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Abstract

Our new model of consumption-based habit generates time-varying risk premia on bonds and stocks from loglinear, homoskedastic macroeconomic dynamics. Consumers' first-order condition for the real risk-free bond generates an exactly loglinear consumption Euler equation, commonly assumed in New Keynesian models. We estimate that the correlation between inflation and the output gap switched from negative to positive in 2001. Higher inflation lowers real bond returns and higher output raises stock returns, explaining why the bond-stock return correlation changed from positive to negative. In the model risk premia amplify this change in bond-stock return comovement, and are crucial for a quantitative explanation.

1 Introduction

This paper develops a novel integration of consumption-based asset pricing with macroeconomics. Asset prices measure agents' forward-looking expectations and are at the heart of consumption and savings decisions. An integrated framework can therefore impose valuable discipline on both macroeconomic and asset pricing models. We develop a new specification of preferences, building on the consumption-based habit formation model of Campbell and Cochrane (1999), and use it to model macroeconomic dynamics jointly with bond and stock returns.

The Campbell-Cochrane habit formation model has become a benchmark for understanding asset prices, and specifically time-varying risk premia, but it has been difficult to apply outside the original setting of exogenous unpredictable consumption growth. We generalize Campbell-Cochrane preferences to more general consumption and interest rate dynamics. The process for habit in our model implies an exact loglinear Euler equation relating consumption to the riskless real interest rate. We assume a simple, empirically realistic link between consumption and the output gap so that we can express the Euler equation in terms of the output gap as is standard in New Keynesian models (e.g. Clarida, Gali, and Gertler 1999 or Woodford 2003).

Because our preferences are consistent with a loglinear Euler equation they are also consistent with loglinear, conditionally homoskedastic processes for macroeconomic variables. We combine the loglinear Euler equation with reduced-form, loglinear, homoskedastic dynamics for inflation and the Federal Funds rate. The resulting model captures the main empirical properties of the output gap, inflation, and the Federal Funds rate in a tractable fashion; and it raises the bar for our preference specification, by requiring it to generate time-varying risk premia even without nonlinear driving processes. We solve for the prices of bonds and stocks, modeled as levered consumption claims whose dividends are cointegrated with consumption. While this paper does not take a stand on the details of macroeconomic frictions or the monetary policy rule, we provide a new tool that can be used to study the asset pricing implications of alternative structural macro models.

We apply our model to understand why nominal Treasury bonds changed from risky (positively correlated with stocks) in the 1980s and 1990s to safe (negatively correlated with stocks) in the first decade of the 2000s. This application demonstrates the usefulness of our approach because it requires an internally consistent macroeconomic and asset pricing framework.

The model explains the qualitative change in Treasury risks with the correlation between inflation and the output gap, which was negative in the first period and positive in the second. This sign switch in correlation, which also occurs in the correlation of five-year average inflation with the lagged output gap and the correlation of the five-year average nominal Federal Funds rate with the lagged output gap, drives our result. If the correlation between inflation and the output gap is negative, as it was during our first period, this means that nominal long-term bond prices decline in periods of high marginal utility and bonds are risky. If this correlation is positive, as in the second period, nominal long-term bond prices decline in periods of low marginal utility, so bonds are hedging assets.

In order to explain the quantitative change in Treasury risks, the model requires an additional element: an endogenously changing correlation between bond and stock risk premia. Habit formation preferences imply that recessions make investors more risk averse, driving down the prices of risky assets and driving up the prices of hedge assets in a “flight to safety”. In the first period, Treasuries are risky assets that suffer from the flight to safety along with stocks, while in the second period Treasuries are hedge assets that benefit from the flight to safety. Thus, time-varying risk aversion amplifies the positive comovement of bonds and stocks in the first period and amplifies the negative comovement in the second.

We start our empirical application by testing for an unknown break date in the relation between inflation and the output gap in US data from 1979Q3 through 2011Q4. We detect a break in 2001Q2, with a negative inflation-output gap correlation before and a positive correlation after. Because nominal bond returns are inversely related to inflation and stock returns are positively related to the output gap, one might expect that the comovement between bonds and stocks should change in the opposite direction around this break date. Figure 1, Panel A shows that indeed the correlation of bond and stock returns was positive on average before 2001Q2 but negative afterwards. Figure 1, Panel B shows a similar change in the beta of nominal bond returns with respect to the stock market. The figure uses daily data to estimate persistent components in the second moments of bond and stock returns.[2]

We estimate our model separately for the two periods 1979Q3–2001Q1 (period 1) and 2001Q2–2011Q4 (period 2) identified by our macroeconomic break test. We calibrate preference parameters following Campbell and Cochrane (1999) and set them equal across subperiods. We estimate the parameters governing macroeconomic dynamics separately for each subperiod using simulated method of moments (SMM). We use no bond or stock returns for the estimation. The moments used for the estimation are the empirical impulse responses of a standard VAR in the output gap, inflation, and the Federal Funds rate, and the correlation

[2]The end-of-quarter bond-stock correlation is the correlation of daily log returns on five-year zero-coupon nominal Treasury bonds with daily log CRSP value-weighted stock market returns including dividends over a rolling three-month window. The end-of-quarter bond beta is the regression coefficient of the same bond returns onto stock returns over the same rolling window. We use a Kalman filter to filter out measurement noise. Specifically, we assume that the bond-stock correlation follows an AR(1) process plus white measurement noise. We use a Kalman filter to estimate the AR(1) parameters by maximum likelihood and then to filter for the unobserved persistent component. Panel A plots the filtered persistent component and its 95% confidence interval. The filtered bond-stock beta and its 95% confidence interval in Panel B are constructed similarly. See the appendix for details of the Kalman filter.

between the five-year average Federal Funds rate and the lagged output gap.

The model is successful at matching the empirical impulse responses, and it generates empirically plausible bond and stock returns. Habit-formation preferences generate volatile and predictable equity returns to address the "equity volatility puzzle," one of the leading puzzles in consumption-based asset pricing (Campbell 2003). In addition, the model generates realistic bond return volatility and matches the predictability of stock returns from the output gap documented in Cooper and Priestley (2009).

Despite not being directly targeted in the estimation, the model matches the changing comovement of quarterly bond and stock returns. In period 1, the model generates a quarterly bond-stock correlation of 0.50 compared to 0.21 in the data. In period 2, the model's bond-stock correlation is -0.66, matching the highly negative correlation in the data of -0.64. Decomposing bond and stock returns into cash-flow news, real-rate news, and risk-premium news, we find that the correlation between bond and stock risk premia switches from highly positive to highly negative and drives the overall bond-stock covariances. The model implies that the magnitude of bond-stock covariances would have been smaller by 30% without time-varying bond risk premia and by 70% without time-varying stock risk premia.

1.1 Literature review and outline

This paper contributes to two main literatures. First, it further integrates the literatures on habit formation in asset pricing and macroeconomics. Habit in macroeconomic models without asset prices, such as Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), serves to generate hump-shaped responses of real economic activity to interest-rate innovations. The macro-finance literature that seeks to model asset prices jointly with macroeconomic outcomes has found it difficult to generate volatile asset returns from consumption-based habit formation preferences without implausibly distorting the dynamics of consumption, output, and the real interest rate. This is particularly true in simple models where habit is proportional to lagged consumption (Heaton 1995, Jermann 1998, Boldrin, Christiano, and Fisher 2001), but is also a problem in models with persistent but linearized habit dynamics (Lettau and Uhlig 2000, Uhlig 2007, Rudebusch and Swanson 2008, Lopez, Lopez-Salido, and Vazquez-Grande 2015).

One response to this problem has been to generate time-varying risk premia from heteroskedasticity in consumption growth (Chen 2017); but this requires large and extremely persistent movements in macroeconomic volatility just as in the long-run risk literature that works with recursive preferences (Bansal, Kiku, and Yaron 2012), and encounters the difficulty that empirical equity risk premia do not vary in proportion with equity volatility (Beeler and Campbell 2012). In contrast to these papers, we assume homoskedastic driving processes for macroeconomic variables and generate time-varying risk premia endogenously from our highly nonlinear preference specification. Our approach is complementary to Bekaert and Engstrom (2017), who model time-variation in the higher moments of consumption growth, while we model time-variation in the conditional mean of consumption growth.

Second, we add to the literature on the term structure of interest rates and macroeconomic factors by modeling risk premia driven by consumption-based habit formation preferences. Within this literature, our paper is most closely related to those that price both bonds and stocks (for example Bekaert, Engstrom, and Grenadier 2010 and Lettau and Wachter 2011), and more specifically to papers that investigate changes in bond-stock comovements over time (Baele, Bekaert, and Inghelbrecht 2010, Campbell, Shiller, and Viceira 2009, Campbell, Sunderam, and Viceira 2017, Gourio and Ngo 2018, Song 2017, Viceira 2012). In contrast to us, these papers do not use consumption-based habit formation preferences, relying either on an exogenous reduced-form stochastic discount factor or recursive preferences combined with stochastic volatility. Our model is also complementary to David and Veronesi (2013), who study bonds and stocks in an endowment economy with learning.

The organization of the paper is as follows. Section 2 describes our model, with the consumption Euler equation, the assumed relation between consumption and the output gap, and our new preference specification (section 2.1), inflation and interest rate dynamics (section 2.2), and the specification for equity dividends (section 2.3). Section 3 explains how we solve the model. Section 3.1 discusses the solution for macroeconomic dynamics, including our procedure for selecting an equilibrium when multiple equilibria exist. Section 3.2 provides intuition for time-variation of risk premia and explains our numerical solution method for asset prices. Section 4 on econometric methodology describes the data (section 4.1), break date tests (section 4.2), calibration of time-invariant parameters (section 4.3), and estimation of subsample-dependent parameters (section 4.4). Section 5 presents our empirical results. Section 5.1 discusses parameter estimates and section 5.2 the implied macroeconomic dynamics. Section 5.3 presents implications for asset prices, and section 5.4 a decomposition into news about real cash flows, real interest rates, and risk premia. Section 6 concludes, and highlights the potential of our framework for future research. An online appendix provides further details of our approach.

2 Model

2.1 Euler equation and preferences

Macroeconomic dynamics in our model satisfy a loglinear Euler equation typical of New Keynesian models, where the log output gap is linked to its own lead and lag and the log real risk-free interest rate (see Woodford, 2003, Chapters 4 and 5):

$$x_t = f^x \mathrm{E}_t x_{t+1} + \rho^x x_{t-1} - \psi r_t. \tag{1}$$

The New Keynesian literature defines the log output gap x_t as log real output minus log potential real output, that is the hypothetical equilibrium without price- and wage-setting frictions (Woodford, 2003, p.245). r_t denotes the log real risk-free interest rate that can be earned from time t to time $t+1$. The coefficients f^x, ρ^x, and ψ are positive parameters. Intuitively, a high real interest rate means that consumers have a strong incentive to save, thereby depressing contemporaneous consumption and output. We model the output gap, inflation, and short-term interest rates relative to a steady state, so the loglinear Euler equation is specified up to a constant.

Our preferences are such that the loglinear Euler equation is indeed the first-order condition for the real risk-free rate and this is what distinguishes our preferences from Campbell-Cochrane and other habit utility functions popular in asset pricing.[3] Our modeling choices ensure that the loglinear Euler equation is exact with no approximation error. Because loglinear Euler equations are pervasive in structural macroeconomic models, this makes our preferences a natural stepping stone to study consumption-based asset prices in researchers' and policy makers' preferred models of the macroeconomy. We now describe what is required to make consumption-based habit formation preferences consistent with a loglinear Euler equation.

[3] In Menzly, Santos, and Veronesi (2004), Wachter (2006), and the working paper version of Campbell and Cochrane (1999) the real risk-free rate is a nonlinear function of current and past consumption shocks, and in Bekaert and Engstrom (2017) the real rate depends in addition on an unobserved state variable governing higher moments of consumption growth.

2.1.1 Consumption and the output gap

The loglinear Euler equation generates endogenous dynamics for the output gap, but not directly for consumption. In order to solve for consumption dynamics, we therefore need a link between the output gap and consumption. We make the simple assumption that the log real output gap, x_t, equals stochastically detrended log real consumption, $\hat{c}_t$:

$$x_t = \hat{c}_t \equiv c_t - (1 - \phi) \sum_{i=0}^{\infty} \phi^i c_{t-1-i}, \tag{2}$$

where ϕ is a smoothing parameter. Here, we again ignore constants, because x_t is specified relative to steady-state.

To see that the consumption-output gap relation is consistent with the New Keynesian macroeconomics literature, one could augment the model with two stylized assumptions: a) consumption equals output, and b) log potential output is a stochastic trend of log output, consistent with how potential output is measured empirically (Staiger, Stock, and Watson 1997, Shackleton 2018). It then follows that the output gap is detrended output, which also equals detrended consumption, i.e. equation (2). Since the interpretation of the output gap is not important for us, it can simply be regarded as stochastically detrended output throughout the paper.

In the appendix, we show empirical evidence that (2) is also a close description of the data when the detrending parameter ϕ is set equal to 0.93 per quarter. We impose this value when we calibrate our model.

2.1.2 Habit preferences

Utility is a power function of the difference between the level of consumption C and external habit H:

$$U_t = \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma} = \frac{(S_t C_t)^{1-\gamma} - 1}{1-\gamma}. \tag{3}$$

Here

$$S_t = \frac{C_t - H_t}{C_t} \tag{4}$$

is the surplus consumption ratio, the fraction of consumption that is available to generate utility, and γ is a curvature parameter that controls risk aversion. Relative risk aversion varies over time as an inverse function of the surplus consumption ratio: $-U_{CC}C/U_C = \gamma/S_t$. Marginal utility in this model is

$$U_t' = (C_t - H_t)^{-\gamma} = (S_t C_t)^{-\gamma}. \tag{5}$$

The consumer first-order condition implies that the gross one-period real return $(1+R_{t+1})$ on any asset satisfies

$$1 = \mathrm{E}_t \left[M_{t+1} \left(1 + R_{t+1} \right) \right], \tag{6}$$

where the stochastic discount factor is related to the log surplus consumption ratio s_{t+1} and log consumption c_{t+1} by

$$M_{t+1} = \frac{\beta U_{t+1}'}{U_t'} = \beta \exp\left(-\gamma(\Delta s_{t+1} + \Delta c_{t+1})\right). \tag{7}$$

2.1.3 Surplus consumption dynamics

We model how habit adjusts to the history of consumption implicitly, by modeling the evolution of the surplus consumption ratio:

$$s_{t+1} = (1-\theta_0)\bar{s} + \theta_0 s_t + \theta_1 x_t + \theta_2 x_{t-1} + \lambda(s_t)\varepsilon_{c,t+1}, \tag{8}$$

$$\varepsilon_{c,t+1} = c_{t+1} - \mathrm{E}_t c_{t+1} = x_{t+1} - \mathrm{E}_t x_{t+1}, \tag{9}$$

where $\bar{s}$ is steady-state log surplus consumption and $\varepsilon_{c,t+1}$ is a conditionally homoskedastic shock to consumption (equivalently, to the output gap) with standard deviation σ_c.

The terms $\theta_1 x_t$ and $\theta_2 x_{t-1}$ are new relative to Campbell and Cochrane (1999)'s surplus consumption dynamics, which correspond to the case $\theta_1 = \theta_2 = 0$. In our calibration, a negative value for θ_1 increases the dependence of habit on the first and second lags of consumption, while a positive value for θ_2 decreases the dependence on the second lag of consumption. The combined effect is that habit loads more on the first and second lags of consumption than in Campbell-Cochrane case.

The sensitivity function $\lambda(s_t)$ is identical to that in Campbell and Cochrane (1999):

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & s_t \leq s_{max} \\ 0 & s_t \geq s_{max} \end{cases}, \tag{10}$$

$$\bar{S} = \sigma_c \sqrt{\frac{\gamma}{1-\theta_0}}, \tag{11}$$

$$\bar{s} = \log(\bar{S}), \tag{12}$$

$$s_{max} = \bar{s} + 0.5(1-\bar{S}^2). \tag{13}$$

The downward-sloping relation between $\lambda(s_t)$ and s_t has the intuitive implication that marginal consumption utility is particularly sensitive to consumption innovations when investors are close to their habit consumption level, as would be the case following an adverse shock.[4]

2.1.4 Deriving the Euler equation from preferences

We now show that the Euler equation (1) holds exactly for our preferences. This exact consistency between the macroeconomic Euler equation and the preferences that govern asset pricing is at the heart of our contribution. Substituting log surplus consumption dynamics (8) into the stochastic discount factor (7) and the no-arbitrage condition (6) for the one-period real risk-free bond gives (up to a constant):

$$r_t = \gamma E_t \Delta c_{t+1} + \gamma E_t \Delta s_{t+1} - \frac{\gamma}{2} \left(1 + \lambda(s_t)\right)^2 \sigma_c^2. \tag{14}$$

Our modeling choices simplify the first-order condition for the real risk-free rate. First, the surplus consumption dynamics imply that we can substitute out for $E_t \Delta s_{t+1}$:

$$r_t = \gamma E_t \Delta c_{t+1} + \gamma \left(\theta_0 - 1\right) s_t + \gamma \theta_1 x_t + \gamma \theta_2 x_{t-1} - \frac{\gamma}{2} \left(1 + \lambda(s_t)\right)^2 \sigma_c^2. \tag{15}$$

Second, the consumption-output gap relation (2) implies that we can write expected consumption growth in terms of the current and expected output gap: $E_t \Delta c_{t+1} = E_t x_{t+1} - \phi x_t$. The real rate first-order condition then becomes

$$r_t = \gamma E_t x_{t+1} - \gamma \phi x_t + \gamma \left(\theta_0 - 1\right) s_t + \gamma \theta_1 x_t + \gamma \theta_2 x_{t-1} - \frac{\gamma}{2} \left(1 + \lambda(s_t)\right)^2 \sigma_c^2. \tag{16}$$

[4] If θ_1 and θ_2 are different from zero, there is the theoretical possibility that the log surplus consumption ratio exceeds the maximal value s_{max}. However, the probability of this event is small in our estimated model (less than 1% per quarter). In this respect our model is similar to Campbell and Cochrane (1999), who also have an upper bound on surplus consumption that is crossed with very low probability in discrete time (and can never be crossed in continuous time).

Third, the sensitivity function has just the right form so that s_t drops out. Substituting in the sensitivity function (8) through (13) and re-arranging, continuing to ignore constants, gives the loglinear Euler equation:

$$x_t = \underbrace{\frac{1}{\phi - \theta_1}}_{f^x} \mathrm{E}_t x_{t+1} + \underbrace{\frac{\theta_2}{\phi - \theta_1}}_{\rho^x} x_{t-1} - \underbrace{\frac{1}{\gamma(\phi - \theta_1)}}_{\psi} r_t. \tag{17}$$

The derivation of the loglinear Euler equation (17) shows that it holds exactly for our preferences. It holds irrespective of the specific microfoundations of a macroeconomic model, provided that consumption is conditionally homoskedastic.

The sensitivity function (10) through (13) is unique such that s_t cancels out of (17) and habit is pre-determined at the steady-state. We need these highly non-linear preferences to obtain a simple first-order condition for the real risk-free rate and linear macroeconomic dynamics. A simpler sensitivity function would complicate the first-order condition for the real risk-free rate (17) by introducing s_t as an additional state variable. For instance, if we were to assume that λ is linear in s_t, there would be s_t^2 terms in (17).

The expressions for the coefficients in equation (17) illustrate the role of the new parameters θ_1 and θ_2. A positive value for the new habit parameter θ_2 is necessary to generate a positive backward-looking component in the Euler equation. Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005), and Smets and Wouters (2007) argue that such a backward-looking component is necessary to capture empirical hump-shaped impulse responses to interest rate shocks. The new parameter θ_1 scales all the coefficients in (17) and controls the sum of the forward-looking and backward-looking coefficients. The sum of coefficients is greater than one when $\theta_1 = 0$ and increases with θ_1.

Three features differentiate (17) from macro Euler equations typically used in New Keynesian models. First, the coefficients on the lagged output gap and the expected future output gap do not generally sum to one. We use a negative value of θ_1 for which the sum of coefficients is slightly greater than one. Second, because of the distinction between consumption and the output gap, the slope of the Euler equation ψ does not equal the representative consumer's elasticity of intertemporal substitution (EIS), as can be seen from the fact that ψ depends on the parameter ϕ linking consumption and output gap dynamics. Third, the loglinear Euler equation holds without shocks. We view the exact consistency between the macroeconomic Euler equation and the asset pricing preferences as a key feature of an internally consistent macro-finance asset pricing model. For this reason, we do not introduce a residual disturbance to the macroeconomic Euler equation.

2.2 Inflation and interest rate dynamics

We now turn to the description of macroeconomic dynamics. We introduce two dynamic equations for inflation and the Federal Funds rate, the minimum state variables needed to model bond prices and inflation dynamics. We assume that log inflation and the log Federal Funds rate share a common stochastic trend (unit root), consistent with the extremely high persistence in US inflation data (Ball and Cecchetti 1990, Stock and Watson 2007) and stationarity of the real interest rate. In addition we assume that log inflation and the log Federal Funds rate are conditionally homoskedastic. Because our assumed consumer preferences are consistent with the loglinear Euler equation, this ensures that both the output gap and consumption are also conditionally homoskedastic—as in fact required by our derivation of the Euler equation.

To make the dynamics of inflation and interest rates tractable, we approximate the log one-period nominal interest rate as the log real rate plus expected log inflation:

$$i_t = r_t + E_t \pi_{t+1}. \tag{18}$$

This approximation improves tractability for the macroeconomic dynamics, because it avoids introducing a small heteroskedastic term. It is standard in New Keynesian models and is the only approximation in our model. At our point estimates, the approximation error has a negligible standard deviation of 4 basis points.

In order to model time-varying risk premia of long-term bonds, we do not not make this approximation for longer-term bonds, and we do not assume the expectations hypothesis of the term structure. Instead, we solve numerically for risk premia on longer-term bonds.

We write the unit-root component of inflation as π_t^* and define inflation and interest-rate gaps as deviations from π_t^*:

$$\hat{\pi}_t = \pi_t - \pi_t^*, \;\; \hat{\imath}_t = i_t - \pi_t^*. \tag{19}$$

We choose a unit root rather than a highly persistent mean-reverting inflation component because it allows us to write macroeconomic dynamics in terms of $\hat{\imath}_t$ and $\hat{\pi}_t$ and reduces the number of macroeconomic state variables by one, simplifying the numerical solution for asset prices. We normalize x_t, $\hat{\pi}_t$, and $\hat{\imath}_t$ to have zero averages.

Macroeconomic dynamics are then described by (17) and the following equations:

$$\hat{\pi}_t = b_{\pi x} x_{t-1} + b_{\pi\pi} \hat{\pi}_{t-1} + b_{\pi i} \hat{\imath}_{t-1} + v_{\pi,t}, \tag{20}$$

$$\hat{\imath}_t = b_{ix} x_{t-1} + b_{i\pi} \hat{\pi}_{t-1} + b_{ii} \hat{\imath}_{t-1} + v_{i,t}, \tag{21}$$

$$\pi_t^* = \pi_{t-1}^* + v_t^*. \tag{22}$$

We interpret (20) and (21) as equilibrium dynamics and not a structural model. The shocks are assumed to have standard deviations σ_π, σ_i, σ_* and cross-correlations $\rho_{\pi i}$, $\rho_{\pi *}$, ρ_{i*}.

2.3 Dividends

We model stocks as a levered claim on consumption, as is common in the asset pricing literature (Abel 1990, Campbell 1986, 2003), while being careful to preserve the cointegration of log consumption and log dividends.

Let P_t^c denote the price of a claim to the consumption stream C_{t+1}, C_{t+2},... We assume that stocks are a claim to all future equity cash flows of a levered firm that invests in the consumption stream. At time t the firm buys P_t^c and sells equity to its investors worth δP_t^c, so its equity financing share is δ which we assume to be constant over time. The remainder of the firm's position is financed by one-period risk-free debt worth $(1-\delta)P_t^c$. We make the simplifying assumption that equity holders give up limited liability.

At time $t+1$, the firm receives a cash flow $C_{t+1} + P_{t+1}^c$, pays $(1-\delta)\,P_t^c \exp(r_t)$ to bond holders, and raises new financing from equity holders δP_{t+1}^c, which we model as a negative dividend. The period $t+1$ gross dividend to equity holders then equals the firm cash flow, minus payments to bond holders and new equity financing:

$$D_{t+1}^\delta = P_{t+1}^c + C_{t+1} - (1-\delta)P_t^c \exp(r_t) - \delta P_{t+1}^c. \tag{23}$$

If P_t^c/C_t and r_t are stationary, (23) implies that D_t^δ/C_t is stationary, so log dividends and log consumption are cointegrated.

The price of the claim to all future cash flows (23) is $P_t^\delta = \delta P_t^c$. From (23) the gross stock return $(1+R_{t+1}^\delta)$ equals the gross consumption claim return $(1+R_{t+1}^c)$ times leverage, less a term reflecting the firm's debt payments:

$$(1 + R^{\delta}_{t+1}) = \frac{D^{\delta}_{t+1} + P^{\delta}_{t+1}}{P^{\delta}_{t}} = \frac{D^{\delta}_{t+1} + \delta P^{c}_{t+1}}{\delta P^{c}_{t}} = \frac{1}{\delta}(1 + R^{c}_{t+1}) - \frac{1-\delta}{\delta}\exp(r_t). \tag{24}$$

Since the levered firm is a pure intermediary and does not add value, the expression for the stock return is independent of whether or not equity investors are required to reinvest.

3 Model solution

3.1 Macroeconomic dynamics and equilibrium selection

In order to highlight the asset pricing properties of our preferences, we keep the macroeconomic dynamics intentionally simple and select an equilibrium of the form:

$$\hat{Y}_t = B\hat{Y}_{t-1} + \Sigma v_t, \tag{25}$$

where

$$\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{\imath}_t]', \;\; v_t = [v_{\pi,t}, v_{i,t}, v^*_t]'. \tag{26}$$

Although we do not have a full New Keynesian model, we have a forward- and backward-looking Euler equation and this leads to well-known equilibrium multiplicity issues (Cochrane 2011). There may exist alternative equilibrium dynamics for $\hat{Y}_t$ with additional lags or sunspot shocks, but characterization of these additional equilibria is beyond this paper.

If there are multiple equilibria of the form (25), we use an empirical procedure to select between them. First, we narrow the set of equilibria by requiring that all eigenvalues of B must be less than one in absolute value, all real eigenvalues must be greater than -0.2, and equilibrium impulse responses must not switch sign within the first four quarters of a shock. Second, if there is still more than one equilibrium, as is the case at our point estimates, we select the equilibrium minimizing the weighted sum of squared differences between model and data impulse response moments. Moments and weights are the same as in the simulated method of moments estimation described in section 4.4. Third, our estimation procedure discards portions of the parameter space where no equilibrium with the properties above exists, where the selected equilibrium does not have asset prices, or where it implies the wrong sign for the inflation-output gap correlation.

Solving for an equilibrium of the form (25) is simplified because the coefficients $b_{\pi x}$, $b_{\pi\pi}$, $b_{\pi i}$, b_{ix}, $b_{i\pi}$, b_{ii} (the last two rows of the matrix B) are given by equations (21) and (22). We solve for the coefficients b_{xx}, $b_{x\pi}$, and b_{xi} (the first row of the matrix B) using the standard Blanchard and Kahn (1980) method. While the Euler equation (17) includes x_{t-1}, x_t, and x_{t+1} and might hence appear inconsistent with an equilibrium of the form (25), the Blanchard-Kahn solution resolves this apparent inconsistency by imposing that shocks to x_t are a particular linear combination of shocks to the other two variables. Given this solution and because there is no shock in our Euler equation, the three-variable system (25) has only two independent shocks. However, $\hat{\pi}_t$ and $\hat{\imath}_t$ are not observable and the shock to the random walk component of inflation, v_t^*, ensures that the variance-covariance matrix for the observable variables x_t, π_t, and i_t is non-singular.

3.2 Solving for asset prices

Asset prices are highly nonlinear functions of the state variables, especially of the surplus consumption ratio, so we need to solve for them numerically. We follow the best practices of Wachter (2005) for solving asset prices with nonlinear habit formation preferences and evaluate expectations iteratively along a grid. For details see the appendix.

To see why asset prices depend non-linearly on surplus consumption, we consider analytic expressions for short-term claims to consumption and nominal dollars provide. Consider a one-period zero-coupon consumption claim that pays aggregate consumption in period $t+1$ and pays nothing in any other period. We denote its log return by $r^c_{1,t+1}$. Since consumption shocks are conditionally perfectly correlated with the output gap, the risk premium, adjusted for a standard Jensen's inequality term, equals the conditional covariance between the negative log SDF and and the output gap:

$$
\begin{aligned}
E_t\left[r^c_{1,t+1} - r_t\right] + \frac{1}{2} Var\left(r^c_{1,t+1}\right) &= \mathrm{Cov}_t\left(-m_{t+1}, x_{t+1}\right), \\
&= \gamma\left(1+\lambda\left(s_t\right)\right)\sigma_x^2. \qquad (27)
\end{aligned}
$$

The time $t+1$ real payoff on a two-period nominal bond equals $\exp\left(-i_{t+1} - \pi_{t+1}\right)$ and is lognormal, so we can solve for the two-period nominal bond risk premium analytically. Denoting the log return on the two-period bond from time t to $t+1$ by $r^\$_{2,t+1}$, the risk premium (again including a Jensen's inequality term) equals:

$$
\begin{aligned}
E_t\left[r^\$_{2,t+1} - r_t\right] + \frac{1}{2} Var_t\left(r^\$_{2,t+1}\right) &= \mathrm{Cov}_t\left(-m_{t+1}, -i_{t+1} - \pi_{t+1}\right), \\
&= \gamma\left(1+\lambda\left(s_t\right)\right)\mathrm{Cov}_t\left(x_{t+1}, -i_{t+1} - \pi_{t+1}\right). \qquad (28)
\end{aligned}
$$

Expressions (27) and (28) show that consumption claim and bond risk premia are proportional to the sensitivity function $\lambda(s_t)$, and hence nonlinear in log surplus consumption.

We use the following recursion to solve for the price-consumption ratio of an n-period zero-coupon consumption claim:

$$\frac{P^c_{nt}}{C_t} = \mathrm{E}_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \frac{P^c_{n-1,t+1}}{C_{t+1}} \right]. \tag{29}$$

The price-consumption ratio for a claim to aggregate consumption is then the infinite sum of zero-coupon consumption claims:

$$\frac{P^c_t}{C_t} = \sum_{n=1}^{\infty} \frac{P^c_{nt}}{C_t}. \tag{30}$$

The price of a levered stock equals $P^\delta_t = \delta P^c_t$, where δ is the firm share financed by equity.

We initialize the bond price recursion by noting that the price of a one-period nominal bond equals:

$$P^\$_{1,t} = \exp(-\hat{\imath}_t - \pi_{*t} - \bar{r}), \tag{31}$$

where $\bar{r}$ denotes the steady-state log risk-free rate. The n-period zero coupon nominal bond price follows the recursion:

$$P^\$_{n,t} = \mathrm{E}_t \left[M_{t+1} \exp(-\pi_{t+1}) P^\$_{n-1,t+1} \right]. \tag{32}$$

To see why we need a flexible numerical solution method for asset prices, consider the consumption-claim recursion (29) for $n = 2$ as an example. The price of the one-period zero-coupon consumption claim $P^c_{1,t+1}$ moves inversely with the required risk premium (27), making $P^c_{1,t+1}/C_{t+1}$ a nonlinear function of log surplus consumption. When evaluating the expectation (29), the crucial covariance between the SDF M_{t+1} and $P^c_{1,t+1}/C_{t+1}$ hence changes with s_t, such that there is no analytic solution and asset prices cannot easily be approximated by standard functions. Iterating along a grid, as opposed to local approximation or global solution methods, is the best practice for this type of numerical problem. When combined with a large grid for surplus consumption, iterating along a grid imposes the least structure on the relation between asset prices and surplus consumption.

Numerical solutions for long-term bonds and stocks inherit important properties from analytical short-term risk premia. Similarly to (27), equity risk premia are positive on average. They increase when surplus consumption is low, because marginal utility is very sensitive to consumption shocks in those states of the world. By contrast, nominal bond risk premia can increase or decrease when surplus consumption is low, depending on how nominal interest rates and inflation covary with consumption. If the real return on a nominal two-period nominal bond covaries negatively with the output gap (that is, if $\mathrm{Cov}_t\left(x_{t+1}, -i_{t+1} - \pi_{t+1}\right) < 0$ in equation (28)), the risk premium on a two-period nominal bond is negative and it decreases (becomes even more negative) when surplus consumption is low. Intuitively, investors are particularly risk averse in states of low surplus consumption, and are therefore particularly willing to hold nominal bonds with low expected returns. This flight-to-safety effect works similarly for longer-term bonds, so when bonds' real returns have hedging value to consumers, the model implies that bond and stock risk premia are negatively correlated.

Different from two-period bonds, long-term nominal bond returns depend inversely on the innovation to nominal short-term interest rates expected over the entire remaining lifetime of the bond. This simple logic suggests that the sign and dynamics of bond risk premia should depend on the correlation between the innovations to the expected Fed Funds rate over the remaining lifetime of the bond with output gap innovations. To ensure that the model dynamics for the output gap, inflation, and interest rates match this particularly informative co-movement in both periods, we target it directly in our estimation.

4 Econometric Methodology

4.1 Data and summary statistics

We use quarterly US data on output, inflation, interest rates, and aggregate bond and stock returns from 1979Q3 to 2011Q4. Our sample period starts with Paul Volcker's appointment as Fed chairman, because of evidence that monetary and macroeconomic dynamics changed at that time (e.g. Clarida, Gali, and Gertler 1999). The goal of our empirical analysis is to illustrate the properties of our model without the additional complications of the "zero lower bound" of close-to-zero short-term nominal interest rates, so we end the sample in 2011Q4.

Real GDP, real consumption for nondurables and services, real potential GDP, and the GDP deflator in 2009 chained dollars are from the FRED database at the St. Louis Federal Reserve.[5] There is an ambiguity with respect to the timing of output and asset prices. Output, just like consumption, is a flow over a quarter, while asset prices are measured at a point in time. Time-averaged output observed in a quarter could therefore reasonably be treated as occurring at the beginning or the end of the quarter. In the consumption-based asset pricing literature, it has been found preferable to treat time-averaged output and consumption as occurring at the beginning of the quarter, because this captures the tendency for measured macroeconomic variables to move more slowly than asset prices. The beginning-of-quarter timing convention implies a higher correlation between stock returns and contemporaneous output and consumption growth, and a close-to-zero correlation between stock returns and lagged output and consumption growth (Campbell 2003). We follow this beginning-of-

[5] Accessed 08/05/2017.

quarter timing convention throughout the paper and align consumption and output reported by FRED for quarter t with asset prices measured at the end of quarter $t-1$. Alternative ways of dealing with time-aggregation include solving and simulating the model at higher frequency (Campbell and Cochrane 1999) or using quarter-averaged rather than quarter-end asset prices (Cochrane 1991).

We use the end-of-quarter Federal Funds rate from the Federal Reserve's H.15 publication. The end-of-quarter bond yield is the CRSP monthly Treasury Fama-Bliss five-year zero-coupon (discount) bond yield. We use the value-weighted combined NYSE/AMEX/Nasdaq stock return including dividends from CRSP, and measure the dividend-price ratio using data for real dividends and the S&P 500 real price. Interest rates, and inflation are in annualized percent, while the log output gap is in natural percent units. All yields and returns are continuously compounded. We consider log returns in excess of the log T-bill rate, where the end-of-quarter three-month T-bill is from the CRSP monthly Treasury Fama risk-free rate files and is based on the average of bid and ask quotes.

4.2 Break date tests

We start our empirical analysis by dividing our sample according to changes in inflation dynamics. We test for a break in the cyclicality of inflation because it is the simplest macroeconomic measure that is inversely related to real cash flows on nominal bonds. We run a Quandt Likelihood Ratio (QLR) test for an unknown break date in the relation between inflation and the output gap on our full sample running from 1979Q3 until 2011Q4. For every quarter τ we estimate a full-sample regression of quarterly log inflation onto a constant, a dummy that takes the value of one if $t \geq \tau$ and zero otherwise, the log output gap, and the log output gap interacted with the dummy:

$$\pi_t = a_\tau + b_\tau I_{t\geq\tau} + c_\tau x_t + d_\tau I_{t\geq\tau} x_t + \varepsilon_t. \tag{33}$$

For each potential break quarter τ, we compute the F-statistic corresponding to d_τ with Newey-West standard errors and one lag. The QLR test statistic is the maximum F-statistic and the estimated break date is the quarter τ with the highest F-statistic (Andrews 2003).

Figure 2 plots the F-statistic against the quarter τ, showing a single-peaked distribution with a statistically significant maximum in 2001Q2. This break date test, which is based only on inflation and output data and does not use asset prices, therefore provides evidence for a change in inflation dynamics in the early 2000s. If we replace inflation in (33) with the nominal Federal Funds rate we estimate a similar break date in 2000Q2, further supporting a change in macroeconomic risks at this time.[6]

A formal test for a break in the relation between daily bond and stock excess returns confirms that the break date from macroeconomic data lines up with changes in bond risks. We run a QLR test for an unknown break date in the relation between bond and stock excess returns.[7] For every date τ within the middle 70% of the sample, we estimate a regression using daily log bond and stock excess returns:

$$r^{\$}_{t+1} = a_\tau + b_\tau I_{t\geq\tau} + c_\tau r^{stock}_t + d_\tau I_{t\geq\tau} x r^{stock}_t + \varepsilon_t. \tag{34}$$

[6] If we replace inflation in (33) with the ex post real interest rate, defined as the nominal Federal Funds rate less realized inflation, the estimated break date is 1989Q2. However the ex post real interest rate is a noisy proxy for the ex ante riskfree real rate which is the object of interest in our model.

[7] We use daily log returns on five-year nominal Treasury bonds and daily log CRSP value-weighted stock market returns, as in Figure 1. To reduce the effect of outliers, we winsorize bond and stock returns at the 0.5% and 99.5% levels.

This QLR test based on asset returns indicates a statistically significant break date on December 6, 2000.

Empirical inflation-output gap betas changed in the opposite direction as bond-stock betas, suggesting that macroeconomic dynamics were at least partly responsible for the change from positive to negative bond-stock betas. At the estimated break dates, the inflation-output beta changed from -0.32 (0.11) before the break to 0.33 (0.07) after (Newey-West standard error with one lag in parentheses). The correlation changed from -0.28 to 0.65. Hence, while prior to the break the US economy was in a stagflationary regime, where inflation increased during periods of low output, after the break inflation has tended to increase during expansions.

4.3 Calibrated parameters

Table 1, Panel A summarizes time-invariant calibrated parameter values. Our selection of parameter blocks is consistent with Smets and Wouters (2007), who find important changes in shock volatilities and parameters driving inflation and Federal Funds rate dynamics, but stable preference parameters. The block of time-invariant parameters includes those governing the relation between the output gap, consumption, and dividends (ϕ, g, δ), and parameters determining investor and consumer preferences (γ, θ_0, θ_1, θ_2, $\bar{r}$).

The parameter ϕ determines the link between the output gap and consumption. We choose $\phi = 0.93$, the value that maximizes the empirical correlation between stochastically detrended consumption and the output gap over our full sample.

The leverage parameter δ scales up the volatility of equity returns, while preserving their Sharpe ratio. We choose a leverage ratio of $\delta = 0.50$ to obtain empirically plausible equity return volatilities. We interpret δ as capturing a broad concept of leverage, including operational leverage.

We set the average consumption growth rate, g, utility curvature γ, surplus consumption persistence, θ_0 (reported after compounding to an annual frequency) and the average real risk-free rate $\bar{r}$ exactly as in Campbell and Cochrane (1999).

We set the new parameters to $\theta_1 = -0.05$ and $\theta_2 = 0.02$ after a simple exploration of the parameter space. We require $\theta_2 \geq 0$ to prevent the backward-looking term in the Euler equation (17) from becoming negative, but we consider negative and positive values for θ_1. We estimate the model for the eight parameter combinations where $\theta_1 = 0.0, 0.02$ and $\theta_2 = -0.05, -0.02, 0.0, 0.02$. We choose the values for θ_1 and θ_2 that generate the best fit to the macroeconomic impulse responses, as measured by the objective function described in the next subsection.[8] Comparing to the impulse responses obtained with $\theta_1 = \theta_2 = 0$ shows that these new habit parameters fulfill the traditional role of habit in macroeconomic DSGE models that do not have asset prices. Similarly to Christiano, Eichenbaum and Evans (2005) and further documented in the appendix, we find that a lagged term in the Euler equation is needed to generate smooth and hump-shaped impulse responses to interest-rate innovations.

The calibrated preference parameters imply an annualized discount factor of $\beta = 0.90$ and an Euler equation with a small backward-looking and a large forward-looking coefficient ($\rho^x = 0.02$, $f^x = 1.02$). The sum of these two coefficients is slightly greater than one, but smaller than it would be in the Campbell-Cochrane case $\theta_1 = \theta_2 = 0$. The coefficient on the real interest rate in the Euler equation is $\psi = 0.14$, within the range of empirical estimates by Yogo (2004) and earlier work by Hall (1988).

[8] To ensure that we are not at a corner solution, we also verified that the objective function deteriorates as we move each parameter individually to $\theta_1 = -0.10$ or $\theta_2 = 0.05$, while holding the other parameter constant.

4.4 Parameter estimation in subperiods

The subperiod-specific parameters are estimated to minimize the distance between model and empirical moments. For each subperiod, we separately estimate the twelve-dimensional parameter vector using simulated method of moments (SMM):

$$params = [b_{\pi i}, b_{\pi\pi}, b_{\pi i}, b_{ix}, b_{i\pi}, b_{ii}, \sigma_\pi, \sigma_i, \sigma_*, \rho_{\pi i}, \rho_{\pi *}, \rho_{i*}]\,. \tag{35}$$

We target impulse responses for the output gap, inflation, and the Federal Funds rate, as well as the correlation between the 20-quarter average Federal Funds rate and the output gap. We do not use further bond or stock return moments for the estimation, other than requiring that asset prices exist. This choice is conservative in the sense that the fit for bond and stock return moments would improve if we targeted them directly in the estimation.

Our estimation methodology for the empirical impulse responses is guided by the desire to remain comparable to the literature. We use a simplified version of the specification in Christiano, Eichenbaum, and Evans (2005), estimating impulse responses from a VAR(1) in levels and with a end-of-quarter timing convention for the output gap. We orthogonalize the VAR innovations such that the innovation to the Fed Funds rate does not enter into contemporaneous inflation or the output gap, and the innovation to inflation does not enter into the contemporaneous output gap. In our notation, we estimate the VAR(1) in $[x_{t-1}, \pi_t, i_t]$. This state vector differs from that in equation (25), which includes stationary deviations $\hat{\pi}_t$ and $\hat{i}_t$ rather than levels π_t and i_t which are nonstationary in the model. Simulated model and empirical samples have the same length to ensure that empirical and model impulse responses reflect the same small-sample effects. In plausible empirical samples the estimated persistence of unit root variables is biased downward, so both the simulated model and the empirical impulse responses converge back to zero. The appendix provides further details and explains the bootstrap procedure used to obtain confidence intervals and standard errors for empirical impulse responses.

In addition to the impulse responses, we target a long-term Fed Funds-output gap correlation to closely match the business cycle properties of the Federal Funds rate. The empirical correlation of the average Federal Funds rate over the next 20 quarters with respect to the output gap is reported in Table 4. Similarly to the sign switch in inflation cyclicality documented in Section 4.2, this correlation moves from -0.38 in the 1979Q3-2001Q1 period to 0.57 in the 2001Q2-2011Q4 period.

The vector $\hat{\Psi} - \Psi(params)$ consists of differences between the data and the model: differences in impulse responses at one (shock period), two, four, 12, 20, and 40 quarters, excluding those that are zero by construction, and the square root of the difference between the empirical 20-quarter Federal Funds rate-output gap correlation minus the analytical model correlation of 20-quarter expected nominal Federal Funds rate innovations with output gap innovations. The estimated parameter vector $\widehat{params}$ minimizes the objective function:

$$Obj(params) = \left[\hat{\Psi} - \Psi(params)\right]' \hat{W} \left[\hat{\Psi} - \Psi(params)\right]. \tag{36}$$

Here, $\hat{W}$ is a data-based, symmetric, positive-definite weighting matrix. To avoid matrix invertibility issues, we follow Christiano, Eichenbaum, and Evans (2005) and take $\hat{W}$ to be a diagonal matrix with inverse sample variances of $\hat{\Psi}$ along the diagonal. We require the model to match the long-term Fed Funds-output gap correlation closely by setting the last element of $\hat{W}$ to 200.

Our twelve-dimensional parameter space is ill-suited for gradient-based optimization methods. We therefore minimize the objective function by grid search over the parameter space. The appendix provides details of the grid search procedure.

5 Empirical Results

5.1 Parameter estimates

Table 1, Panel B shows the estimated macroeconomic parameters for 1979Q3–2001Q1 and 2001Q2–2011Q4. The first part of the panel reports the estimated lag parameters. Two parameters, $b_{\pi x}$ and b_{ix}, switch from negative to positive between periods 1 and 2, with the switch in b_{ix} statistically significant. Keeping in mind that (21) represents equilibrium dynamics and not a structural model, the increase in b_{ix} from negative to positive is suggestive of an increase in the Fed's interest in stabilizing output in period 2, relative to the strong focus on stabilizing inflation in period 1.

Moving to the volatility parameters, the estimated volatility of the unit-root component declines from 0.56 in period 1 to 0.43 in period 2, while the volatility of short-term inflation shocks, σ_π increases. This squares well with long-term inflation surveys, which have been very stable during our second subperiod. These estimates also line up with Stock and Watson (2007), who estimate that the permanent component of inflation has become less volatile while the transitory component has become more volatile in recent decades. The estimated volatility of interest rate shocks, σ_i, is higher during period 1, reflecting potentially the higher volatility of monetary policy during earlier decades.

Among the correlations, none of them are estimated to switch sign across periods. Finally, the implied steady-state and maximum surplus consumption ratios are similar to Campbell and Cochrane (1999) in both subperiods.

5.2 Estimated macroeconomic dynamics

Figures 3 through 5 assess the model's macroeconomic fit through impulse responses. We plot the macroeconomic impulse responses that were used in the SMM estimation, and are described in detail in Section 4.4. Figures 3 through 5 show that model impulse responses are generally within the 95% confidence bands for empirical impulse responses, with the few exceptions being short-lived. The impulse responses are orthogonalized, so if one wanted to assume that output and inflation react to monetary policy with a one quarter lag, the orthogonalized Fed Funds rate innovation would correspond to a structural monetary policy shock (Sims 1986, Christiano, Eichenbaum, and Evans 2005). However, our SMM estimation does not require this stronger structural interpretation.

Figure 4 shows that the output gap response to a one standard deviation inflation innovation switches from negative in period 1 to positive in period 2, consistent with the inflation-output gap correlation changing from negative to positive. Figures 3 and 5 show that inflation responses to output gap innovations and Federal Funds rate innovations are smaller and mostly statistically insignificant, so these innovations appear less important for changing inflation cyclicality. Because our asset pricing preferences imply an Euler equation with both forward- and backward-looking terms, the model generates smooth and hump-shaped impulse responses to Federal Funds rate innovations, as shown in Figure 5.

In addition to matching impulse responses, the model generates unconditional moments for consumption growth and the output gap that are comparable to those in the data. The model's annualized consumption growth volatility is 1.75% in period 1 and 1.59% in period 2, compared to 0.9% in the data. Similarly to the data, real consumption growth is serially correlated but not highly persistent. The AR(1) coefficient of model log real consumption growth onto its own one-quarter lag is 0.03 for period 1 and 0.25 for period 2, compared to an empirical coefficient of 0.26 reported in Beeler and Campbell (2012). The model output gap is highly persistent, similarly to the data. The AR(1) coefficient of the model output gap onto its own one-quarter lag equals 0.91 for period 1 and 0.99 for period 2, compared to 0.92 and 0.94 in the data.

Having seen that the model matches changes in empirical macroeconomic dynamics, we next turn to understanding how changing macroeconomic dynamics contribute to bond risks.

5.3 Asset pricing implications

Table 2 shows that the model replicates the successes of Campbell and Cochrane (1999) for the stock market. The model generates a high stock market Sharpe ratio and volatile and predictable stock returns. To mimic firms' dividend smoothing in the data, we compare empirical moments for the price-dividend ratio to the price of levered equities divided by dividends smoothed over 64 quarters. The model generates a highly persistent price-dividend ratio, though it is somewhat less volatile than in the data. The model generates substantial stock return predictability from the price-dividend ratio. The model coefficient of 1-year stock returns onto the lagged price-dividend ratio matches the longer postwar sample results in Campbell and Cochrane (1999). While the corresponding empirical moment for period 1 might appear to suggest little stock return predictability, this is likely due to the short sample.

Table 3 shows the bond market implications of our model. Bond returns are volatile, with standard deviations of 6.26% in period 1 and and 3.78% in period 2, even though they are somewhat less volatile than in the data. Importantly, the volatility of bond returns declines from period 1 to period 2, just as in the data, even though bond and stock returns were not used in the model estimation. The spread between the five-year nominal log yield and the log Federal Funds rate is persistent, though again less volatile than in the data.

In period 2 the average excess return on long-term bonds is positive even though our model implies a negative term premium. We reconcile the data with the model by noting that this average excess return is imprecisely estimated and statistically indistinguishable from zero. Intuitively, a substantial and arguably unanticipated decline in the short-term Federal Funds rate drove up realized term premia over the relatively short second period relative to the model, where nominal interest rates are constant on average.

The bottom of Table 3 shows that our model generates small but empirically plausible bond excess return predictability from the term spread in period 2, but it fails to match the stronger empirical bond return predictability in period 1. However, bond excess returns do move with the term spread across periods. The regression of subperiod average bond excess returns onto average log yield spreads generates a slope coefficient of 1.80, in line with the slope coefficients reported in long-sample regressions by Campbell and Shiller (1991). In a model extension with time-varying regime probabilities, this cross-regime relation between yield spreads and bond excess returns would presumably generate higher-frequency bond return predictability.

Table 4 turns to our main object of interest: the changing comovement of bond and stock returns. It shows that the model can explain empirical bond-stock return comovements across subperiods, even though these moments were not directly targeted in the estimation. The model replicates the switch from a positive bond-stock return correlation to a negative bond-stock return correlation in the data, with a model bond-stock return correlation of 0.50 in period 1 versus -0.66 in period 2. The model stock-market beta of nominal bond returns also shows an economically significant change from 0.14 in period 1 to -0.16 in period 2.

The middle panel in Table 4 shows that the change from a positive to a negative bond-stock correlation is underpinned by changes in inflation-output and Federal Funds-output correlations in the opposite direction. Our estimation explicitly targets the five-year average Fed Funds-output gap correlation and the inflation-output gap correlation, so it is reassuring that the model fits the changes in these correlations. The model correlation between innovations to five-year inflation expectations with output gap innovations also switches from negative in period 1 to positive in period 2, although the magnitude of the change in the model is smaller than in the data.

The last panel of Table 4 shows that the model generates an empirically reasonable link between the output gap and risk premia. In the model, this link arises because a low output gap tends to go along with low consumption relative to habit, when investors are risk averse. Hence, the model output gap forecasts stock excess returns negatively; and Table 4 shows that the magnitude is similar to the empirical relation documented by Cooper and Priestley (2009). Turning to bonds, the output gap's ability to forecast bond excess returns is mixed, both in the model and the data. In period 1, where bonds have a positive stock beta, their risk properties are similar to stocks', so the model predicts a negative relation between the output gap and future bond excess returns. We confirm this prediction in period 1 data, where the coefficient is statistically indistinguishable from the model and significantly negative at the 90% confidence level. In period 2, the model predicts a positive coefficient, because bonds have negative stock betas, and so their risk properties are the opposite of stocks'. In the data, the period 2 forecasting coefficient increases relative to period 1 and is statistically indistinguishable from the model at conventional significance levels.

Figure 6 explores which model parameters are responsible for the change in the model bond-stock correlation between periods 1 and 2. The figure reports counterfactual changes in the bond-stock correlation, the inflation-output gap correlation, and the correlation between the innovations to five-year Fed Funds rate expectations with output gap innovations. We show changes in these correlations implied by the model from changing subsets of parameters from their period 1 to their period 2 values. It shows that lag parameters generate the largest changes in all three correlations. There is a smaller contribution from shock correlations and a modest offsetting effect from changes in shock standard deviations. Intuitively, lag coefficients are crucial for the changing bond-stock correlation because they determine how long-term expectations of inflation and interest rates, and hence bond returns, correlate with innovations to risk aversion.

To summarize, we have seen that the model links changing bond risks and changing macroeconomic dynamics, and that it generates empirically plausible risk premia from the business cycle. However, Table 4 and Figure 6 do not reveal the importance of time-varying risk premia for changing bond risks. We turn to this next.

5.4 Decomposing model bond and stock returns

Table 5 illustrates the amplifying effect of time-varying risk premia by decomposing model bond and stock returns as in Campbell and Ammer (1993). Panel A decomposes the variance of stock returns, showing the variance-covariance matrix of stock real cash-flow news, real-rate news, risk-premium news, and total stock returns. Because covariances are additive, the first three columns sum to the covariance with total stock returns, shown as a fourth column. The total stock return variance in the bottom-right is the sum of the upper-left 3 $\times$ 3 quadrant. Panel A shows that the majority of the variation in stock returns is due to risk premium news. As in Campbell and Cochrane (1999), the risk premium component of stock returns is highly correlated with changes in the log surplus consumption ratio. Because stocks represent a levered claim on consumption and the surplus consumption ratio resembles stochastically de-trended consumption, the cash flow and risk premium components of stock returns move closely together in both periods. The covariance between real-rate news and total stock returns switches from positive in period 1 to negative in period 2, consistent with the evidence from UK inflation-indexed bonds (Campbell, Shiller, and Viceira 2009).

Table 5, Panel B similarly decomposes the variance of bond returns. If the expectations hypothesis of the term structure held, bond returns would simply equal the sum of the real rate and cash-flow news terms. The majority of the variation in five-year nominal bond returns is attributed to news about real cash flows, or the negative innovation to long-term inflation expectations over the remaining maturity of the bond. The relatively smaller variance for risk-premium news in bonds than in stocks is broadly consistent with the empirical analysis of Campbell and Ammer (1993), even though they considered 10-year nominal bonds and an earlier sample.

Table 5, Panel C however shows that time-varying bond and stock risk premia are both quantitatively important for the covariance between bond and stock returns. The rightmost column of Panel C shows that switching off bond risk premia alone implies a reduction in bond-stock covariances by $22.71/68.94 = 33\%$ in period 1 and $(-11.40)/(-39.48) = 29\%$ in period 2. The bottom row shows that switching off stock risk premia alone implies a reduction in bond-stock covariances by $48.88/68.94 = 71\%$ in period 1 and $(-31.91)/(-39.48) = 81\%$ in period 2. When investors' risk aversion increases following a decrease in the output gap, a flight-to-safety effect arises, driving down bond valuations when bonds are risky according to their real cash flow and real rate components as in period 1, and driving them up when bonds' real cash flow and real rate components are safe as in period 2. Because bond and stock risk premia are highly correlated, either positively or negatively, this amplification effect is large.

The intuition for the amplifying effect goes back to the analytical expression (28). This expression suggests that bond risk premia increase with investors' sensitivity $\lambda(s_t)$, similarly to stock risk premia, if the covariance between the log stochastic discount factor and the sum of bond real cash-flow and real-rate news is positive. The rightmost column in Panel C shows that bonds' real cash-flow and real-rate news changed from risky to safe, thereby explaining why bond risk premia changed from positively to negatively correlated with stock risk premia. We interpret the relative magnitudes of the covariances between bond cash-flow and real-rate rate news, on the one hand, and stock returns, on the other hand, with caution, because the model fits the long-term inflation-output gap correlation less well than the long-term Fed Funds-output gap correlation as shown in Table 4.

In summary, Table 5 demonstrates that the time-variation of risk premia is a crucial amplification mechanism linking macroeconomic dynamics and bond risks.

6 Conclusion

We provide a new framework for understanding how macroeconomic dynamics drive stocks and bonds and apply it to changing bond-stock return comovements. Our model is the first one to combine Campbell and Cochrane (1999) habit-formation preferences with homoskedastic, loglinear macroeconomic dynamics. As such, we hope that it will provide a new tool to researchers studying time-varying risk premia in a wide range of macroeconomic models that give rise to such dynamics either exactly or approximately. We conclude by discussing some of the possibilities for future research along these lines.

A natural extension would be to study time-varying consumption-based risk premia in a structural macroeconomic model. One could start from a standard small scale New Keynesian model and loglinearize firms' optimal price-setting condition to yield a Phillips curve (Woodford 2003). One could then add an interest rate rule for monetary policy as in Taylor (1993), and close the model with our preferences, which will generate both asset prices and the standard consumption Euler equation.

There are some difficulties that this approach will have to confront. Even simple New Keynesian models may have multiple equilibria or explosive solutions (Cochrane, 2011). Since our new preferences are compatible with different macroeconomic equilibria, they open up the possibility to use bond and stock prices to select between macroeconomic equilibria. In addition, there are well documented challenges in modeling households' labor-leisure choice in the presence of habit formation preferences (Lettau and Uhlig, 2000). Similarly to what we have done in this paper, judicious choices in relating the value of leisure to the habit stock may be one possible avenue to resolve this second challenge.

Loglinearized models with additional state variables, such as Smets and Wouters (2007) or models with a fiscal sector, are of even greater interest to policy makers. Our preferences generate a loglinearized Euler equation similar to that in Smets and Wouters (2007), giving reason to be optimistic that inserting our preferences into that model would preserve its desirable macroeconomic properties. For a model with real investment, researchers would need to use the structural relation between consumption, output and investment to relate the output gap to surplus consumption.

While we select our state variables inspired by the New Keynesian literature, our preferences are more widely applicable. For a real business cycle model, it would be typical to have a loglinear Euler equation in terms of consumption rather than the output gap. It is straightforward to substitute the output gap out of the loglinear Euler equation (17) using the link between consumption and the output gap, and thereby write the Euler equation entirely in terms of consumption. In fact, it is not at all necessary to define the output gap to work with our preferences. It would also be possible to regard x_t simply as a stationary function of current and lagged consumption.

It would be conceptually straightforward to add lags or state variables in the macroeconomic dynamics and then solve for asset prices, whether these dynamics result from a structural model or are reduced form. Similarly, one could replace the unit root in inflation and interest rates by a slowly mean-reverting component, at the computational expense of introducing another state variable. Finally, one particularly interesting application of our model will be to study the role of the zero lower bound for bond risks.

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Table 1: Parameters

Panel A: Calibrated Parameters

Consumption Growth Rate	g	1.89
Utility Curvature	γ	2.00
Steady-State Riskfree Rate	$\bar{r}$	0.94
Persistence Surplus Cons.	θ_0	0.87
Dependence Output Gap	θ_1	-0.05
Dependence Lagged Output Gap	θ_2	0.02
Smoothing Parameter Consumption	ϕ	0.93
Leverage	δ	0.50
Implied Parameters		
Discount Rate	β	0.90
Euler Eqn. Lag Coefficient	ρ^x	0.02
Euler Eqn. Forward Coefficient	f^x	1.02
Euler Eqn. Real Rate Slope	ψ	0.13

Table 1: Parameters (continued)

Panel B: Estimated Parameters

Lag Parameters		79Q3-01Q1	01Q2-11Q4
Inflation-Output Gap	$b_{\pi x}$	-0.44	0.44
		(0.56)	(0.24)
Inflation-Inflation	$b_{\pi\pi}$	0.56	0.33
		(0.32)	(0.10)
Inflation-Fed Funds	$b_{\pi i}$	-0.56	-0.33
		(0.63)	(0.18)
Fed Funds-Output Gap***	b_{ix}	-0.44	0.44
		(0.13)	(0.16)
Fed Funds-Inflation	$b_{i\pi}$	-0.11	-0.11
		(0.27)	(0.07)
Fed Funds-Fed Funds	b_{ii}	0.33	0.33
		(0.26)	(0.21)
Std. Shocks (%)			
Std. Infl.	σ_π	0.68	0.94
		(0.29)	(0.13)
Std. Fed Funds	σ_i	0.68	0.56
		(0.17)	(0.14)
Std. Infl. Unit Root	σ_*	0.56	0.43
		(0.08)	(0.16)
Shock Correlations			
Inflation-Fed Funds	$\rho_{\pi i}$	0.11	0.33
		(0.57)	(0.51)
Inflation-Infl. Unit Root	$\rho_{\pi *}$	0.11	0.33
		(0.60)	(0.77)
Fed Funds-Infl. Unit Root	ρ_{i*}	-0.11	-0.77
		(0.35)	(0.78)
Implied Parameters			
Steady-State Surplus Cons. Ratio	$\bar{S}$	0.07	0.06
Max. Surplus Cons. Ratio	s_{max}	0.11	0.09

Panel A shows calibrated parameters, that are held constant across subperiods. Consumption growth and the steady-state risk-free rate are in annualized percent. The discount rate and the persistence of surplus consumption are annualized. The estimated macroeconomic parameters in Panel B are reported in units corresponding to our empirical variables, i.e. the output gap is in percent, and inflation, the Fed Funds rate and the unit root component of inflation are in annualized percent. The implied Euler equation real rate slope is reported in the same units, that is $\frac{1}{4}\frac{1}{\gamma(\phi-\theta_1)}$. We report quarterly standard deviations of shocks to annualized percent inflation, Fed Funds rate, and unit-root component of inflation. We use superscripts *,**, and *** to denote that for a parameter we can reject that it is constant across subperiods at the 10%, 5%, and 1% levels, accounting for estimation uncertainty in both periods.

Table 2: Stocks

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Excess Returns				
Equity Premium	7.97	10.99	4.03	8.11
Volatility	16.42	21.92	20.00	15.95
Sharpe Ratio	0.49	0.50	0.20	0.51
Log Price-Dividend Ratio				
Mean ($exp(mean(pd))$)	34.04	14.43	53.73	20.33
Volatility	0.46	0.14	0.20	0.16
AR(1) Coefficient	1.00	0.95	0.86	0.98
Predictability				
1-YR Excess Return on pd	-0.01	-0.33	-0.43	-0.22
R^2	0.00	0.05	0.22	0.05

Log stock excess returns are quarterly log returns for the value-weighted combined NYSE/AMEX/Nasdaq stock return including dividends from CRSP in excess of the log 3-month T-bill from the CRSP Monthly Treasury file plus one-half times the log excess return variance to adjust for Jensen's inequality. The empirical price-dividend ratio is the S&P 500 real price divided by real dividends from Robert Shiller's website. The model price dividend ratio divides by dividends smoothed over 64 quarters. The last two rows report results from regressing 4-quarter log excess returns onto the lagged log price-dividend ratio: $xr^{stock}_{t\rightarrow t+4} = b_0 + b_1 pd_t + \epsilon_{t+4}$. Model moments are averaged over 2 simulations of length 10000.

Table 3: Bonds

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Excess Returns				
Term Premium	2.31	1.68	3.23	-1.41
Volatility	8.37	6.26	5.98	3.78
Sharpe Ratio	0.28	0.27	0.54	-0.37
Yields				
Mean log Yield Spread	1.16	0.94	1.40	-0.77
Volatility	1.29	0.82	0.93	0.47
AR(1) Coefficient	0.70	0.82	0.79	0.89
Predictability				
1-YR Excess Returns on log Yield Spread	2.78	-0.15	0.39	0.89
R^2	0.19	0.00	0.01	0.01

Log bond excess returns are quarterly log returns on 5-year nominal bonds in excess of the log nominal 3-month T-bill. We compute empirical log returns on the n-quarter nominal bond from log nominal bond yields: $r^{\$}_{n,t+1} = -(n-1)y^{\$}_{n-1,t+1} + ny^{\$}_{n,t}$. We measure quarter-end 5-year bond yields and 3-month T-bill yields with CRSP Monthly Treasury continuously compounded yields based on the average of bid and ask quotes. We approximate 19-quarter yields with 5-year bond yields in the data. The log yield spread is computed as the log 5-year bond yield minus the log nominal 3-month Treasury bill. The term premium is the average log 5-year nominal bond return in excess of the log nominal 3-month T-bill plus one-half times the log excess return variance to adjust for Jensen's inequality. The term premium, volatility of bond excess returns, and the log yield spread are in annualized percent. The last two rows report results from regressing 4-quarter log excess returns onto the lagged log yield spread: $xr^{\$}_{n,t\rightarrow t+4} = b_0 + b_1 spread_t + \epsilon_{t+4}$. Model moments are averaged over 2 simulations of length 10000.

Table 4: Bonds and Stocks

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Bond-Stock Comovement				
Correlation Bond and Stock Returns	0.21	0.50	-0.64	-0.66
Beta Bond Returns on Stock Returns	0.11	0.14	-0.19	-0.16
Nominal-Real Comovement				
Correlation Quarterly Inflation and Output Gap	-0.28	-0.37	0.65	0.35
Correlation 5-Year Average Inflation and Output Gap	-0.15	-0.05	0.20	0.14
Correlation 5-Year Average Federal Funds Rate and Output Gap	-0.38	-0.38	0.57	0.57
Predictability				
1-YR Excess Stock Return on Output Gap	-1.05	-1.56	-4.71	-0.54
R^2	0.02	0.02	0.21	0.04
1-YR Excess Bond Return on Output Gap	-0.89	-0.20	-0.11	0.05
R^2	0.05	0.00	0.00	0.01

The bond-stock correlation is the correlation of quarterly log bond excess returns with log stock excess returns. The bond-stock beta is the slope coefficient from regressing quarterly log bond excess returns onto log stock excess returns: $xr^{\$}_{n,t+1} = b_0 + b_1 xr^{stock}_{t+1} + \epsilon_{t+1}$. "Correlation 5-Year Average Inflation and Output Gap" reports the empirical correlation between x_t and $(\pi_t + \pi_{t+1} + ... + \pi_{t+20})/20$, where t ranges from the first quarter in the subperiod to the last. We compare this to the analytical model correlation between innovations to expected inflation over the next five years with output gap innovations. "Correlation 5-Year Average Federal Funds Rate and Output Gap" is analogous. The last four rows report results from regressing 4-quarter log stock excess returns onto the lagged output gap: $xr^{stock}_{t\to t+4} = b_0 + b_1 x_t + \epsilon_{t+4}$. The regression for 5-year log bond excess returns is analogous. Data for inflation, the Federal Funds rate, and the log output gap are described in section 4.1. Data for quarterly nominal 5-year bond excess returns and stock excess returns are described in Tables 2 and 3. All model moments are averages over 2 simulations of length 10000, with the exception of "Correlation 5-Year Average Inflation and Output Gap" and "Correlation 5-Year Average Federal Funds Rate and Output Gap" which are computed analytically.

Table 5: Decomposing Model Bond and Stock Returns

Panel A: Stock Return Variance Decomposition

	79Q3-01Q1				01Q2-11Q4			
	Cash Flow	Real Rate	Risk Premium	Total	Cash Flow	Real Rate	Risk Premium	Total
Cash Flow	17.39	8.95	61.41	87.75	90.38	-66.52	118.26	142.13
Real Rate	8.95	5.41	33.63	47.99	-66.52	49.68	-85.60	-102.43
Risk Premium	61.41	33.63	249.57	344.61	118.26	-85.60	182.01	214.68
Total	87.75	47.99	344.61	480.35	142.13	-102.43	214.68	254.37

Panel B: Bond Return Variance Decomposition

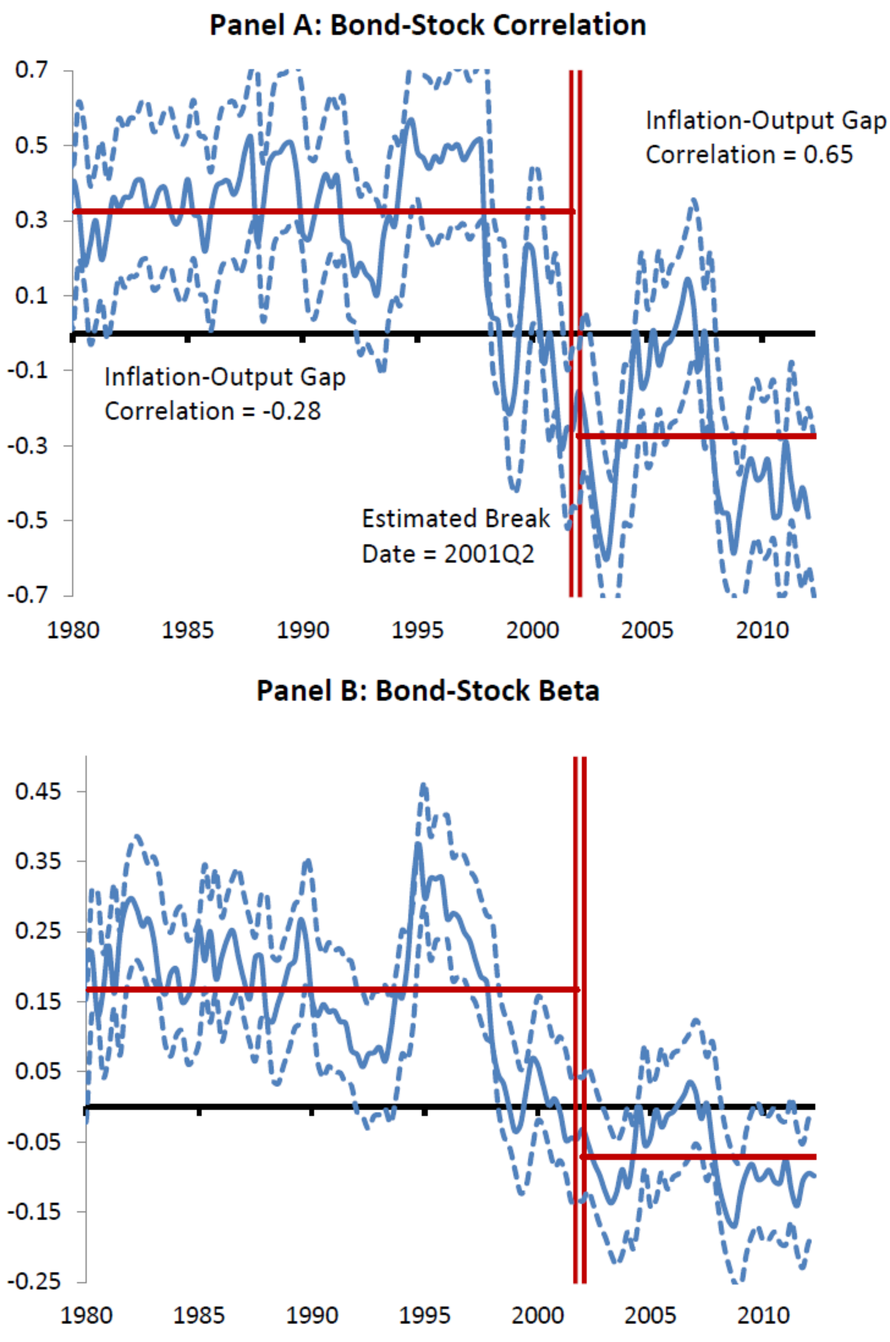
	79Q3-01Q1				01Q2-11Q4			
	Cash Flow	Real Rate	Risk Premium	Total	Cash Flow	Real Rate	Risk Premium	Total
Cash Flow	31.50	-1.33	0.08	30.25	12.41	-1.90	0.19	10.70
Real Rate	-1.33	5.04	2.02	5.73	-1.90	2.77	0.96	1.83
Risk Premium	0.08	2.02	1.17	3.27	0.19	0.96	0.57	1.72
Total	30.25	5.73	3.27	39.25	10.70	1.83	1.72	14.25

Panel C: Bond-Stock Return Covariance

	79Q3-01Q1				01Q2-11Q4			
Bonds $\downarrow$/Stocks $\rightarrow$	Cash Flow	Real Rate	Risk Premium	Total	Cash Flow	Real Rate	Risk Premium	Total
Cash Flow	1.40	-1.03	0.98	1.34	1.48	-3.98	-4.05	-6.54
Real Rate	8.45	5.10	31.34	44.89	-14.53	11.18	-18.19	-21.54
Risk Premium	3.97	2.18	16.56	22.71	-6.25	4.53	-9.68	-11.40
Total	13.82	6.25	48.88	68.94	-19.30	11.73	-31.91	-39.48

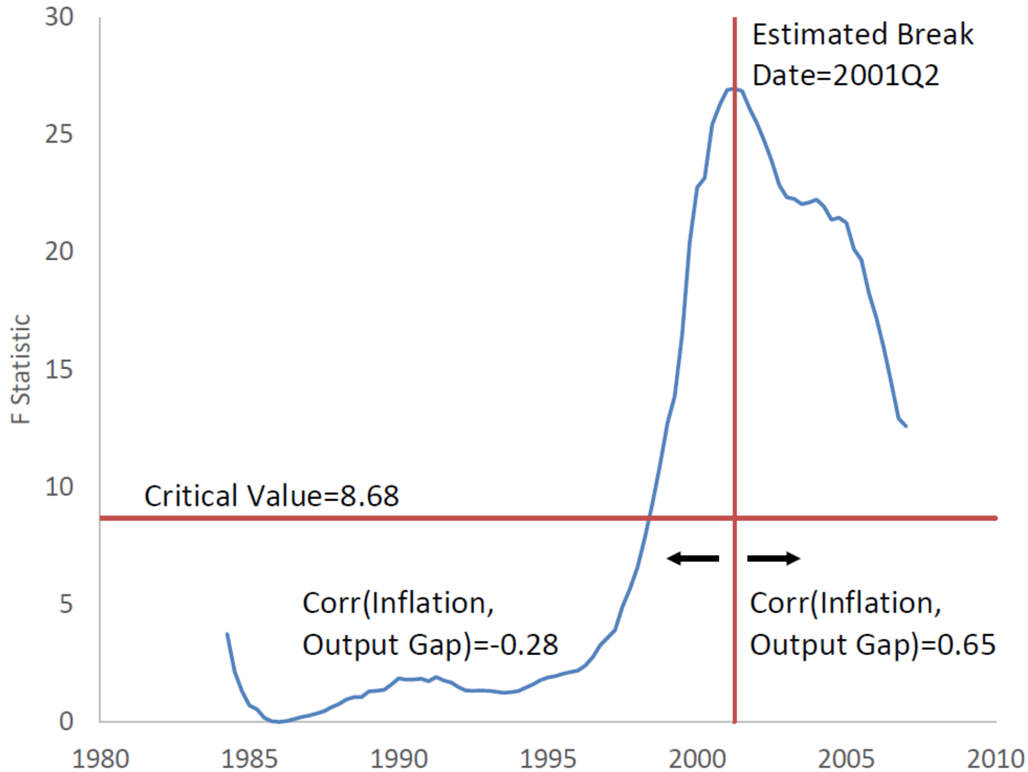
This table decomposes model stock and nominal bond returns into real cash-flow news, real-rate news, and risk-premium news (Campbell and Ammer 1993). For details of this decomposition see the appendix. Panel A shows the variance-covariance matrix of stock real cash-flow news, real-rate news, and risk-premium news. Panel B shows the variance-covariance matrix of bond return real cash-flow news, real-rate news, and risk-premium news. Panel C shows the covariance between bond real cash-flow news, real-rate news, and risk-premium news, on the one hand, with stock real cash-flow news, real-rate news, and risk-premium news, on the other hand. Model moments are averaged over 2 simulations of length 10000.

Figure 1: Rolling Bond-Stock Comovement



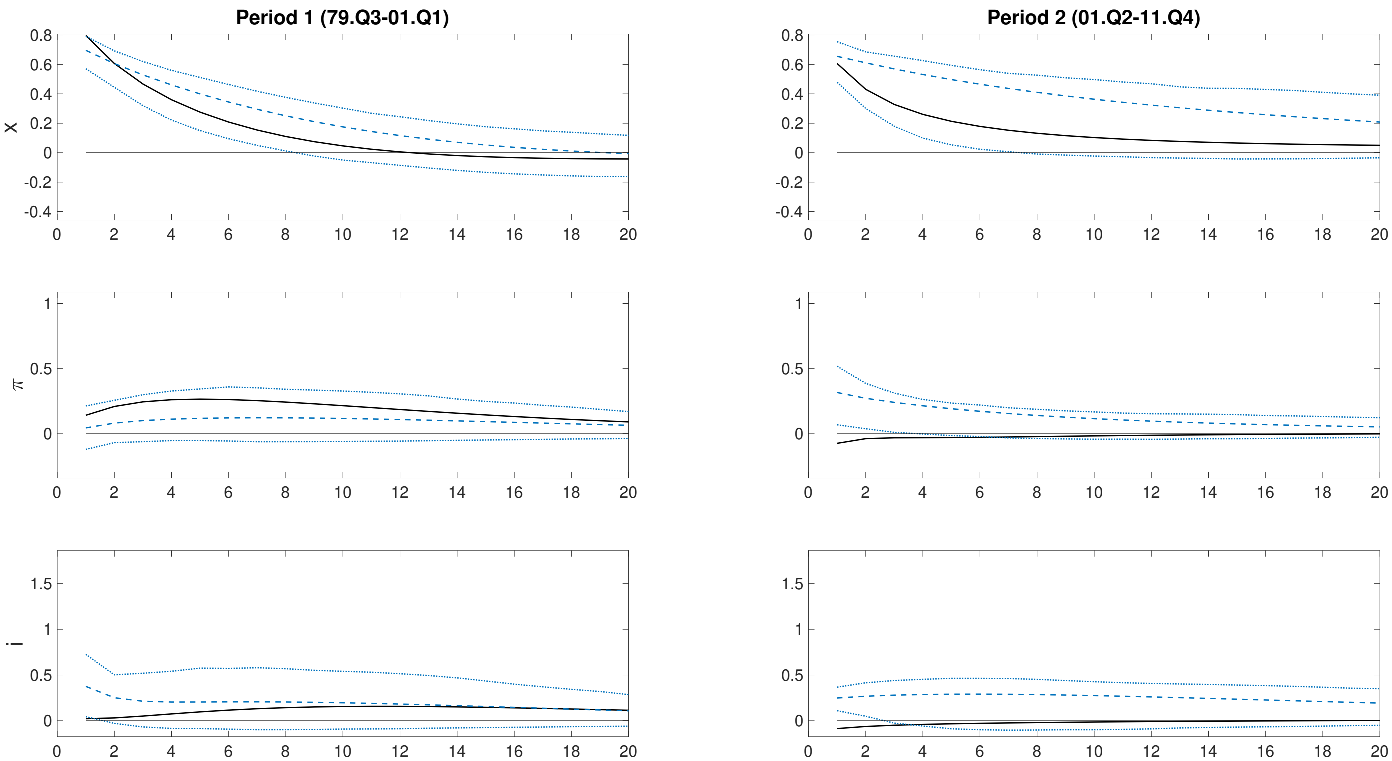
Rolling nominal bond-stock correlations and bond-stock betas use daily log returns on 5-year nominal Treasury bonds and daily log CRSP value-weighted stock market returns including dividends over past three months. We approximate daily nominal bond returns using changes in continuously compounded 5-year bond yields from Gürkaynak, Sack, Wright (2007). We show filtered correlations and betas from a Kalman filter, that assumes that observed correlations and betas follow an AR(1) trend plus white observation noise. 95% confidence intervals are shown in dashed. A red vertical line indicates the estimated break date from the Quandt Likelihood Ratio test for an unknown break date in the slope of quarterly inflation onto the quarterly output gap, described in detail in Section 4.2. Horizontal lines indicate subperiod averages.

Figure 2: Testing for Break in Inflation-Output Relation



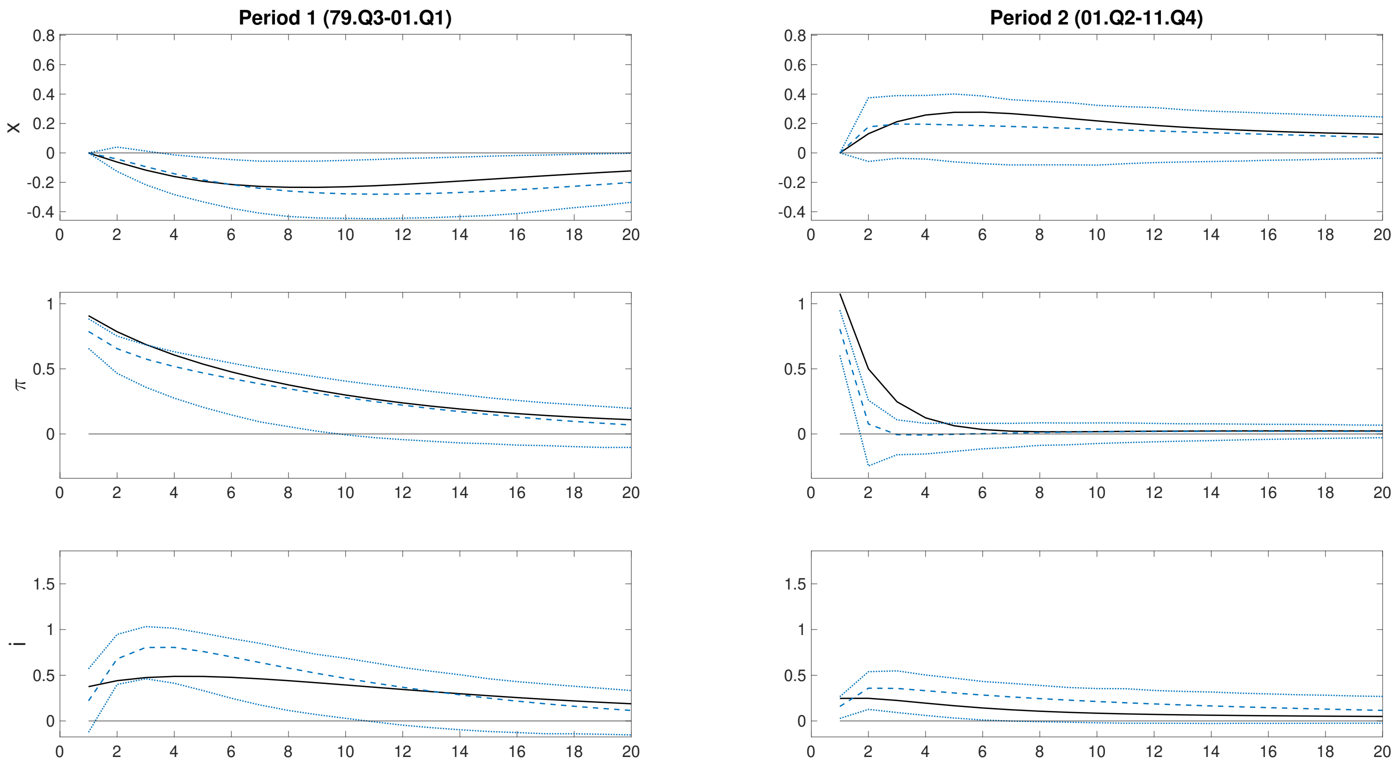
This figure plots the F-statistic for d_τ in the quarterly regression $\pi_t = a_\tau + b_\tau I_{t\geq\tau} + c_\tau x_t + d_\tau I_{t\geq\tau} x_t + \varepsilon_t$ for all τ in the middle 70% of our sample. We reject the null hypothesis of no break in the relation between π_t and x_t if the maximum F-statistic exceeds the 95% critical value with one constraint and 15% trimming (Andrews 2003). The critical value is indicated by the red horizontal line. The vertical line shows the estimated break date (2001Q2), which equals the quarter with the maximum F-statistic.

Figure 3: Empirical and Model Impulse Responses to Output Gap Innovations



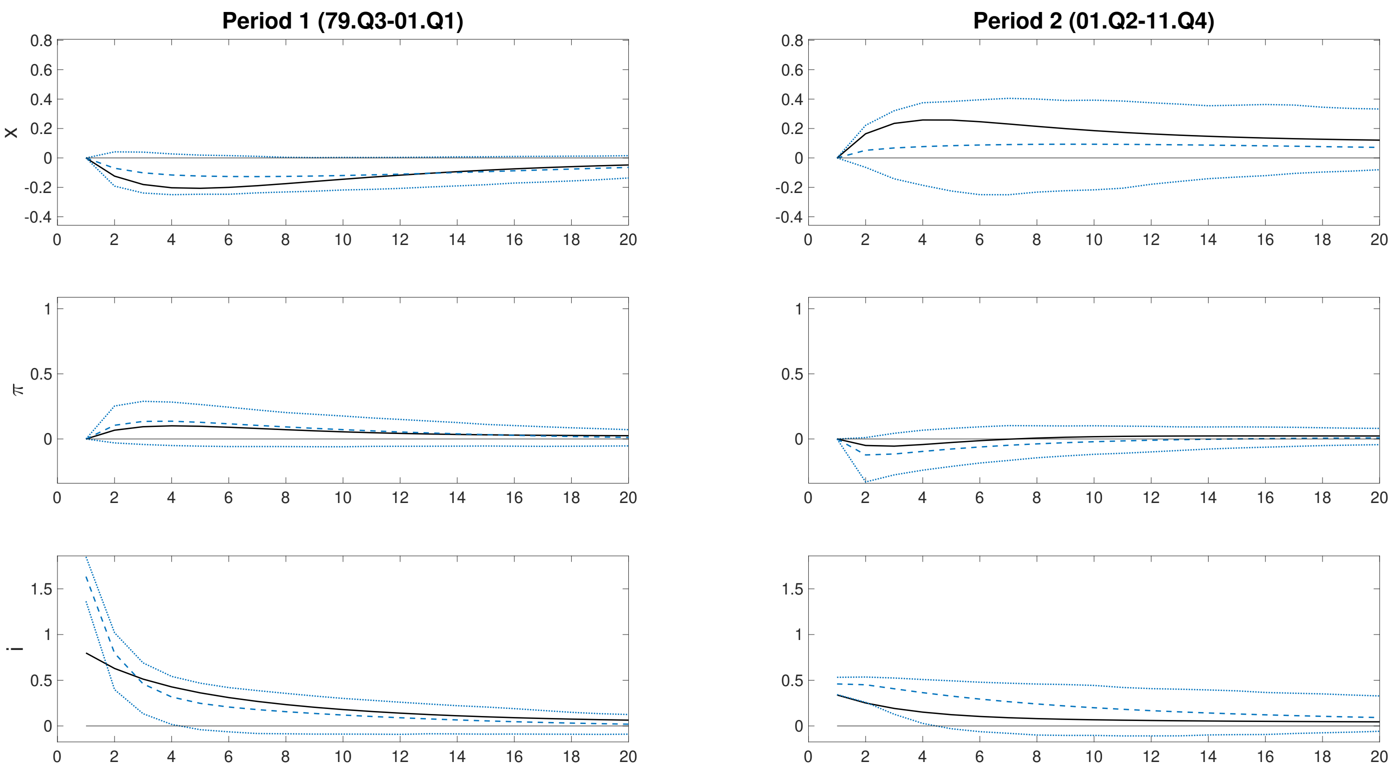
This figure shows model (black) and data (blue with 95% CI) orthogonalized impulse responses for the output gap, inflation, and the Federal Funds rate in response to a one-standard deviation output gap innovation. To provide a unique rotation of impulse responses, shocks are ordered such that an output gap shock affects inflation and the Fed Funds rate contemporaneously, an inflation shock affects the Fed Funds rate but not the output gap contemporaneously, and a Fed Funds rate shock affects neither inflation nor the output gap contemporaneously.

Figure 4: Empirical and Model Impulse Responses to Inflation Innovations



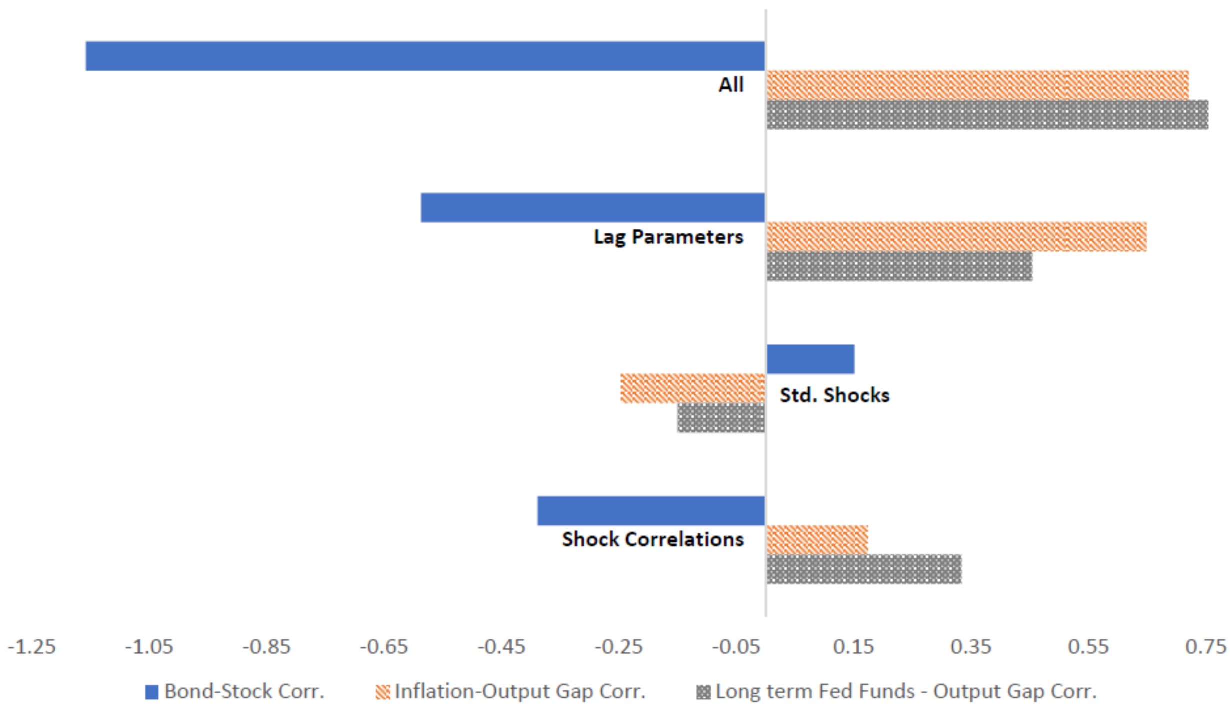
This figure shows model (black) and data (blue with 95% CI) orthogonalized impulse responses for the output gap, inflation, and the Federal Funds rate in response to a one-standard deviation inflation innovation. To provide a unique rotation of impulse responses, shocks are ordered such that an output gap shock affects inflation and the Fed Funds rate contemporaneously, an inflation shock affects the Fed Funds rate but not the output gap contemporaneously, and a Fed Funds rate shock affects neither inflation nor the output gap contemporaneously.

Figure 5: Empirical and Model Impulse Responses to Fed Funds Innovation



This figure shows model (black) and data (blue with 95% CI) orthogonalized impulse responses for the output gap, inflation, and the Federal Funds rate in response to a one-standard deviation Federal Funds rate innovation. To provide a unique rotation of impulse responses, shocks are ordered such that an output gap shock affects inflation and the Fed Funds rate contemporaneously, an inflation shock affects the Fed Funds rate but not the output gap contemporaneously, and a Fed Funds rate shock affects neither inflation nor the output gap contemporaneously.

Figure 6: Counterfactuals



This figure shows counterfactual changes in the bond-stock return correlation, the quarterly inflation-output gap correlation, and the correlation between innovations to the expected nominal Federal Funds rate over the next five years with output gap innovations. For each correlation, we depict the change in the correlation when changing subsets of parameters from their period 1 to their period 2 values. "All" shows the difference between period 2 and period 1 model correlations tabulated in Table 4. "Lag Parameters" shows the change in the model correlation when moving lag parameters $b_{\pi x}, ..., b_{ii}$ from their period 1 to their period 2 values. "Std. Shocks" changes only the standard deviations of shocks while holding all other parameters constant at their period 1 values. "Shock Correlations" changes only the correlations of shocks $\rho_{\pi i}, \rho_{\pi *}, \rho_{i*}$ while holding all other parameters constant at their period 1 values.