

Network Interconnectivity and Entry into Platform Markets

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Abstract

Digital technologies have led to the emergence of many platforms in our economy today. In certain platform networks, buyers in one market purchase services from providers in many other markets, whereas other platform networks may comprise local clusters in which buyers primarily purchase services from providers within the same cluster. Consequently, network interconnectivity varies across different industries. We examine how network interconnectivity affects interactions between an incumbent platform serving multiple markets and an entrant platform seeking to enter one of these markets. Our model yields several interesting results. First, even if the entrant can advertise at no cost, it still may not want to make every user in a local market aware of its service, as doing so may trigger a competitive response from the incumbent. Second, having more mobile buyers, which increases interconnectivity between markets, can reduce the incumbent's incentive to fight and, thus, increase the entrant's incentive to expand. Third, stronger interconnectivity between markets may or may not make the incumbent more defensible: when advertising is not costly and mobile buyers consume in both their local markets and the markets they visit, a large number of mobile buyers will increase the entrant's profitability, thereby making it difficult for the incumbent to deter entry. However, when advertising is costly or mobile buyers only consume in the markets they travel to, a large number of mobile buyers will help the incumbent deter entry. When advertising cost is at an intermediate level, the entrant prefers a market with moderate interconnectivity between markets. Fourth, we find that even if advanced targeting technologies can enable the entrant to also advertise to mobile buyers, the entrant may choose not to do so in order to avoid triggering the incumbent's competitive response. Finally, we find that the presence of network effects is likely to decrease the entrant's profit. Our results offer managerial implications for platform firms and help understand their performance heterogeneity.

Keywords: network interconnectivity, platform competition, market entry

1. INTRODUCTION

Digitalization has led to the emergence of numerous platforms in our economy today (Rochet and Tirole 2003, Iansiti and Levien 2004, Parker and Van Alstyne 2005). Examples of popular platforms include Uber in the transportation industry, Airbnb in the accommodation industry, Craigslist in the classifieds market, and Groupon in the local daily deals market. A growing body of literature on information systems attempts to understand the optimal strategies required for digital platforms to scale and compete. Scholars have examined a variety of issues, including optimal pricing (e.g., Parker and Van Alstyne 2005), interactions between competing platforms (e.g., Koh and Fichman 2014, Niculescu et al. 2018, Zhang et al. 2018), optimal business models (e.g., Parker et al. 2017, Tian et al. 2018, Chen et al. forthcoming), strategies to motivate third-party providers (e.g., Huang et al. forthcoming, Kuang et al. forthcoming), matching efficiency between buyers and sellers (e.g., Hong and Pavlou 2017, Xu forthcoming), platforms' investment decisions (e.g., Anderson et al. 2014), managing multigenerational platforms (e.g., Hann et al. 2016), and contractual relationships or tensions between platform owners and third-party providers (e.g., Huang et al. 2013, Hao et al. 2017, Li and Agarwal 2017).

Our study adds to this literature by examining how network characteristics affect the strategies and performance of competing platforms. All platforms exhibit two-sidedness in that they facilitate matching and transactions between consumers and service providers in their markets, but the interconnectivity of their businesses—which measures the degree to which consumers in one market purchase services from service providers in a different market—varies considerably across industries. For example, Airbnb's network exhibits high interconnectivity among different markets: travelers do not care about the number of hosts in their home cities; they care about the number of hosts in the cities they wish to visit. In contrast, Uber's network consists of local network clusters with some interconnectivity among them: riders transact with drivers in their own city and, except for frequent travelers, they care mostly about the local availability of Uber drivers. We observe similar local network clusters with some interconnectivity in group buying platforms such as Groupon, classifieds sites such as Craigslist, food delivery platforms such as Grubhub,

restaurant-reservation platforms such as OpenTable, and marketplaces that match freelance labor with local demand such as TaskRabbit, Instacart, and Rover.

The network interconnectivity of a platform market has important implications for the profitability and defensibility of incumbent platforms. When the network is strongly interconnected, it is difficult for a new entrant to compete, particularly when consumers in one local market only purchase services when they visit other markets. A platform that enters one city to compete with Airbnb, for example, would waste a significant amount of marketing resources to build awareness among local hosts and local residents without generating a large number of transactions, because local residents are only interested in transacting with hosts in cities they visit. Thus, a competitor would have to enter on a global scale and create brand awareness in many cities simultaneously in order to attract the critical mass of hosts and travelers necessary to build a liquid marketplace. Therefore, for a new platform, entry into highly interconnected markets is very costly. Many of Airbnb's potential competitors exited the market after exhausting their marketing budgets. In contrast, when consumers and service providers mostly transact within their local clusters, it is relatively easy for a new platform to enter, as it can specialize in one local cluster and build awareness from there. In the ride-sharing industry, many entrants have successfully challenged market leaders in local markets. For example, in New York City, Uber faces competitors such as Lyft, Juno, and Via. In China, the incumbent Didi faces competitors in many cities.

In this paper, we adopt a game-theoretical approach to examine how network interconnectivity affects competitive interactions between an incumbent platform and an entrant platform. The incumbent platform has an installed base of buyers and service providers in multiple local markets; the entrant is interested in entering one of these markets. To capture interconnectivity between local markets, we assume that some buyers are mobile: they travel between markets, purchasing services in each. In the first stage, the entrant invests money to build brand awareness in one of these markets. In the second stage, the incumbent and the entrant set prices for buyers and wages for service providers in that market. In the third stage, buyers and service providers in that market choose one platform on which to conduct transactions.

Our model yields several interesting results. First, even if the entrant can advertise at no cost, it still may not want to make every user in a local market aware of its service, as doing so may trigger a competitive response from the incumbent. Second, having more mobile buyers, which increases interconnectivity between markets, can reduce the incumbent's incentive to fight and, thus, increase the entrant's incentive to expand. Third, stronger interconnectivity across markets may or may not make the incumbent more defensible: when advertising is not costly and mobile buyers consume in both their local markets and the markets they visit, a large number of mobile buyers will increase the entrant's profitability, thereby making it difficult for the incumbent to deter entry; however, when advertising is costly or mobile buyers only consume in the markets they travel to, a large number of mobile buyers will help the incumbent deter entry. When advertising cost is at an intermediate level, the entrant prefers a market with moderate interconnectivity between markets. Fourth, we find that even if advanced targeting technologies can enable the entrant to also advertise to mobile buyers, the entrant may choose not to do so to avoid triggering the incumbent's competitive response. Finally, we find that the presence of network effects is likely to decrease the entrant's profit.

In the literature on platform strategies, our paper is closely related to the literature examining entry into platform markets. Studies have identified a number of factors that influence the success or failure of entrants in platform markets, such as the strength of network effects (e.g., Zhu and Iansiti 2012, Niculescu et al. 2018), platform quality (e.g., Liebowitz 2002, Tellis et al. 2009), multi-homing (e.g., Cennamo and Santalo 2013, Koh and Fichman 2014, Anderson et al. 2018), and exclusivity (e.g., Corts and Lederman 2009). All these studies assume a strongly interconnected network. As indicated in Afuah (2013), this assumption does not reflect the actual networks in most industries. Our study extends this literature by examining how network interconnectivity affects the strategies and performance of incumbents and entrants.

Broadly, our paper is related to competitive interactions between incumbents and entrants. Theoretical models in the literature focus on incumbent strategies such as capacity investment to deter or accommodate entry (e.g., Fudenberg and Tirole 1984, Tirole 1988). Empirical studies often find that incumbent reactions to entrants are selective (e.g., Geroski 1995): while some incumbents choose to react

to entrants aggressively, others do not appear to respond to entry. Studies have also shown that this variation in responses often depends on entrant characteristics, such as scale (e.g., Debruyne et al. 2002, Karakaya and Yannopoulos 2011). Our model finds support for these empirical results. Chen and Guo (2014) document a similar result regarding an entrant refraining from over-advertising to avoid competitive response from a competitor but in a very different setting. In their setting, a third-party seller sells the same product as a retail platform and the seller needs to decide how much to advertise through other channels, such as search engines and social media. Our paper differs from these studies by examining how network interconnectivity changes the incumbent's incentives to react, the entrant's incentives to advertise, and the entrant's profit. We show that buyers' consumption behavior matters: when mobile buyers consume in local markets, greater network interconnectivity sometimes increases entrant profits; however, when they do not, greater network interconnectivity reduces entrant profits.

Our paper is also related to studies that examine how network structures affect product diffusion (e.g., Abrahamson and Rosenkopf 1997, Suarez 2005, Sundararajan 2007, Tucker 2008). These studies typically focus on social networks, like instant messaging platforms, and examine questions related to issues such as seeding within these networks (e.g., Galeotti and Goyal 2009, Manshadi et al. 2018), pricing policies to facilitate product diffusion (e.g., Campbell 2013, Leduc et al. 2017), network formation processes and how local network clustering leads to local bias (e.g., Lee et al. 2016), prediction accuracy (e.g., Qiu et al. 2014), and market segmentation (e.g., Banerji and Dutta 2009). These networks have more complicated connections because they depend on individuals' own social networks and, consequently, these studies rely on simulations or descriptive results. We adopt a different perspective to focus on how interconnectivity between local markets affects market entry and derive closed-form solutions.

The remainder of the paper is organized as follows. In Section 2, we introduce the model and analyze the competitive interactions between an incumbent and an entrant. In Section 3, we examine extensions to our main models. In Section 4, we conclude by discussing the implications of our results and potential future research. All proofs are provided in a technical appendix.

2. THE MODEL

2.1 Model setup

Assume that there are multiple local markets each with N buyers who are currently using the incumbent's platform (denoted as I) for transactions. A fraction of buyers in each market are mobile— r percent of them travel between markets. Assume the movement is random, so that in equilibrium, in each market, rN buyers visit other markets and rN additional buyers come from other markets to make purchases. Hence, r measures the interconnectivity between these markets. Each mobile buyer places one order for the service in his local market and another order when he travels. For example, riders use ride-sharing services in their local markets; when they travel, they use ride-sharing services in other markets.¹ Each service provider fulfills one order at most. To accommodate these mobile buyers, each market has $(1 + r)N$ service providers. Table 1 provides a summary of the notations used in the main model.

Before an entrant (denoted as E) enters one of these markets, the incumbent serves the market as a monopoly and all the users (i.e., both service providers and buyers) are aware of the incumbent.² Neither the buyers nor the service providers are aware of the entrant, but the entrant can advertise to build awareness.

The game proceeds as follows, as depicted in Figure 1. In the first stage, the entrant invests to build brand awareness among users in the local market. Advertising is costly, and it costs the entrant $L(n)$ to reach n potential users. The entrant decides on θ , a fraction of the potential users reached through advertising. Because we have N buyers and $(1 + r)N$ service providers, $n = \theta(2N + rN)$. Following the literature (Thompson and Teng 1984, Tirole 1988, Esteves and Resende 2016, Jiang and Srinivasan 2016), we assume the advertising cost is a (weakly) increasing and convex function of n : $L'(n) \geq 0$ and $L''(n) \geq 0$. Note that even with digital technologies, it remains costly to build awareness. While certain platforms may be able to attract their first tranche of customers relatively inexpensively, through word-of-mouth or

¹ We consider the scenario in which mobile buyers do not consume in their local markets in an extension.

² This assumption is relaxed in an extension of the model in which not all buyers and sellers are aware of the incumbent.

other low-cost strategies, the cost typically begins escalating when the platform begins to look for new and somewhat different customers through search advertising, referral fees, and other marketing strategies.³ Consequently, many platforms exit the market after burning too much money on customer acquisition. In our model, we allow advertising cost to vary and examine its implications on platform strategies and performance. In the main model, we also assume that the entrant is not able to advertise to mobile buyers. We relax this assumption in an extension.

In the second stage, the incumbent sets the price to each buyer, denoted as p_I , and the wage to each service provider, denoted as w_I , in the local market. The entrant also sets the price for the service buyers, denoted as p_E , and the wage for the service providers, denoted as w_E . Here, the subscript denotes the platform (I for incumbent and E for entrant). For example, Instacart, decides the prices for users and the wages for shoppers. Uber decides the rates for riders and the commissions it takes before passing on the revenue from riders to drivers, which effectively determines the wages for drivers. Consistent with the practice, we allow firms to set different prices and wages in different markets, but they do not price discriminate based on whether a buyer is local or mobile within a market. We denote each buyer's willingness to pay for the service as v . Further, we normalize the value of outside options to zero and the service providers' marginal cost to zero. Hence, without the entrant, as a monopoly, the incumbent will choose $p_I = v$ and $w_I = 0$.

In the third stage, the rN mobile buyers from other markets arrive. Buyers and service providers choose one platform on which to conduct transactions. Mobile buyers are not exposed to the entrant's advertisements and are, therefore, only aware of the incumbent. Hence, the entrant and the incumbent compete for buyers and service providers from the local market, but the mobile buyers will only use the incumbent platform.

³ See, for example, "Unsustainable customer acquisition costs make much of ecommerce profit proof," Steve Dennis, *Forbes*, August 31, 2017.

The $(1 - \theta)$ portion of users in the local market is only aware of the incumbent and will buy or provide the service on the incumbent platform as long as they receive a non-negative utility from the incumbent. Specifically, a buyer will buy the service as long as $p_I \leq v$, and a service provider will provide the service as long as $w_I \geq 0$. Because $p_I \leq v$ and $w_I \geq 0$ always hold, these users will always use the incumbent's platform.

The θ portion of users in the local market becomes aware of both the incumbent and the entrant and will remain with the incumbent's platform unless the entrant provides a higher utility. If a user elects to switch to the entrant's platform, there is a switching cost that varies across users. We denote this cost for a service provider, i , as c_i and for a buyer, j , as a_j . Similar to Ruiz-Aliseda (2016), we assume that both c_i and a_j follow a uniform distribution between 0 and m , where m captures the difficulty in switching to a new service in the market. To be consistent with real world scenarios, we assume that m is sufficiently large (i.e., there are some users whose switching cost is sufficiently large) so that, in equilibrium, the entrant will not take away the entire segment of users who are aware of both platforms.⁴

Among the θ portion of service providers, a service provider, i , will choose the entrant if the utility from using the entrant's platform ($U_{Ei}^S = w_E - c_i$) is greater than the utility from using the incumbent's platform ($U_{Ii}^S = w_I$). Here, the subscript again denotes the platform (I for incumbent and E for entrant) and the superscript denotes the user (S for service provider and B for buyer). The solution to the equation $U_{Ei}^S = U_{Ii}^S$ is $c^* = w_E - w_I$, describing the switching cost of the indifferent service provider. Thus, service providers with $c_i < c^*$ will choose the entrant and those with $c_i \geq c^*$ will choose the incumbent. Let N_I^S denote the number of service providers selecting the incumbent and N_E^S denote the number of service providers selecting the entrant. Then, we have the following two equations:

⁴ Mathematically, this assumption requires that the distribution of the switching cost be sufficiently sparse—that is, $m > \frac{9(1+r)v}{16(2+r)}$.

$$N_I^S = \left(1 - \frac{c^*}{m}\theta\right)(1+r)N \quad (1)$$

$$N_E^S = \frac{c^*}{m}\theta(1+r)N. \quad (2)$$

Similarly, a buyer, j , will choose the entrant if the utility from using the entrant's platform ($U_{Ej}^B = v - p_E - a_j$) is greater than the utility from using the incumbent's platform ($U_{Ij}^B = v - p_I$). The solution to the equation $U_{Ej}^B = U_{Ij}^B$ is $a^* = p_I - p_E$. Thus, buyers with $a_j < a^*$ will choose the entrant and those with $a_j \geq a^*$ will choose the incumbent. Let N_I^B denote the number of service buyers selecting the incumbent and N_E^B denote the number of service buyers selecting the entrant. We obtain the following two equations:

$$N_I^B = \left(1 - \frac{a^*}{m}\theta + r\right)N. \quad (3)$$

$$N_E^B = \frac{a^*}{m}\theta N. \quad (4)$$

We can then derive the incumbent's profit, π_I , and the entrant's profit, π_E , from the local market as follows:

$$\pi_I = \min(N_I^S, N_I^B)(p_I - w_I). \quad (5)$$

$$\pi_E = \min(N_E^S, N_E^B)(p_E - w_E) - L(\theta(2N + rN)). \quad (6)$$

It is possible that under some prices and wages of the two platforms, the number of buyers is not the same as the number of service providers. In such cases, either some buyers' orders are not fulfilled, or some service providers will not serve any buyers and hence earn no income.

2.2 Equilibrium analysis

We use backward induction to derive the equilibrium. Specifically, we first derive each platform's optimal price and profit given the entrant's advertising decision and then solve for the entrant's optimal advertising decision in the first stage.

To derive each platform's optimal price given the entrant's advertising decision, we recognize that it is often difficult to derive closed-form equilibrium solutions when we allow two competing platforms to set prices on both sides, especially when the platforms are heterogeneous. Prior studies have often had to make simplifying assumptions, such as a fixed price (or royalty rate) on one-side of the market, symmetric pricing, or one platform being an open source platform and, thus, free (e.g., Rochet and Tirole 2003, Economides and Katsamakas 2006, Casadesus-Masanell and Halaburda 2014, Adner et al. forthcoming). In this study, we take advantage of a market clearing condition to derive the optimal prices and wages for the two competing platforms. Specifically, we prove that, in equilibrium, the incumbent and the entrant will always choose their prices and wages so that the number of service providers using a platform equals the number of buyers using the same platform: $N_I^S = N_I^B$ and $N_E^S = N_E^B$. Lemma 1 states this result (proofs of all lemmas and propositions are provided in the appendix).

Lemma 1. The incumbent and the entrant will set their prices and wages so that the number of service providers using a platform equals the number of buyers using the same platform.

The intuition for Lemma 1 is that if the numbers on the two sides are not balanced, a firm can adjust its price or wage to get rid of excess supply or demand to increase its profitability. The lemma suggests that $a^* = (1 + r)c^*$. Hence, $(p_I - p_E) = (1 + r)(w_E - w_I)$. Thus, we can rewrite the profit functions as follows:

$$\pi_I = \left(1 - \frac{p_I - p_E}{m} \theta + r\right) N \left(p_I + \frac{p_I - p_E}{1+r} - w_E\right). \quad (7)$$

$$\pi_E = \frac{p_I - p_E}{m} \theta N \left(p_E - \frac{p_I - p_E}{1+r} - w_I \right) - L(\theta(2N + rN)). \quad (8)$$

We can then derive each platform's optimal price and profit, given the entrant's advertising decision, as shown in Proposition 1.

Proposition 1. *Given the entrant's choice of advertising intensity θ , the optimal prices, number of buyers and service providers, and platform profits can be determined as follows:*

(i) *If $0 \leq \theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$, then $p_I^* = v$, $w_I^* = 0$, $p_E^* = \frac{(3+r)v}{2(2+r)}$, $w_E^* = \frac{v}{2(2+r)}$, $N_I^{B*} = N_I^{S*} = \frac{N(1+r)}{2} \left(2 - \frac{\theta v}{m(2+r)}\right)$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)\theta v}{2m(2+r)}$, $\pi_I^*(\theta) = \frac{N(1+r)v}{2} \left(2 - \frac{\theta v}{m(2+r)}\right)$, and $\pi_E^*(\theta) = \frac{N(1+r)\theta v^2}{4m(2+r)} - L(\theta(2N + rN))$.*

(ii) *If $\min\left(\frac{2m(2+r)}{3v}, 1\right) < \theta \leq 1$, then $p_I^* = \frac{2(2+r)m}{3\theta}$, $w_I^* = 0$, $p_E^* = \frac{(3+r)m}{3\theta}$, $w_E^* = \frac{m}{3\theta}$, $N_I^{S*} = \frac{2N(1+r)}{3}$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)}{3}$, $\pi_I^*(\theta) = \frac{4Nm(1+r)(2+r)}{9\theta}$, and $\pi_E^*(\theta) = \frac{Nm(1+r)(2+r)}{9\theta} - L(\theta(2N + rN))$.*

When θ is smaller than a certain threshold $\left(\min\left(\frac{2m(2+r)}{3v}, 1\right)\right)$, we find that the incumbent platform chooses not to respond to the entrant. It continues to charge the monopoly price, v , and offer the monopoly wage, 0, although its profit does decrease as θ increases because it loses market share to the entrant. The entrant platform incentivizes some buyers and service providers to switch by charging a lower price and offering a higher wage.

The threshold for θ (weakly) increases with r because mobile buyers are only aware of the incumbent platform (i.e., the incumbent platform has monopoly power over them) and their existence reduces the incumbent's incentive to respond to the entrant. It is thus not surprising that the entrant can take advantage of this lack of incentive and increase its advertising intensity. The number of transactions hosted on the incumbent platform increases with r because of mobile buyers from other markets, even though the

incumbent loses more transactions from local buyers to the entrant when r increases. The incumbent platform's profit increases with r because of the increase in transactions at the same monopoly price it charges. The number of transactions the entrant serves also increases with r because it can advertise more aggressively without triggering a competitive response from the incumbent. The entrant's profit increases with r without taking the advertising cost into account. If advertising cost increases significantly with r , the entrant's profit may decrease with r , a scenario which will be examined later.

When θ is larger than the threshold, however, the entrant platform has the potential to steal a large market share from the incumbent. The incumbent platform chooses to respond by lowering its price to buyers. The entrant platform thus lowers its price to buyers as well. Note that the wages offered by the entrant in this case decrease with θ . This is because even though advertising reaches many service providers, there is no demand for all the service providers due to the competitive response from the incumbent on the buyer side, thereby allowing the entrant to offer lower wages.

We again find that because mobile buyers reduce the incumbent's incentive to fight, both the incumbent and the entrant can charge (weakly) higher prices to buyers while maintaining the same wages as r increases. They both have more transactions when r increases. The incumbent's profit increases with r , while the entrant's profit increases with r when its advertising cost does not increase too much with r .

Note that when θ is larger than the threshold, as θ increases, the profits of both platforms decrease due to intense competition even without considering advertising cost. Thus, we expect the entrant's optimal choice of θ to be no more than the threshold $\left(\min\left(\frac{2m(2+r)}{3v}, 1\right)\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response.

Corollary 1. The entrant's optimal choice of advertising intensity θ always satisfies $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$.

The exact optimal level of θ for the entrant depends on the cost of advertising, $L(n) = L(\theta(2N + rN))$. Following the literature (e.g., Thompson and Teng 1984, Tirole 1988, Esteves and Resende 2016, Jiang and Srinivasan 2016), we assume a quadratic cost function, $L(n) = kn^2$, where $k \geq 0$. A large k suggests that advertising is costly, while a small k suggests that it is inexpensive.⁵

[Insert Figure 2 here]

Figure 2 illustrates how the entrant's profit changes with the choice of θ for different values of k . We notice that for a given level of k , the entrant's profit increases and then decreases with θ . Even if advertising has no cost (i.e., $k = 0$), there is an optimal advertising level for the entrant. As k increases (i.e., advertising becomes more expensive), the optimal advertising intensity, θ^* , decreases. However, the incumbent's profit always decreases with θ and is independent of k . The following proposition formalizes the relationship between the optimal advertising intensity, θ^* , and the value of k .

Proposition 2. *The optimal advertising intensity, θ^* , depends on the value of k .*

- (i) *If $k \geq \max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, then $\theta^* = \frac{(1+r)v^2}{8(2+r)^3kNm}$, which decreases with r . The entrant's profit is $\frac{(1+r)^2v^4}{64km^2(2+r)^4}$ and the incumbent's profit is $\frac{N(1+r)v}{2}\left(2 - \frac{(1+r)v^3}{8kNm^2(2+r)^4}\right)$.*
- (ii) *If $0 \leq k < \max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, then $\theta^* = \min\left(\frac{2m(2+r)}{3v}, 1\right)$, which weakly increases r . When $\frac{2m(2+r)}{3v} < 1$, the entrant's profit is $\frac{N(1+r)v}{6} - \frac{4kN^2m^2(2+r)^4}{9v^2}$ and the incumbent's profit is $\frac{2Nv(1+r)}{3}$. When $\frac{2m(2+r)}{3v} \geq 1$, the entrant's profit is $\frac{N(1+r)v^2 - 4kN^2m(2+r)^3}{4m(2+r)}$ and the incumbent's profit is $\frac{N(1+r)v}{2}\left(2 - \frac{v}{m(2+r)}\right)$.*

⁵ If there is no cost for the entrant to reach its fans, we can modify the cost function to be $L(n) = k(\max(n-z, 0))^2$, where z is the total number of fans. Our results hold qualitatively.

We have two cases. When k is large, advertising is costly. In this case, the optimal advertising intensity $\theta^* \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$. The entrant and the incumbent have no strategic interactions with each other and the entrant's optimal advertising intensity is determined by the marginal benefits and marginal cost from reaching another user. Consequently, the entrant's equilibrium profit is independent of the market size, N . This result also highlights the impact of network interconnectivity, independent of the market size.

When k is small, advertising is inexpensive, and the entrant platform, thus, has an incentive to increase advertising intensity θ . The entrant's profit increases with θ until $\theta = \min\left(\frac{2m(2+r)}{3v}, 1\right)$. When $\frac{2m(2+r)}{3v} \geq 1$, the entrant will advertise to everyone in the market. Otherwise, the entrant's profit first increases as θ increases up to $\frac{2m(2+r)}{3v}$ and, then—because of the competitive response from the incumbent discussed in Corollary 1—decreases with θ afterwards. Thus, the entrant will choose $\theta^* = \frac{2m(2+r)}{3v}$. Note that the entrant's optimal choice of θ is independent of market size N but increases with r , which again highlights that market size and network interconnectivity affect equilibrium outcomes differently.

The discussion above leads to the following corollary:

Corollary 2. *Even if the advertising cost is zero (i.e., $L(\mathbf{n}) = 0$ or $k = 0$), the entrant will not necessarily advertise to the entire market but instead choose the optimal advertising intensity $\theta^* = \frac{2m(2+r)}{3v}$ when $\frac{2m(2+r)}{3v} < 1$.*

We then examine how the fraction of mobile buyers, r , affects the optimal θ and the platforms' profits in the two cases in Proposition 2. Figure 3 illustrates the relationships under different values of k . When k is large (in Proposition 2i), advertising is costly. As r increases, the number of service providers, $(1+r)N$, increases in the market, but the number of buyers accessible to the entrant remains the same. Thus, the likelihood that advertising is wasted on some service providers without matched buyers also increases. With a large k , it is optimal for the entrant to reduce θ^* to reduce its advertising cost,

$L(\theta^*(2N + rN))$, even if a large r reduces the incumbent's incentive to respond. This explains the declining curve in Figure 3a for a large k (e.g., $k = 0.0004$). Because the entrant advertises to fewer buyers, the entrant's profit also decreases with r , as depicted in Figure 3b for $k = 0.0004$.

[Insert Figure 3 here]

In contrast, when k is small (in Proposition 2ii), θ^* (weakly) increases with r . This is because when advertising is inexpensive, the advertising wasted on unmatched service providers becomes a less significant issue and the entrant wants to take advantage of the incumbent's disincentive to respond instead. Thus, we observe an increasing curve of θ^* in Figure 3a for a small k (e.g., $k = 0$). The impact of r on the entrant's profit is also positive as long as k is sufficiently small.⁶ This result is consistent with Proposition 1, where we have shown that if the advertising cost is small for the entrant, the entrant's profit will increase with r regardless of θ . When we use k to capture the cost of advertising, as long as k is sufficiently small (e.g., $k = 0$ in Figure 3b), the entrant's profit increases with r . The result shows that when the incumbent has more captive buyers and, therefore, less incentive to fight, the entrant could be more profitable when advertising is not costly.

Note that the threshold of k , $\left(\max\left(\frac{3v^3}{32Nm^2(2+r)^3}, \frac{v^2}{8mN(2+r)^3}\right)\right)$, below which the entrant's profit increases with r , is a decreasing function of r . Therefore, an intermediate value of k may begin from below the threshold when r is small, but then exceed the threshold as r increases. This implies that the equilibrium may switch between the two cases (where k falls above or below the threshold) as r changes. This explains why we may observe a non-monotonic relationship between the optimal advertising intensity (θ^*) and r and the same for the relationship between the entrant's profit and r , as shown by the case of $k = 0.0002$ in Figures 3a and 3b.

⁶ When $\frac{2m(2+r)}{3v} < 1$, $\theta^* = \frac{2m(2+r)}{3v}$ and the entrant's profit increases with r , as long as $k < \frac{3v^3}{32Nm^2(2+r)^3}$. When $\frac{2m(2+r)}{3v} \geq 1$, $\theta^* = 1$ and the entrant's profit increases with r , as long as $k < \frac{v^2}{8mN(2+r)^3}$.

Regardless of the value of k , the incumbent's profit always increases with r , because it has more captive buyers when r is larger (as illustrated in Figure 3c).

The two corollaries below summarize the relationship between the entrant's advertising intensity and the fraction of mobile buyers (Corollary 3) as well as the relationship between the platform profit and the fraction of mobile buyers (Corollary 4):

Corollary 3. When k is small, as the fraction of mobile buyers, r , increases, the entrant has incentive to advertise more (higher θ^) until it reaches the entire market. Conversely, when k is large, as r increases, the entrant has incentive to reduce advertising (lower θ^*). For intermediate values of k , as r increases from zero, the optimal θ^* increases with r first and then decreases with r .*

Corollary 4. The incumbent's profit always increases with the fraction of mobile buyers, r . How the fraction of mobile buyers, r , affects the entrant's profit depends on the value of k . When k is small, the entrant's profit increases with r , and conversely, when k is large, the entrant's profit decreases with r . For intermediate values of k , as r increases from zero, the entrant's profit increases with r first and then decreases with r .

The results suggest that under certain circumstances (e.g., when advertising is cheap), network interconnectivity, which is supposed to benefit the incumbent that operates in multiple interconnected markets may, in fact, encourage the entrant entering into a single market to expand and increase entrant profit. In this case, a higher network interconnectivity will make it even more difficult for an incumbent to deter entry. These results also have important implications for the entrant regarding the choice of market to enter when the fraction of incoming mobile buyers varies across markets; we will examine this further in the next section.

3. EXTENSIONS

3.1 Heterogeneous markets

In our main analysis, we assume that all markets are homogenous. Consequently, the entrant could begin by entering any one of these markets. If these markets have different fractions of mobile buyers visiting from other markets, assuming the entrant only has the budget to enter one market only, which market should the entrant choose to enter?

Suppose there are H markets. Let r_h be the fraction of mobile buyers coming into market h ($h=1, 2, \dots, H$). Hence, market h has N local buyers, $(1 + r_h)N$ service providers, and $r_h N$ incoming mobile buyers. We obtain the following proposition:

Proposition 3. When k is small, the entrant should choose the market with the highest fraction of mobile buyers from other markets, r , to enter; when k is large, the entrant should choose the market with the lowest fraction of mobile buyers from other markets, r , to enter. For an intermediate value of k , the entrant may choose a market where r is also intermediate.⁷

The result echoes Corollary 4, where we find that the entrant's profit increases with r when k is small, decreases with r when k is large, and has a non-monotonic relationship with r for an intermediate value of k . While corollary 4 focuses on how the entrant's profit changes when r in a local market increases, this proposition extends our finding to how the entrant should choose a market among the markets that vary in the fraction of incoming mobile buyers. The proposition suggests that when the fraction of visitors is high, the incumbent is less likely to fight the entrant. At the same time, however, a high fraction of visitors means that a large fraction of the entrant's advertising expenditure will be wasted on unmatched service providers. Hence, the entrant will find such a market attractive when advertising is not costly. For example,

⁷ The thresholds for k are provided in the appendix. We also explored heterogeneous market sizes. As indicated by Proposition 2, when k is sufficiently large, market sizes would not affect the entrant's profit. When k is small, if market size is sufficiently large for a certain market, the fraction of mobile buyers (r) will have a negligible effect and, thus, the entrant should simply choose the largest market to enter; otherwise, the entrant's choice will depend on both r and N .

if Google wants to offer ride-sharing services because it already has a larger number of users from its current services and can build awareness at a low cost (k is small), Google should start offering these services in large cities with a large fraction of visitors. However, a new startup, for which advertising is rather costly, should target small cities with a small fraction of visitors in order to improve advertising efficiency.

3.2 The incumbent does not own the entire market

In our main model, we also assume that the incumbent owns the entire market (i.e., all potential buyers and service providers are aware of the incumbent) before the entrant emerges. In reality, it is possible that not every user in the local market is aware of the incumbent. Thus, it is possible for the entrant to attract users who are not aware of the incumbent. We consider this possibility in this extension. Assume the incumbent's market share before the entrant arrives is s , where $0 < s < 1$. We then have the following proposition:

Proposition 4. *The results from our main model are qualitatively the same when $s \geq \frac{m(2+r)}{2m+mr+v+rv}$. If $s < \frac{m(2+r)}{2m+mr+v+rv}$, both platforms charge buyers $p_I^* = p_E^* = v$ and offer service providers $w_I^* = w_E^* =$*

0.

The results from the main model remain qualitatively the same as long as s is sufficiently large. But when s is below a certain threshold, the results differ from our main results. When the incumbent has a small share of the market, the entrant and the incumbent can effectively avoid direct competition by targeting different segments of that market. Hence, both will charge monopoly prices and offer monopoly wages, and no buyers and service providers will switch from the incumbent to the entrant.

3.3 Mobile buyers only consume when they travel

In our main model, mobile buyers purchase services in both their local markets and the markets they visit. This assumption fits with markets such as those in the ride-sharing industry, where riders hail cars in their

own markets and also in other markets when they travel, or daily local deal markets, where consumers buy deals in their own markets and also in other markets when they travel. The assumption, however, may not hold for markets like the accommodation market, where buyers typically consume only when they travel. In this extension, we examine the scenario in which mobile buyers do not consume in their local markets. We obtain the following result under this assumption:

Proposition 5. The results from the main model are qualitatively the same when mobile buyers do not consume in their local markets, except that the entrant's profit under the optimal θ always decreases with r .

When mobile buyers do not consume in their local market, a local market with a larger fraction of mobile buyers will have fewer potential buyers for the entrant. Although the entrant can continue to take advantage of the incumbent's disincentive to fight and advertise more aggressively, its demand decreases and, hence, its profit decreases with r . This result explains why it is more difficult to challenge an incumbent platform like Airbnb compared to one like Uber.

3.4 The entrant can target mobile buyers

In the main model, we assume that the entrant is not able to advertise to mobile buyers. We make this assumption because mobile buyers often stay in the market they visit briefly. Even if the entrant is continuously advertising in that market, without sufficient exposure to its advertisement, a mobile buyer may not consider the entrant's product. We now relax this assumption and assume that advanced targeting technologies can help the entrant identify mobile buyers and can advertise to them effectively. Let θ and θ_t be the fractions of local and mobile users, respectively, that become aware of the entrant's platform after the entrant's advertising. The demand on the service provider side remains the same as in Equations (1) and (2). The demand on the buyer side becomes

$$N_I^B = \left(1 + r - \frac{a^*}{m} (\theta + \theta_t r) \right) N, \quad (9)$$

$$N_E^B = \frac{a^*}{m} (\theta + \theta_t r) N. \quad (10)$$

In this case, we find that the entrant may not want to advertise to the mobile buyers even if there is no cost of advertising, as summarized by the following proposition.

Proposition 6. *Even if the cost of advertising is zero (i.e., $L(\mathbf{n}) = 0$), the entrant will not necessarily choose to advertise to mobile buyers even if it is able to, that is, $\theta_t^* = 0$ if $\frac{2m(2+r)}{3v} \leq 1$.*

As we show in the main analysis, mobile buyers help deter the incumbent from fighting with the entrant. Hence, the entrant may not want to steal the mobile buyers from the incumbent even when it can target them and advertise to them at zero cost. The entrant is more likely to avoid advertising to the mobile segment when the value of these buyers to the incumbent is high (large v), the mobile segment is not large so the entrant does not lose a huge number of potential buyers (small r), and a small amount of advertising can steal a large number of mobile buyers away from the incumbent and thus trigger its response (small m). When the advertising cost for mobile buyers is higher than the cost for local users, the entrant will be even less likely to advertise to mobile buyers. The only situation in which the entrant will advertise to mobile buyers is when the entrant has advertised to all local buyers and has not triggered competitive responses from the incumbent, which is rarely observed in practice. Thus, Proposition 6 helps justify the assumption in our main model that mobile buyers are only aware of the incumbent.

3.5 The presence of network effects

In our main model, we focus on matching between the buyers and service providers. Similar to other matching models (e.g., Zhang et al. 2018), we do not model network effects. This approach enables us to

separate the network-interconnectivity effect from the network effects, but network effects may have an impact on matching quality or speed. For example, in the case of ride-sharing services, a large number of drivers on a platform can reduce the wait time for riders. Similarly, a large number of riders reduces the idle time for drivers. In the accommodation market, a large number of hosts and travelers on a platform increase the likelihood that each traveler and each host is matched with a party close to his or her personal preference. To capture such benefits, we add a utility component to capture the network effects in the buyers' and service providers' utility functions and allow this utility component to increase with the number of users on the other side of the same platform:

$$U_I^B = eN_I^S + v - p_I. \quad (11)$$

$$U_E^B = eN_E^S + v - p_E - a_i. \quad (12)$$

$$U_I^S = eN_I^B + w_I. \quad (13)$$

$$U_E^S = eN_E^B + w_E - c_i. \quad (14)$$

Here, we use parameter e ($e \geq 0$) to capture the strength of network effects. We first consider the case where e is small and both platform firms can co-exist. To avoid multiple equilibria due to network effects, we assume e to be much smaller than the value of the transaction itself.⁸ This assumption is reasonable because in such markets most benefits to buyers or service providers come from the transaction itself. We find our main results to be qualitatively unchanged, as summarized in the following proposition:

Proposition 7. The results from the main model are qualitatively the same in the presence of network effects when the strength of network effects is small.

We also examine how the strength of network effects affects the profits of both the entrant and incumbent. Given the computational complexity, we explore this effect as the strength of network effects,

⁸ Mathematically, we require $e < \min\left(\frac{v}{2N}, \frac{m}{4N}\right)$.

e , approaches 0. We find that as long as m is sufficiently large (e.g., $m > v$), because the incumbent has a larger market share, network effects make the incumbent more attractive to users, thereby reducing users' tendencies to switch to the entrant. Hence, as the network effects become stronger, the entrant's profit decreases and the incumbent's profit increases.

When e is sufficiently large, we find that the equilibrium in which the entrant has positive demand cannot be sustained and the incumbent becomes the monopoly. This result is expected because when network effects dominate pricing effects, if an entrant enters the market, the incumbent always has the incentive and is able to take advantage of its installed base advantage to drive the entrant out of the market.

Proposition 8. When network effects become sufficiently large, the incumbent can deter the entrant from entering the market and thereby monopolize the market.

3.6 Heterogeneous switching costs

In our model, we assume that buyers and service providers face the same switching costs. In practice, however, their switching costs may differ. For example, in the ride-sharing industry, riders only need to download a new app to switch to a different platform, while drivers may have to undergo background checks and verification processes to switch to a different platform. In order to investigate how heterogeneity in switching costs affects the platforms, we allow buyers' switching cost to be uniformly distributed between 0 and m_b and service providers' switching costs to be uniformly distributed between 0 and m_s . Then, we obtain the following proposition:

Proposition 9. When we allow buyers' switching cost to be uniformly distributed between 0 and m_b and service providers' switching costs to be uniformly distributed between 0 and m_s , we have $\frac{\partial \pi_E^*}{\partial m_b} < \frac{\partial \pi_E^*}{\partial m_s} <$

0 and $\frac{\partial \pi_I^*}{\partial m_b} \geq \frac{\partial \pi_I^*}{\partial m_s} \geq 0$.

Proposition 9 suggests that an increase in switching costs on the buyer side harms the entrant or benefits the incumbent more than the same increase on the service provider side. The intuition is that because of the existence of mobile buyers, we have more service providers than local buyers. Hence, the total number of transactions that the entrant platform serves depends largely on the number of buyers the entrant can incentivize to switch to the entrant platform. Thus, buyers' switching cost affects firm profits more than that of service providers. Note that when the optimal θ , θ^* , reaches the threshold that is just high enough to not trigger the incumbent's response, the incumbent's profit is independent of m_b and m_s . This explains why sometimes $\frac{\partial \pi_I^*}{\partial m_b} = \frac{\partial \pi_I^*}{\partial m_s} = 0$.

4. DISCUSSION AND CONCLUSION

Extant studies in the platform strategy literature typically assume that each participant on one side of a market is (potentially) connected to every participant on the other side of the market. Our paper departs from this assumption to explore the impact of network interconnectivity on the defensibility of an incumbent with presence in multiple markets against an entrant that seeks to enter one of these markets.

As depicted in Figure 4, our model captures heterogeneous network interconnectivity across different industries, ranging from isolated network clusters ($r = 0$) to a strongly connected network ($r = 1$). When we have isolated local clusters (i.e., no mobile buyers), as our results show, an incumbent has low profitability. Examples of such a network include Handy, a marketplace for handyman services, and Instacart, a platform that matches consumers with personal grocery shoppers. In such markets, consumers only buy services in their local markets and do not typically use such services when they travel. At the other end of the spectrum, we have strongly connected networks. This is the case for Airbnb, a platform on which travelers transact with any hosts outside their local clusters, and Upwork, an online outsourcing marketplace, where any clients and freelancers can initiate projects. Between the two extreme scenarios, we have networks that consist of local clusters with some interconnectivity. In the case of Uber, Grubhub, and

Groupon, consumers primarily use their services in their local clusters but also use such services when they travel.

[Insert Figure 4 here]

We find that the greater the interconnectivity, the lower the incumbent's incentive to respond and, hence, the stronger the entrant's incentive to reach more users in a local market. While we find that the incumbent's profit always increases with interconnectivity, the entrant's profit does not always decrease with interconnectivity. When advertising is inexpensive and mobile buyers consume in both their local markets and the markets they travel to, the high interconnectivity between markets also increases the entrant's profit, thereby making it difficult for the incumbent to deter entry; on the other hand, when advertising is costly and/or mobile buyers only consume in the markets they travel to, high interconnectivity reduces the entrant's profit, thereby helping the incumbent deter entry. When the advertising cost is at an intermediate level, the entrant is more likely to survive under moderate interconnectivity between markets. We also extend our model to examine situations where markets are heterogenous, the entrant is able to target mobile buyers, buyers and service providers have different switching costs, and network effects are present. Overall, these results help explain barriers to entry in platform markets and the resulting performance heterogeneity among platform firms in different markets.

These results corroborate empirical observations of many platform markets. For example, we show that it is optimal for an entrant not to trigger incumbent responses. The founders of Fasten, an entrant into the ride-sharing market in Boston, were very clear from the beginning that they did not want to trigger Uber's response by strategically minimizing their advertising activities.⁹ Fasten also chose not to target visitors in Boston: it did not advertise in Boston's Logan Airport or in its South Station Bus Terminals. Indeed, although Fasten grew rapidly in Boston during the period 2015–2017, Uber and Lyft did not change their prices or wages to compete. As a counterexample, when Meituan—a major player in China's online-to-offline services such as food delivery, movie ticketing, and travel bookings—entered the ride-sharing

⁹ Based on the authors' interviews with the founders.

business, it was able to build awareness of its service at almost no cost through its existing app, which had an extensive user base. Meituan's entry into the Shanghai ride-sharing market triggered strong responses from the incumbent, Didi, thereby leading to a subsidy war between the two companies. Meituan subsequently decided to halt ride-sharing expansion in China.

Our results also suggest that Airbnb's and Booking.com's business models are more defensible than those of Uber because most of their customers are travelers and do not typically use the service in their local markets, while Uber's consumers primarily use its services in their local markets. The difference in defensibility is a key aspect for why Airbnb and Booking.com were able to achieve profitability, while Uber has been hemorrhaging money.¹⁰

Our study offers important managerial implications for platform owners. We find that an incumbent's profit increases with interconnectivity regardless of whether or not mobile buyers consume in local markets; thus, incumbent platforms should seek to build strong interconnectivity in their networks. In our model, the level of interconnectivity is given exogenously; however, in practice, how firms design their platforms can influence interconnectivity. For example, while Craigslist is a local classifieds service, its housing and job services attract users from other markets. Our research suggests that such services are important sources of Craigslist's sustainability and, thus, Craigslist should strategically devote more resources to grow these services. As another example, many social networking platforms such as Facebook and WeChat allow companies or influencers to create public accounts that any user can connect with. Such moves increase interconnectivity among their local network clusters.

Our research suggests that an entrant needs to conduct a thorough network analysis to understand the interconnectivity among different markets, the strength of network effects, the capability of its targeting technologies, and whether or not mobile users consume in their local markets. These factors, together with the cost of reaching users, can help inform the entrant's location choice and how aggressively it should

¹⁰ See, for example, <https://techcrunch.com/2019/01/15/ahead-of-ipo-airbnb-achieves-profitability-for-second-year-in-a-row/>, <https://www.macrotrends.net/stocks/charts/BKNG/booking-holdings/gross-profit> and <https://www.reuters.com/article/us-uber-ipo/uber-unveils-ipo-with-warning-it-may-never-make-a-profit-idUSKCN1RN2SK>; accessed October 2019.

build awareness in a new market. The entrant needs to realize that even if advertising incurs little cost, it is not always optimal for it to advertise to every user. The entrant should advertise to the extent that it does not trigger competitive responses from the incumbent. Equally important, it is not always the case that an entrant should choose a market with low interconnectivity. When advertising is inexpensive and mobile buyers consume in local markets, it could be more profitable to enter a market with high interconnectivity.

As one of the first papers that explicitly models network interconnectivity of platform markets, our paper opens a new direction for future research on platform strategies. For example, our model focuses on an entrant's entry strategy and only allows the incumbent to react through pricing. Future research could consider the incumbent's perspective and examine other strategies for entry deterrence.

Our paper focuses on examining an entrant with limited resources (to overcome entry cost in each market) and an incumbent that already exists in many markets. Even if we allow the entrant to enter more than one market, as long as the number of markets the entrant can realistically enter is small compared to the total number of available markets available (which is true in most cases), our results would not change qualitatively because network interconnectivity (or awareness spillover) plays a rather minor role for the entrant relative to the incumbent. Take the Uber and Fasten cases as examples. Even if Fasten enters a second market, the number of Fasten users from that market to Boston is rather small compared to the number of Uber users from the hundreds of cities outside Boston to Boston. This also implies that Fasten's advertising in the second market has little impact, relative to Uber, on the first market that Fasten entered. Future research can extend our analysis to examine cases involving a resourceful entrant that can enter many markets at once, such as in the case of Uber vs. Grab in Southeast Asia.

In our model, one buyer and one service provider are matched during each transaction. In other words, at a given time, a buyer cannot buy from multiple service providers (regardless of whether they are on the same platform or different ones) and a service provider cannot serve multiple buyers (regardless of whether these buyers are on the same or different platforms). This assumption matches with the rides-

sharing industry in that the same rider or the same driver does not show up in multiple cars at a time.¹¹ If we allow multiple transactions for each user, we may observe multi-homing in that a rider may be matched to Uber drivers for certain transactions and Lyft drivers for other transactions. Future research can extend our model to incorporate multiple transactions for each user and allow buyers and service providers to multi-home. In this case, the entrant needs to decide on the entry and advertising strategy based on how many transactions it expects to serve. Buyers and service providers will decide whether to adopt the entrant platform based on their switching costs and expected benefits from future transactions. While the game will be more complicated, we believe that our key insights would continue to hold. For example, in equilibrium, only the buyers and service providers with low switching cost will adopt the entrant platform to multi-home. Incoming mobile buyers will continue to disincentivize the incumbent to respond in each period, which ultimately drives the impact of network interconnectivity on the entrant's advertising strategy and profitability, as illustrated in our model.

Furthermore, to focus on the impact of network interconnectivity, we abstract away many other factors that could influence competitive interactions between incumbents and entrants. For example, in the ride-sharing industry, riders may not care much about vehicle features. However, in the accommodation industry, travelers are likely to care about the features of properties, thereby making it easier for an entrant into the accommodation industry to differentiate itself from an incumbent and reducing the competitive intensity. Future research could explore how these factors affect competitive interactions.

¹¹ This assumption applies to Airbnb as well if we assume that each host has one room to offer at a time and if we treat each host with multiple rooms to offer as multiple service providers.

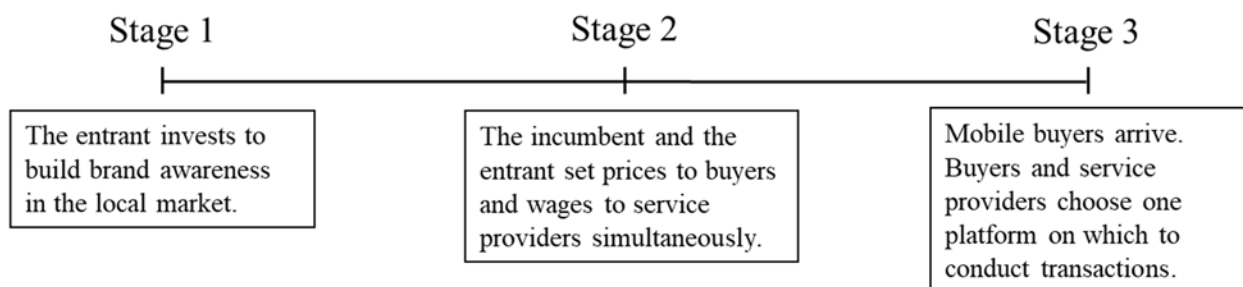
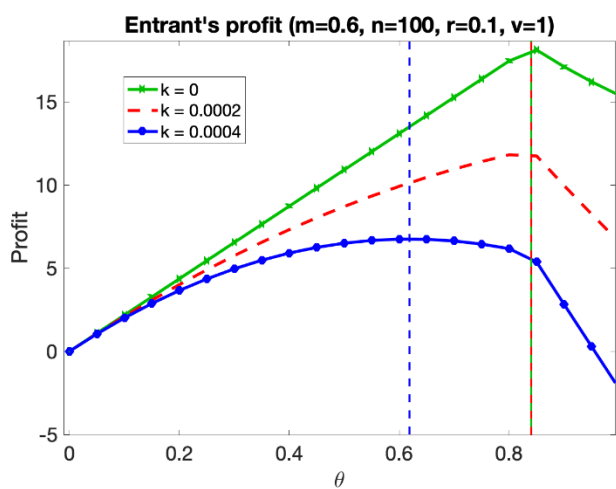
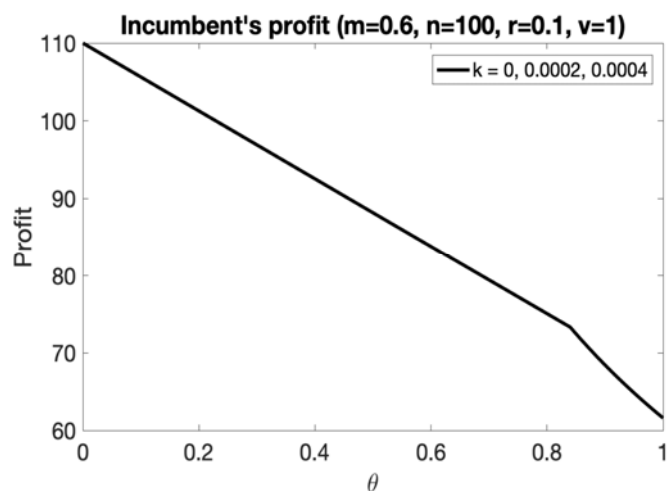


Figure 1: Sequence of the game



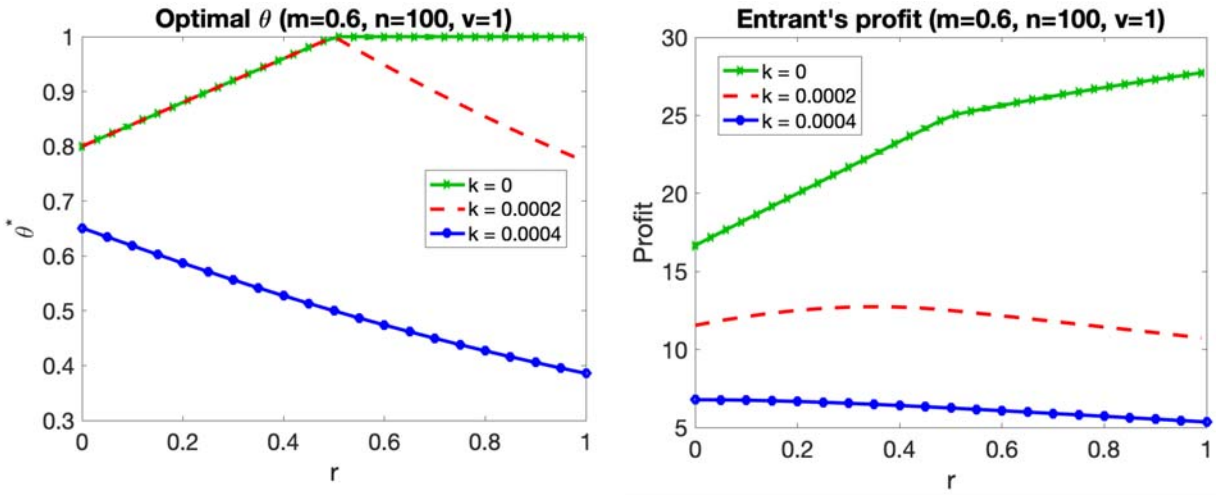
(a)



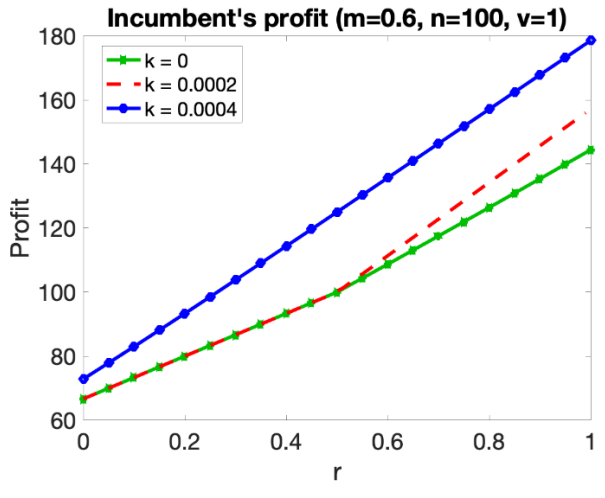
(b)

The vertical lines indicate the optimal θ for each scenario.

Figure 2: Firms' profits vs. advertising intensity θ



(a) (b)



(c)

Figure 3: The entrant's optimal advertising intensity and firms' profits under different values of r

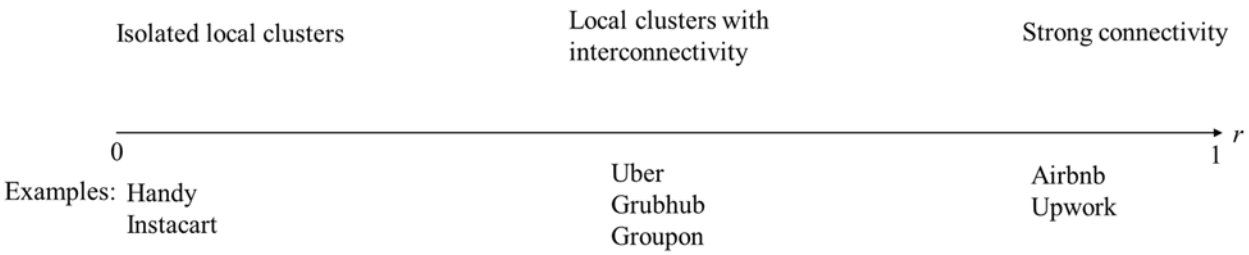


Figure 4: Platform markets with different degrees of network interconnectivity

Table 1: Notations in the main model

Variable	Meaning
N	Total number of local buyers
r	Proportion of buyers who are mobile and travel between markets, $r \in [0,1]$
n	Number of buyers and service providers (users) who are exposed to the entrant's advertising
$L(n)$	Advertising cost for the entrant for reaching n buyers and service providers
k	Advertising cost parameter
θ	Proportion of users that the entrant targets for advertisement, $\theta \in [0,1]$
θ^*	Optimal proportion of potential users that the entrant targets for advertisement, $\theta^* \in [0,1]$
v	Buyer's willingness to pay for the service
m	Maximum switching cost
a_i	Switching cost for buyer i to adopt the entrant's platform, $a_i \sim Uni(0, m)$
c_j	Switching cost for service-provider j to adopt the entrant's platform, $c_j \sim Uni(0, m)$
a^*	The threshold at which buyers with lower switching cost will adopt the entrant's platform.
c^*	The threshold at which service providers with lower switching cost will adopt the entrant's platform.
N_l^B	Number of buyers who use platform $l \in \{I, E\}$
N_l^S	Number of service providers who use platform $l \in \{I, E\}$
$U_{l_i}^B$	Utility of buyer i who uses platform $l \in \{I, E\}$
$U_{l_j}^S$	Utility of service provider j who uses platform $l \in \{I, E\}$
p_l	Price for buyers that is set by platform $l \in \{I, E\}$
w_l	Wage for service providers that is set by platform $l \in \{I, E\}$
π_l	Profit for platform $l \in \{I, E\}$

APPENDIX: PROOFS

Proof of Lemma 1. Given the entrant's prices, p_E and w_E , the incumbent's best response is always to choose p_I and w_I , such that the number of service buyers equals the number of service providers on the incumbent's platform. Otherwise, the incumbent can always increase its profit by increasing p_I or decreasing w_I so that the profit margin $(p_I - w_I)$ goes up without affecting the matched demand (i.e., $\min(N_I^S, N_I^B)$). Similarly, given the incumbent's prices, p_I and w_I , the entrant's best response is always to choose p_E and w_E , such that the number of service buyers equals the number of service providers on the entrant's platform.

Proof of Proposition 1. We first solve for the optimal prices for the interior equilibrium, where $0 < a_i^* < m$. We first confirm that the second-order derivatives are both negative: $\frac{\partial^2 \pi_I}{\partial p_I^2} = \frac{\partial^2 \pi_E}{\partial p_E^2} = -\frac{2N(1+\frac{1}{1+r})\theta}{m} < 0$.

We then derive the first-order conditions:

$$\frac{\partial \pi_I}{\partial p_I} = N \left(2 + r + \frac{(p_E(3+r) - 2p_I(2+r) + (1+r)w_E)\theta}{m(1+r)} \right) = 0 \quad (\text{A1})$$

$$\frac{\partial \pi_E}{\partial p_E} = \frac{N(p_I(3+r) - 2p_E(2+r) + (1+r)w_I)\theta}{m(1+r)} = 0 \quad (\text{A2})$$

And we further obtain the following:

$$p_I = \frac{1}{2} \left(\frac{p_E(3+r) + (1+r)w_E}{2+r} + \frac{m(1+r)}{\theta} \right) \quad (\text{A3})$$

$$p_E = \frac{(3+r)p_I + (1+r)w_I}{4+2r} \quad (\text{A4})$$

Solving (A3) and (A4) along with $(p_I - p_E) = (1+r)(w_E - w_I)$ (according to Lemma 1), we obtain $p_I^* = \frac{2m(2+r)}{3\theta} + w_I$, $p_E^* = \frac{m(3+r)}{3\theta} + w_I$, and $w_E^* = \frac{m}{3\theta} + w_I$. The number of buyers and service providers using each platform under the optimal prices are $N_I^{B*} = N_I^{S*} = \frac{2N(1+r)}{3}$ and $N_E^{B*} = N_E^{S*} = \frac{N(1+r)}{3}$. The profits under the optimal prices are $\pi_I^*(\theta) = \frac{4mN(1+r)(2+r)}{9\theta}$ and $\pi_E^*(\theta) = \frac{mN(1+r)(2+r)}{9\theta} - L(\theta(2N + rN))$.

For this interior equilibrium to hold, we need to ensure that the incumbent has no incentive to deviate from this equilibrium by charging such a high price $p_I = v$ that no one from the overlapped market will transact on its platform but that it gets the most profit from the users who are not aware of the entrant.¹²

¹² This deviation is not adequately captured by the optimization process above because its calculation automatically assigns a negative profit to the overlapped market if $p_I - p_E > m$.

The highest possible deviation profit the incumbent gets in this case is $(1 - \theta + r)Nv$, whereas the incumbent's equilibrium profit is $\frac{4mN(1+r)(2+r)}{9\theta}$. To guarantee that the latter is higher (i.e., $\frac{4mN(1+r)(2+r)}{9\theta} > (1 - \theta + r)Nv$) for all values of θ , we assume that $m > \frac{9(1+r)v}{16(2+r)}$. This condition also ensures that the incumbent will never completely give up the overlapped market—that is, the entrant will never have the entire overlapped market. This is quite realistic because, in practice, there are always users who have a sufficiently high switching cost such that would rather remain with their current platform.

For this interior equilibrium to hold, we must also have $p_I^* = \frac{2m(2+r)}{3\theta} + w_I < v$. Because the choice of w_I does not affect either platform's profit and any border solution is inferior to the interior solution, the incumbent has incentive to ensure that $p_I^* < v$ holds as much as possible by setting $w_I^* = 0$. Consequently, $p_I^* = \frac{2m(2+r)}{3\theta}$, $p_E^* = \frac{m(3+r)}{3\theta}$, and $w_E^* = \frac{m}{3\theta}$, and the condition $p_I^* < v$ requires $\theta > \frac{2m(2+r)}{3v}$.

When $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$, even with $w_I^* = 0$, the optimal p_I^* cannot satisfy $p_I^* < v$. Thus, the incumbent's price is bounded at $p_I^* = v$ to the buyers. Then, based on (A4), $w_I^* = 0$ and $p_I - p_E = (1+r)(w_E - w_I)$, we obtain $p_E^* = \frac{(3+r)v}{2(2+r)}$ and $w_E^* = \frac{v}{2(2+r)}$. The number of buyers and service providers using each platform under the optimal prices are $N_I^{B*} = N_I^{S*} = \frac{N(1+r)}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $N_E^{B*} = N_E^{S*} = \frac{N(1+r)v\theta}{2m(2+r)}$. The profits under the optimal prices are $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L(\theta(2N + rN))$.

Proof of Corollary 1. When $\theta > \frac{2m(2+r)}{3v}$, $\pi_E^*(\theta) = \frac{mN(1+r)(2+r)}{9\theta} - L(\theta(2N + rN))$, which decreases with θ . Thus, the entrant never has incentive to increase θ once $\theta > \frac{2m(2+r)}{3v}$. Therefore, the optimal choice of θ always satisfies $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$.

Proof of Proposition 2. According to Corollary 1, the entrant always chooses $\theta \leq \min\left(\frac{2m(2+r)}{3v}, 1\right)$; then, according to Proposition 1, $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L(\theta(2N + rN))$. Given $L(n) = kn^2$, $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - kN^2(2+r)^2\theta^2$. We first confirm that the second-order derivative is negative: $\pi_E^{*''}(\theta) = -2kN^2(2+r)^2 < 0$. Then, we derive the first-order condition:

$$\pi_E^{*'}(\theta) = \frac{N(1+r)v^2 - 8kN^2m(2+r)^3\theta}{4m(2+r)} = 0 \quad (\text{A5})$$

This yields

$$\theta^* = \frac{(1+r)v^2}{8kNm(2+r)^3} \quad (\text{A6})$$

Because the optimal choice of θ is bounded by $\theta \leq \frac{2m(2+r)}{3v}$ and $\theta \leq 1$, we compare $\frac{(1+r)v^2}{8kNm(2+r)^3}$ with the two bounds and obtain $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq \frac{2m(2+r)}{3v}$ if $k \geq \frac{3(1+r)v^3}{16m^2N(2+r)^4}$ and $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq 1$ if $k \geq \frac{(1+r)v^2}{8mN(2+r)^3}$. Thus, we derive the following two cases:

- (i) When $k \geq \max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, both $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq \frac{2m(2+r)}{3v}$ and $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq 1$ hold. Then $\theta^* = \frac{(1+r)v^2}{8kNm(2+r)^3}$, and it is easy to verify that $\frac{\partial\theta^*}{\partial r} < 0$. By replacing θ with $\frac{(1+r)v^2}{8kNm(2+r)^3}$ in $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\left(2N\left(\theta + \frac{\theta r}{2}\right)\right)$, we obtain $\pi_I^* = \frac{N(1+r)v}{2}\left(2 - \frac{(1+r)v^3}{8kNm^2(2+r)^4}\right)$ and $\pi_E^* = \frac{(1+r)^2v^4}{64km^2(2+r)^4}$.
- (ii) When $k < \max\left(\frac{3(1+r)v^3}{16m^2N(2+r)^4}, \frac{(1+r)v^2}{8mN(2+r)^3}\right)$, either $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq \frac{2m(2+r)}{3v}$ or $\frac{(1+r)v^2}{8kNm(2+r)^3} \leq 1$ does not hold. Then $\theta^* = \min\left(\frac{2m(2+r)}{3v}, 1\right)$, and it is easy to verify that $\frac{\partial\theta^*}{\partial r} \geq 0$. When $\frac{2m(2+r)}{3v} < 1$, by replacing θ with $\frac{2m(2+r)}{3v}$ in $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\left(2N\left(\theta + \frac{\theta r}{2}\right)\right)$, we obtain $\pi_I^* = \frac{2Nv(1+r)}{3}$ and $\pi_E^* = \frac{N(1+r)v}{6} - \frac{4kN^2m^2(2+r)^4}{9v^2}$. When $\frac{2m(2+r)}{3v} \geq 1$, by replacing θ with 1 in $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{v\theta}{m(2+r)}\right)$ and $\pi_E^*(\theta) = \frac{N(1+r)v^2\theta}{4m(2+r)} - L\left(2N\left(\theta + \frac{\theta r}{2}\right)\right)$, we obtain $\pi_I^* = \frac{N(1+r)v}{2}\left(2 - \frac{v}{m(2+r)}\right)$ and $\pi_E^* = \frac{N(1+r)v^2 - 4kN^2m(2+r)^3}{4m(2+r)}$.

Proofs of Corollaries 2, 3, and 4. These corollaries follow directly from Proposition 2.

Proof of Proposition 3. If the entrant enters market h , we can similarly obtain the demand function in market h as follows:

$$N_{Ih}^S = \left(1 - \frac{c_h^*}{m}\theta_h\right)(1+r_h)N, \quad (\text{A7})$$

$$N_{Eh}^S = \frac{c_h^*}{m} \theta_h (1 + r_h) N, \quad (\text{A8})$$

$$N_{Ih}^B = \left(1 + r_h - \frac{a_h^*}{m} \theta_h\right) N, \quad (\text{A9})$$

$$N_{Eh}^B = \frac{a_h^*}{m} \theta_h N, \quad (\text{A10})$$

where $c_h^* = w_{Eh} - w_{Ih}$ and $a_h^* = p_{Ih} - p_{Eh}$.

Following the same procedure as that in the main analysis, we can obtain the following two main propositions in market h with a similar assumption of switching cost ($m > \frac{3v}{5}$). Without loss of generality, let us focus on the case where $\frac{2m(2+r_{max})}{3v} \leq 1$,¹³ where r_{max} is the maximum r_h across all markets.

Proposition 1A: Given the entrant's choice of θ_h , the optimal prices, number of buyers and service providers, and profits are as follows:

$$(i) \text{ If } 0 \leq \theta \leq \frac{2m(2+r_h)}{3v}, \text{ then } p_{Ih}^* = v, \text{ } p_{Eh}^* = \frac{(3+r_h)v}{2(2+r_h)}, \text{ } w_{Eh}^* = \frac{v}{2(2+r_h)}, \text{ } N_{Ih}^{B*} = N_{Ih}^{S*} = \frac{N(1+r_h)}{2} \left(2 - \frac{\theta_h v}{m(2+r_h)}\right), \text{ } N_{Eh}^{B*} = N_{Eh}^{S*} = \frac{N(1+r_h)\theta_h v}{2m(2+r_h)}, \text{ } \pi_{Ih}^*(\theta) = \frac{N(1+r_h)v}{2} \left(2 - \frac{\theta_h v}{m(2+r_h)}\right), \text{ and } \pi_{Eh}^*(\theta) = \frac{N(1+r_h)\theta_h v^2}{4m(2+r_h)} - L(\theta_h(2N + r_h N)).$$

$$(ii) \text{ If } \frac{2m(2+r_h)}{3v} < \theta \leq 1, \text{ then } p_{Ih}^* = \frac{2(2+r_h)m}{3\theta_h}, \text{ } w_{Ih}^* = 0, \text{ } p_{Eh}^* = \frac{(3+r_h)m}{3\theta_h}, \text{ } w_{Eh}^* = \frac{m}{3\theta_h}, \text{ } N_{Ih}^{B*} = N_{Ih}^{S*} = \frac{2N(1+r_h)}{3}, \text{ } N_{Eh}^{B*} = N_{Eh}^{S*} = \frac{N(1+r_h)}{3}, \text{ } \pi_{Ih}^*(\theta) = \frac{4Nm(1+r_h)(2+r_h)}{9\theta_h}, \text{ and } \pi_{Eh}^*(\theta) = \frac{Nm(1+r_h)(2+r_h)}{9\theta_h} - L(\theta_h(2N + r_h N)).$$

We again confirm the entrant's optimal choice of θ_h to be no more than $\frac{2m(2+r_h)}{3v}$. That is, regardless of the market the entrant is in, it is in the best interest of the entrant not to trigger the incumbent's competitive response. Endogenizing θ_h , we obtain the following proposition.

Proposition 2A: The optimal θ_h^ depends on the value of k :*

$$(i) \text{ If } k \geq \frac{3(1+r_h)v^3}{16m^2N(2+r_h)^4}, \text{ then } \theta_h^* = \frac{(1+r_h)v^2}{8(2+r_h)^3 k N m}. \text{ The entrant's profit is } \frac{(1+r_h)^2 v^4}{64km^2(2+r_h)^4} \text{ which decreases with } r_h.$$

¹³ Results are qualitatively the same if $\frac{2m(2+r_{max})}{3v} > 1$.

- (ii) If $0 \leq k < \frac{3(1+r_h)v^3}{16m^2N(2+r_h)^4}$, then $\theta_h^* = \frac{2m(2+r_h)}{3v}$. The entrant's profit is $\frac{N(1+r_h)v}{6} - \frac{4kN^2m^2(2+r_h)^4}{9v^2}$, which increases with r_h if $k < \frac{3v^3}{32m^2N(2+r_h)^3}$

Thus, the entrant will choose the market that yields the highest profit as follows:

- (i) If $0 \leq k \leq \frac{3v^3}{32m^2N(2+r_{max})^3}$, the entrant will choose the market with the largest fraction of incoming mobile users to enter.
- (ii) If $\frac{3v^3}{32m^2N(2+r_{max})^3} < k < \frac{3v^3}{32m^2N(2+r_{min})^3}$, the entrant will choose the market with an intermediate fraction of incoming mobile users to enter.
- (iii) If $k \geq \frac{3v^3}{32m^2N(2+r_{min})^3}$, $\pi_E^* = \frac{(1+r)^2v^4}{64km^2(2+r)^4}$ the entrant will choose the market with the smallest fraction of incoming mobile users to enter.

Proof of Proposition 4. We can similarly obtain the demand function as follows:

$$N_I^S = \left(1 - \frac{c^*}{m}\theta\right) (1+r)sN, \quad (\text{A11})$$

$$N_E^S = \frac{c^*}{m}\theta(1+r)sN + \theta(1-s)N, \quad (\text{A12})$$

$$N_I^B = \left(1 - \frac{a^*}{m}\theta + r\right) sN, \quad (\text{A13})$$

$$N_E^B = \frac{a^*}{m}\theta sN + \theta(1-s)N. \quad (\text{A14})$$

Following the same procedure as in the main analysis, we can obtain the following two main propositions given $m > \frac{3v}{5}$:

Proposition 1B: *Given the entrant's choice of θ , the optimal prices, transaction quantity, and profits for the two platforms are as follows:*

- (i) If $s \geq \frac{m(2+r)}{2m+mr+v+rv}$
- a. If $0 \leq \theta \leq \min\left(\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)}, 1\right)$, then $p_I^* = v$, $p_E^* = \frac{1}{2}\left(m\left(\frac{1}{s} - 1\right) + \frac{(3+r)v}{2+r}\right)$,
- $w_E^* = \frac{1}{2}\left(\frac{v}{2+r} - \frac{m(1-s)}{(1+r)s}\right)$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{N}{2}\left(\theta + s\left(2 + 2r - \theta - \frac{(1+r)v\theta}{m(2+r)}\right)\right)$, $N_E^{B*} =$

$$N_E^{S*} = \frac{N}{2} \left(1 - s \left(1 - \frac{(1+r)v}{m(2+r)} \right) \right) \theta, \quad \pi_I^*(\theta) = \frac{Nv(m(2+r)(s(2+2r-\theta)+\theta) - (1+r)sv\theta)}{2m(2+r)}, \quad \text{and}$$

$$\pi_E^*(\theta) = \frac{N(m(2+r)(1-s) + (1+r)sv)^2 \theta}{4m(1+r)(2+r)s} - L(\theta(2N+rN)).$$

b. If $\min\left(\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)}, 1\right) < \theta \leq 1$, then $p_I^* = \frac{m(2+r)(s(2+2r-\theta)+\theta)}{3(1+r)s\theta}$, $p_E^* = \frac{m}{3} \left(\frac{(3+2r)(1-s)}{(1+r)s} + \frac{3+r}{\theta} \right)$, $w_E^* = \frac{m}{3} \left(\frac{1}{\theta} - \frac{1-s}{(1+r)s} \right)$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{N(s(2(1+r)-\theta)+\theta)}{3}$, $N_E^{B*} = N_E^{S*} = \frac{N(s(1+r-2\theta)+2\theta)}{3}$, $\pi_I^*(\theta) = \frac{mN(2+r)(s(2+2r-\theta)+\theta)^2}{9(1+r)s\theta}$, and $\pi_E^*(\theta) = \frac{mN(2+r)(s(1+r-2\theta)+2\theta)^2}{9(1+r)s\theta} - L(\theta(2N+rN))$.

(ii) If $s < \frac{m(2+r)}{2m+mr+v+rv}$

a. $p_I^* = v$, $p_E^* = v$, $w_E^* = 0$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = (1+r)sN$, $N_E^{B*} = N_E^{S*} = (1-s)N\theta$, $\pi_I^*(\theta) = (1+r)sNv$, and $\pi_E^*(\theta) = (1-s)Nv\theta - L(\theta(2N+rN))$.

Endogenizing θ , we obtain the following proposition.

Proposition 2B: The optimal θ^* depends on the value of k and the value of s .

(i) If $s \geq \frac{m(2+r)}{2m+mr+v+rv}$

a. If $k \geq \max\left(\frac{(m(2+r)(1-s) + (1+r)sv)^2(3(1+r)sv - m(2+r)(1-s))}{16m^2N(1+r)^2(2+r)^4s^2}, \frac{(m(2+r)(1-s) + (1+r)sv)^2}{8mN(1+r)(2+r)^3s}\right)$, then

$$\theta^* = \frac{(m(2+r)(1-s) + (1+r)sv)^2}{8kmN(1+r)(2+r)^3s}, \quad \text{which decreases with } r \text{ and } s. \text{ The entrant's profit is}$$

$$\frac{(m(2+r)(1-s) + (1+r)sv)^4}{64km^2(1+r)^2(2+r)^4s^2} \quad \text{and} \quad \text{the incumbent's profit is}$$

$$\frac{v(m^3(2+r)^3(1-s)^3 - m(1+r)^2(2+r)(1-s)s^2v^2 - (1+r)^3s^3v^3 + m^2(1+r)(2+r)^2s(16kN(1+r)(2+r)^2s + (1-s)^2v))}{16km^2(1+r)(2+r)^4s}.$$

b. If $0 \leq k < \max\left(\frac{(m(2+r)(1-s) + (1+r)sv)^2(3(1+r)sv - m(2+r)(1-s))}{16m^2N(1+r)^2(2+r)^4s^2}, \frac{(m(2+r)(1-s) + (1+r)sv)^2}{8mN(1+r)(2+r)^3s}\right)$, then

$$\theta^* = \min\left(\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)}, 1\right), \quad \text{which weakly increases with } r \text{ and weakly decreases}$$

with s . When $\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)} < 1$, the entrant's profit is

$$\frac{N((m(2+r)(1-s) + (1+r)sv)^2(3(1+r)sv - m(2+r)(1-s)) - 8km^2N(1+r)^2(2+r)^4s^2)}{2(3(1+r)sv - m(2+r)(1-s))^2} \quad \text{and} \quad \text{the}$$

incumbent's profit is $\frac{2N(1+r)^2s^2v^2}{3(1+r)sv - m(2+r)(1-s)}$. When $\frac{2m(1+r)(2+r)s}{3(1+r)sv - m(2+r)(1-s)} \geq 1$, the

entrant's profit is $\frac{N(m(2+r)(1-s) + (1+r)sv)^2}{4m(2+r)(1+r)s} - k((2+r)N)^2$ and the incumbent's profit is

$$\frac{Nv(m(2+r)(1+s+2rs) - (1+r)sv)}{2m(2+r)}.$$

(ii) If $s < \frac{m(2+r)}{2m+mr+v+rv}$,

a. If $k \geq \frac{(1-s)v}{2N(2+r)^2}$, then $\theta^* = \frac{(1-s)v}{2kN(2+r)^2}$, which decreases with s and r . The entrant's profit is

$\frac{(1-s)^2v^2}{4k(2+r)^2}$ and the incumbent's profit is $(1+r)sNv$.

b. If $0 \leq k < \frac{(1-s)v}{2N(2+r)^2}$, then $\theta^* = 1$, the entrant's profit is $N((1-s)v - kN(2+r)^2)$ and the incumbent's profit is $(1+r)sNv$.

Proof of Proposition 5. In this case, we only need N service providers to match N orders in each local market. We can obtain the demand functions as:

$$N_I^S = \left(1 - \frac{c^*}{m}\theta\right)N \quad (\text{A15})$$

$$N_E^S = \frac{c^*}{m}\theta N \quad (\text{A16})$$

$$N_I^B = \left(1 - \frac{a^*}{m}\theta\right)(1-r)N + rN \quad (\text{A17})$$

$$N_E^B = \frac{a^*}{m}\theta(1-r)N \quad (\text{A18})$$

We then follow the same procedure as in our main analysis to derive the following two main propositions given $m > \frac{3v}{5}$:

Proposition 1C: Given the entrant's choice of θ , the optimal prices, number of buyers and service providers, and profits are as follows:

(i) If $0 \leq \theta \leq \min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right)$, then $p_I^* = v$, $p_E^* = \frac{(3-2r)v}{2(2-r)}$, $w_E^* = \frac{(1-r)v}{2(2-r)}$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{N(2(2-r)m - (1-r)\theta v)}{2(2-r)m}$, $N_E^{B*} = N_E^{S*} = \frac{(1-r)N\theta v}{2(2-r)m}$, $\pi_I^* = \frac{Nv(2(2-r)m - (1-r)\theta v)}{2(2-r)m}$, and $\pi_E^* = \frac{(1-r)N\theta v^2}{4(2-r)m} - L(2\theta N)$.

(ii) If $\min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right) < \theta \leq 1$, then $p_I^* = \frac{2(2-r)m}{3(1-r)\theta}$, $p_E^* = \frac{(3-2r)m}{3(1-r)\theta}$, $w_E^* = \frac{m}{3\theta}$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{2N}{3}$, $N_E^{B*} = N_E^{S*} = \frac{N}{3}$, $\pi_I^*(\theta) = \frac{4N(2-r)m}{9(1-r)\theta}$, and $\pi_E^*(\theta) = \frac{N(2-r)m}{9(1-r)\theta} - L(2\theta N)$.

We again confirm the entrant's optimal choice of θ to be no more than $\min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response. Endogenizing θ , we obtain the following proposition.

Proposition 2C: The optimal θ^* depends on the value of k :

- (i) If $k \geq \max\left(\frac{3(1-r)^2v^3}{64m^2N(2-r)^2}, \frac{(1-r)v^2}{32mN(2-r)}\right)$, then $\theta^* = \frac{(1-r)v^2}{32kNm(2-r)}$, which decreases with r . The entrant's profit is $\frac{(1-r)^2v^4}{256km^2(2-r)^2}$ and the incumbent's profit is $Nv - \frac{(1-r)^2v^4}{64km^2(2-r)^2}$.
- (ii) If $0 \leq k < \max\left(\frac{3(1-r)^2v^3}{64m^2N(2-r)^2}, \frac{(1-r)v^2}{32mN(2-r)}\right)$, then $\theta^* = \min\left(\frac{2(2-r)m}{3(1-r)v}, 1\right)$, which weakly increases with r . When $\frac{2(2-r)m}{3(1-r)v} < 1$, the entrant's profit is $\frac{Nv}{6} - \frac{16kN^2m^2(2-r)^2}{9(1-r)^2v^2}$ and the incumbent's profit is $\frac{2Nv}{3}$. When $\frac{4m}{3v} \geq 1$, the entrant's profit is $\frac{N(1-r)v^2}{4m(2-r)} - 4kN^2$ and the incumbent's profit is $\frac{Nv}{2}\left(2 - \frac{(1-r)v}{m(2-r)}\right)$.

One difference between Proposition 2B and Proposition 2 is that in this case $\pi_E^*(\theta)$ always decreases with r .

Proof of Proposition 6. Given the new demand functions, we can follow the same procedure as that in our main analysis to derive the following two main propositions, given $m > \frac{3v}{5}$:

Proposition 1D: Given the entrant's choice of θ and θ_t , the optimal prices, number of buyers and service providers, and profits are as follows:

- (i) If $0 \leq \frac{\theta(r\theta_t+\theta)}{2\theta+r(\theta_t+\theta)} \leq \min\left(\frac{2m}{3v}, \frac{1}{2}\right)$, $p_I^* = v$, $p_E^* = \frac{2r\theta_tv+(3+r)v\theta}{4\theta+2r(\theta_t+\theta)}$, $w_E^* = \frac{v(r\theta_t+\theta)}{4\theta+2r(\theta_t+\theta)}$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{N(1+r)(4m\theta+2mr(\theta_t+\theta)-v\theta(r\theta_t+\theta))}{2m(2\theta+r(\theta_t+\theta))}$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)v\theta(r\theta_t+\theta)}{2m(2\theta+r(\theta_t+\theta))}$, $\pi_I^*(\theta, \theta_t) = \frac{N(1+r)v(4m\theta+2mr(\theta_t+\theta)-v\theta(r\theta_t+\theta))}{2m(2\theta+r(\theta_t+\theta))}$, and $\pi_E^*(\theta, \theta_t) = \frac{N(1+r)v^2\theta(r\theta_t+\theta)}{4m(2\theta+r(\theta_t+\theta))} - L(\theta(2N+rN) + \theta_t rN)$.
- (ii) If $\min\left(\frac{2m}{3v}, \frac{1}{2}\right) < \frac{\theta(r\theta_t+\theta)}{2\theta+r(\theta_t+\theta)}$, $p_I^* = \frac{2m(2\theta+r(\theta_t+\theta))}{3\theta(r\theta_t+\theta)}$, $p_E^* = \frac{m}{3}\left(\frac{2}{\theta} + \frac{(1+r)}{r\theta_t+\theta}\right)$, $w_E^* = \frac{m}{3\theta}$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{2N(1+r)}{3}$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)}{3}$, $\pi_I^*(\theta, \theta_t) = \frac{4mN(1+r)(2\theta+r(\theta_t+\theta))}{9\theta(r\theta_t+\theta)}$, and $\pi_E^*(\theta, \theta_t) = \frac{mN(1+r)(2\theta+r(\theta_t+\theta))}{9\theta(r\theta_t+\theta)} - L(\theta(2N+rN) + \theta_t rN)$.

We again confirm that the entrant's optimal choices of θ and θ_t satisfy the condition $\frac{\theta(r\theta_t+\theta)}{2\theta+r(\theta_t+\theta)} \leq \min\left(\frac{2m}{3v}, \frac{1}{2}\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response. The threshold $\frac{\theta(r\theta_t+\theta)}{2\theta+r(\theta_t+\theta)}$ is an increasing function of θ and θ_t and it increases faster with θ than with θ_t . When this condition is satisfied, the entrant's profit is also an increasing function of θ and θ_t and

it also increases faster with θ than with θ_t . Thus, endogenizing θ and θ_t , we obtain the following proposition for the case of $k = 0$.

Proposition 2D: *When $k = 0$, the optimal θ^* and θ_t^* are*

$$(i) \quad \text{If } \frac{2m(2+r)}{3v} \leq 1, \text{ then } \theta^* = \frac{2m(2+r)}{3v}, \theta_t^* = 0.$$

$$(ii) \quad \text{If } \frac{2m(2+r)}{3v} > 1, \theta^* = 1, \text{ and } \theta_t^* = \frac{2m(2+r)-3v}{r(3v-2m)} \text{ when } 3v > 4m \text{ and } \theta_t^* = 1 \text{ when } 3v \leq 4m.$$

Note that in case (ii), the entrant obtains all local users and has not triggered a response from the incumbent. Therefore, it proceeds to advertise to mobile buyers.

Proof of Proposition 7. Assume that a^* is the switching cost of the indifferent user and c^* is the switching cost of the indifferent service provider. Then, Equations (1)–(4) define the number of buyers and service providers that select the entrant and the incumbent, respectively. Then, given the utility functions in Equations (9)–(12), we derive a^* and c^* by solving the following two equations simultaneously:

$$e \left(1 - \frac{c^*}{m}\right) (1+r)N + v - p_I = e \frac{c^*(1+r)}{m} \theta N + v - p_E - a^*, \quad (\text{A19})$$

$$e \left(1 - \frac{a^*}{m}\theta + r\right) N + w_I = e \frac{a^*}{m} \theta N + w_E - c^*. \quad (\text{A20})$$

Thus, we obtain $a^* = \frac{m(m(p_I - p_E - eN(1+r)) + 2eN(1+r)\theta(w_E - w_I - eN(1+r)))}{m^2 - 4e^2N^2(1+r)\theta^2}$ and

$c^* = \frac{m(m(w_E - w_I - eN(1+r)) + 2eN\theta(p_I - p_E - eN(1+r)))}{m^2 - 4e^2N^2(1+r)\theta^2}$. We can then prove that Lemma 1 holds in this extension, that is, the incumbent and the entrant will always choose their prices and wages so that $a^* = (1+r)c^*$, as long as $e < \frac{m}{4N}$.

We then follow the same procedure as that in our main analysis to derive the two main propositions and find that our key results hold qualitatively under conditions $m > \frac{3v}{5}$ and $e < \frac{v}{2N}$.

Proposition 1E: *Given the entrant's choice of θ , the optimal prices, number of buyers and service providers, and profits are as follows:*

$$(i) \quad \text{If } 0 \leq \theta \leq \min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right), \quad \text{then } p_I^* = \frac{2m(2+r)(eN(1+r)+v)-eN(1+r)(6eN(1+r)+8v)\theta}{2m(2+r)-7eN(1+r)\theta}, p_E^* = \frac{m(eN(1+r)^2+(3+r)v)-eN(1+r)(eN(1+r)+6v)\theta}{2m(2+r)-7eN(1+r)\theta}, w_E^* = \frac{m(eN(1+r)(3+2r)+v)-eN(1+r)(5eN(1+r)+2v)\theta}{2m(2+r)-7eN(1+r)\theta}, w_I^* = 0, N_I^{B^*} = N_I^{S^*} = \frac{N(1+r)(2m(2+r)-(6eN(1+r)+v)\theta)}{2m(2+r)-7eN(1+r)\theta},$$

$$\begin{aligned}
N_E^{B*} = N_E^{S*} &= \frac{N(1+r)(v-eN(1+r))\theta}{2m(2+r)-7eN(1+r)\theta}, & \pi_I^* &= \\
&= \frac{2N(1+r)(2m(2+r)-(6eN(1+r)+v)\theta)(m(2+r)(eN(1+r)+v)-eN(1+r)(3eN(1+r)+4v)\theta)}{(2m(2+r)-7eN(1+r)\theta)^2}, & \text{and } \pi_E^* &= \\
&= \frac{N(1+r)(v-eN(1+r))^2\theta(m(2+r)-4eN(1+r)\theta)}{(2m(2+r)-7eN(1+r)\theta)^2} - L(\theta(2N+rN)).
\end{aligned}$$

(ii) If $\min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right) < \theta \leq 1$, then $p_I^* = \frac{2m(2+r)}{3\theta} - 2eN(1+r)$, $p_E^* = \frac{m^2(2+r)(3+r)-3meN(1+r)(8+3r)\theta+12eN(1+r)(2eN(1+r))\theta^2}{3\theta(m(2+r)-4eN(1+r)\theta)}$, $w_E^* = \frac{m^2(2+r)-meN(4-r)(1+r)\theta}{3\theta(m(2+r)-4eN(1+r)\theta)}$, $w_I^* = 0$, $N_I^{B*} = N_I^{S*} = \frac{2N(1+r)(m(2+r)-3eN(1+r)\theta)}{3(m(2+r)-4eN(1+r)\theta)}$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)(m(2+r)-6eN(1+r)\theta)}{3(m(2+r)-4eN(1+r)\theta)}$, $\pi_I^*(\theta) = \frac{4N(1+r)(m(2+r)-3eN(1+r)\theta)^2}{9\theta(m(2+r)-4eN(1+r)\theta)}$, and $\pi_E^*(\theta) = \frac{N(1+r)(m(2+r)-6eN(1+r)\theta)^2}{9\theta(m(2+r)-4eN(1+r)\theta)} - L(\theta(2N+rN))$.

We again confirm the entrant's optimal choice of θ to be no more than $\min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$. That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response.

Therefore, the entrant will select θ to maximize $\pi_E^*(\theta) = \frac{N(1+r)(v-eN(1+r))^2\theta(m(2+r)-4eN(1+r)\theta)}{(2m(2+r)-7eN(1+r)\theta)^2} - k(\theta(2N+rN))^2$ under the constraint that $\theta \leq \min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$. Because the first term of $\pi_E^*(\theta)$ increases with θ and the second term of $\pi_E^*(\theta)$ decreases with θ , we can conclude that there exists a k^* so that the two scenarios in Proposition 2 hold qualitatively. That is, 1) when $k \geq k^*$, the optimal θ^* is the solution to $\frac{\partial \pi_E^*(\theta)}{\partial \theta} = 0$, and $\theta^* \leq \min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$; and 2) when $k < k^*$, $\theta^* = \min\left(\frac{4m(2+r)}{16eN(1+r)+3v+\sqrt{16e^2N^2(1+r)^2+9v^2}}, 1\right)$.

Proof of Proposition 8. Following the same procedure as that in the proof of Proposition 7, we can prove that if there exists an equilibrium where both the entrant and the incumbent have a positive demand, Lemma 1 holds in this extension when e is very large—that is, the incumbent and the entrant will always choose their prices and wages so that $a^* = (1+r)c^*$ when $e > \frac{m}{2N\theta}$. Then, we show that such an equilibrium cannot be sustained because the incumbent will always deviate to drive the entrant out of the market. This is because when e is very large, the highest possible profit that an incumbent can obtain if giving up xN ($x \leq \theta$ and $xN \geq 1$) local buyers and xN matched service providers to the entrant is lower than the profit

an incumbent can obtain by deviating to prevent these local buyers and service providers from switching to the entrant.

Specifically, if the incumbent gives up xN local buyers and xN matched service providers to the entrant, a buyer's utility from using the incumbent platform is $e(1-x+r)N + v - p_I$ and a service provider's utility from using the incumbent platform is $e(1-x+r)N + w_I$. Thus, the highest possible profit the incumbent can obtain is $(2e(1-x+r)N + v)(1-x+r)N$, by charging $p_I = e(1-x+r)N + v$ and $w_I = -e(1-x+r)N$. Its actual profit can be lower because the incumbent may have to lower its margin to compete with the entrant, so that its $(1-x+r)N$ buyers and $(1-x+r)N$ service providers will not switch to the entrant.

In contrast, if the incumbent deviates to charge $p_I = e(1-x+r)N + w_E$ and $w_I = w_E - e(1-x+r)N$, it can prevent these xN local buyers and xN service providers from switching to the entrant. This can be shown as follows. The highest possible utility of a buyer switching to the entrant platform, assuming xN service providers will switch together, is $exN + v - p_E$, which is not higher than the utility of remaining with the incumbent, which is $e(1+r)N + v - p_I$, given that $p_I = e(1-x+r)N + w_E \leq e(1-x+r)N + p_E$. Similarly, the highest possible utility of a buyer switching to the entrant platform, assuming xN buyers will switch together, is $exN + w_E$, which is not higher than the utility of remaining with the incumbent, which is $e(1+r)N + w_I$, given that $w_I = w_E - e(1-x+r)N$. In this case, the profit the incumbent can obtain is $2e(1-x+r)N(1+r)N$, which is higher than the highest possible profit that the incumbent can obtain once it gives up these local buyers and service providers to the entrant (which is $(2e(1-x+r)N + v)(1-x+r)N$) when $e > \frac{v}{2Nx}$.

Thus, we find that the condition under which the equilibrium in which the entrant has positive demand does not exist and the incumbent will take the entire market: $e > \max\left(\frac{m}{2N\theta}, \frac{v}{2Nx}\right)$. As $xN \geq 1$ and $x \leq \theta$, the sufficient condition is $e > \max\left(\frac{m}{2}, \frac{v}{2}\right)$.

Proof of Proposition 9. The demand functions are now changed to

$$N_I^S = \left(1 - \frac{c^*}{m_s}\theta + r\right)N, \quad (\text{A21})$$

$$N_E^S = \frac{c^*}{m_s}\theta N, \quad (\text{A22})$$

$$N_I^B = \left(1 - \frac{a^*}{m_b}\theta + r\right)N, \quad (\text{A23})$$

$$N_E^B = \frac{a^*}{m_b}\theta N. \quad (\text{A24})$$

Then, we follow the same procedure as that in our main analysis to derive the following two main propositions given $m > \frac{3v}{5}$:

Proposition 1F: *Given that the entrant's choice of θ , the optimal prices, number of buyers and service providers, and profits are as follows:*

- (i) If $0 \leq \theta \leq \min\left(\frac{2(m_b+m_s+m_br)}{3v}, 1\right)$, $p_I^* = v$, $p_E^* = \frac{(m_b+2m_s+m_br)v}{2(m_b+m_s+m_br)}$, $w_E^* = \frac{m_sv}{2(m_b+m_s+m_br)}$, $N_I^{B*} = N_I^{S*} = \frac{N(1+r)}{2}\left(2 - \frac{\theta v}{m_b+m_s+m_br}\right)$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)\theta v}{2(m_b+m_s+m_br)}$, $\pi_I^*(\theta) = \frac{N(1+r)v}{2}\left(2 - \frac{\theta v}{m_b+m_s+m_br}\right)$, and $\pi_E^*(\theta) = \frac{N(1+r)\theta v^2}{4(m_b+m_s+m_br)} - L(\theta(2N + rN))$.
- (ii) If $\min\left(\frac{2(m_b+m_s+m_br)}{3v}, 1\right) < \theta \leq 1$, then $p_I^* = \frac{2(m_b+m_s+m_br)}{3\theta}$, $w_I^* = 0$, $p_E^* = \frac{m_b+2m_s+m_br}{3\theta}$, $w_E^* = \frac{m_s}{3\theta}$, $N_I^{B*} = N_I^{S*} = \frac{2N(1+r)}{3}$, $N_E^{B*} = N_E^{S*} = \frac{N(1+r)}{3}$, $\pi_I^*(\theta) = \frac{4N(1+r)(m_b+m_s+m_br)}{9\theta}$, and $\pi_E^*(\theta) = \frac{N(1+r)(m_b+m_s+m_br)}{9\theta} - L(\theta(2N + rN))$.

We again confirm the entrant's optimal choice of θ to be no more than $\min\left(\frac{2(m_b+m_s+m_br)}{3v}, 1\right)$.

That is, it is in the best interest of the entrant not to trigger the incumbent's competitive response. Endogenizing θ , we obtain the following proposition.

Proposition 2F: *The optimal θ^* depends on the value of k :*

- (i) If $k \geq \max\left(\frac{3(1+r)v^3}{16N(2+r)^2(m_b+m_s+m_br)^2}, \frac{(1+r)v^2}{8N(2+r)^2(m_b+m_s+m_br)}\right)$, then $\theta^* = \frac{(1+r)v^2}{8kN(2+r)^2(m_b+m_s+m_br)}$, which decreases with m_b , m_s , and r . The entrant's profit is $\frac{(1+r)^2v^4}{64k(2+r)^2(m_b+m_s+m_br)^2}$ and the incumbent's profit is $\frac{N(1+r)v}{2}\left(2 - \frac{(1+r)v^3}{8kN(2+r)^2(m_b+m_s+m_br)^2}\right)$.
- (ii) If $0 \leq k < \max\left(\frac{3(1+r)v^3}{16N(2+r)^2(m_b+m_s+m_br)^2}, \frac{(1+r)v^2}{8N(2+r)^2(m_b+m_s+m_br)}\right)$, then $\theta^* = \min\left(\frac{2(m_b+m_s+m_br)}{3v}, 1\right)$, which weakly increases with m_b , m_s , and r . When $\frac{2(m_b+m_s+m_br)}{3v} < 1$, the entrant's profit is $\frac{N(1+r)v}{6} - \frac{4kN^2(2+r)^2(m_b+m_s+m_br)^2}{9v^2}$ and the incumbent's profit is $\frac{2Nv(1+r)}{3}$. When $\frac{2(m_b+m_s+m_br)}{3v} \geq 1$, the entrant's profit is

$$\frac{N(1+r)v^2 - 4kN^2(2+r)^2(m_b + m_s + m_b r)}{4(m_b + m_s + m_b r)} \text{ and the incumbent's profit is } \frac{N(1+r)v}{2} \left(2 - \frac{v}{m_b + m_s + m_b r} \right).$$

Comparative statics suggest that $\frac{\partial \pi_E^*}{\partial m_b} < \frac{\partial \pi_E^*}{\partial m_s} < 0$ and $\frac{\partial \pi_I^*}{\partial m_b} \geq \frac{\partial \pi_I^*}{\partial m_s} \geq 0$.