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Contract Duration and the Costs of Market Transactions*

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Abstract

The optimal duration of a supply contract balances the costs of re-selecting a supplier against the costs of being matched to an inefficient supplier when the contract lasts too long. I develop a structural model of contract duration that captures this tradeoff and provide an empirical strategy for quantifying (unobserved) transaction costs. I estimate the model using federal supply contracts for a standardized product, where suppliers are selected by procurement auctions. The estimated transaction costs are substantially greater than consumer switching costs and a significant portion of total buyer costs. Counterfactuals illustrate the importance of accounting for the duration margin.

JEL Codes: D22, D44, H57, L13, L14

Keywords: Supply Contracts, Intermediate Goods, Switching Costs, Auctions

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1 Introduction

When buyers select sellers, they select not only who but also how long. For the supply of services, the duration of a buyer-seller relationship is often formalized in a contract. At the expiration of the contract, the buyer returns to the market to re-select a supplier. The buyer bears some costs for doing so: each time the buyer goes to market, he must identify potential sellers, negotiate, determine the new seller, and draw up a contract (Coase, 1937, 1960). These *market transaction costs* all occur before an agreement is reached.[1] As noted by Coase (1937), a key motivation for a longer contract is to avoid these costs. Choosing a two-year contract instead of sequential one-year contracts can cut these ex ante costs in half.

Though going to the market frequently can be costly, there are typically gains from doing so. Consider a procurement auction that selects the efficient (lowest-cost) supplier. Over a longer contract, the lowest-cost supplier at the beginning of the contract may not have the lowest cost by the end. By running more frequent auctions with shorter contracts, the buyer can switch among the lowest-cost suppliers and pay a lower price. In the absence of transaction costs, the buyer would prefer a spot market that allocates the lowest-cost supplier in every instant.

In the exchange of intermediate goods, market transactions can be especially costly. Unlike the retail sector, markets for intermediate goods are typically not well established. When running a procurement auction, a buyer brings together potential sellers and administers a mechanism to determine the winner and the price. In a sense, the buyer bears the cost of creating his market. Even for standardized goods and services (e.g., raw materials, electricity, paper products, and accounting services), transaction costs can be large, as exact specifications vary among buyers. Indeed, as inputs, these products are typically sold on fixed-price, fixed-duration contracts, rather than in spot markets.[2] Thus, we may infer that market transaction costs are meaningful for a wide variety of goods and services.

Despite their importance, quantifying transaction costs has proven challenging, as they are typically unobserved. To address this, I develop an empirical model where a buyer selects a seller from an imperfectly competitive market. The buyer chooses the duration of the contract to balance ex ante transaction costs against the benefit from selecting more efficient suppliers. This tradeoff is intuitive and corresponds to a common real-world contracting problem.[3] The model also provides a novel strategy for the direct estimation of transaction costs. By revealed preference, transaction costs may be identified from the duration of the contract and the price

[1] Market transaction costs correspond to the first two categories of the taxonomy of transaction costs suggested by Dahlman (1979): (1) search and information costs and (2) bargaining and decision costs. The third category, (3) policing and enforcement costs, relates to the principal-agent problem and incomplete contracts, which have been the focus of the literature on transaction costs following Williamson (1979).

[2] In their seminal NBER survey, Stigler and Kindahl (1970) found that about half of the commodities in their sample were purchased with fixed-term contracts. A more recent comprehensive survey has not been conducted and would be welcome. Fixed-price contracts constitute over 90 percent of U.S. federal government contracts (source: Federal Procurement Data System), and, anecdotally, remain predominant in the private sector.

[3] As described to the author by several procurement and purchasing personnel.

schedule faced by the buyer. I apply the model in the context of federal procurement for a standardized product, and I find that market transaction costs are large and economically meaningful, comprising 10.9 percent of total buyer costs. Thus, I provide the first estimates of transaction costs in intermediate goods markets.[4]

The model of optimal contract duration and its main implications are presented in Section 2. One novel prediction of the model is that, in equilibrium, a longer contract would result in a higher price. This arises from straightforward economic logic: if a longer duration would reduce the price, the buyer would prefer it, because it would also reduce the burden of transaction costs.[5] A price schedule that is increasing with duration is the natural result of time-varying supply costs among suppliers, and it may arise from, e.g., capacity constraints or idiosyncratic outside options. Therefore, the model provides an economic justification for shorter contracts.

I focus on a setting in which goods and services are standardized, there is little uncertainty, and relationship-specific investments are negligible. Even in these straightforward economic environments, the duration decision is non-trivial, depending on (1) the magnitude of the transaction costs, (2) the degree of competition, and (3) the stochastic properties of the underlying supply costs. I present a simplified version of the model to provide some intuition about these features. Perhaps surprisingly, the optimal duration is non-monotonic in the degree of competition. With few suppliers, long-term contracts are optimal, as the benefit of re-selecting a supplier is small. This benefit increases as the number of suppliers increases, resulting in short-term contracts at moderate levels of competition. With many suppliers, the buyer can secure a low-enough price for a long-term contract that they are once again optimal. Likewise, higher variance in supply costs could lead to longer or shorter contracts. These ambiguous predictions help motivate a structural approach to estimation.

For an empirical application, I select a specialized setting that allows me to isolate the tradeoff described above and recover estimates of market transaction costs. I construct a unique dataset of 1,046 contracts for building cleaning services for the U.S. federal government. Consistent with the model, duration is determined ex ante by the local government agency, and, for the contracts I analyze, the government is required to go to market at the expiration of the previous contract. Importantly, building cleaning services are standardized, supply-side conditions are stable, and relationship-specific investments are small.[6] This suggests that abstracting away

[4]Economists studying the effects of transaction costs have primarily pursued the testable implications of these costs, rather than their direct estimation (see, e.g., Monteverde and Teece, 1982; Walker and Weber, 1984). One recent exception is Atalay et al. (2017), who construct a measurement of external transaction costs by examining input flows between integrated and non-integrated firms across sectors. Likewise, the literature on contract duration has also focused on testable implications rather than structural modeling.

[5]By contrast, in the transaction costs literature, longer contracts tend to reduce supply costs by solving ex post incentive problems.

[6]Ex post incentive problems, which are a large focus of the contract literature, are not a first-order concern here. Performance is observable, contracts are rarely canceled, and the suppliers in this market are generally well-established firms. Hyytinen et al. (2018), who study cleaning contracts in Sweden, make similar observations and also assume complete contracts. The detailed specifications of these contracts leave little room for noncontractible performance. As is common in the auction literature, Hyytinen et al. (2018) do not analyze the duration margin.

from other contracting concerns may be reasonable, and it allows me to focus on identifying the direct (Coasian) costs of going to the market.

For context, each year the federal government manages over one million contracts with an annual value of less than $1 million. These constitute 97 percent of all federal contracts and are disproportionally made with fixed-price, fixed-duration contracts through competitive procedures.[7] In prices, the estimation sample is roughly comparable to these contracts and closely resembles the full set of building cleaning contracts. The sample is presented in Section 3, along with descriptive regressions that are used to motivate the structural model. I verify a core prediction of the model: in the data, longer contracts are more expensive. Therefore, time-varying supply costs appear to outweigh potential supplier-side benefits from a longer contract (e.g., learning), which are likely small in this setting.

Section 4 presents the specific modeling assumptions and parameterizations used to take the model to data. Consistent with the empirical setting, the model takes three stages: (1) the buyer's duration decision, (2) a participation decision by suppliers, and (3) a first-price auction. Thus, compared to a standard auction model with endogenous entry, the model also incorporates a strategic decision by the buyer (duration). As in Krasnokutskaya (2011), I allow for unobserved heterogeneity across auctions. I show that the joint distributions of private costs and unobserved heterogeneity are identified when only the winning bid and the number of bidders are observed, thus extending identification to data that are more broadly available.[8] Intuitively, variation in the number of bids shifts the distribution of the private component in a known way, while the distribution of auction-specific heterogeneity is unaffected.

Section 5 presents the model estimates. The median estimated transaction cost is $10,400, representing a meaningful portion of total buyer costs. In magnitude, the estimates are roughly comparable to back-of-the-envelope calculations of the labor costs of procurement specialists. Further, I find that these costs are correlated with other observables in ways that align with our intuition. For example, the costs are positively correlated with the complexity of the facility: contracts for medical buildings have much higher transaction costs than those for offices.

As the model illustrates, the impact of transaction costs on prices and welfare depends on both the magnitude of these costs and the buyer's ability to respond by changing the duration of the contract. I consider the value to the buyer of this strategic response through a counterfactual analysis in Section 6. Instead of optimizing for each contract, I consider an alternative policy where all contracts are issued with a standard duration. Mandating more frequent transactions could be costly: issuing only one-year contracts would increase total costs by 37 percent. Of

[7]Source: Federal Procurement Data System.

[8]Previous approaches relied on observing either multiple bids per auction (Krasnokutskaya, 2011; Hu et al., 2013) or a reservation price (Roberts, 2013). Aradillas-López et al. (2013) exploit variation in the number bids for second-price auctions, though the identification results of their paper are limited to constructing bounds on surplus. Concurrent work by Quint (2015) exploits variation in the number of bidders in a model with additively separable unobserved heterogeneity. That identification strategy does not translate to the more common multiplicative structure examined here.

standard contracts with full-year durations, the four-year standard term has the lowest impact, increasing total costs by 1.4 percent. Therefore, a poorly chosen standard could substantially increase costs, but an informed standard may have modest effects.

As a second counterfactual, I consider the impact of endogenous contracts on the estimation of welfare effects. To illustrate the importance of this margin, I consider the pass-through of a reduction in supply costs. When buyers can adjust duration, the pass-through to prices is reduced by 10 percent, compared to a world in which duration is held fixed. Thus, appropriately modeling contract duration can change the interpretation of observed price changes and the estimation of welfare effects. Further, transaction costs may be a sizable portion of total costs and should be accounted for in addition to any price effects.[9]

A novel contribution of this paper is an empirical model of optimal contract duration. To the best of my knowledge, this is the first model to illustrate a general cost of longer contracts, which arises from suboptimal buyer-seller matching over time. The previous literature on contract duration has focused on ex post coordination problems, primarily through costly renegotiation (Masten and Crocker, 1985) and relationship-specific investments (Joskow, 1987). Recent empirical work on these features (e.g., Decarolis, 2014; Bajari et al., 2014) focuses on one-time projects and therefore does not model repeated demand. As discussed above, I abstract away from such ex post incentive problems and focus on "recurrent spot contracting," in the terminology of Williamson (1979). For commodities and simple products, finding the lowest-cost supplier is often more important than whether buyer and seller incentives are properly aligned. I am also able to test for and abstract away from incumbency advantage, which is often a concern in settings with repeated contracts (see, e.g., Greenstein, 1993). My work is complementary to models with these features.[10]

A related empirical literature measures switching costs in consumer markets (e.g., Dubé et al., 2010; Handel, 2013; Honka, 2014; Luco, 2019). These studies also use a revealed-preference approach to recover switching costs, using a different identification strategy that is made possible by the economic environment. Conceptually, switching costs in consumer markets can be inferred from posted prices,[11] whereas contract prices for intermediate goods are idiosyncratic to the buyer-seller match. Additionally, the switching costs literature tends to take supply costs as exogenous, whereas variation in supply costs is a key factor in the decision to switch suppliers in my setting. As one might expect, I find that transaction costs in intermediate goods markets are substantially higher than consumer switching costs.[12]

[9]Carlton and Keating (2015) emphasize the role of transaction costs in welfare analysis when the affected variable is not simply the price level, through the effect on a firm's ability to implement nonlinear pricing.

[10]The tradeoff in this paper between transaction costs and price is closely related to the models of contract duration of Dye (1985) and Gray (1978), who take the stochastic price process as given. An innovation of this paper is to use tools of industrial organization to model primitives of the price process and explore its implications.

[11]See, for example, Dubé et al. (2010) for orange juice and margarine or Elzinga and Mills (1998) for wholesale cigarettes. The wholesale market in the analysis of Elzinga and Mills (1998) mirrors a consumer market in that pricing, though nonlinear, is uniformly applied.

[12]A key feature of consumer markets is an inability to contract on future prices, leading to models that weigh an

The theoretical and empirical analysis of the costs of market transactions has typically been cast in light of the decision to vertically integrate (for a summary, see Lafontaine and Slade, 2007). As integration often induces additional benefits from the alignment of ex post incentives, disentangling the direct cost of transacting on the market from, for example, the cost of hold-up can be challenging. By focusing on the question of contract duration, I isolate the direct costs of transacting on the market while abstracting away from other features of vertical integration. Of course, an increase in these costs should also increase the propensity to vertically integrate, as noted by Coase (1960).

2 Model

I introduce the model of a buyer with demand for a good or service for many periods. The buyer chooses the duration of the contract, balancing the per-period payment to suppliers with an ex ante cost realized at the beginning of each contract. I provide a numerical example to illustrate this key tradeoff. A central empirical implication of the model is that it may be applied to recover unobserved transaction costs. The model does not capture every real-world consideration, but its representation of a key contracting tradeoff provides the basis for an empirical investigation into the magnitudes of transaction costs.

2.1 The Buyer's Problem

A risk-neutral buyer has inelastic demand for a good or service over many (infinite) future periods. The buyer selects a single seller and can commit to buy from that seller for multiple periods with a contract. The buyer announces the duration of the contract (T) in advance of implementing a market mechanism, which is used to select the seller and determine price. Each time the buyer uses the mechanism, it costs the buyer δ.

The game proceeds in three stages. First, the buyer determines duration T after observing characteristics of the service $\boldsymbol{x}$, market conditions $\boldsymbol{m}$, and the mechanism cost. Second, N suppliers decide to participate in the market mechanism after observing $(T, \boldsymbol{x}, \boldsymbol{m})$. Contract characteristics $(T, \boldsymbol{x})$ affect the per-period supply costs, while market conditions affect entry costs (through outside options). Third, a supplier is selected with a per-period stochastic price $P(N, T, \boldsymbol{x}, \boldsymbol{m})$, where the price distribution may depend on the duration of the contract and the number of sellers.

Let $\overline{P}$ denote the ex ante expected price conditional on $(T, \boldsymbol{x}, \boldsymbol{m})$, so that $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m}) = \sum_{n=1}^{\mathbb{N}} \left(E[P(n, T, \boldsymbol{x}, \boldsymbol{m})] \cdot \Pr(N = n | T, \boldsymbol{x}, \boldsymbol{m})\right)$. The buyer expects market conditions and mechanism costs to remain the same in future periods. With this assumption, we use $\overline{P}(T)$ in the exposition below as shorthand, suppressing $(\boldsymbol{x}, \boldsymbol{m})$.

"investing" effect versus a "harvesting" effect (Klemperer, 1995; Rhodes, 2014; Cabral, 2016). When buyers and sellers agree on future prices, as in this paper, these effects are competed away.

The value function for the buyer in period τ who has not yet chosen a seller can be expressed as

$$V(\tau) = \min_{T} \delta + \sum_{k=1}^{T} \beta^{k-1} \overline{P}(T) + \beta^{T} V(\tau + T). \tag{1}$$

After incurring the cost to determine the seller and the price (δ), the buyer pays $\overline{P}(T)$ for T periods and returns to the decision problem in period $\tau + T$. The buyer discounts future periods at rate β.

For an optimal T, it must be that, for any other duration S:

$$\delta + \sum_{l=1}^{T} \beta^{T-1} \overline{P}(T) + \beta^{T} V(\tau + T) \leq \delta + \sum_{l=1}^{S} \beta^{S-1} \overline{P}(S) + \beta^{S} V(\tau + S). \tag{2}$$

We can expand each side of the equation by iterating forward to period $\tau + T \cdot S$. As the buyer expects market conditions to persist, the problem is stationary. If T is optimal in period τ, the buyer expects T to be optimal at the expiration of a contract in a future period, e.g., in period $\tau + T$. Plugging in a sequence of contracts of duration T and S, we obtain

$$\begin{aligned} &\sum_{l=1}^{S} \beta^{T(l-1)} \left(\delta + \sum_{k=1}^{T} \beta^{k-1} \overline{P}(T) \right) + \beta^{T \cdot S} V(\tau + T \cdot S) \\ \leq &\sum_{l=1}^{T} \beta^{S(l-1)} \left(\delta + \sum_{k=1}^{S} \beta^{k-1} \overline{P}(S) \right) + \beta^{T \cdot S} V(\tau + T \cdot S). \end{aligned} \tag{3}$$

That is, the buyer may pay a per-period price of $\overline{P}(T)$ while running the market mechanism S times in $S \cdot T$ periods, or a per-period price of $\overline{P}(S)$ while running the mechanism T times over the same horizon.

Rearranging,[13] we obtain the optimality condition

$$\overline{P}(T) - \overline{P}(S) \leq \frac{\delta}{\sum_{k=1}^{S} \beta^{k-1}} - \frac{\delta}{\sum_{k=1}^{T} \beta^{k-1}}. \tag{4}$$

This formulation has straightforward interpretation. The left-hand side is the per-period savings by choosing contract S instead of T. The right-hand side is the increase in amortized transaction costs from choosing S instead of T. Thus, at the optimal contract, potential savings in the per-period price by selecting a different (shorter) duration are less than increased transaction costs

[13] We can cancel out the continuation value component from the inequality, and, with the substitutions $\delta = \sum_{k=1}^{T} \beta^{k-1} \frac{\delta}{\sum_{k=1}^{T} \beta^{k-1}}$ on the left-hand side and $\delta = \sum_{k=1}^{S} \beta^{k-1} \frac{\delta}{\sum_{k=1}^{S} \beta^{k-1}}$ on the right-hand side, we can factor out the common aggregate discount factor $\sum_{k=1}^{TS} \beta^{k-1} = \sum_{l=1}^{S} \beta^{T(l-1)} \sum_{k=1}^{T} \beta^{k-1} = \sum_{l=1}^{T} \beta^{S(l-1)} \sum_{k=1}^{S} \beta^{k-1}$. This results in

$$\left(\sum_{k=1}^{TS} \beta^{k-1} \right) \left(\frac{\delta}{\sum_{k=1}^{T} \beta^{k-1}} + \overline{P}(T) \right) \leq \left(\sum_{k=1}^{TS} \beta^{k-1} \right) \left(\frac{\delta}{\sum_{k=1}^{S} \beta^{k-1}} + \overline{P}(S) \right),$$

and equation (4) follows.

from using the market mechanism more frequently.

Given realizations for contract and market characteristics $\boldsymbol{x}$ and $\boldsymbol{m}$, the optimal duration, T^*, is therefore given by

$$T^* = \arg\min_{T \in \mathbb{T}} \overline{P}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{\sum_{k=1}^{T} \beta^{k-1}} \tag{5}$$

where $\mathbb{T}$ is the set of allowable durations. Intuitively, this expression shows the buyer's objective is to minimize the sum of the per-period supply price and amortized transactions cost.

The optimality condition generates two fundamental results, which we express as our first propositions:

Proposition 1. *If the optimal contract is not the maximum allowable duration (i.e., an interior solution exits), then the expected per-period price is increasing with the duration of the contract*

Proposition 2. *If an interior solution exits, then the optimal duration is increasing with transaction costs.*

Proof. See Appendix A. □

The second proposition is intuitive. The first proposition is a direct result of having ex ante costs for the market mechanism. The buyer can always reduce these (amortized) costs by choosing a longer contract. Therefore, if the buyer chooses something other than the maximum duration, it must be that the buyer expects the marginal increase in the per-period price to offset the decline in transaction costs. As illustrated below, the per-period price will be increasing when suppliers have idiosyncratic variation in supply costs. This variation causes the low-cost supplier changes over time and provides a benefit of shorter contracts.

2.2 Illustrative Example

To illustrate the key tradeoff of this model and its implications, consider a stylized example. Suppose there are N symmetric suppliers in the market, and the set of suppliers stays the same in every period. The buyer can only issue single-period or two-period contracts. Under these conditions, the buyer only has to consider the effects of his decision over the next two periods. Thus, the analysis of the infinite-horizon problem in this example is equivalent to that of a two-period problem, and I describe it as such for clarity.

Suppliers are risk-neutral and participate in an auction to win the contract. Every supplier participates in the auction (entry is exogenous). Thus, the mechanism is efficient. The per-period cost to each supplier is the random variable c. The distribution of c is stable across periods, but the realizations for each supplier may vary over time. When the buyer issues single-period contracts, the per-period cost of the winning supplier is $c_{1:N}$, which is the minimum of N draws of c. When the buyer issues a two-period contract, the average per-period costs for each supplier is the average of two draws, $\tilde{c} = \frac{1}{2}(c^{(1)} + c^{(2)})$, and the cost to the winning supplier is $\tilde{c}_{1:N}$.

This brings us to a key feature about costs in the model:

Remark 1 By changing the duration of the contract, the buyer changes the effective per-period cost structure faced by suppliers.

As long as the per-period costs c are not perfectly correlated across periods, $\tilde{c} \neq c$ and $Var(\tilde{c}) < Var(c)$. As suppliers' bids will reflect the average cost over the duration of the contract, the distribution of per-period costs changes with contract duration. When the distribution of supply costs is stable over time, this serves to reduce the variance of cost draws. The cost of a longer contract is that the low-cost supplier may not be selected in each period. In the absence of transaction costs, short-term contracts would be optimal.

Risk-neutrality and symmetry generate the standard auction result that the expected winning bid is equal to the second-order statistic from the cost draws. Thus, the buyer-optimal contract solves

$$\min\{\underbrace{2E[c_{2:N}] + 2\delta}_{\textit{short-term}}, \underbrace{2E[\tilde{c}_{2:N}] + \delta}_{\textit{long-term}}\} \tag{6}$$

The buyer will pick the long-term contract if the increase in expected supply costs is less than the reduction in (amortized) transaction costs, i.e., if

$$E[\tilde{c}_{2:N}] - E[c_{2:N}] < \frac{\delta}{2}. \tag{7}$$

This condition mirrors the optimality condition in equation (4).

This simple example illustrates a second key feature of the model:

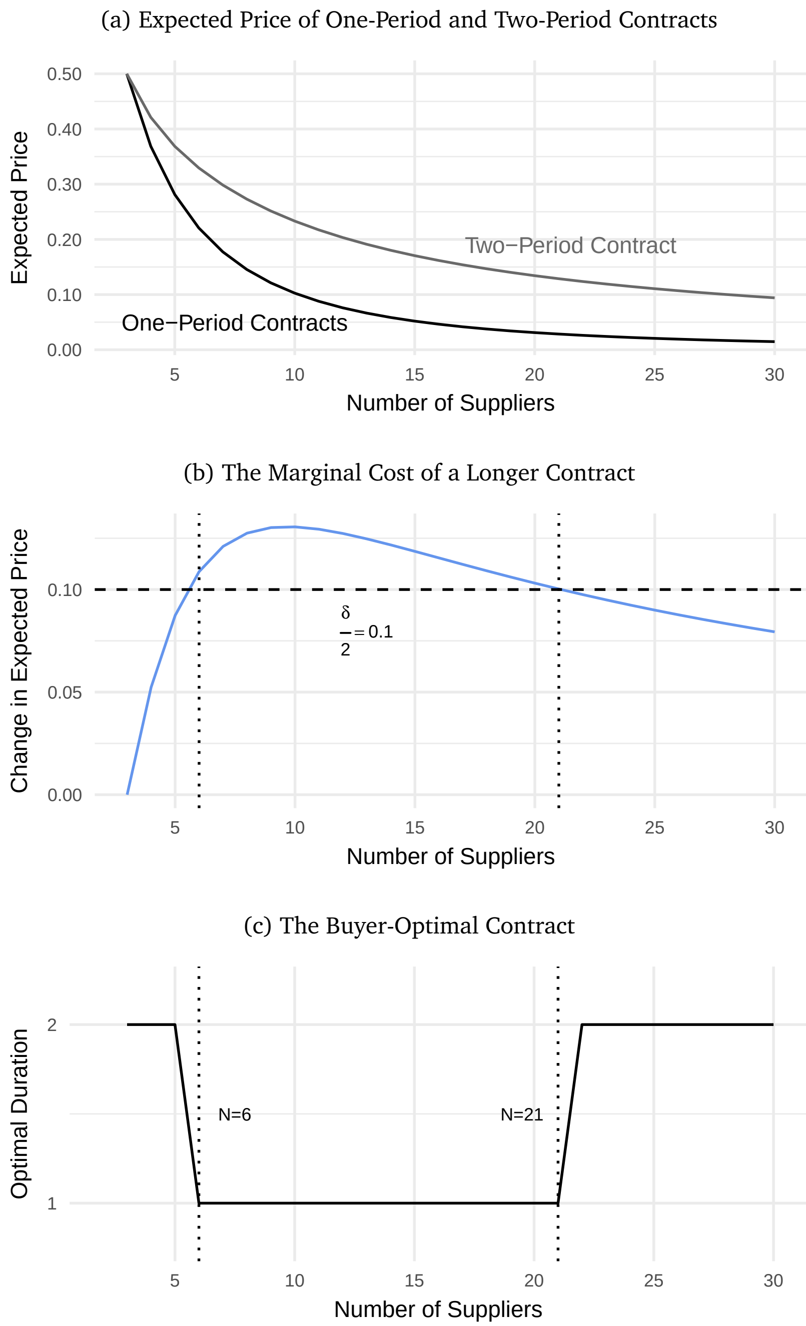
Remark 2 The intensity of supply-side competition, in terms of the number of participating suppliers, affects the optimal contract by changing the per-period cost structure.

Variation in N affects the left-hand side of (7), changing the marginal effect of a longer contract on the per-period price. This marginal effect is non-monotonic in N, so an increase in the number of suppliers has an ambiguous effect on equilibrium contracts. Therefore, the optimal duration may be decreasing, increasing, or U-shaped with N. I describe the intuition for this result below along with the numerical example.

To illustrate the above features, I present a numerical example in which the per-period costs are drawn independently over time from a beta distribution with shape parameters $(0.5, 0.5)$. Recall that the beta distribution has support $[0, 1]$. With shape parameters $(1, 1)$ it is equivalent to a uniform distribution, and as the shape parameters approach zero it approaches a Bernoulli distribution.

Figure 1 illustrates how expected buyer costs vary with contract duration and the degree of competition. Panel (a) plots the expected supply price for one-period contracts and a two-

Figure 1: Competition, Costs, and Contract Duration: A Numerical Example



Notes: Panel (a) plots the expected per-period costs for separate one-period contracts and a bundled two-period contract, as a function of the number of bids. The blue line in panel (b) is the difference between the two, which is the expected price increase to the buyer. The dashed line in panel (b) reflects a transaction cost of 0.2 amortized over two periods, which is the amount saved by issuing a two-period bundled contract. For values of N where the blue line is above the dashed line ($N \in \{6, \ldots, 21\}$), short-term contracts are optimal, as the increase in supply costs from the long-term contract is greater than the savings in transaction costs. Panel (c) plots the buyer-optimal contract duration.

period contract. For $N = 3$, the expected prices are the same, and for $N > 3$ the single-period contracts always have a lower expected price. The blue line in panel (b) plots the difference between these two lines. This difference is equivalent to the left-hand side of equation 7 and is non-monotonic in the number of suppliers. The dashed line indicates a transaction cost of 0.20, which is amortized by two periods. When the blue line falls above this dashed line, the increase in the expected supply price exceeds the savings in transaction costs, and one-period contracts are optimal. Panel (c) plots the U-shaped buyer-optimal duration as a function of N. Short-term contracts are optimal for moderate level of competition; in this case, when $N \in \{6, ..., 21\}$.

This stylized example conveys a general insight from the model. For low levels of competition, the benefit of switching suppliers is low, and long-term contracts are preferred. At moderate levels of competition, there is an increased benefit of switching among suppliers more frequently. When competition is intense, the expected costs of both long-term and short-term contracts approach the lower bound of costs, and therefore long-term contracts, which minimize transaction costs, are optimal.

Thus, even in a stylized example, the directional predictions of the model are empirical, depending on particular contract or market conditions. This finding further motivates the use of a structural approach to assess the impacts of transaction costs. The model also generates a set of predictions related to the stochastic properties of per-period costs to suppliers. Higher autocorrelation in supply costs will lead to longer contracts, while higher variance in per-period supply costs can have effects in either direction. As the remainder of the paper focuses on the structural approach, I discuss these in more detail in Appendix A.

2.3 Identification of Transaction Costs via Revealed Preference

I now discuss the empirical strategy for recovering unobserved translation costs. The optimality condition for the buyer generates a contract-specific implied value for δ that rationalizes the observed duration. Thus, by revealed preference, we can recover δ for each contract.

If a contract T is optimal, then, relative to contract S, we have the following optimality condition, which is a re-expression of equation (4):

$$\left(\sum_{k=1}^{T} \beta^{k-1}\right)\left(\sum_{k=1}^{S} \beta^{k-1}\right)\left(\overline{P}(T) - \overline{P}(S)\right) \leq \sum_{k=1}^{T} \beta^{k-1}\delta - \sum_{k=1}^{S} \beta^{k-1}\delta. \tag{8}$$

By comparing the optimal contract to one that is one period shorter ($S = T - 1$), we obtain the inequality

$$\delta \geq \frac{1}{\beta^{T-1}}\left(\sum_{k=1}^{T} \beta^{k-1}\right)\left(\sum_{k=1}^{T-1} \beta^{k-1}\right)\left(\overline{P}(T) - \overline{P}(T-1)\right). \tag{9}$$

Likewise, a comparison of of T to $S = T + 1$ obtains

$$\delta \leq \frac{1}{\beta^T} \left(\sum_{k=1}^{T+1} \beta^{k-1} \right) \left(\sum_{k=1}^{T} \beta^{k-1} \right) \left(\overline{P}(T+1) - \overline{P}(T) \right). \tag{10}$$

In principle, one could generate an inequality for every element in the duration choice set $\mathbb{T}$, excluding the chosen contract. The minimum upper bound and the maximum lower bound are sufficient bounds on δ. When $\overline{P}(T)$ is monotonic, the two inequalities above will capture the full information about δ.

The right-hand sides of the inequalities depend only on the discount rate and the expected per-period price function. The next result follows immediately:

Proposition 3. *When β and $\overline{P}(T)$ are known, bounds for contract-specific realizations of δ are identified.*

The tightness of the bounds depends on the slope of the function $\overline{P}(T)$ and the discount rate β. Effectively, these parameters are governed by the length of each period. Thus far, we have used a discrete formulation where 1 unit separated each period. If the length of this unit became infinitesimally short, then T is chosen from a continuous set and we obtain point identification of δ. We present this as a corollary:

Corollary 1. *When T is continuous (i.e., the duration of each period approaches zero), then δ is point identified for each contract.*

Proof. See Appendix A. □

Even in the discrete case, the full distribution of δ can be identified from additional assumptions on the relationship between δ and $\boldsymbol{x}$ or $\boldsymbol{m}$. This distribution can be used as a prior over the bounds. Recall that $\overline{P}(T)$ is shorthand for $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$.

Proposition 4. *Assume that there exists a special covariate w that is an element of $\boldsymbol{x}$ or $\boldsymbol{m}$. This covariate meets the following two conditions: (i) δ and w are independent and (ii) $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$ varies continuously with w. Local variation in w provides local identification of the distribution of δ. When sufficient variation in $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$ is generated by variation in w, the distribution of δ is identified.*

Proof. Under the above conditions, the bounds (9) and (10) vary continuously with w. Therefore, the cumulative distribution function of δ is identified. □

The model provides a straightforward way to recover unobserved contract-specific transaction costs. A key input to this procedure is the duration-dependent function $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$. Intuitively, observed variation in contract duration will be necessary to estimate this function when not known a priori. I discuss the empirical strategy to estimate this object in Section 4.

2.4 Discussion

Setting

The model captures a setting in which the buyer chooses fixed-price, fixed-duration contracts. This type of contract is quite common. They constitute the vast majority of government service contracts, and, anecdotally, are used quite frequently in the private sector. Why might a buyer choose this type of contract instead of allowing for options to renew or a less formal structure? Some possible reasons are (a) to maximize the number of suppliers that bid on the next contract and (b) to protect against favoritism by the buyer's agent (at the expense of the buyer) through increased transparency. I take the contract structure as given; an analysis of why fixed-duration contracts are prevalent lies outside the scope of this paper.

The model abstracts away from ex post transaction costs that arise from the principal-agent problem and incomplete contracts. These considerations have been studied in detail by the transaction costs literature. The focus of this paper is on illustrating a new fundamental mechanism, which may be important even when ex post transaction costs are negligible or complete contracts are possible. In some settings, a richer model that accounts for both ex ante and ex post transaction costs may be appropriate. Ex post transaction costs tend to suggest longer contracts, as longer contracts can offer a greater return and better align incentives. By contrast, the model of this paper illuminates that time-varying supply costs make longer contracts more costly, providing a reason why shorter contracts may be preferred. Even with ex post transaction costs, the per-period price should be increasing in duration at the equilibrium. If not, the buyer will opt for a longer contract that reduces the supply price and (both ex ante and ex post) transaction costs.

One simplifying assumption of the model is that the market transaction costs are fixed. In some settings, we may expect that these cost vary with the duration of the contract. Longer contracts could require more market research by the buyer or additional costs of drawing up the contract. This is not a first-order consideration for simple goods and services. The language of the contracts in the empirical application is not duration-dependent, likely reflecting the standardized nature of the service and stable market conditions.

Another simplification is that suppliers have perfect foresight about future costs. A more general setup with imperfect information shares the same qualitative features of this model, though there is an additional ex post inefficiency arising from imperfect information. However, fixed-price contracts eliminate this risk to the buyer.

Efficiency

The duration of a fixed-price, fixed-duration contract is typically determined by the buyer. Thus, the analysis of this paper focuses on buyer-optimal contracts, though the intuition translates to efficient contracts as well. The buyer is concerned with the supply price, which shares similar stochastic properties to supply costs in most settings. For example, in the illustrative example above, expected supply cost and expected price are the first and second order statistics from the cost distribution, which generate similar directional predictions. In the efficient case, supply-side frictions such as entry costs should also be incorporated for when analyzing welfare.

Though the qualitative features of the buyer-optimal and the efficient contract are similar, they are not identical. This raises a questions about property rights: should the buyer or the sellers be endowed with the ability to decide duration? In Appendix B, I analyze the allocation of term-setting rights. Contracts that are determined by market participants (buyers and sellers) may be too long or too short, resulting in wasteful social costs. Counterintuitively, these extra costs may increase as a market becomes more competitive. Therefore, when ex ante costs are taken into account, highly competitive markets may be of more concern for regulators than those that are more concentrated.[14]

Bundling

The contract duration decision is theoretically linked to a simultaneous bundling problem, where the contract bundles demand over time. The analysis here could be reinterpreted to allow T to represent the bundle size and δ represent the transaction cost for each bundle. Thus, we obtain predictions relating the underlying variance of costs (or valuations) to the optimal bundle size, as well as the effect of competition on optimal bundling. In the bundling literature, Zhou (2017) and Palfrey (1983) provide the most closely related analogs and generate complementary insights. In this paper, I demonstrate that the smaller variance induced by bundling reduces total surplus when there are no transaction costs.[15]

3 Empirical Application: Data and Reduced-Form Analysis

3.1 Data

To estimate the costs of market transactions, I construct a dataset of 1,046 competitive contracts for building cleaning services for the United States federal government. This market provides a relatively clean case study to analyze the duration decision and estimate costs. For many commodity products and services, the buyer (a government agency) is compelled to run a

[14]This result occurs because market participants care about price rather than cost, and the price responds more quickly to a change in contract duration when the number of bidders is large. If we think of expected price as the expected second-order statistic, and the cost as the first-order statistic, then we have some intuition for why this could be true. The second-order statistic responds more strongly to a change in variance (or mean) than the first-order statistic when the number of draws is large and the cost distribution is bounded from below. The buyer (or seller) internalizes the duration's effect on the second-order statistic rather than its effect on the first-order statistic.

[15]Compared to Zhou (2017) and Palfrey (1983), I allow for intermediate degrees of bundling and introduce transaction costs. Salinger (1995) and Bakos and Brynjolfsson (1999) note that bundling affects prices by reducing the variance of average valuations. Cantillon and Pesendorfer (2006) share this insight in their analysis of combination bidding for multi-unit auctions.

sealed-bid auction for a contract of a pre-determined duration at the expiration of the previous contract. Thus, for many federal procurement contracts, the empirical setting is aligned with the model of the previous section. The sample period is October 2003 to May 2017.

A general empirical challenge is that procured goods and services may have heterogeneity that is multi-dimensional and difficult to quantify. Thus, I focus on commodity-like goods and services with standard cost structures. Indeed, products of this sort are numerous in procurement and make up a significant portion of all transactions.[16] Of the set of commodity-like products, building cleaning services were chosen because they are numerous, cost factors are easily quantified, and there is a lot of variation in contract duration. Finally, demand is inelastic, as there were no significant substitutes. The market for such services is sizable; the federal government spent $1.2 billion annually on such services. In addition, the standardized nature of the work, along with the fact that the contracts are rarely terminated, mitigates concerns about the impact of relationship-specific investments in this setting.

To the best of the author's knowledge, this is the first dataset on contracts to combine measures price, duration, and competition, which are the key outcomes of the model. To construct this dataset, I combined detailed location, price, and vendor information maintained in the Federal Procurement Data System (FPDS)[17] with contract-specific documents downloaded from the Federal Business Opportunities (FedBizOpps) website. By law, the FPDS keeps public records of all contracts for the U.S. federal government, and its data has been used in recent empirical work (e.g., Kang and Miller, 2017; Bhattacharya, 2018; Decarolis et al., 2018). I was able to identify 4,119 contracts that appeared in both sources. The final sample was restricted to competitive contracts in the United States that received more than one bid, had an annual price of less than $1 million, and included square footage in the text of the contract documents, which is a key cost factor. These contracts span different types of facilities and government agencies. For additional details on the construction of the sample, see Appendix C.1.

I matched the contract-specific dataset with auxiliary datasets of (1) government contracting expenditures at the same location in related products and (2) local labor market conditions. Local labor market conditions include county-level unemployment from the Local Area Unemployment Statistics and the number of NAICS-code level establishments in the same 3-digit ZIP code from the County Business Patterns data.

[16]For context, 97 percent of federal government contracts during the period had an annual price of under $1 million. A counter-example of the ideal setting for this sort of analysis might be a customized, large-scale computer software system for an agency.

[17]These data were obtained from USASpending.gov.

3.2 Institutional Details

Competitive contracts are posted publicly and allow open competition from registered vendors.[18] Many of these contracts are posted on the centralized web portal FedBizOpps.gov, from which I collected the data in this analysis. On the website, a prospective supplier can view the contract details, including contract duration and the square footage of the building, requirements for the job, and a list of interested suppliers. From the portal, a supplier submits a bid to the contracting office that includes the total price over the duration of the contract. The contracting office determines the winning supplier primarily based on the lowest price. By law, the contracting office must justify selecting other than the lowest-price offer.[19]

Importantly, contract duration is determined by the local contracting office and varies from contract to contract, even within an agency. As several industry personnel described to the author, contract duration is a balance between minimizing the administrative costs of re-contracting and realizing the benefits from re-competing more frequently. Costs may be increasing with duration because suppliers charge a premium or because the buyer ends up locked in to a high-cost supplier. Transaction costs and competition are key motivating factors for the duration choice. These details motivate using this market as a case study.

Contracts include specifications for the tasks to be done and their frequencies. For building cleaning, tasks include mopping, vacuuming carpets, picking up debris, dusting, and emptying trash cans. For an example list of specifications, see Appendix C.5. Contract documents are extensive, and multiple documents are often posted for each solicitation. The median primary document runs 49 pages.

The buildings to be cleaned are categorized into offices (694), research facilities (111), medical facilities (61), service centers (59), visitor centers (41), airports (30), technical facilities (19), accommodations (18), and industrial facilities (13). Offices are split into standard offices (424) and field offices (270), which also have an auxiliary building, such as an exercise room, a bunkhouse, or a small warehouse. Appendix C.3 provides a breakdown by the issuing agency and by subcategory.

3.3 Summary Statistics

Summary statistics for the contracts are displayed in Table 1. Contracts vary in price, duration, and the number of bids. As shown later in this section, much of the variation in price can be captured by the square footage of the building and the weekly cleaning frequency. For the

[18] These contracts fall under three categories: Full and Open Competition, Full and Open Competition after the Exclusion of Sources, and Competed Under Simplified Acquisition. 86 percent of the contracts deemed Full and Open Competition after the Exclusion of Sources are listed as a small business set-aside. As 96 percent of the contracts are won by small businesses (as determined by the contracting officer), I ignore this distinction for the purposes of analysis. See Federal Acquisition Regulation (FAR) Part 5.

[19] Based on the guidelines established by FAR and conversations with local contracting officers, one reason the contracting office might request an exception is to select a bidder that has an established history.

Table 1: Summary Statistics

	Mean	Min	p25	Median	p75	Max
Price (Annual, $)	43,870	1,112	7,259	13,180	26,731	976,538
Contract Value ($)	190,200	2,914	28,500	50,550	102,000	4,882,692
Duration (Years)	4.2	0.4	3.0	5.0	5.0	6.5
Square Footage	25,701	145	3,700	7,000	14,500	2,031,842
Price per Square Foot	2.91	0.16	1.32	2.01	3.14	33.02
Number of Bids	6.5	2.0	4.0	5.0	8.0	40.0
Weekly Frequency	3.5	0.1	2.0	3.0	5.0	7.0
Num. Employees (Winner)	61.5	1.0	3.0	14.0	75.0	650.0
Observations	1046					

Notes: The table displays summary statistics for key variables in the contract data. Included are outcomes (price, duration, and number of bids), as well as cost characteristics such as the number of square feet and the frequency of cleaning. The last variable is the size of the winning firm, in terms of number of employees.

sample, which removes contracts greater than $1 million per year, the mean annual contract price is $43,870 and the median is $13,180. The sample contains 76 contracts with an annual price greater than $100,000.

Overall, the estimation sample compares favorably to the broader set of cleaning contracts in the FPDS. For fiscal years 2004 to 2016, an average of 6,366 cleaning contracts are in effect each year. 95 percent of these have an annual value of less than $1 million. Within this subset, the mean annual value is $70,322 and the median is $11,200. Cleaning contracts are larger in value than the average contract. For all contracts with an annual value of less than $1 million, the mean annual value is $35,469 and the median is $4,731. These contracts comprise 97 percent of all contracts in the relevant period, or an average of 1,004,991 each year.

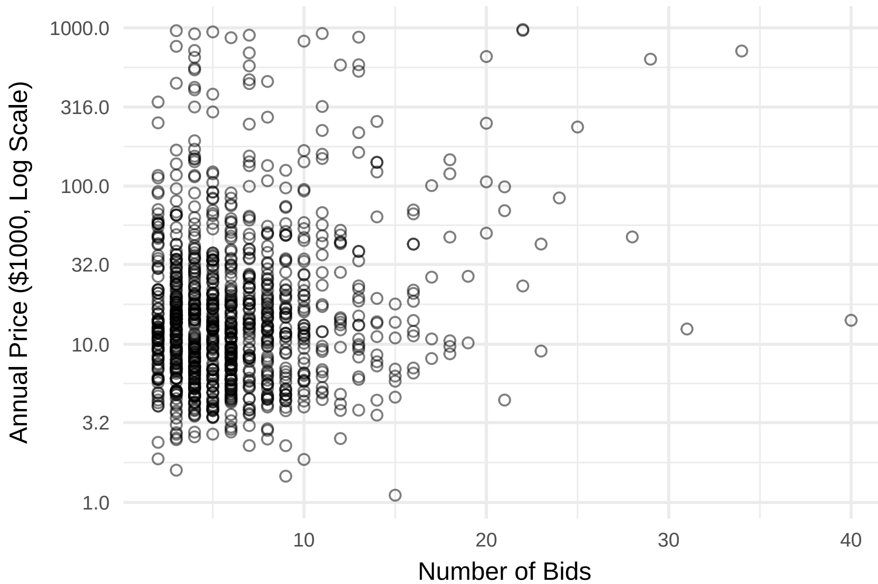
One important source of variation in the analysis is in the number of bids received. The median is 5 bids, and the maximum is 40. Thus, there is a good deal of competition for these contracts. The variation in the number of bids will help to disentangle the effect of private costs from unobserved heterogeneity in the structural analysis.

In the last row, the table provides the number of employees for the winning firms. The winning firms in this dataset are typically small, with a median of 14 employees. Over 25 percent of the winning suppliers have 3 or fewer employees.

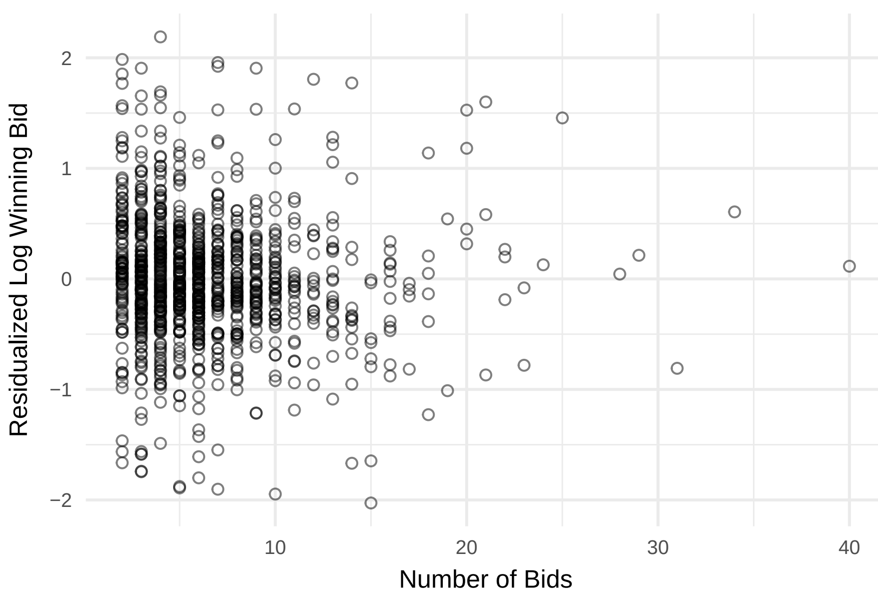
Figure 2 plots the logged values of the winning bids on the y-axis against the number of bidders on the x-axis. The second panel displays residualized values for the (log) winning bids. The residuals were constructed from a regression of price on duration, square footage, cleaning frequency, baseline unemployment, and fixed effects for facility type. Even after controlling for observable characteristics, there is large variation in prices for auctions with many bidders. The pattern observed in the figure—large variation in prices with clustering at the median price, rather than the minimum—motivates the assumption of unobserved auction-specific heterogeneity

Figure 2: Price versus Number of Bids

(a) Annual Price

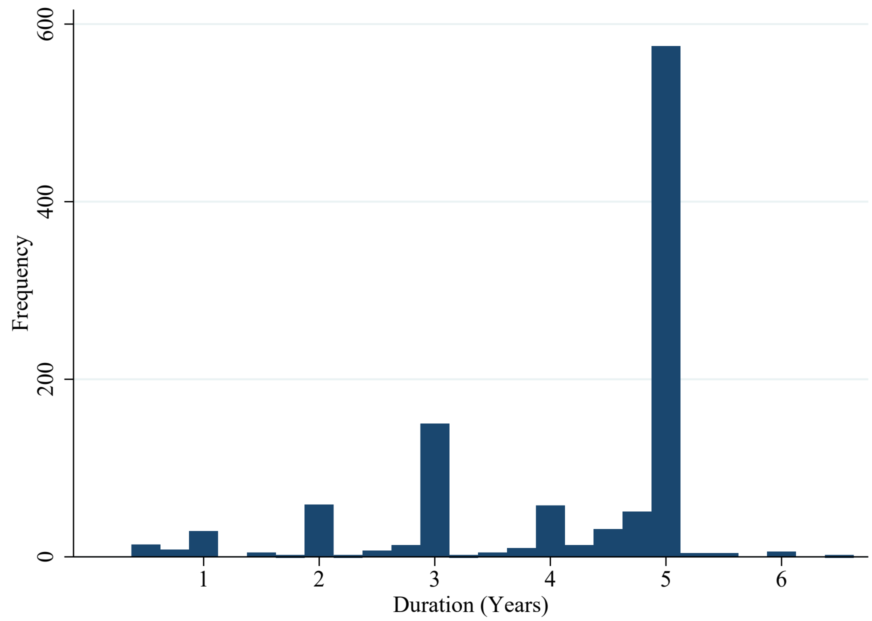


(b) Residualized Annual Price



Notes: The figure plots the log annual price against the number of bids received for each contract. There is a great deal of variation in the annual price, much of which cannot be explained by observable variables. This is illustrated by the residualized bids in the lower panel. The R^2 of the regression used to construct the residuals, which includes duration, square footage, frequency, baseline unemployment, and fixed effects for facility type, is 0.74. It is notable that some of the highest and lowest prices are realized with few bidders.

Figure 3: Contract Duration



Notes: The figure displays a histogram of contract duration in 3-month bins. Over half of the contracts have a five-year duration, which is the maximum duration (by regulation) without specifically requesting an extension. Contracts are clustered in yearly intervals, though the support in between full years is relatively well-covered.

used in the model. Though much of the variation in prices can be explained by observables, there is still residual variation that is inconsistent with an independent private values model; the model with multiplicative common costs fits far better.

The contracts in the dataset have a good deal of variation in duration. Figure 3 provides a histogram of duration in three-month intervals. There is a good deal of variation in duration, ranging from 5 months to 6.5 years, though contracts tend to cluster at yearly increments. Empirical variation in contract duration occurs within and across government agencies.[20] Notably, 53 percent of contracts are for 5 years, which is the typical maximum contract duration imposed by federal budgeting regulations. Longer durations require the contracting officer to request and justify an extension. The observed variation in duration, combined with the presence of a five-year cap on contract duration, help motivate the counterfactual analysis of Section 6, where I consider the value of the duration decision compared to standard-duration contracts.

3.4 Descriptive Regressions

To demonstrate the fit between the model and the data and to motivate specific assumptions made in estimation, I present descriptive regressions. Table 2 provides regressions of the log annual price on the number of bids, duration, and controls. The first three columns display the

[20] For summary statistics by agency, see Appendix C.4.

Table 2: Descriptive Regressions: ln(Annual Price)

	OLS-1	OLS-2	OLS-3	IV-1	IV-2
ln(Square Footage)	0.730***	0.658***	0.658***	0.689***	0.687***
	(0.018)	(0.017)	(0.017)	(0.024)	(0.024)
Number of Bids		−0.014***	−0.009*	−0.053**	−0.047**
		(0.005)	(0.005)	(0.022)	(0.022)
Duration (Years)		0.041***	0.032**	0.043***	0.033**
		(0.015)	(0.015)	(0.016)	(0.015)
ln(Weekly Frequency)		0.459***	0.394***	0.467***	0.407***
		(0.039)	(0.038)	(0.041)	(0.040)
ln(2004 Unemp.)		0.054***	0.037***	0.080***	0.060***
		(0.012)	(0.012)	(0.019)	(0.018)
High-Intensity Cleaning		0.586***		0.559***	
		(0.071)		(0.075)	
Building Type FEs			X		X
Observations	1046	1046	1046	1046	1046
R^2	0.62	0.71	0.74	0.69	0.73

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays estimated coefficients from regressions of log annual price on auction characteristics. The variables from specification IV-1 are included in the structural model. These regressions show that square footage, cleaning frequency, and market characteristics explain much of the variation in prices. Once square footage, cleaning frequency, and market characteristics are accounted for, fixed effects for location type add little explanatory power. Specifications IV-1 and IV-2 are two-stage least squares regressions, where the instruments for the number of bids are monthly (log) county-level unemployment relative to 2004, the (log) number of NAICS code 561720 establishments in the same 3-digit ZIP code in 2004, and an indicator for whether the set-aside was for generic small businesses.

results from ordinary least squares regressions. Square footage alone, as reported in the first specification, captures 62 percent of the variation in prices.

To account for endogenous entry, I instrument for the number of bidders using time-series and cross-sectional variation in local labor market conditions, as well as variation in the type of bidders permitted to compete for the contract. The first instrument is the (log) ratio of county-level unemployment relative to a 2004 baseline. This generates a time-varying county-specific unemployment shock. The second instrument is the number of establishments for NAICS code 561720 (corresponding to building cleaning services) in the same 3-digit ZIP code.[21] It is plausible that an increase in unemployment or the presence of more firms in the broader geographic area are not driven by unobservable characteristics of these contracts, yet they are likely to generate increased entry.

A third instrument is developed from the federal government practice of "setting aside" cer-

[21] I add 1 to the raw value to use the logged value in estimation, as a few contracts have zero in the raw value.

Table 3: Descriptive Regressions: Number of Bids

	(1)	(2)	(3)	(4)
Duration (Years)	0.104	−0.017	−0.002	−0.002
	(0.104)	(0.099)	(0.099)	(0.100)
ln(Square Footage)	0.760***	0.779***	0.834***	0.825***
	(0.111)	(0.106)	(0.106)	(0.112)
ln(Weekly Frequency)	0.487*	−0.081	0.009	0.137
	(0.254)	(0.247)	(0.253)	(0.257)
ln(2004 Unemp.)		−0.832***	−0.794***	−0.793***
		(0.239)	(0.238)	(0.238)
ln(Unemployment)		1.415***	1.420***	1.356***
		(0.232)	(0.231)	(0.231)
ln(Num. Firms in Zip3)		0.241	0.257*	0.276*
		(0.148)	(0.148)	(0.147)
Generic Set-Aside			1.134***	0.987***
			(0.350)	(0.361)
High-Intensity Cleaning			−0.294	
			(0.475)	
Building Type FEs				X
Observations	1046	1046	1046	1046
R^2	0.06	0.16	0.17	0.19
F-statistic	22.2	32.0	25.9	14.7

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays estimated coefficients from regressions of the number of bids on auction characteristics and local labor market variables. Specification (3) is equivalent to the first-stage regression of IV-1 in Table 2. Specification (4) includes fixed effects for each building type.

tain contracts for firms with particular types of owners. Specialized set-asides include women-owned and veteran-owned small businesses. As we have removed economically disadvantaged set-asides (e.g., for Economically Disadvantaged Women-Owned Small Business) from the sample, it is plausible that the ownership type is uncorrelated with the underlying cost structure of the participating firms. If the cost structure is independent of ownership for these firms, then the type of set-aside is a valid instrument for price (by affecting entry). This instrument is implemented as a binary variable with the value of 1 if the contract is open to all small businesses, i.e., the set-aside does not restrict entry based on characteristics of the owner.

The last two columns report the estimated coefficients from instrument variables regressions. Consistent with endogenous entry, I find a larger negative effect of the number of bidders on price compared to the corresponding OLS specifications. In the structural model of Section 5, I explicitly model entry to account for this endogeneity. The main motivating specification is IV-1, which uses square footage, weekly cleaning frequency, and baseline (2004) unemployment as controls. To capture variation in the types of buildings and cleaning required, IV-1 includes an indicator for "high intensity" cleaning of airports and medical buildings.[22] IV-2 includes indicators for all building types. The inclusion of fixed effects for all types have low in specifications OLS-3 and IV-2 have a low per-variable impact on R^2 and do not have a substantial effect on the estimated coefficients. Therefore, I omit them from the structural estimation and proceed with the variables used in IV-1.

Though the linear model does not account for the offsetting effects of duration on price and (via profits) on entry, the regressions capture a positive relationship between price and duration. This reduced-form correlation is consistent with Proposition 1, which predicts that a longer duration generates a higher per-period price. Thus, the data match a key distinguishing feature of the model of optimal duration. In the structural estimation, I also find a positive and significant direct relationship between duration and price.

In Table 3, I display regressions of the number of bids on auction characteristics and local measures of unemployment. Specification (3) is equivalent to the first-stage regression in IV-1, with an F-statistic of 25.9. All three instruments—the unemployment shock, the presence of existing firms, and a generic set-aside—have the expected positive signs. Though current unemployment is associated with more bids, higher baseline levels, which are used as a control, are associated with fewer bids. I interpret the negative correlation between higher 2004 unemployment and fewer bids as a reflection of local labor market frictions, leading to reduced competition and higher wages.

4 Empirical Implementation

4.1 Supplier Participation, Bidding, and Equilibrium

In the general model of Section 2, the buyer decides on the contract T with knowledge of $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$, the expected price conditional on contract and market characteristics. The buyer's expectation is taken over the number of bidders N and the cost realizations for these bidders.

For the empirical analysis, we separately model the participation decision that determines $\Pr(N = n|T, \boldsymbol{x}, \boldsymbol{m})$ and the market mechanism that determines $E[P(n, T, \boldsymbol{x}, \boldsymbol{m})|N = n]$. The model thus proceeds in three stages: the first stage reflects the buyer's problem, the second stage is the participation decision of suppliers, and the third stage is the market mechanism that determines the chosen supplier and the price.

[22] If separate indicators are estimated for medical buildings and airports, the coefficients on the indicators are very similar and the coefficients on the other variables are unchanged.

1st Stage: Duration Decision The buyer observes $(\boldsymbol{x}, \boldsymbol{m}, \delta)$ and sets T to minimize the expected per-period price plus the amortized transaction cost. The buyer's objective function is:

$$\min_{T\in\mathbb{T}} \sum_{n=1}^{\mathbb{N}} \left(E[P(n,T,\boldsymbol{x},\boldsymbol{m})] \cdot \Pr(N=n|T,\boldsymbol{x},\boldsymbol{m})\right) + \frac{\delta}{\sum_{k=1}^{T} \beta^{k-1}}. \tag{11}$$

2nd Stage: Participation Potential entrants observe $(T, \boldsymbol{x}, \boldsymbol{m})$, entry costs $k(\boldsymbol{m}) \cdot \varepsilon$, and contract cost-shifters $h(\boldsymbol{x})$. These costs are common across bidders. $k(\boldsymbol{m})$ and $h(\boldsymbol{x})$ are observed by all parties (including the econometrician), but the entry shock ε is only observed by suppliers.

Bidders enter if expected profits exceed entry costs. Profits conditional on participation depend on total contract costs $C_i \cdot U \cdot h(\boldsymbol{x})$. Thus, we make the usual assumption that cost components are multiplicative. Bidders do not observe the private cost C_i or the common cost U until after they decide to participate. In this context, U captures unobserved auction-specific heterogeneity.

Let π_n denote proportional profits for the n^{th} marginal entrant, which depends on C_i, the distribution of C, and the number of participating suppliers. Total profits are $\pi_n \cdot U \cdot h(\boldsymbol{x})$. The entry condition is given by

$$E[\pi_n \cdot U \cdot h(\boldsymbol{x})|n, T] - k(\boldsymbol{m}) \cdot \varepsilon > 0 \iff N \geq n. \tag{12}$$

3rd Stage: Bidding Participating suppliers realize their private (proportional) cost C_i and the common cost U. They then engage in a supplier selection mechanism. For the empirical application, we model the mechanism as a first-price auction, though the model can be generalized to other structures.

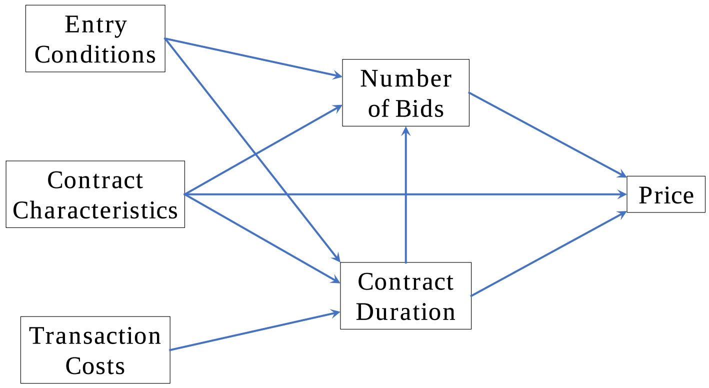
We assume bidders are risk neutral. Therefore, in equilibrium, each bidder submits a bid of $b_i \cdot U \cdot h(\boldsymbol{x})$, where b_i represents the proportional bid for bidder i. The lowest proportional bid, B, is the winning bid.

Equilibrium is characterized by the buyer choosing duration to minimize expected buyer costs, potential suppliers entering if expected profits exceed entry costs, and participating suppliers bidding optimally in the market mechanism. The model is summarized in Figure 4. Relative to a standard auction model with entry, the model also allows for a strategic decision by the buyer (duration).

4.2 Identification of Participation and Bidding

As shown in Section 2, contract-specific transaction costs are identified conditional on $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$, using the optimality condition of the buyer. The function $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$ is identified separately. Thus, identification of the participation and bidding game is valid even if T is not set opti-

Figure 4: Summary of Model



Notes: The figure summarizes the causal assumptions embedded in the empirical model. The three sets of variables on the left: entry conditions, contract characteristics, and transaction costs, are taken as given. Price, number of bids, and contract duration are jointly determined in the model. Arrows indicate the direction of causality.

mally.[23]

4.2.1 Identification of Supply Price

The econometrician observes the transaction price $P = B \cdot U \cdot h(\boldsymbol{x})$ as well as $(N, T, \boldsymbol{x}, \boldsymbol{m})$. The cost shocks U, ε, and C are unobserved by the buyer and the econometrician, but their distributions are common knowledge. To achieve nonparametric identification, we make restrictions on the distributions of unobservables. Assume

(i) *Independence of Unobservables:* C_i, U, and ε are independent conditional on $(N, T, \boldsymbol{x}, \boldsymbol{m})$.

(ii) *Mean Independence of Common Shocks:* $E[\varepsilon|T, \boldsymbol{x}, \boldsymbol{m}] = E[\varepsilon]$ and $E[U|T, \boldsymbol{x}, \boldsymbol{m}] = E[U]$. $E[U]$ is normalized to 1.

(iii) $h(\cdot)$ and $k(\cdot)$ are continuous, and the range of $h(\cdot)$ or $k(\cdot)$ has broad support. $h(\boldsymbol{x}_0)$ and $k(\boldsymbol{m}_0)$ are normalized to 1 for specific values of $\boldsymbol{x}$ and $\boldsymbol{m}$.

Under these assumptions, it is straightforward to obtain identification of the expected proportional bid (B), the cost-shifter functions, and relative profits:

Proposition 5. *When $(P, N, T, \boldsymbol{x}, \boldsymbol{m})$ is observed, the following components of the model are identified:*

1. $E[B|N, T, \boldsymbol{x}, \boldsymbol{m}]$
2. $h(\boldsymbol{x})$ *and* $k(\boldsymbol{m})$

[23]Moreover, the participation and bidding model may be applied in other settings where the buyer does not choose duration.

3. *Relative profits for N and N' participants:* $\frac{E[\pi_N|N,T]}{E[\pi_{N'}|N',T]}$
4. *Relative profits for T and T' with N participants:* $\frac{E[\pi_N|N,T]}{E[\pi_N|N,T']}$

Proof. See Appendix D. □

The first two components are sufficient to identify $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$ and transaction costs, without using the auction structure for the third stage of the game. Thus, even when the underlying selection mechanism is unknown, the model can be useful for counterfactual analysis of the impact of duration and transaction costs.

4.2.2 Identification of Profits and the Joint Distribution of Costs

With additional structure on the final mechanism, seller surplus can be identified. This can be useful in analyzing efficiency and for identification of the full joint distribution of costs. To this end, in addition to (i)-(iii) we further assume:

(iv) Bidders are symmetric: $C_i \sim F_i$, with $F_i = F$ for all i.

(v) F is continuous with positive support. $U \sim G$, where G has positive support.

(vi) Auctions with sequential values of $N \in \{\underline{N}, ..., \overline{N}\}$ are observed, with $\underline{N} < \overline{N}$.

Proposition 6. *When the supplier selection mechanism is an auction with symmetric bidders, seller surplus is identified.*

Proof. See Appendix D. □

Variation in N, combined with identification of relative profits, allows for identification of seller surplus in the auction model.

Once seller surplus (or expected profit) is identified, the distribution of ε is identified from equation (12), using variation in h or k. Further, we can pin down properties of the private cost distribution.

Proposition 7. *The distribution of private costs is identified up to the first $(\overline{N} - \underline{N} + 2)$ expected order statistics of $\overline{N}$ draws from F.*

Proof. See Appendix D. □

Intuitively, exogenous variation in N shifts the private cost component of the winning bid, but not the common costs. Observe that if $\underline{N} = 2$ and $\overline{N} \to \infty$, the restrictions on expected order statistics approximate the quantile function, and F is exactly identified. The restrictions have additional power in that they may reject many classes of flexible distributions with $(\overline{N} - \underline{N} + 2)$ parameters.

Corollary 2. *The distribution of unobserved heterogeneity is obtained after F is identified.*

Proof. By independence, we can use the characteristic function transform to write $\varphi_{\ln W_N}(z) = \varphi_{\ln B_N}(z) \cdot \varphi_{\ln U}$, where $W_N = B_N \cdot U$ is the observed winning bid scaled by the observables and B_N is the distribution of the proportional winning bid conditional on N. We can construct this function for two different values of N. Once the characteristic function of F is obtained, either by exact identification ($\overline{N} \to \infty$) or by flexible estimation methods, G is pinned down. □

4.2.3 Discussion of Identification Assumptions

In my empirical setting, it is important to account for unobserved auction-specific heterogeneity. The baseline set of assumptions map to the setting of Krasnokutskaya (2011), while allowing for endogenous entry. Symmetry is a typical assumption in auction models of unobserved heterogeneity (see also, e.g., Aradillas-López et al., 2013). To relax symmetry, one could consider alternative restrictions to pin down costs and the joint distribution of outcomes. One alternative would be to employ supplementary data on profits for one (N, T) pair. This would identify the expected profit function, which could then be used to identify the joint distribution of costs.[24]

Another common assumption is that bidders realize some auction-specific costs after deciding to participate.[25] One could relax this assumption by allow bidders to select into the auction based observing U beforehand. In this case, it is straightforward to extend the identification results. Additional steps would be required if potential bidders observed their private cost draw before deciding to participate.

These assumptions are sufficient for the nonparametric identification of the auction model. In effect, the entry cost shifters $\boldsymbol{m}$ serve as instruments for the number of bidders. Exogenous variation in N is then used to separately identify private costs from unobserved heterogeneity. With no instruments, these distributions can still be separately identified. Therefore, with unobserved heterogeneity and conditionally independent private values (i.e., the setting of Krasnokutskaya, 2011), the model is identified with data on only the winning bid and variation in the number of bidders. I provide this alternative identification approach in Appendix D.5.

4.3 Parameterizations

For the empirical application, I estimate the model of Section 4.2, where bidders are symmetric and participate in a first-price auction. I employ a parametric approach for parsimony. The nonparametric identification results provided earlier, along with robustness checks, suggest that first-order features of the estimated distributions are not entirely driven by functional form. In this application, there is an added complication of estimating a duration-dependent distribution of private costs, which would increase the number of parameters needed for any nonparametric approach.

[24]I test for the presence of asymmetry in Section 5.4. The results are consistent with symmetric bidders.

[25]This assumption may a reasonable approximation because observables explain roughly 70 percent of the variation in prices.

Table 4: Empirical Parameterizations

Cost Component	Notation	Parameterization
Private Costs	C_i	$\sim Weibull(\mu_0 + \mu_1 T, \alpha_0 + \alpha_1 T)$
Unobserved Heterogeneity	U	$\sim \ln \mathcal{N}(-\frac{\sigma_U^2}{2}, \sigma_U^2)$
Entry Shock	ε	$\sim \ln \mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon^2)$
Observed Heterogeneity	$h(\boldsymbol{x})$	$= square\_footage^{\gamma_1} \cdot weekly\_frequency^{\gamma_2}$ $\cdot 2004\_unemployment^{\gamma_3} \cdot \gamma_4^{\mathbb{1}[high\text{-}intensity\_cleaning]}$
Entry Costs	$k(\boldsymbol{m})$	$= T \cdot square\_footage^{\kappa_1} \cdot weekly\_frequency^{\kappa_2}$ $\cdot unemployment\_shock^{\kappa_3} \cdot establishments^{\kappa_4}$ $\cdot \kappa_5^{\mathbb{1}[generic\_set\text{-}aside]}$

The parameterizations are given in Table 4. A central consideration of this paper is that the distribution of the average per-period private cost, C_i, changes with the duration of the contract. One approach to estimation would be to estimate a microfounded model where the per-period cost shocks are governed by an autocorrelation parameter. Instead, I estimate the average per-period cost distribution as a primitive, allowing the mean of the average per-period cost and the variance to vary with T. As I am not taking a stand on the underlying cost process, I estimate a "reduced-form" primitive for the cost distribution. By picking an appropriately flexible distribution, this approach may better approximate a wider range of per-period distributional families.[26]

For private costs, the Weibull distribution is chosen for tractability and flexibility, as it allows the estimated probability density functions to be either convex or concave. It is governed by the mean parameter $\mu(T) = \mu_0 + \mu_1 T$ and the shape parameter $\alpha(T) = \alpha_0 + \alpha_1 T$. I allow the parameters of the private cost distribution to vary linearly with duration to capture the first-order effects of interest in this model. A finding of $\alpha_1 > 0$ is consistent with autocorrelation in cost shocks, which results in reduced variance in average per-period costs over longer contracts. For the distribution of unobserved heterogeneity, the log-normal distribution was chosen because it best fit the model out of several choices.[27]

In the first step, I estimate the parameters for participation and bidding using maximum likelihood. Entry costs shocks are parameterized as increasing linearly with the duration of the contract.[28] Note that square footage and weekly frequency affect the entry decision by both increasing supply costs (price) and affecting entry costs.

One of the challenges in the estimation of auction models arises from the computational

[26]For an example microfounded model, see Appendix E.

[27]Other estimated distributions of unobserved heterogeneity were the gamma distribution and the Weibull distribution. Both have the desirable properties of support on $(0, \infty)$ and can be normalized to have a mean of 1.

[28]This has the interpretation that entry costs are borne annually and could reflect the opportunity costs of other contracts. Allowing a free parameter on the entry costs in estimation generates a coefficient close to one.

burden of inverting the bid function. I employ a simple innovation—a change of variables—which circumvents this step and greatly speeds up estimation in the presence of unobserved heterogeneity. This innovation and details of the likelihood function are in Appendix F.

In a second step, I used the estimated parameters in the buyer's objective function (equation (11)) to generate nonparametric bounds on δ for each contract, following the steps in Section 2.3. In my data, contracts are either set to the nearest monthly or nearest yearly increment, providing a set of tight and loose bounds, respectively. That is, if I observe a 16-month contract, I assume it was preferred to 15-month and 17-month contracts, whereas I assume that a 24-month contract was preferred to other yearly increments (12-month and 36-month contracts). I estimate these transaction costs with an annual discount rate of $\beta = 0.97$. A smaller value of β would imply higher transactions costs. This is because the β-dependent components of the bounds given by equations (9) and (10) are decreasing in β, for $\beta < 1$. Intuitively, a smaller β corresponds to a smaller marginal benefit for a longer contract. This, in turn, implies that a greater transaction cost is needed to rationalize the chosen duration.

Finally, to construct expected market transaction costs and conduct counterfactuals, I apply a uniform prior for the density between the distribution-free bounds.[29] Using the prior, I construct point estimates by taking the expectation. In practice, many of the contracts in my data face a cap on maximum duration of five years, due to federal regulation. For contracts affected by the cap, only a lower bound for δ can be obtained without additional assumptions. I make the assumption that the chosen duration at five years is optimal. This generates a relatively conservative upper bound on transaction costs. Many of the optimal contracts under a higher cap would be likely be longer, implying a larger upper bound and larger point estimates for the transaction costs.

5 Results

Estimation of the structural model proceeds in three steps. First, I use a parametric maximum likelihood to perform joint estimation of entry and bidding. Second, using the duration decision of the buyer and estimated parameters from the first step, I construct distribution-free bounds for transaction costs. Third, I construct estimates of transaction costs by applying a prior over the bounds. These estimates are inputs to the policy counterfactuals in Section 6. The estimation steps are described in more detail in Section 4.

Table 5: Parameter Estimates

Group	Parameter	Variable	Estimate	95 Percent C.I.
Private Costs	μ_0	-	18.521	[16.658, 20.861]
	μ_1	Duration	0.546	[0.085, 0.979]
	α_0	-	4.807	[3.527, 6.969]
	α_1	Duration	0.386	[0.053, 0.674]
Heterogeneity	σ_U	-	0.608	[0.572, 0.646]
	γ_1	Square Footage	0.664	[0.627, 0.701]
	γ_2	Weekly Frequency	0.488	[0.411, 0.556]
	γ_3	2004 Unemployment	0.087	[0.070, 0.106]
	γ_4	High-Intensity Cleaning	0.302	[0.187, 0.437]
Entry	μ_ε	-	-0.459	[-0.749, -0.173]
	σ_ε	-	0.649	[0.608, 0.687]
	κ_1	Square Footage	0.543	[0.488, 0.599]
	κ_2	Weekly Frequency	0.542	[0.420, 0.650]
	κ_3	Unemployment Shock	-0.309	[-0.449, -0.208]
	κ_4	Establishments	-0.066	[-0.108, -0.025]
	κ_5	Generic Set-Aside	-0.262	[-0.390, -0.148]

Notes: The table displays maximum likelihood parameter estimates from the structural model. The first group of coefficients indicate how the mean and shape of the private cost distribution change with the duration of the contract. The second set of coefficients indicate the distribution of unobserved auction-specific heterogeneity and how auction-specific common costs vary with observable cost characteristics. The third set of coefficients pertain to entry costs in the model. 95 percent confidence intervals are displayed in the last column. As minor data cleaning steps (de-meaning) are data-dependent, confidence intervals are constructed via 500 bootstrap samples.

5.1 Estimated Supply Costs and Entry Costs

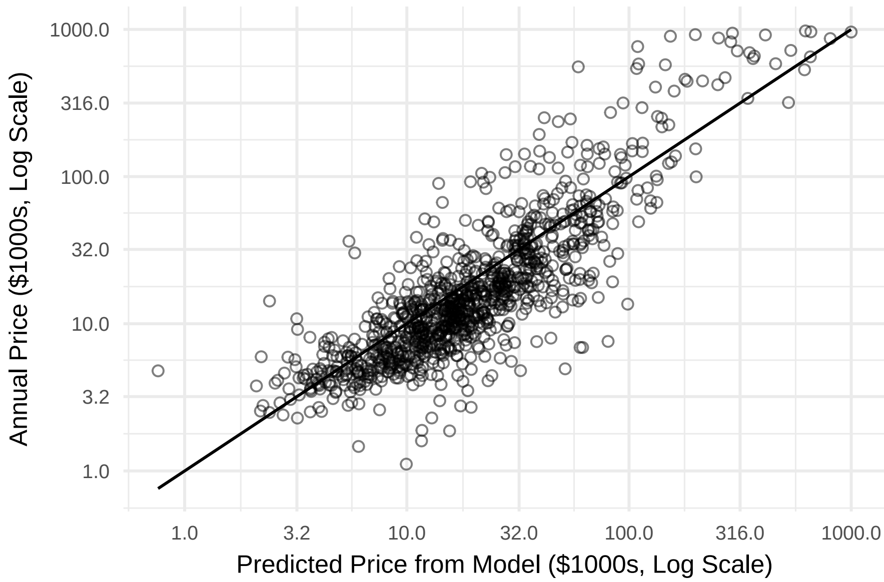
Table 5 displays the parameter estimates from the first step. Square footage, weekly frequency, and 2004 unemployment are scaled by the mean, so that the estimate of μ_0 is interpreted as the mean annual private cost draw for a zero-duration contract at a typical location. The mean annual private cost is $18,521 and increases by 2.9 percent per contract year (μ_1/μ_0).[30] Thus, I find that the data are consistent with the equilibrium prediction from Proposition 1. Prices increase with duration due to both the increase in mean costs and the reduction in variance, as we would expect if cost shocks are not perfectly correlated over time. Reduced variance is captured by the positive coefficient α_1.

As expected, higher values for square footage and weekly frequency increase costs. Con-

[29]The uniform prior is appealing for its transparency and also for the reason that the observed duration is optimal at the mean transaction cost when buyers can issue contracts in monthly increments. If a left triangular prior were used instead, the optimal monthly-increment contract would be shorter than the observed value for contracts observed in yearly increments.

[30]For a visual representation of how costs depend on duration, I plot the density of private cost draws for a one-year and a five-year contract in Appendix G.1.

Figure 5: Model Fit: Actual Versus Predicted Annual Price



Notes: The figure plots observed prices against predicted prices from the model. The R^2 of the predicted values is 0.71, which compares favorably to the R^2 of 0.69 from the linear instrumental variables model.

sistent with the findings from the descriptive regressions, baseline unemployment and high-intensity buildings have higher costs. For entry, higher current unemployment and the presence of more local establishments lower entry costs. Generic small business set-asides also have lower entry costs, relative to demographic-specific set-asides. Square footage has a net positive effect on entry, as $\gamma_1 > \kappa_1$. Supply costs, which are positively correlated with profits, increase by more than entry costs for square footage. Weekly frequency, on the other hand, has a net negative effect on entry, as $\gamma_2 < \kappa_2$. This is consistent with capacity constraints, as some firms may be limited in the days they are available to clean. The mean per-bidder entry cost estimate is $573.

The model fits the data well. In Figure 5, I display actual values for annual prices compared to the predicted values. The R^2 for the structural model is 0.71, which compares favorably to the linear model IV-1 in Table 2. Unobserved heterogeneity is important to match the distribution of prices. Unobserved common costs are economically meaningful, in that they capture approximately 30 percent of the variance of log prices.

5.2 Estimated Transaction Costs

Market transaction costs are significant in this setting, comprising 10.9 percent of annual costs. These costs capture the marginal costs to the federal government of running a procurement auction, and they are summarized in Table 6. To obtain the aggregate share of costs attributable

Table 6: Estimated Market Transaction Costs ($)

Contract-Specific Measure	Median	95 Percent C.I.	p25	p75	Mean
Transaction Costs	10,400	[3,000, 17,400]	5,100	21,400	24,500
Annualized	2,500	[700, 4,100]	1,300	4,800	5,400
Contract Value	50,500	[46,900, 54,100]	28,500	102,000	190,200
Price (Annual)	13,200	[12,300, 13,900]	7,300	26,700	43,900
Percent Share of Costs	15.2	[5.0, 22.0]	9.9	20.8	16.3
Aggregate Measure	**Estimate**	**95 Percent C.I.**			
Percent Share of Costs	10.9	[3.8, 20.3]			

Notes: Estimated transaction costs are the expectation taken with a uniform prior over the distribution-free bounds identified from the duration decision of the buyer. For $T = 5$, conservative upper bounds are projected by assuming that the duration is optimally chosen. Transaction costs are also expressed as a share of total (buyer) costs. The aggregate share of total costs attributable to transaction costs is 10.9 percent, which is calculated by comparing the mean annualized transaction costs to the mean price. Confidence intervals are constructed via the bootstrap. For display, values are rounded to the nearest 100.

to transaction costs, I divide the mean annualized transaction costs by the mean total annual cost (the sum of annualized transaction costs and the price). Also displayed in the table are the median values for transaction costs, the annualized values, and the corresponding medians for contract value and price. The median transaction cost is estimated to be $10,400, and the median share of costs attributable to transaction costs is 15 percent across contracts. The estimated magnitudes seem reasonable based on some back-of-the-envelope calculations, which I discuss in the following section.

The sequential, revealed-preference approach has the benefit of providing testable implications of the model via the unconstrained estimates presented here. A finding of negative transaction costs, which would arise with private costs that fall with duration, would suggest that the tradeoff in this paper is not first-order to contract duration. Instead, the 95 percent confidence intervals of μ_1 and α_1 have positive support, implying positive transaction costs only, which is consistent with the model. As previously, the premium on duration can arise simply from averaging cost draws across multiple periods. The premium captures opportunity costs as well, reflecting the seller's beliefs about the arrival rate of more profitable options.

In some cases, the market transaction costs are quite large as a percent of total costs. The 95th percentile of share of costs attributable to market transaction costs is 32.3 percent. For these estimates, this is driven by moderate transaction costs realized by low-price projects, rather than very high absolute costs. For example, contracts with a portion of transaction costs in the 95th percentile or above (greater than 32.3 percent) have a mean price of $9,000, which is much smaller than the full-sample mean of $43,900.

5.3 Verifying the Estimated Transaction Costs

To check the magnitudes of the estimated transaction costs, we can perform a back-of-the-envelope calculation by looking at the labor costs of employees who specialize in contracting, purchasing, and procurement.[31] Considering only these specialists will understate the full labor costs of managing contracts, as employees in several other categories are involved in issuing contracts. These other categories include supply program managers, logistics managers, and support service administrators, who perform other tasks in addition to procurement, and specialists with industrial, engineering, or scientific knowledge that assist in determining the requirements for the contract.[32]

The labor costs for contract specialists provide a reasonable benchmark for relatively simple products and services. Large and complex contracts require more input from subject-matter experts and from consumers of the product or service, which will not be captured by the measure. Another consideration is that some contracts require ongoing maintenance by the contracting agency, so a portion of labor costs do not reflect the one-time costs of new contracts. For simple contracts, the labor costs of contracting officers is likely a close approximation of these otherwise hidden costs, as most of the effort is made up front.

For a new contract, the contracting officer must draft the requirements,[33] survey the market and decide terms, ensure compliance with existing regulations, post the solicitation, communicate with interested bidders, determine the winning bidder, and conclude the contract. A senior contracting officer for the federal government estimated that the simplest cleaning contract would take about three weeks of full-time work for a contracting employee. For fiscal years 2004 to 2016, the average salary for contracting specialists was $78,253. Three weeks of full-time work, assuming a 50-week year, provides a cost estimate of $4,695, which is roughly in line with the 25th percentile estimate.

Looking at the contracts in aggregate provides another rough benchmark for transaction costs. To compare to the estimation sample, I examine all contracts for fiscal years 2004 to 2016 that had an annual price of less than $1 million. Approximately 750,000 of these contracts are issued each year, or 99 percent of all new contracts. If we assume that new contracts over $1 million required five times the contracting specialists compared to these smaller contracts, then the labor cost per new contract is $3,805, or 7.4 percent of total buyer costs. These contracts are modestly less expensive than the estimation sample, with a mean annual price of $35,469 and a median of $4,731.[34] These two calculations provide benchmarks that are the same order

[31] These correspond to GS-1102, GS-1105, and GS-1106 in the federal government's classification system.

[32] The classifications corresponding to the administrative roles are GS-2003, GS-0346, GS-0342. Those pertaining to industrial, engineering, or scientific knowledge are GS-1150, GS-0800, GS-1300, and GS-0400. The standards may be obtained here: https://www.opm.gov/policy-data-oversight/classification-qualifications/classifying-general-schedule-positions/.

[33] The main document for the median contract runs 49 pages. See Appendix C.5 for example pages.

[34] 75 percent of contracts under $1 million and 29 percent of larger contracts are new each year, based on the latter half of the data (FY 2011-16). Larger contracts likely involve significant resources from other employee

Table 7: Estimated Market Transaction Costs by Category

(a) Median Values by Location Type

Type	Transaction Costs	Contract Value	Square Footage	Count
Medical	38,900	206,000	10,000	61
Airport	32,850	254,300	7,850	30
Technical	30,600	84,000	15,600	19
Industrial	28,500	60,000	27,900	13
Accommodations	28,350	121,200	32,000	18
Services	18,400	87,700	8,300	59
Research	13,400	58,500	6,000	111
Visitors	13,000	189,700	6,500	41
Field Office	9,250	48,500	8,450	270
Office	7,300	35,550	4,750	424

(b) Median Values by Department

Department	Transaction Costs	Contract Value	Square Footage	Count
Homeland Security	40,000	269,300	14,600	45
GSA	29,350	223,100	12,750	40
Veterans Affairs	24,450	143,550	8,800	80
Other	20,900	69,200	11,650	24
Commerce	14,150	59,750	5,500	78
Interior	11,900	71,000	8,900	43
Agriculture	9,500	46,700	9,300	347
Defense	7,300	35,900	4,200	389

Notes: The table displays the median estimated market transaction costs. Also displayed are the median contract value, the median square footage of the facility, and the count of observations. In panel (a), observations are grouped by location type. In panel (b), observations are grouped by contracting department. For display, underlying values are rounded to the nearest 100.

of magnitude of the transaction costs recovered in estimation.

As an additional exercise to check the plausibility of the estimated market transaction costs, I project the estimates on other variables not used in the structural estimation. First, I calculate the median transaction costs by facility type and by department in Table 7. As expected, the highest transaction costs are among facilities with relatively complicated or technical requirements, such as medical centers, airports, and technical facilities (e.g., power plants). Simpler settings such as office cleaning have the lowest estimated transaction costs. In the second panel, I calculate the median by government department. The Department of Homeland Security has the highest median transaction costs, at $40,000 per contract. This might be expected given the high levels of security required at their facilities and the relative lack of institutional knowledge at the recently-formed department.[35] Conversely, Agriculture and Defense have low median

classifications. The 3 percent of contracts over $1 million annually (31,929 each year on average) comprise over 90 percent of all dollars obligated. When including this long tail, the aggregate labor cost per contract from contracting specialists is less than one percent of total costs. Larger contracts likely involve significant resources from other employee classifications.

[35]Contracting employees in Homeland Security had lower length of service and were less likely to have a bachelor degree compared to the average contracting employee.

Table 8: Projecting Market Transaction Costs on Variables Outside of the Model

	(1)	(2)	(3)	(4)	(5)	(6)
High-Intensity Cleaning	1.448***	1.189***	1.140***	1.022***	1.158***	0.612***
	(0.144)	(0.133)	(0.134)	(0.138)	(0.138)	(0.108)
ln(Word Count)	0.085***				0.124***	0.043**
	(0.024)				(0.023)	(0.018)
ln(Related Expenditures)		0.096***			0.073***	0.054***
		(0.011)			(0.023)	(0.017)
ln(Related Modifications)			0.283***		0.047	−0.141**
			(0.035)		(0.071)	(0.055)
Simplified Acquisition Ind.				−0.692***	−0.715***	−0.429***
				(0.090)	(0.089)	(0.069)
ln(Square Footage)						0.577***
						(0.026)
ln(Weekly Frequency)						0.510***
						(0.057)
Observations	1046	1046	1046	1046	1046	1046
R^2	0.09	0.14	0.13	0.13	0.20	0.54

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays estimated coefficients from regressing estimated log transaction costs on variables outside of the model. These variables are (i) the (log) number of pages in the contract, (ii) log government procurement expenditures at the same 9-digit ZIP for maintenance, office furniture, and other housekeeping services, (iii) the count of contract actions for these expenditures, and (iv) an indicator for whether the contract falls under the federal government's simplified acquisition protocol.

transaction costs, at $9,500 and $7,300, respectively. After controlling for square footage, cleaning frequency, and facility type in a regression, Homeland Security has the highest fixed effect for (log) transaction costs, 91 percent larger than Defense. Agriculture has lowest fixed effect, 17 percent smaller than Defense. The regression table is provided in Appendix G.2.

In Table 8, I regress the estimated market transaction costs on variables excluded from the structural model. Included covariates are the number of pages in the contract, related expenditures and contract modifications[36] in the same 9-digit ZIP, and an indicator for whether the contract falls under the simplified acquisition protocol. One would expect that lengthier contracts and busier agencies are reflective of higher opportunity costs, and that the simplified acquisition label would reflect lower market transaction costs. Indeed, all four enter with the expected sign. After controlling for square footage and cleaning frequency, high-expenditure locations are associated with higher transaction costs. Economic theory could rationalize a sign in either direction, as economies of scale lead to a positive association and capacity constraints produce a negative one. The negative coefficient on contract modifications in the fifth specifica-

[36]Spending and contract actions for other housekeeping services, maintenance, and office furniture.

Table 9: Test for Asymmetry: Do Incumbents Have an Advantage?

Follow-On Contracts	Symmetric Win Rate	Incumbent Win Rate	*N*	*t*-Statistic
Estimation Sample	0.224	0.217	175	(0.20)
Extended FPDS Sample	0.278	0.263	845	(1.00)

Notes: The table displays the results of a test for asymmetry in performance by incumbent bidders. The expected win rate for symmetric bidders, based on the number of bids, is compared to the observed win rate by incumbent bidders. The *t*-statistics indicate no significant difference in either sample. The first sample is follow-on contracts in the estimation sample, and the second sample uses the same criteria for all FPDS building cleaning contracts. Follow-on contracts are identified as contracts that have a single leading contract for the same agency in the same nine-digit zip code. A leading contract is one that is active in the year prior to the start of the follow-on contract and begins at least thirty days prior to the start of the follow-on contract.

tion may reflect economics of scale or simply that lower transaction costs lead to more contract modifications.

5.4 Robustness

To estimate the model, I have followed typical practice in the empirical literature, such as assuming multiplicative separability in cost components. For the discount rate and for five-year contracts, I have made assumptions that generate conservative estimates of transaction costs. For a discussion of these choices and sensitivity to specific parameterizations, see Section 4.3.

For the empirical model, we have proceeded under the assumption that bidders are symmetric with respect to private supply costs. In a dynamic setting, the procurement process might result in asymmetry between bidders that would invalidate this assumption. One common source of asymmetry in procurement is the presence of an incumbent bidder who may have an advantage via a relationship-specific investment (e.g., through learning-by-doing or lowered transaction costs of retaining the same supplier). Additionally, competing bidders may retain some information about competitors if costs are correlated over time.[37]

I check for the presence of asymmetries by comparing the expected win rate under symmetry (based on the number of bidders) to the win rate for incumbent suppliers in follow-on contract. I identify follow-on contracts in the analysis sample by finding contracts that have a single active supplier on another contract in the same 9-digit ZIP code within the prior year (and starting at least thirty days before). 9-digit ZIP codes are geographically narrow, typically corresponding to a city block or an individual company. The prior contract may be any of the approximately 11,000 cleaning contracts in the FPDS data. I also construct a broader set of follow-on contracts from the extended FPDS sample.

Table 9 compares the expected win rate for symmetric bidders to the actual win rate for incumbent bidders in identified follow on contracts. There is no significant difference between

[37]Saini (2012) discusses the literature on endogenous asymmetries and evaluates a model in which capacity constraints hurt the winning bidder.

the two, suggesting that the incumbency advantage is not first-order in this setting. I obtain similar results for the 175 contracts in the estimation sample and the 845 contracts from the broader FPDS sample.[38] There are a priori reasons to believe that the incumbency advantage is not large for competitive federal procurement, as, per regulation, the agencies are mandated to seriously consider all qualified bidders and, in most cases, select the lowest price. The degree of relationship-specific investments in facility cleaning is likely to be low, as the menu of services tend to be standardized.

Transaction costs do not appear to depend on whether the winning bidder is an incumbent. For follow-on contracts, there is no difference between the mean (log) transaction costs between contracts that are won by an incumbent ($2.277, N = 38$) and those that are not ($2.274, N = 137$). As an additional test, I include a dummy for whether the contract is an identified follow-on contract in the descriptive regressions from Section 3 to determine if variation in prices and entry are explained by the presence of an incumbent bidder. None of the coefficients on the dummy are significant, and its inclusion does not meaningfully change any of the coefficients of interest. For these regressions, see Appendix G.3. The results of these tests are consistent with the maintained assumption of no endogenous asymmetries.

An additional general concern might be that there is heterogeneity in supplier types. The above tests for endogenous asymmetries are also valid tests for exogenous asymmetries in supplier types. Lower-cost types would be more likely to win the first contract in the identified set of follow-on contracts, generating a correlation in win rates over time. Thus, the above findings are consistent with symmetry across suppliers more generally. In contrast to many other industries, there is no great distinguishing factor that separates types of building-cleaning firms, and it is reasonable to expect that production is roughly constant returns-to-scale. This makes the empirical setting a nice fit for the model.

5.5 Generalizability

The estimated transaction costs are obtained in a specialized setting which allows for them to be isolated from other factors. It may be reasonable to translate these costs to federal contracts for other standardized products, where the buyer's problem is similar. As discussed in Section 3, cleaning contracts are comparable in magnitudes to the 97 percent of federal contracts under \$1 million per year. An appealing feature about the application is that the large variation in observables allows us to account for how these costs vary with project scale and complexity.

We might expect that transaction costs are somewhat lower for buyers in the private sector, who face similar sets of suppliers but may be able to avoid some of the requirements mandated by regulation. On the other hand, the size of government procurement may provide some economies of scale and reduce these costs. The federal government can use an existing platform

[38] As I only observe winning bidders, I am unable to adjust for when a supplier does not bid on a follow-on to the supplier's current contract.

(FedBizOpps) to post each solicitation, so the estimated transaction costs exclude the fixed costs of setting up such a platform. Based on conversations with procurement officers from several organizations, transaction costs are, anecdotally, of similar magnitudes in the private sector.

In other contexts, buyers often have the option to choose what type of competitive procedure to engage in and the type of contractual arrangement, such as choosing to negotiate with the incumbent supplier or signing an evergreen contract. These options provide means to further mitigate transaction costs, though this may come at the expense of greater supply costs.

The nature of supply costs is an important consideration in more general contexts. With building cleaning services, supply is stable over time, and the service represents a small fraction of overall expenditures. In other settings, such as technology-dependent firms, the per-period supply costs may be falling over time. Whether or not this feature leads to shorter contracts depends on whether cost-reducing innovations are predictable and whether they are driven by high-cost or low-cost firms. Furthermore, buyers may not be risk-neutral with respect to a primary input that is specialized to their business, which is another factor to consider when taking the model to other settings. Even so, the tradeoff I identify here remains relevant.

6 Counterfactuals: The Impact of the Duration Margin

I now consider the implications of the endogenous duration decision on welfare. In the first counterfactual, I analyze the cost to the buyer of removing the ability to adjust duration through standard contracting terms. In the second counterfactual, I show how duration provides a margin of adjustment that mediates the pass-through of cost shocks to prices.

6.1 Strategic Value of the Duration Decision, Compared to Standardization

When the buyer can adjust the non-price terms of a contract, the buyer can minimize expected costs for each transaction. This flexibility provides a cost-minimizing advantage to the firm. In many settings, contract terms are standardized. For example, a three-year contract is the industry standard for office supplies. The structural model allows us to estimate how costly it is to remove the buyer's strategic option to adjust duration. One may also interpret these impacts as the costs of going to market more or less frequently.

Table 10 reports the impact on aggregate buyer costs by moving to standardized terms of yearly increments. Total costs would increase substantially, by 37 percent, if all contracts were issued in one-year terms. This is not surprising, as the median duration in the data is 5 years and standardization would result in much more frequent contracting. On the other hand, standard durations of 4 years or 5 years would have a small impact, increasing buyer costs by less than two percent.

Though standardization increases costs, one might expect that, by eliminating the buyer's effort to determine the optimal duration, market transaction costs might fall. Thus, it is worthw-

Table 10: Effects of Standardized Terms (Percent)

$\bar{T}$	Total Cost	95 Percent C.I.	Price	Trans. Cost	Affected	Change to δ
1	36.7	[12.1, 55.3]	−11.9	354.0	1018	−60.9
2	9.9	[3.2, 15.0]	−8.0	127.0	992	−33.0
3	3.2	[1.0, 4.8]	−4.2	51.3	907	−16.0
4	1.4	[0.5, 2.4]	−0.4	13.5	992	−9.4
5	1.6	[0.5, 2.8]	3.2	−9.2	496	−13.0
6	2.7	[0.8, 4.4]	6.8	−24.3	1041	−26.6

Notes: The table displays the resulting percent changes in total costs, prices, and annualized transaction costs when all contracts are issued in standardized durations corresponding to $\bar{T}$. For a uniform duration policy of 4 years or less, the average price paid decreases and the amount spent on transaction costs increases. Affected contracts are the count of those that are displaced from the optimal duration. The final column displays the reduction in transaction costs that would render a uniform policy equivalent to the existing policy in terms of buyer costs. Confidence intervals are reported for total costs and are constructed via the bootstrap.

hile investigating the reduction in market transaction costs necessary to offset the increased costs arising from a standardized policy. This compensating transaction cost would make the firm (the government) indifferent between flexible-duration and standard-duration policies.

In the final column of Table 10, I report the change in transaction costs that would make the standardized term policy equivalent to the flexible term policy. For a four-year standard term, the necessary reduction in transaction costs is modest. If the government could reduce transaction costs by 10 percent by implementing a standardized four-year duration policy for these services, the results indicate that it would be beneficial to do so.

These results suggest that flexible terms may be quite valuable, compared to a poorly-chosen standard (e.g., one year or two years in this setting). Thus, knowledge of the relevant cost structure and transaction costs is important for setting non-price terms. However, an intelligently-chosen standard may be cost-effective, as the required reduction in transaction costs to offset the costs of standardization is modest for a four-year standard.

A related question to standardized terms is that of a cap on maximum duration, similar to the five-year cap imposed by government-wide budgeting regulations in my data. This is analogous to the imposition of standard terms on only a subset of contracts. In Appendix G.4, I provide a detailed breakdown of the effects by whether duration is increased or decreased by the standard, which provides insight into the cost of the cap.[39]

6.2 Contract Duration and Welfare Analysis

Transaction costs are important to welfare analysis as they can constitute a substantial portion of total costs and affect how equilibrium prices respond to a change in the economic environ-

[39] Table 19 reports averages by contract, rather than in aggregate, which is why the numbers differ slightly from those in Table 10.

Table 11: Effects of a 10 Percent Reduction in Supply Costs (Percent)

Specification	Price	Duration	Total Cost
No Response in Entry or Duration	-10.00	0.00	-8.67
	[-10.00, -10.00]	[0.00, 0.00]	[-9.53, -8.01]
Endogenous Entry Only	-8.59	0.00	-7.45
	[-8.92, -8.14]	[0.00, 0.00]	[-8.23, -6.85]
Endogenous Entry and Duration	-7.70	6.64	-7.50
	[-8.33, -7.04]	[5.86, 9.63]	[-8.26, -6.92]

Notes: The table reports the equilibrium changes to prices, duration, and total costs when supply costs fall by 10 percent. In the first row, entry and contract duration are treated as exogenous. In the second row, entry by potential bidders changes in response to supply costs. In the last row, both entry and contract duration respond endogenously. The first column shows the effects on price. The estimate divided by -10 percent captures the pass-through of supply costs to prices in each scenario. Confidence intervals are constructed via the bootstrap.

ment. When transaction costs are unaffected by a policy change, a welfare analysis that omits transaction costs will misstate the impact for two reasons. First, the measured impact on prices should be weighted by the share of total costs attributable to prices. That is, the impact should be discounted toward zero by the share attributable to (unaffected) transaction costs. Second, market participants adjust equilibrium behavior in response to the change. The choice of duration provides an additional margin of adjustment, mitigating the effect on prices but improving welfare compared to an analysis that takes duration as fixed.

To demonstrate these effects, I simulate a 10 percent reduction in supply costs for all of the contracts in my data. For comparison, I provide three specifications: one in which entry and duration are taken as given, a second in which entry is endogenous, and a third in which both entry and duration respond to the cost shock.

The results of the counterfactual are reported in Table 11. In the first specification, the aggregate price falls by exactly 10 percent, as the equilibrium bids in the model are proportional to supply costs. However, total costs to the buyer fall by only 8.7 percent, as transaction costs represent a substantial portion of total costs. Thus, these costs that are otherwise hidden may be important to account for when measuring welfare impacts.

In the second specification, I allow the participation of the bidders to respond to the change in costs. Based on the estimated parameters, lower supply costs results in less entry. Due to reduced competition, the impact on prices is 14 percent lower (8.6 percent compared to 10 percent). The impact of a reduction in supply costs is not perfectly passed through to prices because firms adjust their participation decision on the margin.

Likewise, the duration margin also mitigates the pass-through of supply costs to prices. In the third specification, buyers can adjust duration. In response to lower supply costs, the average contract duration increases by 6.6 percent. As supply costs increase with duration, this adjustment partially offsets the reduced supply costs. While allowing for both endogenous entry and duration to respond, supply prices fall by only 7.7 percent. Thus, the marginal effect of endogenous duration is to reduce pass-through by 10 percent. In contrast to the previous counterfactual, here the duration margin provides only a small reduction in total costs.

This counterfactual exercise illustrates that the duration margin can mitigate the pass-through of costs to prices. It can be important in a welfare analysis, as observed changes to prices may reflect a number of endogenous levers. Further, prices may only capture a portion of the total costs, so accounting for transaction costs can be important.

When transaction costs are affected by a policy change, the above two forces also affect welfare estimates. Changes to transaction costs should be directly accounted for in the welfare calculation, and any such changes allow for new duration and price choices that may improve welfare.

7 Conclusion

In this paper, I develop a model of optimal contract duration arising from underlying supply costs and market transaction costs. I show how latent transaction costs may be recovered from the duration decision of the buyer. Using a dataset of federal supply contracts, I find that the costs of going to the market can be a significant portion of total costs for intermediate goods. The methods developed in this paper may prove useful for welfare analysis, especially in industries where supply contracts are prevalent. In many settings, the tradeoff presented in this paper may complement other concerns arising from ex post incentive problems and incomplete contracts. An appropriate model should be tailored to the industry in question.

The analysis presented here offers, albeit indirectly, one novel prediction regarding the theory of the firm. Supply contracts lie in between arms-length transactions and vertical integration. As is known, conditions favorable for long-term contracts are also favorable for vertical integration, as the end is similar and integration may result in additional benefits. I demonstrate here that long-term contracts arise when competition is sufficiently low, and also when competition is very intense. Likewise, vertical integration may be most likely for low levels and high levels of competition. When the industry is moderately competitive, a downstream firm realizes a large benefit by switching among suppliers and may have the smallest incentive to integrate upstream.

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Appendices

A Model Proofs and Additional Predictions

A.1 Proof of Propositions 1 and 2

By assumption, an interior solution exists. Let the set of possible contracts be order in increasing duration: $\mathbb{T} = \{1, ..., R, S, T, ...\}$. At an interior solution S, it must be that

$$\overline{P}(R) \geq \overline{P}(S) - \left(\frac{\delta}{\sum_{k=1}^{R} \beta^{k-1}} - \frac{\delta}{\sum_{k=1}^{S} \beta^{k-1}} \right) \tag{13}$$

$$\overline{P}(T) \geq \overline{P}(S) + \left(\frac{\delta}{\sum_{k=1}^{S} \beta^{k-1}} - \frac{\delta}{\sum_{k=1}^{T} \beta^{k-1}} \right). \tag{14}$$

These are obtained by simple rearrangements of equation (4). Because $\beta \in (0, 1]$ and $R \leq S \leq T$, both terms in parentheses are positive. Therefore, the function $\overline{P}$ is locally increasing the duration of the contract, i.e., $\overline{P}(S) \leq \overline{P}(T)$.

Further, an increase in δ decreases the right-hand side of equation (13), so that the inequality remains satisfied. An increase in δ increases the right-hand side of equation (14). No increase in δ could make the shorter contract R preferred to S, but a large enough increase in δ will flip the inequality in (14), and the longer contract T will be preferred to S. QED.

A.2 Additional Predictions from Model

Recall the decision rule for the buyer from the illustrative example. The buyer will choose a long-term contract if and only if

$$E[\tilde{c}_{2:N}] - E[c_{2:N}] < \frac{\delta}{2} \tag{15}$$

This simple decision rule generates a number of comparative statics. I discuss two of these in the main text, and I present additional ones here.

Remark 3 Higher marginal costs lead to shorter contracts.

Higher marginal costs increase the left-hand side of equation (15). This increases the cost of long-term contracts relative to the savings in transaction costs, which shifts buyers to long-term contracts at the margin. Similarly, this an increase in transaction costs affects the right-hand side only and leads to longer contracts.

Remark 4 The optimal duration is increasing with autocorrelation in supply costs.

This prediction is intuitive. As the autocorrelation in marginal costs increases, there is less of a benefit from switching suppliers, and longer-term contracts are preferred. Suppose that d is a cost process with lower autocorrelation than c, but the same per-period marginal distribution, i.e. $E[d_{2:N}] = E[c_{2:N}]$. Let $\tilde{d}$ denote the average cost across two periods. Then it follows that, for $N > 3$,

$$\begin{aligned} E[\tilde{d}_{2:N}] &> E[\tilde{c}_{2:N}] \\ \implies E[\tilde{d}_{2:N}] - E[d_{2:N}] &> E[\tilde{c}_{2:N}] - E[c_{2:N}]. \end{aligned}$$

The marginal cost of long-term contracts is decreasing with the autocorrelation of the cost process. With greater autocorrelation, long-term contracts are preferred.

Remark 5 The optimal duration is decreasing in the variance of costs across suppliers, provided there is sufficient competition ($N > 3$).

For a simple case, consider location-scale transformations of c, such that $d = a + bc$ and $E[d] = E[c]$. Under the marginal cost structure d, a longer contract is chosen if

$$b \cdot (E[\tilde{c}_{2:N}] - E[c_{2:N}]) < \frac{\delta}{2}.$$

As b increases, shorter contracts become more desirable.

Remark 6 When costs are bounded from below, the optimal duration is U-shaped in the variance in costs, provided there is sufficient competition ($N > 3$).

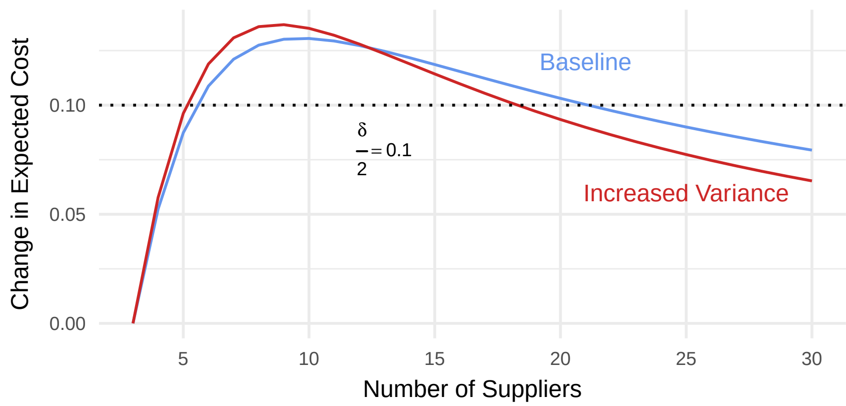
From a starting point of zero variance across suppliers, increasing the variance of marginal costs leads to shorter contracts, as there is more to gain from selecting the low-cost supplier in each period. This holds for the buyer-optimal contract as long as there are more than three suppliers, in which case the expected second-order statistic falls below the median. When costs are bounded from below, eventually both $E[\tilde{c}_{2:N}]$ and $E[c_{2:N}]$ approach zero, and the cost of a longer duration falls with respect to transaction costs. After a certain threshold, contract duration increases.

This occurs because the expected average price and expected per-period price approach the lower bound. Let c denote per-period marginal costs with a lower bound at 0, and let σ represent its standard deviation. Then, when $N > 3$,

$$\lim_{\sigma \to \infty} E[\tilde{c}_{2:N}] = \lim_{\sigma \to \infty} E[c_{2:N}] = 0.$$

As $E[\tilde{c}_{2:N}] - E[c_{2:N}] \to 0$, long-term contracts are optimal in the limit. This effect tends to dominate as N gets large, as more draws brings the minimum price closer to the lower bound.

Figure 6: Increased Variance in Cost



Notes: The blue line shows the marginal cost to the buyer of a two-period contract relative to one-period contracts and is equivalent to the blue line in Figure 1. The red line shows the marginal cost of a longer contract when the costs are draw from the same distributional family (the beta distribution) with 11 percent greater variance.

This tradeoff is illustrated in Figure 6. The blue line displays the baseline marginal cost of longer contracts, corresponding to the blue line in panel (b) of Figure 1 in the main text. Costs drawn from a beta distribution with shape parameters $(0.5, 0.5)$. The red line displays the change in marginal costs when variance of the cost distribution increases by 11 percent, corresponding to a beta distributions with shape parameters $(0.4, 0.4)$.

For $N < 12$, greater variance increases the cost of long-term contracts, reflecting the intuition of Remark 5. When $N \geq 13$, the winning supplier's price is close enough to the lower bound to reduce the cost, reflecting the prediction of Remark 6. Thus, the figure illustrates how an increase in variance leads to shorter contracts when competition is lower and longer contracts when competition is intense.

B Efficiency and Allocation of Rights (For Online Publication)

In this section, I explore the relationship between optimal and efficient contract duration. It should be noted that the analysis here is not restricted to the special case of the duration-setting problem, rather, any transaction characteristic that has a "scale" effect (as duration does on transaction costs) can be related to this framework. One of the natural extensions is to bundling, where T is the size of a bundle (determined by the buyer or seller) and δ is the transaction cost for the bundle.

B.1 A Framework Relating Optimal and Efficient Contract Duration

In contrast to the buyer, whose problem was presented in Section 2, the social planner's concern is minimizing expected costs.[40] Let $\overline{C}$ denote the ex ante expected cost conditional on $(T, \boldsymbol{x}, \boldsymbol{m})$, so that $\overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) = \sum_{n=1}^{\mathbb{N}} (E[C(n, T, \boldsymbol{x}, \boldsymbol{m})] \cdot \Pr(N = n | T, \boldsymbol{x}, \boldsymbol{m}))$.

For ease of exposition, assume that T is continuous and $\beta = 1$. Thus, the ex ante efficient $\tilde{T}$ contract is given by

$$\tilde{T} = \arg\min_{T \in \mathbb{T}} \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T} \tag{16}$$

with the first-order condition

$$\frac{d\overline{C}(T, \boldsymbol{x}, \boldsymbol{m})}{dT}|_{T=\tilde{T}} = \frac{\delta}{\tilde{T}^2}. \tag{17}$$

In general, $\frac{d\overline{C}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=\tilde{T}} \neq \frac{d\overline{P}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=\tilde{T}}$, which will result in an inefficiency when the contract is determined by the buyer. As long as interior solutions exist (see Proposition 9), we have the result that the efficient contract $\tilde{T}$ will be longer than the buyer-optimal contract T^* when $\frac{d\overline{C}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=\tilde{T}} < \frac{d\overline{P}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=\tilde{T}}$

Defining the expected seller surplus as $E[\pi(T, \boldsymbol{x}, \boldsymbol{m})] = \overline{P}(T, \boldsymbol{x}, \boldsymbol{m}) - \overline{C}(T, \boldsymbol{x}, \boldsymbol{m})]$, we have the following result:

Proposition 8. *When interior solutions to the buyer's problem and the social planner's problem exist, the efficient contract will be longer than the equilibrium (buyer-optimal) contract if and only if the expected seller surplus is increasing at $\tilde{T}$:*

$$\begin{aligned} \tilde{T} > T^* &\iff \left(\frac{d\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})}{dT}|_{T=\tilde{T}} - \frac{d\overline{C}(T, \boldsymbol{x}, \boldsymbol{m})}{dT}|_{T=\tilde{T}} \right) > 0 \\ &\iff \frac{dE[\pi(T, \boldsymbol{x}, \boldsymbol{m})]}{dT}|_{T=\tilde{T}} > 0 \end{aligned}$$

The existence of interior solutions depends on the concavity of the expected price function.

[40] In this setting, I assume the social planner is limited by information constraints; in this setting the social planner cannot observe the private information about sellers' costs. This reflects the idea that the mechanism (and the associated transaction costs) are important to the truthful revelation of information. A third party with full information would solve a different problem, awarding the contract to the lowest-cost seller at every instant and switching when the net savings outweigh the transaction cost.

Proposition 9. *Interior solutions to the buyer's problem and social planner's problem exist as long as the first-order conditions can be satisfied and $\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})$ and $\overline{C}(T, \boldsymbol{x}, \boldsymbol{m})$ are not too concave. In particular, $\frac{d^2\overline{P}(T,\boldsymbol{x},\boldsymbol{m})}{dT^2}|_{T=T^*} > -\frac{2}{T^*}\frac{\overline{P}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=T^*}$ and $\frac{d^2\overline{C}(T,\boldsymbol{x},\boldsymbol{m})}{dT^2}|_{T=\tilde{T}} > -\frac{2}{\tilde{T}}\frac{\overline{C}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=\tilde{T}}$. These are the second-order conditions to ensure that first-order conditions achieve a minimum.*

B.2 Numerical Illustration

To illustrate the difference in contracts, we replicate the illustrative example from section 2.2. The social planner's problem is similar to the buyers problem, except that the social planner will choose a long-term contract if

$$E[\tilde{c}_{1:N}] - E[c_{1:N}] < \frac{\delta}{2}.$$

Thus, the social planner's decision depends on the first-order statistic of cost draws, rather than the second-order statistic that generates the expected price. For $N > 3$, these order statistics have similar qualitative behavior, so the comparative static predictions follow for the efficient case. However, the efficient contract will not necessarily coincide with the buyer-optimal contract, raising the question of the allocation of rights to non-price terms of the transaction.

Figure 7 provides a comparison of the buyer-optimal and the efficient contract. Panel (a) displays the marginal impact on the price to the buyer of a longer contract with a blue line. The orange line displays the marginal impact on supply costs. Once N is large enough ($N > 5$ in the example), longer contracts have a greater marginal effect on price than cost. This is intuitive, as the first-order statistic approaches the lower bound faster than the second-order statistic.

Panel (b) plots the efficient contract with an orange dashed line. It has similar qualitative features to the buyer-optimal contract, displaying the inverse U shape. The efficient and buyer-optimal contract coincide only when $N \in \{6, 7\}$. When $N = 4$, the buyer would choose a long-term contract when the short-term contract is efficient, and when $N \in 8, ..., 21$ the buyer would choose a short-term contract when a long-term contract is efficient. Thus, the buyer-optimal contract may be longer or shorter than the efficient contract. Information rents from private costs drive a wedge between the buyer-optimal contract and the efficient contract.

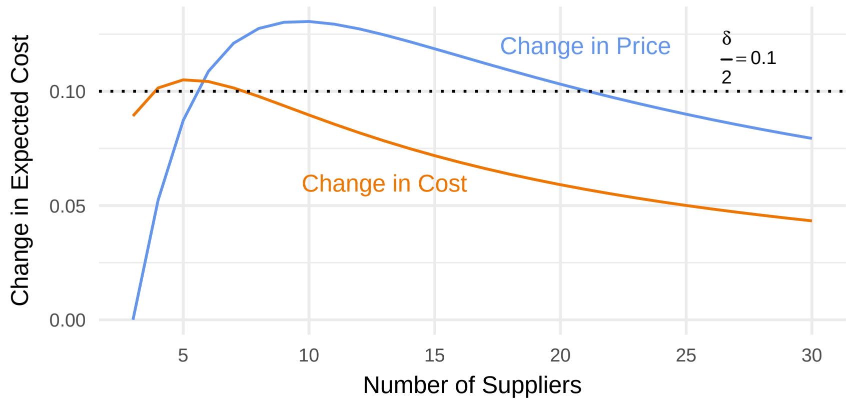
Reflecting Proposition 8, the buyer-optimal contract is (weakly) shorter than the efficient contract when seller surplus is increasing with the longer contract, i.e., when the blue line is above the orange line in panel (a).

B.3 Allocation of Term-Setting Rights

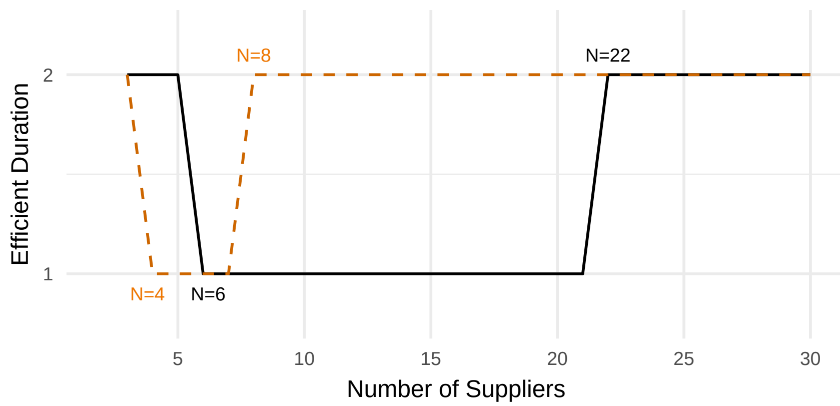
Given the general model, we can identify settings in which inefficiency arising from market power over contract length may be of first-order importance. In this section, I provide some intuition and a heuristic guide to the assignment of term-setting rights to limit such inefficiencies.

Figure 7: Comparing Buyer-Optimal and Efficient Contracts

(a) The Marginal Costs of Longer Contracts



(b) Efficient versus Optimal Contracts



Notes: The figure shows the relationship between competition, the marginal costs of longer contracts, and the effect on buyer-optimal and efficient durations. The blue line in panel (a) shows the marginal cost to the buyer of a two-period contract and is equivalent to the blue line in Figure 1. The orange line shows the increase in marginal social costs of a longer contract. The dash line reflects a transaction cost of 0.20 amortized over two periods. For values of N where the blue line is above the dashed line, $N \in \{6, ..., 21\}$, the buyer would prefer to issue one-period contracts, as the increase in price is greater than the savings in transaction costs. This range does not coincide with the efficient contract, which is plotted with the orange dashed line in panel (b). One-period contracts are efficient for $N \in \{4, .., 7\}$. The buyer will select the efficient contract in this example only if $N \in \{6, 7\}$.

The buyer's problem can be written in the following form:

$$\min_T \overline{P}(T, \boldsymbol{x}, \boldsymbol{m}) - \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T}$$
$$= \min_T E[\pi(T, \boldsymbol{x}, \boldsymbol{m})] + \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T}$$

Notice that when $\frac{dE[\pi(T,\boldsymbol{x},\boldsymbol{m})]}{dT} = 0$, this problem is equivalent to the social planner's problem. Therefore, when the buyer sets the duration of the contract, these contracts will be efficient when the seller surplus does not change with the length of the contract. The more sensitive buyer surplus is to the duration of the contract, the greater the potential for inefficiency.

What about assigning contract term-setting power along with the transaction costs to the sellers? Sellers solve the problem:

$$\begin{aligned} & \max_T \overline{P}(T, \boldsymbol{x}, \boldsymbol{m}) - \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) - \frac{\delta}{T} \\ = \quad & \min_T -\overline{P}(T, \boldsymbol{x}, \boldsymbol{m}) + \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T} \end{aligned}$$

Sellers solve the social planner problem when $\frac{d\overline{P}(T,\boldsymbol{x},\boldsymbol{m})}{dT} = 0$. Therefore, if price is not sensitive to contract duration, it is efficient to let the sellers determine the length of the contract.[41]

If either price or buyer surplus changes with the duration of the contract, there is potential for inefficiency arising from market power. A simple heuristic to mitigate efficiency loss is to let sellers determine contract duration when the duration affects price more than buyer surplus, and to let buyers determine contract duration otherwise.

These heuristics, combined with Proposition 8, provide insight into which settings may allow for substantive inefficiencies and whether the efficient contract is longer or shorter. Below, I provide a simple example to illustrate how changing the allocation of rights over non-price terms, such as duration, may lead to vastly different outcomes.

Example: Markup Pricing Suppose sellers in equilibrium follow a simple markup pricing rule, $P = \mu C$. Then the buyer's problem is

$$\min_T \mu\overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T}$$

and the seller's problem is

$$\min_T (1-\mu)\overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T}$$

As $\mu \geq 1$ in equilibrium, the seller's problem reverses the sign that expected costs enter in the objective function. By increasing costs, sellers increase total profits. In this setting, the buyer should determine the duration. The greater the markup, the more that the equilibrium contract will diverge from the efficient contract.

B.4 Achieving Efficiency with a Tax

The efficient contract can be achieved with a per-transaction tax (or subsidy) when either side of the transaction holds the term-setting rights. When the buyer determines the length of the

[41] Sellers have an equivalent rule to Proposition 8: $t_S > \tilde{T} \iff \frac{d\overline{P}(T,\boldsymbol{x},\boldsymbol{m})}{dT}|_{T=\tilde{T}} > 0$. This means that either 1) $t_S \geq \tilde{T} \geq t$, 2) $t \geq \tilde{T} \geq t_S$, or 3) $t_S \geq \tilde{T} \cap t \geq \tilde{T}$. The case where both the buyer-optimal and seller-optimal contract are shorter than the efficient contract is ruled out by the fact that per-period costs must be increasing at the efficient contract for an interior solution.

contract, the efficient per-transaction tax τ_B solves

$$\tau_B = \tilde{T}^2 \frac{dE[\pi(T, \boldsymbol{x}, \boldsymbol{m})]}{dT}|_{T=\tilde{T}}$$

This tax equates the buyer's problem with the social planner's problem. Note below how the tax causes the externality on the seller to drop out at the efficient contract.

$$\begin{aligned}
\tilde{T} &= \arg\min_T E[\pi(T, \boldsymbol{x}, \boldsymbol{m})] + \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta + \tau_B}{T} \\
&= \arg\min_T E[\pi(T, \boldsymbol{x}, \boldsymbol{m})] + \frac{\tau_B}{T} + \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T} \\
&= \arg\min_T \overline{C}(T, \boldsymbol{x}, \boldsymbol{m}) + \frac{\delta}{T}
\end{aligned}$$

Analogously, the efficient tax on the seller (when the seller has term-setting rights) is given by

$$\tau_S = -\tilde{T}^2 \frac{d\overline{P}(T, \boldsymbol{x}, \boldsymbol{m})}{dT}|_{T=\tilde{T}}$$

In general, $\tau_S \neq \tau_B$. A policymaker has a choice between two efficient taxes, with different effects on tax revenue.

C Data Details (For Online Publication)

C.1 Details of Data Cleaning

To construct this dataset, I combined detailed location, price, and vendor information maintained in the Federal Procurement Data System (FPDS)[42] with contract-specific documents downloaded from the Federal Business Opportunities (FedBizOpps) website. By law, the FPDS keeps public records of all contracts for the U.S. federal government. The FedBizOpps website is the most common posting location for competitive contracts, which must be posted publicly. From October 2003 through May 2017, I identified 11,210 unique solicitations in the FPDS data and 7,984 unique solicitations in the FedBizOpps data. I was able to merge 4,119 of these contracts. Unique contracts were identified in FPDS from the variables IDVPIID and PIID.

From the solicitations found in both systems, I selected competitive, non-zero value contracts in the United States that had documents with relevant cost information (i.e., square footage).[43] I obtained the relevant contract documents (request for proposal, cleaning frequency charts, maps, etc.), and constructed detailed contract information directly from the documents. The resulting 1,427 contracts were further processed by hand to construct key variables, including the square footage of the site to be cleaned, the frequency of service,[44] and the facility type. Contracts that were restricted to economically disadvantaged businesses were removed from the sample. After identifying contracts for regular cleaning service, I restricted the sample to contracts that received more than one bid and had an annual price of less than $1 million. Table 12 summarizes the construction of the dataset.

I matched the contract-specific dataset with auxiliary datasets of (1) government contracting expenditures at the same location in related products and (2) local labor market conditions. Local labor market conditions include county-level unemployment from the Local Area Unemployment Statistics and the number of NAICS-code level establishments in the same 3-digit ZIP code from the County Business Patterns data.

C.2 Measurement Error in FPDS

Though the FPDS data are broadly appealing for research, there is a great deal of measurement error in the data, likely due to user (input) error. As most contracts have multiple entries and multiple indicators of duration and value within each entry, different assumptions about data quality could lead to widely different measures of price. As I obtained high-quality measures of price and duration from a second data source, FedBizOpps, I was able to cross-validate the data and construct preferred measures from the FPDS.

[42] These data were obtained from USASpending.gov.

[43] The candidate solicitations were identified with a computational text analysis of documents from all matched contracts.

[44] Cleaning frequency is encoded as the maximum required weekly frequency.

Table 12: Construction of Sample

	Criterion	Observations	Portion
(1)	FedBizOpps Solicitation IDs	7,984	
(2)	FPDS Solicitation IDs	11,210	
	Matched (1) and (2)	4,119	
(3)	In United States	3,818	0.93
(4)	Competitive Procurement	3,584	0.94
(5)	Non-Zero FPDS Value	4,064	0.99
(6)	Square Footage Indicators	1,654	0.40
	Intersection of (3)-(6)	1,427	0.35
(7)	US, Excluding Territories	1,409	0.99
(8)	Regular Cleaning Service	1,405	0.98
(9)	Measurable Square Footage	1,301	0.91
(10)	No Economic Disadvantage Preference	1,289	0.90
(11)	Single Auction, More Than 1 Bid	1,339	0.94
(12)	Annual Price Less Than $1,000,000	1,338	0.94
	Estimation Sample Intersection of (7)-(12)	1,046	0.73

Notes: The table describes the construction of the estimation sample from two data sources for facility cleaning contracts for the U.S. federal government. The relevant range is from October 1, 2003 through May 1, 2017 for the Federal Procurement Data System and though February 3, 2017 for FedBizOpps. After cleaning identification variables, 4,119 of the solicitations were matched. Of these, 1,046 met the criteria needed for analysis, including the availability of square footage data, which is a key cost indicator, non-zero value, and receiving more than one bid from the solicitation.

In supplemental work, I detail the steps to cross-check the data and different candidate measures for price and duration. These comparisons result in the following recommendations:

Duration *The maximum observed date in the contract, minus the start date in the first entry within a contract.*

Price *The price is the value of obligated dollars if it is the same (or within 10 percent) in consecutive years. If this is not observable, use the maximum value of the three (summed) measures of dollar amounts for the total value of the contract. Divide this by the duration measure above to obtain the price.*

Any missing values of price or duration in the FedBizOpps data are imputed with the above values constructed from FPDS. Researchers interested working with the FPDS data may contact the author for a short paper that details the measurement error in the data and the accuracy of variables constructed under alternative assumptions.

C.3 Count of Sites by Location Type and Government Agency

Table 13: Count of Sites by Contracting Agency

Agency	Count	Percent
Defense	389	37.2
Agriculture	347	33.2
Veterans Affairs	80	7.7
Commerce	78	7.5
Homeland Security	45	4.3
Interior	43	4.1
GSA	40	3.8
Energy	5	0.5
Labor	5	0.5
Transportation	4	0.4
EPA	2	0.2
State	2	0.2
National Archives	2	0.2
CNCS	1	0.1
Health And Human Services	1	0.1
OPIC	1	0.1
Railroad Retirement Board	1	0.1
Total	1,046	100.0

Notes: The table lists the count of contracts in the estimation sample by government department or agency.

Table 14: Count of Contracts by Location Type

Category	Sub-Category	Count
Office (424)		
	Office	221
	Recruiting Office	203
Field Office (270)		
	Ranger District Office	171
	Field Office	46
	Ranger Station	43
	Work Center	7
	Reserve Fleet	3
Research (111)		
	Weather Station	43
	Laboratory	28
	Research Center	28
	Plant Materials Center	12
Medical (61)		
	Clinic	36
	Medical Center	25
Services (59)		
	Service Center	38
	Vet Center	21
Visitors (41)		
	Recreation Area	18
	Cemetery	9
	Visitor Center	7
	Restroom	4
	Museum	3
Airport (30)		
	Airport	30
Technical (19)		
	Power Plant	14
	Surveillance Center	4
	Data Center	1
Accommodations (18)		
	Housing	14
	Dormitories	4
Industrial (13)		
	Equipment Center	6
	Warehouse	6
	Gym	2
Total		1,046

Notes: The table lists the count of contracts in the estimation sample by facility type. Types were hand-coded after reading the contract documents.

C.4 Summary Statistics by Department

Table 15: Summary Statistics by Department

Department	Duration Mean	Duration S.D.	Price Mean	Square Footage Mean	Count
Defense	4.1	1.3	38,782	27,517	389
Agriculture	3.9	1.2	18,685	13,285	347
Veterans Affairs	4.6	1.2	61,040	24,647	80
Commerce	4.7	0.8	15,846	9,578	78
Homeland Security	4.8	0.4	93,738	21,746	45
Interior	4.1	1.4	28,746	15,709	43
GSA	4.6	1.1	222,045	144,749	40
Other	4.5	1.1	160,972	58,578	24

Notes: The table displays summary statistics for the contract characteristics of the estimation sample grouped by federal department. The mean and standard deviation for contract duration are provided, along with the mean annual price and the mean square footage. The final column reports the count of contracts in each department.

C.5 Contract Documents

The following page is an example first page from a building cleaning service contract. The subsequent pages contain an example description of the required services and their respective frequencies.

CONTRACT DOCUMENTS, EXHIBITS OR ATTACHMENTS

C.1 SCOPE OF CONTRACT

Description of Work: The intent of this contract is to secure services (inclusive of supplies) for normal custodial (janitorial) and routine maintenance service at the Georgetown Ranger District of the Eldorado National Forest.

2 Project Location & Description

Location: The project is located on the Georgetown Ranger District, 7600 Wentworth Springs Road, Georgetown, CA 95634.

Description: The headquarters office of the Georgetown Ranger District is located at 7600 Wentworth Springs Road, Georgetown, California. Winter working hours are 6:00 a.m. through 5:30 p.m. Monday through Friday from November through May. Summer hours are 7:00 a.m. through 6:00 p.m. Sunday through Saturday.

The office building contains approximately 6,376 gross square feet of space. The office is carpeted throughout, expect for restrooms and front reception area. There are 6 restrooms in the building.

Any prospective contractor desiring an explanation or interpretation of the solicitation, drawings, specifications, etc., must request it in writing from the Contracting Officer soon enough to allow a reply to reach all prospective contractors before the solicitation closing date. Oral explanations or instructions given before the award of a contract will not be binding.

3 Estimated Start Date & Contract Time

Start: January 1, 2010
Time: 9 Months

4 Cleaning Schedule

Work Days and Hours. Work shall be performed during Monday through Friday, provided that no work is performed between 7 a.m. and 4:30 p.m. on normal Federal workdays. Regularly scheduled twice weekly work will not be on consecutive days. The contractor may work in the building on weekends and Federal holidays without restrictions to hours.

Quarterly cleaning items will be performed the first week (preferably on Friday) of December, March, June, and September. Annual cleaning shall be performed during the first 2 weeks of May.

5 Licenses and Insurance

Contractor shall provide proof of Workman's Compensation. If the contractor is working alone, with no employees, no Workman's Compensation is required.
.

6 Contractor-Furnished Materials and Services

6-1. The Contractor shall provide everything--including, but not limited to, all equipment, supplies (listed below), transportation, labor, and supervision--necessary to complete the project, except for that which the contract clearly states is to be furnished by the Government.

18. TECHNICAL SPECIFICATIONS

The janitorial services shall be performed in accordance with the following specifications at the frequencies prescribed.

1. Services Performed Daily - Bid Item #0001

   a. Restrooms
      - Clean and sanitize all surfaces including sinks, counters, toilet bowls, toilet seats, urinals, etc.
      - Clean and sanitize tile walls adjacent to and behind urinals and water closets.
      - Clean and sanitize sanitary napkin receptacles and replace liners.
      - Sweep, mop and sanitize tile floors.
      - Clean and polish mirrors, dispensers and chrome fixtures
      - Empty, clean and sanitize all wastebaskets.
      - Spot clean all other surfaces and dust horizontal surfaces including tops of partitions and mirrors.
      - Re-stock restroom supplies.

   b. Front Foyer and Doors
      - Wash inside and outside of all glass surfaces on entrance doors. Remove dust and soil from metal frames surrounding entrance glass doors.
      - Vacuum rugs.
      - Sweep and mop tile floors and clean baseboards.

   c. Reception Area
      - Vacuum all reception carpeted areas and rugs including edges.
      - Clean and polish all counter surfaces.

   d. Drinking Fountains
      - Clean and sanitize drinking fountains.

   e. Breakroom Waste Receptacles
      - Empty all waste receptacles, wash if needed with a sanitizing cleaner.

2. Services Performed Weekly – Bid Item #0002

   a. Waste Receptacles
      - Empty all waste receptacles unless needed more frequently. Wash if needed with a sanitizing cleaner. Change liners only if needed.

   b. Breakroom
      - Sweep and mop, use a cleaner that doesn't require rinsing and is a sanitizer and will not damage the wax. Mop under table, chairs, coffeemaker cabinet, trash can and wheeled carts.
      - Clean Formica countertops.

- Spot clean walls and doors.

c. Back Door Foyers
   - Sweep and mop, use a cleaner that doesn’t require rinsing and is a sanitizer and will not damage the wad. Vacuum rug and clean baseboards.
   - Spot clean walls and doors.

d. Hallways
   - Vacuum all carpeted areas, including wall edges.
   - Spot clean anytime a stain or soiled area needs cleaning.
   - Tile floors sweep and mop, use a cleaner that doesn’t require rinsing and is a sanitizer and will not damage the wax.
   - Spot clean walls, doors and partitions that appears to be soiled.

e. Outdoor Waste Receptacles
   - Empty all outdoor waste receptacles and ash trays at the front entrance and two back entrances**.** Wash if needed with a sanitizing cleaner. Change liners if needed.

f. Conference Room
   - Clean and polish conference room tables.
   - Vacuum all carpeted areas, including wall edges and around the edges of all furniture which is not easily moveable, this includes under desks, tables, chairs etc. All light weight furniture must be moved and vacuumed under. All electrical cords must be picked up and vacuumed under.
   - Spot clean anytime a stain or soiled area needs cleaning.
   - Vacuum chalk dust out of chalk tray. Wash chalkboard only if it has been erased by the Forest Service.

g. Copy Machine and Mail room area
   - Vacuum all carpeted areas, including wall edges and around the edges of all furniture which is not easily moveable, this includes under desks, tables, chairs etc. All light weight furniture must be moved and vacuumed under. All electrical cords must be picked up and vacuumed under.
   - Spot clean anytime a stain or soiled area needs cleaning.
   - Clean and polish table and counter tops.

3. <u>Services Performed Monthly - Bid Item #0003</u>

a. Dusting
   - Dust below a 5 foot level. Dust all horizontal and vertical surfaces including but not limited to furniture, baseboards, wood molding, windowsills, bookcases, ledges, signs, wall hangings, photographs, fire alarm boxes, exhibits, top edge of privacy partitions, excluding desktops and computers.

b. Offices
   - Vacuum all carpeted areas, including wall edges and around the edges of all furniture which is not easily moveable, this includes under desks, tables,

chairs etc. All light weight furniture must be moved and vacuumed under. All electrical cords must be picked up and vacuumed under.

- Spot clean anytime a stain or soiled area needs cleaning.
- Tile floors sweep and mop, use a cleaner that doesn't require rinsing and is a sanitizer and will not damage the wax.

c. Outside Foyer and Adjacent Areas

- Sweep outside area around all outside doors and adjacent area.
- Pick up any trash laying within 100 feet on the outside of the office building and parking area. This includes all the bushes and trees.

4. <u>Services Performed Annually - - Bid Item #0004</u>

a. Dusting above 5 feet

- All horizontal and vertical dust catching surfaces shall be kept free of obvious dust, dirt, and cobwebs. Dust furniture in all offices above the 5 foot level, including, but not limited to tops of high bookcases and top edge of privacy partitions.

b. Windows

- Clean all windows and screens inside and outside of building, with an appropriate glass cleaner. Removing screens on windows that have screens for cleaning.

c. Blinds

- Dust, clean and/or vacuum all window blinds. Vinyl blinds may require a liquid cleaner and blinds with fabric may require vacuuming. Clean in accordance with manufacturer's recommendations by type of fabric or material.

d. Chairs

- Vacuum all upholstered chairs.
- Clean all vinyl covered chairs with an appropriate cleaner for vinyl.
- Clean chair legs and/or pedestal bases on all the chairs in the office.
- Wood chairs use an oil, such as lemon oil.

e. Door and Door Frames

- Clean with appropriate wood/metal cleaner and apply a good penetrating oil to the wood doors.

D Identification Proofs

D.1 Some Lemmas

To demonstrate the following proofs, it will be useful to first introduce several lemmas.

Lemma 1. *For symmetric auctions with independent private values,* $E[b_{1:N}] = E[c_{2:N}]$.

This is a standard result and can be obtained directly by taking the expectation given the equilibrium bid function. I omit the proof here.

Lemma 2. $\min b_{1:N} = E[c_{1:(N-1)}]$ *for the IPV model when the support of* c *is bounded from below by* $\underline{c} > -\infty$.

Proof. The equilibrium bid function is given by

$$\beta(c; N) = c + \frac{\int_c^\infty [1 - F(\xi)]^{N-1} d\xi}{[1 - F(c)]^{N-1}}$$

Then the minimum bid is

$$\begin{aligned}
\beta(\underline{c}; N) &= \underline{c} + \frac{\int_{\underline{c}}^\infty [1 - F(\xi)]^{N-1} d\xi}{[1 - F(\underline{c})]^{N-1}} \\
&= \underline{c} + \int_{\underline{c}}^\infty [1 - F(\xi)]^{N-1} d\xi \\
&= \underline{c} + \xi[1 - F(\xi)]^{N-1}|_{\underline{c}}^\infty + \int_{\underline{c}}^\infty \xi(N-1)f(\xi)[1 - F(\xi)]^{N-2} d\xi \\
&= \underline{c} + (0 - \underline{c}) + \int_{\underline{c}}^\infty \xi(N-1)f(\xi)[1 - F(\xi)]^{N-2} d\xi \\
&= E[c_{1:(N-1)}]
\end{aligned}$$

Where the third line comes from integration by parts. Here we require the assumption that $\lim_{\xi\to\infty} f(\xi)[1 - F(\xi)]^N = 0$, so that

$$\begin{aligned}
\xi[1 - F(\xi)]^{N-1}|_{\underline{c}}^\infty &= \lim_{\gamma\to 0} \frac{[1 - F(\frac{1}{\gamma})]^{N-1}}{\gamma} - \underline{c}[1 - F(\underline{c})]^{N-1} \\
&= \lim_{\gamma\to 0} \frac{-(N-1)f(\frac{1}{\gamma})[1 - F(\frac{1}{\gamma})]^{N-2}}{1} - \underline{c} \\
&= 0 - \underline{c}
\end{aligned}$$

□

Lemma 3. *The expected* k*-th order statistic of* N *draws can be written in terms of the expected* k*-th and* $(k+1) - th$ *order statistics from* $N + 1$ *draws:* $E[c_{k:N}] = \frac{k}{N+1} E[c_{(k+1):(N+1)}] + \frac{N+1-k}{N+1} E[c_{k:(N+1)}]$

Proof. First, examining the difference between the k-th order statistics of N and $N+1$ draws. Expressing $E[c_{k:N}] - E[c_{k:(N+1)}]$ and rearranging terms gives:

$$
\begin{aligned}
& E[c_{k:N}] - E[c_{k:(N+1)}] \\
= & \int \frac{N!}{(k-1)!(N-k)!} cf(c)F(c)^{k-1}[1-F(c)]^{N-k} dc - \int \frac{(N+1)!}{(k-1)!(N+1-k)!} cf(c)F(c)^{k-1}[1-F(c)]^{N+1-k} dc \\
= & \int \left(\frac{N!(N+1-k)}{(k-1)!(N+1-k)!} - \frac{(N+1)!}{(k-1)!(N+1-k)!}[1-F(c)] \right) cf(c)F(c)^{k-1}[1-F(c)]^{N-k} dc \\
= & \int \frac{(N+1)!}{(k-1)!(N+1-k)!} cf(c)F(c)^{k}[1-F(c)]^{N-k} dc - \int \frac{kN!}{(k-1)!(N+1-k)!} cf(c)F(c)^{k-1}[1-F(c)]^{N-k} dc \\
= & \frac{k}{(N+1-k)} \left(E[c_{(k+1):(N+1)}] - E[c_{k:N}] \right)
\end{aligned}
$$

Rearranging, we obtain
$E[c_{k:N}] = \frac{k}{N+1} E[c_{(k+1):(N+1)}] + \frac{N+1-k}{N+1} E[c_{k:(N+1)}]$. □

D.2 Proof of Proposition 5

Consider the entry equation

$$E[\pi_n \cdot U \cdot h(\boldsymbol{x})|n,T] - k(\boldsymbol{m})\cdot > 0 \iff N \geq n \tag{18}$$

$$\implies E[\pi_n|n,T] \cdot \frac{h(\boldsymbol{x})}{k(\boldsymbol{m})} > \varepsilon \iff N \geq n \tag{19}$$

For any realization $(T, \boldsymbol{x}, \boldsymbol{m})$, there exists $(T, \boldsymbol{x}', \boldsymbol{m}')$ such that $\Pr(N \geq n|T, \boldsymbol{x}, \boldsymbol{m}) = \Pr(N \geq n|T, \boldsymbol{x}', \boldsymbol{m}')$ for all N.[45] Using these values $\boldsymbol{x}'$ and $\boldsymbol{m}'$ that provide the same conditional distribution of N, calculate:

$$\frac{E[B \cdot U \cdot h(\boldsymbol{x})|N, T, \boldsymbol{x}, \boldsymbol{m}]}{E[B \cdot U \cdot h(\boldsymbol{x}')|N, t, \boldsymbol{x}', \boldsymbol{m}']} = \frac{E[B|N,t] \cdot E[U|N,T,\boldsymbol{x},\boldsymbol{m}] \cdot h(\boldsymbol{x})}{E[B|N,t] \cdot E[U|N,t,\boldsymbol{x}',\boldsymbol{m}'] \cdot h(\boldsymbol{x}')} = \frac{h(\boldsymbol{x})}{h(\boldsymbol{x}')}. \tag{20}$$

As $h(\boldsymbol{x})$ is normalized to 1 at $\boldsymbol{x} = \boldsymbol{x}_0$, $h(\cdot)$ is identified.

Once $h(\cdot)$ is identified, $E[B|N, T, \boldsymbol{x}, \boldsymbol{m}]$ is identified by calculating the mean of the scaled (conditional) winning bid $E[\frac{1}{h(\boldsymbol{x})} B \cdot U \cdot h(\boldsymbol{x})|N, T, \boldsymbol{x}, \boldsymbol{m}]$. Recall that $B \cdot U \cdot h(\boldsymbol{x})$ is the observed winning bid.

$k(\cdot)$ is identified by the relation

$$\frac{h(\boldsymbol{x})}{k(\boldsymbol{m})} = \frac{h(\boldsymbol{x}')}{k(\boldsymbol{m}')} \tag{21}$$

based on the identification of $h(\cdot)$ and the normalization $k(\boldsymbol{m}_0) = 1$ for an arbitrary $\boldsymbol{m}_0$.

Relative profits are identified by considering different values of $\boldsymbol{x}$ and $\boldsymbol{m}$ that generate N and $N' \neq N$ with the same probability. For every $(N, T, \boldsymbol{x}, \boldsymbol{m})$, there exists $(N', T, \boldsymbol{x}', \boldsymbol{m}')$ for

[45] Here, and once more in the proof, I rely on either $h(\cdot)$ or $k(\cdot)$ having broad support.

which $\Pr(N \geq n|T, \boldsymbol{x}, \boldsymbol{m}) = \Pr(N' \geq n|T, \boldsymbol{x}', \boldsymbol{m}')$. Here, I re-use notation; $\boldsymbol{x}'$ and $\boldsymbol{m}'$ are different from the first part of this section. Thus, the entry condition

$$E[\pi_N|N,T] \cdot \frac{h(\boldsymbol{x})}{k(\boldsymbol{m})} = E[\pi_{N'}|N',t] \cdot \frac{h(\boldsymbol{x}')}{k(\boldsymbol{m}')} \tag{22}$$

can be used to solve for $\frac{E[\pi_N|N,T]}{E[\pi_{N'}|N',T]}$, as $h(\cdot)$ and $k(\cdot)$ are identified. Analogously, relative profits $\frac{E[\pi_N|N,T]}{E[\pi_N|N,T']}$ are identified.

It is straightforward to extend these identification results to setting in which sellers observe U prior to entry. This is possible because the distribution of U conditional on $(T, \boldsymbol{x}, \boldsymbol{m})$ will be equal to the distribution of U conditional on $(T, \boldsymbol{x}', \boldsymbol{m}')$.

D.3 Proof of Proposition 6

The ratio of expected profits for n and n' conditional on T is given by

$$R = \frac{E[\pi_n|n,T]}{E[\pi_{n'}|n',T]} = \frac{\frac{1}{n}\left(E[B|n,T] - E[C|n,T]\right)}{\frac{1}{n'}\left(E[B|n',T] - E[C|n',T]\right)} \tag{23}$$

As shown by Proposition 5, R is identified. Let $n' = n+1$.

When the selection mechanism is a symmetric auction. $E[B|n,T] = E[C_{2:n}|T]$ and $E[C|n,T] = E[C_{1:n}|T]$. From here on I suppress notation indicating that costs are conditional on T. From Lemma (3), we have $E[C_{1:n}] = \frac{1}{n+1}E[C_{2:(n+1)}] + \frac{n}{n+1}E[C_{1:(n+1)}]$. Plugging this into the equation for R obtains

$$\begin{aligned} R\left(E[C_{2:(n+1)}] - E[C_{1:(n+1)}]\right) &= E[C_{2:n}] - \frac{1}{n+1}E[C_{2:(n+1)}] - \frac{n}{n+1}E[C_{1:(n+1)}] \\ \left(R + \frac{n}{n+1}\right)E[C_{1:(n+1)}] &= E[C_{2:n}] - \left(R + \frac{1}{n+1}\right)E[C_{2:(n+1)}] \end{aligned}$$

$E[C_{2:(n+1)}]$ and $E[C_{2:(n+1)}]$ are equivalent to $E[B|N=n]$ and $E[B|N=n+1]$, both of which are identified by Proposition 5. Therefore, $E[C_{1:(n+1)}]$ is identified. $E[C_{1:n}]$ is obtained from equation (23). These are sufficient to identify seller surplus.

Once seller surplus is identified, the distribution of ε is identified from equation (19) by using variation in $h(\cdot)$ or $k(\cdot)$.

D.4 Proof of Proposition 7

For each observed sequential value of $N \in \{\underline{N}, ..., \overline{N}\}$, the first-order and second-order statistics of N draws from the cost distribution are identified (see Propositions 5 and 6). Using the recursive relationship of order statistics shown in Lemma 3, these are equivalent to identifying the first $\overline{N} - \underline{N} + 2$ expected order statistics from $\overline{N}$ draws of C.

D.5 Alternative Identification and a Note on Independent Private Values

The model in this paper allows for endogenous entry. To separate the private cost distribution from unobservable heterogeneity, I make use of a entry cost shifter $\boldsymbol{m}$ to generate exogenous variation in N. Without endogenous entry, there is no entry cost shifter and the entry equation cannot be used to identify the model. When N is purely exogenous, variation in N can still be used to separately identify the private and common cost distributions.

Proposition 10. *First-price, symmetric auctions with unobserved heterogeneity and conditionally independent private values are identified when only the winning bid and the number of bidders is observed. In particular, seller surplus and the first* $(\overline{N} - \underline{N} + 2)$ *expected order statistics of* $\overline{N}$ *draws from* F *are identified. Identification is obtained without modeling entry as long as there is no selection on unobservables.*

The result can be generalized to auction settings that are independent of the duration-setting problem. Thus, this identification result may prove practical. With only the winning bid and variation in the number of bidders, researchers can estimate a model with unobserved heterogeneity, which is far less restrictive than the assumption of independent private values (IPV) that is common in such settings with limited data.

This implies that in any setting where estimation is motivated by IPV, one could also estimate a conditional independent private values model with unobserved heterogeneity. The econometrician may expect that unobserved heterogeneity is present, and this provides a theoretical background to test for its importance. In Appendix F, I detail a computational approach that greatly speeds up the maximum likelihood estimation of these models.

Proof. The ratio of second-order statistics is identified by comparing winning bids $B \cdot U \cdot h(\boldsymbol{x})$ for different values of n and n'.

$$\frac{E[B|n, T, \boldsymbol{x}, \boldsymbol{m}] \cdot E[U|n, T, \boldsymbol{x}, \boldsymbol{m}] \cdot h(\boldsymbol{x})}{E[B|n', T, \boldsymbol{x}, \boldsymbol{m}] \cdot E[U|n', T, \boldsymbol{x}, \boldsymbol{m}] \cdot h(\boldsymbol{x})} = \frac{E[C_{2:n}|T, \boldsymbol{x}, \boldsymbol{m}]}{E[C_{2:n'}|T, \boldsymbol{x}, \boldsymbol{m}]} \tag{24}$$

where $E[U|n, T, \boldsymbol{x}, \boldsymbol{m}] = E[U|n', T, \boldsymbol{x}, \boldsymbol{m}] = E[U|T, \boldsymbol{x}, \boldsymbol{m}]$ by independence and no selection on unobservables.

From here on, C_i and U may be conditional on $(T, \boldsymbol{x}, \boldsymbol{m})$. I suppress this in my notation for clarity. Normalizing $E[U] = 1$ pins down the scale of $E[C_{2:n}]$. [46]

Suppose that another $(\hat{F}, \hat{G})$ rationalizes the data. Then

$$\begin{aligned} B_n \cdot U &\overset{d}{=} \hat{B}_n \cdot \hat{U} \\ B_{n'} \cdot U &\overset{d}{=} \hat{B}_{n'} \cdot \hat{U} \end{aligned}$$

[46] Note that, in practice, we may normalize $E[U|T, \boldsymbol{x}, \boldsymbol{m}] = 1$ for all $(T, \boldsymbol{x}, \boldsymbol{m})$ realizations. How the mean of $C_{2:n} \cdot U$ changes is captures in changes to the mean of C.

Construct $\tilde{b}_{n'}$, $\tilde{\hat{b}}_{n'}$, $\tilde{U}$, and $\tilde{\hat{U}}$ as random variables that are independent of and have the same conditional distributions as their tilde-free counterparts. Then it follows that

$$\begin{aligned}(B_n \cdot U) \cdot \left(\tilde{\hat{B}}_{n'} \cdot \tilde{\hat{U}}\right) &\overset{d}{=} \left(\hat{B}_n \cdot \hat{U}\right) \cdot \left(\tilde{B}_{n'} \cdot \tilde{U}\right) \\ \implies B_n \cdot \tilde{\hat{B}}_{n'} &\overset{d}{=} \hat{B}_n \cdot \tilde{B}_{n'}\end{aligned}$$

From this relation, we may take the minimum on both sides. By independence and Lemma 2, I obtain

$$\begin{aligned}E[C_{1:(n-1)}] \cdot E[\hat{C}_{1:(n'-1)}] &= E[\hat{C}_{1:(n-1)}] \cdot E[C_{1:(n'-1)}] \\ \frac{E[C_{1:(n-1)}]}{E[C_{1:(n'-1)}]} &= \frac{E[\hat{C}_{1:(n-1)}]}{E[\hat{C}_{1:(n'-1)}]}\end{aligned}$$

That is, any $(\hat{F}, \hat{G})$ that rationalizes the data has a private cost distribution with the same ratio of first order statistics.

Finally, using the fact that $E[C_{1:(n-1)}] = \frac{1}{n}E[C_{2:n}] + \frac{n-1}{n}E[C_{1:n}]$, we can link together these ratios when $n' = n + 1$.

$$\begin{aligned}\frac{\frac{1}{n}E[C_{2:n}] + \frac{n-1}{n}E[C_{1:n}]}{E[C_{1:n}]} &= \frac{\frac{1}{n}E[\hat{C}_{2:n}] + \frac{n-1}{n}E[\hat{C}_{1:n}]}{E[\hat{C}_{1:n}]} \\ \implies \frac{E[C_{2:n}]}{E[C_{1:n}]} &= \frac{E[\hat{C}_{2:n}]}{E[\hat{C}_{1:n}]}\end{aligned}$$

As we have identified $E[C_{2:n}]$, $E[C_{1:n}]$ and $E[C_{1:(n-1)}]$ is also identified. Therefore, $\hat{F}$ and F have the same ratio of second-order statistics. With sequential values of $N \in \{\underline{N}, ..., \overline{N}\}$, we can iterate forward from the from the identified first-order and second-order statistics using the recursive relationship between order statistics from Lemma 3. Therefore, $\hat{F}$ and F and identical up to the first $\overline{N} - \underline{N} + 2$ expected order statistics from $\overline{N}$ draws of C. □

E A Model with Microfoundations (For Online Publication)

In the empirical application of this paper, I employ a "reduced-form" approach to a capturing how the distribution of private costs changes with T. Here, I provide a model of underlying costs that generates both the distribution of costs and how duration affects the distribution. Suppose that instantaneous costs follow an Ornstein-Uhlenbeck diffusion process. The continuous-time cost process X_t is governed by the differential equation

$$dx_t = \theta(\mu - x_t) + \gamma dW_t$$

where W_t is a Wiener process. This process is stationary over t. That is, any contract with duration T will have the same unconditional distribution as any other contract with duration T. Define the average cost over time T as

$$c_T = \frac{1}{T}\int X_t dt$$

Then c_T is Gaussian with mean μ and variance $\sigma^2 = \frac{1}{T^2}\frac{\gamma^2}{\theta^3}\left(\theta T + e^{-\theta T} - 1\right)$. When costs are Gaussian, $E[c_{1:N}(\sigma)] = E[z_{1:N}]\sigma + \mu$, where z is a standard normal.

First, consider the efficient contract. Let $\xi(T) = \sigma = \sqrt{\frac{1}{T^2}\frac{\gamma^2}{\theta^3}\left(\theta T + e^{-\theta T} - 1\right)}$. For ease of exposition, assume $\beta = 1$. The efficient contract T solves

$$\min_T E[z_{1:N}]\xi(T) + \mu + \frac{\delta}{T}.$$

As $E[z_{1:N}]$ is negative and the variance of c_T is decreasing with T, the average expected supply cost $\mu + E[z_{1:N}]\xi(T)$ is increasing with T. In this microfounded model, the increasing supply costs over many periods is due the idiosyncratic variation over time. The mean expected supply cost for each bidder, μ, is constant over time.

Likewise, the same analysis applies to the buyer-optimal contract when $N > 3$. The buyer solves the same problem where the second-order statistic $E[z_{2:N}]$ is substituted for $E[z_{1:N}]$. For $N \in \{2, 3\}$, $E[z_{2:N}] > 0$.

E.1 Relating Competition to Contract Duration

The first-order condition from the problem above is

$$\begin{aligned} E[z_{1:N}]\xi'(T) &= \frac{\delta}{T^2} \\ -\xi'(T)T^2 &= -\frac{\delta}{E[z_{1:N}]}. \end{aligned} \tag{25}$$

In this case, we obtain a monotonic relationship between the number of bidders and the optimal duration, as N has a monotonic effect on the right-hand side. Unlike the U-shape models, the microfounded model here does not have a lower bound on costs.

Proposition 11. *The efficient duration is decreasing in the number of bidders.*

Proof. $\frac{d}{dT}\left(-\xi'(T)T^2\right) = -2T \cdot \xi'(T) - T^2\xi''(T)$. Combining the second-order conditions and first-order conditions, we obtain.

$$\begin{aligned} E[z_{1:N}]\xi''(T) &> -\frac{2}{T}E[z_{1:N}]\xi'(T) \\ \implies T^2 \cdot \xi''(T) &< -2T\xi'(T) \end{aligned}$$

An increase in N increases the RHS of equation 25. As $\frac{d}{dT}\left(-\xi'(T)T^2\right) < 0$, the optimal T falls. □

We now turn to the buyer-optimal contract, which solves the same problem where the second-order statistic $E[z_{2:N}]$ is substituted for $E[z_{1:N}]$.

Proposition 12. *The buyer-optimal duration is decreasing in the number of bidders. It is optimal for the buyer to issue a permanent contract for $N \in \{2, 3\}$.*

The permanent contract result follows from the fact that the second-order statistic is greater than zero with a small N.

Additionally, we have that $E[z_{1:N}] < E[z_{2:N}]$. Therefore,

Proposition 13. *The efficient duration is less than the buyer-optimal duration.*

F Likelihood Function (For Online Publication)

For estimation, we obtain the likelihoods for Y_n and N given by

$$f_{Y_n|N,X,T,M} = \int f_{B_n|T,N}(\frac{y}{U}\frac{1}{h(\boldsymbol{x})})\frac{1}{U}\frac{1}{h(\boldsymbol{x})}f_{U|N,T,\boldsymbol{x},\boldsymbol{m}}(U)dU$$

$$\Pr(N = n|T, \boldsymbol{x}, \boldsymbol{m}) = \int \Pr(N = n|U, T, \boldsymbol{x}, \boldsymbol{m})f_{U|T,\boldsymbol{x},\boldsymbol{m}}(U)dU$$

For estimation, I make the assumption that $U \perp\!\!\!\perp (X, M)$. As U is not observed by the buyer when setting T, $U \perp\!\!\!\perp (T, \boldsymbol{x}, \boldsymbol{m})$. This simplifies the problem so that $f_{U|T,\boldsymbol{x},\boldsymbol{m}}(U) = f_U(U)$. The conditional distribution of U used in the likelihood of Y_N is given by $f_{U|N,T,\boldsymbol{x},\boldsymbol{m}}(u) = \frac{\Pr(N=n|U,T,\boldsymbol{x},\boldsymbol{m})f_U(u)}{\Pr(N=n|T,\boldsymbol{x},\boldsymbol{m})}$. This simplifies so that the joint contribution is given by

$$\begin{aligned}
f_{Y_n|N,X,T,M}(y_n) \cdot \Pr(N = n|T, \boldsymbol{x}, \boldsymbol{m}) &= \left(\int f_{B_n|T,N}(\frac{y}{u}\frac{1}{h(\boldsymbol{x})})\frac{1}{u}\frac{1}{h(\boldsymbol{x})}f_{U|N,T,\boldsymbol{x},\boldsymbol{m}}(u)du\right)\Pr(N = n|T, \boldsymbol{x}, \boldsymbol{m}) \\
&= \left(\int f_{B_n|T,N}(\frac{y}{u}\frac{1}{h(\boldsymbol{x})})\frac{1}{u}\frac{1}{h(\boldsymbol{x})}\frac{\Pr(N = n|u, T, \boldsymbol{x}, \boldsymbol{m})f_U(u)}{\Pr(N = n|T, \boldsymbol{x}, \boldsymbol{m})}du\right)\Pr(N = n|T, \boldsymbol{x}, \boldsymbol{m}) \\
&= \int f_{B_n|T,N}(\frac{y}{u}\frac{1}{h(\boldsymbol{x})})\frac{1}{u}\frac{1}{h(\boldsymbol{x})}\Pr(N = n|u, T, \boldsymbol{x}, \boldsymbol{m})f_U(u)du
\end{aligned}$$

With the assumption that the shock ε is independent of $(U, T, \boldsymbol{x}, \boldsymbol{m})$, we have the following expression for conditional probability of N.

$$\begin{aligned}
\Pr(N = n|U, T, \boldsymbol{x}, \boldsymbol{m}) &= F_{\ln\varepsilon}\left(\ln E[\pi_n|T] + \ln h(\boldsymbol{x}) + \ln U - \ln k(\boldsymbol{m})\right) \\
&\quad - F_{\ln\varepsilon}\left(\ln E[\pi_{n+1}|T] + \ln h(\boldsymbol{x}) + \ln U - \ln k(\boldsymbol{m})\right)
\end{aligned}$$

I use the joint likelihood of Y_n and N to obtain estimates for cost and entry parameters.

F.1 A Computational Innovation

In this setting, there is a symmetric equilibrium in which each bidder has a monotone bid function $\beta(\cdot; n)$ mapping private costs to the submitted bid. The density of an observed bid is given by

$$f_{B_n}(b) = f_c(\beta^{-1}(b; n))\frac{1}{\beta'(\beta^{-1}(b; n))}$$

In maximum likelihood estimation of the cost distribution, it is necessary to invert the bid function to calculate the density. This can be computationally intensive when β does not have a closed-form solution.

In the presence of unobserved heterogeneity, the density of the observed bid $\tilde{B} = B \cdot U$ is given by the convolution when $B \perp\!\!\!\perp U$.

$$
\begin{aligned}
f_{\tilde{B}}(\tilde{b}) &= \int_{\underline{\mathbf{u}}}^{\bar{u}} f_B\left(\frac{\tilde{b}}{u}\right)\frac{1}{u} f_u(u)du \\
&= \int_{\underline{\mathbf{u}}}^{\bar{u}} f_c\left(\beta^{-1}\left(\frac{\tilde{b}}{u};n\right)\right)\frac{1}{\beta'\left(\beta^{-1}\left(\frac{\tilde{b}}{u};n\right)\right)}\frac{1}{u} f_u(u)du
\end{aligned}
$$

Here, the computational burden increases greatly. Integrating out the unobserved heterogeneity means that the bid function must be inverted for each value of u within the integral, in order to calculate $\beta^{-1}\left(\frac{\tilde{b}}{u};n\right)$. As the inverse bid function has an analytic solution for only a few specialized cases, in practice this computation relies on a non-linear equation solver or an approximation. Thus, the calculations are constrained by the efficiency and accuracy of such an approach.

One easy-to-implement solution that makes maximum likelihood significantly more tractable is to use a change-of-variables to calculate the density. Instead of integrating out the unobserved heterogeneity by integrating over u, replace u with $u = \frac{\tilde{b}}{\beta(c)}$ and integrate over c. The density then becomes:

$$
\begin{aligned}
f_{\tilde{B}}(\tilde{b}) &= \int_{\underline{\mathbf{u}}}^{\bar{u}} f_C\left(\beta^{-1}\left(\frac{\tilde{b}}{u}\right)\right)\frac{1}{\beta'\left(\beta^{-1}\left(\frac{\tilde{b}}{u}\right)\right)}\frac{1}{u} f_u(u)du \\
&= \int_{\psi^{-1}(\underline{\mathbf{u}})}^{\psi^{-1}(\bar{u})} f_C\left(\beta^{-1}\left(\beta(c)\right)\right)\frac{1}{\beta'\left(\beta^{-1}\left(\beta(c)\right)\right)}\frac{\beta(c)}{\tilde{b}} f_u\left(\frac{\tilde{b}}{\beta(c)}\right)\left(-\frac{\tilde{b}}{\beta(c)^2}\beta'(c)\right)dc \\
&= \int_{\underline{\mathbf{c}}}^{\bar{c}} f_C\left(c\right) f_u\left(\frac{\tilde{b}}{\beta(c)}\right)\frac{1}{\beta(c)}dc
\end{aligned}
$$

Note that in this form, there is no need to invert the bid function. As the general form for the symmetric equilibrium bid function is

$$
\beta(c) = c + \frac{\int_c^{\infty}[1-F(z)]^{n-1}}{[1-F(c)]^{n-1}},
$$

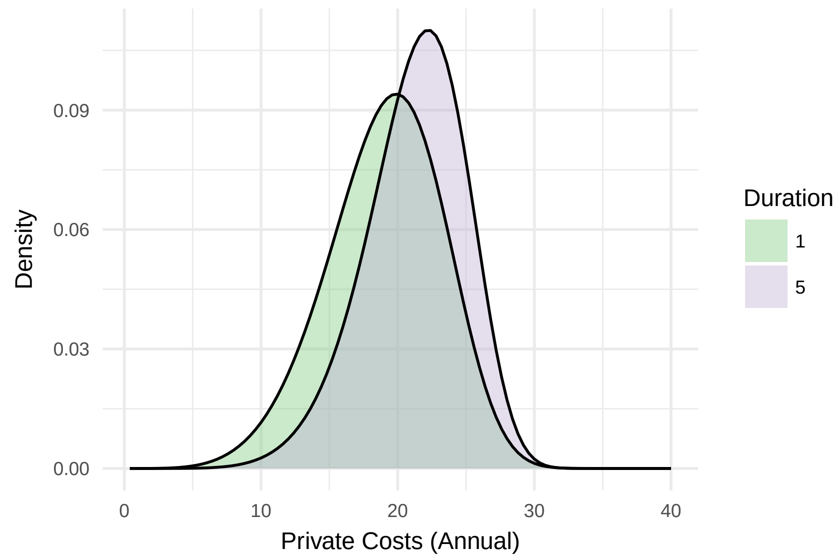
the primary computational cost is a numerical integration routine. Therefore, the model is computationally tractable for a vast class of parametric distributions of C and U, as well as nonparametric approximations such as B-splines.

G Supplemental Empirical Results (For Online Publication)

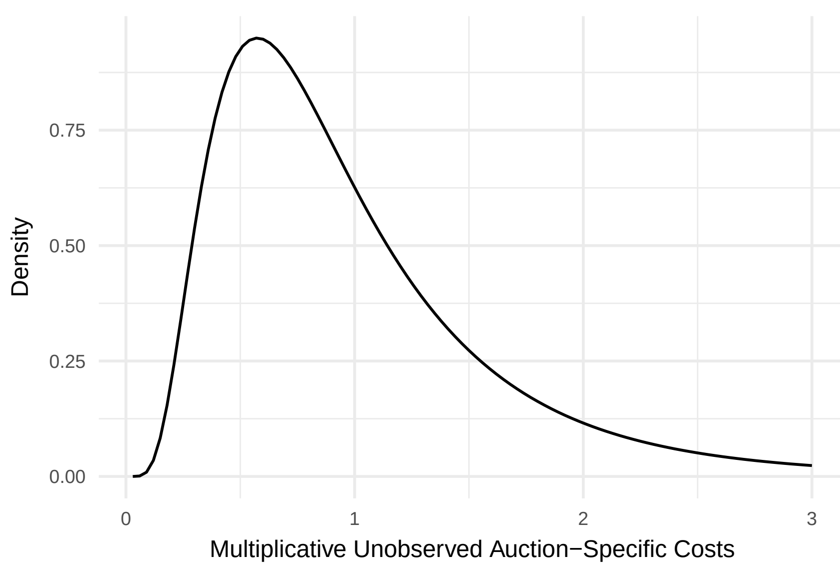
G.1 Distributions of Bidder Costs

Figure 8: Distribution of Bidder Costs

(a) Duration-Dependent Private Costs



(b) Unobservable Auction-Specific Heterogeneity



Notes: The figure plots the distributions of the unobservable components of bidder costs. Private costs are displayed in panel (a), and the density of unobserved auction-specific heterogeneity is displayed in panel (b). In panel (a), the density is plotted for a one-year contract and a five-year contract. The estimated parameters indicate an increasing mean and a decreasing variance in private costs with contract duration. The density shifts smoothly between these functions for intermediate values of duration.

G.2 Projecting Transaction Costs on Location Type and Agency

Table 16: Dependent Variable: ln(Transaction Costs)

	(1)		(2)	
Location Type: Accommodations	1.436^{***}	(0.278)	0.249	(0.219)
Location Type: Airport	0.414	(0.264)	0.543^{***}	(0.203)
Location Type: Field Office	0.027	(0.125)	0.068	(0.095)
Location Type: Industrial	1.150^{***}	(0.325)	0.331	(0.254)
Location Type: Medical	1.530^{***}	(0.256)	0.294	(0.201)
Location Type: Office	0.000	(.)	0.000	(.)
Location Type: Research	0.371^{**}	(0.157)	0.232^{*}	(0.120)
Location Type: Services	0.180	(0.207)	0.005	(0.158)
Location Type: Technical	1.403^{***}	(0.272)	0.285	(0.212)
Location Type: Visitors	0.784^{***}	(0.196)	0.319^{**}	(0.152)
Department: Agriculture	0.309^{**}	(0.123)	-0.183^{*}	(0.096)
Department: Commerce	0.559^{***}	(0.177)	0.292^{**}	(0.137)
Department: Defense	0.000	(.)	0.000	(.)
Department: GSA	1.577^{***}	(0.240)	0.418^{**}	(0.189)
Department: Homeland Security	1.645^{***}	(0.217)	0.650^{***}	(0.171)
Department: Interior	0.440^{**}	(0.193)	-0.095	(0.149)
Department: Other	1.071^{***}	(0.252)	0.256	(0.195)
Department: Veterans Affairs	0.177	(0.235)	0.404^{**}	(0.180)
ln(Square Footage)			0.601^{***}	(0.026)
ln(Weekly Frequency)			0.431^{***}	(0.059)

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays results for regressions of estimated (log) transaction costs on location type and department, with additional controls in specification (2). N=1,046.

G.3 Incumbency and Asymmetries

In this section, I present regressions for the dependent variables of price and the number of bids, including an indicator for whether or not a single incumbent bidder was identified from a previous contract. That is, the indicator equals one if building cleaning services for the same agency and 9-digit ZIP were performed by a single supplier in the previous year. The coefficient on this variable is not significant, and its inclusion does not meaningfully impact the estimated coefficients.

Table 17: Descriptive Regressions: Incumbency Check, Price

	IV-1 (a)	IV-1 (b)	IV-1 (c)	IV-2 (a)	IV-2 (b)	IV-2 (c)
Number of Bids	-0.053^{**}	-0.052^{**}	-0.052^{**}	-0.047^{**}	-0.046^{**}	-0.046^{**}
	(0.022)	(0.022)	(0.022)	(0.022)	(0.022)	(0.022)
Duration (Years)	0.043^{***}	0.043^{***}	0.043^{***}	0.033^{**}	0.033^{**}	0.033^{**}
	(0.016)	(0.016)	(0.016)	(0.015)	(0.015)	(0.015)
ln(Square Footage)	0.689^{***}	0.688^{***}	0.688^{***}	0.687^{***}	0.686^{***}	0.686^{***}
	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)	(0.024)
ln(Weekly Frequency)	0.467^{***}	0.467^{***}	0.467^{***}	0.407^{***}	0.407^{***}	0.407^{***}
	(0.041)	(0.041)	(0.041)	(0.040)	(0.040)	(0.040)
ln(2004 Unemp.)	0.080^{***}	0.080^{***}	0.080^{***}	0.060^{***}	0.060^{***}	0.060^{***}
	(0.019)	(0.019)	(0.019)	(0.018)	(0.018)	(0.018)
High-Intensity Cleaning	0.559^{***}	0.559^{***}	0.559^{***}	-0.076	-0.076	-0.077
	(0.075)	(0.075)	(0.075)	(0.125)	(0.125)	(0.125)
Follow-On Contract		0.018			-0.003	
		(0.053)			(0.050)	
Incumbent Winner			0.012			-0.008
			(0.106)			(0.100)
Site Type FEs				X	X	X
Observations	1046	1046	1046	1046	1046	1046
R^2	0.69	0.69	0.69	0.73	0.73	0.73

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays regression results for regressions of log annual price on auction characteristics and local market characteristics. Specifications IV-1 (a) and IV-2 (a) are two-stage least squares regressions and are identical to the descriptive regressions in Table 2. The (b) specifications include an additional regressor indicating whether the contract is a follow-on contract and the (c) specifications include an indicator for whether the contract was won by an incumbent bidder in a follow-on contract.

Table 18: Descriptive Regressions: Incumbency Check, Number of Bids

	(1)	(2)	(3)	(4)	(5)	(6)
Duration (Years)	−0.002	−0.005	−0.009	−0.002	−0.005	−0.009
	(0.099)	(0.099)	(0.099)	(0.100)	(0.100)	(0.100)
ln(Square Footage)	0.834***	0.835***	0.840***	0.825***	0.824***	0.829***
	(0.106)	(0.106)	(0.106)	(0.112)	(0.112)	(0.112)
ln(Weekly Frequency)	0.009	0.014	0.010	0.137	0.146	0.141
	(0.253)	(0.253)	(0.253)	(0.257)	(0.258)	(0.257)
ln(2004 Unemp.)	−0.794***	−0.809***	−0.813***	−0.793***	−0.808***	−0.811***
	(0.238)	(0.239)	(0.239)	(0.238)	(0.238)	(0.238)
ln(Unemployment)	1.420***	1.432***	1.436***	1.356***	1.366***	1.370***
	(0.231)	(0.231)	(0.231)	(0.231)	(0.231)	(0.231)
ln(Num. Firms in Zip3)	0.257*	0.248*	0.250*	0.276*	0.267*	0.269*
	(0.148)	(0.148)	(0.148)	(0.147)	(0.147)	(0.147)
Generic Set-Aside	1.134***	1.125***	1.131***	0.987***	0.982***	0.985***
	(0.350)	(0.350)	(0.350)	(0.361)	(0.361)	(0.361)
High-Intensity Cleaning	−0.294	−0.303	−0.305			
	(0.475)	(0.475)	(0.475)			
Follow-On Contract		−0.351			−0.353	
		(0.326)			(0.326)	
Incumbent Winner			−0.836			−0.814
			(0.650)			(0.646)
Site Type FEs				X	X	X
Observations	1046	1046	1046	1046	1046	1046
R^2	0.17	0.17	0.17	0.19	0.19	0.19

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table displays regression results for regressions of number of bids on auction characteristics and local labor market variables. Specifications (1) and (4) are equivalent to the descriptive regressions (3) and (4) in Table 3. The additional specifications included indicators for whether the contract is a follow-on contract or whether the contract was won by an incumbent bidder in a follow-on contract.

G.4 Detailed Impacts of Standardized Duration

Table 19: Percent Impact of Uniform Term Policies

$\bar{T}$	Affected	Price	Trans. Cost	Total Cost	Count
1	All	−11.2	317.1	33.7	1046
	$T > \bar{T}$	−11.8	334.2	35.5	995
	$T < \bar{T}$	1.5	−36.6	0.6	23
2	All	−7.5	108.5	9.0	1046
	$T > \bar{T}$	−8.7	125.4	10.0	930
	$T < \bar{T}$	4.1	−50.5	2.3	62
3	All	−3.9	39.0	2.9	1046
	$T > \bar{T}$	−6.3	61.8	3.4	761
	$T < \bar{T}$	5.1	−42.6	2.7	146
4	All	−0.4	4.3	1.3	1046
	$T > \bar{T}$	−3.2	24.0	0.7	686
	$T < \bar{T}$	6.0	−39.1	2.9	306
5	All	3.1	−16.6	1.5	1046
	$T > \bar{T}$	−2.0	12.2	0.3	18
	$T < \bar{T}$	6.9	−36.8	3.3	478

Notes: The table displays the average percent changes (by contract, not in aggregate) in total costs, prices, and annualized transaction costs when all contracts are issued in standardized durations corresponding to $\bar{T}$. For a uniform duration policy of 4 years or less, the average price paid decreases and the amount spent on transaction costs increases. The final column lists the count of the affected contracts. The first column indicates the group affected by the policy. Rows corresponding to $T > \bar{T}$ pertain to all contracts that see a reduction in duration, and the reported effects are equivalent to a policy that caps duration at $\bar{T}$.