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Working Paper 19-063

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December 20, 2018

Abstract

The scalability of a marketplace depends on the operations of the marketplace platform as well as its sellers' cost structures and capacities. When fixed costs of entry are high, sellers with small capacities may be deterred from entering the market because of their inability to leverage economies of scale. In this study, we explore one strategy that a marketplace platform can use to enhance its scalability: providing an ancillary service to sellers to reduce their fixed costs. In our model, a platform can choose whether and when to provide this service to sellers. When the platform provides the service, it encourages the entry of small sellers. However, it diminishes large sellers' incentives to make their own investment, thus reducing their potential output. When the output reduction by the large sellers is substantial, the platform may not want to provide the ancillary service even if it could do so at no cost. To encourage entry while mitigating output reduction, the platform may choose to strategically delay providing the service.

Keywords: marketplace, scalability, platform strategy, platform investment

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1 Introduction

Marketplace platforms such as Airbnb, Craigslist, eBay, Uber, and Upwork have become increasingly influential in our economy (e.g., Rochet and Tirole 2003; Iansiti and Levien 2004; Parker and Van Alstyne 2005; McIntyre and Srinivasan 2017). These platforms attract and facilitate transactions between buyers and sellers. One well-known feature of such marketplaces is their scalability. Previous research has focused on two sources of scalability. The first is rooted in indirect network effects, whereby a large number of buyers attracts more sellers and vice versa. The second is technological and hinges on the classic leveraging of fixed costs; once the requisite technological infrastructure is deployed (e.g., software and servers), a marketplace platform can serve additional transactions at trivial marginal costs.

The scalability of a marketplace, however, does not just depend on the operations of the platform. The cost structures and capacities of sellers can also constrain its scalability. Consider, for instance, a large e-commerce platform like eBay. The platform's operation is highly scalable; the marginal cost of serving a new buyer or seller is almost zero. Because of indirect network effects, eBay also enjoys increasing returns as buyers and sellers attract each other.

However, eBay sellers themselves are not necessarily as scalable. Small sellers (e.g. individuals selling products out of their homes) may have limited capacity to increase their supply. Furthermore, sellers incur fixed costs, such as renting storage space and purchasing software to manage logistics and after-sales services. When fixed costs are high, sellers may be deterred from entering the market. This effect is particularly strong for small sellers because of their inability to leverage economies of scale by spreading fixed costs over many products. Marketplace scalability will be constrained when a large number of sellers are deterred by high fixed costs to meet growing volume and variety demand, even in circumstances where the platform itself has a highly scalable operation. As most platforms' business models depend critically on the overall volume of transactions flowing through them, it is in their interest to improve their scalability.

In this paper, we explore one strategy that a marketplace platform might use to enhance its scalability: providing an ancillary service to sellers that helps reduce their fixed costs of running the business. For example, Amazon, Walmart and Alibaba offer logistics and distribution services to third-party sellers, allowing smaller third-party sellers to use those platforms without incurring the fixed costs required to invest in warehouses and distribution capabilities. Google provides Android developers with a software development kit for mobile ads. As a result, Android developers can generate ad revenue from their apps without having to look for advertisers themselves. Didi, the dominant ride-sharing platform in China, leases vehicles to individuals who wish to drive for it. In contrast, some marketplace platforms, such as eBay, do not offer such services. For example, after testing out a low-cost cleaning service for its hosts in San Francisco, Los Angeles, and New York City, Airbnb decided not to offer such services.

Why do some marketplace platforms choose to offer such services, but not others, and how do such decisions impact scalability? We build a game-theoretical model to explore these questions. In our model, a continuum of sellers is interested in selling their products on a marketplace platform.

These sellers differ in terms of the maximum number of products they can carry. The platform can choose to invest in the provision of an ancillary service that is essential for transactions between buyers and sellers. The ancillary service eliminates fixed costs for sellers. Depending on the platform's choice, sellers choose whether or not to enter, and, if it enters, whether to invest in the ancillary service on its own or use the platform's service if it is provided.

Our results show that when a platform provides the ancillary service, it induces more entries because small sellers are no longer deterred by fixed costs. Offering the service, however, may diminish large sellers' incentives to invest in the service themselves, thus reducing their potential output. A marketplace platform, therefore, faces a trade-off between an extensive margin of entry and an intensive margin of output levels. In a one-period model, when the output reduction effect by large sellers is substantial, the platform may not want to provide the ancillary service even if it could do so at no cost. In a two-period model, it can be optimal for the platform to delay offering the service in the second period only. Such strategic delay induces large sellers to invest in the ancillary service on their own in the first period, thereby encouraging them to fully exploit economies of scale in both periods.

Overall, our research shows that platform investment in ancillary services can change the composition of sellers that participate on the platform and their output levels by altering sellers' cost structures. These changes affect the scalability of the marketplace. Our research also highlights that the timing of such platform investment is an important consideration for maximizing marketplace scalability.

Our work adds to the literature on platform strategies. Early work in this area has focused primarily on two-sided pricing strategies (e.g., Caillaud and Jullien 2003; Rochet and Tirole 2003; Parker and Van Alstyne 2005; Armstrong 2006; Jin and Rysman 2015). More recent studies have examined a variety of non-price strategic levers platforms can use to grow their businesses such as the strategic revelation of information (e.g., Tucker and Zhang 2010; Chellappa and Mukherjee 2017; Niculescu et al. 2018), the use of different business models (e.g., Economides and Katsamakas 2006; Chen et al. 2016), product versioning (e.g., Bhargava et al. 2013), contractual relationships with third parties (e.g., Lee 2013; Hao et al. 2017), direct entry into third-parties' spaces (e.g., Gawer and Henderson 2007; Jiang et al. 2011; Huang et al. 2013), and diversification into adjacent markets (e.g., Eisenmann et al. 2011). A subset of this literature has examined platform investment decisions. Anderson et al. (2014) point out that in industries such as the video game industry, a platform may not want to invest in platform performance because a high-performance platform discourages developers from participating. Tan et al. (2018) explore how platform investment in integration tools interacts with a platform's pricing decisions. Basu et al. (forthcoming) and Chellappa and Mukherjee (2018) examine a platform's decision to offer authentication services. Huang et al. (2018) show that a platform sponsor's investment in knowledge seeding increases its users' knowledge contribution. Cui et al. (2018) show the value of investing in logistic services for e-commerce platforms. A few studies on net neutrality have examined internet service providers' incentives to expand network capacity and how such expansion affects content providers and consumers (e.g., Choi and Kim 2010; Cheng et al. 2011; Krämer and Wiewiorra 2012). Our paper differs from these studies in that in our setting both a platform and sellers can invest in the same service.

Our paper's focus on whether a platform should invest in ancillary services to encourage entry is also related to a few recent studies on entry inducement and deterrence. Chen and Wu (2012) show that conversion of fixed cost to variable cost enables small firms to enter existing markets. In our setting, a platform's provision of ancillary services also converts fixed cost to variable cost. Hagiu and Wright (2018) examine when a platform should steer its buyers to try products from new sellers, even though these products involve more risks. Nagaraj and Piezunka (2018) show that the entry of Google Maps into different countries deters potential new members from contributing to OpenStreetMap. Luo et al. (2018) find that a platform's use of ratings to rank sellers may increase new sellers' entry costs and thus discourage their entry. Our paper complements these studies by identifying the strategic trade-off when a platform makes an investment to empower existing and potential sellers.

The remainder of the paper is organized as follows. Section 2 presents a numerical example to illustrate our main insight: even if a platform's cost of offering an ancillary service is zero, the platform may prefer to not offer it. Section 3 presents a one-period model to illustrate this result in general. Next we consider a two-period model, presented with a numerical example in Section 4 and then a general model in Section 5, to show that the timing of offering such a service is also a strategic consideration for the platform. After discussing a few extensions in Section 6, we conclude by discussing managerial implications and future research opportunities in Section 7. We provide all technical proofs in an appendix.

2 A Numerical Example for the One-Period Game

We first provide a numerical example to build intuition for our main results and highlight the key effects. Consider three differentiated sellers: A, B, and C. A is a small seller with a capacity of 1 unit. B is a medium-sized seller with a capacity of 2 units. C is a large seller with a capacity of 4 units.

The marginal revenue for each seller is given in column 2 of Table 1. The pattern captures diminishing returns, where the first unit generates the most revenue. Without loss of generality, we assume the marginal cost of producing each good to be zero.

To sell the good to consumers, either the platform or the sellers need to provide a service (e.g., delivery). Columns 3 and 4 describe a seller's marginal cost of a seller using its own service or the platform's service, respectively. By using its own service, a seller incurs a fixed cost of \$12 and a marginal cost of \$5 per unit. By using the platform's service, a seller incurs no fixed cost and a marginal cost of \$8 per unit. Notice that the marginal cost of a seller using its own service is lower than using the platform's service, reflecting the idea that a seller can optimize its service for its own need and the platform's service converts some fixed cost to variable cost (Chen and Wu 2012).[1]

[1]For example, sellers that use Amazon's fulfillment service need to ship products to Amazon's fulfillment centers

Table 1: Marginal revenue and marginal cost

Unit	MR ($)	MC (Own) ($)	MC (Platform) ($)
1	12	17	8
2	11	5	8
3	7	5	8
4	6	5	8

For simplicity, we assume that the platform prices its service equal to its marginal cost. In other words, the platform breaks even in providing the service.[2] We also assume that the platform makes a fixed commission on the sales of the three sellers. Hence, the platform's objective is to maximize the total revenue of three sellers.

Consider the following sequence of actions:

- First, the platform decides whether to provide the service itself (d_1) or not (d_0).
- Second, each seller decides whether to enter the market. If so, each then decides whether to invest in such a service itself or not.
- Third, each seller decides on the number of units to sell.

To determine whether the platform should offer the service or not, first suppose it does not provide the service (d_0). In this case, seller A does not enter the market, because it can only sell one unit and the marginal revenue ($12) is smaller than the marginal cost ($17). Seller B will enter and sell two units. Its total revenue is $12 + $11 = $23, and its total cost is $17 + $5 = $22, yielding a profit of $1. Seller C will sell four units. Its total revenue is $12 + $11 + $7 + $6 = $36, and its total cost is $17 + $5 + $5 + $5 = $32, yielding a profit of $4. The total revenue generated in this case is $59.

When the platform provides the service (d_1), in addition to the entry decision, sellers need to decide whether to invest in the service themselves or not. Seller A enters the market and uses the platform's service. It sells one unit, generating a revenue of $12, incurring a cost of $8, and making a profit of $4.

Seller B also uses the platform's service. It sells two units as in the d_0 case, generating a revenue of $23, incurring a cost of $16, and making a profit of $7. Notice that seller B is better off using the platform's service than providing the service itself, which only generates a profit of $1.

and pay Amazon for storage, picking and packing, and shipping and handling. Many sellers choose not to use Amazon's fulfillment service because their own services are more efficient. Amazon, as a result, created the Seller Fulfilled Prime program to allow some of its sellers to sell products with the Prime badge using their own fulfillment services (source: `https://services.amazon.com/services/seller-fulfilled-prime.html`, accessed November 2018).

[2]This assumption is consistent with Amazon's practice in offering its logistics services.

Seller C also uses the platform's service. It sells two units, generating a revenue of \$23, incurring a cost of \$16, and making a profit of \$7. Again, notice that seller C is better off using the platform's service rather than providing the service itself, which only generates a profit of \$4.

As a result, the total revenue generated is \$58. Therefore, it is better for the platform not to provide the service. Table 2 summarizes the results.

Table 2: Outcomes for the one-period game

	d_0			d_1		
Seller	Units Sold	Revenue (\$)	Profits (\$)	Units Sold	Revenue (\$)	Profits (\$)
A	0	0	0	1	12	4
B	2	23	1	2	23	7
C	4	36	4	2	23	7
Total	**6**	**59**	**5**	**5**	**58**	**18**

This example illustrates two opposing forces that occur when the platform provides the service. First, the platform's service reduces the entry barrier by eliminating the fixed cost. As a result, seller A enters the market. This is the *entry inducement effect*. Second, the platform's service reduces the output by discouraging some sellers from investing in their own service. Seller C now sells two units instead of four units. This is the *output reduction effect*. In this example, the output reduction effect dominates the entry inducement effect. As a result, the platform will not offer the service even if the cost of offering the service is zero. There are, of course, parameter values for which the entry inducement effect dominates. The next section provides a model that analyzes which effect dominates in general.

3 A One-Period Model

3.1 Setup

There is a continuum of sellers with different capacities, k, where $0 < k < \bar{k}$. The probability density function of sellers with capacity k is given by $f(k)$. If a seller produces q, its total revenue is given by $R(q)$. Thus the marginal revenue is $MR(q) = R'(q)$, where $MR(q) > 0$ and $MR'(q) < 0$ for all $q \in (0, \bar{k})$.

A seller can provide an ancillary service itself by incurring a fixed cost F. By using its own service, the seller incurs a constant marginal cost, c. The seller can also use the platform's service if available. By using the platform's service, the seller incurs a constant marginal cost $C > c$. The seller's objective is to maximize its profit, that is, total revenue minus total cost. In practice, some third parties could also provide this service to the sellers. Our results hold qualitatively when these

Table 3: List of variables and their definitions

Variables	Definitions
k	Capacity of a seller.
$\bar{k}$	The upper bound for capacity k.
α	The commission rate.
c	The marginal cost of a seller's own service.
C	The marginal cost of the platform's service.
F	The fixed cost of setting up a seller's own service.
M	The fixed cost of maintaining a seller's own service.
q_c	The profit-maximizing quantity for using a seller's own service without capacity constraint, which is defined by $MR(q_c) = c$.
q_C	The profit-maximizing quantity for using the platform's service without capacity constraint, which is defined by $MR(q_C) = C$.
q_0	The entry-capacity threshold under d_0, i.e., $\pi_{own}(q_0) = 0$.
q_1	The indifference-capacity threshold under d_1, i.e., $\pi_{own}(q_1) = \pi_{plat}(q_1)$.
q_{00}	The entry-capacity threshold under d_{00} in both periods, i.e., $\pi_{own,own}(q_{00}) = 0$.
q_{11}	The indifference-capacity threshold under d_{11}, i.e., $\pi_{own,own}(q_{11}) = \pi_{plat,plat}(q_{11})$.
q_{01}	The entry-capacity threshold under d_{01} in the first period, i.e., $\pi_{own,plat}(q_{01}) = 0$.
q_s	The indifference-capacity threshold between switching to the platform's service and maintaining a seller's own service, satisfied by $q_s = M/(C - c)$.

third parties' cost structure is less efficient than the platform's, an assumption that is likely to hold in practice. As a result, we abstract away from this possibility in our model.

The platform decides whether to provide the service (d_1) or not (d_0). For simplicity, we abstract away the fixed cost for the platform to provide the service. We also assume that if the platform provides the service, the price it charges (C) equals its marginal cost. Consistent with the practices in many marketplaces, the platform charges a fixed commission rate on sales; hence, its objective is to maximize the total revenue of all sellers on the platform. Denote α as the fixed commission rate. The platform profit can be expressed as follows:

$$\Pi = \alpha \int_{k=0}^{\bar{k}} R(q(k,d)) f(k) dk,$$

where $q(k,d)$ is the quantity chosen by a seller with capacity k given the platform's choice, $d \in \{d_0, d_1\}$.

The timeline of the game is the same as in the numerical example in Section 2. We assume that $MR(0) > C > c > MR(\bar{k})$. This assumption implies that all sellers will enter the market when the platform provides the service, and not all sellers will produce at their capacities when using their own services.

For convenience, Table 3 provides the definitions of all variables in this paper.

3.2 Analysis

We analyze the game by backward induction. First, for each of the platform's actions (i.e., whether to provide the service), we determine each seller's optimal response. For a seller with capacity k, given the platform choice, the seller chooses whether to enter or not and, if so, whether to provide its own service and what the output level, q, will be. Next, given the seller's response, we determine the platform's optimal action.

3.2.1 The platform does not provide the service

When the platform does not provide the service, the seller's profit is zero if it does not enter, or enters but does not provide the service itself. If it enters and provides the service, the seller with capacity k chooses quantity q to maximize $R(q) - cq - F$, subject to $q \leq k$. We define a seller's profit if it uses its own service as follows:

$$\pi_{own}(k) = \max_q \{R(q) - cq - F \text{ s.t. } q \leq k\}.$$

To ensure that some sellers will enter the market, we assume that $\pi_{own}(\bar{k}) > 0$. Let q_c be the quantity that satisfies $MR(q_c) = c$. The seller will then produce either q_c or its capacity when $q_c > k$. Therefore,

$$\pi_{own}(k) = \begin{cases} R(k) - ck - F, \text{ if } k < q_c \\ R(q_c) - cq_c - F, \text{ if } k \geq q_c. \end{cases}$$

Notice that $\pi_{own}(k)$ increases with k. Moreover, $\pi_{own}(0) = -F < 0$. We define q_0 as the quantity that satisfies $\pi_{own}(q_0) = 0$. Therefore, sellers will only enter if and only if $k > q_0$.

The optimal quantity each seller produces is illustrated in Figure 1. Sellers with capacities smaller than q_0 do not enter and produce 0. Sellers with capacities between q_0 and q_c produce at their capacities. Sellers with capacities above q_c produce q_c.

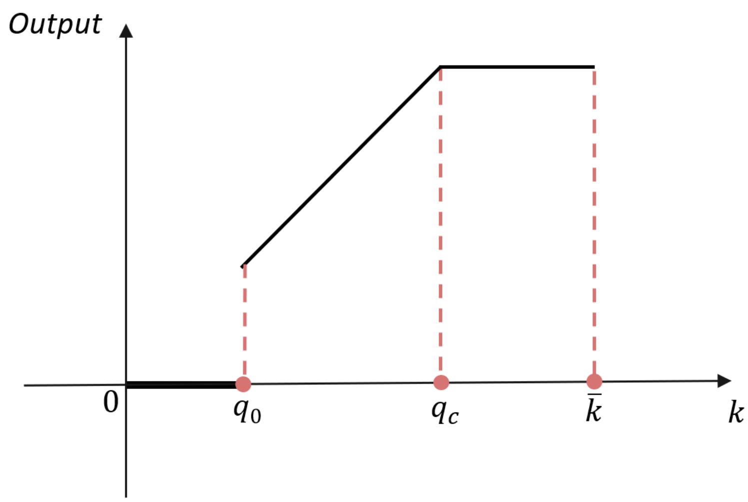


Figure 1: Optimal quantity when the platform does not offer the service.

3.2.2 The platform provides the service

When the platform provides the service, all sellers will enter and they have two choices. First, if a seller uses the platform's service, the seller with capacity k chooses quantity q to maximize $R(q) - Cq$, subject to $q \leq k$. The seller's profit is defined as follows:

$$\pi_{plat}(k) = \max_{q}\{R(q) - Cq \text{ s.t. } q \leq k\}.$$

Let q_C be the quantity that satisfies $MR(q_C) = C$. Therefore,

$$\pi_{plat}(k) = \begin{cases} R(k) - Ck, \text{ if } k < q_C \\ R(q_C) - Cq_C, \text{ if } k \geq q_C. \end{cases}$$

We can similarly illustrate the optimal quantity provided by each seller in Figure 2. Sellers with capacities less than q_C will produce at their capacities. Sellers with capacities greater than q_C will produce q_C. Notice that $q_C < q_c$ because marginal revenue is decreasing in q and $C > c$.

If a seller uses its own service, its profit is again $\pi_{own}(k)$. For ease of exposition, we assume that $\pi_{own}(k) < \pi_{plat}(k)$ for all k.[3] In other words, when the platform provides the service, all sellers will

[3] This assumption is equivalent to $F > [R(q_c) - cq_c] - [R(q_C) - Cq_C]$.

choose the platform's service and enter the market because there is no fixed cost. This assumption is relaxed in Section 6.3. Our main results do not change qualitatively.

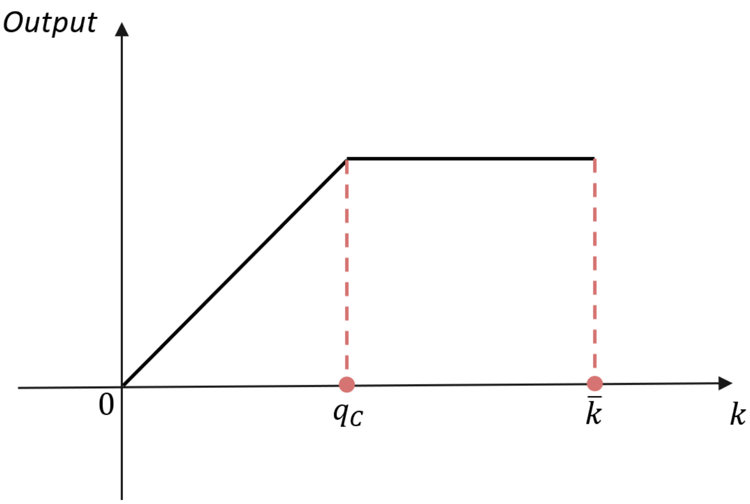


Figure 2: Optimal quantity when the platform provides the service.

3.2.3 Comparing the two cases

To compare the platform profits in the two cases, it is easier to examine the platform profit loss to its *maximal profit*. Note that no seller will choose to produce above q_c, the quantity at which the seller's marginal revenue equals c. The platform's maximal profit is therefore realized when all sellers with capacities below q_c produce at their capacities and those with capacities above q_c produce at q_c.

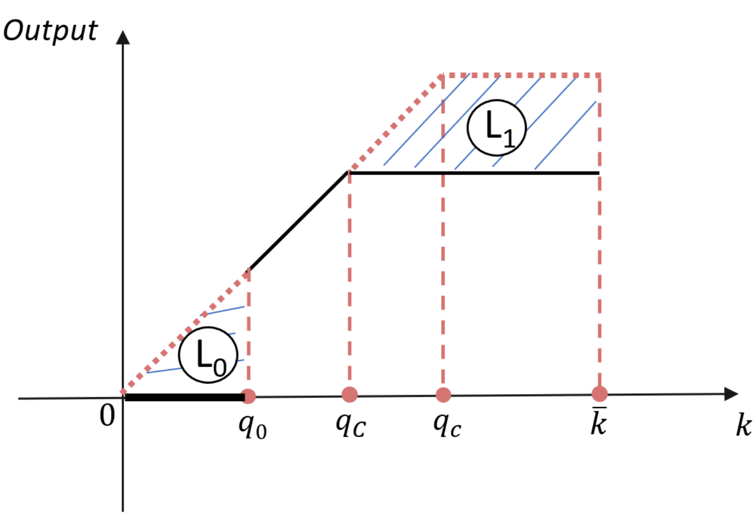


Figure 3: Profit loss relative to the case with the maximal profit.

Figure 3 illustrates the profit loss of the cases relative to the maximal profit. Recall that when the platform does not provide the service, sellers with capacities below q_0 will not enter. Relative to the maximal profit, the profit loss of the platform in this case is captured by $L_0 = \alpha \int_0^{q_0} R(k)f(k)dk$, which describes the entry reduction effect. Notice that q_0 increases with the fixed cost, F. Therefore, the entry reduction effect increases with F. The entry reduction effect is also larger when there are many sellers with capacities below q_0.

When the platform provides the service, sellers with capacity above q_C will only produce at q_C. Relative to the output level that achieves maximal profit, sellers with capacity k between q_C and q_c reduce their output by $k - q_C$ and those with capacity k between q_c and $\bar{k}$ reduce their output by $q_c - q_C$. The profit loss of the platform in this case is therefore captured by $L_1 = \alpha \int_{q_C}^{\bar{k}} (R(min(q_c, k)) - R(q_C))f(k)dk$, which describes the output reduction effect. Note that the amount of output reduction increases weakly with seller capacity, suggesting that when there are more sellers with large capacities (i.e., when there is a fat tail), the output reduction effect is likely to be bigger. The next proposition summarizes our discussion.

Proposition 1. *The following holds:*

1. *The platform will not provide the service (d_0) if and only if $L_0 < L_1$.*

2. *The platform will not provide the service (d_0) if $F(q_c)$ is sufficiently small (i.e., fat tail).*

The next proposition shows how the cost structures of the sellers or the platform affects the likelihood that the platform's service will not be provided.

Proposition 2. *The platform is less likely to provide the service (that is, d_0 is more likely to be chosen) when the sellers' fixed cost (F) decreases, the marginal cost of the platform's service (C) increases, or the marginal cost of the sellers' own service (c) decreases.*

When the fixed cost (F) decreases, fewer sellers will be deterred from entering the market. Therefore, the profit loss from not providing the service is smaller, making d_0 more likely. When C increases or c decreases, the relative efficiency of a seller having its own service increases. In other words, when sellers switch from using their own service to the platform's service, the output reduction effect is larger, reducing the platform's incentive to provide the service.

Note that Figure 3 illustrates the case where $q_0 < q_C$. It is also possible that $q_0 \geq q_C$. But the relationship between q_0 and q_C does not affect our propositions. In the discussion below, we proceed with the assumption that $q_0 < q_C$ for simplicity.

4 A Numerical Example for the Two-Period Game

Our analysis so far has highlighted the trade-off between the entry inducement effect and the output reduction effect. In practice, the platform's decision is not just whether or not to provide the service but also when to provide the service. To explore the timing of providing the service, we consider a two-period model. As in the one-period case, we first illustrate the intuition using a numerical example.

This example builds on the one-period example in Section 2. The first period is the same as in the one-period example. For simplicity, we assume that if the platform provides the service in the first period, it will continue to do so in the second period.[4] In the second period, the same sequence of actions is repeated. If a seller does not invest in its own service in the first period, the second period is identical to the first period. However, if a seller chooses to provide its own service in the first period, then in the second period, they incur a maintenance cost of \$8 instead of incurring the same fixed cost. Table 4 summarizes the second-period marginal revenue and marginal cost for a seller with first-period investment.

Table 4: The second-period marginal revenue and marginal cost for a seller with first-period investment

Unit	MR	MC (Own) (\$)	MC (Platform) (\$)
1	12	13	8
2	11	5	8
3	7	5	8
4	6	5	8

Proposition 3. *There is a unique equilibrium where the following holds:*

1. *The optimal platform choice is to provide the service only in the second period.*
2. *Seller A does not enter in the first period but enters in the second period using the service provided by the platform. It produces one unit.*
3. *Seller B invests in the first period but switches to the service provided by the platform in the second period. It produces two units in each period.*
4. *Seller C invests in the first period and continues to use its own service in the second period. It produces four units in each period.*

Denote d_{ij} as the platform's choice, where i and $j \in \{0, 1\}$ represents the platform's action in the first and second period, respectively. To see why it is optimal for the platform to provide the service only in the second period (d_{01}), we first show that the platform prefers d_{01} to d_{00}. If the platform chooses d_{01}, it can readily be shown that sellers will respond accordingly to the description in the proposition, generating a total revenue of \$59 (first period) + \$71 (second period) = \$130. When the platform chooses d_{00}, note that seller A will not enter in either period. Seller B will invest in the first period and use its own service in both periods, generating a revenue of \$23 in each period (a total of \$46). Seller C will again invest in the first period and use its own service in both periods, generating a revenue of \$36 in each period (a total of \$72). Therefore, the total

[4] As shown in our general model, this assumption is without loss of generality.

revenue is $\$0 + \$46 + \$72 = \$118$, which is less than \$130. Comparing the two cases, the quantities produced by sellers B and C are the same. Seller A is able to enter in the second period in d_{01} but not in d_{00}. Providing the service in the second period therefore generates a profit gain for the platform by eliminating the entry reduction effect in the second period.

When the platform chooses d_{11}, note that all sellers will use the platform's service. Seller A will produce 1 unit in both periods, and sellers B and C will produce 2 units each in both periods. Compared to d_{01}, seller C reduces output from 4 to 2 units, reflecting the output reduction effect. There is no change for seller B. Seller A now enters in the first period and increases output by 1 unit. This benefit from entry, however, is smaller than the loss from the output reduction. Therefore, the platform prefers d_{01} to d_{11}.

This example illustrates that the timing of providing the service is a strategic decision. When the output reduction effect is large, it can be optimal for the platform to delay provision of the service. The next section provides a general analysis for the platform's strategic decision.

5 A Two-Period Model

5.1 Setup

In this model, the first period is the same as the one-period model in Section 3. In the second period, the same sequence of actions is repeated. If a seller does not invest in its own service in the first period, the second period will be identical to the first period. However, if a seller chooses to provide its own service in the first period, then in the second period, they incur a maintenance cost of $M \in (0, F)$ instead of the same fixed cost F. Our solution concept continues to be a subgame-perfect Nash equilibrium. Given this solution concept, the platform cannot commit to its actions across the two periods. We discuss this possibility in the extension.

To analyze this game, we proceed in the same way as in the one-period model by first considering the sellers' best responses under a given platform action. There are four cases to consider.

5.2 Best responses of the sellers given platform actions

5.2.1 The platform does not provide the service in either period (d_{00})

When the platform does not provide the service, each seller must provide the service on its own in order to enter the market. Once a seller incurs a fixed cost of F in the first period, it will use its own service in the second period and incur a maintenance cost of M. Therefore, the *de facto* fixed cost for each seller is $(F + M)/2$ per period and the same analysis from the one-period model can be directly applied.

Define a seller's profit if it uses its own service in both periods as follows:

$$\pi_{own,own}(k) = \max_{q_t}\{\sum_{t=1}^{2}(R(q_t) - cq_t) - F - M \text{ s.t. } q_t \leq k\}.$$

Recall that q_c is the quantity that satisfies $MR(q_c) = c$. Therefore, in each period the seller will produce either q_c or its capacity when $q_c > k$. Therefore,

$$\pi_{own,own}(k) = \begin{cases} 2(R(k) - ck) - F - M, \text{ if } k < q_c \\ 2(R(q_c) - cq_c) - F - M, \text{ if } k \geq q_c. \end{cases}$$

Recall that, in the one-period model, q_0 is the quantity that satisfies $\pi_{own}(q_0) = 0$. Define the corresponding q_{00} as the quantity that satisfies $\pi_{own,own}(q_{00}) = 0$. Because the de facto fixed cost in the two-period model is lower than that in the one-period model ($(F + M)/2 < F$), fewer sellers are deterred from entering the market; that is, $q_{00} < q_0$. The lemma below summarizes our discussion.

Lemma 1. *If the platform does not offer the service in either period (d_{00}), the following holds:*

1. *Sellers with capacity $k < q_{00}$ will stay out of the market in both periods.*
2. *Sellers with capacity $k \geq q_{00}$ will enter the market and use their own services in both periods.*
3. *$q_{00} < q_0$.*

5.2.2 The platform provides the service in both periods (d_{11})

Similar to the previous subsection, the same analysis in the one-period model can be directly applied, with the modification that the seller's de facto fixed cost becomes $(F + M)/2$ per period. Recall that q_C is the quantity satisfying $MR(q_C) = C$. We have the following lemma.

Lemma 2. *If the platform provides its service in both periods (d_{11}), then*

1. *All sellers will enter.*
2. *Sellers with capacity $k < q_C$ will produce k.*
3. *Sellers with capacity $k \geq q_C$ will produce q_C.*

5.2.3 The platform provides the service in the second period only (d_{01})

In this case, all sellers will enter the market in the second period. In the first period, some sellers will not enter. Among those that enter by investing in their own services, some will continue to use their own services and the others will switch to the platform's service in the second period. The incentive to stay with their own services is stronger for sellers with larger capacities because they can spread the maintenance cost over more units. This discussion suggests there are three types of sellers depending on their capacities, as summarized in the lemma below.

Lemma 3. *If the platform provides its service in the second period only (d_{01}), there exist two thresholds, q_{01} and q_s, such that:*

1. *Sellers with capacity $k < q_{01}$ will stay out of the market in the first period. In the second period, these sellers will enter and use the platform's service and produce to their capacity k;*

2. *Sellers with capacity $k \in (q_{01}, q_s)$ will enter the market in both periods, and will use their own services in the first period and the platform's service in the second period. If $q_s \leq q_C$, sellers produce their capacity k in both periods. If $q_s > q_C$, sellers produce their capacity k in the first period, and produce $min(k, q_C)$ in the second period;*

3. *Sellers with capacity $k \geq q_s$ will enter and use their own services in both periods, and will produce $min(k, q_c)$ in each period;*

4. *$q_{01} = q_0$ and $q_s = M/(C - c)$.*

Because all sellers will use the platform's service in the second period, the marginal seller that chooses to enter must break even in the first period. Therefore, $q_{01} = q_0$. The output level at which a seller is indifferent between using its own service and using the platform's service is q_s, which can be derived from $R(q_s) - cq_s - M = R(q_s) - Cq_s$.

Part 2 of Lemma 3 highlights the type of sellers that will switch to the platform's service in the second period. This switch, however, may or may not lead to a reduction in their output levels. Sellers reduce their output when their capacities $k > q_C$, which occurs only when $q_s > q_C$. When $q_s \leq q_C$, all sellers that switch will continue to produce at their capacities. For simplicity, we continue our analysis below under the assumption that $q_s \leq q_C$. In Section 6.3.2, we describe the case where $q_s > q_C$ and show that the main results remain the same.

5.2.4 The platform provides the service in the first period only (d_{10})

This case is a mirror image of d_{01}. In particular, instead of entering in the second period only, small sellers now enter and stay for the first period only. Sellers with intermediate capacities now use the platform's service in the first period and their own services in the second period. Large sellers behave in the same way. We summarize these results in the following lemma.

Lemma 4. *If the platform provides its service in the first period only (d_{10}), there exists two thresholds that are identical to the ones in Lemma 3, q_{01} and q_s, such that:*

1. *Sellers with capacity $k < q_{01}$ will enter and use the platform's service in the first period and produce k, and will stay out of the market in the second period;*

2. *Sellers with capacity $k \in (q_{01}, q_s)$ will enter the market in both periods, and will use the platform's service in the first period and their own services in the second period. If $q_s \leq q_C$, sellers produce their capacity k in both periods. If $q_s > q_C$, sellers produce $min(k, q_C)$ in the first period, and their capacity k in the second period;*

3. *Sellers with capacity $k \geq q_s$ will enter and use their own services in both periods, and will produce $min(k, q_c)$ in each period.*

5.3 Equilibrium outcomes

To determine equilibrium outcomes, we first rule out d_{00} and d_{10} as equilibrium outcomes and then compare platform profits under d_{11} and d_{01}.

5.3.1 Ruling out d_{00} and d_{10}

Proposition 4. *The platform actions, d_{00} and d_{10}, cannot be part of the equilibrium outcomes.*

This proposition results from the lack of commitment from the platform. To see why d_{00} cannot be part of the equilibrium, suppose that the platform chooses d_{00}. Sellers with capacity $k > q_{00}$ will then enter in the first period and invest in their own services. Once this happens, however, the platform has an incentive to deviate at the beginning of the second period by providing its service. In doing so, the platform gains by attracting small sellers to enter in the second period without reducing output from existing sellers. Note that some sellers may switch to the platform's service after this deviation, but under the assumption $q_s < q_C$, they will continue to produce at their capacities so that such switches do not affect the platforms profit. Our discussion therefore implies that platform can benefit from switching to d_{10}, making it impossible for d_{00} to be part of an equilibrium.

Similarly, to see why d_{10} cannot be part of the equilibrium outcomes, suppose that the platform chooses d_{10}. All sellers will enter in the first period, and those with capacity $k > q_{01}$ will invest in their own services, anticipating that the platform will cease to provide its service in the second period. At the beginning of the second period, knowing that those sellers have made their investment, the platform will deviate by continuing to provide its service. Such deviation is profitable because the platform retains the small sellers without reducing the output of the other sellers. In fact, under the assumption $q_s < q_C$, such deviation leads to maximum output levels in both periods: all sellers enter in both periods and will produce $max(k, q_c)$. The platform therefore gains from deviating to d_{11}, so that d_{10} cannot be part of an equilibrium.

In general, commitment makes a difference when the optimal strategy involves ex-post inefficiency. That is, by committing to making a loss in the future, the players can make higher ex-ante profits (see, for example, Fudenberg and Tirole 1984). In our setting, the source of this inefficiency is the heterogeneity in seller types. To see why heterogeneity matters, consider d_{00} for example. Suppose there were only large sellers. If the platform does not provide the service and the large sellers have invested in their services in the first period, the platform does not gain from providing the service in the second period. Because of the existence of small sellers, however, the platform improves its profit by providing the service in the second period.

We next compare the platform's profits in the remaining two cases. As in the one-period model, it is easier to compare the platform's profit loss relative to its maximal profit.

5.3.2 Profit losses in d_{11} and d_{01}

Suppose the platform provides the service in both periods (d_{11}). Notice that in each period, the profit loss is identical to that of d_1 in the one-period model. The shaded area of Figure 4, $L_{11} = \alpha \int_{q_C}^{\bar{k}} (R(min(q_c, k)) - R(q_C))f(k)dk$, is the same as the profit loss in the one-period model, illustrated in Figure 3, i.e., $L_{11} = L_1$. Hence, the total loss in both periods is $2L_{11}$.

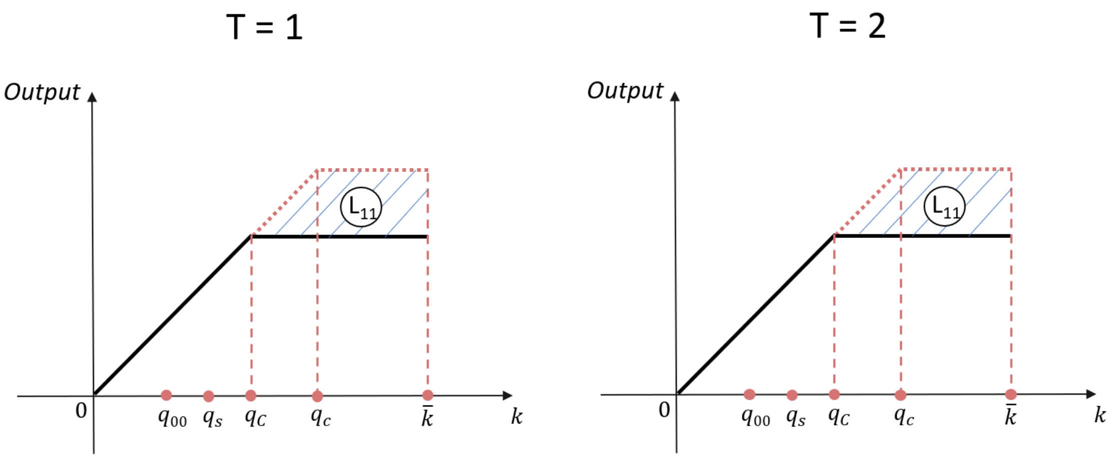


Figure 4: Profit losses in d_{11} relative to the case with maximal profits.

Now, suppose the platform provides the service in the second period only (d_{01}). In this case, there is no profit loss in the second period, as Figure 5 illustrates. All sellers enter the market in the second period. Sellers with capacity $k < q_s$ will produce up to their capacities and use the platform's service. Sellers with capacity $q_s \leq k < q_c$ will produce up to their capacities and use their own services. Sellers with capacity $k \geq q_c$ will produce q_c and use their own services. Unlike the d_{11} case, we do not observe an output reduction effect for sellers with capacity $k > q_C$ because these sellers have to invest in their service in the first period and will continue to use their own services in the second period.

In the first period, sellers with capacity $k < q_{01}$ do not enter. The profit loss is captured by $L_{01} = \alpha \int_0^{q_{01}} R(k)f(k)dk$. Because there is no profit loss in the second period, L_{01} is the total profit loss for both periods. Because $q_{01} = q_0$ (by Lemma 3), we have $L_{01} = L_0$.

5.3.3 Optimal platform choice

Comparing the profit losses in the two cases above, we derive the following proposition.

Proposition 5. *The following holds:*

1. *The platform will provide the service only in the second period (d_{01}) if and only if $L_{01} < 2L_{11}$. Otherwise, the platform will provide the service in both periods (d_{11}).*

2. *The platform will provide the service only in the second period (d_{01}) if $F(q_c)$ is sufficiently small (i.e., there is a fat tail).*

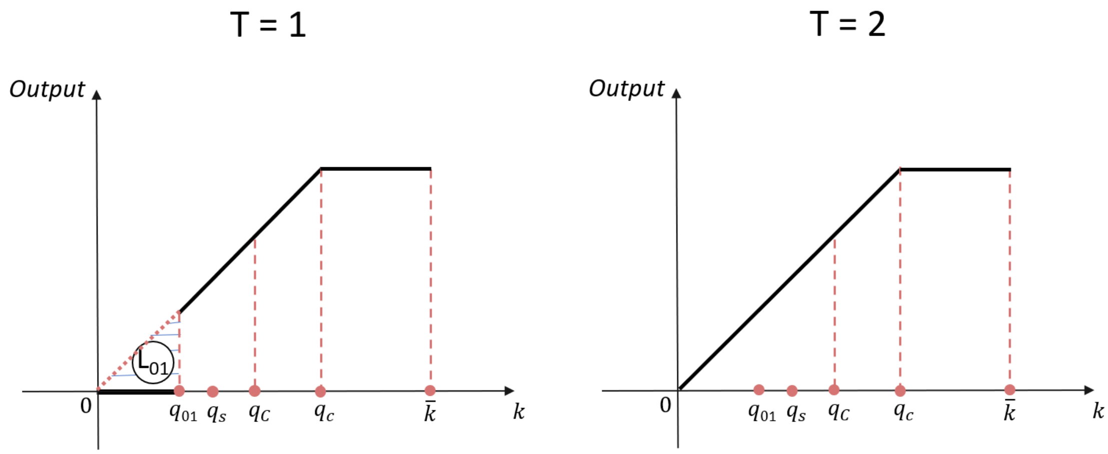


Figure 5: Profit losses in d_{01} relative to the case with maximal profits.

Part 1 of Proposition 5 shows that, in the two-period model, the platform's choice depends on comparing the same two effects as in the one-period model: the entry reduction (L_{01}) and output reduction ($2L_{11}$) effects. The condition for the comparison, however, differs. Recall that $L_{01} = L_0$ and $L_{11} = L_1$; therefore, the profit loss from output reduction is twice as much as in the one-period model, but the profit loss from entry reduction remains the same. Under d_{01}, the platform incurs profit loss due to the entry reduction effect in the first period, but there is no output reduction effect in the second period. The absence of an output reduction effect arises because once sellers invest in their own services in the first period, they will continue to produce at the same output level in the second period, regardless of whether they switch to the platform's service or not.

An implication of this result is that if the platform prefers d_0 to d_1 in the one-period model, it will also prefer d_{01} to d_{11}. The condition under which the platform will prefer d_0 to d_1 in the one-period model (i.e., the fat tail) will guarantee that the platform will prefer d_{01} to d_{11} in the two-period model, as Part 2 of Proposition 5 illustrates.

However, if the platform prefers d_1 to d_0 in the one-period model, it may still prefer d_{01} to d_{11}, suggesting that the timing of providing the service is an important strategic tool in a dynamic environment.

As in the one-period model, we next describe the conditions under which it is more likely for the platform to delay providing the service using the primitives of the model.

Proposition 6. *The platform is more likely to provide the service in the second period only (that is, d_{01} is more likely to be chosen) when the fixed cost (F) decreases, the marginal cost of the platform's service (C) increases, or the marginal cost of the sellers' own service (c) decreases.*

The effect of the cost structure is identical to the one in Proposition 2 because the platform's decision to provide the service depends on a similar set of conditions as described in Propositions 1 and 5. The change in the cost structure affects the relative importance of the entry reduction and output reduction effects in the same way. Therefore, whenever they make the platform less likely to provide the service in the one-period model, they also make the platform more likely to delay providing the service in the two-period model.

6 Extensions

Our results depend on the assumptions that sellers have heterogeneous capacities and that the platform cannot commit to its actions across the two periods. In this section, we describe how the results change when we change these assumptions. We then describe the equilibrium outcome under a more general cost structure.

6.1 Sellers with homogeneous capacity

In our model, sellers differ in their capacities. This assumption is consistent with empirical observations of almost all marketplaces, but is also important from a theoretical perspective. To highlight the importance of this assumption, we now analyze the situation in which sellers are homogeneous in their capacities, which we assume to be a constant, k. Recall from our discussion in Section 5.3.1 that d_{00} cannot be part of the equilibrium outcome precisely because sellers are heterogenous in their capacities. When sellers are homogenous, it is possible for d_{00} to emerge as the equilibrium outcome. Moreover, the emergence of d_{00} implies that there is no gain from strategic delay (i.e., d_{01} does not need to become the equilibrium outcome), as illustrated by the following proposition.

Proposition 7. *If sellers are homogeneous in their capacities, the platform does not benefit from delaying the provision of the service. In other words, the platform's optimal strategy can be implemented by either providing the service immediately or never providing it.*

Proposition 7 follows from comparing the platform's payoffs under all possible strategies. The general intuition is that when sellers are small (i.e, $k < q_0$), the entry reduction effect dominates and providing services in both periods (i.e., d_{11}) is (weakly) optimal. When sellers are large (i.e., $k > q_0$), the output reduction effect dominates. In this case, if the platform does not provide the service in the first period, all sellers will provide their own services and will produce up to their capacity in both periods. This implies that the platform is indifferent between d_{01} and d_{00} so that there is no gain from delaying the service.

6.2 Commitment

Another important assumption is that the platform cannot commit to its actions. Although this assumption matches well with real-world scenarios, it is also possible that the platform can take actions to credibly signal its commitment. We now analyze how commitment changes the platform's decision and profitability.

When the platform can commit, we also need to consider d_{00} and d_{10} as equilibrium outcomes. It can be shown that the profit losses from d_{10} and d_{01} are the same. For simplicity, we will not consider d_{10} and only need to calculate the profit loss from d_{00}.

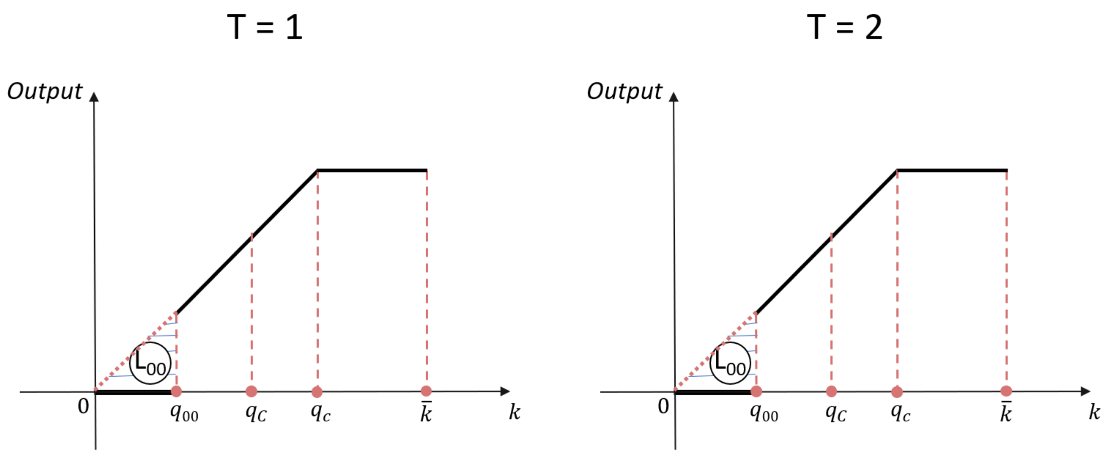


Figure 6: Profit losses in d_{00} relative to the case with maximal profits.

As in the one-period model, the platform's maximal profit in each period is realized when all sellers with capacity below q_c produce at their capacities and sellers with capacity above q_c produce at q_c. When the platform does not offer the service in either period, according to Lemma 1, sellers with capacity $k < q_{00}$ will not enter the market. Therefore, the profit loss in each period is $L_{00} = \alpha \int_0^{q_{00}} R(k)f(k)dk$, the shaded part in Figure 6. The total profit loss across the two periods is $2L_{00}$. Recall that $q_{00} < q_0$ (Lemma 1), so we have $L_{00} < L_0$.

Comparing the three cases, we have the following proposition.

Proposition 8. *When the platform can commit to its actions across the two periods, the following holds:*

1. *The platform will not provide the service in either period (d_{00}) if and only if $L_{00} < min(L_{01}/2, L_{11})$.*

2. *The platform will provide the service in both periods (d_{11}) if and only if $L_{11} < min(L_{01}/2, L_{00})$.*

3. *The platform will provide the service only in the second period (d_{01}) if and only if $L_{01} < min(2L_{00}, 2L_{11})$.*

Part 1 of this proposition shows that, compared to the case without commitment, it is possible for d_{00} to become the equilibrium outcome. In this case, the total profit loss from d_{00} ($2L_{00}$) must be smaller than that in d_{01} (L_{01}) and d_{11} ($2L_{11}$). This condition can be rewritten as $L_{00} < min(L_{11}, L_{01} - L_{00})$. Note that L_{11} captures the output reduction effect and $L_{01} - L_{00}$ can be interpreted as the *incremental* entry reduction effect since, under d_{01}, a greater number of smaller sellers will not enter the market in the first period. Part 1 therefore implies that d_{00} is likely to be optimal if the output reduction effect is large and the incremental entry reduction effect is large. Parts 2 and 3 of Proposition 8 can be interpreted similarly. In particular, d_{11} is likely to be optimal when the output reduction effect is small, and d_{01} is likely to be optimal when the output reduction effect is large and the incremental entry reduction effect is small.

Comparing platform profits with and without commitment, note that the platform makes the same profit when d_{01} or d_{11} are chosen. When d_{00} is chosen, which occurs when the output reduction effect is large and the incremental entry reduction effect is large, the platform makes a greater profit. The extra profit reflects the gain from commitment.

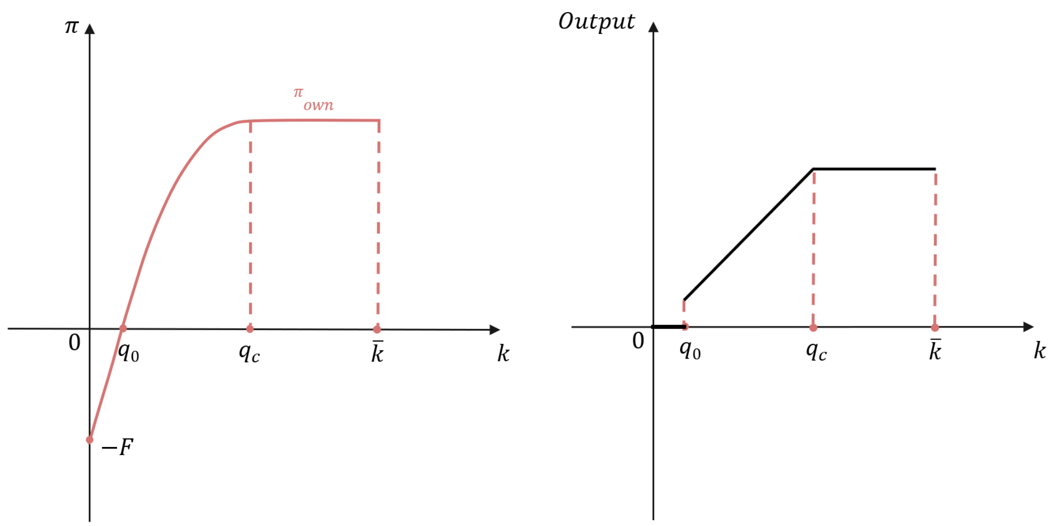
6.3 General cost structure for sellers

In this subsection, we provide a complete analysis by considering the general cost structure of sellers. We start the analysis with the one-period model and then move to the two-period model.

6.3.1 One-period model

In Section 3, we assumed that all sellers will use the platform's service when it is provided (i.e., $F > [R(q_c) - cq_c] - [R(q_C) - Cq_C]$ as in footnote 3). Now we complete the analysis by examining the scenario where $F \leq [R(q_c) - cq_c] - [R(q_C) - Cq_C]$. The qualitative results remain the same.

When the platform does not provide the service (d_0), sellers' decisions are the same as in our previous analysis. Figure 7a plots the sellers' profits from providing their own services as a function of their capacities. Sellers with capacities smaller than q_0 have negative profits and therefore will not enter the market, reflecting the entry reduction effect. Figure 7b plots the optimal output of sellers as a function of their capacities.

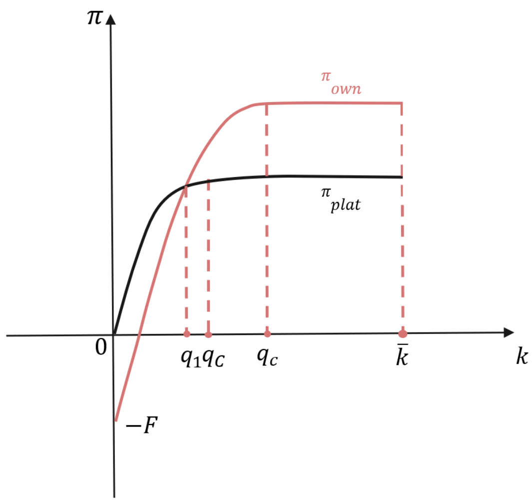


(a) Profit curve

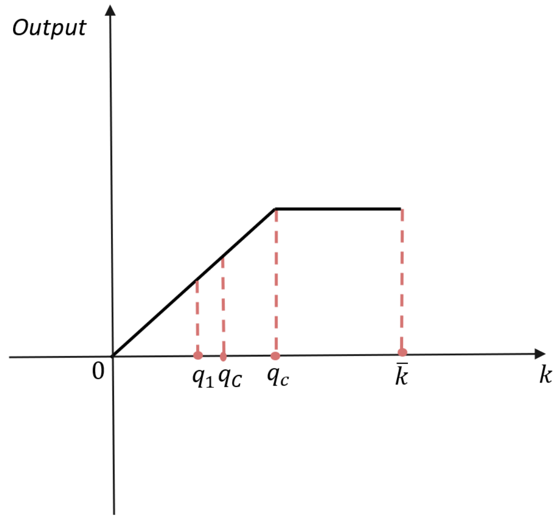
(b) Optimal output

Figure 7: Profit curve and optimal output in d_0.

When the platform provides the service (d_1), sellers' profit curve of using their own services intersects with the one of using the platform's service, unlike the analysis in Section 3. Let q_1 be the point at which the two profit curves intersect. Figure 8 illustrates the case where $q_1 < q_C$ and Figure 9 illustrates the case where $q_1 > q_C$.

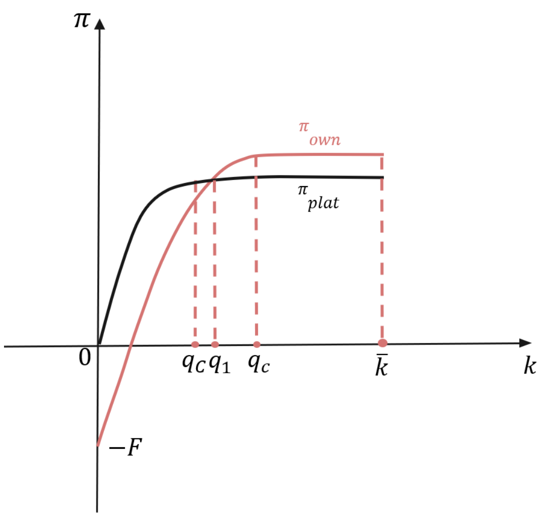


(a) Profit curve

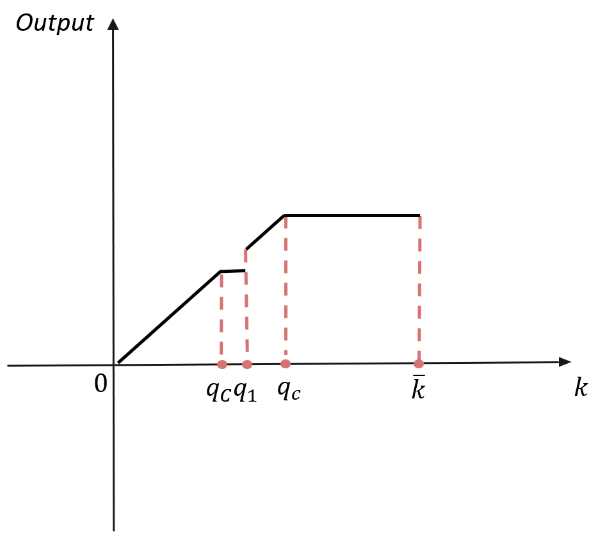


(b) Optimal output

Figure 8: The case when $q_1 \leq q_C$ in d_1.



(a) Profit curve



(b) Optimal output

Figure 9: The case when $q_1 > q_C$ in d_1.

As shown in Figure 8b, when $q_1 \le q_C$, sellers with capacities $k \le q_1$ use the platform's service and produce up to their capacities. Sellers with capacities $q_1 < k < q_c$ use their own services and produce up to their capacities. Sellers with capacities $k \ge q_c$ produce q_c. As a result, the output produced in this case reaches the first best, and there is no output reduction effect. In this case, d_1 dominates d_0.

As shown in Figure 9b, when $q_1 > q_C$, sellers with capacities $k \le q_1$ use the platform's service. Unlike Figure 8b, only sellers with capacities $k \le q_C$ produce up to their capacities. Sellers with capacities $q_C < k < q_1$ produce only q_C, which is smaller than their capacities, resulting in output reduction. Sellers with capacities $q_1 < k < q_c$ use their own services and produce up to their capacities. Sellers with capacities $k \ge q_c$ produce q_c. As a result, except for sellers with capacity $q_C < k < q_1$, the output produced in this case reaches the first best.

To determine whether the platform should provide the service when $q_1 > q_C$, we again compare the profit losses resulting from the entry reduction effect and the output reduction effect. Figure 10 plots these two profit losses. In this case, the profit loss from the entry reduction effect is

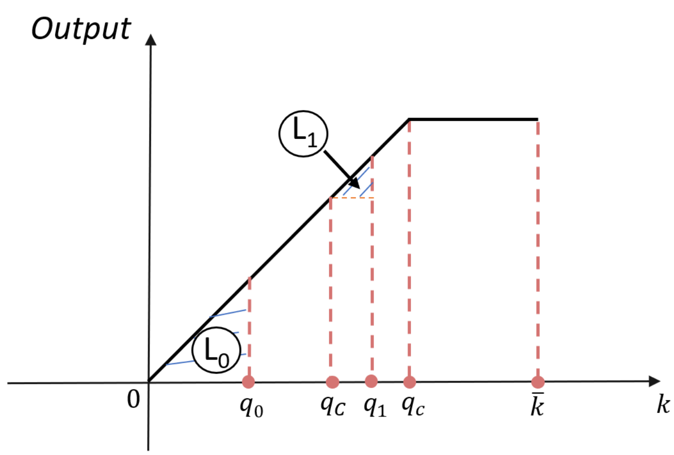


Figure 10: Profit loss in d_1 relative to the case with maximal profit.

given by $L_0 = \alpha \int_0^{q_0} R(k)f(k)dk$, and the profit loss from the output reduction is given by $L_1 = \alpha \int_{q_C}^{q_1} (R(k) - R(q_C))f(k)dk$. Note that the area in which the profit loss occurs is a triangle.

Combining both cases ($q_1 > q_C$ and $q_1 \le q_C$), the following proposition describes the platform's choice of providing the service.

Proposition 9. *The platform will not provide the service (d_0) if and only if $q_1 > q_C$ and $L_0 < L_1$.*

6.3.2 Two-period model

In this subsection, we analyze the platform's entry decision for a more general cost structure in a two-period model. Recall that in our main analysis, we have assumed that $q_C \ge q_s$. We now complete the analysis by incorporating the possibility that $q_C < q_s$. Again, our qualitative results remain the same, but we need to consider more cases. In particular, we can no longer rule out d_{00}

as part of the equilibrium outcome. This is because unlike the analysis in Section 5.3.1, output reduction can happen when the platform deviates from d_{00} to d_{01}. This possibility implies that d_{00} may become an equilibrium outcome, and so we need to compare the profit loss under each of the three possibilities: d_{00}, d_{11} and d_{01}.

First, consider the case where the platform does not provide the service in either period (d_{00}). In this case, the two-period model is the same as the one in Section 5.2.1. For completeness, we illustrate the profit loss in each period in Figure 11. The total profit loss across the two periods is thus $2L_{00} = 2\alpha \int_0^{q_{00}} R(k)f(k)dk$.

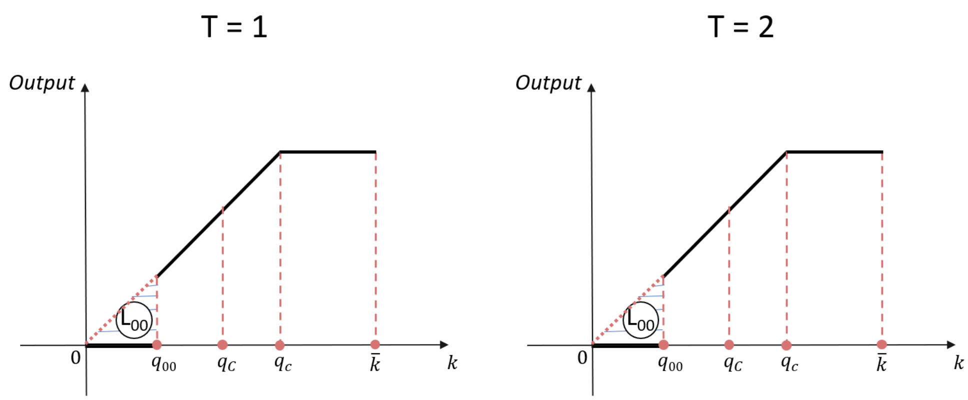


Figure 11: Profit losses in d_{00} relative to the case with maximal profits.

Now, consider the case where the platform provides the service in both periods (d_{11}). In this case, it can be shown that, once a seller invests in its own service in the first period, it will use its service in both periods. Let q_{11} be the capacity level at which the seller is indifferent between using its own service (paying a de facto fixed cost of $(F+M)/2$) and using the platform's service. There are two possibilities. When $q_{11} \leq q_C$, the optimal output achieves the first best outcome. When $q_{11} > q_C$, profit loss does occur. In particular, for sellers with capacities $q_C < q < q_{11}$, instead of producing up to their capacities, they produce at a lower output level, q_C. The total profit loss of the sellers, denoted by the triangular areas in Figure 12, is therefore $2L_{11} = 2\alpha \int_{q_C}^{q_{11}} (R(k) - R(q_C))f(k)dk$.

Finally, we consider the case where the platform provides the service in the second period only (d_{01}). Suppose that sellers have already invested in their own services in the first period. Recall that q_s is the capacity level at which a seller is indifferent between using its own service (paying a maintenance cost of M) and using the platform's service in the second period. When $q_s \leq q_C$, the profit loss (illustrated Figure 13) is identical to the one in the main analysis. There is an entry reduction effect in the first period but no output reduction effect in the second period. The total loss is again $L_{01} = \alpha \int_0^{q_0} R(k)f(k)dk$.

When $q_s > q_C$, there is an output reduction effect in the second period. In particular, sellers with capacities $q_C < k < q_s$ produce output q_C rather than their capacities. The total loss over the two periods (illustrated in Figure 14) is $L_{01} + L_{01}^* = \alpha \int_0^{q_0} R(k)f(k)dk + \alpha \int_{q_C}^{q_s} (R(k) - R(q_C))f(k)dk$.

Comparing all these cases, we obtain the following proposition.

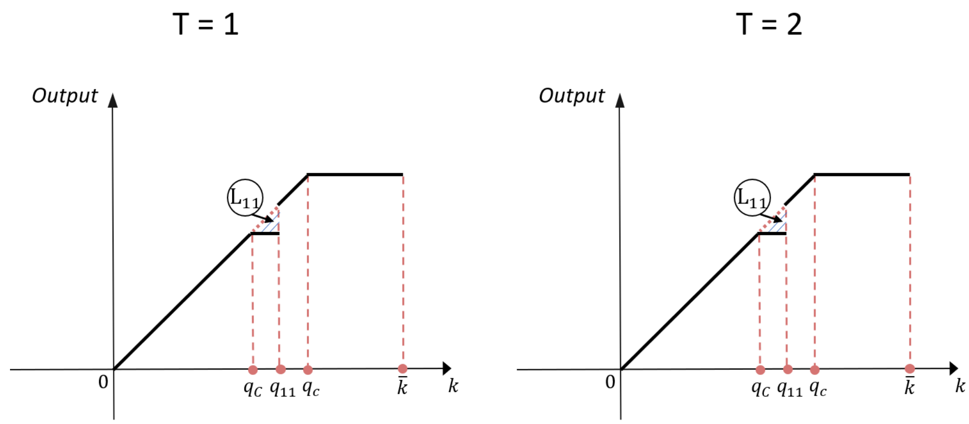


Figure 12: Profit losses in d_{11} when $q_{11} > q_C$ relative to the case with maximal profits.

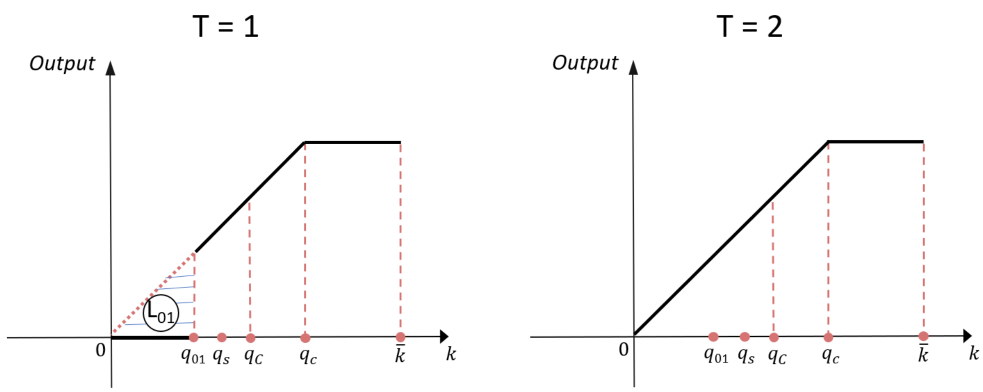


Figure 13: Profit losses in d_{01} when $q^* \leq q_C$ relative to the case with maximal profits.

Proposition 10. *Under the general cost structure, the following holds:*

1. *If $q_{11} \leq q_C$, the platform will provide the service in both periods (d_{11}).*

2. *If $q_{11} > q_C \geq q_s$, then*

   (a) *If $L_{01} \leq 2L_{11}$, the platform will provide the service only in the second period (d_{01}).*

   (b) *If $L_{01} > 2L_{11}$, the platform will provide the service in both periods (d_{11}).*

3. *If $q_{11} > q_s > q_C$, then*

   (a) *If $L_{00} \leq L^*_{01}$, the platform will not provide the service in either period (d_{00}).*

   (b) *If $L_{00} > L^*_{01}$ and $L_{01} + L^*_{01} \leq 2L_{11}$, the platform will provide the service only in the second period (d_{01}).*

   (c) *If $L_{00} > L^*_{01}$ and $L_{01} + L^*_{01} > 2L_{11}$, the platform will provide the service in both periods (d_{11}).*

Proposition 10 provides a complete characterization of the equilibrium outcome. Part 1 describes the parameter range under which there is no output reduction effect. As a result, the

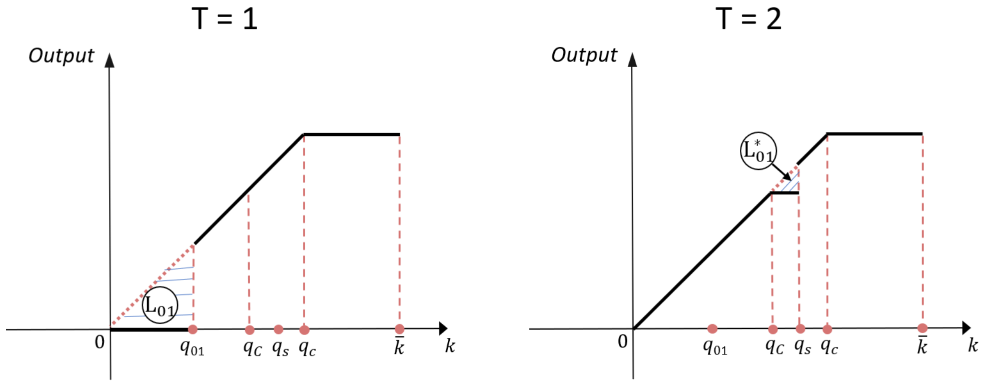


Figure 14: Profit losses in d_{01} when $q^* > q_C$ relative to the case with maximal profits.

platform will always provide the service to encourage entry in both periods. Part 2 corresponds to our analysis in the main body of the paper (i.e., Section 5.3.1). The condition for d_{01} to become the equilibrium outcome is the same as that in Proposition 5. In Part 3, d_{00} can become part of the equilibrium outcome due to the existence of L^*_{01} (i.e., the output reduction can occur for sellers with k between q_C and q_s). As a result, the equilibrium outcome is determined by comparing the profit losses under all three cases. Notice that even if the profit loss of a particular strategy is the smallest of the three cases, it may fail to become part of the equilibrium outcome, because without commitment, the platform may have an incentive to deviate. In other words, to carry out such a comparison we first need to determine the parameter range for each of the three strategies under which the platform will not deviate. Although the calculation is cumbersome, the equilibrium outcome is still determined by comparing the entry reduction effect and the output reduction effect.

7 Conclusions

In this paper, we study a marketplace platform's use of investments in ancillary services to influence the size distribution and scalability of the sellers it attracts. We identify two important effects from such actions: an entry inducement effect that could encourage entry of small sellers and an output reduction effect that reduces the output of large sellers. We also find that the platform can increase its profits by strategically delaying the provision of ancillary services.

Our study has important implications for platform owners. First, we show that their scalability depends on the scalability of the whole ecosystem and not just on themselves. As a result, marketplace platforms need to develop a deep understanding of the economics of their sellers. In our study, the main driver of output reduction effect is that sellers' marginal cost from using their own services is lower than that from using the platform's service. The assumption is likely to hold when a platform's service converts some fixed cost to marginal cost or when such services involve specialization that tailors to sellers' individual needs. There are also services (e.g., insurance and payment) in which platforms do not make such conversions and sellers' individual needs are not important. In such cases, sellers do not reduce their marginal cost by providing the service themselves. Hence, there will not be output reduction effect. As a result, it is always optimal for the platform to provide these services.

Second, we provide a framework to illustrate that the composition of sellers is as important as the sheer number of sellers. Seller composition affects a platform's optimal action and, at the same time, will be influenced by its action. In addition, it is not always optimal for a platform to provide help because such actions shift the composition of its sellers and may consequently reduce its profit. For example, because eBay does not provide fulfillment services to its sellers, it has many large sellers. In contrast, by providing fulfillment services, Amazon sellers may have reduced their output levels. Amazon also induces many small sellers to join its marketplace.

Finally, the timing of the investment into ancillary services depends on the composition and investment decisions of sellers. For example, if most large hosts (i.e., hosts with many properties) on Airbnb have already made fixed-cost investments, it might be the right time for Airbnb to offer hosts ancillary services such as apartment cleaning and centralized laundry facilities.

We have made a few simplifying assumptions in developing our model. Our analysis implicitly assumes that changes in seller size have negligible effects on their bargaining power over the platform. When this assumption does not hold, the platform needs to take sellers' bargaining power into consideration. For example, as sellers expand their output levels, they gain greater bargaining power over the platform and can reduce the platform's profitability. At the same time, they may also have greater bargaining power over their suppliers, resulting greater revenue for themselves and hence greater profit for the platform. In such cases, the platform could use the ancillary service as a strategic tool to influence relative bargaining power of sellers.

In our analysis, we only consider one tool that a platform can use to scale its marketplace, but in practice, a platform can use a number of other instruments. For example, a platform may consider using non-linear pricing such that service fees depend on sellers' capacities. Its pricing strategy may also change over time. The platform may choose to take a loss from providing the ancillary services to increase total transactions in the marketplace. In general, it will be useful to understand the interaction between the pricing policy and the decision to provide an ancillary service.

In addition, some third-party service providers may provide ancillary services to help scale the marketplace. The current analysis implicitly assumes that this market is inefficient (that is, the prices charged by third parties are high). When third-party services are as efficient as the platform's service, sellers have three options: their own service, the platform's service, and the third-party services. Furthermore, the platform's decision to offer the service itself or not affects the entry decision of these third parties. Incorporating these efficient third parties into our model therefore introduces another layer of dynamic strategic interactions.

Future research could also examine how firms strategically use their investments to affect others' fixed costs in broader settings. For example, some airlines (like Lufthansa) provide aircraftmaintenance services to other airlines. While this strategy may reduce entry barriers of new airlines, it also reduces small airlines' imperatives to grow. Platform investment in this case turns competing firms into frenemies—they cooperate while competing with each other. We believe that how firms strategically use investments to manage their relationships with each other, in both marketplace and non-marketplace settings, would be a fascinating research area.

References

Anderson, Edward G., Geoffrey G. Parker, Burcu Tan. 2014. Platform performance investment in the presence of network externalities. *Information Systems Research* **25**(1) 152–172.

Armstrong, Mark. 2006. Competition in two-sided markets. *Rand Journal of Economics* **37**(3) 668–691.

Basu, Amit, Sreekumar R. Bhaskaran, Rajiv Mukherjee. forthcoming. An analysis of search and authentication strategies for online matching platforms. *Management Science* .

Bhargava, Hemant K., Daewon Sun, Byung Cho Kim. 2013. Commercialization of platform technologies: Launch timing and versioning strategy. *Production and Operations Management* **22**(6) 1374–1388.

Caillaud, Bernard, Bruno Jullien. 2003. Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics* **34**(2) 309–328.

Chellappa, Ramnath K., Rajiv Mukherjee. 2017. Platform preannouncement strategies: The strategic role of information in two-sided markets competition. Available at SSRN: https://ssrn.com/abstract=2705238.

Chellappa, Ramnath K., Rajiv Mukherjee. 2018. Platforms, pricing and piracy. Working paper.

Chen, Jianqing, Ming Fan, Mingzhi Li. 2016. Advertising versus brokerage model for online trading platform. *MIS Quarterly* **40**(3) 575–596.

Chen, Pei-yu, Shin-yi Wu. 2012. The impact and implications of on-demand services on market structure. *Information Systems Research* **24**(3) 750–767.

Cheng, Hsing Kenneth, Subhajyoti Bandyopadhyay, Hong Guo. 2011. The debate on net neutrality: A policy perspective. *Information Systems Research* **22**(1) 60–82.

Choi, Jay Pil, Byung-Cheol Kim. 2010. Net neutrality and investment incentives. *RAND Journal of Economics* **41**(3) 446–471.

Cui, Ruomeng, Meng Li, Qiang Li. 2018. Value of high-quality logistics: Evidence from a clash between SF Express and Alibaba. Available at SSRN: https://ssrn.com/abstract=3234023.

Economides, Nicholas, Evangelos Katsamakas. 2006. Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry. *Management Science* **52**(7) 1057–1071.

Eisenmann, Thomas R., Geoffrey Parker, Marshall W. Van Alstyne. 2011. Platform envelopment. *Strategic Management Journal* **32**(12) 1270–1285.

Fudenberg, Drew, Jean Tirole. 1984. The fat-cat effect, the puppy-dog ploy, and the lean and hungry look. *American Economic Review* **74**(2) 361–366.

Gawer, Annabelle, Rebecca Henderson. 2007. Platform owner entry and innovation in complementary markets: Evidence from Intel. *Journal of Economics & Management Strategy* **16**(1) 1–34.

Hagiu, Andrei, Julian Wright. 2018. Platforms and the exploration of new products. Working paper.

Hao, Lin, Hong Guo, Robert F. Easley. 2017. A mobile platform's inapp advertising contract under agency pricing for app sales. *Production and Operations Management* **26**(2) 189–202.

Huang, Peng, Marco Ceccagnoli, Chris Forman, D.J. Wu. 2013. Appropriability mechanisms and the platform partnership decision: Evidence from enterprise software. *Management Science* **59**(1) 102–121.

Huang, Peng, Ali Tafti, Sunil Mithas. 2018. Platform sponsor investments and user contributions in knowledge communities: The role of knowledge seeding. *MIS Quarterly* **42**(1) 213–240.

Iansiti, Marco, Roy Levien. 2004. *The Keystone Advantage: What the New Dynamics of Business Ecosystems Mean for Strategy, Innovation, and Sustainability*. Harvard Business School Press.

Jiang, Baojun, Kinshuk Jerath, Kannan Srinivasan. 2011. Firm strategies in the "mid tail" of platform-based retailing. *Marketing Science* **30**(5) 757–775.

Jin, Ginger Zhe, Marc Rysman. 2015. Platform pricing at sports card conventions. *Journal of Industrial Economics* **63**(4) 704–735.

Krämer, Jan, Lukas Wiewiorra. 2012. Network neutrality and congestion sensitive content providers: Implications for content variety, broadband investment, and regulation. *Information Systems Research* **23**(4) 1303–1321.

Lee, Robin S. 2013. Vertical integration and exclusivity in platform and two-sided markets. *American Economic Review* **103**(7) 2960–3000.

Luo, Xueming, Zhijie Lin, Tedi Skiti. 2018. Review rating-based platform screening and new complementor entry: Evidence from natural experiment and machine learning of a sharing economy. Working paper.

McIntyre, David P., Arati Srinivasan. 2017. Networks, platforms, and strategy: Emerging views and next steps. *Strategic Management Journal* **38**(1) 141–160.

Nagaraj, Abhishek, Henning Piezunka. 2018. Motivating the established, deterring the new the divergent effect of platform competition on member contributions in digital mapping communities. Working paper.

Niculescu, Marius F., D.J. Wu, Lizhen Xu. 2018. Strategic intellectual property sharing: Competition on an open technology platform under network effects. *Information Systems Research* **29**(2) 498–519.

Parker, Geoffrey, Marshall W. Van Alstyne. 2005. Two-sided network effects: A theory of information product design. *Management Science* **51**(10) 1494–1504.

Rochet, Jean-Charles, Jean Tirole. 2003. Platform competition in two-sided markets. *Journal of European Economic Association* **1**(4) 990–1029.

Tan, Burcu, Edward G. Anderson, Geoffrey Parker. 2018. Platform pricing and investment to drive third party value creation in two-sided networks. Working paper.

Tucker, Catherine, Juanjuan Zhang. 2010. Growing two-sided networks by advertising the user base: A field experiment. *Marketing Science* **29**(5) 783–961.

Appendix

Proof of Proposition 1. Part 1 of the proposition follows directly from the discussion above the proposition.

For part 2, recall that $L_0 = \alpha \int_0^{q_0} R(k)f(k)dk$. We can rewrite L_1 as $L_1 = \alpha(\int_{q_C}^{q_c}[R(k) - R(q_C)]f(k)dk + \int_{q_c}^{\infty}[R(q_c) - R(q_C)]f(k)dk)$. To obtain a sufficient condition for $L_0 < L_1$, we use the facts that $R(\cdot)$ is monotonically increasing and both $R(\cdot)$ and $f(\cdot)$ are non-negative functions to derive the following inequalities:

$$L_0/\alpha \leq R(q_0) \int_0^{q_0} f(k)dk = R(q_0)F(q_0),$$

$$L_1/\alpha \geq \int_{q_c}^{\infty} [R(q_c) - R(q_C)]f(k)dk = [R(q_c) - R(q_C)][1 - F(q_c)].$$

A sufficient condition would therefore require

$$R(q_0)F(q_0) < [R(q_c) - R(q_C)][1 - F(q_c)].$$

Rearranging the inequality, we have

$$\frac{F(q_0)}{1 - F(q_c)} < \frac{R(q_c) - R(q_C)}{R(q_0)}.$$

Since $F(\cdot)$ is monotonically increasing, we can relax the condition as

$$\frac{F(q_c)}{1 - F(q_c)} < \frac{R(q_c) - R(q_C)}{R(q_0)},$$

which can be reexpressed as

$$F(q_c) < \frac{R(q_c) - R(q_C)}{R(q_0) + [R(q_c) - R(q_C)]}.$$

Note that the right hand side of the above inequality is independent of the distribution of the sellers' capacities. It follows that a sufficient condition for $L_0 < L_1$ is that the distribution has a fat tail. $\square$

Proof of Proposition 2.

$$L_0 = \alpha \int_0^{q_0} R(k)f(k)dk$$

$$L_1 = \alpha \int_{q_C}^{\bar{k}} [R(\min\{q_c, k\}) - R(q_C)]f(k)dk$$

When F decreases, q_0 decreases, and L_0 will decrease. When C increases, q_C decreases. The integrand of L_1 increases and the interval for integrating is extended. Both will help L_1 to increase. When c decreases, q_c increases, and the integrand weakly increases, which in turn makes L_1 increase.

In all these scenarios, it is more likely that $L_0 < L_1$ holds and d_0 becomes the equilibrium. $\square$

Proof of Proposition 3. Given the platform's choice, we first show that none of the three sellers have incentives to deviate. For seller A, given d_{01}, it is obvious that it never wants to invest and could only enter the market when the platform provides the service.

For seller B, there are two ways it can deviate. It could choose not to invest in the first period and therefore not entering, or it could choose to invest in the second period instead of using the platform's service. Notice that under the equilibrium strategy, seller B makes a profit of \$1 in the first period and \$7 in the second period. Suppose seller B does not invest in the first period, its first period profit is then zero. In the second period, it could make a profit of \$1 (if it invests) or \$7 (if it uses the platform's service). Therefore, it will use the platform's service in the second period. The maximum payoff of seller B by not investing in the first period is \$7, which is smaller than the equilibrium payoff. This shows that seller B will not deviate to not investing in the first period. Next, we check whether seller B would deviate in the second period by not using the platform's service. This is obvious, as its payoff from using its own service is \$5, which is less than \$7.

For seller C, again there are two ways it can deviate. It could choose not to invest in the first period and therefore not entering, or it could choose to use the platform's service instead of its own service. Notice that under the equilibrium strategy, seller C makes a profit of \$4 (by producing four units) in the first period and \$8 in the second period (by producing four units). Suppose seller C does not invest in the first period, its first period profit is the zero. In the second period, the maximum profit it can make is \$8. Therefore, it will invest in the first period. Next, we check whether seller C would deviate in the second period by using the platform's service. This is obvious, as its payoff from using the platform's service is \$7, which is less than \$8.

Now, we will make sure that the platform will not deviate. There are two possibilities for deviation: d_{11} or d_{00}. Under the equilibrium, the total revenue for the first period from the three sellers is \$59 (\$0 from seller A, \$23 from seller B and \$36 from seller C), and for the second period is \$71 (\$12 from seller A, \$23 from seller B, and \$36 from seller C).

Consider the first deviation scenario, d_{11}. Notice that in this case, seller A will produce 1 unit in both periods using the platform's service. For seller C, it can be shown that its optimal response is to use the platform's service in both periods, which yields a profit of \$7 in each period and \$14 in total. (Notice that if it uses its own service in both periods, it will produce four units in both periods, making a profit of \$4 in the first period and \$8 in the second period, yielding a total of \$12).

For seller B, it can be similarly shown that its optimal response is to use the platform's service in both periods, which yields a profit of \$14. (Notice that we checked C before checking B because B's profit using its own service is always smaller than C's. Therefore, if C chooses to use the platform's service, B will always do the same).

The total revenue of the three sellers will be: seller A: $\$12+\$12 = \$24$; seller B: $\$23+\$23 = \$46$; and seller C: $\$23 + \$23 = \$46$. The total is \$116, which is less than \$130.

Next consider the second deviation (d_{00}). In this case, seller A will not enter in either period. Seller B will invest in the first period and use its own service in both periods, generating a revenue of \$23 in each period (a total of \$46). Seller C will invest in the first period and use its own service in both periods, generating a revenue of \$36 in each period (a total of \$72). Hence, the total revenue is $\$0 + \$46 + \$72 = \$118$, which is less than \$130.

The above analysis shows that no player will deviate. Notice that for each seller, its payoff depends only on its own strategy and the platform's strategy. Essentially, we have three two-player games. Following Zermelo's Theorem, the subgame perfect Nash equilibrium is unique. □

Proof of Lemma 1-4. Lemma 1 and 2 follow directly from the discussion in the paper. Since Lemma 4 is the mirror image of Lemma 3, we only need to prove Lemma 3 here.

Given the platform's choice (d_{01}), it is clear that all sellers will enter in the second period. For the first period, the sellers' entry decision is monotone in their capacities. Therefore, there exists a threshold, q_{01}, such that sellers with capacities $k < q_{01}$ will not enter in the first period. For these sellers, they will enter in the second period and use the platform's service, producing up to their capacities k. This gives the results in part 1.

For sellers with capacities $k \geq q_{01}$, they will enter in both periods and all of them will invest in their own services in the first period. However, some of them may switch to the platform's service in the second period. The decision to switch is monotone in their capacities. Therefore, there exists a threshold, q_s, such that sellers with capacities $q_{01} \leq k < q_s$ will switch to the platform's service. Their output level is given by $\min(k, q_C)$. Note that $q_C \geq q_s$, $\min(k, q_C) = k$. This gives part 2 of the lemma.

Sellers with capacities $k \geq q_s$ will continue to use their own services in the second period. They will always produce to $\min(k, q_c)$. This gives part 3 of the lemma.

Part 4 follows directly from the discussion in the paper. □

Proof of Proposition 4. Suppose the sellers' responses in the first period follow the d_{00} case, we are at the beginning of the second period and the platform has to choose between d_{01} and d_{00}. Sellers with $0 < k < q_{00}$ stay out of the market, while sellers with $k \geq q_{00}$ use their own services. Suppose the platform decides to deviate to d_{01} in the second period. Because $q_{00} < q_0 < q_s$, we know that the deviation would result in the "first best" total revenue for the platform in the second period, which is higher than the total revenue if the platform stays with d_{00}. Therefore, d_{00} cannot be supported as an equilibrium outcome.

Now, let us turn to d_{10}. If the platform does not deviate, the loss would have been L_{01} in the second period. Suppose the platform deviates to d_{11}. Recall that in the first period, sellers with $k < q_s$ use the platform's service and sellers with $k \geq q_s$ use their own services. In the second period, it is more beneficial for the latter group of sellers to continue using their own services. It is also more beneficial for the former group to continue using the platform's service. Therefore, the "first best" total revenue is restored in the second period should the platform deviate, which means d_{10} cannot be supported as an equilibrium outcome. □

Proof of Proposition 5. For part 1, as discussed in the paper, the profit loss under d_{01} is smaller than d_{11} if and only if $L_{01} < 2L_{11}$. It remains to be checked that when $L_{01} < 2L_{11}$, the platform will not deviate from d_{01}, and when $L_{01} \geq 2L_{11}$, the platform will not deviate from d_{11}. For d_{01}, because choosing d_{01} would realize the "first best" total revenue in the second period for the platform, it cannot be beneficial for the platform to deviate to d_{00} even if the sellers choose their best responses accordingly. In other words, the platform will not deviate from d_{01}.

For d_{11}, suppose the platform deviates. Because all sellers are using the platform's service in the first period, the sellers' best responses in the second period would be the same as in the d_0 case in the one-period model. That is, sellers with $0 < k < q_0$ stay out of the market and sellers with $q_0 < k < \infty$ use their own services. The loss for the platform would be L_{01}. If the platform stays with d_{11}, the loss would be L_{11}. Therefore, the platform will not deviate from d_{11} if $L_{01} > L_{11}$, which holds because $L_{01} \geq 2L_{11}$.

For part 2, recall from Proposition 1 that when $F(q_c) < \frac{R(q_c)-R(q_C)}{R(q_0)+[R(q_c)-R(q_C)]}$, $L_0 < L_1$. Since $L_{01} = L_0$ and $L_{11} = L_1$, it follows that the same condition guarantees that $L_{01} < 2L_{11}$. □

Proof of Proposition 6. Noticing that $L_{01} = L_0$ and $L_{11} = L_1$, the proof is thus essentially the same as the one for Proposition 2. □

Proof of Proposition 7. To show that the platform does not gain from providing the service only in the second period when sellers are homogenous, we need to compute the platform's revenue under the four possible strategies for all possible capacity levels.

First, for $k < q_{00}$, it can be shown that only d_{01} and d_{11} can be supported as the equilibrium strategies. When d_{01} is chosen, the platform revenue is $R(k)$. When d_{11} is chosen, the platform revenue is $2R(k)$.

Second, for $q_{00} \leq k < q_0$, it can be shown that d_{00}, d_{01} and d_{11} can supported as the equilibrium strategies. When d_{00} is chosen, the platform revenue is $2R(k)$. When d_{01} is chosen, the platform revenue is $R(k)$. When d_{11} is chosen, the platform revenue is $2R(k)$.

Third, for $q_0 \leq k < q_C$, all four strategies can be supported as the equilibrium strategies and they all generate the same revenue, which equals $2R(k)$.

Fourth, for $q_C \leq k < q_c$, it can be shown that d_{00}, d_{01} and d_{10} can supported as the equilibrium strategies, and they all generate the same revenue, which equals $2R(k)$.

Finally, for $k \geq q_c$, it can be shown that d_{00}, d_{01} and d_{10} can supported as the equilibrium strategies, and they all generate the same revenue, which equals $2R(q_c)$.

Summarizing all cases, we observe that when $k \leq q_0$, the platform revenue is maximized when it chooses d_{11}; otherwise, the platform revenue is maximized when it chooses d_{00}. Hence, when sellers are homogenous, the platform does not gain from choosing d_{01}. □

Proof of Proposition 8. The proof of the proposition follows directly from the comparison of the profit losses in the three cases, as discussed in the paper. □

Proof of Proposition 9. The result follows directly from the discussion in the paper. □

Proof of Proposition 10. We first consider part 1 of the proposition. When $q_{11} \leq q_C$, it can be shown that only d_{01} and d_{11} can be supported as sub-game perfect equilibrium outcomes. Since there is no profit loss in d_{11} in either period, the platform would choose d_{11}.

When $q_{11} > q_C$, there are two possibilities: $q_{11} > q_C \geq q_s$ and $q_{11} > q_s > q_C$. When $q_{11} > q_C \geq q_s$, the analysis is the same as in Proposition 5, which gives the results in part 2 of the proposition.

When $q_{11} > q_s > q_C$, recall that $L_{00} = \alpha \int_0^{q_{00}} R(k)f(k)dk$, $L_{01} = \alpha \int_0^{q_0} R(k)f(k)dk$, $L_{01}^* = \alpha \int_{q_C}^{q_s} R(k)f(k)dk$, and $L_{11} = \alpha \int_{q_C}^{q_{11}} R(k)f(k)dk$, and note that

$$q_{00} < q_0 \Rightarrow L_{00} < L_{01}$$

$$q_s < q_{11} \Rightarrow L_{01}^* < L_{11}.$$

We now explore the conditions under which each of the platform's strategies can be supported as a sub-game perfect equilibrium outcome. Since the platform has only two choices in each period, and that sellers are infinitesimal, we only need to ensure that the platform will not deviate to the other choice in each period. Recall that the platform induces a loss of $2L_{00}$ with d_{00}, $L_{01} + L_{01}^*$ with d_{01} or d_{10}, and $2L_{11}$ with d_{11}.

For d_{00}, the platform will induce a loss of L_{00} in the second period. If the platform deviates to d_{01}, the loss will be L_{01}^*. Therefore, to support d_{00}, we need $L_{00} \leq L_{01}^*$. In the first period, if the platform deviates to d_1, it will choose either d_{11} or d_{10}. Note when $L_{00} \leq L_{01}^*$, we have $2L_{00} < 2L_{11}$ and $2L_{00} < L_{01} + L_{01}^*$ (since $L_{01}^* < L_{11}$). Therefore, the platform will not deviate to d_1 in the first period. In summary, the condition for d_{00} to be the sub-game perfect equilibrium outcome is $L_{00} \leq L_{01}^*$.

For d_{01}, the platform will induce a loss of L_{01}^* in the second period. If the platform deviates to d_{00}, the loss will be L_{01}. Therefore, to support d_{01}, we need $L_{01}^* \leq L_{01}$. In the first period, if the platform deviates to d_1, it will choose either d_{11} or d_{10}. Since d_{10} results in the same loss as d_{01}, the platform will not gain from deviation. To ensure the platform does not deviate to d_{11}, we need $L_{01} + L_{01}^* \leq 2L_{11}$. Therefore, the condition for d_{01} to be the sub-game perfect equilibrium outcome is $L_{01}^* \leq L_{01}$ and $L_{01}^* + L_{01} \leq 2L_{11}$.

For d_{10}, the platform will induce a loss of L_{01} in the second period. If the platform deviates to d_{11}, the loss will be L_{01}^*. Therefore, to support d_{10}, we need $L_{01} \leq L_{01}^*$. In the first period, if the platform deviates to d_0, it will choose either d_{00} or d_{01}. Since $2L_{00} < L_{01} + L_{01}^*$, the loss from d_{00} is smaller than d_{10}. In other words, if the platform prefers d_{00} to d_{01}, then d_{10} cannot become the equilibrium outcome. Now suppose the platform prefers d_{01} to d_{00}. This implies that $L_{01} \geq L_{01}^*$. Combining with the condition that $L_{01} \leq L_{01}^*$, we must have $L_{01} = L_{01}^*$. Because this is a zero probability event and d_{10} generates the same loss as d_{01}, it is dominated by d_{01}.

For d_{11}, the platform will induce a loss of L_{11} in the second period. If the platform deviates to d_{10}, the loss will be L_{01}. Therefore, to support d_{11}, we need $L_{11} \leq L_{01}$. In the first period, if the platform deviates to d_0, it will choose either d_{01} or d_{00}. Since d_{00} always induces a smaller total loss than d_{11}, for d_{11} to be the equilibrium outcome, the platform needs to prefer d_{01} to d_{00}, which requires $L_{01}^* \leq L_{01}$. In addition, we need $2L_{11} \leq L_{01} + L_{01}^*$ for the platform not to deviate to d_{01}. Note that these two conditions ensure $L_{11} \leq L_{01}$. In summary, the condition for d_{11} to be the sub-game perfect equilibrium outcome is $L_{01}^* \leq L_{01}$ and $2L_{11} \leq L_{01} + L_{01}^*$.

Given our discussion above, part (a) follows because when $L_{00} \leq L_{01}^*$, the platform will choose d_{00}. When $L_{00} > L_{01}^*$, we must have $L_{01} > L_{01}^*$ since $L_{00} < L_{01}$. Parts (b) and (c) therefore follow from the discussion above. $\square$