HARVARD | BUSINESS | SCHOOL

First-Party Content, Commitment and Coordination in Two-Sided Markets

Andrei Hagiu
Daniel Spulber

Working Paper

11-123

October 23, 2011

First-Party Content, Commitment and Coordination in Two-Sided Markets*

Andrei Hagiu† and Daniel Spulber‡

October 23, 2011

Abstract

The strategic use of first-party content by two-sided platforms is driven by two key factors: the nature of buyer and seller expectations (favorable vs. unfavorable) and the nature of the relationship between first-party content and third-party content (complements or substitutes). As a result, first-party content is a strategic instrument that plays a dual role. On the one hand, it enables platforms facing unfavorable expectations to compensate for their difficulty in attracting third-party sellers. They should *over-invest* in first-party content which substitutes for third-party content relative to platforms benefitting from favorable expectations. On the other hand, platforms that benefit from favorable expectations capture a larger share of total surplus from buyers and sellers. They derive a higher return on investment in first-party content that complements third-party content relative to platforms facing unfavorable expectations. As a result, the latter *under-invest* in complementary first-party content. These results hold with both simultaneous and sequential entry of the the two sides.

With two competing platforms - incumbent facing favorable expectations and entrant facing unfavorable expectations - and singlehoming on one side of the market, the incumbent always invests (weakly) more in first-party content relative to the case in which it is a monopolist.

"Content is king" (Gates (1996))

1 Introduction

Two-sided platforms face a challenging coordination problem that consists of attracting both buyers and sellers. Buyers and sellers derive cross-market benefits, which may be the result of product variety and scale effects, market liquidity, and connectivity of communications networks (cf. Spulber, 2010). As a result, participation by buyers and by sellers depends on their expectations of participation on the other side of the market. In order to solve this problem, many firms provide "first-party

*We are grateful to Ramon Casadesus-Masanell and participants at the Second Annual Conference on Internet Search and Innovation (Northwestern Law School) for very helpful comments and feedback.

†Harvard University, ahagiu@hbs.edu.

‡Northwestern University, jems@kellogg.northwestern.edu.

content," which makes participation more attractive to one side (typically, buyers), independently of the presence of the other side (typically, sellers). The first-party content that platforms provide competes with or complements content provided by sellers, which we also call "third-party content providers."[1] Examples of platforms providing first-party content include: videogame console manufacturers offering games (e.g. Microsoft's *Halo*, Sony's *Gran Turismo,* Nintendo's *Wii Sports*), motion-activated controllers (e.g. Microsoft's Kinnect) and online gaming portals (e.g. Sony's PlayStation Store); operating system vendors producing software applications (e.g. Microsoft's Office) and online marketplaces (e.g. Apple's App Store); e-commerce sites offering payment systems (e.g. eBay's PayPal), product information and expedited shipping services (e.g. Amazon's *Prime*); search engines and Internet portals (e.g. Google, Bing) providing objective search results, maps, news, weather, and entertainment.

What exactly drives the strategic use of first-party content? In this paper we show that two-sided platforms' incentives to invest in first-party content are closely related to their optimal pricing strategy (which is endogenous) and determined by two key factors, which are exogenous in our analysis: i) the nature of buyer and seller expectations; ii) the nature of the relationship between first-party content and seller participation (or third-party content).

Because of the inter-dependence between buyer and seller decisions to participate in any given two-sided platform, there are typically multiple coordination equilibria: buyer and seller expectations play a key role in determining which equilibrium prevails. If buyers and sellers *always* coordinate on the equilibrium with the highest participation rates given the prices and first-party content chosen by a platform, we say that expectations are *favorable*. Conversely, if buyers and sellers *always* coordinate on the equilibrium with the lowest (or zero) participation rates on both sides, expectations are said to be *unfavorable*. In this context, platforms facing unfavorable expectations typically have a choice between two pricing strategies. One is to subsidize sellers in order to secure their participation irrespective of buyers' decisions, and then extract profits from the buyer side (the Seller Subsidy strategy). The other is to charge low prices to buyers and extract most of their profits from sellers (the Buyer Attraction Strategy). These two pricing strategies are common in practice (cf. Eisenmann et al. (2006)) and standard in most analyses of two-sided platforms.

The novelty of our analysis is to introduce investment in first-party content as a strategic variable for two-sided platforms. The platform uses first-party content as a key coordination device in addition to prices. We show that the key property of first-party content that matters for platform strategy is whether it is a *substitute* or a *complement* for seller participation (or third-party content). For instance, first-party videogames offered by console makers are *substitutes* for games provided by third-party publishers like Electronic Arts and Take Two Interactive. This means that the presence

[1] This terminology follows the software and videogame industries, where first-party games or applications are supplied by platform providers themselves and third-party games or applications are supplied by independent developers.

of first-party games reduces the marginal utility users derive from third-party games. The same goes for the apps provided by Apple, HTC, Samsung and other smart phone manufacturers on their respective phones. On the other hand, Microsoft's Kinnect motion-activated game controller and Xbox Live online gaming system are *complements* to third-party developers' games for Xbox, in the sense that they enhance these games' value to users. Similarly, when Amazon and eBay offer expedited shipping, product information and payment systems, they are enhancing the value of the products sold by affiliated sellers. The following table contains these and other examples of first-party content and their relationship with third-party content.

Platform	**Sellers**	**Substitute first-party content**	**Complementary first-party content**
Xbox 360	independent game developers	Halo	Kinnect, Xbox Live
PlayStation 3	independent game developers	Gran Turismo	PlayStation Store
Amazon	3rd-party sellers	products sold by Amazon	Amazon Prime, expedited shipping
Google	advertisers	maps, news, blogs, images, video, books	maps, news, blogs, images, video, books
iPhone	3rd-party app developers	iTunes and other Apple apps	the App Store and its billing system
Facebook	3rd-party app developers	news feeds, games, digital gifts, e-mail notifications, friend suggestions	information sharing, ability to comment, notifications from fan sites, Facebook Credits

The highest-level insight that emerges from our analysis and that should be of particular managerial interest is that first-party content plays a dual strategic role for two-sided platforms. First, by increasing buyer participation, first-party content is an instrument allowing platforms that face unfavorable expectations to make up for their greater difficulty in attracting third-party sellers. This suggests that they should *over-invest* in first-party content relative to platforms benefitting from favorable expectations. Second, first-party content can be interpreted as a measure of platform quality from the perspective of buyers, which also impacts the surplus of sellers (at least indirectly, through buyer demand). Since platforms facing unfavorable expectations capture a *smaller* share of total surplus from buyers and sellers, they usually derive a *lower* return on investment in first-party content, therefore they should *under-invest* in first-party content relative to platforms benefitting from favorable expectations. Whether the first or the second role dominates depends on whether first- and third-party content are substitutes or complements.

Related literature

This paper contributes to the strategy and economics literature on two-sided platforms (Armstrong (2006), Caillaud and Jullien (2003), Economides and Katsamakas (2006), Eisenmann et al. (2006), Parker and Van Alstyne (2005), Rochet and Tirole (2006), Weyl (2010), Zhu and Iansiti (2011)). Our key contribution is to introduce first-party content as a third strategic instrument chosen by two-sided platforms in addition to prices. In this way, our work is related to recent efforts to expand the formal study of two-sided platforms beyond pricing (e.g. Boudreau (2010), Parker and Van Alstyne (2008) who study openness decisions).

At a very general level, the provision of first-party content can be viewed as a form of diversification by two-sided platforms (cf. Kim and Bogut (1996)). More specifically, several broad business strategy studies of platforms (Evans et al. (2006), Gawer (2009), Gawer and Cusumano (2002), Iansiti and Levien (2004)) have informally discussed the importance of providing first-party content or applications as a tool to get platform adoption off the ground. This is one aspect of the dual role played by first-party content that we analyze here. Our formal model allows us to provide a more nuanced view of first-party content and link it to pricing decisions and user expectations.

We build upon the models of Caillaud and Jullien (2003) and Hagiu (2006) by using the same notion of favorable vs. unfavorable expectations for two-sided platforms connecting buyers and sellers (cf. also Halaburda and Yehezkel (2011)). The novelty relative to this work is that we allow for investments in first-party content. Our work also extends Hagiu (2006)'s analysis of commitment by two-sided platforms facing sequential entry. The model and the insights we obtain are richer here for two reasons: i) the presence of first-party content, and ii) the linearity of platform demand on the buyer side (in contrast, both Caillaud and Jullien (2003) and Hagiu (2006) assume members of each side are identical). Furthermore, the insights we obtain regarding the interplay between optimal pricing strategies and investments in first-party content extend the results on two-sided platform pricing structures (Armstrong (2006), Parker and Van Alstyne (2005), Rochet and Tirole (2006), Spulber (1999), Hagiu (2009)).

The interpretation of first-party content as a strategic instrument facilitating two-sided coordination also builds on the discussion in Spulber (2010). In game theory terms, when there is more than one Nash equilibrium in the participation subgame, and buyers and sellers cannot coordinate directly, players need not wind up at one of the equilibria because there is no convergence of equilibrium expectations. Players may experience confusion and split moves so that almost every combination of player strategies is possible (Farrell and Klemperer (2009)). Our model with favorable vs. unfavorable expectations is designed to capture this real uncertainty in a stylized way.

2 Model Set-Up

This section sets up the monopoly version of our model. A monopoly two-sided platform firm connects a continuum of buyers i uniformly distributed over $[0, 1]$, and a continuum of sellers j uniformly distributed over $[0, M]$. The platform charges participation fees p to buyers and w to sellers. In addition, the platform can offer buyers an amount x of *first-party* content (hereafter content) at cost $C(x)$, which is increasing and convex in x. $C(x)$ is the cost of building or acquiring the capability of offering x. In contrast, we will refer to the products or services provided by sellers to buyers as "third-party content". Depending on the context, the platform's content can be products, features, information, quality of service, ease of use and other non-pecuniary benefits provided to buyers. The nature of x is discussed in more detail below.

Buyer and Seller Preferences

Buyer i's net benefit from joining the platform is:

$$U_i(m, x, p = u(m, x) - i - p.$$

The utility function $u(m, x)$ represents buyers' preferences over combinations of first-party content x and seller participation on the platform $m \in [0, M]$, the latter of which we equate with third-party content. Buyer i incurs a personal cost of adopting the platform equal to i, so that buyers have different willingness to pay for participation. Let the buyer's utility function $u(.,.)$ be increasing, twice continuously differentiable and concave or linear in each of its arguments. Thus, as is natural, buyer utility is increasing in both types of content - first-party and third-party. A buyer joins the platform if and only if her expected net benefit is non-negative.

Sellers are assumed to be *identical* and each seller's net benefit from joining the platform is:

$$V(n, w) = \pi(x)\, n - w - \phi,$$

where $\pi(x) > 0$ is the profit per buyer made by each seller, $n \in [0, 1]$ is the number of buyers who participate on the platform and $\phi > 0$ is a seller's fixed cost of "porting" his product to the platform. The function $\pi(.)$ can be increasing or decreasing depending on the relationship between first and third-party content (cf. discussion below), while ϕ is an exogenous parameter. A seller chooses to join if expected benefits are non-negative. We assume that ϕ is small enough so that: i) no platform finds it profitable to be simply a content provider to buyers without any sellers, and ii) platforms are viable (i.e. make non-negative profits) under all types of expectations.

First-Party Content

A key factor in our analysis turns out to be the relationship between first-party content x and third-party content m. Loosely speaking, if the platform and the sellers provide similar information or entertainment services of the same nature, then x and m are likely to be substitutes. On the other hand, if the platform's content consists of features or services that enhance the value of third-party sellers' products, then x and m are likely to be complements. Let us precisely define the notions of subtitutability and complementarity that we will use throughout the paper as follows:

Definition 1 *Content x and seller participation m are said to be strong complements (substitutes) if $\partial^2 u/\partial m \partial x > 0$ and $d\pi/dx \geq 0$ (respectively if $\partial^2 u/\partial m \partial x < 0$ and $d\pi/dx \leq 0$). They are said to be independent if $\partial^2 u/\partial m \partial x = 0$ and $d\pi/dx = 0$.*

Complementarity and substitutability as defined here are stronger notions than the ones commonly used. Indeed, these terms usually only refer to the relationship between x and m in the buyer utility function. Because of the two-sided nature of our set-up however, we include a restriction on $\pi(x)$ (the precise reasons will become clear in the analysis that follows in the next sections). For strong complements (substitutes) we only require $d\pi/dx \geq 0$ (≤ 0) instead of strict inequality. Consequently, when sellers' profits per buyer are independent of x, first-party and third-party content are strong complements (substitutes) if and only if they are strict complements (substitutes) in buyer demand in the usual sense. Sellers' profits per buyer are independent of x if for example sellers have an advertising-based business model; the advertising rates they receive per user are unlikely to be influenced by the platform's investments in first-party content. More generally, this holds whenever sellers are unable to extract the increase in buyer surplus created by first-party content, which is not uncommon. For instance, it is not clear that Electronic Arts will be able to raise the price of its Xbox games when Microsoft increases the quality and functionality of Xbox Live (the latter arguably enhances the user experience for all Xbox games that take advantage of it). In the specific functional examples we use below to illustrate this, we always have $d\pi/dx = 0$. This restriction in the examples is used only for clarity of exposition: all of our main results are stated generally (cf. Propositions below) and do not depend on this simplification.

Finally, we will not analyze the theoretically possible but unlikely scenarios in which $\partial^2 u/\partial m \partial x$ and $d\pi/dx$ have opposite signs. While our framework does allow for the analysis of these cases, it seems reasonable to assume that when first-party content is complementary to third-party content in buyer demand, third-party sellers should be no worse off when the platform increases its investment in first-party content (and vice versa with substitutes).

We introduce three specific functional forms that illustrate the three scenarios in the definition above and that will be used below to derive closed-form solutions. In section 1 of the online appendix, we provide the detailed micro-foundations underlying each of these examples.

Example 1 (independent) $u(m,x) = ms + x$, $\pi(x) = \pi$ *and* $C(x) = c\frac{x^2}{2}$

In this example, x and m can be thought of applications, games or other features offered to buyers by the platform (x) and by sellers (m), all of which are independent of one other. In other words, there is neither competition nor complementarity among them. π and s are constants which result from the division of economic surplus created by a product between its seller and each participating buyer.

Example 2 (strong complements) $u(m,x) = u_0 + msx$, $\pi(x) = \pi$ *and* $C(x) = c\frac{x^2}{2}$

In this example, u_0 is the (fixed) stand-alone utility offered by the platform to each buyer and π is a seller's profit per buyer (constant). m and x are strong complements: x can be thought of as a set of platform features that *enhance* the value of third-party seller products (the net utility derived by a buyer from a seller product is sx). Illustrations include eBay's PayPal payment system that makes it easier for sellers to transact with buyers; Microsoft's Xbox Live service that makes it easier for third-party developers to endow their games with online gaming capabilities.

Example 3 (strong substitutes) $u(m,x) = ms(1-x) + M_0 sx$, $\pi(x) = \pi$ *and* $C(x) = c\frac{x^2}{2}$

In this example, $M_0 > M$ can be interpreted as the total number of product categories (e.g. applications, services, etc.) that can be potentially offered on the platform. The platform offers one basic product in each available category (i.e. covers the entire spectrum $[0, M_0]$), each of which is of quality $x \leq 1$. In contrast, each seller offers one product in the range $[0, M]$ and all third-party sellers' products are of quality 1. Thus, for each product category, the product offered by the platform is an inferior substitute for the product offered by the corresponding third-party seller. A buyer joining the platform obtains utility sx from each of the M_0 products offered by the platform and an *additional* $s(1-x)$ from each of the m products offered by third-party sellers. Sellers' profits per buyer are constant and equal to π.

Observations

Some observations regarding this modelling set-up are in order. First, we only allow the platform to charge fixed participation fees on both sides. This turns out to be the richest scenario in our model. If the platform were to charge variable fees (royalties) to sellers instead of or in addition to fixed access fees, the set of its optimal pricing strategies would be identical or strictly smaller, resulting in identical or strictly lower profits (cf. section 2 of the online appendix).

Second, we have assumed that π is independent of m. Introducing competition among sellers (i.e. allowing π to be decreasing in m) would make the derivation of the various equilibria (favorable and unfavorable expectations) more difficult but their identity and nature would remain unchanged.[2]

[2] Two-sided platform pricing with competition among members on at least one side has been studied at length elsewhere (cf. Belleflamme and Toulemonde (2009), Hagiu (2009), Nocke et al. (2007)).

Third, the linearity of buyer demand in price is by no means restrictive: our analysis carries over (with more complicated calculations) to more general formulations of buyer demand. The same is true if, instead of assuming identical sellers, we allowed for elastic seller demand; the tractability of our model would be greatly reduced but the results would continue to hold.

3 Monopoly platform with simultaneous entry

The game with simultaneous entry of the two sides - buyers and sellers - has two stages. In the first stage, the profit-maximizing platform chooses the amount of first-party content x, as well as participation prices p and w for buyers and sellers respectively: (p, w, x) are publicly announced and observed by all players. In the second stage, individual buyers and sellers simultaneously choose whether or not to join the platform, based on the content and prices chosen by the firm and their expectations about market participation by others. There is no communication or coordination among buyers or sellers. Given the preferences defined above, buyers' and sellers' actions only depend on *total* participation on the other side of the market; furthermore, we assume that expectations are *consistent* across buyers and across sellers, and that they are *fulfilled* in equilibrium.

The equilibrium of the full game consists of the first-stage choices of the platform, (p^*, w^*, x^*), and the resulting Nash equilibrium of buyer and seller participation decisions in the second-stage subgame, (n^*, m^*). As usual in the presence of network effects, there can be multiple participation equilibria even with consistent and fulfilled expectations. Indeed, for any vector (p, w, x) of platform choices, the equilibrium buyer participation $n = n(x, p, w)$ and the equilibrium seller participation $m = m(x, p, w)$ solve the following two equations:

$$n = \max\{u(m, x) - p, 0\},$$

$$m = MH[\pi(x)n - w - \phi] = \begin{cases} M & \text{if} \quad \pi(x)n - w - \phi \geq 0 \\ \\ 0 & \text{if} \quad \pi(x)n - w - \phi < 0, \end{cases}$$

where $H[y]$ is a Heaviside step function, with $H[y] = 1$ for $y \geq 0$ and $H[y] = 0$ otherwise.

In our model, the concavity of buyer benefits in seller participation and step-function shape of seller benefits in buyer participation narrow the number of possible stable equilibria to two: a low-participation equilibrium and a high-participation equilibrium. For any vector of prices and content (p, w, x), the configuration of buyer participation and seller participation typically looks like Figure 1.

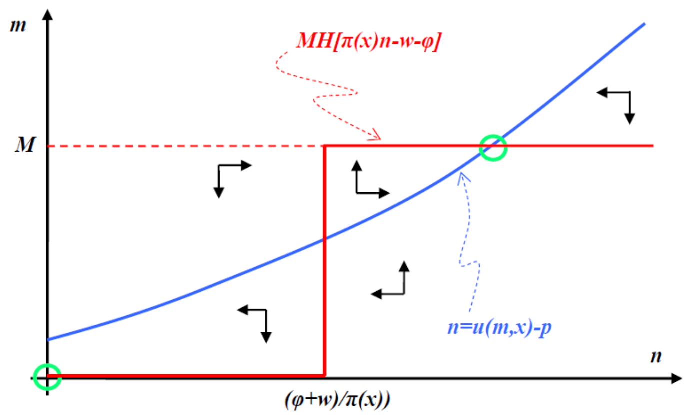

Figure 1

The circles identify the stable equilibria, with arrows indicating dynamic adjustment. Denote the low-participation Nash equilibrium (0 in this case) as the "unfavorable expectations" equilibrium and the high-participation equilibrium as the "favorable expectations" equilibrium - given prices and content (p, w, x).

The platform firm's profits are:

$$\Pi(p, w, x) = pn(x, p, w) + wm(x, p, w) - C(x),$$

The firm chooses content x and prices p and w to maximize profits given the effects of its actions on the outcome of the participation subgame played by buyers and sellers. Note that content x changes the *shape* of each buyer's net benefit curve $u(m, x) - p$, while the price p simply shifts the net benefit curve up or down without altering its shape.

In theory, even with at most two possible participation equilibria for any given prices and content vector (p, w, x), there can be an infinity of distinct, two-sided demand correspondences $(p, w, x) \rightarrow (m, n)$. To simplify things however, we will focus our analysis on two polar types of platforms, characterized by two specific demand correspondences:

Definition 2 *A UF platform faces unfavorable expectations for any (p, w, x), i.e. it is such that buyers and sellers always coordinate on the equilibrium with the lowest possible levels of participation on both sides. In contrast, a F platform faces favorable expectations for any (p, w, x), i.e. it is such that buyers and sellers always coordinate on the equilibrium with the highest possible levels of participation on both sides.*

The unfavorable expectations outcome describes the problems faced by entrepreneurial entrants without established brands and name recognition. In contrast, favorable expectations describes the situation of established firms with well-known brands that are extending existing businesses through diversification. Transaction costs suggest another interpretation of market expectations. Unfavorable expectations describe markets with high transaction costs that make it difficult for buyers and sellers to engage in pre-play communication, while favorable expectations describe markets with low transaction costs in which buyers and sellers can coordinate through some pre-play communication.

Needless to say, the reason we focus on the two polar types of expectations (favorable vs. unfavorable) is concision. In some situations, platforms facing unfavorable expectations can overcome them by using "insulating tariffs" (cf. Weyl (2010)). Nevertheless, they have to work harder and - most important for our purposes - will be able to extract lower profits relative to platforms benefitting from favorable expectations, even though they may be able to achieve the desired "market allocation".

3.1 Favorable expectations

Consider first the F platform. It maximizes profits given buyer participation subject to the sellers' individual rationality (participation) constraint:

$$\max_{p,w,x} \{p\,[u\,(M,x)-p]+wM-C\,(x)\}$$

$$\text{subject to } \pi\,(x)\,[u\,(M,x)-p]-w-\phi\geq 0.$$

The constraint is binding, so the platform's maximization problem becomes:

$$\max_{p,x} \{[p+M\pi\,(x)]\,[u\,(M,x)-p]-C\,(x)-M\phi\}\,. \tag{1}$$

The first-order conditions determining the F platform's optimal choices of buyer price and first-party content (p_f, x_f) are:

$$p_f=\frac{u\,(M,x_f)-M\pi\,(x_f)}{2}$$
$$\frac{[u_x\,(M,x_f)+M\pi_x\,(x_f)]\,[u\,(M,x_f)+M\pi\,(x_f)]}{2}=C'\,(x_f)\,, \tag{2}$$

where we have used the notation:

$$u_x\,(m,x)\equiv\frac{\partial u}{\partial x}\,(m,x)\quad\text{and}\quad \pi_x\,(x)\equiv\frac{d\pi}{dx}\,(x)\,.$$

The price to sellers is given by the sellers' participation condition: the platform extracts the entire seller surplus.

3.2 Unfavorable expectations

Let us now turn to the UF platform. To have a chance of making positive profits, the UF platform must set its prices so as to eliminate the unfavorable expectations equilibrium. The corresponding seller participation condition is:

$$\pi(x)\max[u(0,x)-p,0]-w-\phi\geq 0,$$

which means that prices must be such that an individual seller finds it profitable to join even when (s)he expects the platform will atract no other sellers. The resulting platform profits are:

$$\max_{p,x}\{p[u(M,x)-p]+M\pi(x)\max[u(0,x)-p,0]-M\phi-C(x)\} \tag{3}$$

The optimal buyer price and first-party content (p_{uf},x_{uf}) cannot be such that $u(0,x_{uf})=p_{uf}$. Indeed, if this were the case, the optimality of p_{uf} given x_{uf} would require $u(M,x_{uf})\leq 2u(0,x_{uf})$ and $u(M,x_{uf})-M\pi(x_{uf})\geq 2u(0,x_{uf})$, which cannot be satisfied simultaneously. Thus, the buyer's benefit from first-party content without seller participation, $u(0,x_{uf})$, is either strictly lower than or strictly greater than the buyer price, p_{uf}. This implies that there are only two *possible* solutions to the platform's optimization problem. Depending on parameter values, only one or both of these solutions are feasible. When both are feasible, the platform chooses the one yielding higher profits.

The first possible solution, (p_{uf1},x_{uf1}), is defined by the two first-order conditions:

$$\begin{aligned} p &= \frac{u(M,x)}{2} \\ \frac{u_x(M,x)\,u(M,x)}{2} &= C'(x), \end{aligned} \tag{4}$$

and must satisfy $p_{uf1}>u(0,x_{uf1})$ to be viable. This implies that the seller price fully subsidizes the sellers' fixed costs, i.e. $w_{uf1}=-\phi$. The platform therefore generates all of its profits from the buyer side of the market. We refer to this solution as the Seller Subsidy Strategy. Referring back to Figure 1, it is apparent that this strategy eliminates the low participation equilibrium by moving the seller participation curve, $m=MH[\pi(x)n-w-\phi]$, to the *left*.

The second possible solution, (p_{uf2},x_{uf2}), is defined by:

$$p=\frac{u(M,x)-M\pi(x)}{2}$$

$$\frac{[u_x(M,x)-M\pi_x(x)][u(M,x)-M\pi(x)]}{2}+M\pi(x)\,u_x(0,x)+M\pi_x(x)\,u(0,x)=C'(x), \tag{5}$$

and must satisfy $p_{uf2}<u(0,x_{uf2})$ to be viable, implying that $w_{uf2}=\pi(x_{uf2})[u(0,x_{uf2})-p_{uf2}]-\phi$,

which is greater than $-\phi$. In other words, the price to buyers is relatively low in order to ensure that even when each individual seller expects no sellers to join, there is sufficient buyer demand to make it profitable for that seller to join. We refer to the second solution as the Buyer Attraction Strategy. Referring back to Figure 1, this strategy eliminates the low participation equilibrium by moving the buyer participation curve, $n = u(m, x) - p$, to the *right*.

These two solutions correspond to different pricing and content strategies for solving the coordination problem by the UF platform. Both strategies rely on convincing sellers to join despite unfavorable expectations. The first strategy simply subsidizes sellers' fixed costs and then charges a high price to buyers, whereas the second strategy charges a low price to buyers and then charges a higher price to sellers. The following lemma formalizes the platform's optimal choice of pricing strategies (all proofs are in the appendix):

Lemma 1 *With unfavorable expectations and simultaneous entry, the platform's optimization problem is equivalent to solving* $\max_x \Pi(x)$*, where:*

$$\Pi(x) = \begin{cases} \frac{u(M,x)^2}{4} - C(x) - M\phi & \text{if} \quad u(M,x) \geq 2u(0,x) + \frac{M\pi(x)}{2} \\ \\ \frac{[u(M,x) - M\pi(x)]^2}{4} + M\pi(x)\,u(0,x) - C(x) - M\phi & \text{if} \quad u(M,x) \leq 2u(0,x) + \frac{M\pi(x)}{2} \end{cases} \tag{6}$$

The resulting solution is either x_{uf1}, defined in (4) and corresponding to the Seller Subsidy strategy, or x_{uf2}, defined in (5) and corresponding to the Buyer Attraction Strategy.

An important implication of Lemma 1 is that the optimal pricing strategy for the UF platform depends on its level of investment in first-party content x. In turn, whether the Seller Subsidy Strategy or the Buyer Attraction Strategy is optimal for higher x depends on the nature of buyer preferences over first- and third-party content, i.e. on whether m and x are complements or substitutes, as we will see below.

In the next subsections, we will compare the optimal levels of investment in first-party content chosen by the F and the UF platforms. The goal is to identify conditions under which one type of platform over-invests in first-party content relative to the other.

3.3 Complementarity and substitutability

We can now derive our first main result regarding platforms' levels of investment in first-party content:

Proposition 1 *If first-party content x and seller participation m are strong complements or independent, then the UF platform underinvests in first-party content relative to the F platform regardless of the pricing strategy chosen ($x_{uf1} < x_f$ and $x_{uf2} \leq x_f$). If x and m are strong substitutes then: i) when the UF platform chooses the Buyer Attraction Strategy, it overinvests in x relative to the F platform ($x_{uf2} > x_f$); ii) when the UF platform chooses the Seller Subsidy Strategy, it underinvests in x relative to the F platform ($x_{uf1} \leq x_f$) if seller profits per buyer are constant or only moderately decreasing in x, otherwise it overinvests.*

The fact that either one of the two types of platforms may end up investing more in x is due to the dual strategic role played by first-party content. On the one hand, it is an instrument for overcoming unfavorable expectations by attracting buyers *in spite* of sellers' reluctance to participate (indeed, both $u(M,x)$ and $u(0,x)$ are increasing in x). On the other hand, x is a measure of platform "quality" for buyers, which in turn also affects seller participation. Since the F platform captures a larger share of total surplus from buyers and sellers, it views investments in x as a way to increase the rents it is able to extract from both sides.

To see how these two roles play out, let us use (1) and (3) in order to compare the expressions of buyer and seller surplus extracted by the two types of platforms given the same level of investment in first-party content x:

	Rents extracted from buyers	Rents extracted from sellers
F platform	$\frac{[u(M,x)]^2-[M\pi(x)]^2}{4}$	$M\pi(x)\frac{[u(M,x)+M\pi(x)]}{2}$
UF platform (Seller Subsidy)	$\frac{[u(M,x)]^2}{4}$	0
UF platform (Buyer Attraction)	$\frac{[u(M,x)]^2-[M\pi(x)]^2}{4}$	$M\pi(x)\frac{[2u(0,x)+M\pi(x)-u(M,x)]}{2}$

The F platform *always* extracts more total surplus from buyers and sellers relative to the UF platform. But what determines investment incentives is not total surplus extracted but its *responsiveness* to an additional dollar of investment x. When x and M are complements, the F platform's return on a dollar of investment in first-party content is higher than for the UF platform. Indeed, compared to the UF platform that chooses the Seller Subsidy strategy, the F platform sacrifices $[M\pi(x)]^2/4$ surplus on the buyer side (through a lower buyer price) but extracts an additional $M\pi(x)[u(M,x)+M\pi(x)]/2$ on the seller side, for a net gain of $M\pi(x)[2u(M,x)+M\pi(x)]/4$, which is clearly *increasing* in x. Compared to the UF platform that chooses the Buyer Attraction strategy, the F platform obtains the same surplus from the buyer side but extracts an additional $M\pi(x)[u(M,x)-u(0,x)]$ on the seller side. This difference is also positive and increasing in x

when x and M are complements. Thus, the total surplus extracted by the F platform from buyers and sellers is the most responsive to increases in x. This is because the UF platform has a *lower* estimate of the synergies between first- and third-party content. As a result, in the case of complementarity the rent extraction role of first-party content carries the day.

When x and M are substitutes however, the UF platform may over-invest in x for two reasons. First and most obvious, when it chooses the Buyer Attraction strategy, the difference between the total surplus it extracts and that extracted by the F platform is now *decreasing* in x. This is because, if first-party content is a substitute for third-party content, *more* first-party content is needed by the platform when sellers are *absent* than when they are *present*. Second, if the UF platform chooses the Seller Subsidy strategy, it completely ignores the effect of first-party content on seller surplus because it extracts 0 surplus from them. Ignoring the *positive* effect of x on seller demand $(u(M,x) - p)$ tends to lead the UF platform to under-invest in first-party content, while ignoring the *negative* effect of x on seller profits per buyer $\pi(x)$ tends to lead UF to *over-invest* in x. If substitutability is sufficiently strong, i.e. if $\pi(x)$ is strongly decreasing in x, the latter effect may dominate the former.

Note that regardless of the pricing strategy chosen by the UF platform (Buyer Attraction or Seller Subsidy), $\pi_x < 0$ tends to lead to over-investment by the UF platform relative to the F platform, while $\pi_x > 0$ tends to lead to under-investment by the UF platform. This is simply because in all cases the F platform extracts a higher share of seller surplus per buyer $\pi(x)$ relative to the UF platform. The latter is therefore always less responsive to the effect of first-party content on sellers' profits: it over-invests when first-party crowds out third-party content and under-invests when the two are complementary.

The following table summarizes the results in Proposition 1 by taking the perspective of the UF platform relative to the F platform:

UF platform strategy	x and m are strong complements	x and m are strong substitutes
Buyer Attraction	*underinvest*	*overinvest*
Seller Subsidy	*underinvest*	*underinvest*

These results have implications for the evolution of vertical scope of two-sided platforms over time. Consider platforms which tend to rely on buyer attraction strategies. Our analysis predicts that such platforms would start off highly integrated in first-party content (x) which substitutes for third-party content (m) in order to overcome unfavorable expectations and then, as they become established in the marketplace, to rely less and less on substitute x investments but instead move towards investing more in complementary x investments. Casual empiricism lends support to these predictions. A good illustration is provided by the videogame industry, where console manufacturers (Microsoft, Nintendo and Sony), which sell their consoles at or below cost and derive most of their profits from royalties charged to game developers (cf. chapter 5 in Evans, Hagiu and Schmalensee (2006)). Throughout the history of videogames, every new entrant in the industry (Sega in 1988, Sony in 1994, Microsoft in 1999) has relied on a strong line-up of first-party content - either through in-house development or acquisition of independent development studios. Conversely, significant investments in first-party content that complements third-party games typically are made by established platform firms: examples include Microsoft's Xbox Live service (introduced towards the end of the first Xbox console cycle), Sony's PlayStation Store (launched only with the PlayStation 3), Sony's Moov and Microsoft's Kinnect motion-sensing controllers (introduced several years after the launch of their respective PlayStation 3 and Xbox 360 consoles).

3.4 Examples and further interpretation

To gain further insights into the drivers of platforms' investments in first-party content, let us derive closed form solutions corresponding to the specific examples introduced in the previous section.

Example 1 *Let* $u(m,x) = sm + x$, $\pi(x) = \pi$ *and* $C(x) = c\frac{x^2}{2}$, *where*[3] $s > 0$ *and* $2c > 1$*:*

- *The F platform chooses* $x_f = \frac{M(s+\pi)}{2c-1}$.
- *The UF platform chooses the Seller Subsidy Strategy with* $x_{uf1} = \frac{Ms}{2c-1} < x_f$ *if* $c\left(1 - \frac{\pi}{2s}\right) \geq 1$ *and the Buyer Attraction Strategy with* $x_{uf2} = \frac{M(s+\pi)}{2c-1} = x_f$ *if* $c\left(1 - \frac{\pi}{2s}\right) \leq 1$.

Example 2 *Let* $u(m,x) = u_0 + msx$, $\pi(x) = \pi$ *and* $C(x) = c\frac{x^2}{2}$, *where* $2u_0 > M\pi$ *and* $2c > M^2s^2$. *Then:*

- *The F platform chooses* $x_f = \frac{Ms(u_0+M\pi)}{2c-M^2s^2}$
- *The UF platform chooses the Seller Subsidy Strategy with* $x_{uf1} = \frac{Msu_0}{2c-M^2s^2} < x_f$ *if* $c \leq \frac{2u_0s^2M^2}{2u_0+M\pi}$ *and the Buyer Attraction Strategy with* $x_{uf2} = \frac{Ms(u_0-M\pi)}{2c-M^2s^2} < x_f$ *if* $c \geq \frac{2u_0s^2M^2}{2u_0+M\pi}$.

Example 3 *Let* $u(m,x) = ms(1-x) + M_0sx$, $\pi(x) = \pi$ *and* $C(x) = c\frac{x^2}{2}$, *where* $M_0 > M$ *and* $2c > (M_0 - M)^2 s^2$. *Then:*

- *The F platform chooses* $x_f = \frac{(M_0-M)Ms(s+\pi)}{2c-(M_0-M)^2s^2}$

[3] These conditions ensure concavity of all profit expressions in (p, x), as well as existence and uniqueness of all solutions derived below.

- *The UF platform chooses the Seller Subsidy Strategy with* $x_{uf1} = \frac{(M_0-M)Ms^2}{2c-(M_0-M)^2 s^2} < x_f$ *if* $\frac{c(2s-\pi)}{2s^2 M_0} \geq (M_0 - M)\, s + M\pi$ *and the Buyer Attraction Strategy with* $x_{uf2} = \frac{[(M_0-M)s+(M_0+M)\pi]Ms}{2c-(M_0-M)^2 s^2} > x_f$ *if* $\frac{c(2s-\pi)}{2s^2 M_0} \leq (M_0 - M)\, s + M\pi$.

Let us now interpret the results obtained with these examples. First, note that they clearly illustrate the general results from Proposition 1. The only case in which the UF platform invests more in first-party content relative to the F platform is when it chooses the Buyer Attraction strategy in example 3, where first-party content and third-party products are strict substitutes ($x_{uf1} < x_f < x_{uf2}$).

Impact of investment cost (c)

Second, an important aspect of our analysis is the way in which the investment cost c of first-party content impacts the choice of pricing strategy for the UF platform. Note that in example 2, the UF platform chooses the Seller Subsidy Strategy when c is *low*. By contrast, in examples 1 and 3, the UF platform chooses the Seller Subsidy Strategy when c is *high* (assuming $\pi < 2s$), so that both strategies are profitable on non-empty sets of c values). Consider example 2. A lower c increases the platform's incentives to invest in first-party content x. Because first-party content and third-party products are strict complements, raising x increases the benefits obtained by a buyer from the presence of a seller on the platform (they are equal to sx). On the other hand, a seller's benefits from the presence of a buyer remain unchanged (equal to π). Thus, the *net* effect of an increase in x is to make the buyer side easier to attract *relative to* the seller side. Standard two-sided market logic (cf. Armstrong 2006, Caillaud and Jullien 2003, Hagiu 2009) implies then that the platform is more likely to focus its subsidization efforts on the seller side and extract more profits from the buyer side, i.e. the platform will choose the Seller Subsidy Strategy. This is confirmed by plugging $u(m, x) = u_0 + msx$ in expression (6): in this case, the condition for the UF platform to choose the Seller Subsidy Strategy holds for *high* x.

Example 3 is interpreted in a similar way. Since first-party content and third-party products are strict substitutes, a higher x decreases the net benefit obtained by a buyer from the presence of a seller on the platform (it is equal to $s(1-x)$). Consequently, the *net* effect of a lower c is to make the buyer side *harder* to attract relative to the seller side, which in turn makes the UF platform more likely to choose the Buyer Attraction Strategy. Note however that the same reasoning does not quite work for example 1 because there x has no impact on the benefits s obtained by buyers from the presence of sellers. For that example, the fact that a lower c makes the UF platform more likely to choose the Buyer Attraction Strategy is happenstance, determined by the specific structure of our model.

Impact of seller profits (π)

Finally, note that in all three examples the UF platform is more likely to choose the Buyer Attraction Strategy over the Seller Subsidy Strategy when π is larger (and vice-versa). This is a general result with a standard intepretation in the two-sided market literature: the Buyer Attraction Strategy involves offering a lower price to the buyer side and extracting more profits from the seller side, therefore it is naturally more attractive when the benefits that sellers obtain from the presence of buyers are larger.

Impact of buyer standalone utility (s, u_0 or M_0)

Conversely, the Seller Subsidy Strategy should be more attractive when buyers' stand-alone utilities, or the benefits they derive from the presence of sellers, are larger. This is indeed the case in examples 1 and 2: the Seller Subsidy Strategy is more likely to be chosen when s (example 1) and u_0 (example 2) are larger. In example 3 however, the Seller Subsidy Strategy is more likely to be chosen for *lower* M_0. This is somewhat surprising given that M_0 can be interpreted as a measure of standalone utility offered to buyers: one would expect that higher M_0 should make buyers easier to attract and therefore make the platform more likely to subsidize sellers. The reason this intuition breaks down is the presence of first-party content, x, which changes the platform's optimal strategy in subtle ways. An increase in M_0 raises the effectiveness of investments in x, which means the platform has stronger incentives to invest in first-party content. But given the substitutability with third-party products, higher x decreases the *net* benefits obtained by buyers from the presence of sellers (they are equal to $(1-x)\,s$) on the platform. Since seller benefits from the presence of buyers remain unchanged (equal to π), this means that the *net* effect of an increase in M_0 is to make the buyer side harder to attract *relative to* the seller side. Standard two-sided market logic implies then that the platform is more likely to focus its subsidization efforts on the buyer side, i.e. to choose the Buyer Attraction Strategy.

4 Monopoly platform with sequential participation decisions

In many two-sided markets, one side - typically sellers - "arrives" or has to be secured by the platform before the other side - typically buyers - can make its adoption decisions. This can be either for exogenously given technological reasons (e.g. videogame consoles have to approach independent game publishers 1-2 years before the planned launch of their gaming systems in order to allow enough time for game development) or a strategic choice by the platforms, in an effort to secure participation by one side earlier.

Consequently, in this section we modify the basic model laid out above by assuming sellers make their platform adoption decisions *before* buyers and the latter can observe the sellers' adoption decisions prior to making their own. Since buyers can observe sellers' decisions (i.e. the realization of m), they face no uncertainty and no coordination problem. On the other hand, sellers now face a coordination problem among themselves. Thus, going from simultaneous to sequential adoption by the two sides has transformed a *two-sided* coordination problem with *indirect* network effects into a *one-sided* coordination problem with *direct* network effects (among sellers).

There is however one key difference with standard pricing problems in the presence of direct network effects. Since buyers arrive after sellers, the platform has two options: it can either choose to commit (if possible) to the price it will charge buyers at the time it announces its price for sellers or it can wait until after sellers have made their adoption decisions and announce its price to buyers afterwards (which is of course factored in the sellers' decisions). We thus have two possible timings:

With commitment	**Without commitment**
1.a) Platform sets w, p and x	1.a) Platform sets w and x
1.b) Sellers decide whether or not to adopt	1.b) Sellers decide whether or not to adopt
2.a) Buyers observe everything and decide whether or not to adopt	2.a) Platform observes sellers' decisions and sets p
	2.b) Buyers observe everything and decide whether or not to adopt

If the platform chooses to commit, then we denote by p^c the buyer price it commits to in stage 1. If it does not commit, then in stage 2 it will choose a buyer price $p(m)$ which is a function of the number m of sellers who have adopted in stage 1. That price maximizes the platform's stage 2 profits, therefore it is given by:

$$p(m) = \arg\max_p \{p[u(m,x) - p]\} = \frac{u(m,x)}{2}$$

In the commitment scenario, the net profits derived by individual sellers when they adopt are:

$$H(m,w) = \pi(x) \max[u(m,x) - p^c, 0] - \phi - w$$

Indeed, note that these profits now depend directly on the number m of sellers who adopt, since that number determines in turn total buyer participation on the platform in the second stage. It is therefore clear that sellers' decisions whether or not to adopt the platform now exhibit *direct* network effects. These network effects are positive since $u(m,x)$ is increasing in m.

In the no-commitment scenario, individual sellers' net profits when they adopt are:

$$H(m,w) = \pi(x)[u(m,x) - p(m)] - \phi - w = \pi(x)\frac{u(m,x)}{2} - \phi - w$$

Again, there are positive, direct network effects between sellers' adoption decisions.

Even though the indirect network effects have now been transformed in direct network effects among sellers, the platform can still face two types of expectations - favorable and unfavorable.

4.1 Favorable expectations

If expectations are favorable, then in the first stage all M sellers will coordinate on the fulfilled-expectations equilibrium with highest adoption for the platform. In this case, if the F platform commits to p^c then all M sellers adopt if and only if:

$$w \leq \pi(x)\left[u(M,x) - p^c\right] - \phi$$

Then the F platform's optimization problem is:

$$\max_{p^c,x} \left\{\left[\pi(x) M + p^c\right]\left[u(M,x) - p^c\right] - C(x) - M\phi\right\} \tag{7}$$

which is exactly the same as in the case with simultaneous entry and favorable expectations (cf. (1) above). Thus, with commitment, the F platform obtains the same profits and chooses the same level of investment in first-party content x_f (cf. (2) above) as in the case with simultaneous entry.

If on the other hand the F platform does not commit to p^c then sellers anticipate it will charge $p = u(m,x)/2$ in the second period and therefore adopt if and only if $w \leq \pi(x) u(M,x)/2 - \phi$. Thus, the F platform's optimization problem becomes (its profits are the sum of first stage profits from sellers and second stage profits from buyers):

$$\max_{x} \left\{\left[\pi(x) M + \frac{u(M,x)}{2}\right]\frac{u(M,x)}{2} - C(x) - M\phi\right\} \tag{8}$$

Comparing (7) with (8), it is apparent that with commitment the F platform has an additional degree of freedom (p^c), so that:

Remark 1 *When sellers arrive before buyers, the F platform always prefers to commit to its buyer price upfront, resulting in the same profits and the same investment in first-party content (x_f) as in the case with simultaneous entry.*

The fact that commitment is always the dominant strategy for the F platform should not be surprising in a monopoly setting. If the platform does not commit, it suffers from a time inconsistency (or hold-up) problem: sellers correctly anticipate that in the second stage the platform will choose $p = u(M,x)/2$ to maximize its own second stage profits, whereas the optimal buyer price from the first stage perspective (for the platform since it extracts all seller surplus) is the one that maximizes joint profits, i.e. $p^c = [u(M,x) - \pi(x)M]/2$. As we will see below however, this logic breaks down for a platform facing unfavorable expectations, so that it may find it profitable not to commit.[4]

4.2 Unfavorable expectations

If expectations are unfavorable, then in the first stage all M sellers will coordinate on the fulfilled-expectations equilibrium with lowest adoption for the UF platform. In this case, if the UF platform commits to p^c then sellers adopt if:

$$w \leq \pi(x) \max[u(0,x) - p^c, 0] - \phi$$

Otherwise, no seller adopts. The platform will set w such that this constraint binds, so that its resulting optimization problem is:

$$\max_{p^c,x} \{M\pi(x) \max[u(0,x) - p^c, 0] + p^c [u(M,x) - p^c] - C(x) - M\phi\}$$

which is identical to the UF platform optimization in the case with simultaneous entry (cf. (3) above). Thus, just like for the F platform, commitment to the price charged to buyers in the case with sequential entry replicates the outcome of the case with simultaneous entry.

If on the other hand the UF platform does not commit to p then sellers adopt if and only if $w \leq \pi(x) u(0,x)/2 - \phi$, so that the UF platform's optimization problem becomes:

$$\max_x \underbrace{M\pi(x)\frac{u(0,x)}{2}}_{\substack{\text{rents extracted} \\ \text{from sellers}}} + \underbrace{\frac{u(M,x)^2}{4}}_{\substack{\text{rents extracted} \\ \text{from buyers}}} - C(x) - M\phi \tag{9}$$

yielding x_{uf3}, which is the solution to:

$$\frac{u_x(M,x)\,u(M,x)}{2} + \frac{M\pi_x(x)\,u(0,x) + M\pi(x)\,u_x(0,x)}{2} = C'(x) \tag{10}$$

Recall that with commitment (just like in the simultaneous entry case), there are only two possible solutions, corresponding to two distinct pricing strategies: one with $p_{uf} > u(0, x_{uf})$ and

[4] A version of this result was first proven by Hagiu (2006) in a simpler model. Here however, our focus is ultimately on the levels of investment in first-party content made by platforms.

$w_{uf} = -\phi$ and the other with $p_{uf} < u(0, x_{uf})$ and $w_{uf} = \pi(x)[u(0, x_{uf}) - p_{uf}] - \phi$. But now note that no commitment *always* dominates the first of these strategies. Indeed, when feasible, the latter yields profits equal to $\max_x \{u(M, x)^2/4 - C(x) - M\phi\}$, which is strictly lower than (9). This is understood in the following way: if the platform is to subsidize sellers, it is better not to commit, in which case it maintains the flexibility to charge a higher price to buyers, which in turn allows it to also charge a higher price to sellers ($w = \pi(x)u(0, x)/2 - \phi$ instead of $w = -\phi$).

Consequently, the only relevant strategy with commitment has $p_c = [u(M, x) - M\pi(x)]/2 < u(0, x)$, yielding profits equal to:

$$\max_x \underbrace{M\pi(x)\left[u(0, x) - \frac{u(M, x) - M\pi(x)}{2}\right]}_{\substack{\text{rents extracted} \\ \text{from sellers}}} + \underbrace{\frac{u(M, x)^2 - [M\pi(x)]^2}{4}}_{\substack{\text{rents extracted} \\ \text{from buyers}}} - C(x) - M\phi \tag{11}$$

Comparing (11) with (9), it is not clear whether the platform prefers to commit or not. The two expressions above make it clear that for a platform facing unfavorable expectations, commitment involves giving up the ability to extract higher rents from the buyer side (since the platform must commit to a low buyer price to convince sellers to join) in order to extract higher rents from the seller side. This can only be profitable if the surplus that can be extracted from the seller side is sufficiently large relative to the surplus that can be extracted on the buyer side. Otherwise, the platform is better off attracting sellers with a low price and maintaining the flexibility to charge a high price to buyers once sellers have adopted (no commitment). This is in stark contrast with the platform facing favorable expectations.

The following lemma characterizes the optimal pricing strategy for the platform facing unfavorable expectations as a function of its level of investment in first-party content.

Lemma 2 *Given x, a platform facing unfavorable expectations and sequential entry commits to its buyer price in stage 1 if and only if $u(M, x) \geq u(0, x) + M\pi(x)/2$. Its optimization problem is equivalent to solving $\max_x \Pi(x)$, where:*

$$\Pi(x) = \begin{cases} M\pi(x)\frac{u(0,x)}{2} + \frac{u(M,x)^2}{4} - C(x) - M\phi & \text{if} \quad u(M, x) \geq u(0, x) + \frac{M\pi(x)}{2} \\ \\ \frac{[u(M,x) - M\pi(x)]^2}{4} + M\pi(x)u(0, x) - C(x) - M\phi & \text{if} \quad u(M, x) \leq u(0, x) + \frac{M\pi(x)}{2} \end{cases} \tag{12}$$

The resulting solution is either x_{uf3} defined by (9) or x_{uf2} defined in (5).

The choice of optimal pricing strategies for a given x defined in Lemma 2 is similar to the one defined in Lemma 1, but with two important differences. First, the Seller Subsidy Strategy has been replaced by the no commitment strategy, which yields higher profits. Although these strategies yield different total profits, they both rely on subsidizing the participation of the seller side[5] and extracting more rents from the buyer side. Second, the Buyer Attraction Strategy is chosen *less* often. Overall, the UF platform is better off when the two sides arrive sequentially than when they arrive simultaneously. This is because sequential arrival allows the UF platform to extract significant surplus from the side arriving later (buyers) *after* it has secured the participation of sellers. In contrast, when the two sides arrive simultaneously, the UF platform can never afford to charge prices to one side which *assume* the other side will participate.

4.3 Substitutability and complementarity

Let us now compare the levels of investments in first party content made by platforms facing favorable, respectively unfavorable expectations for the case with sequential entry. The following proposition summarizes this comparison (assuming second order conditions are satisfied):

Proposition 2 *If content and seller participation are strong complements then the UF platform always underinvests in first-party content relative to the F platform, i.e.* $x_{uf2} < x_f$ *and* $x_{uf3} < x_f$*. If content and seller participation are independent then* $x_{uf2} = x_f = x_{uf3}$*. If content and seller participation are strong substitutes then the UF platform always underinvests in first-party content relative to the F platform, i.e..* $x_{uf2} > x_f$ *and* $x_{uf3} > x_f$*.*

The result contained in Proposition 2 is similar to - but cleaner than - the one from Proposition 1: the UF platform invests less (more) in first-party content relative to the F platform facing favorable expectations whenever content and seller participation are strong complements (substitutes).

UF platform strategy	x and m are strong complements	x and m are strong substitutes
Buyer Attraction (commitment)	*underinvest*	*overinvest*
Seller Subsidy (no commitment)	*underinvest*	*overinvest*

[5] In the case with sequential entry, the no commitment strategy involves $w = u(0, x)/2 - \phi$, which may not be a subsidy strictly speaking if $u(0, x)$ is sufficiently large. What we mean by subsidization is charging one side a price lower than the price which would maximize profits conditional on participation by the other side.

The profits extracted by the F platform and by the UF platform when it chooses the Buyer Attraction strategy (which implies commitment) are exactly the same as in the scenario with simultaneous entry, which is why the first row in the table above is the same as in the corresponding table from the previous section. The only change is that when the UF platform chooses the "new" Seller Subsidy strategy with no commitment, it now extracts a larger surplus from sellers ($M\pi(x)\,u(0,x)/2$) relative to what the Seller Subsidy yielded under simultaneous entry (0):

	Rents extracted from buyers	Rents extracted from sellers
F platform	$\frac{[u(M,x)]^2-[M\pi(x)]^2}{4}$	$M\pi(x)\frac{[u(M,x)+M\pi(x)]}{2}$
UF platform (no commitment)	$\frac{[u(M,x)]^2}{4}$	$M\pi(x)\frac{u(0,x)}{2}$
UF platform (commitment)	$\frac{[u(M,x)]^2-[M\pi(x)]^2}{4}$	$M\pi(x)\frac{[2u(0,x)+M\pi(x)-u(M,x)]}{2}$

As a result, the incentives to invest in first-party content for the UF platform that chooses to subsidize sellers are now stronger: in particular, when x and M are substitutes, it now *over-invests* relative to the F platform.

4.4 Examples

Let us now return to the three examples used in the previous section (the same restrictions on parameters are assumed to hold so we do not repeat them here).

Example 1 *Let $u(m,x)=ms+x$, $\pi(x)=\pi$ and $C(x)=c\frac{x^2}{2}$. Then the UF platform commits if $\pi \geq 2s$; it does not commit if $\pi \leq 2s$. In both cases, the UF platform sets $x_{uf2}=x_{uf3}=\frac{M(s+\pi)}{2c-1}=x_f$.*

Example 2 *Let $u(m,x)=u_0+msx$, $\pi(x)=\pi$ and $C(x)=c\frac{x^2}{2}$. Then:*

- *the UF platform commits and sets $x_{uf2}=\frac{Ms(u_0-M\pi)}{2c-M^2s^2}<x_f$ if $c\geq\frac{Ms^2u_0}{\pi}$*
- *it does not commit and sets $x_{uf3}=\frac{Msu_0}{2c-M^2s^2}<x_f$ if $c\leq\frac{Ms^2u_0}{\pi}$.*

Example 3 *Let $u(m,x)=ms(1-x)+M_0sx$, $\pi(x)=\pi$ and $C(x)=c\frac{x^2}{2}$. Then:*

- *the UF platform commits and sets $x_{uf2}=\frac{[(M_0-M)s+(M_0+M)\pi]Ms}{2c-(M_0-M)^2s^2}>x_f$ if $\frac{2c(2s-\pi)}{M_0s^2}\leq 2(M_0-M)s+(4M-M_0)\pi$*

- *it does not commit and sets* $x_{uf3} = \frac{[(M_0-M)s+M_0\pi]Ms}{2c-(M_0-M)^2s^2} < x_{uf2}$ *if* $\frac{2c(2s-\pi)}{M_0s^2} \geq 2(M_0-M)s + (4M-M_0)\pi$.

The way in which the cost c of first-party content impacts the choice of pricing strategy in examples 2 and 3 is the same as with simultaneous entry, replacing the Seller Subsidy strategy by the no-commitment strategy (both rely on subsidizing sellers and making more profits on buyers). In example 2, no commitment is chosen when the cost of providing first-party content is *low*. By contrast, in example 3 no commitment is chosen when the cost of providing first-party content is *high* (assuming $\pi < 2s$).

In example 1 however, first-party content has no effect on the choice between the two strategies, which is solely determined by π and s, the respective benefits that each side derives from the presence of the other side. Specifically, the UF platform chooses not to commit if the surplus derived by sellers is sufficiently small relative to the surplus derived by buyers ($\pi < 2s$). This is intuitive: the no-commitment strategy aims to extract relatively more surplus from buyers, therefore it is more attractive when s is larger and π is smaller. As first-party content and third-party products are neither complements nor substitutes, it turns out that c has no impact on the comparison between the two strategies (they both result in the same level of first-party content).

All of the other comparative statics (effects of π, s, u_0 and M_0) and corresponding discussion from the case with simultaneous entry go through unchanged.

5 Competition between homogeneous platforms

In this section we turn to the analysis of platform competition with simultaneous entry of the two sides. There are two identical platforms, which we denote I (incumbent) and E (entrant). Each platform chooses (p_k, w_k, x_k) where $k = \{I, E\}$. If both platform faced the same kind of user expectations, they would engage in Bertrand competition, which would lead to 0 profits. We therefore focus on the more interesting case in which the incumbent faces favorable expectations while the entrant faces unfavorable expectations. The way in which these expectations determine market outcomes is the same as in Caillaud and Jullien (2003) and Hagiu (2006): market participants always coordinate on the equilibrium which maximizes adoption on both sides for I and minimizes adoption for E.

We analyze two alternative scenarios: in the first one sellers single-home (i.e. join at most one platform) and buyers multi-home (i.e. can join both platforms); in the second one buyers single-home and sellers multi-home. The third scenario, in which both sides singlehome, is technically complex and does not add any further insights: we treat an example in the online appendix.

5.1 Sellers single-home and buyers multi-home

Suppose first that sellers can only join one platform at most, whereas buyers may join both. In this context, buyer i joins platform $k \in \{I, E\}$ if and only if $u(m_k^e, x_k) - p_k - i \geq 0$, where m_k^e is the expected number of sellers who join k and, as in the monopoly section, i is uniformly distributed over $[0, 1]$. A seller who joins platform $k \in \{I, E\}$ derives payoff $\pi(x_k) n_k^e - w_k - \phi$, where $n_k^e = u(m_k^e, x_k) - p_k$ is the expected number of buyers who join k. All M sellers are identical, therefore they all make the same platform adoption decision: they join the platform $k \in \{I, E\}$ which offers the higher payoff (we assume they join I if indifferent). The following proposition characterizes the competitive outcome for this scenario:

Proposition 3 *When sellers single-home and buyers multi-home, the incumbent keeps the entrant out of the market and makes the same investment in first-party content as if it were a monopolist facing favorable expectations.*

Proof. The proof is an extension of the main results in Caillaud and Jullien (2003) and Hagiu (2006). We relegate it to the online appendix (section 3.1). ■

Note that unlike in the monopoly analysis, this result does not depend on complementarity or substitutability between first- and third-party content. It is due to the fact that here competition between the entrant and the incumbent focuses on attracting sellers (buyers multihome). Indeed, I can simply discount w_I by a fixed amount to prevent sellers from joining E.

This has an interesting policy implication. Suppose that the incumbent firm were to merge with or acquire the prospective entrant, making the incumbent a monopolist. In evaluating such a merger or acquisition, the proper comparison would be between competition and a monopolist that faced favorable expectations. The merger or acquisition would have no effect on the incumbent's investment in first-party content or the buyer price. So, when sellers single-home and buyers multi-home, such a merger or acquisition need not raise antitrust concerns. The outcome differs when competition focuses on buyers as the next section shows.

5.2 Buyers single-home and sellers multi-home

Suppose now that buyers single-home whereas sellers multi-home. Buyer i's utility from joining platform k is $u(m_k^e, x_k) - p_k - i$. A seller who joins platform $k \in \{I, E\}$ obtains payoffs $\pi(x_k) n_k^e - w_k - \phi$, whereas a seller who multihomes obtains payoffs $[\pi(x_E) n_E^e + \pi(x_I) n_I^e] - w_E - w_I - 2\phi$. The following proposition (proven in section 3.2 of the online appendix) characterizes the competitive outcome for this scenario:

Proposition 4 *When buyers single-home and sellers multi-home, the incumbent firm facing entry invests more in first-party content than when it faces no competition.*

Thus, in contrast to the previous scenario, competitive pressure from the entrant induces higher investments in first-party content by the incumbent. This is because competition is for buyers (sellers multihome), which means the incumbent has to offer higher net utility to buyers than it would if it were a monopolist. Since buyer net utility is increasing in x, the result follows. Once again, note that nothing depends on whether first-party content and seller participation are complements or substitutes.

For public policy purposes, competition between the incumbent and the entrant should be compared with a hypothetical monopolist facing favorable expectations. If the incumbent firm were to merge with or acquire the entrant, the effect would be to lower content provided to buyers. Overall, when buyers single-home and sellers multi-home, buyer benefits are increased by the effects of competition. This suggests that if the incumbent firm were to merge with or acquire the entrant, buyers would be made worse off.

6 Conclusion and managerial implications

Our study of two-sided platforms' incentives to invest in first-party content has yielded several important insights that should be of managerial interest. First, the strategic use of first-party content is determined by its relationship with third-party content. When first- and third-party content are substitutes, first-party content acts as a coordination instrument for platforms facing unfavorable expectations to compensate for their difficulty in attracting sellers. When first- and third-party content are complements, first-party content is an instrument used by platforms benefitting from favorable expectations to improve their power to extract rents from buyers and sellers. These points enrich the conventional wisdom, according to which platform businesses should produce first-party content in the early stages in order to stimulate user adoption and later in order to differentiate from competing platforms (cf. Evans et al. (2006) and Reisinger (2009)). By distinguishing between complementary and substitutable first-party content, managers should be better able to adjust platform companies' investments in content, depending on their development stage.

Second, for a platform facing unfavorable expectations, investments in first-party content are a third strategic variable, closely linked to the choice of the pricing strategy. Here, our model yields straightforward - and, to the best of our knowledge, novel - recommendations for managers. If first- and third-party content are substitutes (complements) then larger investments in first-party content decrease (increase) the benefits derived by buyers from the presence of third-party sellers, so the platform should charge less to buyers (sellers) and make more profits on sellers (buyers).

All of these results hold for monopoly platforms, both under simultaneous and under sequential entry of the two sides, i.e. when sellers arrive before buyers. In the latter scenario, our analysis yields an additional, important implication for practitioners. Two-sided platforms facing favorable expectations should *always* commit to the price charged to the side arriving later (buyers) at the time they are courting the side arriving earlier (sellers). This helps eliminate a potential hold-up problem vis-a-vis sellers, who worry that once they are on board, the platform might charge too much to buyers. In contrast, platforms facing unfavorable expectations should commit only if the surplus derived by sellers from trading on the platform is sufficiently large relative to the surplus derived by buyers. The reason no commitment may be profitable for such platforms is that it allows the flexibility to charge a higher price to buyers, *after* unfavorable expectations have been overcome.

We have also analyzed two scenarios with two homogeneous competing platforms, one facing favorable expectations, the other facing unfavorable expectations. Comparing the competitive outcome with the monopolist that faces favorable expectations has implications both for platform strategy and for antitrust policies toward mergers and acquisitions among platform firms. From a strategy perspective, it is noteworthy that incumbent platforms need not invest more in first-party content when they face more competition: they can keep entrants out and maintain positive profits simply by adjusting their pricing structure. This is true when sellers single-home and buyers multi-home. The competition policy implication is that, in this case, a merger between the incumbent and the entrant need not raise antitrust concerns because it would have no effect on the incumbent's investment in first-party content or the buyer price. If on the other hand buyers single-home and sellers multi-home, the incumbent platform always invests more when it faces competition than it does when it is an unconstrained monopolist. Then, the merger considered earlier would raise antitrust concerns because it would result in lower investments in first-party content as well as in lower buyer suplus (higher buyer prices).

There is significant scope to build upon our analysis, both theoretically and empirically. On the theory side, an in-depth analysis of a model with a monopoly platform and one or two strategic third-party content providers is warranted (as opposed to the continuum of content providers we have assumed here). The goal would be to abstract away from the issue of expectations and explore instead the role of first-party vs. third-party content in resolving investment risks. Furthermore, one could build a dynamic version of our model in order to explore how the optimal intensity and nature of platforms' investments in first-party content evolve over time. On the empirical side, an interesting avenue of research would be to rigorously measure the allocation of platform investment resources between first-party content that is substitutable for and first-party content that is complementary to third-party content.

References

[1] Armstrong, M. (2006) "Competition in two-sided markets," *RAND Journal of Economics* 37, 668-691.

[2] Belleflamme P. and Toulemonde E. (2009): "Negative Intra-group Externalities in Two-sided Markets", *International Economic Review* 50(1), 245-272.

[3] Boudreau, K.J. (2010) "Open Platform Strategies and Innovation: Granting Access vs. Devolving Control," *Management Science* 56(10), 1849-1872.

[4] Caillaud, B., and B. Jullien (2003) "Chicken and Egg: Competition Among Intermediation Service Providers," *Rand Journal of Economics,* 34(2), 309-328.

[5] Economides, N. and E. Katsamakas (2006) "Two-Sided Competition of Proprietary vs. Open Source Technology Platforms and the Implications for the Software Industry," *Management Science* 52(7), 1057-1071.

[6] Eisenmann, T., G. Parker and M. Van Alstyne (2006) "Strategies for Two-Sided Markets," *Harvard Business Review* 84(10), 92-101.

[7] Evans, D. S., A. Hagiu and R. Schmalensee (2006) *Invisible Engines: How Software Platforms Drive Innovation and Transform Industries*, Cambridge, MA: The MIT Press.

[8] Farrell, J. and P. Klemperer (2007) "Coordination and Lock-In: Competition with Switching Costs and Network Effects," *Handbook of Industrial Organization,* vol. 3, Amsterdam: North Holland, 1967-2072.

[9] Gates, B. (1996) "Content is king," http://www.craigbailey.net/content-is-king-by-bill-gates/ (21 October 2011)

[10] Gawer, A. (2009) "Platform Dynamics and Strategies," *Platforms, Markets and Innovation*, Cheltenham, UK and Northampton, MA: Edward Elgar, 45-76.

[11] Gawer, A. and M. Cusumano (2002) *Platform Leadership: How Intel, Microsoft, and Cisco Drive Industry Innovation*, Boston, MA: Harvard Business School Press.

[12] Hagiu, A. (2006) "Pricing and Commitment by Two-Sided Platforms," *Rand Journal of Economics*, Vol. 37 (3), 720-737.

[13] Hagiu, A. (2009) "Two-Sided Platforms: Product Variety and Pricing Structures," *Journal of Economics & Management Strategy*, Vol. 18(4), 1011-1043.

[14] Halaburda, H. and Y. Yehezkel (2011) "Platform Competition Under Asymmetric Information," Harvard Business School Working Paper 11-080.

[15] Iansiti, M. and R. Levien (2004) *The Keystone Advantage: What the New Dynamics of Business Ecosystems Mean for Strategy, Innovation and Sustainability*, Boston, MA: Harvard Business School Press.

[16] Kim, D. and B. Bogut (1996) "Technological Platforms and Diversification," *Organization Science*, Vol 7(3), 283-301.

[17] Nocke, V., M. Peitz and K. Stahl (2007) "Platform Ownership," *Journal of the European Economic Association*, 5(6), 1130-1160.

[18] Parker, G. G. and M. W. Van Alstyne (2005) "Two-Sided Network Effects: A Theory of Information Product Design," *Management Science,* 51: 1494-1504.

[19] Parker, G. and Van Alstyne, M. W. "Innovation, Openness and Platform Control." Working Paper, Tulane University, Boston University and MIT.

[20] Reisinger, D. (2009) "Why First-Party Video Games Matter," *CNET News*, May 12, http://news.cnet.com/8301-13506_3-10236513-17.html (21 October 2011)

[21] Rochet, J.-C., and J. Tirole (2006) "Two-Sided Markets: Where We Stand," *Rand Journal of Economics*, Vol. 37 (3), 645-666.

[22] Spulber, D. F. (1999) *Market Microstructure: Intermediaries and the Theory of the Firm*, Cambridge: Cambridge University Press.

[23] Spulber, D. F. (2010) "Solving the Circular Conundrum: Communication and Coordination in Two-Sided Networks," *Northwestern University Law Review*, Vol. 104, 537-591.

[24] Weyl, G. (2010) "A Price Theory of Multi-Sided Platforms," *American Economic Review*, 100, pp. 1642-1672.

[25] Zhu, F. and M. Iansiti (2011) "Entry Into Platform-Based Markets," *Strategic Management Journal*.

7 APPENDIX

Proof. of Lemma 1.

Fix x. For $p \geq u(0,x)$, the expression of platform profits is:

$$\Pi_1(p,x) = p\left[u(M,x) - p\right] - M\phi - C(x)$$

which attains its maximum in p at $p_1(x) = u(M,x)/2$.

For $p \leq u(0,x)$, the expression of platform profits is:

$$\Pi_2(p,x) = p\left[u(M,x) - p\right] + M\pi(x)\left[u(0,x) - p\right] - M\phi - C(x)$$

which attains its maximum in p at $p_2(x) = \left[u(M,x) - M\pi(x)\right]/2 < p_1(x)$.

There are thus three possibilities:

- if $u(M,x) < 2u(0,x)$ then the platform's optimal choice of p is $p_2(x)$
- if $u(M,x) > 2u(0,x) + M\pi(x)$ then the platform's optimal choice of p is $p_1(x)$
- if $2u(0,x) \leq u(M,x) \leq 2u(0,x) + M\pi(x)$ then the platform's optimal choice of p is $p_1(x)$ if $\Pi_1(p_1(x),x) \geq \Pi_2(p_2(x),x)$ and $p_2(x)$ otherwise.

But $\Pi_1(p_1(x),x) \geq \Pi_2(p_2(x),x)$ is equivalent to $u(M,x) \geq 2u(0,x) + M\pi(x)/2$. Thus:

$$\Pi(x) = \begin{cases} \frac{u(M,x)^2}{4} - C(x) - M\phi & \text{if} \quad u(M,x) \geq 2u(0,x) + \frac{M\pi(x)}{2} \\ \frac{\left[u(M,x) - M\pi(x)\right]^2}{4} + M\pi(x)\,u(0,x) - C(x) - M\phi & \text{if} \quad u(M,x) \leq 2u(0,x) + \frac{M\pi(x)}{2} \end{cases}$$

as stated in the text of the lemma.

The only thing left to verify is that the corner solution x_0 defined by $u(M,x_0) = 2u(0,x_0) + M\pi(x_0)/2$ cannot maximize $\Pi(x)$ unless x_0 is a maximizer of $\Pi_1(p_1(x),x)$ or $\Pi_2(p_2(x),x)$, i.e. $x_0 = x_{uf1}$ (defined in 4) or $x_0 = x_{uf2}$ (defined in 5). Suppose by contradiction that x_0 maximizes $\Pi(x)$ but $x_0 \neq x_{uf1}$ and $x_0 \neq x_{uf2}$. Then the first derivatives of $\Pi_1(p_1(x),x)$ and $\Pi_2(p_2(x),x)$ in x evaluated at x_0 must have opposite signs. There are two possibilities:

1. Either:

$$\begin{aligned} \frac{\left[u_x(M,x_0) - M\pi_x(x_0)\right]\left[u(M,x_0) - M\pi(x_0)\right]}{2} + M\pi(x_0)\,u_x(0,x_0) + M\pi_x(x_0)\,u(0,x_0) - C'(x_0) &> 0 \\ \text{and} \quad \frac{u_x(M,x_0)\,u(M,x_0)}{2} - C'(x_0) &< 0 \end{aligned}$$

Subtracting the first from the second inequality and re-arranging we obtain:

$$\frac{\pi(x_0)}{2}\left[u_x(M,x_0)-2u_x(0,x_0)-\frac{M\pi_x(x_0)}{2}\right]+\frac{\pi_x(x_0)}{2}\left[u(M,x_0)-2u(0,x_0)-\frac{M\pi(x_0)}{2}\right]<0$$

But note that the left-hand side is the derivative in x of $\frac{\pi(x)}{2}\left[u(M,x)-2u(0,x)-\frac{M\pi(x)}{2}\right]$ evaluated at x_0. This means that $\frac{\pi(x)}{2}\left[u(M,x)-2u(0,x)-\frac{M\pi(x)}{2}\right]$ is decreasing in x for all x smaller than but sufficiently close to x_0. In turn, this implies that $u(M,x)-2u(0,x)-\frac{M\pi(x)}{2}>0$ for all such x, so that $\Pi(x)=\Pi_1(p_1(x),x)$. But then, since x_0 is a maximizer of $\Pi(x)$, the derivative of $\Pi_1(p_1(x),x)$ in x evaluated at x_0 must be non-negative, i.e. $\frac{u_x(M,x_0)u(M,x_0)}{2}-C'(x_0)\geq 0$, which is a contradiction.

2. Or:

$$\begin{aligned}\frac{[u_x(M,x_0)-M\pi_x(x_0)][u(M,x_0)-M\pi(x_0)]}{2}+M\pi(x_0)u_x(0,x_0)+M\pi_x(x_0)u(0,x_0)-C'(x_0) &< 0\\ \text{and}\quad \frac{u_x(M,x_0)u(M,x_0)}{2}-C'(x_0) &> 0\end{aligned}$$

Using the exact same reasoning as for the previous case and focusing on x larger than but sufficiently close to x_0, we obtain once again a contradiction. ■

Proof. of Proposition 1.

Assume second-order conditions are verified so that x_f, x_{uf1} and x_{uf2} are uniquely defined.

Compare then the first-order condition determining x_f (equation 2) with the one determining x_{uf2} (equation 5), the investment in first-party content by the UF platform when it chooses the Buyer Attraction strategy. Specifically, let us plug x_{uf2} instead of x_f in (2) taking (5) into account:

$$\begin{aligned}&\frac{[u_x(M,x_{uf2})+M\pi_x(x_{uf2})][u(M,x_{uf2})+M\pi(x_{uf2})]}{2}-C'(x_{uf2})\\ =\;& M\{\pi(x_{uf2})[u_x(M,x_{uf2})-u_x(0,x_{uf2})]+\pi_x(x_{uf2})[u(M,x_{uf2})-u(0,x_{uf2})]\}\end{aligned}$$

If x and m are strong complements (strong substitutes) then the last expression is positive (negative) because $u_x(M,x)-u_x(0,x)>0$ (<0) and $\pi_x\geq 0$ (≤ 0). Therefore, if x and m are strong complements (strong substitutes) then $x_{uf2}<x_f$ ($x_{uf2}>x_f$). If they are independent, then $x_{uf2}=x_f$.

Similarly, compare (2) with (4) and let us plug x_{uf1} instead of x_f in (2) taking (4) into account:

$$\begin{aligned}&\frac{[u_x(M,x_{uf1})+M\pi_x(x_{uf1})][u(M,x_{uf1})+M\pi(x_{uf1})]}{2}-C'(x_{uf1})\\ =\;& \frac{M}{2}\{\pi(x_{uf1})u_x(M,x_{uf1})+\pi_x(x_{uf1})[u(M,x_{uf1})+M\pi(x_{uf1})]\}\end{aligned}$$

If x and m are strong complements or independent then the last expression is positive, so that $x_{uf1}<x_f$.

If on the other hand x and m are strong substitutes, then the sign of the last expression is ambiguous. Clearly, if $\pi_x(x)$ is equal to 0 or sufficiently small for all x then the last expression remains positive, so $x_{uf1} \leq x_f$. Otherwise, it is possible that the expression is negative, so that $x_{uf1} > x_f$. ■

Proof. of Lemma 2.

If x is such that $u(M,x) \geq 2u(0,x) + M\pi(x)/2$ then we know from Lemma 1 and expression (9) that the platform's profits with commitment are strictly lower than the profits without commitment. Suppose then that x is such that $u(M,x) \leq 2u(0,x) + M\pi(x)/2$. Then the profits obtained by the platform when it commits are higher than when it does not commit if and only if:

$$\frac{[u(M,x) - M\pi(x)]^2}{4} + M\pi(x)\,u(0,x) \geq M\pi(x)\frac{u(0,x)}{2} + \frac{u(M,x)^2}{4}$$

which is equivalent to:

$$\frac{M\pi(x)}{2} \geq u(M,x) - u(0,x)$$

Note that this last inequality holds only if $u(M,x) \leq 2u(0,x) + M\pi(x)/2$ holds, which leads to expression (12) in the text of the lemma.

As in Lemma 1, we need to verify that the corner solution x_0 defined by $u(M,x_0) = u(0,x_0) + M\pi(x_0)/2$ cannot maximize $\Pi(x)$ unless x_0 is a maximizer of $\frac{[u(M,x)-M\pi(x)]^2}{4} + M\pi(x)\,u(0,x) - C(x)$ or of $M\pi(x)\frac{u(0,x)}{2} + \frac{u(M,x)^2}{4} - C(x)$, i.e. $x_0 = x_{uf2}$ (defined in 5) or $x_0 = x_{uf3}$ (defined in 10). This part of the proof is identical to Lemma 1 (reasoning by contradiction), therefore we omit it here. ■

Proof. of Proposition 2

The comparison of x_{uf2} with x_f is unchanged from the case with simultaneous entry (Proposition 1).

Let us now compare x_f with x_{uf3}. Since second-order conditions are satisfied, they are uniquely determined by the first-order conditions (2) and 10) respectively and $x_{uf3} < x_f$ if and only if the first-order condition determining x_f (2) evaluated at x_{uf3} is positive, i.e. if and only if:

$$\frac{[u_x(M,x_{uf3}) + M\pi_x(x_{uf3})][u(M,x_{uf3}) + M\pi(x_{uf3})]}{2} - C'(x_{uf3}) > 0$$

which is equivalent to:

$$\pi_x(x_{uf3})[M\pi(x_{uf3}) + u(M,x_{uf3}) - u(0,x_{uf3})] + \pi(x_{uf3})[u_x(M,x_{uf3}) - u_x(0,x_{uf3})] > 0$$

If x and m are strong complements, the last inequality holds strictly ($\pi_x \geq 0$ and $u_x(M,x) - u_x(0,x) > 0$), so $x_{uf3} < x_f$. If on the other hand x and m are strong substitutes, the opposite inequality holds strictly, so $x_{uf3} > x_f$. Finally, when x and m are independent ($\pi_x = 0$ and $u_x(M,x) - u_x(0,x) = 0$), the inequality becomes an equality so $x_{uf3} = x_f$. ■