting with a risk-averse agent, finds that the optimal incentive pay combines salary and nonlinear commission. More granularly, Hölmstrom and Milgrom (1987) and Lal and Srinivasan (1993) show that, in a multi-period setting, only a linear contract can achieve the first-best outcome. In contrast, Oyer (2000) finds that quota-based bonus compensation, along with linear over-achievement commissions, is uniquely optimal when the participation constraints are unbinding. More recently, Schöttner (2016) derives the conditions under which commission dominates a bonus plan and vice versa, depending on the degree of the agent's responsiveness to incentives.

The findings from empirical studies on compensation also reflect the discrepancy over differential effects on motivation and productivity. Oyer (1998), using aggregate sales data, finds that quota-bonus pay can induce salespeople to manipulate the timing of sales, thereby negatively affecting productivity. On the contrary, analyzing at the individual-level, Steenburgh (2008) concludes that quota-bonus pay induces additional effort that provides net improvement in sales. A similar disparity is reported using dynamic models; for example, Misra and Nair (2011) and Chung et al. (2014) report contrasting findings regarding the impact of quota-based bonuses on sales performance.

This paper contributes to the sales compensation literature in two ways. First, we expand the scope of compensation plans to discuss the dynamic selection of sales agents. By empirically analyzing sales force recruiting and attrition, we provide a better understanding of how a firm's compensation plans could facilitate the restructuring of its existing portfolio of sales agents. In addition, we link agents' training records to their performance. Both sales training and compensation serve as significant investments for a firm, and, thus, this study allows us to evaluate the relative effectiveness of these different sales management instruments. To the best of our knowledge, this research is the first to jointly examine the effect of multiple sales management instruments on sales force selection and performance.

As selection, by definition, accompanies employee turnover, the paper relates to the strand of literature on the antecedents of sales force turnover. In marketing, studies have put greater emphasis on the negative aspects of sales force departure. Richardson (1999) derives managerial measures for assessing the direct and indirect costs of turnovers, and Darmon (2008) proposes a cost/benefit analysis of turnover to provide management efficiency. Using empirical analyses, Shi et al. (2017) find that the negative effects vary, and Sunder et al. (2017) find that turnover risk is the greatest for salespeople with moderate performance.

The abovementioned studies, however, are limited to evaluating the short-term effect of territory absence and potentially overlook the selection process that simultaneously takes place. That is, if a firm can select the right salespeople, then despite the short-term loss, turnover may result in greater profitability in the long run. Hence, our study contributes to the appropriate valuation of turnover by investigating salespeople's latent future potential. In addition, our structural approach allows for various counterfactual policy simulations, whereas descriptive studies limit this applicability.

This study also relates to the strand of literature on sales training effectiveness. Although various studies have emphasized the pivotal role of sales training on performance and have proposed conceptual frameworks (Walker et al., 1977; El-Ansary, 1993; Honeycutt et al., 1995; Attia et al., 2005), only a handful of empirical studies have followed this footprint, likely due to difficulties in collecting data. Early empirical studies have generated strikingly mixed findings, ranging from enhancing performance up to 50% (Martin & Collins, 1991; Roman et al., 2003) to being largely uninfluential (Christiansen et al., 1996; Dubinsky, 1996). However, the findings in these studies have limited managerial implications, in that they rely primarily on survey measures over aggregate performance outcomes of a limited number of participating firms.

More recently, Kumar et al. (2014), investigating the effect of voluntary training opportunity on the salesforce lifetime value (net present value of a salesperson's output), show that sales training indeed has a positive effect in both the short term and the long term. However, the study is limited to evaluating the correlation between agents' self-selected training and their outcomes, refraining from developing causal inference. To identify the causal effect of training, Atefi et al. (2018) conduct a controlled field experiment that varies training policies across retail stores. They find a positive relation between the proportion of salespeople who receive training and sales outcomes; however, they analyze it only at the aggregate (store) level due to institutional and experimental settings.

Our study provides several insights into the literature by measuring the true effectiveness of sales force training. First, by analyzing the training records at the individual salesperson level, we are able to link different types of agents to the effectiveness towards training—thereby investigating the differential impact across heterogeneous salespeople. Second, our behavioral model allows us to analyze the comprehensive impact of training, in which it affects not only intertemporal performance outcomes, but also subsequent turnover and selection of the firm's sales force. Lastly, the structural formulation of the model allows us to evaluate the compensation-equivalent monetary value of training, providing guidance to firms on resource allocation for sales training.

Finally, this study relates to the economics and psychology literatures on time preferences and intertemporal decision making. The discounted utility model, established by Samuelson (1937) and further developed by Thaler (1981), posits that individuals discount future payoffs relative to immediate payoffs. To understand the behavioral responses to immediate versus delayed outcomes, we consider two models of time preferences: exponential and (quasi-) hyperbolic discounting. The exponential discounting model assumes that discounting occurs at a fixed rate per unit of time (Samuelson, 1937; Dhami, 2016), representing stationarity and time-invariance. Meanwhile, a hyperbolic discounting model posits that the longer the time horizon before a reward is received, the lower the per-period discount rate becomes (Ainslie, 1975; Ainslie & Herrnstein, 1981; Loewenstein & Prelec, 1992; Laibson, 1997; O'Donoghue & Rabin, 1999), implying present-bias and time-inconsistent behavior. That is, when the time delay is short, the per-period discount rate is high, and individuals are more likely to become impatient by selecting smaller but earlier rewards; but when the time delay is long, the per-period discount rate declines, and individuals exhibit greater patience for larger but later rewards.

In terms of identifying agents' time preferences, Rust (1994) shows that the discount factor is generally not identified from naturally occurring data without imposing further restrictions. Magnac and Thesmar (2002) generalize this idea to provide conditions on exclusion restrictions, the existence of instruments that do not affect an agent's current payoff but only his or her future payoff. This allows for identification of the discount factor. Empirical studies in economics and marketing apply this exclusion restriction to identify time preferences (discount factor) across various contexts, including new and used durable goods (Chevalier & Goolsbee, 2009; Ishihara & Ching, 2019); cellular phone usage (Yao et al., 2012); hardware and software platforms (Lee, 2013); sales force compensation (Chung et al., 2014); and consumer learning and inventory (Ching et al. 2014; Akca & Otter, 2015; Ching & Osborne 2017).

In a recent debate, Fang and Wang (2015) and Abbring and Daljord (2019) discuss in depth the identification of time preferences in dynamic discrete choice models. Fang and Wang (2015), by extending the above exclusion restriction arguments (Magnac & Thesmar 2002), consider conditions to identify various discounting behaviors, including exponential, quasi-hyperbolic, and naïve time preferences. Abbring and Daljord (2019) consider exclusion restrictions on model primitives and argue that the arguments presented in Fang and Wang (2015), under weaker conditions, may not allow for point identification of the discount factor.

Our paper's contribution to this stream of literature is threefold. First, we expand the scope of identification to incorporate *continuous* choice of the agent's actions. We discuss the limitations in applying the results of a discrete choice framework (Magnac & Thesmar, 2002; Fang & Wang, 2015; Abbring & Daljord, 2019) to the continuous choice setting and provide proper regularity conditions for identification—for both exponential and quasi-hyperbolic discounting time preferences.

Second, our identification argument allows for the agent's actions to be *unobserved* (e.g., effort) and indirectly measured by the observable output (e.g., sales). This unobservability, when combined with individual heterogeneity and random shocks in the production function, brings about additional concerns for identification. Most of the existing research assume that choices are fully observed and unobserved state variables are serially uncorrelated. Several exceptions include Norets (2009), Arcidiacono and Miller (2011), Hu and Shum (2012) and Chou, Derdenger and Kumar (2019), who consider persistent or serially correlated unobserved state variables. However, none of them allows for unobserved choice variables. To the best of our knowledge, Hu and Xin (2019) is the only other paper that allows for unobserved choice variables in a dynamic model, but their setting is only limited to unobserved discrete choices. This paper considers unobserved continuous choice variables.

Third, we provide empirical evidence of heterogeneity in both present-biased and discounting behavior using naturally occurring field data, bringing additional insights to the handful of empirical studies that estimate time preferences.

3. Model

We propose a comprehensive model of an agent's behavior based on the sales management framework illustrated in **Figure 1**. The discussion proceeds in three parts: (i) the agent's per-period utility and performance response functions; (ii) the dynamic allocation of effort and stay-or-leave decisions; and (iii) time preferences (discount factors); exponential and quasi-hyperbolic discounting models.

An agent derives utility and disutility from compensation and effort, respectively, and faces intertemporal employment (stay-or-leave) decisions. Compensation can be non-linear and dependent on past history of performance (e.g., quarterly sales outcomes). Thus, an agent exhibits forward-looking behavior and dynamically allocates effort over a specific time horizon.

3.1. Per-Period Utility and Performance

Agent i in period t derives per-period utility based on the pecuniary benefits conditional on his or her choice of actions—whether to stay with the firm (d_{it}) and, (if so), how much effort to exert (e_{it})—such that

$$\tilde{U}(W_{it}, d_{it}, e_{it}, \varepsilon_{dit}) = \begin{cases} M(W_{it}) - C(e_{it}) + \varepsilon_{1it} & \text{if } d_{it} = 1, \\ \rho_i + \varepsilon_{0it} & \text{otherwise.} \end{cases} \tag{1}$$

If the agent decides to stay with the firm ($d_{it} = 1$), he or she receives positive pecuniary utility $M(W_{it})$ as a function of compensation $W_{it} = W(q_{it}, s_{it}; \psi_{it})$, the level of which is determined by the agent's performance q_{it} and state s_{it}, given the firm's compensation plan ψ_{it}. Concurrently, the agent incurs disutility $C(e_{it})$ from exerting effort e_{it}, which affects the performance outcomes in the contemporaneous period. On the other hand, if the agent decides to quit ($d_{it} = 0$), he or she receives reservation utility ρ_i in perpetuity. This reservation utility represents the value of the agent's outside option. The decision to leave the firm is an absorbing state (i.e., permanent), and, thus, once an action is taken, the agent cannot return to the firm.[1]

In addition to the deterministic element, the per-period utility includes a structural error term ε_{dit}, specific to the agent's stay/leave decision (d_{it}). The structural error represents the states observed by the agent (but unobserved by the researcher) in his or her decision making. As is standard in the literature, the error term is assumed to satisfy the conditional independence assumption (Rust, 1987), in that, in a given period, it is not a function of an agent's effort allocation decision (e_{it}). That is, the error term (unobserved state) is assumed to be determined ex-ante of the current-period effort decision.

We focus on a class of non-linear compensation schemes in which payout depends on aggregate performance outcome over a certain time horizon. These schemes typically include components such as quarterly/yearly bonuses or end-of-year commissions, which are commonly administered in practice (Joseph & Kalwani, 1998). By providing a benefit at the end of the quota evaluation cycle consisting of multiple periods, the compensation plan stimulates the sales agent's forward-looking behavior, as the agent's effort exerted today accumulates to influence his or her future payoff. This cumulative effort is captured by a subset of the state variables in s_{it}, whose subsequent-period values $s_{i,t+1}$ evolve as a function of current-period performance q_{it} and state s_{it}.

We model the agent's per-period performance, q_{it}, as a monotonically increasing function of effort e_{it} such that

$$q_{it} = q(e_{it}, \alpha_i, \xi_{it}), \tag{2}$$

where α_i denotes unobserved individual heterogeneity and ξ_{it} is a random performance shock, not yet realized but whose distribution is known, to the agent when determining the level of effort.[2] In this manner, the agent's *unobserved* effort e_{it}, is linked to the *observed* performance q_{it}, in a given period. The performance outcome q_{it}, in turn, has both (i) a direct effect on contemporaneous compensation $W_{it} = W(q_{it}, s_{it}; \psi_{it})$ in bonus/commission periods; and (ii) an indirect effect on future compensation

[1] In our empirical application, no agent returned after departing from the firm.

[2] In terms of interpretation, individual heterogeneity α_i could represent either (i) the agent's baseline productivity (performance attained without exerting any effort); or (ii) the minimal level of effort necessary to maintain the job. As we do not directly observe the agent's effort, these effects cannot be separately identified. Hence, our definition of effort e_{it} represents the agent's additional contribution to performance from the baseline.

through the evolution of the subsequent state variables $s_{i,t+1} = f(q_{it}, s_{it}; \psi_{it})$, where $f(\cdot)$ is the state transition function conditional on the firm's compensation plan ψ_{it}.

The utility function given by **Equation (1)** represents the *ex-post* utility of the agent, as performance shock ξ_{it} in **Equation (2)**, which determines q_{it} and W_{it} in a given period, has yet to be realized when making the stay/leave and effort decisions. To form the basis of decision making, the agent takes expectation over his or her compensation $W_{it} = W(q_{it}, s_{it}; \psi_{it})$, given the amount of effort exerted e_{it} (that determines performance outcome q_{it}) under current state s_{it}. In this manner, the *ex-ante* utility function of the agent can be represented as

$$U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) = \begin{cases} \mathrm{E}[M(W_{it}) \mid e_{it}, s_{it}] - C(e_{it}) + \varepsilon_{1it} & \text{if } d_{it} = 1, \\ \rho_i + \varepsilon_{0it} & \text{otherwise.} \end{cases} \tag{3}$$

The ex-ante utility function in **Equation (3)**, when linked with the aforementioned course of actions, outcomes, and state transitions, naturally leads to a dynamic formulation of the model.

3.2. Dynamic Allocation of Actions

Given the per-period utility function, an agent chooses an action that solves the dynamic optimization problem, maximizing the sum of current and future payoffs. An agent i receives a stream of utility over discrete time periods (t=1,2,...,∞). Let us define the value function as the agent's discounted present value of the expected utility stream such that

$$\begin{aligned} \tilde{V}(s_{it}, \varepsilon_{dit}) &= \mathrm{E}\left[\sum_{\tau=t}^{\infty} \phi(\tau - t)\left\{\max_{d_{i\tau}, e_{i\tau}} U(d_{i\tau}, e_{i\tau}, s_{i\tau}, \varepsilon_{di\tau})\right\}\Big| s_{it}, \varepsilon_{dit}\right] \\ &= \max_{d_{it}, e_{it}}\left\{U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) + \mathrm{E}\left[\sum_{\tau=t+1}^{\infty} \phi(\tau - t)\left\{\max_{d_{i\tau}, e_{i\tau}} U(d_{i\tau}, e_{i\tau}, s_{i\tau}, \varepsilon_{di\tau})\right\}\Big| d_{it}, e_{it}, s_{it}, \varepsilon_{dit}\right]\right\}, \end{aligned}$$

where $\phi(j)$ denotes the agent's discount function for the utility from future j-periods forward ($j = 0, 1, 2, 3, \ldots$) and $\phi(0)$=1, and $U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit})$ represents the *ex-ante* per-period utility function in **Equation (3).** Hence, the present value is represented by the expected utility flow upon making an infinite sequence of optimal decisions ($d_{i\tau}, e_{i\tau} : \tau$=$t$, t+1, ...), governed by the discount function $\phi(\cdot)$. The expectation is taken with regard to both the idiosyncratic performance shock ξ and the structural error ε for each period $\tau \geq t$+1.

The *choice-specific* value function with respect to each action pair (d_{it}, e_{it}), representing the discounted present value when the agent chooses actions d_{it} and e_{it}, given state variables s_{it} and ε_{dit}, is defined as

$$V(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) = U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) + \mathrm{E}\left[\sum_{\tau=t+1}^{\infty} \phi(\tau - t)\left\{\max_{d_{i\tau}, e_{i\tau}} U(d_{i\tau}, e_{i\tau}, s_{i\tau}, \varepsilon_{di\tau})\right\} \middle| d_{it}, e_{it}, s_{it}, \varepsilon_{dit}\right], \tag{4}$$

In each period, the agent incorporates the information contained in the current states $(s_{it}, \varepsilon_{dit})$ to evaluate the future outcome of current-period actions: employment and effort.

The agent's effort policy function is given as

$$e_{it} = \begin{cases} \arg\max_{e} \left\{V(1, e, s_{it}, \varepsilon_{1it})\right\} & \text{if } d_{it} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

That is, the agent chooses the *optimal* level of effort e_{it}, which maximizes the discounted stream of expected utility flow, conditional on the current states and on staying with the firm.[3] The temporal trade-off of exerting effort e_{it} (in non-bonus/commission periods) arises between the per-period disutility $C(e_{it})$ in **Equation (3)**, and the state-transition $s_{i,t+1}$ (updated through the performance outcome q_{it} in **Equation (2)**) towards a higher probability of future pecuniary benefit at the end of the compensation cycle.

The agent decides to continue with the firm if the choice-specific value $V(1, e_{it}, s_{it}, \varepsilon_{1it})$ of staying and exerting effort is greater than the reservation value $V(0, 0, s_{it}, \varepsilon_{0it})$. That is,

$$d_{it} = \begin{cases} 1 & \text{if } V(1, e_{it}, s_{it}, \varepsilon_{1it}) \geq V(0, 0, s_{it}, \varepsilon_{0it}), \\ 0 & \text{otherwise.} \end{cases}$$

The summary of the model structure is as follows: After observing his or her current state (with regard to future pecuniary benefit), an agent exerts effort and incurs disutility. Exerted effort, in combination with an idiosyncratic shock, determines the agent's current-period sales performance. This performance affects both the current-period payoff and the probability distribution of state variables in the subsequent period. Hence, the agent's effort helps preserve the states in healthy conditions, increasing the chance of receiving a monetary payoff in later periods. However, if the current state shows a limited chance of receiving future payoffs (e.g., after several periods of low performance), the agent may give up

[3] For brevity, we suppress the optimality (*) notation throughout the paper.

exerting effort in order to reduce current-period disutility. Furthermore, if the value of staying becomes lower than the outside option, the agent will decide to leave the firm.

3.3. Time Preferences

The above forward-looking model naturally leads to a conceptual question: How does an agent discount the stream of future utility to derive the optimal policy? In other words, what is the agent's time preference, the degree to which immediate utility is favorable over delayed utility? The question can be addressed through varying the structure of $\phi(j)$, the discount function of the dynamic optimization problem in **Equation (4)**. We consider two models of time preference: exponential discounting and quasi-hyperbolic discounting.

3.3.1. Exponential Discounting

The exponential discounting model (Samuelson, 1937) postulates that an agent's discount function for the jth future period takes the form

$$\phi(j) = \delta^j \text{ for } j = 0, 1, 2, ...,$$

where $\delta \in (0,1)$. The model implies time-consistent behavior by featuring stationary discounting where expected future utility geometrically decays. Because of its analytical convenience, exponential discounting is frequently assumed in the economics and marketing literatures.

The dynamic optimization problem can be decomposed into an infinite sequence of single-period decisions. Assuming exponential discounting, the infinite sum of the discounted future utility flow in **Equation (4)** can be replaced by the subsequent period value function such that

$$\begin{aligned} V(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) &= U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) + \mathrm{E}\left[\sum_{\tau=t+1}^{\infty} \delta^{\tau-t}\left\{\max_{d_{i\tau}, e_{i\tau}} U(d_{i\tau}, e_{i\tau}, s_{i\tau}, \varepsilon_{di\tau})\right\}\middle| d_{it}, e_{it}, s_{it}, \varepsilon_{dit}\right] \\ &= U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) + \delta\mathrm{E}\left[\max_{d_{i,t+1}, e_{i,t+1}} V(d_{i,t+1}, e_{i,t+1}, s_{i,t+1}, \varepsilon_{di,t+1})\middle| d_{it}, e_{it}, s_{it}, \varepsilon_{dit}\right]. \end{aligned}$$

Henceforth, for brevity of exposition, subscripts i and t are suppressed, and the subsequent period $(t+1)$ is denoted by a prime $(')$ symbol when possible.

Let v and u be the *deterministic portion* of value and utility functions (i.e., $v(\cdot)=V(\cdot)-\varepsilon_{dit}$ and $u(\cdot)=U(\cdot)-\varepsilon_{dit}$). Assuming additive separability and serial independence of the structural errors, the above equation further simplifies to

$$v(d,e,s) = u(d,e,s) + \delta\mathrm{E}\left[\max_{d',e'}\left\{v(d',e',s') + \varepsilon_d'\right\}\middle| d,e,s\right]. \tag{5}$$

3.3.2. Quasi-Hyperbolic Discounting

The quasi-hyperbolic discounting model (Phelps & Pollak, 1968; Laibson, 1997) posits that an agent's discount function for the jth future period takes the form

$$\phi(j)=\begin{cases}1 & \text{if } j=0,\\ \beta\delta^{j} & \text{if } j=1,2,3,\ldots,\end{cases}$$

where $\delta\in(0,1)$ is the standard discount factor, and $\beta\in(0,1]$ is the present-bias factor. Often referred to as the Beta-Delta preferences, the model parsimoniously captures present-bias and, thus, time-inconsistency. The standard discount factor δ captures long-run, time-consistent discounting, and the present-bias factor β captures short-term impatience and the discontinuity between the present and the future (O'Donoghue & Rabin, 1999). Note that exponential discounting is a special case of quasi-hyperbolic discounting when β=1—i.e., the agent is not present-biased.

Given quasi-hyperbolic discounting, the choice-specific value function in **Equation (4)** can be rewritten as

$$v(d,e,s)=u(d,e,s)+\beta\delta\mathrm{E}\left[\max_{d',e'}\left\{\tilde{v}(d',e',s')+\varepsilon_d'\right\}\middle|d,e,s\right], \tag{6}$$

where

$$\tilde{v}(d,e,s)=u(d,e,s)+\delta\mathrm{E}\left[\max_{d',e'}\left\{\tilde{v}(d',e',s')+\varepsilon_d'\right\}\middle|d,e,s\right]. \tag{7}$$

Unlike the case of exponential discounting, however, the quasi-hyperbolic discounting model does not allow a recursive representation of a single value function. The flow of future utility involves an additional value function $\tilde{v}(\cdot)$ due to the agent's time-inconsistency. Hence, the optimal choice of effort e in the present becomes different from that of the future (i.e., present-biased).

The structure of the choice-specific value functions in **Equations (6)** and **(7)** requires solving two equations for two functions. This leads to a challenge in identification, which we discuss, in detail, in the following section.

4. Identification

We begin by discussing the identification of the primitives of the performance response and utility functions. Then, we discuss identification with regard to the discount function, both exponential and quasi-hyperbolic. Our arguments build upon those of Magnac and Thesmar (2002), who propose exclusion restrictions to identify the standard (exponential) discount factor. We expand identification of time preferences to (i) incorporate continuous and unobserved choice variables; and (ii) consider identification of present-bias factor in a quasi-hyperbolic discounting model.

4.1. Performance Response Function

Suppose that the data $(d_{it}, s_{it}, q_{it}, \psi_{it})$ for agents $i = 1, 2, \ldots, N$ over time $t = 1, 2, \ldots, T$ are observed and that these observations are i.i.d. (independent and identically distributed) across agents. To consider the identification of the performance response function in **Equation (2)**, we impose the following assumptions on the components of the performance response function.

Assumption 1 (Performance Response Function): *The performance response function is linear in effort e_{it}, individual heterogeneity α_i and idiosyncratic performance shock ξ_{it}, such that $q(e_{it}, \alpha_i, \xi_{it}) = e_{it} + \alpha_i + \xi_{it}$. The components of the performance response function exhibit the following properties.*

(i) *Effort e_{it} is not a function of individual heterogeneity α_i.*

(ii) *Idiosyncratic performance shock ξ_{it} is independent of $(s_{is}, \alpha_i, \varepsilon_{is})$ for any $s \leq t$, and is i.i.d. across agents and over time. Furthermore, $E(\xi_{it}) = 0$.*

(iii) *The agent has rational expectations about the law of motion: the distribution of the performance shock ξ_{it}, which affects the transition probability of future states, is known to the agents.*

Assumption 1 governs the relation among unobserved effort, individual heterogeneity, and observed performance, and stipulates the forward-looking behavior of the agents. The challenge in identifying models of unobserved effort is in disentangling the individual heterogeneity. As we do not directly observe either construct, separately identifying the individual heterogeneity from effort remains infeasible without further restrictions. Hence, Assumption 1(i) restricts the effect of individual heterogeneity α_i on performance that occurs independently of effort e_{it}. This assumption is satisfied (as in our context) when effort e_{it} reflects the agent's incremental contribution to performance.

Given Assumption 1, we characterize the identification of the performance response function:

Proposition 1. *Under Assumption* 1, *(i) the agent's effort e_{it}, (ii) individual heterogeneity (fixed effect) α_i, and (iii) the distribution of performance shock ξ_{it} are identified.*

Proof. Because the agent chooses effort after observing the state variables, the effort e_{it} is a function of s_{it} and ε_{dit}—i.e., $e_{it}{=}e(s_{it}, \varepsilon_{dit})$. However, ε_{dit} does not affect effort because it is invariant to the effort choice conditional on the stay-or-leave decision, d_{it}. Hence, the performance response function can be represented as

$$q_{it} = \alpha_i + e(s_{it}) + \xi_{it},$$

where q_{it} and s_{it} are observed, but α_i and ξ_{it} are not. This function becomes a nonparametric panel data fixed effects model, whose identification arguments follow those of Evdokimov (2010) and Henderson et al. (2008): $e(s_{it})$ is a regression function of q_{it} on s_{it} with fixed effect α_i. The fixed effect α_i is consistently estimated as $T\to\infty$. The distribution of the residuals is a consistent estimator for the distribution of ξ_{it}. (Q.E.D.)

The above proof is based on a nonparametric regression approach, in a similar vein to Hu and Xin (2019). Although Hu and Xin (2019) also consider the identification of dynamic models with unobserved choices, our setting differs in at least three dimensions: (i) we allow for continuous choices, while they consider only discrete choices; (ii) we incorporate unobserved heterogeneity in α_i, thereby exploiting the panel data structure; and (iii) we have an observed variable q_{it} as a function of the unobserved choice, which is not covered in Hu and Xin (2019).

From Proposition 1, the effort policy function $e(s_{it})$ and the fixed effect α_i are identified. Henceforth, we treat effort e_{it} as if it is observed while subsuming fixed effect α_i within the state variables s_{it} of the agent.

4.2. Exponential Discounting Model

With regard to the identification of the exponential discounting model in **Equation (5),** we make an additional but standard assumption on the instantaneous utility function (see, e.g., Rust 1994; Fang & Wang, 2015).

Assumption 2 (Utility Function): *Instantaneous utility function $U(d, e, s, \varepsilon)$ is additively separable in the utility shock, such that $U(d, e, s, \varepsilon) = u(d, e, s) + \varepsilon$. The components of the utility function exhibit the following properties.*

(i) *The utility of choosing to leave the firm is normalized to zero.*

(ii) *The utility shock ε_{dit} is i.i.d. across individuals and over time and is independent of s_{it}. The joint distribution $F_\varepsilon(.)$ of $\varepsilon_{dit} = (\varepsilon_{0it}, \varepsilon_{1it})$ is known.*

Without loss of generality, we can rewrite the per-period utility function in **Equation (3)** to satisfy Assumption 2(i), such that

$$U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) = \begin{cases} \mathrm{E}[M'(W_{it}) \mid e_{it}, s_{it}] - C(e_{it}) + \varepsilon_{1it} & \text{if } d_{it} = 1, \\ \varepsilon_{0it} & \text{otherwise,} \end{cases} \tag{8}$$

where ρ_i is normalized to zero. Hence, the identified pecuniary utility $M'(W_{it})$ represents the difference relative to the reservation value. For brevity of illustration, we use the normalized utility function given by **Equation (8)** for the remainder of the section.

Because the agent's optimal effort conditional on staying with the firm is identified in Proposition 1, effort is treated as an observed variable for an agent who stays with the firm. Even if an agent leaves the firm, we can identify the optimal effort $e_{it} = e(s_{it})$ during the period in which the agent stayed with the firm.

Lemma 1. *Suppose that the agent's true time preferences follow the exponential discounting model. Under Assumptions* 1-2, *the choice-specific value function v is non-parametrically identified.*

Proof.[4] Consider an agent with $(s_{it}, \varepsilon_{0it}, \varepsilon_{1it})$. The agent chooses to stay with the firm only if

$$v(1, e, s_{it}) + \varepsilon_{1it} \geq \varepsilon_{0it}.$$

Although ε_{0it} and ε_{1it} are unobserved, their joint distribution is assumed to be known. Thus, the probability of staying with the firm can be written as

$$\begin{aligned} \Pr(d_{it} = 1 \mid s_{it}) &= \Pr(v(1, e, s_{it}) \geq \varepsilon_{0it} - \varepsilon_{1it}) \\ &= F_{\varepsilon_0 - \varepsilon_1}(v(1, e, s_{it})), \end{aligned}$$

where $F_{\varepsilon_0 - \varepsilon_1}(.)$ is the cumulative distribution function of $\varepsilon_{0it} - \varepsilon_{1it}$. As the probability of staying with the firm can be computed from the observable data (d_{it}), we obtain the employment-choice-specific value function via

$$v(1, e, s_{it}) = F^{-1}_{\varepsilon_0 - \varepsilon_1}(\Pr(d_{it} = 1 \mid s_{it})).$$

(Q.E.D)

Hence, the choice-specific value function v is identified at the optimal level of effort (identified in Proposition 1). What remains for identification of the exponential discounting model are the primitives of the utility function and the discount factor. We impose the following assumption (Magnac & Thesmar, 2002) on the state variables within the data.

[4] The proof builds upon that of Lemma 1 in Magnac and Thesmar (2002).

Assumption 3 (Exclusion Restriction): *Suppose that the state variables s can be partitioned into two vectors, s_1 and s_2, where s_1 is the vector of variables that satisfies the following condition: there exists a subset S_1 in the support of s_1 such that if $s_1 \in S_1$,*

(i) $u(d,e,s_1,s_2) = u(d,e,s_1,\tilde{s}_2)$ *for any* s_1 *and* $\tilde{s}_2$; *and*

(ii) $v(d,e,s_1,s_2) \neq v(d,e,s_1,\tilde{s}_2)$ *for some* s_1 *and* $\tilde{s}_2$.

That is, if s_1 takes a value in S_1, s_2 does not affect the present utility. In our empirical application, the variables month type and an agent's distance-to-quota (DTQ) play the role of s_1 and s_2, respectively. For example, there is no performance-based lump-sum payment in October or November, so the DTQ does not affect the per-period utility. In these months, the per-period utility depends only on the disutility of effort, which does not include s. However, the future expected utilities would differ according to the DTQ.

Proposition 2. *Suppose that the agent's true time preferences follow the exponential discounting model. Under assumptions* 1-3, *the instantaneous utility function u at the optimal effort and the discount factor δ are identified.*

Proof.[5] Suppose that, from Lemma 1, the choice-specific value function is identified (at the optimal effort). Then, the value functions, at state variables ($s_1 \in S_1$, s_2) and ($s_1 \in S_1$, $\tilde{s}_2$) that satisfy Assumption 3 (ii), can be evaluated such that

$$v(d,e,s_1,s_2) = u(d,e,s_1,s_2) + \delta \mathrm{E}[\max_{d',e'}\{v(d',e',s') + \varepsilon'_d\} \mid d,e,s_1,s_2], \tag{9}$$

$$v(d,e,s_1,\tilde{s}_2) = u(d,e,s_1,\tilde{s}_2) + \delta \mathrm{E}[\max_{d',e'}\{v(d',e',s') + \varepsilon'_d\} \mid d,e,s_1,\tilde{s}_2]. \tag{10}$$

By the exclusion restriction and, thus, subtracting **Equation (10)** from **Equation (9)**, the per-period utility cancels out such that

$$v(d,e,s_1,s_2) - v(d,e,s_1,\tilde{s}_2) = \delta\left\{\mathrm{E}[\max_{d',e'}\{v(d',e',s') + \varepsilon'_d\} \mid d,e,s_1,s_2] - \mathrm{E}[\max_{d',e'}\{v(d',e',s') + \varepsilon'_d\} \mid d,e,s_1,\tilde{s}_2]\right\}.$$

(Q.E.D.)

Because the expected future utility can be computed utilizing the (identified) value functions and the law of motion, the discount factor δ is the only unknown in this equation. Hence, the discount factor is uniquely identified. The per-period utility u is identified from either **Equation (9)** or **Equation (10)**, given the value function and the discount factor. Once u is identified, the identification of pecuniary

[5] The proof uses a similar argument in the proofs for Corollary 3 and Proposition 4 in Magnac and Thesmar (2002).

utility $M'(W)$ and disutility of effort $C(e)$ within u is straightforward. As there is no variation in pecuniary utility during non-bonus periods, we have $C(e)$ identified in those months. The remaining $M'(W)$ is identified using $M'(W) = u(d, e, s) + C(e)$ during the bonus-paying months.

4.3. Quasi-Hyperbolic Discounting Model

Under the quasi-hyperbolic discounting model in **Equations (6)** and **(7)**, there exist two value functions, v and $\tilde{v}$, and two discounting parameters, β and δ. The quasi-hyperbolic discounting model is more general than the exponential discounting model, as the latter is a special case of the quasi-hyperbolic discounting model of when β=1.

As Lemma 1 equally applies to the quasi-hyperbolic discounting model, the value function v is non-parametrically identified. However, identification of $\tilde{v}$ is not as straightforward, as **Equations (6)** and **(7)** form a system of equations. By multiplying **Equation (7)** by β and subtracting it from **Equation (6)**, we obtain

$$v(d,e,s) - \beta\tilde{v}(d,e,s) = (1-\beta)u(d,e,s).$$

Because $\beta > 0$, the above equation simplifies to

$$\tilde{v}(d,e,s) = \frac{\beta - 1}{\beta}u(d,e,s) + \frac{1}{\beta}v(d,e,s).$$

Finally, inserting the above equation into **Equation (6)** establishes that

$$v(d,e,s) = u(d,e,s) + \delta \mathrm{E}\left[\max_{d',e'}\left\{(\beta-1)u(d',e',s') + v(d',e',s') + \beta\varepsilon_d'\right\}\middle| d,e,s\right]. \tag{11}$$

In **Equation (11)**, the distribution of ε_d' and the value function v are known, whereas the per-period utility u and the discount factors, δ and β, are unknown. Because this equation summarizes the system of equations, (u, δ, β) are identified if there exists a unique solution to **Equation (11)**.

In Mathematics, the structure of **Equation (11)** is known as a nonlinear Fredholm integral equation of the second kind (Arfken & Weber, 1999; Polyanin & Manzhirov, 1998; Vetterling et al., 1992). Solving the integral equation for the unknown utility function $u(d, e, s)$ is an ill-posed inverse problem due to the maximum function and integration taken over the utility function. Because a lot of information is "integrated out" and naturally lost during the process, it is well-known that the solution to this ill-posed inverse problem may not exist, or even if a solution exists, it may not be unique.[6]

[6] Conceptually, obtaining the solution to this problem is equivalent to finding an inverse mapping of the non-linear integral. Even if there exists a unique solution, it is known to be extremely difficult, if not impossible, to obtain.

The essence of this problem arises due to the *continuity* of the choice variable. If the choice variable is discrete, the integral equation in **Equation (11)** can be replaced by a matrix algebra, and the problem is simplified to finding the inverse of the matrix. For example, Abbring and Daljord (2019) and Fang and Wang (2015), in a discrete choice setting, rely on matrix algebra to find the inverse for identification. This is not applicable to our setting—in which the choice variable is continuous—as solving for the inverse of an integral equation is ill-posed. Thus, without further restrictions, the utility function and the discount factors cannot be non-parametrically identified, even if the aforementioned exclusion restrictions hold. Intuitively, the ill-posed problem in our case is due to the fact that the continuous choice in the model requires the utility function to be an infinite dimensional object (absent parametric assumptions), whereas in a discrete choice model, the utility function is represented by a finite dimensional vector (as in Fang and Wang (2015)). Because of this difference, a finite number of exclusion restrictions is insufficient to non-parametrically identify the utility function of a continuous choice hyperbolic discounting model.

The exponential discounting model bypasses this issue because the utility function does not enter the integral due to its recursive nature. That is, the value function for the future payoffs is identified directly from the choice probabilities. In contrast, in the quasi-hyperbolic discounting model, the utility function enters the integral as in **Equation (11)**. This change in the value function creates complications in solving for the equation, leading to uncertainty about the existence of the solution and, if it does exist, its uniqueness.

A typical solution for an ill-posed inverse problem is "regularization." In a broad sense, to regularize is to provide additional assumptions that can aid the existence, uniqueness, and numerical stability of a solution. Some common examples include discretization of variables (Magnac & Thesmar, 2002; Fang & Wang, 2015; Abbring & Daljord, 2019); parameterization of functions; and Lasso-type penalization methods. In the following, we regularize by imposing a parametric assumption on the per-period utility function u to identify quasi-hyperbolic time preferences in the presence of a continuous choice variable.

Assumption 4 (Parametric Assumption): *The instantaneous utility function is known up to a parameter vector* $\mu = (\gamma, \theta)$. *That is,* $M'(W) = M'(W \mid \gamma)$ *and* $C(e) = C(e \mid \theta)$. *In addition, the joint distribution* $F_\varepsilon(.)$ *of* $\varepsilon_d = (\varepsilon_0, \varepsilon_1)$ *follows a type-*1 *extreme value distribution.*

For illustration, we use the parametric assumptions $M'(W \mid \gamma) = \gamma_0 + \gamma_1 W - \gamma_2 W^2$ and $C(e \mid \theta) = \theta e^2$, for $\gamma_1, \gamma_2, \theta > 0$. Given the parameter vector μ, we can compute the agent's optimal effort (in the subsequent period), conditional on then staying with the firm, using

$$e'(s \mid \mu) = \arg\max_{e} \left\{(\beta - 1)u(1, e, s) + v(1, e, s)\right\}.$$

Note that prior to parametrization, this optimal effort for the subsequent period was intractable.

Given the extreme value distribution assumption, the future payoff component within the expectation in **Equation (11)**, conditional on s', can now be written as

$$\max_{d',e'} \left\{(\beta - 1)u(d', e', s' \mid \mu) + v(d', e', s') + \beta\varepsilon'_{d'}\right\}$$
$$= \beta \log\left\{\exp\left(\frac{(\beta - 1)u(d', e', s' \mid \mu) + v(d', e', s')}{\beta}\right) + 1\right\} \equiv \Lambda(s' \mid \mu, \beta).$$

By Assumption 1, we can compute the expectation of the above future payoff over s', given the current period state and choice variables. That is,

$$\mathrm{E}\left[\max_{d',e'} \left\{(\beta - 1)u(d', e', s' \mid \mu) + v(d', e', s') + \beta\varepsilon'\right\} \mid d, e, s\right] = \int \Lambda(s' \mid \mu, \beta) f(s' \mid d, e, s) ds' .$$

Thus, the identification criteria in **Equation (11)** simplify to a function of the parameters (μ, δ, β), where

$$u(d, e, s \mid \mu) - v(d, e, s) + \delta \int \Lambda(s' \mid \mu, \beta) f(s' \mid d, e, s) ds' = \Pi(d, e, s \mid \mu, \delta, \beta).$$

The true parameter vector (μ, δ, β) solves the above equation $\Pi(d, e, s \mid \mu, \delta, \beta) = 0$ for all (d, e, s). Thus, for identification, a full-rank condition assumption is sufficient.

Assumption 5 (Rank Condition): *Denote the agent's decision and state variables by $x = (d, e, s)$. There exists a subset $X = \{x_j : j = 1, 2, \ldots, J\}$ in support of x such that $\left\{\frac{\partial \Pi(x_j)}{\partial(\mu, \delta, \beta)'} : j = 1, 2, \ldots, J\right\}$ has a rank greater than or equal to the number of parameters.*

Theorem 1. *Under Assumptions* 1-5, *the parameters (μ, δ, β) under the quasi-hyperbolic discounting model are identified.*

Proof. Let $\omega = (\mu, \delta, \beta)$ denote the vector of parameters, and suppose that ω_0 is the true value of these parameters. The vector of true parameters ω_0 is said to be locally identified[7] if there exists a positive number ζ such that no other parameter value $\tilde{\omega} \neq \omega_0$ satisfies $\Pi(\tilde{\omega}) = 0$ and $\| \tilde{\omega} - \omega_0 \| < \zeta$. That is, ω_0 is the unique solution to Π within a certain radius.

[7] The local identification approach is the standard definition of identification in the economics literature (e.g., Chen et al., 2014). Global identification can be achieved by assuming that the second derivative is globally convex or concave.

The true parameter ω_0 solves $\Pi(x \mid \omega_0) = 0$ for all x. Although x has infinite support, the information necessary for identification is up to the number of the parameters. Let $\{x_1, x_2, ..., x_J\}$ be a subset of the support of x satisfying Assumption 5. Let us denote the equations evaluated at the subset by

$$\pi(\omega) = \begin{pmatrix} \Pi(x_1 \mid \omega) \\ \Pi(x_2 \mid \omega) \\ \vdots \\ \Pi(x_J \mid \omega) \end{pmatrix}$$

and $\pi' = \frac{\partial \pi(\omega_0)}{\partial \omega'}$. A sufficient condition for local identification of α_0 is $rank(\pi') = \dim(\omega)$ (Chen et al. 2014), which follows from Assumption 5. Sufficient conditions for the full rank condition and algebraic derivation of the derivatives are provided in the appendix.

(Q.E.D.)

5. Institutional Details and Descriptive Analysis

We first describe the details of the institution and its compensation plan for our empirical application. We then provide model-free evidence on forward-looking behavior and allocation of effort, justifying our dynamic structural formulation of the model.

5.1. Sales Environment

The focal firm under study is a multinational generic pharmaceutical company, offering a portfolio of branded prescription products through its own direct sales force. The data come from the firm's sales operations in Turkey. Some notable aspects of the Turkish pharmaceutical market are worth mentioning. First, the government heavily regulates prices. Second, the nation's universal healthcare system induces a high level of competition among the generics companies. Third, direct-to-consumer (DTC) advertising is prohibited, as is the case in most other markets.[8] As such, personal selling plays an important and *only* role in the firm's marketing and go-to-market strategies. Thus, recruiting and maintaining a sustainable pool of sales agents and training and motivating them properly are critical factors for success.

The data consist of salespeople's turnover, hours of training, and performance records during a three-year period (2015-2017). **Table 1** shows the number of employees who joined and departed and the corresponding turnover rate for each year. The firm has an average (voluntary) turnover rate of 14.60%

[8] As of 2018, direct-to-consumer advertising was allowed only in Brazil, New Zealand, and the United States, with varying restrictions on mode and content.

over the three years.[9] We focus our attention to those individuals who have remained in the firm (stay), and who have voluntarily separated (quit). In addition, to minimize the effect of the initial learning curve, we drop individuals with observations of fewer than or equal to three months since being hired (i.e., who joined on or after October 2017). The data-cleaning process leaves us with 554 salespeople, and the corresponding descriptive statistics are shown in **Table 2**. Employees who have decided to stay with the firm tend to perform better with higher variable pay and to have longer tenure.

The firm offers three types of sales training programs: primary training session; year-end sales session; and new rep sales orientation. The 12-hour primary training session took place twice during the data observation period: once in January 2015, targeted at the reps from the primary care division (representing half of the entire sales force), and the other in April 2016, targeted towards senior reps across all divisions. The three-hour year-end sales session, which took place in December 2016, was mandatory for all sales reps. In 2017, a new rep sales orientation program was introduced; this session ran for three hours and was designed for agents with less than one year of tenure.[10] The hourly cost of sales training per rep is estimated to have been $37 in 2016.

The firm operates its sales activity by route call sales: each sales agent has a preplanned series of meetings with either physicians or pharmacists over their exclusive territory. On average, an agent makes 20 calls per day. During each meeting, the agents exert effort to promote the firm's products over a wide range of competitors' products.

5.2. The Compensation Plan

The firm's compensation plan is comprised of three components: a base salary, a quota-based bonus, and an overachievement commission. An overview of the plan is illustrated in **Figure 2**, and the specifics of the quota-bonus payment schedule are described in **Table 3**. The agents receive an average fixed monthly salary of $1,500.[11] At the end of the first three quarters, the agents receive a $1,700 bonus if the respective quotas are met.[12] At the end of the year, a $3,400 bonus is paid if the agent has met the

[9] To focus on the agent's behavior towards selection (voluntary turnover), we treat layoffs as an idiosyncratic decision by the firm and do not consider the case of involuntary departures. The involuntary turnover rate of the focal firm was 7% in 2017; the majority of those who left were agents in their probation period (less than one year since hire).

[10] Because participants in the primary training sessions were chosen based on an entire division or seniority, there exists unique variation in training hours that is exogenous to individual performance. Also, the agents that joined the firm during the observation period add to this variation, as they missed the training opportunity in the earlier periods.

[11] Converted to U.S. dollars using the exchange rate at the beginning of our data sample (January 2015).

[12] As shown in the **Table 3**, the agent receives a diminishing fraction of the bonus when her performance is at 90-99% of the quota, starting from the second quarter. Hence, in strict terms, 'to meet the quota' would mean the sales/quota level is at or above 90%. However, the firm avoids using this definition to discourage underachievers from believing that they

annual quota. In addition, the sales agents receive an overachievement commission of approximately \$170 (2% of the combined bonuses of \$8,500) per any excess percentage points above the annual quota. The overachievement commission is capped at \$8,500, attained when the agent's performance (sales/quota realization) reaches 150%.

In setting and updating the quota for each territory, the firm uses a prominent market research firm (that possesses a majority of the pharmaceutical sales data per region, including the focal firm's competitors') to incorporate information on market-level growth and potential (rather than the sales outcomes of previous years) to mitigate possible ratcheting concerns. Some features of the firm's quota-based bonus system are noteworthy. First, the quotas are set to be cumulative from the beginning of the year. Second, the firm allows unearned bonus amounts in each quarter to be deferred to the subsequent quarter. That is, if an agent misses the quota in a given quarter, the respective bonus amount is added to the total amount attainable in the next quarter. For example, if a salesperson meets both Q1 and Q2 quotas, he or she would receive \$1,700 in both March and June. However, if a salesperson met only the Q2 quota and not the Q1 quota, he or she would receive \$3,400 only in June.

This payout structure creates unique dynamics in the forward-looking behavior of the agents. On the one hand, it motivates agents to keep up the pace from the beginning of the year. When an agent performs adequately and achieves bonus in a given period, he or she remains motivated due to the attainable quarterly bonus in the next period. Even if the agent does not meet quota in a given quarter, the motivation to exert effort becomes greater in the subsequent periods, as the total bonus amount increases due to the deferred bonus amounts from the previous quarters.

On the other hand, however, the cumulative nature of the performance evaluation raises potential concerns that poor performers lose motivation and give up. Because the sales/quota realization is computed cumulatively from the beginning of the year, the effect of several negative sales shocks can have a lasting effect throughout the year. This could demotivate the agents with poor-performance outcomes during the early part of the year, whereas they would have received a fresh start under an independent quarterly-quota system.

5.3. Model-Free Evidence: Forward-Looking Behavior

If agents' performances (in non-bonus periods) are affected by variables related to the proximity to bonuses (DTQ), it suggests forward-looking behavior (Chung et al., 2014). Specifically, the state of

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are performing adequately. In this paper, we follow the focal firm's terminology indicating that the quota is met when the agent's performance is at or above 100%.

proximity to quota would impact the performance of those who have a reasonable chance of achieving it. Hence, to show evidence of forward-looking behavior, we divide agents by their cumulative quota achieved (%QA). When %QA>0.8, agents have a reasonable probability of achieving quota by the quarterly-bonus periods and by the end of year, whereas when %QA<0.8, there is little chance. In addition, the probability likely decreases as time passes within the year. **Table 4** reports the results of a regression analysis, with each column showing monthly performance[13] as the dependent variable and the %QA up to the corresponding month as the explanatory variable, for each group of reps who are %QA>0.8 and %QA<0.8.

Consistent with forward-looking behavior, the (state) variable %QA remains significant throughout the year for agents with %QA>0.8. However, for agents with %QA<0.8, %QA is significant only during earlier periods of the year. This is the case because, despite some bad outcomes during the earlier months of the year, there still exists some probability of meeting the annual quota by achieving high performance for the remainder of the year. However, the chance of achieving the annual quota decreases as months with low performance are accumulated, and, by mid-year, low-performing agents (%QA<0.8) start to give up.

For a graphical illustration of forward-looking behavior, **Figure 3** displays the scatterplot and the best-fitting non-parametric smoothed polynomial (and its 95% confidence interval) of the agents' performance in March, June, September, and December, against %QA by the previous month. Three items stand out from **Figure 3**. First, from March through September, a considerable number of agents with states %QA<0.8 achieve monthly performance (sales/quota) greater than 100%. However, in December, very few in the lower group exhibit excess performance. Consistent with the results in **Table 4**, agents likely give up in December when they are far from quota because there is no way of achieving the annual quota with simply one month of superior performance.

Second, an agent's effort increases as he or she is on track to meet quota but flattens once quota is met. The proximity to achieving quota motivates an agent, but, once he or she surpasses quota (%QA>1), the motivation is no longer intact. Lastly, an agent's marginal effort with regard to his or her state (%QA of the previous month) increases with time in a calendar year. That is, the slope of the fitted line is steeper in December than in March. There are likely two reasons. First, the presence of the overachievement commission in December motivates agents to exert greater effort towards the end of

[13] Hereafter, the term 'performance' denotes sales normalized by the agent's corresponding monthly quota, which are used to construct the cumulative interim and annual quotas. As indicated in Section 5.2, quotas are set by a well-established outside consulting firm, taking into account territorial and seasonal fluctuations in demand.

the year. Second, the large end-of-the-year bonus (including the overachievement commission) is less discounted due to temporal proximity and, thus, motivates agents more towards the end of the year.

6. Empirical Application, Estimation, and Identification

We first formalize the firm's compensation plan and then define the evolution of the state transitions. Next, the per-period utility and performance response functions that facilitate identification are specified. Finally, we describe the details of the estimation procedure.

6.1. The Agent's Compensation

The firm's incentive scheme (ψ_{it}) includes the following components: (i) individual specific monthly base salary w_{it}; (ii) the maximum attainable quarterly-bonus amount Q_t (including the deferred amount from previous periods), common across all agents but varying across years; (iii) quarterly-bonus payout rate R_{qt}; and (iv) the end-of-year overachievement commission rate R_{yt}.

The components Q_t, R_{qt}, and R_{yt} can be expressed as follows:

$$Q_t = \begin{cases} 1{,}700 & \text{if } s_{i1t} = 3, \\ 3{,}400 & \text{if } s_{i1t} = 6, \\ 5{,}100 & \text{if } s_{i1t} = 9, \\ 8{,}500 & \text{if } s_{i1t} = 12, \\ 0 & \text{otherwise}, \end{cases}$$

$$R_{qt} = \begin{cases} 0.35 & \text{if } 0.90 \leq s_{i2,t+1} < 0.91, \text{ and } s_{i1t} = 12 \\ 0.41 & \text{if } 0.91 \leq s_{i2,t+1} < 0.92, \text{ and } s_{i1t} = 12 \\ 0.47 & \text{if } 0.92 \leq s_{i2,t+1} < 0.93, \text{ and } s_{i1t} = 12 \\ 0.53 & \text{if } 0.93 \leq s_{i2,t+1} < 0.94, \text{ and } s_{i1t} = 12 \\ 0.59 & \text{if } 0.94 \leq s_{i2,t+1} < 0.95, \text{ and } s_{i1t} = 12 \\ 0.65 & \text{if } 0.95 \leq s_{i2,t+1} < 0.96, \text{ and } s_{i1t} \in \{6,9,12\} \\ 0.72 & \text{if } 0.96 \leq s_{i2,t+1} < 0.97, \text{ and } s_{i1t} \in \{6,9,12\} \\ 0.79 & \text{if } 0.97 \leq s_{i2,t+1} < 0.98, \text{ and } s_{i1t} \in \{6,9,12\} \\ 0.86 & \text{if } 0.98 \leq s_{i2,t+1} < 0.96, \text{ and } s_{i1t} \in \{6,9,12\} \\ 0.93 & \text{if } 0.99 \leq s_{i2,t+1} < 1.00, \text{ and } s_{i1t} \in \{6,9,12\} \\ 1.00 & \text{if } 1.00 \leq s_{i2,t+1}, \text{ and } s_{i1t} \in \{3,6,9,12\} \\ 0 & \text{otherwise}, \end{cases}$$

$$R_{yt}=\begin{cases}1.00 & \text{if } 1.00\le s_{i2,t+1}<1.01, \text{ and } s_{i1t}=12,\\ 1.02 & \text{if } 1.01\le s_{i2,t+1}<1.02, \text{ and } s_{i1t}=12,\\ & \vdots \\ 1.98 & \text{if } 1.49\le s_{i2,t+1}<1.50, \text{ and } s_{i1t}=12,\\ 2.00 & \text{if } 1.50\le s_{i2,t+1}, \text{ and } s_{i1t}=12,\\ 0 & \text{otherwise,}\end{cases}$$

where the state variable s_{i1t} denotes the month-type ({1, 2, ..., 12}) and s_{i2t} denotes and percentage of the quota achieved (%QA) by the end of the previous month. The above components are collected to represent the firm's incentive scheme by the vector $\psi_{it}=\{w_{it},\ Q_t,\ R_{qt},\ R_{yt}\}$.

Given the incentive scheme ψ_{it}, an agent i receives compensation $W_{it}=W(q_{it},\ s_{it};\ \psi_{it})$ at time t, conditional on performance q_{it} and state s_{it}. Compensation W_{it} is comprised of three components: (i) monthly base salary w_{it}; (ii) quarterly (and annual) bonus QB_{it}; and (iii) end-of-year overachievement commission OC_{it}, in the following form:

$$W_{it} = w_{it} + QB_{it} + OC_{it},$$

whose elements QB_{it} and OC_{it} are expressed as follows:

$$QB_{it} = \max\left\{Q_t \cdot R_{qt}\left(\frac{(s_{i1t}-1)\cdot s_{i2t}+q_{it}}{s_{i1t}}\right)-s_{i3t},0\right\},$$
$$OC_{it} = Q_t \cdot R_{yt}\left(\frac{(s_{i1t}-1)\cdot s_{i2t}+q_{it}}{s_{i1t}}\right),$$

where s_{i3t} represents the amount of bonus accrued (%BA) in previous quarters (hence limiting the maximum quarterly-bonus amount attainable for those agents who previously received the bonus). Note that in non-bonus periods, QB_{it}=0, and, thus, W_{it} depends solely on w_{it}; note also that OC_{it} distributes only in December.

6.2. Evolution of State Variables

The state variables directly linked to compensation include: (i) the month-type within the year, s_{i1t}; (ii) the percentage of cumulative quota achieved (%QA), s_{i2t}; and (iii) the amount of annual bonus accrued (%BA), s_{i3t}.

The state variables evolve as follows:

1. Month-type

$$s_{i1t}=\begin{cases}1 & \text{if } t \text{ is the start of the year,}\\ s_{i1(t-1)}+1 & \text{otherwise.}\end{cases}$$

2. Percentage of cumulative quota achieved (%QA)

$$s_{i2t} = \begin{cases} 0 & \text{if } t \text{ is the start of the year,} \\ \dfrac{s_{i1(t-2)} \cdot s_{i2(t-1)} + q_{i(t-1)}}{s_{i1(t-1)}} & \text{otherwise.} \end{cases}$$

3. Percentage of annual bonus accrued (%BA)

$$s_{i3t} = \begin{cases} 0 & \text{if } t \text{ is the start of the year,} \\ \max\left\{ Q_{(t-1)} \cdot R_{q(t-1)} \left(\dfrac{s_{i1(t-2)} \cdot s_{i2(t-1)} + q_{i(t-1)}}{s_{i1(t-1)}} \right), s_{i3(t-1)} \right\} & \text{otherwise.} \end{cases}$$

Whereas the month-type evolves in a purely deterministic manner, the latter two state variables evolve stochastically. The percentage of cumulative quota achieved (%QA) evolves every month, conditional on the performance in the previous periods. The percentage of annual bonus accrued (%BA) evolves stepwise every quarter, conditional on receiving the quarterly-bonus. The state variables that directly affect compensation are represented by the vector s_{it}=$\{s_{i1t},\ s_{i2t},\ s_{i3t}\}$.

6.3. Parametric Specifications

Based on the identification argument in Section 4, we impose a parametric functional form on the instantaneous utility function in **Equation (3)**—namely, on the pecuniary utility M and disutility C. We specify the pecuniary utility M of wealth W_{it} to take the form of constant absolute risk-aversion (CARA) and approximate it by the mean-variance utility

$$\mathrm{E}[\, M(W_{it}) \mid e_{it}, s_{it}] = \mathrm{E}[\, W_{it} \mid e_{it}, s_{it}] - \gamma_i \mathrm{Var}[\, W_{it} \mid e_{it}, s_{it}],$$

where $\gamma_i > 0$ represents the agent's risk preference. The additional benefit of the above specification is that it provides, by construction, scale and location normalization for the pecuniary benefit, conditional on the agent staying with the firm (through the base salary). This allows us to estimate, rather than to normalize, the agent's reservation value ρ_i and the distribution of structural error ε_{dit}, which is crucial when evaluating selection policies (see Section 7).

We specify the disutility C to be convex in effort e_{it}, such that

$$C(e_{it}) = C(e_{it}; \theta_i) = \theta_i e_{it}^2\ ,$$

where $\theta_i > 0$ denotes the agent's sensitivity to effort.

Given the above parametric specification, the instantaneous utility function in **Equation (3)** can be written as

$$U(d_{it}, e_{it}, s_{it}, \varepsilon_{dit}) = \begin{cases} \mathrm{E}[W_{it} \mid e_{it}, s_{it}] - \gamma_i \mathrm{Var}[W_{it} \mid e_{it}, s_{it}] - \theta_i e_{it}^2 + \sigma_{\varepsilon,i} \varepsilon_{1it} & \text{if } d_{it} = 1, \\ \rho_i + \sigma_{\varepsilon,i} \varepsilon_{0it} & \text{otherwise.} \end{cases} \tag{12}$$

We assume that the structural errors ε_{dit} are distributed i.i.d. across choices ($d_{it} \in \{0,1\}$), over time and across agents, according to the Type-I extreme value distribution. In estimating the agent's reservation value ρ_i, we incorporate reservation value shifters z_i such that

$$\rho_i = \exp(\rho_{0i} + \rho_{1i} z_i),$$

where ρ_{0i} represents the baseline reservation value of agent i, and reservation value shifters z_i include tenure and higher education.

With regard to productivity, we model the agent's per-period performance to take a multiplicative form such that

$$q_{it} = q(e_{it}, \alpha_i, \xi_{it}) = \exp(\alpha_i + e_{it} + \xi_{it}), \tag{13}$$

or in logarithmic terms, $\ln(q_{it}) = \alpha_i + e_{it} + \xi_{it}$. The idiosyncratic performance shock, ξ_{it}, is assumed to follow $N(0,\sigma_{\xi,i})$.

To incorporate observed heterogeneity in performance, we augment individual fixed effects α_i with performance shifters x_{it} such that

$$\alpha_i = \alpha_{0i} + \alpha_{1i} x_{it},$$

where x_{it} includes the agent's tenure, training, tenure-training interaction, and higher education. To capture the long-run persistence effect of training, we characterize the training variable using the cumulative hours of sales training obtained by the agent. Under this formulation, each training hour accumulates to form the agent's stock of expertise, which carries over to the subsequent periods and affects the agent's performance in the long run.

6.4. Estimation

We utilize the full-solution method (Rust, 1987) using maximum likelihood to estimate our model rather than the conditional choice probability approach (Hotz & Miller, 1993; Bajari et al., 2007), as the two-step estimation procedure can generate biases if the state variables in the policy function are correlated with the first-stage errors. In addition, the maximum likelihood approach has the minimum variance achievable by a consistent and asymptotically normally distributed estimator.

6.4.1. Individual Likelihood

Given the value function (**Equation (4)**) and the empirical specification of the per-period utility function (**Equation (12)**), we obtain the expected value function through the inner loop in the conventional nested fixed-point algorithm (NFXP).

Let the expected value function be $\mathrm{EV}(d,e,s) \equiv \mathrm{E}_{\varepsilon,\xi}[V(s',\varepsilon_{d'} \mid d,e,s)]$. By iterating over the equation

$$\begin{aligned}\mathrm{EV}(d,e,s) &= \int_{\xi'} \log\left\{\sum_{d\in\{0,1\}} \exp\left[\max_{e\in C_e}\{u(d',e',s') + \phi(1)\mathrm{EV}(d',e',s')\}\right]\right\} d\xi' \\ &= \int_{\xi'} \log\left\{\exp\left[\max_{e\in C_e}\{u(d',e',s') + \phi(1)\mathrm{EV}(d',e',s')\}\right] + \exp\left[\tfrac{\rho}{1-\delta}\right]\right\} d\xi',\end{aligned}$$

we obtain its value over all possible states and effort levels. Then, the choice probability of stay-or-leave, $\pi_{d\in\{0,1\}}$, conditional on the agent's state, is obtained by solving the agent's dynamic optimization problem

$$\pi_{dit} = \Pr(d_{it} \mid s_{it}) = \frac{\exp\left(\max_{e\in C_e}\{u(d,e,s) + \phi(1)\mathrm{EV}(d,e,s)\} / \sigma_\varepsilon\right)}{\sum_{d\in\{0,1\}} \exp\left(\max_{e\in C_e}\{u(d,e,s) + \phi(1)\mathrm{EV}(d,e,s)\} / \sigma_\varepsilon\right)}. \tag{14}$$

In the process, the optimal level of effort e_{it}, given agent i's state in period t, is inferred by the level at which the expected value function is maximized. The attained effort enters the performance response function in **Equation (13)**.

By combining **Equation (13)** and **Equation (14)**, we can compute the likelihood of the agent's observations. Given the history (data) of an agent with observations over T-periods, we write the likelihood as

$$L_i(\Omega_i; \mathbf{q_i}, \mathbf{d_i}, \mathbf{s_i}) = \prod_{t=1}^{T} (\phi_{\xi,i}(\ln(\hat{q}_{it}) - \ln(q_{it})) \cdot \pi_{1it})^{\hat{d}_{it}} \cdot \pi_{0it}^{(1-\hat{d}_{it})},$$

where the vector $\Omega_i = \{\delta_i, \beta_i, \gamma_i, \theta_i, \rho_i, \alpha_i, \sigma_{\varepsilon,i}, \sigma_{\xi,i}\}$ is the set of parameters of the time preferences, utility and performance response functions; $\hat{d}_{it}$ denotes the observed stay-or-leave decision, and $\hat{q}_{it}$ is the observed per-period performance for agent i at time t; and $\phi_{\xi,i}$ denotes the probability density function of a normal distribution with mean 0 and variance $\sigma^2_{\xi,i}$.

6.4.2. Unobserved Heterogeneity

We accommodate unobserved heterogeneity[14] by allowing for discrete segments (Kamakura & Russell, 1989). Assume that salesperson i belongs to one of K segments $k \in \{1, \ldots, K\}$, with relative probabilities

$$m_k = \frac{\exp(\lambda_k)}{\sum_{k'} \exp(\lambda_{k'})}.$$

[14] In line with footnote 3, heterogeneity in the disutility parameter θ_i (agent's type) and individual fixed effects term α_i cannot be separately identified. Hence, we keep the performance response function parameters homogeneous.

Let $L_{ikt} = L(\Omega_k \mid k\,;\, q_{it},\, d_{it}\,,\, s_{it})$ be the likelihood of parameters for individual i at time t, conditional on unobservable segment k, given the agent's data. Then, the likelihood of the segment-level parameters upon observing an individual's history is given as

$$L_k(\Omega_k; \mathbf{q_i}, \mathbf{d_i}, \mathbf{s_i}) = m_k \left(\prod_{t=1}^{T} L_{ikt} \right).$$

By summing over all of the unobserved states $k \in \{1, \ldots, K\}$, we obtain the overall likelihood of individual i:

$$L(\Omega; \mathbf{q_i}, \mathbf{d_i}, \mathbf{s_i}) = \sum_{k=1}^{K} L_k(\Omega_k; \mathbf{q_i}, \mathbf{d_i}, \mathbf{s_i}),$$

where $\Omega = \{\Omega_1, \ldots, \Omega_k\}$ contains the segment-level parameters. Hence, the log-likelihood over the N sample of individuals becomes:

$$\sum_{i=1}^{N} \log\Big(L(\Omega; \mathbf{q_i}, \mathbf{d_i}, \mathbf{s_i}) \Big) = \sum_{i=1}^{N} \log\left(\sum_{k=1}^{K} m_k \left(\prod_{t=1}^{T} L_{ikt} \right) \right).$$

6.5. Model Identification

In addition to the formal identification arguments described in Section 4, we briefly discuss model identification in our empirical context. The main challenge is identifying unobserved effort and utility parameters with limited variations in the agent's compensation contract. There exists some variation in the compensation contract—specifically in the quarterly and annual bonus amounts across years. Thus, variations in performance across the different compensation regimes enable identification. Also, the relation between an agent's sales performance and his or her state variables help identification. The agent likely exerts more effort when close to quota versus when far from quota. Thus, systematic differences in sales performance at different DTQs identify effort and, thus, facilitate identification of the disutility of effort (Misra & Nair, 2011; Chung et al., 2014). Suppose that there are two agents with the same states, but one has higher performance than the other. Then, we can infer that the agent with higher performance has lower disutility of effort. The extent to which an agent over- or under-performs on quota identifies the risk-aversion parameter. A risk-averse agent would constantly over- or under-achieve (especially in March, June, and September, when no overachievement incentives are offered), whereas a risk-neutral agent would just meet quota. The variation in sales and turnover in the same states within an agent identifies the distribution of the performance shocks and the structural error. The variation in sales with variation in performance shifters identifies the performance response parameters. The variation in the attrition behavior of agents with similar characteristics identifies the reservation value relative to their fixed salary, and the variation in reservation shifters identifies the corresponding parameters.

As explained in Section 4.2, an agent's DTQ in non-bonus periods act as an exclusion restriction to identify discount factor(s). Suppose that there are two agents with the same characteristics who display same behavior (and, thus, performance) at the end of the year (final period of a compensation cycle). But suppose that, in non-bonus periods, one agent performs better than the other, even though both are in the same state (DTQ). We can infer that the agent with high performance in non-bonus periods has a higher discount factor (or a lower discount rate). The hyperbolic discounting model, under a functional form specification, is identified if there exist more than two periods with exclusion restrictions. The performance of an exponential discounter would be more consistent and smooth throughout the year as compared to that of a hyperbolic discounter.

7. Results

First, we report the parameter estimates of the exponential and quasi-hyperbolic discounting models and discuss their implications. Next, we show how the changes in compensation plan lead to sales force selection across heterogeneous agents. Finally, we perform a series of counterfactual simulations to address the substantive questions we seek to answer.

7.1. Parameter Estimates

The parameter estimates of the exponential and quasi-hyperbolic discounting models are reported in **Table 5**. We estimate the model under different numbers of segments and find that the three-segment model shows the best fit under the Bayesian information criterion (BIC).

In terms of time preference parameters, under the exponential discounting model, the discount factor (δ) is estimated to be 0.832, 0.977, and 0.966, respectively, for segments 1, 2, and 3. The range of the standard discount factor is consistent with the behavioral and empirical studies on time preferences (Frederick et al., 2002; Yao et al., 2012; Chung et al., 2014). In the quasi-hyperbolic discounting model, the standard discount factor (δ) is estimated to be 0.903, 0.990, and 0.967, and the present-bias factor (β) is estimated to be 0.574, 0.996, and 0.904, respectively, for segments 1, 2, and 3. **Figure 4** depicts a graphical illustration of time preferences for each segment. The graph shows each segment and time preference model (solid lines represent exponential, while dotted lines represent quasi-hyperbolic discounting) and the amount of discounting towards the future. We observe that segment 1 is both myopic and present-biased, and segments 2 and 3 are forward-looking and exhibit relatively time-consistent behavior.

The Bayesian Information Criterion (BIC) value of the two models reveals that the quasi-hyperbolic discounting model fits the data better, implying that sales agents are present-biased in their time preferences. Hence, for model inference, we turn our attention to the parameter estimates under the quasi-hyperbolic discounting model. For the structural parameters of the utility function, the disutility parameter ranges from 5.521 to 16.834. The disutility parameter is small for segment 3 (hereafter referred to as the high type), representing the agents' ease and flexibility in exerting effort. Conversely, the estimate is large for segments 1 and 2 (hereafter referred to as the low type). Hence, we observe the following pattern in terms of segmentation. Segment 3 exhibits forward-looking behavior, which is expected of high-performing agents who seek the end-of-year bonus and commission. The low-type agents, on the other hand, are divided into two groups: segment 1 tends to be myopic, whereas segment 2 exhibits forward-looking behavior.

The variance of the utility shock is high for segment 3, implying the potential of opportunities outside the firm. The reservation value is low for segment 1, reflecting limited outside opportunities for these type of agents. For reservation value shifters, education positively affects the agent's outside option. However, tenure is found to be statistically insignificant, potentially reflecting the nature of personal selling, where interpersonal and relational skills are of greater importance than such observable characteristics.[15]

With regard to the parameters of the performance response function, tenure (both the mid-level and the senior dummy variables) is found to have positive effect on performance. We find that training improves performance; however, the interaction effect of mid-level and senior dummies with training is found to be negative and significant. Hence, training may not benefit the senior salespeople as much as the firm intended.

7.2. Selection

We examine how the change in the focal firm's compensation plan has led to selection of its sales force. **Table 6** shows the share of the three segments and their descriptive characteristics, across the full three years of data observation. Segment 3, the high type, is the smallest, with a share of 17.05%; segments 1 and 2 represent greater shares, 36.62% and 46.33%, respectively. Consistent with the parameter estimates in **Table 5**, segment 3 achieves the highest average performance outcomes, with the largest portion meeting the annual quota. Average base salary is higher for this segment, reflecting their tenure. Segment 2, the forward-looking low type, falls short on performance compared to segment 3 and is comprised by those with slightly lower tenure. The myopic low type, segment 1, falls short in every performance dimension and exhibits a stark difference in annual variable pay compared to the other segments.

In terms of selection induced over time, **Table 7** shows the segment portfolio (in percentages) of the total sales force by each year-end—reflecting how the portfolio of salespeople has changed over time. The share of segment 2, the forward-looking low type, has constantly increased, which could cause concern about the firm's productivity in the long run. As low-type agents are less likely to be actively poached by the focal firm's competitors, they tend to remain with the firm despite subpar performance. Also notable is the relatively smaller share of Segment 1 over the years, compared to the full observation

[15] In addition, the tenure information contained in our data is within the focal firm, which may lead to underestimating its effect on the outside option (compared to industry tenure).

period in **Table 6**. This reflects the frequent joining-and-quitting within the segment, likely due to, among other items, insufficient compensation from lack of productivity.

7.3. Counterfactual Simulations

With the structural parameters obtained in Section 7.1, we perform a series of counterfactual simulations to address the key substantive question: how to manage, motivate, and sustain a healthy sales force using the sales management instruments outlined in **Figure 1**. Agents' performance is evaluated according to changes in: (i) compensation structure; (ii) recruiting and termination policy; and (iii) training hours. For each counterfactual scenario, we evaluate how the change affects performance and selection across different segments within the firm's sales force.

In running the counterfactual studies, we suppose that the firm is undertaking its annual compensation plan design at the beginning of 2018 (i.e., following the data observation period). We focus on the remaining portfolio of salespeople (N=400) and set the basis of changes from the 2017-plan components. For each new regime, we simulate 200 paths per each individual-segment pair, using the parameter estimates of the quasi-hyperbolic discounting model. Then, we allocate individuals into each segment, based on the segment probability obtained during the estimation. Finally, we aggregate performance and selection.

7.3.1. Alternative Compensation Structures

The challenge in designing a compensation plan is to determine the optimal ratio of fixed and variable pay. The theory predicts that when a firm increases the portion of fixed pay, employee turnover will likely decrease. But how would heterogeneous segments react differently to the change in terms of both selection and sales performance? Hence, in the first counterfactual exercise, we change the amount of fixed versus variable pay, while keeping other components constant, and compare the changes in productivity and turnover. The increase in compensation for fixed versus variable pay is set at 1:3, to reflect the data showing that approximately a third of salespeople meet the annual quota. **Table 8** depicts the performance and turnover outcomes of the counterfactual simulations under the new regimes.

First, we increase the base salary by 5% and keep everything else constant. As anticipated, the turnover rate decreases across all segments. However, a notable aspect is that sales productivity also decreases. This is driven mainly by the retention effect: being granted higher rent, the low- performing agents, who otherwise would have left the firm, are now more likely to stay with the firm. The retention effect is greater for the low types, reflected by the more pronounced decrease in productivity. Next, we increase the bonus amount by 15% and keep everything else constant. Again, we observe a decrease in employee turnover; however, compared to the case in which we increase the base salary, the reduction is smaller. Moreover, the impact on productivity is found to be positive across the board, with the high-type agent's being more pronounced, as an increase in the bonus amount helps motivate the agents to a greater extent throughout the year.

The experiment demonstrates the trade-off between adjusting fixed versus variable pay on performance and selection. While increasing the fixed salary could serve as a simple remedy to reduce employee turnover, this could, on average, hurt the overall performance of the sales force. On the contrary, an increase in variable pay does not harm performance but has a smaller effect on employee turnover. As can be seen, the effect of policy changes applies heterogeneously across segments, which affects the resulting portfolio of the remaining salespeople.

7.3.2. Recruiting and Termination Policy

A firm can induce selection of the sales force through its recruiting and termination policy. We consider two cases: (i) changes in the firm's recruiting policy; and (ii) changes in its termination policy. First, the recruiting policy relates to the type of agents that the firm should target during its hiring process. Should the firm focus on targeting the high types, who are more likely to be skilled but require greater compensation to keep? Or should the firm target the low types at lower costs? To evaluate the outcomes of the recruiting policy, we simulate the firm to have hired 150 new agents—50 of each of low, mid, and high types—and compare their differences in performance and turnover over a five-year horizon. **Figure 5a** depicts the annual performance (solid lines) and cumulative turnover rate (columns) by segment.[16] As anticipated, the high types show better performance than the low types. However, the high types' high performance comes at a cost: these agents are more apt to depart the firm due to better outside opportunities, leaving their territories vacant. Hence, in **Figure 5b**, we report the turnover-adjusted performance, which accounts for the territory vacancy (treated as zero outcome). We observe that, although in the short run, hiring high-type agents leads to greater performance, in the long run, territory vacancy can be detrimental to the firm's objectives. Thus, without any changes in effort to retain high-type salespeople, simply recruiting a large number of them can have limited positive effects on the firm's productivity.

On the flip side of recruiting is the firm's policy for terminating its sales agents. In most nations, including Turkey, firm-initiated employee termination (layoff) is limited due to labor force regulations.

[16] The performance figures tend to be lower than the segment characteristics in **Table 6**, as the respective new hires have zero tenure and training records.

Hence, to terminate the agents by discretion, the firm must provide a leave package that those agents will agree upon. We evaluate the effect of a leave package by providing a lump sum of $1,500 (equivalent to a month's base salary) that agents can opt into. The results are depicted in **Table 9**, which shows that, in terms of turnover rate, it is mainly the low-type myopic agent (segment 1) that is affected by the leave package. This is expected, as the marginal value of the leave package is higher for that segment. However, the average firm productivity increases, as agents with less potential are the ones to accept the package and leave first. Hence, the termination policy experiment reveals that strategically providing leave packages can lead to better outcomes and to the firm's desired selection of salespeople.

7.3.3. Sales Training

The final counterfactual simulation involves changes in sales training. As discussed in Section 7.1, sales training positively affects productivity, but mostly for junior salespeople. The increase in performance affects not only current period utility but also future outcomes, which brings about changes in the dynamic optimization of effort over time. To evaluate the role of sales training in performance and selection, we provide 24 hours of sales training (cost-wise equivalent to an annual 5% salary increase) in January for all agents. **Table 10** shows the results.

As anticipated, sales training leads to increased performance across all segments. We also observe a decrease in turnover rate across all segments. Overall, the general trend of providing sales training is similar to that of increasing the bonus amount—training helps agents obtain better performance, which, in turn, raises the probability of attaining the bonus. Hence, our model provides a practical tool for firms in the cost-benefit evaluation of compensation and training.

8. Conclusion

Managing a sales force is an intricate task with multidimensional outcomes. If properly managed, firms can induce greater performance from their sales force while keeping their top performers from leaving the firm. This paper develops and estimates a dynamic structural model of a comprehensive response to multiple sales management instruments, including compensation, training and recruiting/termination policies. The model takes into account many key elements that constitute a realistic sales force setting: allocation of continuous effort; forward-looking behavior, including present-bias; effectiveness of sales training; and employee attrition. Substantively, we seek to provide guidance to firms on (i) evaluating the differential outcomes of compensation policy changes; (ii) assessing the selection of different types of employees with regard to changes in the recruiting and termination policies; and (iii) addressing the value of sales force training.

Overall, we find that an increase in fixed salary positively affects employee retention but may decrease aggregate sales because low-type agents, who otherwise would have left the firm, are likely to stay. In contrast, an increase in variable pay enhances sales productivity but has limited effect on employee retention. Because of the focal firm's selection process over time, high performers have steadily left the firm, while mid-to-low performers have remained. We find, however, that if the firm were to focus mainly on recruiting high-performing salespeople, sales would increase in the short-term but would likely decrease in the long-term due to the territory vacancies created by salespeople's attrition. Hence, firms should focus on retention efforts in addition to their recruiting efforts of high performers. Furthermore, we find that providing adequate leave packages can lead to an appropriate selection of salespeople to maintain a healthy sales force. Finally, we find that sales training, a novel instrument that has been overlooked in both the academic literature and practice, is an effective long-term performance driver that aids salespeople in their early careers to improve their performance, which, in turn, helps the firm increase sales performance and employee retention.

Methodologically, we introduce a new insight to the marketing and economics literature. We provide formal proof regarding the identification of discount factors in a model accompanying continuous and unobserved choices. The key to identification is the aggregation of performance outcomes over a specific time horizon when evaluating compensation: an agent's distance-to-quota for obtaining a bonus payment (in non-bonus periods) serves as an exclusion restriction that affects only future utility and not current utility. We provide conditions under which both an exponential and a hyperbolic discounting model are identified, and through our empirical application, find strong evidence of present-bias in salespeople's behavior.

This study has certain limitations that open up avenues for future research. First, we do not consider multidimensional effort with regard to different products (Chung et al., 2018) or customer types (Kim et al., 2017), whereby agents could exhibit dynamic substitution across products and customers or both. For example, in the early periods of a quota-evaluation cycle, an agent might focus on high-ticketed products that, if sold, could satisfy a large portion of his or her quota. However, as periods pass, an agent might gradually shift to low-ticketed and easy-to-sell products. Second, free goods as a sales promotion tool, which is common in the pharmaceutical industry, can induce additional dynamics in a sales agent's behavior. While free goods reduce agents' short-term returns on performance, they can induce greater long-term outcomes by building a stronger relationship with a customer. Although not addressed in this research due to data limitations and model parsimony, the abovementioned topics would provide exciting avenues for future research.

In summary, this research offers a comprehensive, practical, yet rigorous application to understand the roles of multiple sales management instruments—compensation, training, recruiting and termination—in the selection and performance of salespeople. We believe that our results will guide firms in their sales management practices to help recruit, compensate, train, and, thus, maintain a healthy sales force to achieve their desired outcomes.

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Appendix

Sufficient Conditions for Full-Rank Condition.

We provide below the sufficient conditions for full-rank condition. Observe that the derivatives of Π with respect to the parameters are given by

$$\frac{\partial \Pi(x \mid \omega)}{\partial \theta} = -e^2 - \delta(\beta-1)\int \left\{ \frac{(e')^2 \cdot \exp\left[\frac{(\beta-1)}{\beta} u(1,e',s' \mid \mu) + \frac{1}{\beta} v(1,e',s')\right]}{\exp[\Lambda(s' \mid \mu,\beta)]} f(s' \mid d,e,s) \right\} ds'$$

$$\frac{\partial \Pi(x \mid \omega)}{\partial \gamma_0} = 1 + \delta(\beta-1)\int \left\{ \frac{\exp\left[\frac{(\beta-1)}{\beta} u(1,e',s' \mid \mu) + \frac{1}{\beta} v(1,e',s')\right]}{\exp[\Lambda(s' \mid \mu,\beta)]} f(s' \mid d,e,s) \right\} ds'$$

$$\frac{\partial \Pi(x \mid \omega)}{\partial \gamma_1} = \mathrm{E}(W \mid d,e,s) + \delta(\beta-1)\int \left\{ \frac{\mathrm{E}(W' \mid d',e',s') \cdot \exp\left[\frac{(\beta-1)}{\beta} u(1,e',s' \mid \mu) + \frac{1}{\beta} v(1,e',s')\right]}{\exp[\Lambda(s' \mid \mu,\beta)]} f(s' \mid d,e,s) \right\} ds'$$

$$\begin{aligned}\frac{\partial \Pi(x \mid \omega)}{\partial \gamma_2} &= -\mathrm{Var}(W \mid d,e,s) \\ &\quad - \delta(\beta-1)\int \left\{ \frac{\mathrm{Var}(W' \mid d',e',s') \cdot \exp\left[\frac{(\beta-1)}{\beta} u(1,e',s' \mid \mu) + \frac{1}{\beta} v(1,e',s')\right]}{\exp[\Lambda(s' \mid \mu,\beta)]} f(s' \mid d,e,s) \right\} ds'\end{aligned}$$

$$\frac{\partial \Pi(x \mid \omega)}{\partial \delta} = \int \left\{ \Lambda(s' \mid \mu,\beta) f(s' \mid d,e,s) \right\} ds'$$

$$\frac{\partial \Pi(x \mid \omega)}{\partial \beta} = \int \left\{ \frac{u(1,e',s' \mid \mu) \cdot \exp\left[\frac{(\beta-1)}{\beta} u(1,e',s' \mid \mu) + \frac{1}{\beta} v(1,e',s')\right]}{\exp[\Lambda(s' \mid \mu,\beta)]} f(s' \mid d,e,s) \right\} ds'.$$

We assume that the set $X = \{x_j : j = 1,2,\ldots,J\}$ satisfying Assumption 5 is given. The following provides sufficient conditions for the rank condition:

(i) For any x and x' in X, there is a first-order stochastic dominance relationship between $f(s' \mid x)$ and $f(s' \mid \tilde{x})$.

(ii) E(W| d, e, s) and Var(W| d, e, s) are weakly monotone in s, but are not constant functions on X.

(iii) e^2, E(W| d, e, s), and Var(W| d, e, s) are linearly independent on X.

(iv) $\Lambda(s \mid \mu,\beta)$ is strictly increasing in s on X.

If $f(s' \mid x)$ first-order stochastically dominates $f(s' \mid \tilde{x})$, we have that $f(a(s')f(s' \mid x)ds' \geq f(a(s')f(s' \mid \tilde{x})ds'$ for any weakly increasing function a. Thus, conditions (i) and (ii) jointly imply that the expected future payoffs vary across x. The derivatives with respect to θ, γ, β are linearly independent by condition (iii). By condition (iv), the derivative with respect to δ is linearly independent of the other derivatives.

Table 1: Sales Force Turnover

	Number of Salespeople		
	2015	2016	2017
Beginning	303	330	367
Joined	58	102	91
Departed	31	65	58
Year-end	330	367	400
Turnover Rate	9.79%	18.65%	15.12%

Table 2: Descriptive Statistics

		All	Decision	
			Stay	Quit
Number of Salespeople		554	400	154
Monthly Base Salary (USD)	Mean	1,513.58	1,549.13	1,421.27
	SD	268.01	271.49	235.65
Annual Variable Pay (USD)	Mean	5,611.07	6,242.38	3,294.32
	SD	3,796.17	3,474.77	4,035.43
Tenure (Years)	Mean	4.08	4.63	2.66
	SD	4.56	4.75	3.67
Sales Training (Hours Per Year)	Mean	3.46	3.79	2.61
	SD	0.66	0.63	0.74
Higher Education (%)	Mean	93.68	93.50	94.16
Annual Performance (%)	Mean	95.12	97.50	86.40
	SD	15.35	13.01	19.64
Meet 100% Quota (%)				
Q1	Mean	29.13	30.73	24.82
Q2	Mean	28.74	30.15	24.94
Q3	Mean	26.93	29.50	18.78
Annual	Mean	28.36	29.71	23.39

Notes. Base salary and variable pay are converted to U.S. dollars. The numbers are approximate for confidentiality.

Table 3: Variable Compensation Payout Ratio (2017)

	Q1	Q2	Q3	Q4
Performance (Sales/Quota)				
0 – 89.99 %	-	-	-	-
90 – 90.99 %	-	-	-	35 %
91 – 91.99 %	-	-	-	41 %
92 – 92.99 %	-	-	-	47 %
93 – 93.99 %	-	-	-	53 %
94 – 94.99 %	-	-	-	59 %
95 – 95.99 %	-	65 %	65 %	65 %
96 – 96.99 %	-	72 %	72 %	72 %
97 – 97.99 %	-	79 %	79 %	79 %
98 – 98.99 %	-	86 %	86 %	86 %
99 – 99.99 %	-	93 %	93 %	93 %
100 %	100 %	100 %	100 %	100 %
Greater than 100 %	100 %	100 %	100 %	101-200 %
Quota Evaluation Period	Jan-Mar	Jan-Jun	Jan-Sep	Jan-Dec
Bonus Amount	$1,700	$1,700	$1,700	$3,400

Notes. The quarterly variable compensation is determined by multiplying the allocated bonus amount (bottom row) by the payout rate, respective to the performance (sales/quota) over the evaluation period.

Table 4: The Relation between Sales Performance and Distance-to-Quota

State	Variable	Monthly Performance											
		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
%QA <0.8	Intercept	-	**0.71**	**0.83**	**0.63**	**0.49**	**0.47**	**0.84**	**0.67**	**0.49**	**0.50**	**0.43**	**0.38**
	%QA	-	**0.14**	0.13	**0.25**	**0.29**	**0.40**	-0.16	0.15	0.14	0.22	0.29	0.31
%QA >0.8	Intercept	-	**0.70**	**0.74**	**0.55**	**0.39**	**0.49**	**0.26**	0.12	**0.60**	**0.21**	**-0.27**	-0.04
	%QA	-	**0.31**	**0.30**	**0.37**	**0.56**	**0.50**	**0.70**	**0.98**	**0.28**	**0.71**	**1.24**	**1.10**

Notes. In each column, monthly performance is regressed on the cumulative annual quota (%QA) attained by the beginning of the corresponding month (respectively for each group of reps who are %QA>0.8 and %QA<0.8). Significance (at the 0.05 level) appears in boldface. Standard errors are omitted for brevity.

Table 5: Parameter Estimates

	Exponential Discounting			Quasi-Hyperbolic Discounting		
	Seg 1	Seg 2	Seg 3	Seg 1	Seg 2	Seg 3
Time Preferences						
Standard Discount Factor	**0.8317**	**0.9770**	**0.9658**	**0.9028**	**0.9902**	**0.9669**
Present-Bias Factor				**0.5744**	**0.9960**	**0.9041**
Utility Function						
Disutility of Effort	**17.2713**	**15.1860**	**5.7196**	**16.6693**	**16.8336**	**5.5208**
Risk-Aversion	**0.0693**	**0.0345**	**0.0126**	**0.0810**	**0.0325**	**0.0241**
S.D. of Utility Shock	**4.0384**	2.3048	**12.9100**	**6.0345**	**1.2814**	**24.2619**
Reservation Value						
Baseline	0.1680	**0.9878**	**0.8811**	**−1.0244**	**0.8734**	−0.4531
Mid-Level (Tenure 3-7 Years)		−0.1418			−1.8880	
Senior (Tenure > 7 Years)		−0.2257			−11.8024	
Higher Education		**0.1055**			**0.2847**	
Performance Response Function						
Baseline Productivity		**−0.2959**			**−0.2803**	
Mid-Level (Tenure 3-7 Years)		**0.0085**			**0.0113**	
Senior (Tenure > 7 Years)		**0.0171**			**0.0182**	
Training		**0.0029**			**0.0029**	
Mid-Level × Training		**−0.0007**			**−0.0010**	
Senior × Training		**−0.0014**			**−0.0016**	
Higher Education		**0.0024**			0.0002	
S.D. of Performance Shock		**0.3412**			**0.3395**	
Log-likelihood		−5535.226			−5518.982	
BIC		11336.802			11332.852	

Notes. Significance (at the 0.05 level) appears in boldface. Standard errors are omitted for brevity.

Table 6: Segment Characteristics

	Segment 1	Segment 2	Segment 3
Segment Size (%)	36.62	46.33	17.05
Monthly Base Salary	1,455.16	1,502.59	1,668.93
Annual Variable Pay	2,621.19	5,839.50	10,635.20
Tenure (Years)	3.43	4.21	5.13
Sales Training (Hours Per Year)	3.09	3.43	4.37
Higher Education	0.95	0.93	0.94
Annual Performance (%)	83.97	95.59	114.95
Meet 100% Quota (%)			
Q1	21.14	28.88	45.96
Q2	18.92	28.99	48.26
Q3	13.35	29.48	47.15
Annual	14.14	29.92	50.87

Notes. Base salary and variable pay are converted to U.S. dollars. The numbers are approximate for confidentiality.

Table 7: Selection of Sales Force over Time

	N	Sales Force Portfolio		
		Segment 1	Segment 2	Segment 3
Beginning of 2015	303	33.94%	44.74%	21.32%
End of 2015	330	30.40%	48.14%	21.47%
End of 2016	367	26.97%	55.20%	17.82%
End of 2017	400	20.35%	60.83%	18.82%

Table 8: Impact of Alternative Compensation Structures

Counterfactual Simulation (Percentage Point Change)	Total	Within Segment		
		Segment 1	Segment 2	Segment 3
1. Increase Salary (+5%)				
Average Performance	−0.055	−0.137	−0.003	−0.020
Turnover Rate	−0.913	−1.991	−0.003	−1.074
2. Increase Bonus (+15%)				
Average Performance	2.934	1.383	2.834	6.640
Turnover Rate	−0.785	−1.064	−0.003	−2.310

Table 9: Impact of Termination Policy

Counterfactual Simulation (Percentage Point Change)	Total	Within Segment		
		Segment 1	Segment 2	Segment 3
Leave Package				
Average Performance	0.231	0.598	0.000	0.060
Turnover Rate	3.040	7.780	0.019	1.071

Table 10: Impact of Sales Training

Counterfactual Simulation (Percentage Point Change)	Total	Within Segment		
		Segment 1	Segment 2	Segment 3
Increase Sales Training (+24Hrs)				
Average Performance	6.364	7.162	5.948	5.780
Turnover Rate	−1.741	−3.828	−0.003	−1.979

Figure 1: Sales Management Instruments

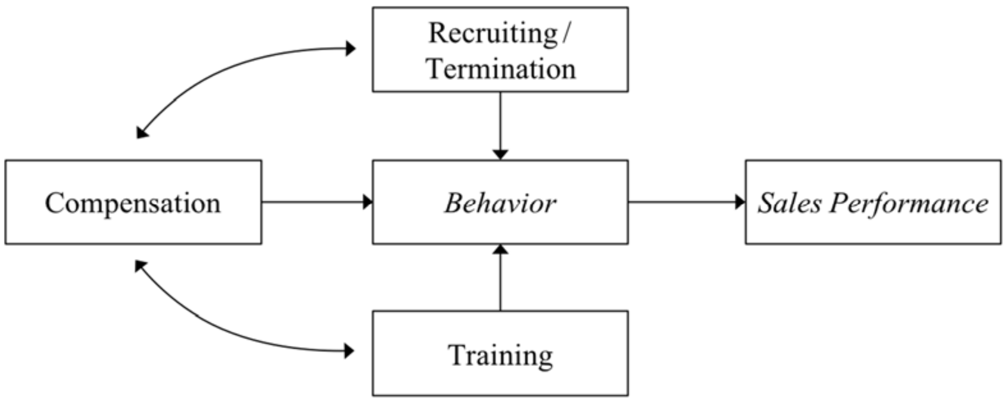
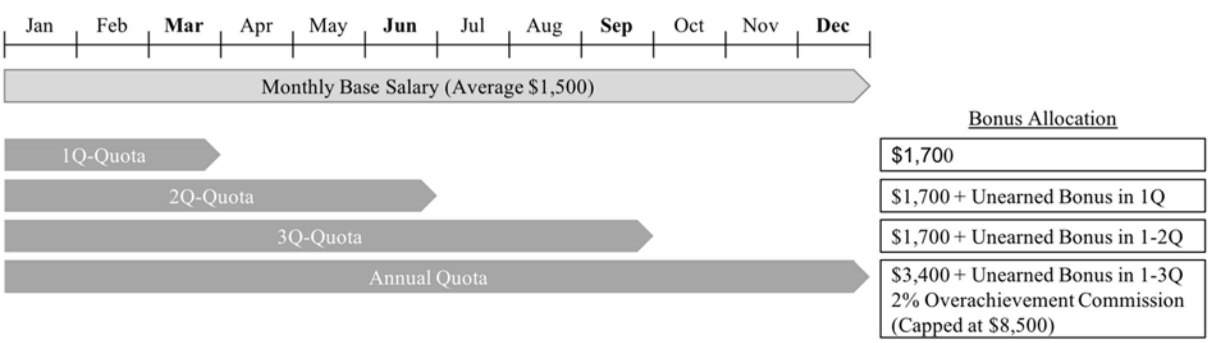


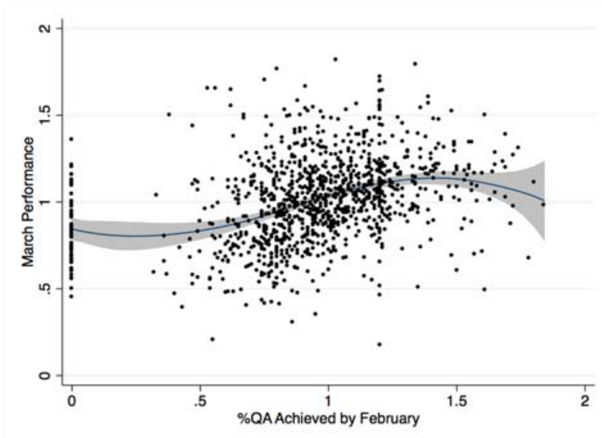
Figure 2: Firm's Compensation Plan



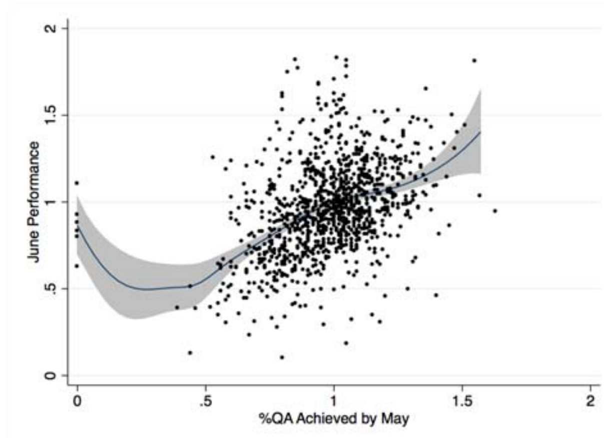
Notes. Base salary and variable pay are converted to U.S. dollars. The numbers are approximate for confidentiality.

Figure 3: Performance and Quota Achieved by Month (%QA)

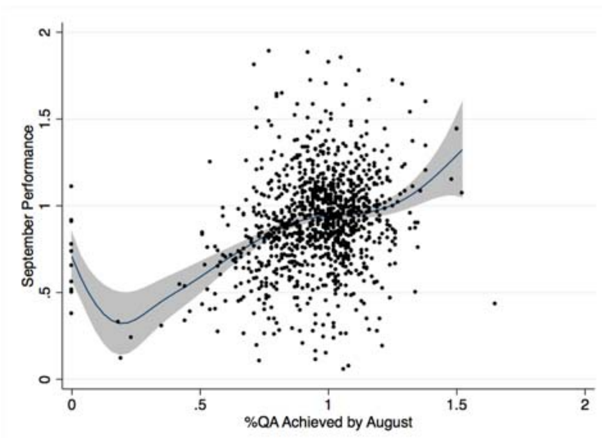
(a) March



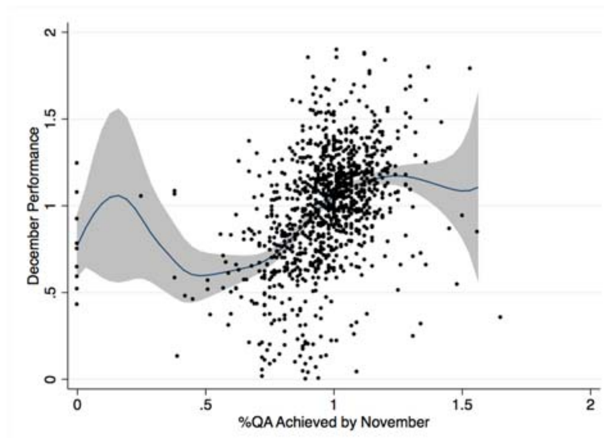
(b) June



(c) September

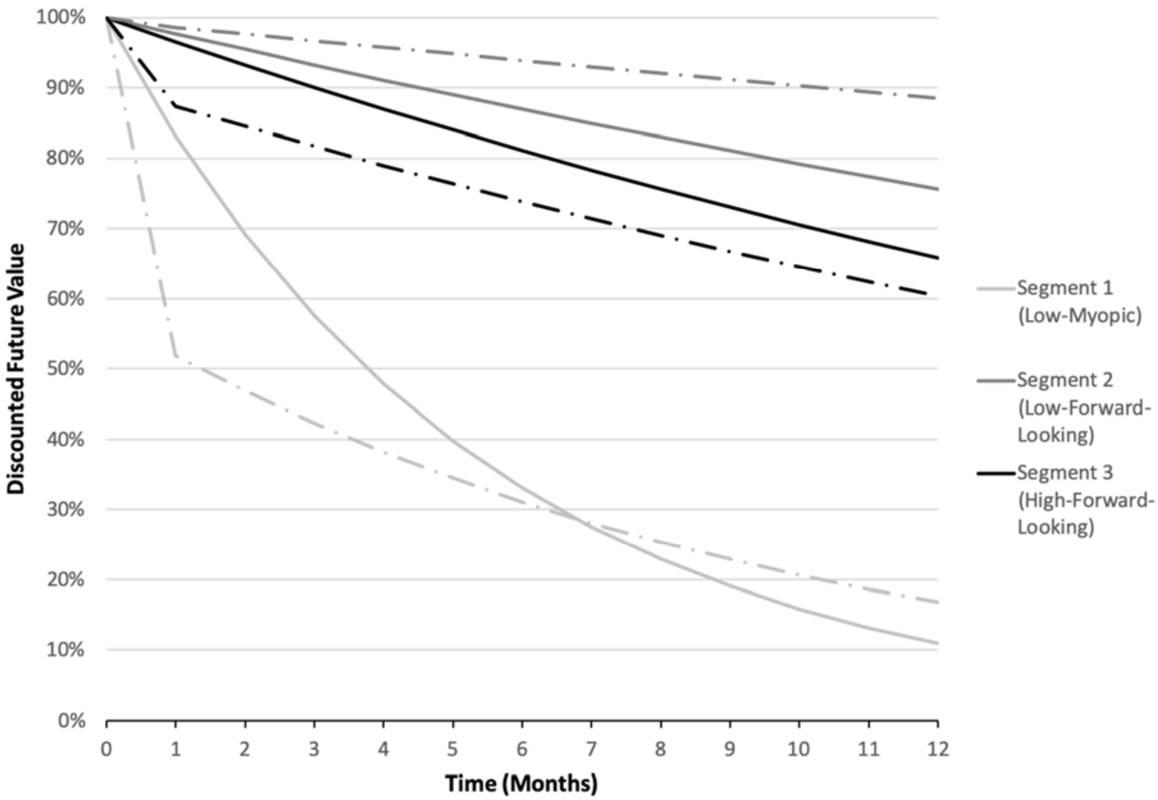


(d) December



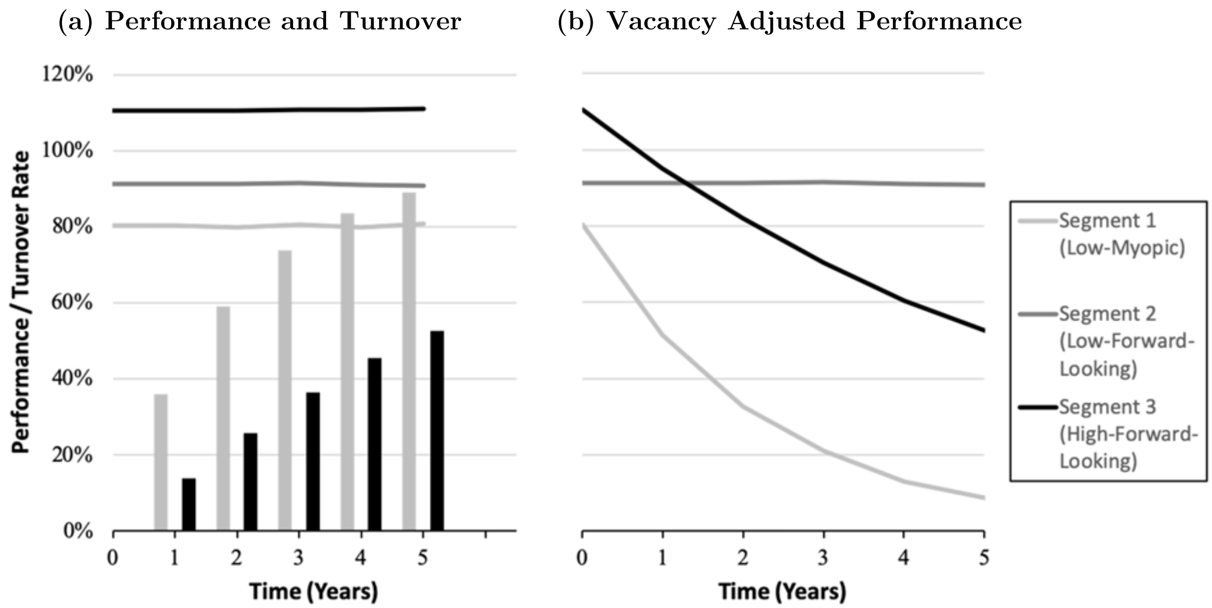
The y-axis depicts performance (i.e., sales/quota) of the quarterly and yearly bonus months, and the x-axis shows the corresponding agent's cumulative annual quota achieved (%QA) until the previous month.

Figure 4: Time Discounting by Segment



Notes. The solid lines represent exponential discounting and the dotted lines represent hyperbolic discounting, respective to each segment. The y-axis depicts the rate of discounted future value (as compared to the present value) and the x-axis depicts time horizon (j-period forward).

Figure 5: Differential Impact of Recruiting



Notes. The y-axis shows annual performance (lines) and cumulative turnover rate (columns) by segment. The x-axis depicts time horizon (i.e., end of year).