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Working Paper 14-031

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First draft: March 2012
This draft: August 2018

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Abstract

Our new model of consumption-based habit formation preferences generates loglinear, homoskedastic macroeconomic dynamics and time-varying risk premia on bonds and stocks. Consumers' first-order condition for the real risk-free interest rate takes the form of an exactly loglinear consumption Euler equation, commonly assumed in New Keynesian models. Estimating the model separately for 1979-2001 and 2001-2011 explains why the exposure of US Treasury bonds to the stock market changed from positive to negative. A change in the comovement between inflation and the output gap explains changing bond risks, but only when risk premia change endogenously as predicted by the model.

1 Introduction

This paper develops a novel integration of consumption-based asset pricing with macroeconomics. Asset prices measure agents' forward-looking expectations and are at the heart of consumption and savings decisions. An integrated framework can therefore impose valuable discipline on both macroeconomic and asset pricing models. We develop a new specification of preferences, building on the consumption-based habit formation model of Campbell and Cochrane (CC 1999), and use it to model macroeconomic dynamics jointly with bond and stock returns.

The CC habit formation model has become a benchmark for understanding asset prices, and specifically time-varying risk premia, but it has been difficult to apply outside the original setting of exogenous unpredictable consumption growth. We generalize CC preferences to more general consumption and interest rate dynamics. The process for habit in our model implies an exact loglinear Euler equation relating consumption to the riskless real interest rate. We assume a simple, empirically realistic link between consumption and the output gap so that we can express the Euler equation in terms of the output gap as is standard in New Keynesian models (e.g. Clarida, Gali, and Gertler 1999 or Woodford 2003).

Because our preferences are consistent with a loglinear Euler equation they are also consistent with loglinear, conditionally homoskedastic processes for macroeconomic variables. We combine the loglinear Euler equation with reduced-form, loglinear, homoskedastic dynamics for inflation and the Federal Funds rate. The resulting model captures the main empirical properties of the output gap, inflation, and the funds rate in a tractable fashion; and it raises the bar for our preference specification, by requiring it to generate time-varying risk premia even without nonlinear driving processes. We solve for the prices of nominal and real bonds and stocks, modeled as levered consumption claims whose dividends are cointegrated with consumption. While this paper does not take a stand on the details of macroeconomic frictions or the monetary policy rule, we provide a new tool that can be used to study the asset pricing implications of alternative structural macro models.

To demonstrate the usefulness of our approach, we apply our model to understand why nominal Treasury bonds changed from risky (comoving positively with stocks) in the 1980s and 1990s to safe (comoving negatively with stocks) in the first decade of the 2000s. This application is especially suitable because it requires an internally consistent macroeconomic and asset pricing framework of the sort that our model provides. The model explains the qualitative change in Treasury risks by a change in the comovement between inflation and the output gap, but a full quantitative explanation requires that risk premia change endogenously as predicted by the model. This is a striking result because our model tightly restricts time-variation in risk premia and does not allow them to change independently of cash flows.

We start our empirical application by testing for an unknown break date in the relation between inflation and the output gap in US data from 1979Q3 through 2011Q4. We detect a break in 2001Q2, with a negative inflation-output gap correlation before and a positive correlation after. Because nominal bond returns are inversely related to inflation and stock returns are positively related to the output gap, one might expect that the comovement between bonds and stocks should change in the opposite direction around this break date. Figure 1, Panel A shows that indeed the correlation of bond and stock returns was positive on average before 2001Q2 but negative afterwards. Figure 1, Panel B shows a similar change in the beta of nominal bond returns with respect to the stock market. Figure 1 uses daily data to estimate persistent components in the second moments of bond and stock returns.[2]

[2]The end-of-quarter bond-stock correlation is the correlation of daily log returns on 5-year nominal Treasury bonds with daily log CRSP value-weighted stock market returns including dividends over a rolling three-month window. The end-of-quarter bond beta is the regression coefficient of the same bond returns onto stock returns over the same rolling window. We use a Kalman filter to filter out measurement noise.

We estimate our model separately for the two periods 1979Q3–2001Q1 (period 1) and 2001Q2–2011Q4 (period 2) identified by our macroeconomic break test. We calibrate preference parameters following CC and set them equal across subperiods. We estimate the parameters governing macroeconomic dynamics separately for each subperiod using simulated method of moments (SMM). We use only macroeconomic dynamics and no bond or stock returns for the estimation. The moments used for the estimation are the empirical impulse responses of a standard VAR in the output gap, inflation, and the Federal Funds rate, and the correlation between the 5-year average Federal Funds rate and the output gap.

The model is broadly successful at matching the empirical impulse responses, and it generates empirically plausible bond and stock returns. As in CC, habit-formation preferences generate volatile and predictable equity returns to address the "equity volatility puzzle," one of the leading puzzles in consumption-based asset pricing (Campbell 2003). Unlike CC, the model generates realistic bond return volatility and predictability of both bond and stock returns from the output gap.

Despite being estimated only on macroeconomic dynamics, the model matches the changing comovement of quarterly bond and stock returns. In period 1, the model generates a quarterly bond-stock correlation of 0.50 compared to 0.21 in the data. In period 2, the model's bond-stock correlation is -0.72, closely matching the highly negative correlation in the data of -0.64. Decomposing bond and stock returns into cash-flow news, real-rate news, and risk-premium news, we find that the correlation between bond and stock cash-flow news switches from positive in period 1 to negative. However, the cash-flow news correlations are

Specifically, we assume that the bond-stock correlation follows an AR(1) process plus white measurement noise. We use a Kalman filter to estimate the AR(1) parameters by maximum likelihood and then to filter for the unobserved persistent component. Panel A plots the filtered persistent component and its 95% confidence interval. The filtered bond-stock beta and its 95% confidence interval in Panel B are constructed similarly. See the appendix for details of the Kalman filter.

only 0.12 in period 1 and -0.14 in period 2. The correlation between bond and stock risk premia switches from almost perfectly positive to almost perfectly negative and drives the quantitatively larger overall bond-stock correlations.

The interplay between the dynamics of inflation and the behavior of risk premia is both quantitatively important and intuitive. The sign switch in the correlation between inflation and the output gap moves the correlation between bond cash flows and stock cash flows from positive in period 1 to negative in period 2, and changes the sign of Treasury risk premia from positive to negative as emphasized by Campbell, Sunderam, and Viceira (2017). But habit formation preferences imply that recessions make investors more risk averse, driving down the prices of risky assets and driving up the prices of hedge assets in a "flight to safety". In period 1, Treasuries are risky assets that suffer from the flight to safety along with stocks, while in period 2 Treasuries are hedge assets that benefit from the flight to safety. Thus, time-varying risk aversion amplifies the positive comovement of bonds and stocks in period 1 and amplifies the negative comovement in period 2. Our model implies that time-varying macroeconomic dynamics and risk premia are not separate explanations for the change in bond risks, but instead are linked when consumers have consumption-based habit formation preferences.

1.1 Literature review and outline

This paper contributes to two main literatures. First, it further integrates the literatures on habit formation in asset pricing and macroeconomics. Habit in macroeconomic models without asset prices, such as Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), serves to generate persistence in macroeconomic fluctuations. The macro-finance literature has found it difficult to generate volatile asset returns from consumption-based habit formation preferences without implausibly distorting the dynamics of consumption, output, and the real interest rate. This is particularly true in simple models where habit is proportional to lagged consumption (Heaton 1995, Jermann 1998, Boldrin, Christiano, and Fisher 2001), but is also a problem in models with persistent but linearized habit dynamics (Lettau and Uhlig 2000, Uhlig 2007, Rudebusch and Swanson 2008, Lopez, Lopez-Salido, and Vazquez-Grande 2015).

One response to this problem has been to generate time-varying risk premia from heteroskedasticity in consumption growth (Chen 2017); but this requires large and extremely persistent movements in macroeconomic volatility just as in the long-run risk literature that works with recursive preferences (Bansal, Kiku, and Yaron 2012), and encounters the difficulty that empirical equity risk premia do not vary in proportion with equity volatility (Beeler and Campbell 2012). In contrast to these papers, we assume homoskedastic driving processes for macroeconomic variables and generate time-varying risk premia endogenously from CC's highly nonlinear preference specification. Our approach is complementary to Bekaert and Engstrom (2017), who model time-variation in the higher moments of consumption growth within the CC framework, while we model time-variation in the conditional mean of consumption growth.

Second, we add to the literature on the term structure of interest rates and macroeconomic factors by modeling risk premia from consumption-based habit formation preferences. Within this literature, our paper is most closely related to those that price both bonds and stocks (for example Bekaert, Engstrom, and Grenadier 2010 and Lettau and Wachter 2011), and more specifically to papers that investigate changes in bond-stock comovements over time (Baele, Bekaert, and Inghelbrecht 2010, Campbell, Shiller, and Viceira 2009, Campbell, Sunderam, and Viceira 2017, Gourio and Ngo 2018, Song 2017, Viceira 2012). In contrast to us, these papers do not use consumption-based habit formation preferences, relying either on an exogenous reduced-form stochastic discount factor or recursive preferences combined with stochastic volatility. Our model is also complementary to David and Veronesi (2013), who study bonds and stocks in an endowment economy with learning.

The organization of the paper is as follows. Section 2 describes our model, with the consumption Euler equation, the assumed relation between consumption and the output gap, and our new preference specification (section 2.1), inflation and interest rate dynamics (section 2.2), and the specification for equity dividends (section 2.3). Section 3 explains how we solve the model. Section 3.1 discusses the solution for macroeconomic dynamics, including our procedure for selecting an equilibrium when multiple equilibria exist. Section 3.2 provides intuition for time-variation of risk premia and explains our numerical solution method for asset prices. Section 4 on econometric methodology describes the data (section 4.1), break date tests (section 4.2), calibration of time-invariant parameters (section 4.3), and estimation of subsample-dependent parameters (section 4.4). Section 5 presents our empirical results. Section 5.1 discusses parameter estimates and section 5.2 the implied macroeconomic dynamics. Section 5.3 presents implications for asset prices, and section 5.4 a decomposition into news about cash flows, real interest rates, and risk premia. Section 6 concludes, and highlights the potential of our framework for future research.

2 Model

2.1 Euler equation and preferences

Macroeconomic dynamics in our model satisfy a loglinear Euler equation typical of New Keynesian models, where the log output gap is linked to its own lead and lag and the log real risk-free interest rate (see Woodford, 2003, Chapters 4 and 5):

$$x_t = f^x \mathrm{E}_t x_{t+1} + \rho^x x_{t-1} - \psi r_t.$$

The New Keynesian literature defines the log output gap x_t as log real output minus log potential real output, where potential output is the hypothetical equilibrium without price- and wage-setting frictions (Woodford, 2003, p.245). r_t denotes the log real risk-free interest rate that can be earned from time t to time $t+1$. The coefficients f^x, ρ^x, and ψ are positive parameters. Intuitively, a high real interest rate means that consumers have a strong incentive to save, thereby depressing contemporaneous consumption and output. We model the output gap, inflation, and short-term interest rates relative to a steady state, so the loglinear Euler equation is specified up to a constant.

Our preferences are such that the loglinear Euler equation is indeed the first-order condition for the real risk-free rate and this is what distinguishes our preferences from CC and other habit utility functions popular in asset pricing.[3] Our modeling choices ensure that the loglinear Euler equation is exact with no approximation error. Because loglinear Euler equations are pervasive in structural macroeconomic models, this makes our preferences a natural stepping stone to study consumption-based asset prices in researchers' and policy makers' preferred models of the macroeconomy. Next, we describe what is required to make consumption-based habit formation preferences consistent with a loglinear Euler equation.

[3]CC is not consistent with a loglinear Euler equation, because it implies a constant real rate. In Menzly, Santos, and Veronesi (2004) and Wachter (2006) the real risk-free rate is a nonlinear function of current and past consumption shocks, and in Bekaert and Engstrom (2017) the real rate depends in addition on an unobserved state variable governing higher moments of consumption growth.

2.1.1 Consumption and the output gap

The loglinear Euler equation generates endogenous dynamics for the output gap, but not directly for consumption. We therefore need a link between the output gap and consumption, so we can solve for consumption dynamics from the output gap. We make the simple assumption that the log real output gap, x_t, equals stochastically detrended log real consumption, $\hat{c}_t$:

$$x_t = \hat{c}_t \equiv c_t - (1-\phi)\sum_{i=0}^{\infty}\phi^i c_{t-1-i}, \tag{1}$$

where ϕ is a smoothing parameter. Here, we again ignore constants, because x_t is specified relative to steady-state.

To see that the consumption-output gap relation is consistent with the New Keynesian macroeconomics literature, one could augment the model with two stylized assumptions: a) consumption equals output, and b) log potential output is a stochastic trend of log output, consistent with how potential output is measured empirically (Staiger, Stock, and Watson 1997, Shackleton 2018). It then follows that the output gap is detrended output, which also equals detrended consumption, i.e. equation (1). Since the interpretation of the output gap is not important for us, it can simply be regarded as stochastically detrended output throughout the paper.

In the appendix, we show empirical evidence that (1) is also a strikingly close description of the data when the detrending parameter ϕ is set equal to 0.93 per quarter. We impose this value when we calibrate our model.

2.1.2 Habit preferences

As in CC, utility is a power function of the difference between the level of consumption C and external habit H:

$$U_t = \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma} = \frac{(S_t C_t)^{1-\gamma} - 1}{1-\gamma}. \tag{2}$$

Here

$$S_t = \frac{C_t - H_t}{C_t} \tag{3}$$

is the surplus consumption ratio, the fraction of consumption that is available to generate utility, and γ is a curvature parameter that controls risk aversion. Relative risk aversion varies over time as an inverse function of the surplus consumption ratio: $-U_{CC}C/U_C = \gamma/S_t$. Marginal utility in this model is

$$U'_t = (C_t - H_t)^{-\gamma} = (S_t C_t)^{-\gamma}. \tag{4}$$

Standard no-arbitrage conditions in asset pricing imply that the gross one-period real return $(1 + R_{t+1})$ on any asset satisfies

$$1 = \mathrm{E}_t \left[M_{t+1} \left(1 + R_{t+1}\right)\right], \tag{5}$$

where the stochastic discount factor is related to the log surplus consumption ratio s_{t+1} and log consumption c_{t+1} by

$$M_{t+1} = \frac{\beta U'_{t+1}}{U'_t} = \beta \exp\left(-\gamma(\Delta s_{t+1} + \Delta c_{t+1})\right). \tag{6}$$

2.1.3 Surplus consumption dynamics

The dynamics of the log surplus consumption ratio are given by:

$$s_{t+1} = (1-\theta_0)\bar{s} + \theta_0 s_t + \theta_1 x_t + \theta_2 x_{t-1} + \lambda(s_t)\varepsilon_{c,t+1}, \tag{7}$$

$$\varepsilon_{c,t+1} = c_{t+1} - \mathrm{E}_t c_{t+1} = x_{t+1} - \mathrm{E}_t x_{t+1}, \tag{8}$$

where $\bar{s}$ is the steady-state log surplus consumption ratio and $\varepsilon_{c,t+1}$ is a conditionally homoskedastic shock to consumption (equivalently, to the output gap) with standard deviation σ_c.

Equation (7) takes the same form as in CC, but with two new terms $\theta_1 x_t$ and $\theta_2 x_{t-1}$. If $\theta_1 = \theta_2 = 0$ our model is identical to CC. Equation (7) implies that log habit is approximately a distributed lag of consumption. Positive θ_1 and θ_2 imply that habit loads less strongly on the two most recent lags of consumption but more strongly on medium-term lags compared to CC. We derive the approximate relation between habit and lagged consumption near the steady state in the appendix.

The sensitivity function $\lambda(s_t)$ is identical to that in CC:

$$\lambda(s_t) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & s_t \leq s_{max} \\ 0 & s_t \geq s_{max} \end{cases}, \tag{9}$$

$$\bar{S} = \sigma_c \sqrt{\frac{\gamma}{1-\theta_0}}, \tag{10}$$

$$\bar{s} = \log(\bar{S}), \tag{11}$$

$$s_{max} = \bar{s} + 0.5(1-\bar{S}^2). \tag{12}$$

Just as in CC, the downward-sloping relation between $\lambda(s_t)$ and s_t has the intuitive implication that marginal consumption utility is particularly sensitive to consumption innovations when investors are too close to their habit consumption level, as would be the case following an adverse shock.[4]

2.1.4 Deriving the Euler equation from preferences

Substituting log surplus consumption dynamics (7) into the stochastic discount factor (6) and the no-arbitrage condition (5) for the one-period real risk-free bond gives (up to a constant):

$$r_t = \gamma E_t \Delta c_{t+1} + \gamma E_t \Delta s_{t+1} - \frac{\gamma}{2} \left(1 + \lambda(s_t)\right)^2 \sigma_c^2.$$

Our modeling choices simplify the first-order condition for the real risk-free rate. First, the surplus consumption dynamics imply that we can substitute out for $E_t \Delta s_{t+1}$:

$$r_t = \gamma E_t \Delta c_{t+1} + \gamma \left(\theta_0 - 1\right) s_t + \gamma \theta_1 x_t + \gamma \theta_2 x_{t-1} - \frac{\gamma}{2} \left(1 + \lambda(s_t)\right)^2 \sigma_c^2.$$

Second, the consumption-output gap relation (1) implies that we can write expected consumption growth in terms of the current and expected output gap: $E_t \Delta c_{t+1} = E_t x_{t+1} - \phi x_t$. The real rate first-order condition then becomes

$$r_t = \gamma E_t x_{t+1} - \gamma \phi x_t + \gamma \left(\theta_0 - 1\right) s_t + \gamma \theta_1 x_t + \gamma \theta_2 x_{t-1} - \frac{\gamma}{2} \left(1 + \lambda(s_t)\right)^2 \sigma_c^2.$$

Third, the sensitivity function has just the right form so that s_t drops out. Substituting in

[4] If θ_1 and θ_2 are different from zero, there is the theoretical possibility that the log surplus consumption ratio exceeds the maximal value s_{max}. However, the probability of this event is small in our estimated model (less than 1% per quarter). In this respect our model is similar to CC, who also have an upper bound on surplus consumption that is crossed with very low probability in discrete time (and can never be crossed in continuous time).

the sensitivity function (7) through (12) and re-arranging, continuing to ignore constants, gives the loglinear Euler equation:

$$x_t = \underbrace{\frac{1}{\phi - \theta_1}}_{f^x} \mathrm{E}_t x_{t+1} + \underbrace{\frac{\theta_2}{\phi - \theta_1}}_{\rho^x} x_{t-1} - \underbrace{\frac{1}{\gamma(\phi - \theta_1)}}_{\psi} r_t. \tag{13}$$

The expressions for the coefficients in equation (13) illustrate the role of the new parameters θ_1 and θ_2 that we have introduced into the CC model. A positive value of θ_2 is needed to obtain a positive coefficient on the lagged output gap that previous papers have found important to capture the hump-shaped response of output to shocks (Fuhrer 2000, Christiano, Eichenbaum, and Evans 2005, Smets and Wouters 2007). A positive value of θ_1 increases all the coefficients in (13).

As in CC, we specify the unique sensitivity function such that s_t cancels out of (13) and habit is pre-determined at the steady-state. We need these highly non-linear preferences to obtain a simple first-order condition for the real risk-free rate and linear macroeconomic dynamics. A simpler sensitivity function would complicate the first-order condition for the real risk-free rate (13) by introducing s_t as an additional state variable. For instance, if we were to assume that λ is linear in s_t, there would be s_t^2 terms in (13).

The derivation of the loglinear Euler equation (13) shows that it holds exactly for our preferences. Further, (13) follows from our preferences and the consumption-output gap relation irrespective of the specific microfoundations of a macroeconomic model, as long as consumption is conditionally homoskedastic.

Three features differentiate (13) from macro Euler equations typically used in New Keynesian models. First, the coefficients on the lagged output gap and the expected future output gap do not generally sum to one. With positive θ_1 and θ_2 as we assume, the sum of the coefficients is greater than one. Second, because of the distinction between consumption and the output gap, the slope of the Euler equation ψ does not equal the representative consumer's elasticity of intertemporal substitution (EIS), as can be seen from the fact that ψ depends on the parameter ϕ linking consumption and output gap dynamics. Third, the loglinear Euler equation holds without shocks.

2.2 Inflation and interest rate dynamics

We now turn to the description of macroeconomic dynamics. We introduce two dynamic equations for inflation and the Federal Funds rate, the minimum state variables needed to price nominal and real bonds. We assume that log inflation and the log Federal Funds rate share a common stochastic trend (unit root), consistent with the extremely high persistence in US inflation data (Ball and Cecchetti 1990, Stock and Watson 2007) and stationarity of the real interest rate. In addition we assume that log inflation and the log Federal Funds rate are conditionally homoskedastic. Because our assumed consumer preferences are consistent with the loglinear Euler equation, this ensures that both the output gap and consumption are also conditionally homoskedastic—as in fact required by our derivation of the Euler equation.

To make the dynamics of inflation and interest rates tractable, we approximate the log one-period nominal interest rate as the log real rate plus expected log inflation:

$$i_t = r_t + E_t \pi_{t+1}.$$

This approximation improves tractability for the macroeconomic dynamics, because it avoids introducing a small heteroskedastic term. The approximation is standard in New Keynesian models. For our preferences and at our point estimates, the approximation error has a negligible standard deviation of 2 basis points.

Because we are interested in bond risk premia, it is important that we do not make this approximation for longer-term bonds. In our model risk premia are time-varying and the expectations hypothesis of the term structure of interest rates does not hold. We solve numerically for the risk premia on longer-term bonds.

We write the unit-root component of inflation as π_{*t} and define inflation and interest-rate gaps as deviations from π_{*t}:

$$\hat{\pi}_t = \pi_t - \pi_{*t}, \ \hat{\imath}_t = i_t - \pi_{*t}. \tag{14}$$

We choose a unit root specification rather than a highly persistent mean-reverting inflation component, because this way we can write the macroeconomic dynamics in terms of $\hat{\imath}_t$ and $\hat{\pi}_t$, thereby reducing the dimensionality of the state space. We normalize x_t, $\hat{\pi}_t$, and $\hat{\imath}_t$ to have zero averages.

Macroeconomic dynamics are then described by (13) and the following equations:

$$\hat{\pi}_t = p_{\pi x} x_{t-1} + p_{\pi\pi} \hat{\pi}_{t-1} + p_{\pi i} \hat{\imath}_{t-1} + v_{\pi,t}, \tag{15}$$

$$\hat{\imath}_t = p_{ix} x_{t-1} + p_{i\pi} \hat{\pi}_{t-1} + p_{ix} \hat{\imath}_{t-1} + v_{i,t}, \tag{16}$$

$$\pi_{*t} = \pi_{*t-1} + v_{*t}. \tag{17}$$

It is important to note that (15) and (16) represent equilibrium dynamics and not a structural model. The standard deviations of shocks are denoted σ_π, σ_i, and σ_* and their cross-correlations are given by $\rho_{\pi i}$, $\rho_{\pi *}$, and ρ_{i*}.

2.3 Dividends

We model stocks as a levered claim on consumption, as is common in the asset pricing literature (Abel 1990, Campbell 1986, 2003), while being careful to preserve the cointegration of log consumption and log dividends.

Let P_t^c denote the price of a claim to the consumption stream C_{t+1}, C_{t+2},... We assume that stocks are a claim to all future equity cash flows of a levered firm that invests in the consumption stream. At time t the firm buys P_t^c and sells equity to its investors worth δP_t^c, so its equity financing share is δ which we assume to be constant over time. The remainder of the firm's position is financed by one-period risk-free debt worth $(1-\delta)P_t^c$. We make the simplifying assumption that equity holders give up limited liability.

At time $t+1$, the firm receives a cash flow $C_{t+1} + P_{t+1}^c$, pays $(1-\delta)\, P_t^c \exp(r_t)$ to bond holders, and raises new financing from equity holders δP_{t+1}^c, which we model as a negative dividend. The period $t+1$ gross dividend to equity holders then equals the firm cash flow, minus payments to bond holders and new equity financing:

$$D_{t+1}^{\delta} = P_{t+1}^c + C_{t+1} - (1-\delta)P_t^c \exp(r_t) - \delta P_{t+1}^c. \tag{18}$$

If P_t^c/C_t and r_t are stationary, (18) implies that D_t^{δ}/C_t is stationary, so log dividends and log consumption are cointegrated.

The price of the claim to all future cash flows (18) is $P_t^{\delta} = \delta P_t^c$. From (18) the gross stock return $(1+R_{t+1}^{\delta})$ equals the gross consumption claim return $(1+R_{t+1}^c)$ times leverage, less a term reflecting the firm's debt payments:

$$(1+R_{t+1}^{\delta}) = \frac{D_{t+1}^{\delta} + P_{t+1}^{\delta}}{P_t^{\delta}} = \frac{D_{t+1}^{\delta} + \delta P_{t+1}^c}{\delta P_t^c} = \frac{1}{\delta}(1+R_{t+1}^c) - \frac{1-\delta}{\delta}\exp(r_t). \tag{19}$$

Since the levered firm is a pure intermediary and does not add value, the expression for the stock return is independent of whether or not equity investors are required to reinvest.

3 Model solution

This section describes the solution methods for macroeconomic dynamics and asset prices. Further details are available in the appendix.

3.1 Macroeconomic dynamics and equilibrium selection

In order to illustrate the properties of our preferences, we need to choose a macroeconomic equilibrium. Although we do not have a full structural New Keynesian model, the forward- and backward-looking Euler equation (13) means that we face a version of the well-known issue of equilibrium multiplicity in New Keynesian models (Cochrane 2011). Because this multiplicity is still an unresolved question, we follow a simple transparent equilibrium selection procedure, which we describe next.

We solve for equilibrium dynamics of the form:

$$\hat{Y}_t = P\hat{Y}_{t-1} + \Sigma v_t, \tag{20}$$

where

$$\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{\imath}_t]'. \tag{21}$$

We solve for $P \in \mathbb{R}^{3\times 3}$ and $Q \in \mathbb{R}^{3\times 4}$ using the method of generalized eigenvectors (Uhlig 1999). We do not pursue solutions with more complicated dynamics, such as dependence on higher-order lags of $\hat{Y}_t$.

In our estimation, we only consider parameter values that have an equilibrium of the form (20) and where the equilibrium satisfies a set of reasonable requirements. Parameter values that do not have an equilibrium of the form (20) are excluded from the estimation. We require eigenvalues of P to be less than one in absolute value and to occur in complex conjugate pairs, so equilibrium dynamics are non-explosive and real-valued. We also require that all real eigenvalues of P are greater than -0.2 and that equilibrium impulse responses do not switch sign within the first four quarters of a shock, thereby ruling out strongly oscillating impulse responses.

If only one equilibrium satisfies these requirements at a set of parameter values, we pick that equilibrium. If there are more than one, we select the equilibrium that minimizes the weighted sum of squared differences between model and data impulse responses, where we use the same impulse response moments and weights as in the SMM estimation. At our point estimates there exist multiple real-valued non-explosive equilibria, so we do employ this last empirical selection criterion.

3.2 Solving for asset prices

Asset prices are highly nonlinear and we need to solve for them numerically. We follow the best practices of Wachter (2005) for solving asset prices with highly nonlinear habit formation preferences and evaluate expectations iteratively along a grid. For details see the appendix.

To illustrate the sources of nonlinear risk premia, we start by showing analytically that the risk premia for near-term claims to consumption and nominal dollars are nonlinear. Consider a one-period zero-coupon consumption claim that pays aggregate consumption in period $t+1$ and pays nothing in any other period. We denote its log return by $r^c_{1,t+1}$.

Since consumption shocks are conditionally perfectly correlated with the output gap, the risk premium, adjusted for a standard Jensen's inequality term, equals the conditional covariance between the negative log SDF and and the output gap:

$$\begin{aligned} E_1\left[r^c_{1,t+1} - r_t\right] + \frac{1}{2}Var\left(r^c_{1,t+1}\right) &= \operatorname{Cov}_t\left(-m_{t+1}, x_{t+1}\right), \\ &= \gamma\left(1+\lambda\left(s_t\right)\right)\sigma_x^2. \end{aligned} \tag{22}$$

The time $t+1$ real payoff on a two-period nominal bond equals $\exp\left(-i_{t+1} - \pi_{t+1}\right)$ and is lognormal, so we can solve for the two-period nominal bond risk premium analytically. Denoting the log return on the two-period bond from time t to $t+1$ by $r^{\$}_{2,t+1}$, the risk premium (again including a Jensen's inequality term) equals:

$$\begin{aligned} E_1\left[r^{\$}_{2,t+1} - r_t\right] + \frac{1}{2}Var_t\left(r^{\$}_{2,t+1}\right) &= \operatorname{Cov}_t\left(-m_{t+1}, -i_{t+1} - \pi_{t+1}\right), \\ &= \gamma\left(1+\lambda\left(s_t\right)\right)\operatorname{Cov}_t\left(x_{t+1}, -i_{t+1} - \pi_{t+1}\right). \end{aligned} \tag{23}$$

Expressions (22) and (23) show that consumption claim and bond risk premia are proportional to the sensitivity function $\lambda(s_t)$, and hence nonlinear in log surplus consumption.

We use the following recursion to solve for the price-consumption ratio of an n-period zero-coupon consumption claim:

$$\frac{P^c_{nt}}{C_t} = \mathrm{E}_t\left[M_{t+1}\frac{C_{t+1}}{C_t}\frac{P^c_{n-1,t+1}}{C_{t+1}}\right]. \tag{24}$$

The price-consumption ratio for a claim to aggregate consumption is then the infinite sum of zero-coupon consumption claims:

$$\frac{P_t^c}{C_t} = \sum_{n=1}^{\infty} \frac{P_{nt}^c}{C_t}. \tag{25}$$

The price of a levered stock equals $P_t^\delta = \delta P_t^c$, where δ is the firm share financed by equity.

We initialize the bond price recursion by noting that the price of a one-period nominal bond equals:

$$P_{1,t}^{\$} = \exp(-\hat{\imath}_t - \pi_{*t} - \bar{r}), \tag{26}$$

where $\bar{r}$ denotes the steady-state log risk-free rate. The n-period zero coupon nominal bond price follows the recursion:

$$P_{n,t}^{\$} = \mathrm{E}_t \left[M_{t+1} \exp(-\pi_{t+1}) P_{n-1,t+1}^{\$} \right]. \tag{27}$$

To see why we need a flexible numerical solution method for asset prices, consider the consumption-claim recursion (24) for $n = 2$ as an example. The price of the one-period zero-coupon consumption claim $P_{1,t+1}^c$ moves inversely with the required risk premium (22), making $P_{1,t+1}^c/C_{t+1}$ a nonlinear function of log surplus consumption. When evaluating the expectation (24), the crucial covariance between the SDF M_{t+1} and $P_{1,t+1}^c/C_{t+1}$ hence changes with s_t, such that there is no analytic solution and asset prices cannot easily be approximated by standard functions. Iterating along a grid, as opposed to local approximation or global solution methods, is the best practice for this type of numerical problem. The reason is that, when combined with a large grid for surplus consumption, iterating along a grid imposes the least structure on the relation between asset prices and surplus consumption.

Numerical solutions for bond and stock risk premia inherit important properties from analytical near-term risk premia. Similarly to (22), equity risk premia are positive on average. They increase when surplus consumption is low, because marginal utility is very sensitive to consumption shocks in those states of the world. By contrast, nominal bond risk premia can increase or decrease when surplus consumption is low, depending on how bond cash flows covary with consumption. If two-period nominal bond cash flows covary negatively with the output gap (that is, if $\text{Cov}_t\left(x_{t+1}, -i_{t+1} - \pi_{t+1}\right) < 0$ in equation (23)), two-period nominal bond risk premia are negative and they decrease (become even more negative) when surplus consumption is low. Intuitively, investors are particularly risk averse in states of low surplus consumption, and are therefore particularly willing to hold nominal bonds with low expected returns. This flight-to-safety effect works similarly for longer-term bonds, so when bonds' cash flows have hedging value to consumers, the model implies that bond and stock risk premia are negatively correlated.

4 Econometric Methodology

4.1 Data and summary statistics

We use quarterly US data on output, inflation, interest rates, and aggregate bond and stock returns from 1979Q3 to 2011Q4. Our sample period starts with Paul Volcker's appointment as Fed chairman, because of evidence that monetary and macroeconomic dynamics changed at that time (e.g. Clarida, Gali, and Gertler 1999). The goal of our empirical analysis is to illustrate the properties of our model without the additional complications of the "zero lower bound" of close-to-zero short-term nominal interest rates, so we end the sample in 2011Q4.

Real GDP, real consumption for nondurables and services, real potential GDP, and the

GDP deflator in 2009 chained dollars are from the FRED database at the St. Louis Federal Reserve.[5] There is an ambiguity with respect to the timing of output and asset prices. Output, just like consumption, is a flow over a quarter, while asset prices are measured at a point in time. Time-averaged output observed in a quarter could therefore reasonably be treated as occurring at the beginning or the end of the quarter. In the consumption-based asset pricing literature, it has been found preferable to treat time-averaged output and consumption as occurring at the beginning of the quarter, because correlations with stock returns tend to be higher (Campbell 2003). We follow this beginning-of-quarter timing convention throughout the paper and align consumption and output reported by FRED for quarter t with asset prices measured at the end of quarter $t-1$.

We use the end-of-quarter Federal Funds rate from the Federal Reserve's H.15 publication. The end-of-quarter five year bond yield is from the CRSP monthly Treasury Fama-Bliss discount bond yields. We use the value-weighted combined NYSE/AMEX/Nasdaq stock return including dividends from CRSP, and measure the dividend-price ratio using data for real dividends and the S&P 500 real price. Interest rates, and inflation are in annualized percent, while the log output gap is in natural percent units. All yields and returns are continuously compounded. We consider log returns in excess of the log T-bill rate, where the end-of-quarter three-month T-bill is from the CRSP monthly Treasury Fama risk-free rate files and is based on the average of bid and ask quotes.

4.2 Break date tests

We start by dividing our sample according to changes in inflation dynamics. We run a Quandt Likelihood Ratio (QLR) test for an unknown break date in the relation between inflation

[5] Accessed 08/05/2017.

and the output gap on our full sample running from 1979Q3 until 2011Q4. For every quarter τ we estimate a full-sample regression of quarterly log inflation onto a constant, a dummy that takes the value of one if $t \geq \tau$ and zero otherwise, the log output gap, and the log output gap interacted with the dummy:

$$\pi_t = a_\tau + b_\tau I_{t \geq \tau} + c_\tau x_t + d_\tau I_{t \geq \tau} x_t + \varepsilon_t. \tag{28}$$

For each potential break quarter τ, we compute the F-statistic corresponding to d_τ with Newey-West standard errors and one lag. The QLR test statistic is the maximum F-statistic, and the break is statistically significant at the 95% level if the QLR statistic exceeds 8.68, which is the critical value for one constraint and 15% trimming by Andrews (2003). The estimated break date is the quarter τ with the highest F-statistic.

Figure 2 plots the F-statistic against the quarter τ, showing a single-peaked distribution with a statistically significant maximum in 2001Q2. This break date test, which is based only on inflation and output data and does not use asset prices, therefore provides clear evidence for a change in inflation dynamics in the early 2000s.

Empirical inflation-output gap betas suggest that inflation and output dynamics contributed to changing bond risks around the break date. The slope coefficient of inflation onto the log real output gap changed sign in the opposite direction as bond-stock betas, which are reported in Figure 1 and Table 5. Before the break we estimate a significantly negative inflation-output gap beta of -0.32 (0.11) and the correlation equals -0.28 (Newey-West standard error with one lag in parentheses). After the break we estimate a significantly positive inflation-output gap beta of 0.33 (0.07) and the corresponding correlation is 0.65. Hence, while prior to the break the US economy was in a stagflationary regime, where inflation increased during periods of low output, after the break inflation has tended to increase during expansions.

A formal test for a break in the relation between daily bond and stock excess returns confirms that the break date from macroeconomic data lines up closely with changes in bond risks. We run a QLR test for an unknown break date in the relation between bond and stock excess returns.[6] For every date τ within the middle 70% of the sample, we estimate a regression using daily log bond and stock excess returns:

$$r_{t+1}^{\$} = a_\tau + b_\tau I_{t\geq\tau} + c_\tau r_t^{stock} + d_\tau I_{t\geq\tau} x r_t^{stock} + \varepsilon_t. \tag{29}$$

This QLR test based on asset returns indicates a break date on December 6, 2000. This break date is statistically significant at the 95% level and remarkably close to the break date estimated with inflation and output data. The regression beta of daily 5-year bond returns onto stock returns switches sign around the estimated break date and is highly significant in each subperiod, consistent with the sign-switch in rolling bond betas in Figure 1.

4.3 Calibrated parameters

Time-invariant calibrated parameter values are summarized in Panel A of Table 1. Our selection of parameter blocks is consistent with Smets and Wouters (2007), who find important changes in shock volatilities and parameters driving inflation and Federal Funds rate dynamics, but stable preference parameters. The block of time-invariant parameters includes those governing the relation between the output gap, consumption, and dividends (ϕ, g, δ), and parameters determining investor and consumer preferences (γ, θ_0, θ_1, θ_2, $\bar{r}$).

[6] We use daily log returns on 5-year nominal Treasury bonds and daily log CRSP value-weighted stock market returns, as in Figure 1. To reduce the effect of outliers, we winsorize bond and stock returns at the 0.5% and 99.5% levels.

We set the average consumption growth rate, g, utility curvature γ, surplus consumption persistence, θ_0 (reported after compounding to an annual frequency) and the average real risk-free rate $\bar{r}$ exactly as in CC. We set our new habit-formation parameters, θ_1 and θ_2, equal for simplicity. As discussed earlier, we need θ_2 to be positive to obtain a backward-looking term in the Euler equation (13). We choose a small value for $\theta_1 = \theta_2 = 0.02$ to limit the magnitude of the deviations from the well-understood CC preferences.

The parameter ϕ determines the link between the output gap and consumption. We choose $\phi = 0.93$, the value that maximizes the empirical correlation between stochastically detrended consumption and the output gap over our full sample.

The leverage parameter δ scales up the volatility of equity returns, while preserving their Sharpe ratio. We choose a leverage ratio of $\delta = 0.50$ to obtain empirically plausible equity return volatilities. We interpret δ as capturing a broad concept of leverage, including operational leverage.

The calibrated preference parameters imply an annualized discount factor of $\beta = 0.90$ and an Euler equation with a large forward-looking and a small backward-looking component ($\rho^x = 0.02$, $f^x = 1.10$). The real rate slope in the Euler equation is $\psi = 0.14$, within the range of empirical estimates by Yogo (2004) and earlier work by Hall (1988).

4.4 Parameter estimation in subperiods

The remaining parameters are estimated to minimize the distance between model and empirical moments describing macroeconomic dynamics. For each subperiod, we separately estimate the twelve-dimensional parameter vector using simulated method of moments (SMM):

$$params = [p_{\pi i}, p_{\pi\pi}, p_{\pi i}, p_{ix}, p_{i\pi}, p_{ii}, \sigma_\pi, \sigma_i, \sigma_*, \rho_{\pi i}, \rho_{\pi *}, \rho_{i*}] \,. \tag{30}$$

We target VAR(1) impulse responses for the output gap, inflation, and the Federal Funds rate, as well as the correlation between the 20-quarter average Federal Funds rate and the output gap. We do not use bond or stock returns for the estimation. We follow a simple methodology for estimating impulse responses that is identical in actual and model-simulated data and that generates a unique mapping between model and empirical moments. For comparability with the literature, we follow Christiano, Eichenbaum, and Evans (CEE 2005) and estimate the VAR in levels and with a end-of-quarter timing convention for the output gap. In our notation, this means that we estimate the VAR(1) in $[x_{t-1}, \pi_t, i_t]$. This state vector differs from that in equation (20), which includes stationary deviations $\hat{\pi}_t$ and $\hat{\imath}_t$ rather than levels π_t and i_t. Simulated model and empirical samples have the same length to ensure that empirical and model impulse responses reflect the same small sample effects. Because samples are relatively short, impulse responses cannot detect the unit root in inflation, so model and empirical impulse responses converge back to zero.

To obtain a unique mapping between model and empirical impulse responses, we orthogonalize the VAR innovations such that the innovation to the Fed Funds rate does not enter into contemporaneous inflation or the output gap, and the innovation to inflation does not enter into the contemporaneous output gap. If one wanted to assume that output and inflation react to monetary policy with a one quarter lag, the orthogonalized Fed Funds rate innovation would correspond to a structural monetary policy shock (Sims 1986, CEE). However, our SMM estimation does not require this stronger structural interpretation. The appendix provides further details and explains the bootstrap procedure used to obtain confidence intervals and standard errors for empirical impulse responses.

In addition to the impulse responses, we target a long-term Fed Funds-output gap correlation to closely match the business cycle properties of the Federal Funds rate. The empirical correlation of the average expected Federal Funds rate over the next 20 quarters with respect to the output gap is reported in Table 5. Similarly to the sign switch in inflation cyclicality documented in Section 4.2, this correlation moves from -0.38 in the 1979Q3-2001Q1 period to 0.57 in the 2001Q2-2011Q4 period.

The vector $\hat{\Psi} - \Psi(params)$ consists of differences between the data and the model: differences in impulse responses at one (shock period), two, four, 12, 20, and 40 quarters, excluding those that are zero by construction, and the square root of the absolute difference in the 20-quarter Federal Funds rate-output gap correlation. The estimated parameter vector $\widehat{params}$ minimizes the objective function:

$$Obj(params) = \left[\hat{\Psi} - \Psi(params)\right]' \hat{W} \left[\hat{\Psi} - \Psi(params)\right]. \tag{31}$$

Here, $\hat{W}$ is a data-based, symmetric, positive-definite weighting matrix. To avoid matrix invertibility issues, we follow CEE and take $\hat{W}$ to be a diagonal matrix with inverse sample variances of $\hat{\Psi}$ along the diagonal. We require the model to match the Fed Funds-output gap correlation closely by setting the last element of $\hat{W}$ to 200.

Our twelve-dimensional parameter space is ill-suited for gradient-based optimization methods. We therefore minimize the objective function by grid search over the parameter space. The appendix provides details of the grid search procedure.

5 Empirical Results

5.1 Parameter estimates

Table 1, Panel B shows the estimated macroeconomic parameters for 1979Q3–2001Q1 and 2001Q2–2011Q4. The first part of the panel reports the estimated lag parameters. Three parameters, $p_{\pi x}$, p_{ix}, and $p_{i\pi}$, switch sign between periods 1 and 2, even though these changes are not statistically significant. Keeping in mind that (15) and (16) represent equilibrium dynamics and not a structural model, the increase in $p_{\pi x}$ is suggestive of an increase in a Phillips curve-type relation between slack in the economy and inflation. The increase in p_{ix} from negative to positive and the corresponding decrease in $p_{i\pi}$ are suggestive of an increase in the Fed's interest in stabilizing output in period 2, relative to the strong focus on stabilizing inflation in period 1.

Moving to the volatility parameters, the most notable change from period 1 to period 2 is the economically and statistically significant decrease in the volatility of shocks to the unit-root component of inflation from 0.56 to 0.28. This squares well with long-term inflation surveys, which have been very stable during our second subperiod.

Among the correlations, the most notable change is that the correlation ρ_{i*} between interest rate innovations and shocks to the unit-root component of inflation switches sign from positive in period 1 to negative in period 2. Finally, panel B reports the implied steady-state and maximum surplus consumption ratios, which are similar to CC in both subperiods.

5.2 Estimated macroeconomic dynamics

Table 2 shows that the model provides a good fit for nominal and real macroeconomic dynamics. We start by discussing nominal moments, because those are especially important for bond risks. The model generates declining nominal rate volatility and stable inflation volatility from period 1 to period 2, just as in the data. The persistence of inflation changes is similar in the model and the data, but the model implies that interest rate changes are close to serially uncorrelated, failing to fit the empirical negative autocorrelation in period 1 and positive autocorrelation in period 2.

The model generates low volatilities for consumption growth and the output gap, broadly consistent with the data. In both subperiods the annualized consumption growth volatility is 1.6% in the model compared to 0.9% in the data. The standard deviation of the model output gap is even closer, averaging 1.8% across subperiods compared to 2.0% in the data. The model generates a persistent output gap and close to serially uncorrelated consumption growth, whereas in the data, consumption growth appears slightly positively autocorrelated. This difference between the model and the data may arise from persistent changes to empirical growth rates, which our model does not aim to capture.

Figures 3 through 5 show that the model fits the empirical impulse responses used for the model estimation, as described in section 4.4. Model impulse responses are generally within the 95% confidence bands for empirical impulse responses, with the few exceptions being short-lived.

Figure 4 illustrates the important role of inflation innovations for the changing inflation-output gap correlation. It shows that output gap responses to inflation innovations switch from negative in period 1 to positive in period 2, while inflation and Federal Funds rate responses are positive and economically meaningful in both periods. Figures 3 and 5 show that inflation and Federal Funds rate responses to output gap innovations and Federal Funds rate innovations are smaller and mostly statistically insignificant, so these innovations appear less important for changing inflation cyclicality. We note however that the output response to Federal Funds rate innovations does switch sign across the two subperiods, and even though this response is imprecisely estimated our model replicates this sign switch.

Having seen that the model matches changing macroeconomic dynamics in the data, we next turn to understanding how changing macroeconomic dynamics contribute to bond risks.

5.3 Asset pricing implications

Table 3 shows that the model replicates the successes of CC for the stock market. The model generates a high stock market Sharpe ratio and volatile and predictable stock returns. To mimic firms' dividend smoothing in the data, we compare empirical moments for the price-dividend ratio to the price of levered equities divided by dividends smoothed over 64 quarters. The model generates a highly persistent price-dividend ratio, though it is slightly less volatile than in the data. The model generates an empirically plausible degree of stock return predictability, as can be seen from the coefficient of 1-year stock returns onto the lagged price-dividend ratio. While we undershoot equity volatility in period 2, we are not concerned about this because we generate a plausible Sharpe ratio. The model could easily accommodate higher equity volatility without changing the Sharpe ratio by adjusting equity leverage.

Table 4 shows the bond market implications of our model. Results in this table are new relative to CC. Bond returns are volatile, with standard deviations of 6.07% in period 1 and and 3.72% in period 2, even though they are somewhat smaller than in the data. Importantly, the volatility of bond returns declines from period 1 to period 2, just as in the data, even though no asset prices were used in the model estimation. The spread between the 5-year nominal log yield and the log Federal Funds rate is persistent, though somewhat less volatile than in the data. We reconcile the negative model term premium in period 2 by noting that this was a period of declining interest rates that could hardly have been predictable, driving up the realized term premium relative to ex ante expectations.

The bottom of Table 4 shows that our model does not generate the return predictability regressions of Campbell and Shiller (CS 1991) within periods, even though bond excess returns do move with the term spread across periods. The regression of subperiod average bond excess returns onto average log yield spreads generates a slope coefficient of 1.77, in line with the slope coefficients reported in CS. In a model extension with time-varying regime probabilities, this cross-regime relation between yield spreads and bond excess returns would presumably generate higher-frequency bond return predictability.

Table 5 turns to our main object of interest: the changing comovement of bond and stock returns. In the model, the bond-stock return correlation switches from positive 0.50 in period 1 to negative -0.72 in period 2 . The close fit of empirical bond-stock correlations is remarkable, since the model was estimated only on macroeconomic moments. The model stock-market beta of nominal bond returns also shows an economically significant change from 0.16 in period 1 to -0.36 in period 2.

The middle panel in Table 5 shows that changing bond-stock return comovements are underpinned by changing nominal-real comovements. In the absence of risk premia, nominal bond returns fall with inflation and stock dividends rise with the business cycle, so a change in the inflation-output gap correlation should coincide with the opposite change in the bond-stock return correlation. Consistent with this intuition, Table 5 shows that the empirical inflation-output gap correlation switched from negative to positive between periods 1 and

2, and that this change was economically significant. Output gap correlations with 5-year average inflation and the 5-year average Federal Funds rate show similar sign-switches in the data. Our estimation explicitly targets the 5-year Fed Funds-output gap correlation, so it is reassuring that the model fits the changes in this correlation and, in addition, changes in empirical inflation-output gap correlations.

The last panel of Table 5 shows that the model generates an empirically reasonable link between the output gap and risk premia. In the model, this link arises because low output means low consumption relative to habit, so investors become risk averse. Hence, the model output gap forecasts stock excess returns negatively; and Table 5 shows that the magnitude is similar to the empirical relation documented by Cooper and Priestley (2009). Turning to bonds, the output gap's ability to forecast bond excess returns is mixed, both in the model and the data. In period 1, where bonds have a positive stock beta, their risk properties are similar to stocks', so the model predicts a negative relation between the output gap and future bond excess returns. We confirm this prediction in period 1 data, where the coefficient is statistically indistinguishable from the model and significantly negative at the 90% confidence level. In period 2, the model predicts a positive coefficient, because bonds have negative stock betas, and so their risk properties are the opposite of stocks'. In the data, the period 2 forecasting coefficient increases relative to period 1 and is statistically indistinguishable from the model at conventional significance levels.

Figure 6 explores which model parameters are responsible for the change in the model bond-stock correlation between periods 1 and 2. The figure reports changes in the bond-stock correlation and the inflation-output gap correlation while changing subsets of parameters from their period 1 to their period 2 values. It shows that lag parameters generate the largest changes in both bond-stock and inflation-output gap correlations and are sufficient to generate almost the entire effect. There is a much smaller contribution from shock correlations and a modest offsetting effect from changes in shock standard deviations.

Table 6 shows model implications for real bonds. Model-implied real bond-stock return correlations changed from positive in period 1 to negative in period 2. US inflation-indexed bonds (TIPS) only became available during the second half of our sample and even then remained illiquid. With this caveat, the model real bond beta is in line with the data. In period 2, the stock market beta of empirical quarterly log TIPS excess returns was -0.08 in the data, compared to -0.27 in the model.[7] The empirical correlation between log TIPS excess returns and stock returns was large and negative at -0.33, compared to -0.89 in the model. The direction of change in the model-implied real bond-stock correlation is also empirically plausible. US inflation-indexed bonds were not available in our first period, but UK inflation-indexed bonds were. Campbell, Shiller, and Viceira (2009) show that the UK inflation-indexed bond-stock correlation was positive before 2000 and became negative thereafter, similarly to the change implied by our model.

To summarize, we have seen that the model links changing bond risks and changing macroeconomic dynamics, and that it generates empirically plausible risk premia from the business cycle. However, Table 5 does not reveal the importance of time-varying risk premia for changing bond risks. We turn to this next.

5.4 The role of risk premia

Table 7, Panel A extends the impulse response analysis of Figures 3 through 5 to model stock and bond returns, decomposed into news about cash flows, real rates, and risk premia as in

[7] We measure the quarterly log TIPS excess return as $-(n-1)y_{n,t+1} + ny_{n,t} - i_t + \pi_{t+1}$, where $y_{n,t}$ is measured as the five year continuously compounded TIPS yield. We obtain 5-year TIPS yields from Bloomberg ticker "USGGT05Y Index".

Campbell and Ammer (1993). The first row regresses simulated log equity excess returns, xr_t^δ, onto the same orthogonalized one-standard-deviation innovations used in Figures 3 through 5. We report slope coefficients averaged across 100 independent model simulations. The slope coefficients for the components of log equity excess returns and bond equity excess returns are computed analogously.

Table 7, Panel A shows that stocks—like the output gap—respond positively to output gap innovations in both periods. By contrast, the stock return responses to inflation and Federal Funds rate innovations change sign between periods 1 and 2—again like the output gap. Because stocks are a levered claim on consumption and consumption is linked to the output gap, equity cash-flow news behaves like the output gap. Equity risk-premium news amplifies cash-flow news and accounts for the majority of stock return volatility. Intuitively, higher output means higher surplus consumption, leading to higher risk tolerance and stock valuations.

The key takeaway from the second half of Panel A is the changing role of bond risk premia from period 1 to period 2. Bond cash flow responses are straightforward, being simply the mirror image of inflation responses in Figures 3 through 5. Bond risk premia respond to output gap innovations in the same direction as equity risk premia in period 1, but in the opposite direction in period 2. The intuition for this change goes back to the analytic expressions (22) and (23). When investors' risk aversion increases following an adverse output shock, a flight-to-safety effect arises, driving down bond valuations when bonds are risky as in period 1, and driving them up when bonds are safe as in period 2. Bond risk premium responses to Federal Funds rate and inflation innovations are small. Taken together, the comovement between bond and stock risk premia switches sign from period 1 to period 2 along with the comovement of cash-flow news.

Table 7, Panel B shows that risk-premium news generates a large change in the comovement of bond and stock returns from a small change in bonds' cash-flow risks. For period 1, we see that the substantially positive bond-stock return correlation reflects a small positive correlation of cash-flow news, and a much larger positive correlation of risk-premium news. The risk-premium effect is also the dominant contributor to the positive bond-stock covariance, reflecting the importance of time-varying risk premia for asset returns and especially for stock returns. By contrast, the period 2 decomposition shows a small negative cash-flow news correlation, and a much larger negative risk-premium news correlation. Again, the risk-premium effect dominates the negative bond-stock covariance. Real rate news contributes to a positive bond-stock correlation in both periods, so it is not a driver of the sign switch. This is intuitive, because both bonds and stocks are long-term assets and real rates therefore affect them similarly.

In summary, Table 7 shows that changing macroeconomic dynamics—in particular changes in the responses of the output gap to news about inflation and the Federal Funds rate—explain why bonds' cash flows were risky in period 1 but safe in period 2. But it also shows that understanding changes in cash flow risk is not sufficient, as quantitatively large changes in bond risks can only be understood from the time-varying role of bond risk premia.

6 Conclusion

We provide a new framework for understanding how macroeconomic dynamics drive stocks and bonds and apply it to changing bond-stock return comovements. Our model is the first one to combine Campbell and Cochrane (1999) habit-formation preferences with homoskedastic, loglinear macroeconomic dynamics. As such, it should serve as a useful tool to study the role of time-varying risk premia in a wide range of macroeconomic models that give rise to such dynamics either exactly or approximately. We conclude by discussing some of the possibilities for future research along these lines.

A natural extension would be to study time-varying consumption-based risk premia in a structural macroeconomic model. One could start from a standard small scale New Keynesian model and loglinearize firms' optimal price-setting condition to yield a Phillips curve (Woodford 2003). One could then add an interest rate rule for monetary policy as in Taylor (1993), and close the model with our preferences, which will generate both asset prices and the standard consumption Euler equation. Using the method of generalized eigenvectors (Uhlig 1999) one could then solve for inflation and interest rate dynamics of the form (15) through (17).

There are some difficulties that this approach will have to confront. Even simple New Keynesian models may have multiple equilibria or explosive solutions (Cochrane, 2011). In addition, there are well documented challenges in modeling households' labor-leisure choice in the presence of habit formation preferences (Lettau and Uhlig, 2000). Similarly to what we have done in this paper, judicious choices in relating the value of leisure to the habit stock may be one possible avenue to resolve this second challenge.

Loglinearized models with additional state variables, such as Smets and Wouters (2007) or models with a fiscal sector, are of even greater interest to policy makers. Our preferences generate a loglinearized Euler equation similar to that in Smets and Wouters (2007), giving reason to be optimistic that inserting our preferences into that model would preserve its desirable macroeconomic properties. For a model with real investment, researchers would need to use the structural relation between consumption, output and investment to relate the output gap to surplus consumption.

While we select our state variables inspired by the New Keynesian literature, our preferences are more widely applicable. For a real business cycle model, it would be typical to have a loglinear Euler equation in terms of consumption rather than the output gap. It is straightforward to substitute the output gap out of the loglinear Euler equation (13) using the link between consumption and the output gap, and thereby write the Euler equation entirely in terms of consumption. In fact, it is not at all necessary to define the output gap to work with our preferences. It would also be possible to regard x_t simply as a stationary function of current and lagged consumption.

It would be conceptually straightforward to add lags or state variables in the macroeconomic dynamics and then solve for asset prices, whether these dynamics result from a structural model or are reduced form. Similarly, one could replace the unit root in inflation and interest rates by a slowly mean-reverting component, at the expense of introducing another state variable. The key challenge here would not be conceptual but computational, potentially requiring the use of a lower-level programming language. Finally, one particularly interesting application of our model will be to study the role of the zero lower bound for bond risks.

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Table 1: Parameters

Panel A: Calibrated Parameters

Consumption Growth Rate	g	1.89
Utility Curvature	γ	2.00
Steady-State Riskfree Rate	$\bar{r}$	0.94
Persistence Surplus Cons.	θ_0	0.87
Dependence Output Gap	θ_1	0.02
Dependence Lagged Output Gap	θ_2	0.02
Smoothing Parameter Consumption	ϕ	0.93
Leverage	δ	0.50
Implied Parameters		
Discount Rate	β	0.90
Euler Eqn. Lag Coefficient	ρ^x	0.02
Euler Eqn. Forward Coefficient	f^x	1.10
Euler Eqn. Real Rate Slope	ψ	0.14

Table 1: Parameters (continued)

Panel B: Estimated Parameters

Lag Parameters		79Q3-01Q1	01Q2-11Q4
Inflation-Output Gap	$p_{\pi x}$	-0.44	0.44
		(0.71)	(1.28)
Inflation-Inflation	$p_{\pi\pi}$	-0.78	-0.33
		(0.59)	(0.72)
Inflation-Fed Funds	$p_{\pi i}$	-0.78	-0.56
		(0.22)	(0.33)
Fed Funds-Output Gap	p_{ix}	-0.44	0.44
		(0.82)	(0.75)
Fed Funds-Inflation	$p_{i\pi}$	0.33	-0.11
		(0.58)	(0.46)
Fed Funds-Fed Funds	p_{ii}	0.33	0.11
		(0.18)	(0.35)
Std. Shocks (%)			
Std. Infl.	σ_π	0.28	0.44
		(0.10)	(0.15)
Std. Fed Funds	σ_i	0.44	0.44
		(0.21)	(0.09)
Std. Infl. Unit Root**	σ_*	0.56	0.28
		(0.07)	(0.05)
Shock Correlations			
Inflation-Fed Funds	$\rho_{\pi i}$	-0.11	-0.33
		(0.46)	(0.55)
Inflation-Infl. Unit Root	$\rho_{\pi *}$	0.11	0.55
		(0.31)	(0.70)
Fed Funds-Infl. Unit Root*	ρ_{i*}	0.11	-0.55
		(0.25)	(0.30)
Implied Parameters			
Steady-State Surplus Cons. Ratio	$\bar{S}$	0.06	0.06
Max. Surplus Cons. Ratio	s_{max}	0.10	0.10

Panel A shows calibrated parameters, that are held constant across subperiods. Consumption growth and the steady-state risk-free rate are in annualized percent. The discount rate and the persistence of surplus consumption are annualized. The estimated macroeconomic parameters in Panel B are reported in units corresponding to our empirical variables, i.e. the output gap is in percent, and inflation, the Fed Funds rate and the unit root component of inflation are in annualized percent. We report quarterly standard deviations of shocks to annualized percent inflation, Fed Funds rate, and inflation target. We use superscripts *,**, and *** to denote that for a parameter we can reject that it is constant across subperiods at the 10%, 5%, and 1% levels, accounting for estimation uncertainty in both periods.

Table 2: Macroeconomic Dynamics

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Nominal Short Rate Changes				
Volatility	1.94	0.82	0.53	0.46
AR(1) Coefficient	-0.36	0.04	0.60	-0.01
Inflation Changes				
Volatility (%)	0.83	0.71	1.03	0.73
AR(1) Coefficient	-0.27	-0.16	-0.40	-0.34
Consumption growth				
Volatility (%)	0.90	1.59	0.90	1.59
AR(1) Coefficient	0.21	-0.03	0.60	0.01
Output gap				
Volatility (%)	1.93	1.30	2.04	2.34
AR(1) Coefficient	0.92	0.80	0.94	0.94

The empirical nominal rate change equals the one-quarter change in the log end-of-quarter Federal Funds rate from the Federal Reserve's H.15 publication. The log Federal Funds rate is averaged over the last five business days of each quarter and expressed in annualized percent. The inflation change equals the one-quarter change in log quarterly inflation. Log quarterly inflation is the log change in the seasonally adjusted GDP deflator in annualized percent. Log real quarterly consumption growth and the log real output gap are in percent. The standard deviation of consumption growth is annualized. We use real consumption expenditures data for nondurables and services from the Bureau of Economic Analysis National Income and Product Accounts Tables. The output gap is log real seasonally adjusted GDP minus log potential real GDP from the Congressional Budget Office. Consumption, the GDP deflator, and real output variables are in chained 2009 dollars and obtained via the St. Louis FRED. Model moments are averaged over 2 simulations of length 10000.

Table 3: Stocks

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Excess Returns				
Equity Premium	7.97	9.33	4.03	4.27
Volatility	16.42	18.65	20.00	7.36
Sharpe Ratio	0.49	0.50	0.20	0.58
Log Price-Dividend Ratio				
Mean $(exp(mean(pd)))$	34.04	17.19	53.73	46.72
Volatility	0.46	0.11	0.20	0.07
AR(1) Coefficient	1.00	0.95	0.86	0.99
Predictability				
1-YR Excess Return on pd	-0.01	-0.37	-0.43	-0.22
R^2	0.00	0.05	0.22	0.05

Log stock excess returns are quarterly log returns for the value-weighted combined NYSE/AMEX/Nasdaq stock return including dividends from CRSP in excess of the log 3-month T-bill from the CRSP Monthly Treasury file plus one-half times the log excess return variance to adjust for Jensen's inequality. The empirical price-dividend ratio is the S&P 500 real price divided by real dividends from Robert Shiller's website. The model price dividend ratio divides by dividends smoothed over 64 quarters. The last two rows report results from regressing 4-quarter log excess returns onto the lagged log price-dividend ratio: $xr^{stock}_{t\rightarrow t+4} = b_0 + b_1 pd_t + \epsilon_{t+4}$. Model moments are averaged over 2 simulations of length 10000.

Table 4: Bonds

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Excess Returns				
Term Premium	2.31	1.57	3.23	-1.40
Volatility	8.37	6.07	5.98	3.72
Sharpe Ratio	0.28	0.26	0.54	-0.38
Yields				
Mean log Yield Spread	1.16	0.84	1.40	-0.78
Volatility	1.29	0.62	0.93	0.41
AR(1) Coefficient	0.70	0.73	0.79	0.53
Predictability				
1-YR Excess Returns on log Yield Spread	2.78	0.01	0.39	-0.29
R^2	0.19	0.00	0.01	0.00

Log bond excess returns are quarterly log returns on 5-year nominal bonds in excess of the log nominal 3-month T-bill. We compute empirical log returns on the n-quarter nominal bond from log nominal bond yields: $r^{\$}_{n,t+1} = -(n-1)y^{\$}_{n-1,t+1} + ny^{\$}_{n,t}$. We measure quarter-end 5-year bond yields and 3-month T-bill yields with CRSP Monthly Treasury continuously compounded yields based on the average of bid and ask quotes. We approximate 19-quarter yields with 5-year bond yields in the data. The log yield spread is computed as the log 5-year bond yield minus the log nominal 3-month Treasury bill. The term premium is the average log 5-year nominal bond return in excess of the log nominal 3-month T-bill plus one-half times the log excess return variance to adjust for Jensen's inequality. The term premium, volatility of real bond excess returns, and the log yield spread are in annualized percent. The last two rows report results from regressing 4-quarter log excess returns onto the lagged log yield spread: $xr^{\$}_{n,t\rightarrow t+4} = b_0 + b_1 spread_t + \epsilon_{t+4}$. Model moments are averaged over 2 simulations of length 10000.

Table 5: Bonds and Stocks

	79Q3-01Q1		01Q2-11Q4	
	Empirical	Model	Empirical	Model
Bond-Stock Comovement				
Correlation Bond and Stock Returns	0.21	0.50	-0.64	-0.72
Beta Bond Returns on Stock Returns	0.11	0.16	-0.19	-0.36
Nominal-Real Comovement				
Correlation Quarterly Inflation and Output Gap	-0.28	-0.37	0.65	0.31
Correlation 5-Year Average Inflation and Output Gap	-0.15	-0.14	0.20	0.16
Correlation 5-Year Average Federal Funds Rate and Output Gap	-0.38	-0.38	0.57	0.65
Predictability				
1-YR Excess Stock Return on Output Gap	-1.05	-1.92	-4.71	-0.84
R^2	0.02	0.02	0.21	0.07
1-YR Excess Bond Return on Output Gap	-0.89	-0.28	-0.11	0.26
R^2	0.05	0.00	0.00	0.03

The bond-stock correlation is the correlation of quarterly log bond excess returns with log stock excess returns. The bond-stock beta is the slope coefficient from regressing quarterly log bond excess returns onto log stock excess returns: $xr^{\$}_{n,t+1} = b_0 + b_1 xr^{stock}_{t+1} + \epsilon_{t+1}$. The bond-stock covariance is for the same returns in quarterly percent. "Correlation 5-Year Average Inflation and Output Gap" reports the empirical correlation between x_t and $(\pi_t + \pi_{t+1} + ... + \pi_{t+20})/20$, where t ranges from the first quarter in the subperiod to the last. "Correlation 5-Year Average Federal Funds Rate and Output Gap" is computed similarly. The last four rows report results from regressing 4-quarter log stock excess returns onto the lagged output gap: $xr^{stock}_{t\rightarrow t+4} = b_0 + b_1 x_t + \epsilon_{t+4}$. The regression for 5-year log bond excess returns is analogous. Data for inflation, the Federal Funds rate, and the log output gap are described in Table 2. Data for quarterly 5-year nominal bond excess returns and stock excess returns are described in Tables 3 and 4. All model moments are averages over 2 simulations of length 10000, with the exception of the conditional inflation-output gap and Federal Funds rate-output gap correlations, which are computed analytically.

Table 6: Real Bonds

	79Q3-01Q1	01Q2-11Q4
Excess Returns		
Term Premium	1.07	-1.08
Volatility	2.20	2.28
Sharpe Ratio	0.49	-0.48
Yields		
Mean log Yield Spread	0.69	-0.57
Volatility	0.75	0.43
Bond-Stock Comovement		
Bond-Stock Beta	0.12	-0.27
Bond-Stock Correlation	0.98	-0.89

This table reports model moments for 5-year real zero coupon bonds. The quarterly 5-year real bond excess return is defined as the quarterly log return on a 5-year real zero-coupon bond in excess of the 3-month log real risk-free rate. All moments for real bonds are computed analogously to nominal bond moments in Table 4. The term premium, volatility of real bond excess returns, and the log yield spread are in annualized percent. Model moments are averaged over 2 simulations of length 10000.

Table 7: Decomposition of Model Stock and Bond Returns

Panel A: Return Loadings on Estimated VAR(1) Shocks

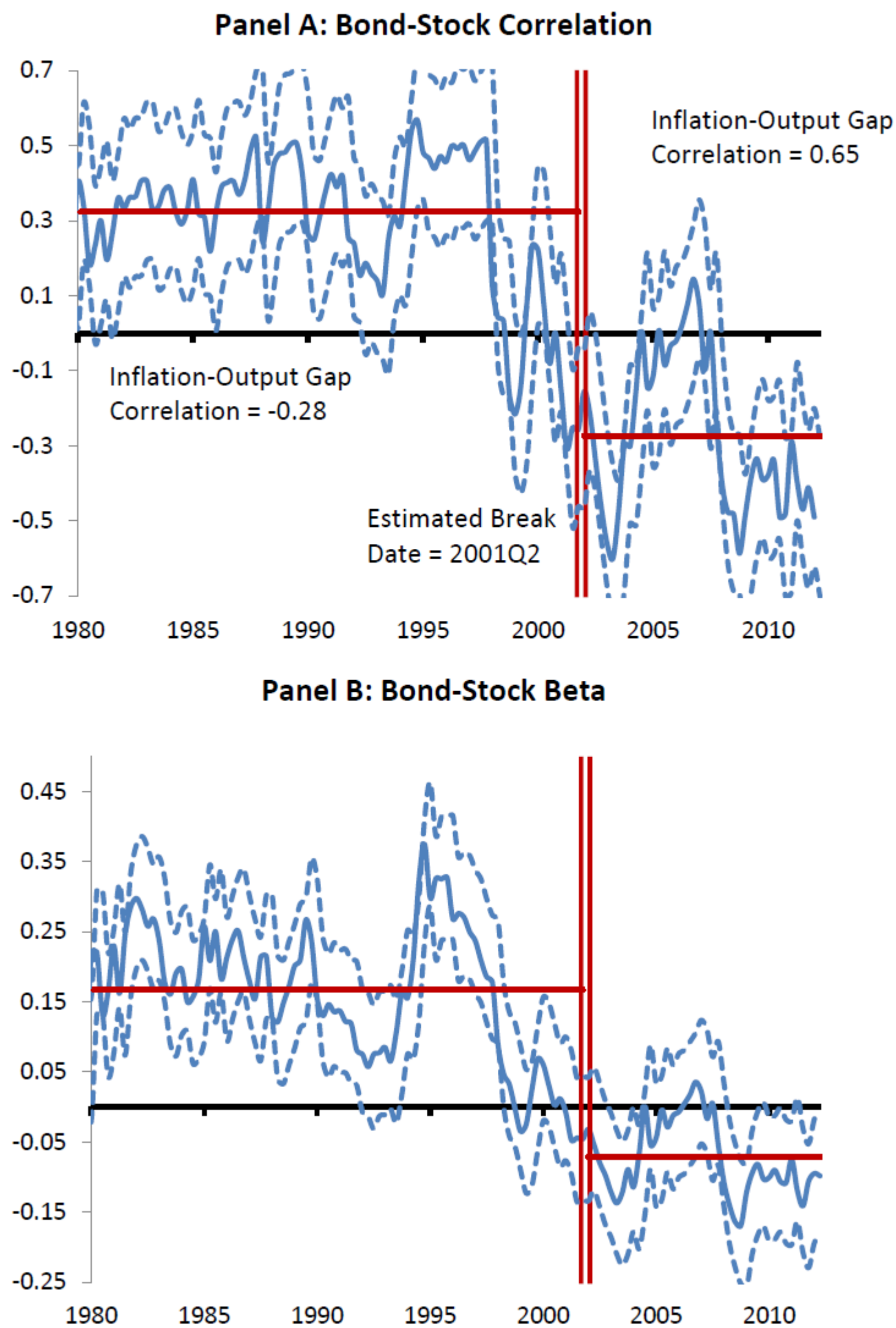
	Output Gap		Inflation		Fed Funds	
Stocks	79Q3-01Q1	01Q2-11Q4	79Q3-01Q1	01Q2-11Q4	79Q3-01Q1	01Q2-11Q4
Full	7.98	2.49	-0.13	0.69	-2.99	0.89
Cash Flow	0.91	0.44	-0.02	0.14	-0.36	0.15
Real Rate	0.65	-0.78	0.00	0.01	-0.29	-0.43
Risk Premium	6.42	2.83	-0.11	0.54	-2.34	1.17
Bonds						
Full	-0.08	-0.45	-2.63	-1.17	-1.05	-0.87
Cash Flow	-0.86	0.37	-2.63	-1.15	-0.71	-0.44
Real Rate	0.36	-0.55	0.00	0.03	-0.19	-0.32
Risk Premium	0.41	-0.27	0.00	-0.05	-0.15	-0.11

Panel B: Covariances and Correlations

	79Q3-01Q1			01Q2-11Q4		
	Cash Flow	Real Rate	Risk Premium	Cash Flow	Real Rate	Risk Premium
Bond-Stock Correlation	0.12	0.98	0.96	-0.14	0.98	-0.94
Bond-Stock Covariance	0.41	0.57	3.57	-0.15	1.00	-1.64

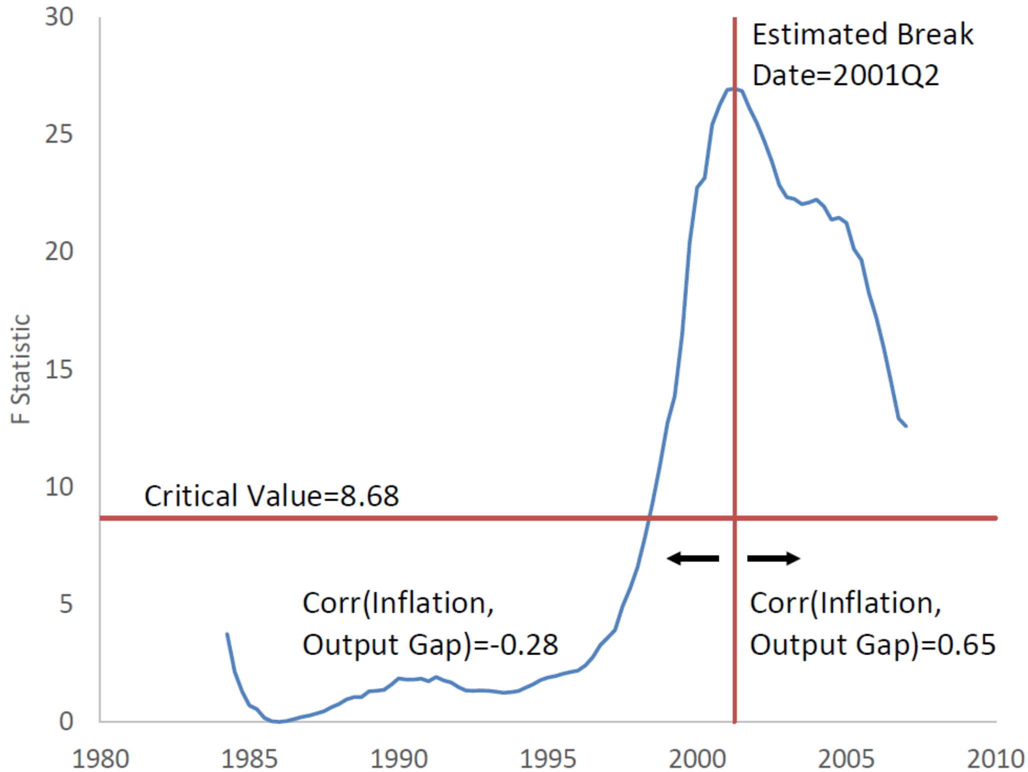
This table decomposes model stock and nominal bond returns into cash flow news, real rate news, and risk premium excess returns (Campbell and Ammer 1993). We solve for risk-neutral bond and stock returns with the risk-neutral pricing kernel $M_{t+1}^{rn} = exp(-r_t)$. We use the loglinear approximation of Campbell and Shiller (1988) to compute real rate news for model stock and bond returns. Cash flow news are computed as risk-neutral excess returns minus real rate news. Risk premium excess returns are the difference between stock or bond log excess returns and risk neutral log excess returns. For details of this decomposition see the appendix. Panel A shows the regression coefficients of model cash flew news, real rate news, and risk premium excess returns onto estimated orthogonalized one-standard-deviation shocks used for impulse responses in Figures 3, 4, and 5. Panel B shows the correlation between bond cash flow news and stock cash flow news, the correlation between bond real rate news and stock real rate news, and the correlation between bond risk premium excess returns and stock risk premium excess returns. The table shows covariances of returns in quarterly percent.

Figure 1: Rolling Bond-Stock Comovement



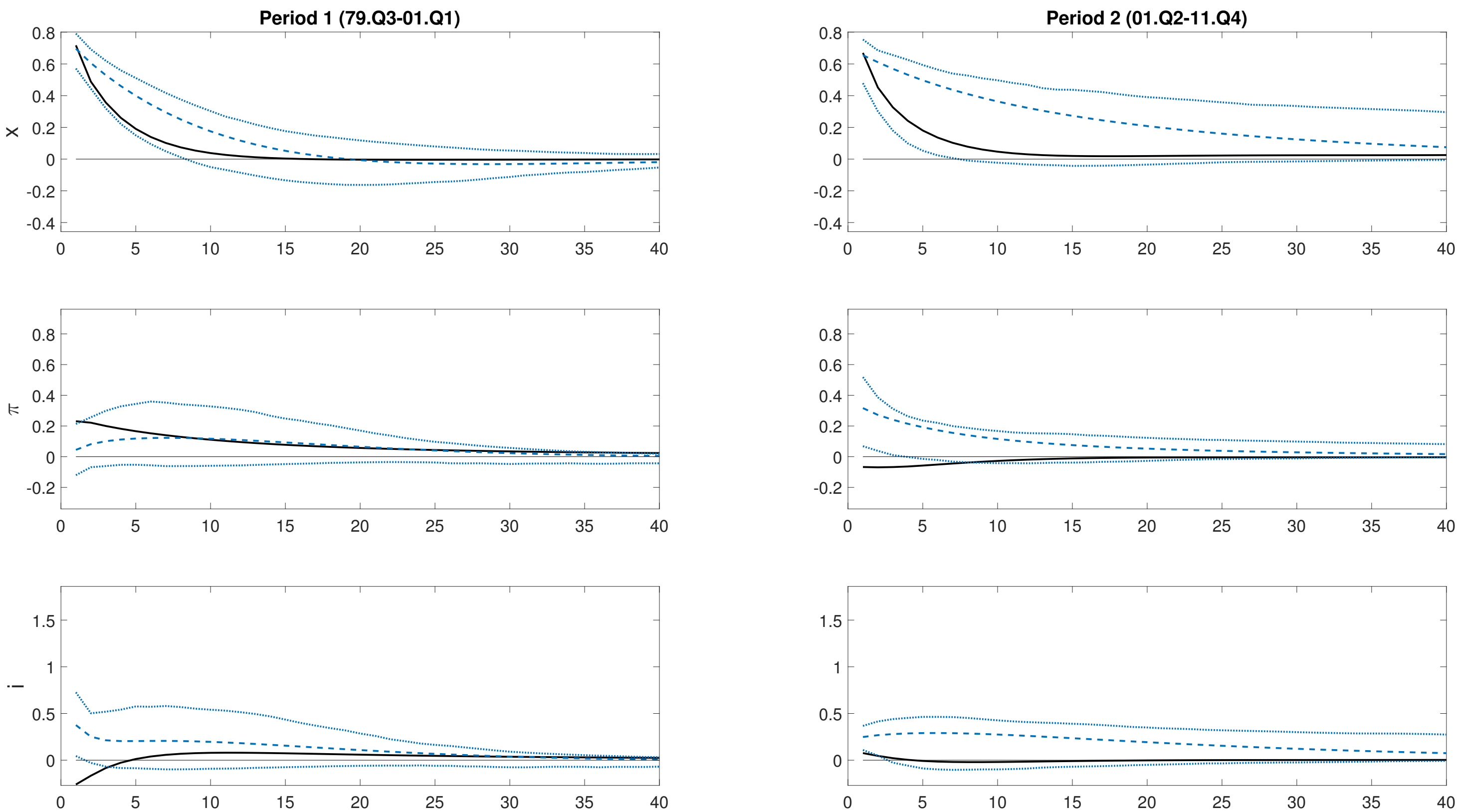
Rolling nominal bond-stock correlations and bond-stock betas use daily log returns on 5-year nominal Treasury bonds and daily log CRSP value-weighted stock market returns including dividends over past three months. We approximate daily nominal bond returns using changes in continuously compounded 5-year bond yields from Gürkaynak, Sack, Wright (2007). We show filtered correlations and betas from a Kalman filter, that assumes that observed correlations and betas follow an AR(1) trend plus white observation noise. 95% confidence intervals are shown in dashed. A red vertical line indicates the estimated break date from the Quandt Likelihood Ratio test for an unknown break date in the slope of quarterly inflation onto the quarterly output gap, described in detail in Section 4.2. Horizontal lines indicate subperiod averages.

Figure 2: Testing for Break in Inflation-Output Relation



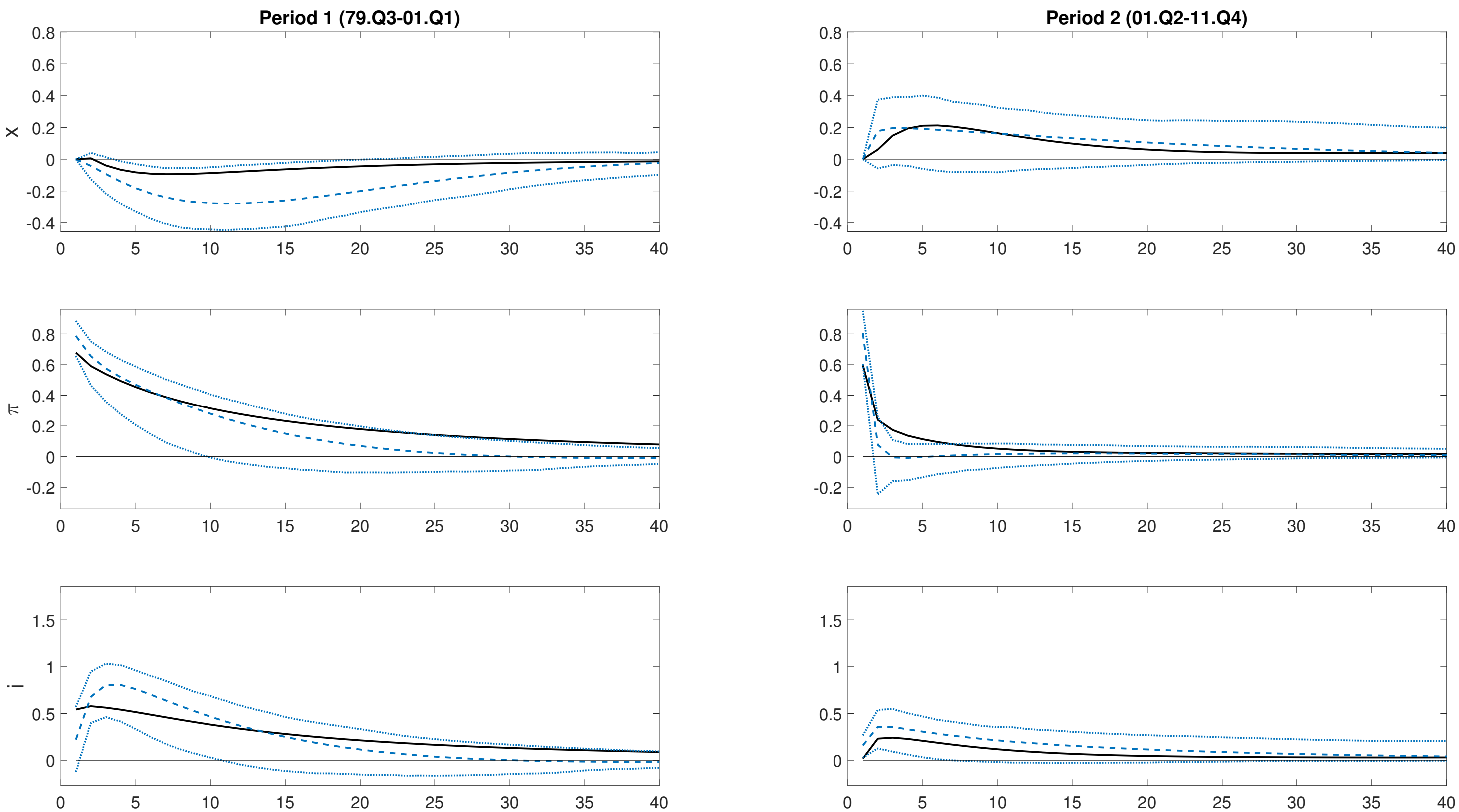
This figure plots the F-statistic for d_τ in the quarterly regression $\pi_t = a_\tau + b_\tau I_{t\geq\tau} + c_\tau x_t + d_\tau I_{t\geq\tau} x_t + \varepsilon_t$ for all τ in the middle 70% of our sample. We reject the null hypothesis of no break in the relation between π_t and x_t if the maximum F-statistic exceeds the 95% critical value with one constraint and 15% trimming (Andrews 2003). The critical value is indicated by the red horizontal line. The vertical line shows the estimated break date (2001Q2), which equals the quarter with the maximum F-statistic.

Figure 3: Empirical and Model Impulse Responses to Output Gap Innovations



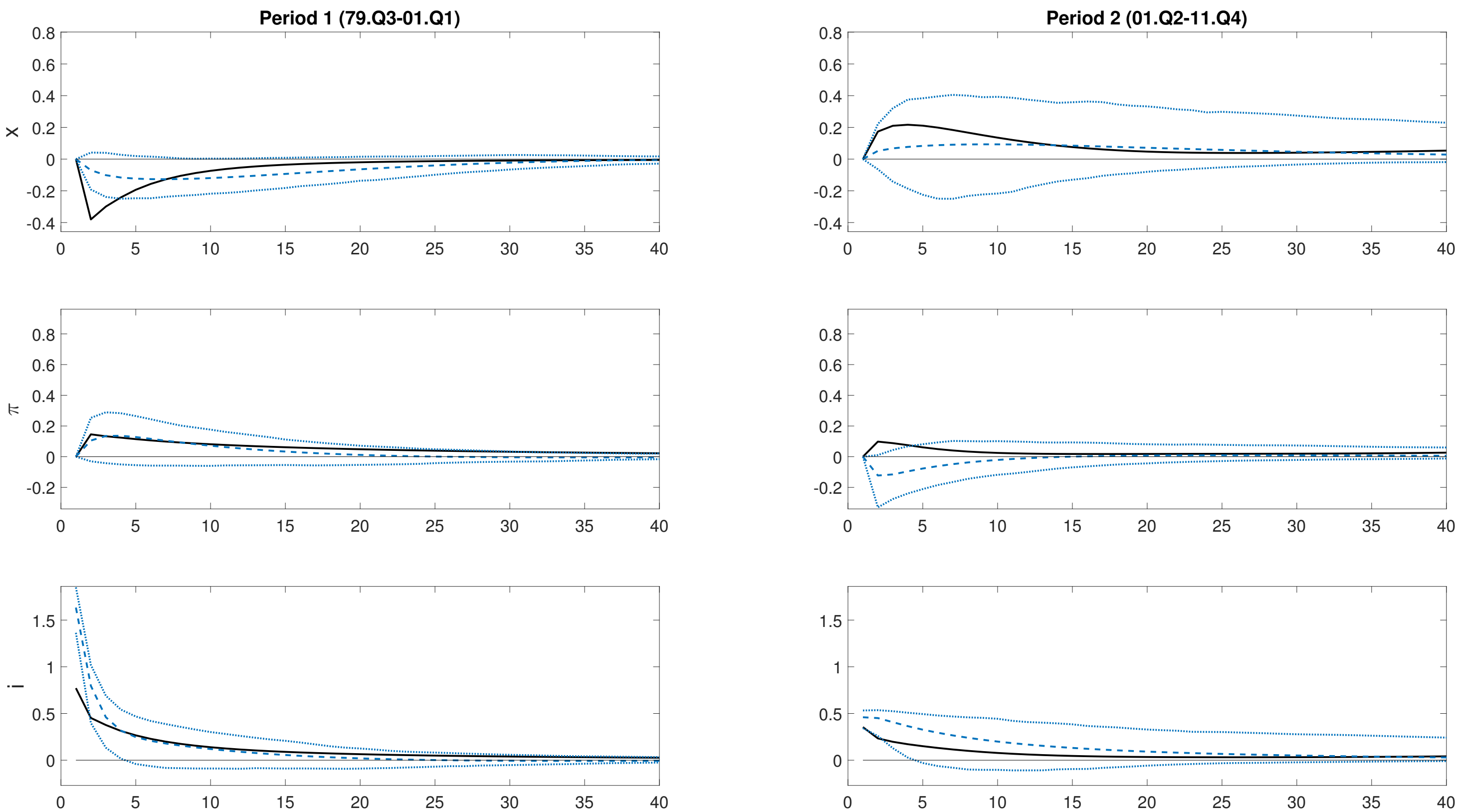
This figure shows model (black) and data (blue with 95% CI) orthogonalized impulse responses for the output gap, inflation, and the Federal Funds rate in response to a one-standard deviation output gap innovation. To provide a unique rotation of impulse responses, shocks are ordered such that an output gap shock affects inflation and the Fed Funds rate contemporaneously, an inflation shock affects the Fed Funds rate but not the output gap contemporaneously, and a Fed Funds rate shock affects neither inflation nor the output gap contemporaneously.

Figure 4: Empirical and Model Impulse Responses to Inflation Innovations



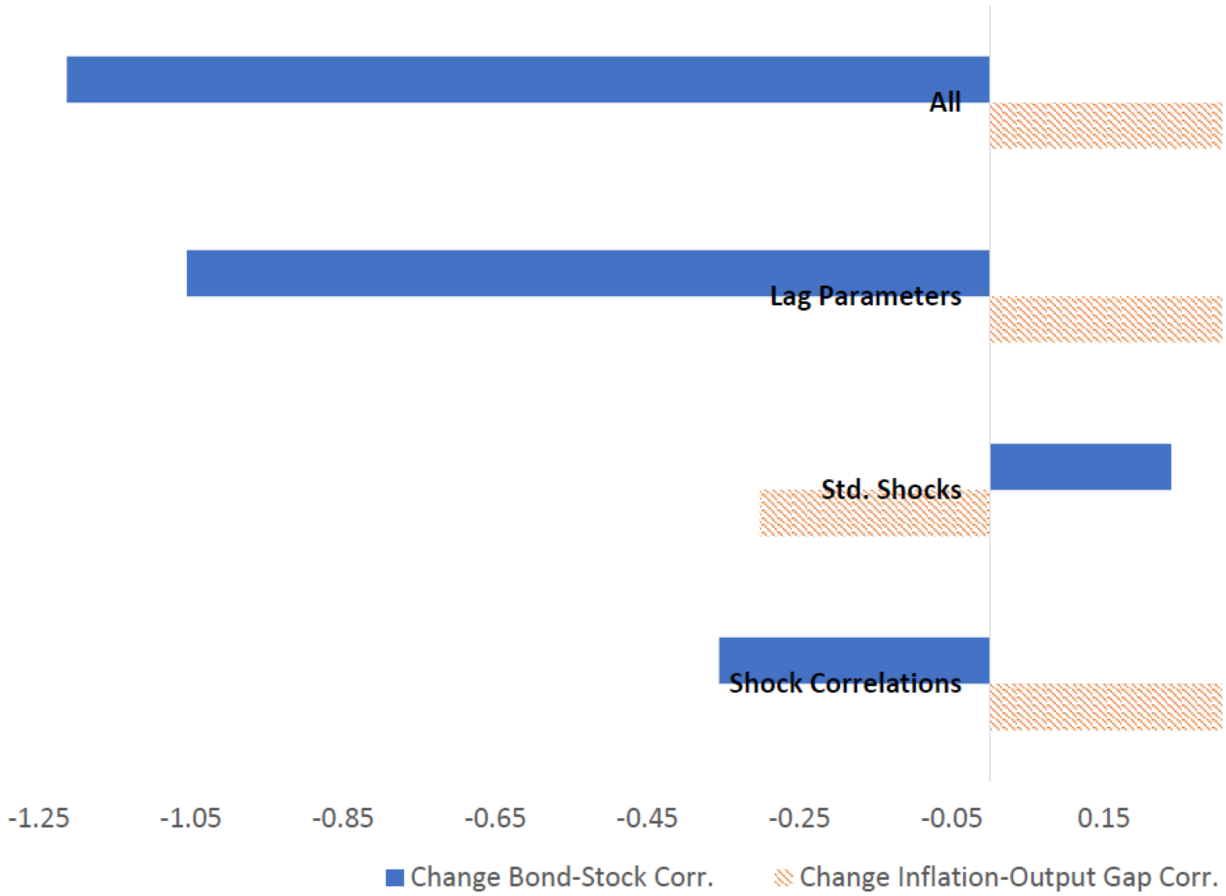
This figure shows model (black) and data (blue with 95% CI) orthogonalized impulse responses for the output gap, inflation, and the Federal Funds rate in response to a one-standard deviation inflation innovation. To provide a unique rotation of impulse responses, shocks are ordered such that an output gap shock affects inflation and the Fed Funds rate contemporaneously, an inflation shock affects the Fed Funds rate but not the output gap contemporaneously, and a Fed Funds rate shock affects neither inflation nor the output gap contemporaneously.

Figure 5: Empirical and Model Impulse Responses to Fed Funds Innovation



This figure shows model (black) and data (blue with 95% CI) orthogonalized impulse responses for the output gap, inflation, and the Federal Funds rate in response to a one-standard deviation Federal Funds rate innovation. To provide a unique rotation of impulse responses, shocks are ordered such that an output gap shock affects inflation and the Fed Funds rate contemporaneously, an inflation shock affects the Fed Funds rate but not the output gap contemporaneously, and a Fed Funds rate shock affects neither inflation nor the output gap contemporaneously.

Figure 6: Counterfactuals



This figure shows changes in the bond-stock return correlation and in the quarterly inflation-output gap correlation when changing subsets of parameters from their period 1 to their period 2 values. "All" shows the difference between period 2 and period 1 model correlations tabulated in Table 5. "Lag Parameters" shows the change in the model correlation when moving lag parameters $p_{\pi x}, ..., p_{ii}$ from their period 1 to their period 2 values. "Std. Shocks" changes only the standard deviations of shocks while holding all other parameters constant at their period 1 values. "Shock Correlations" changes only the correlations of shocks $\rho_{\pi i}, \rho\pi*, \rho_{i*}$ while holding all other parameters constant at their period 1 values.