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Working Paper

10-098

March 26, 2013

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January 12, 2013

Abstract

Seminal papers recommend that platforms in two-sided markets increase the number of complements available. We show that a two-sided platform can successfully compete by limiting the choice of potential matches it offers to its customers while charging higher prices than platforms with unrestricted choice. Starting from microfoundations, we find that increasing the number of potential matches not only has a positive effect due to larger choice, but also a negative effect due to competition between agents on the same side. Agents with heterogeneous outside options resolve the trade-off between the two effects differently. For agents with a lower outside option, the competitive effect is stronger than the choice effect. Hence, these agents have higher willingness to pay for a platform restricting choice. Agents with a higher outside option prefer a platform offering unrestricted choice. Therefore, the two platforms may coexist without the market tipping. Our model helps explain why platforms with different business models coexist in markets, including on-line dating, housing and labor markets.

Keywords: matching platform; indirect network effects; limits to network effects

1 Introduction

Seminal papers recommend that platforms in two-sided markets increase the number of complements available (Caillaud and Jullien, 2003; Rochet and Tirole 2003, 2006). For example, the more game developers there are for a particular video console or the more retailers accept a particular payment card, the more valuable these platforms become to consumers.

*Special thanks to Bharat Anand, Robert Aumann, Ramon Casadesus-Masanell, Ettore Damiano, Eddie Dekel, Ulrich Doraszelski, Drew Fudenberg, Constance Helfat, Elon Kohlberg, Greg Lewis, Bart Lipman, Glenn MacDonald, Cynthia Montgomery, Felix Oberholzer-Gee, Ariel Pakes, Motty Perry, Łukasz Pomorski, Phil Renny, Jan Rivkin, Al Roth, Mike Ryall and Dennis Yao for helpful comments and suggestions. We also thank the participants of Games 2008 conference, seminars at Harvard Business School, at University of Toronto, at Universitat Autònoma de Barcelona, at Hebrew University's Center for the Study of Rationality, and IO workshop in Economics Department at Harvard University.

As the literature progressed, scholars recognized that increasing the number of complements also increases the competition between then. This effect is particularly salient in matching markets, such as labor, real estate or dating markets. For example, men prefer a dating site with more women, but know that this site will also attract a lot of other men, who will reduce their chances of getting a date, thus reducing their preference for largest platform. This implies that we can no longer assume that agents value is a monotone function of the number of agents on the other side of that platform, giving rise to the possibility that we will observe platforms offering fewer choices that co-exist with those offering more.

To understand when such co-existence occurs, scholars have focused on relative prices charged and the quality of candidates offered by platforms offering more and fewer choices. One set of papers showed that coexistence is possible as long as agents getting fewer choices are also charged less (e.g., Ambrus and Argenziano, 2009). Another set established that coexistence could occur as long as platform charge more for access to agents of higher quality (e.g., Damiano and Li, 2009). In our paper, we show that neither of these conditions is necessary for co-existence, and platforms offering fewer choices can be profitable even if they charge more and offer access to matches of no higher quality than the competitor.

The model in this paper helps us understand the economic forces driving this result and the kinds of environments in which it will arise. We assume that people differ in their utility of staying alone. When choosing platforms agents with low value of staying alone will opt for a platform with fewer choices even if it costs more, because it gives them a higher chance of getting matched. In contrast, agents who are content to stay alone will opt for a platform offering larger choice, even if their chances of being rejected are higher there.

The model allows us to account for the co-existence of more expensive limited-choice platforms with cheaper ones offering more choice. For example, in on-line dating, sites such as eHarmony, pursue active member growth, but limit the number of new candidates that any member can see, and yet charges a 25% premium over competitors providing unlimited access. In labor markets, more expensive headhunting firms, offering limited choice, co-exist with unrestricted search platforms. In real estate markets more expensive, but also more limited, brokerage services coexist with unrestricted searches on For Sale By Owner databases.

Our explanation of this phenomenon is based on the interplay of two opposite effects that arise when the number of candidates increases. On the one hand, a man is more likely to find an attractive candidate if the dating site offers more women. On the other hand, the man is less likely to be accepted by his chosen candidate if there are more men available to that woman. This is because as the probability of being inferior to another candidate increases in the number of candidates the woman sees. We call the former a *choice effect* and the latter a *competition effect*. Interestingly, the trade-off between the two effects is resolved differently by different individuals.

For men who have high cost of staying alone, or have low outside options, the competition effect is stronger than the choice effect. These men will prefer on-line dating sites such as eHarmony, where both men and women see only a limited number of candidates. There, they can improve the probability of being accepted, even at the cost of seeing fewer women. In contrast, men who have higher value of staying alone (or better outside options) do not find rejection as costly and hence opt to join sites which offer a larger selection of candidates. The differences in the value of staying alone, which determine the sensitivity to the positive choice effect and the negative competition effect, can explain the coexistence of firms competing with different business models: those that offer more choice, and firms that actively limit choice. Different business models appeal to different types of customers. Contrary to what would be expected, the site that offers fewer complements could be the one creating and capturing more value.

We build a model and formalize our insight in three steps. First, we define a stylized two-sided matching environment with men and women on its two sides and derive properties of indirect network effects in this environment.[1] These properties play an important role in the platform's strategy via the choice effect and the competition effect outlined above. Our model shows that the relative magnitudes of the two effects change as the number of candidates offered increases. When only few candidates are offered, there is also little competition, and the choice effect dominates. This leads to a positive network effect. As the number of candidates offered grows, competition increases, and the competition effect overwhelms the choice effect. The network effect then becomes negative.

Second, we recognize that agents are heterogeneous in their outside options. Agents who have high costs of staying alone, or have low outside options, are more concerned about higher competition. Competition increases the likelihood that they will not be matched with anyone, in which case they would obtain their unattractive outside option. For them, the competitive effect will domi-

[1] Following seminal papers on network effects (e.g., Katz and Shapiro, 1985), we say that there is a positive network effect when the value of joining the platform increases in the number of agents participating in the platform on the same side. This definition applies to both direct and indirect network effects. For the indirect network effect, in a two-sided platform, the benefit arises because having more agents on the same side increases the participation of agents on the other side. With more people available on the other side, the platform brings a higher payoff to the original agent.

nate. Conversely, agents with high outside options are less concerned about not being matched, so for them the positive choice effect plays a more important role.

Finally, we study how these properties of network effects influence strategies of competing platforms and we characterize possible equilibria in such a market. We assume that agents can participate in one platform at most. Specifically, we show that a platform such as eHarmony, where both men and women see only a limited number of candidates, attracts agents with low outside options. These agents are willing to pay a premium to participate in such a platform. Therefore the platform is active in the market and is profitable, even if it competes with a platform that offers more candidates and does not charge a fee to participate. This implies that platforms may compete not only in prices, but also by offering different numbers of candidates and explains why platforms such as eHarmony and Match can coexist in spite of the fact that the former restricts choice and charges a higher fee relative to the latter. The mechanism we describe suggests that this situation may well be an equilibrium and that the market may not tip to the platform that offers more choice. Our explanation of the existence and popularity of restricted-choice platforms does not assume any additional superior matching skills of the platform or psychological aversion to abundant choices, but does not preclude them. If the platform that restricts choice also offers higher quality services, or if people have distaste for excessive choice, then platforms restricting choice will be even more successful.

While we focus on the dating market to make the exposition more clear, our model applies to other markets as well. Given the model's assumptions, it is best suited for two-sided markets with users who have heterogeneous outside options and whose taste for complements is subjective.[2] Consider for example the labor market, where headhunting companies coexist with unrestricted search platforms. The former are the more expensive option, and yet they offer fewer candidates to firms and expose a candidate only to a limited number of firms. We observe similar effects in the housing market, where buyers and sellers have the choice of using a broker's services or the For Sale By Owner database (FSBO). A broker usually shows only a few houses to a buyer and exposes every house to a limited number of clients. Nonetheless, many people use this restricted method rather than the FSBO database which gives a broader exposure to the market. This is surprising as there is little evidence that brokers help their clients secure better deals. For example, Hendel, Nevo, and Ortalo-Magne (2009) show that broker-mediated transactions and FSBO transactions resulted in

[2] Of course, there are markets for which the model does not apply, for example, a smartphone platform connecting users and apps. Apps do not have differentiated outside options and subjective taste for users, so the tradeoffs we described will not arise.

similar house sale prices in the Madison housing market. The combination of similar house prices and the typical 6% broker fee makes broker-mediated transactions the more expensive option. This makes explaining why people resort to brokers' services challenging for existing theories. In contrast, this behavior is consistent with the predictions of our model.

Our model not only explains the stylized facts of these industries, but also captures the mechanism through which they arise. The key intuition from our analysis is that platforms that restrict choice attract people with lower outside options or less patience. Less patient agents join the platform with restricted choice because it increases their chance of matching sooner, even at the cost of a relatively worse match. More patient agents join the platform with more choice because it increases the value of the match, even if it extends the wait time until a successful match. In the on-line dating market, eHarmony advertises itself as a website for people who are looking for a serious relationship, including marriage. This may be interpreted as targeting people with lower utility of being alone who want to get married quickly. In the labor market, headhunters are used primarily by employers or candidates for whom the cost of not finding an acceptable match quickly is high (Khurana, 2004). In the real estate market, it is accepted that agents who opt for real estate brokers (as opposed to FSBO) are those who assign more value to quickly finalizing the transaction. In all these markets, people are willing to pay a premium for immediacy, which makes platforms that restrict choice more costly, as predicted by our model.

The remainder of the paper is structured in following way: Section **??** provides a review of related literature. Section **??** sets up the model. Section **??** analyzes the strength of and limit to network effects, and how they depend on an agent's type. We show that as the number of candidates on both sides increases, positive network effects disappear for agents with lower outside option. Section **??** investigates a market with a matching platform, and shows that there always exists an equilibrium where agents pay to participate in a platform that offers to its participants fewer candidates than the outside market, which is accessible for free. Section **??** discusses the importance of some of the assumptions in the model, and how relaxing these assumptions influences the results.

2 Related Literature

Our paper contributes to a large literature on network competition. Seminal works in this literature suggests that when platforms compete with each other, the platform offering the largest choice should take over the market (e.g., Katz and Shapiro, 1985). Moreover, previous work on network effects often assumed that the presence of other agents on the platform exogenously increases utility, usually in a linear form (e.g., Rochet and Tirole, 2003). As a consequence, every additional agent on the platform increases payoff to others by the same amount, no matter how many other agents are already available. We depart from such assumptions and derive the network effects from the microfoundations in the defined matching environment. We find that aside the positive choice effect (opposite-side) there is also a negative competitive effect (same-side). Our model shows that a trade-off between the two effects allows for coexistence of multiple platforms offering different number of candidates to choose from.

More recently, a number of papers examined the trade-off between the positive opposite-side effect and negative same-side effect to show how multiple firms can coexist in environments with network effects. These papers make different assumptions from ours, and so their results are determined by different factors. For example, Ellison and Fudenberg (2003) and Ellison, Fudenberg, and Möbius (2004) study competition between two auction sites. Similarly to our paper, they assume that agents are heterogeneous. In contrast, however, their agents choose auction sites (platforms) before they know their type, while ours are aware of their type prior to choosing their platform. Furthermore, they assume that the clearing price on every platform is determined by the ratio of buyers to sellers. Then, they show that multiple auction sites can coexist as long as they have the same buyer-to-seller ratio. Although this ratio is appropriate for auctions, it cannot be assumed to be the only crucial factor in other environments. Thus, our model builds microfoundations of the trade-off between choice and competition to show that the number of candidates offered on a platform explains why multiple platforms can coexist in matching environments.

Our model is perhaps closest to Damiano and Li (2007), who examine why a revenue-maximizing monopolist would establish many platforms with different entry prices to separate and match different types of agents. Their model assumes that agents are heterogeneous in productivity and have different reservation utilities. They find that platforms can charge different prices to separate high productivity agents and allow them to match with each other. This is similar to our result whereby price separates agents with low utility of being unmatched from others. However, they assume that on every platform established by the monopolist agents can only consider one candidate. We relax this assumption to show that platforms that reduce the number of candidates are valuable to agents with low utility of staying alone. In the discussion section we examine the relationship between this paper and Damiano and Li (2007) in more detail.

Our paper also contributes to an emerging literature in strategy that explores competitive interaction between organizations with different business models. Casadesus-Masanell and Ghemawat (2006) and Economides and Katsamakas (2006), for example, study duopoly models in which a profit-maximizing competitor interacts with an open-source competitor. Casadesus-Masanell and Zhu (2010) study competitive interaction between a high-quality incumbent that faces a low-quality ad-sponsored competitor. Finally, Casadesus-Masanell and Hervas-Drane (2010) analyze competitive interactions between a free peer-to-peer file-sharing network and a profit-maximizing firm that sells the same content at positive price, and distributes digital files through an efficient client-server architecture. In our paper, a matching platform that deliberately limits the choice is competing against one that offers unlimited choice within its data base. We study forces in the market that allow such competition to be successful.

3 The Model

We consider a model of two-sided market. We describe the model in the context of a stylized heterosexual dating market, for stability of reference. We call one side of the market 'men', and the other side 'women'. On both sides of the market there is a continuum of agents of measure 1. Every agent has an exogenously fixed value of being alone, $\mathfrak{a}$. This value is drawn from a uniform distribution on the interval $[0,1]$, $\mathfrak{a} \sim U[0,1]$, and is private information.

There are two stages in the game. In the first stage, every agent meets some fixed number N of agents from the other side of the market.[3] The number of candidates, N, is the comparative statistics parameter in this model: We consider different values for N throughout the paper, and we study how it influences the expected payoff of agents. When a man m meets a woman w,[4] he learns $\Lambda^m(w) \in [0,1]$ — how much he will like being in a relationship with w.[5] Similarly every woman w learns $\Lambda^w(m) \in [0,1]$ about every man m she meets. In the second stage of the game all agents simultaneously make at most one offer.[6] If two agents made their offers to each other,

[3]We consider markets where the two sides are treated symmetrically. Platforms literature has shown the potential of asymmetric treatment of the two-sides (e.g., Parker and van Alstyne, 2005). However, in many markets firms are restricted to treat both sides symmetrically, for legal or technical reasons.

[4]If m meets w, then it must be that w meets m.

[5]Where there is no risk of confusion, the notation is simplified by dropping superscripts. For example, $\Lambda^m(w)$ may be simplified to Λ^m or Λ.

[6]The assumption that limits agents to only one offer is meant to reflect the fact that people are able to pursuit only limited number of possible relationships. Because this is potentially restrictive assumption. Appendix **??** considers other offer-making procedures and shows that the results of this restrictive assumption hold also under more realistic procedures. The one-offer assumption made throughout the paper simplifies the analysis and the intuition behind the results.

we say that such offers have been reciprocated or accepted, and the two agents are matched. They receive their respective payoffs of $\Lambda^m(w)$ and $\Lambda^w(m)$. If an offer was not reciprocated (i.e., it is "rejected"), the agent who made the offer remains unmatched, and he receives his or her $\mathfrak{a}$. The game ends with these payoffs.

Values of parameters Λ are drawn from a uniform distribution on the interval $[0, 1]$. Each value Λ is independent of any other value Λ and of any values of $\mathfrak{a}$. That is, the extend to which a man likes a particular woman is independent of how much he likes other women that he meets and how much other men like the woman.[7]

3.1 One Candidate

As a benchmark, let us first consider the case where every agent meets exactly one candidate on the other side. In such a case, after man m and woman w meet, the man wants to enter the relationship if and only if $\Lambda^m(w) > \mathfrak{a}^m$, and the woman wants to enter the relationship if and only if $\Lambda^w(m) > \mathfrak{a}^w$. When both those conditions are satisfied, then both agents make offers to each other and the relationship is realized; the woman gets the payoff of $\Lambda^w(m)$ and the man gets $\Lambda^m(w)$. If at least one of those conditions is violated, the agents remain single, and they get the payoff of $\mathfrak{a}^m$ and $\mathfrak{a}^w$, respectively.

When a man meets a woman, he does not know her $\mathfrak{a}^w$ or how much she likes him, $\Lambda^w(m)$. He only knows that both of those values are drawn from a uniform distribution on the interval $[0, 1]$ and that they are independent. Therefore, the man assigns probability $\frac{1}{2}$ to the event that the woman likes him more than being alone:

$$Pr(\Lambda^w(m) > \mathfrak{a}^w) = \int_0^1 \left(\int_{\mathfrak{a}}^1 d\Lambda \right) d\mathfrak{a} = \frac{1}{2}\,. \tag{1}$$

Before he meets the woman, the man m characterized by $\mathfrak{a}^m$ expects to match successfully with probability $Pr(\Lambda^w(m) > \mathfrak{a}^w) \cdot Pr(\Lambda^m(w) > \mathfrak{a}^m | \mathfrak{a}^m) = \frac{1}{2}(1 - \mathfrak{a}^m)$. The expected value of a match, conditional on successful matching is $\frac{1}{2}(1 + \mathfrak{a}^m)$:

$$\frac{Pr(\Lambda^w(m) > \mathfrak{a}^w) \cdot \int_{\mathfrak{a}^m}^1 \Lambda^m d\Lambda^m}{\frac{1}{2}(1 - \mathfrak{a}^m)} = \frac{1}{2}(1 + \mathfrak{a}^m)\,. \tag{2}$$

Combining the above formulas with the remaining probability of $\frac{1}{2}(1 + \mathfrak{a}^m)$ that the man remains

[7]Section **??** discusses the importance of this assumption.

alone, we find that the total expected payoff of agent characterized by $\mathfrak{a}^m$ is $\frac{1}{4}(1+\mathfrak{a}^m)^2$:

$$EU(\mathfrak{a}^m) = \frac{1}{2}(1+\mathfrak{a}^m)\mathfrak{a}^m + \frac{1}{2}(1+\mathfrak{a}^m) \cdot \frac{1}{2}(1-\mathfrak{a}^m) = \frac{1}{4}(1+\mathfrak{a}^m)^2 \,. \tag{3}$$

Observation 1. Notice that if the man could choose, he would prefer to meet a woman with low $\mathfrak{a}^w$. Similarly, a woman would prefer to meet a man with low $\mathfrak{a}^m$. This is because a candidate with lower $\mathfrak{a}$ is more likely to want to match. This increased probability of matching results in increasing expected payoff for the agent. Moreover, there is no downside to this increased probability of matching, since the agent is always free to reject an undesirable candidate. To see that formally, consider the expected payoff of a man $\mathfrak{a}^m$ when he meets a woman with known parameter $\mathfrak{a}^w$. Then the probability that the woman wants to enter the relationship is $Pr(\Lambda^w(m) > \mathfrak{a}^w | \mathfrak{a}^w) = 1 - \mathfrak{a}^w$. The expected payoff of a man $\mathfrak{a}^m$ is then

$$EU(\mathfrak{a}^m|\mathfrak{a}^w) = \int_0^{\mathfrak{a}^m} \mathfrak{a}^m d\Lambda^m + \int_{\mathfrak{a}^m}^1 (\mathfrak{a}^w \cdot \mathfrak{a}^m + (1-\mathfrak{a}^w)\Lambda^m)\, d\Lambda^m = \frac{1}{2}(1+(\mathfrak{a}^m)^2) - \frac{1}{2}\mathfrak{a}^w(1-\mathfrak{a}^m)^2 \,.$$

Payoff $EU(\mathfrak{a}^m|\mathfrak{a}^w)$ decreases as $\mathfrak{a}^w$ increases. Therefore, agent m prefers to meet a woman with lower $\mathfrak{a}^w$.

This preference for candidates with lower $\mathfrak{a}$ plays a role later in the analysis of matching platform.

4 Limits to Positive Network Effects

Consider a market just as the one in the example above, except that every agent meets N candidates. As before, at the offer stage (where offers are made simultaneously) every agent can make at most one offer. A match between man m and woman w is made if m made his offer to w and w has made her offer to m.

Before going to the general case of N candidates, let us as an example consider first the market with $N = 2$, and compare it to the previously described market with $N = 1$. If the expected payoff for an agent increases as N increases from 1 to 2, there is a positive indirect network effect: Having more agents on the same side increases the agent's utility, because it is combined with increasing the number of candidates on the other side of the market. As we show below, for some agents there is a positive network effect in this situation, but for others there is not.

When man m meets two women, w_1 and w_2, he learns $\Lambda^m(w_1)$ and $\Lambda^m(w_2)$. Without more information, both women seem in all other aspects the same. The man therefore makes his offer to the woman that he likes more, i.e., with higher Λ^m (provided it is above $\mathfrak{a}^m$).

Higher expected value of a match (for all agents). With two candidates the expected maximal value of Λ^m is larger than with only one candidate. This is because with two independent draws from the uniform distribution the expected largest drawn value increases.

The expected value of the maximum of two random variables $\Lambda^m(w_1)$ and $\Lambda^m(w_2)$ is $\frac{2}{3}$. Compare this value to $\frac{1}{2}$ when there is only one candidate: $\int_0^1 \Lambda d\Lambda = \frac{1}{2}$. Notice that the conditional expected value of a match (if the match is successful) is not the same as the expected value of maximum Λ^m. Reading from equation (**??**), with one candidate, the expected value of a match if the man successfully entered the relationship is $\frac{1}{2}(1+\mathfrak{a}^m)$, not $\frac{1}{2}$. This is because the man is only matched if $\Lambda^m > \mathfrak{a}^m$ (but not always when $\Lambda^m > \mathfrak{a}^m$). Therefore, with two candidates the expected value of the match conditional on successfully matching is $\frac{2}{3}\left(\frac{1+\mathfrak{a}^m+(\mathfrak{a}^m)^2}{1+\mathfrak{a}^m}\right)$.

Notice that $\frac{2}{3}\left(\frac{1+\mathfrak{a}+\mathfrak{a}^2}{1+\mathfrak{a}}\right) > \frac{1}{2}(1+\mathfrak{a})$ for any $\mathfrak{a} \in [0,1)$; and for $\mathfrak{a}=1$, the two values are equal. Thus, the expected value of a match, upon matching successfully, is higher for any agent when he or she meets two candidates. We call this a "choice effect".[8]

Lower probability of matching (for some agents). For agents with lower $\mathfrak{a}$, the probability of successful matching is lower when $N=2$ than when $N=1$. With $N=2$, each woman has two men to choose from, and thus, every man has more competition. It decreases the probability that any particular woman w wants to match with man m when m wants to match with w, as compared to the case with $N=1$. This "competition effect" holds for all $\mathfrak{a}$'s. However, as we show below, for large $\mathfrak{a}$'s it is offset by the fact that with more candidates, the agent is more likely to meet at least one acceptable candidate (i.e., preferred to staying alone).

Man m gets an offer from woman w if she likes him more than being alone ($\Lambda^w(m) > \mathfrak{a}^w$), and more than she likes the other man she meets, m_2 ($\Lambda^w(m) > \Lambda^w(m_2)$). If $\mathfrak{a}^w$ is known, those conditions are satisfied with probability $\frac{1}{2}\left(1-(\mathfrak{a}^w)^2\right)$:

$$\int_0^{\mathfrak{a}^w} 0 d\Lambda^w(m) + \int_{\mathfrak{a}^w}^1 \left(\int_0^{\Lambda^w(m)} 1 d\Lambda^w(m_2) + \int_{\Lambda^w(m)}^1 0 d\Lambda^w(m_2)\right) d\Lambda^m(m) \;=\; \frac{1}{2}\left(1-(\mathfrak{a}^w)^2\right) .$$

But since man m does not know $\mathfrak{a}^w$, he assigns probability of $\frac{1}{3}$ that he gets an offer from

[8] In literature this effect is also known as positive opposite-side effect.

woman w:

$$\int_0^1 \frac{1}{2}\left(1-(\mathfrak{a}^w)^2\right) d\mathfrak{a}^w = \frac{1}{3}\,.$$

Notice that this probability is the same for any of the women that man m meets. This probability is lower than the probability of $\frac{1}{2}$ in market with only one candidate (cf. (??)). This is the "competition effect" of having more men on the same side of the market.[9]

When man m likes both women less than being alone, then he does not want to match with any of them. This happens when $\Lambda^m(w_1) < \mathfrak{a}^m$ and $\Lambda^m(w_2) < \mathfrak{a}^m$; that is, with probability $(\mathfrak{a}^m)^2$. Otherwise, the man makes an offer to one of the women, and the offer is reciprocated with probability $\frac{1}{3}$. Thus, the man successfully matches with probability $\frac{1}{3}(1-(\mathfrak{a}^m)^2)$ when he meets two candidates. Compare it with the probability of successfully matching in the market with only one candidate: $\frac{1}{2}(1-\mathfrak{a}^m)$. For $\mathfrak{a}^m > \frac{1}{2}$, the probability of successfully matching is higher with two candidates than with one. However, agents with $\mathfrak{a}^m < \frac{1}{2}$ match successfully with higher probability in market with one candidate than with two candidates.

It may seem puzzling that the probability of successfully matching depends so much on $\mathfrak{a}$ when the probability of having an offer reciprocated is the same for all agents. This is because successful matching also depends on whether there is at least one candidate who is above the reservation value $\mathfrak{a}$. The probability of such an event decreases with $\mathfrak{a}$, and increases with N.

For agents with low $\mathfrak{a}$ it is already likely that one candidate provides matching value above $\mathfrak{a}$. Meeting more candidates does not increase this probability enough to offset the decreased probability of having an offer reciprocated. However, for agents with high $\mathfrak{a}$, the increase in the probability that at least one candidate is better than $\mathfrak{a}$ offsets the decreased probability of having an offer reciprocated, as the agent meets an additional candidate.

Total expected payoff. From the formulas derived above, we see that in a market with two candidates, an agent characterized by $\mathfrak{a}$ successfully matches with probability $\frac{1}{3}\left(1-\mathfrak{a}^2\right)$. When this happens, the expected value of the match is $\frac{2}{3}\left(\frac{1+\mathfrak{a}+\mathfrak{a}^2}{1+\mathfrak{a}}\right)$. Otherwise, the agent remains alone and achieves payoff $\mathfrak{a}$.

Using the same approach as in formula (??), we find that the total expected payoff of agent $\mathfrak{a}$

[9] In the literature, it is also known as negative same-side effect.

in market with two candidates is

$$EU(\mathfrak{a}|N=2) = \left[1 - \frac{1}{3}\left(1-\mathfrak{a}^2\right)\right] \cdot \mathfrak{a} + \frac{1}{3}(1-\mathfrak{a}^2) \cdot \frac{2}{3}\,\frac{1+\mathfrak{a}+\mathfrak{a}^2}{1+\mathfrak{a}} = \frac{1}{9}\mathfrak{a}^3 + \frac{2}{3}\mathfrak{a} + \frac{2}{9}\,.$$

Are there positive indirect network effects? Given both the choice and the competition effects, we ask now whether the market with $N = 2$ results in higher expected payoffs than the market with $N = 1$. If there are positive indirect network effects, then having more people on the same side increases the overall payoff of an agent because there is also more people on the other side of the market, i.e., the choice effect outweighs the competition effect. There are positive indirect network effects at $N = 1$ when

$$EU(\mathfrak{a}|N=2) > EU(\mathfrak{a}|N=1) \iff \frac{1}{9}\mathfrak{a}^3 + \frac{2}{3}\mathfrak{a} + \frac{2}{9} > \frac{1}{4}(1+\mathfrak{a})^2 \iff \mathfrak{a} > \frac{1}{4}\,.$$

Thus, for agents with utility of being alone larger than $\frac{1}{4}$ there is a positive indirect network effect at $N = 1$. Those agents prefer to have one more agent on the same side if it implies having one more candidate on the opposite side of the market. It is not surprising for agents with $\mathfrak{a} > \frac{1}{2}$, since for them it is more likely to match *and* the match has higher expected payoff under $N = 2$. But for agents with $\mathfrak{a} \in (\frac{1}{4}, \frac{1}{2})$, probability of successfully matching is lower under $N = 2$. Nonetheless, the choice effect outweighs the competition effect for those agents.

It is not true for agents with the utility of being alone *smaller* than $\frac{1}{4}$. For those agents, the competition effect outweighs the choice effect, and adding one more agent on both sides of the market decreases overall payoff. There are no positive indirect network effects for agents with $\mathfrak{a} < \frac{1}{4}$ at $N = 1$.

The more expected payoff an agent gains by having one more person on the same side (and thus one more on the opposite side) the stronger the network effect. Therefore, the strength of the network effect for agent with $\mathfrak{a}$, at $N = 1$ is given by

$$EU(\mathfrak{a}|N=2) - EU(\mathfrak{a}|N=1) = \frac{1}{36}(4\mathfrak{a}-1)(1-\mathfrak{a})^2\,.$$

Not only there are no positive network effects for $\mathfrak{a} < \frac{1}{4}$, but also the strength of the positive effects for $\mathfrak{a} > \frac{1}{4}$ varies for different values of $\mathfrak{a}$. Therefore, we may expect that the effect of further increasing N on the network effects will also vary for different $\mathfrak{a}$.

The results in the example above generalize for any N, and are stated in Lemmas **??** and **??**. For any N, the expected value of a successful match increases with N.

Lemma 1. *For any $\mathfrak{a} < 1$, expected value of a successful match strictly increases with N.*

Proof. See the Appendix, page **??**.

For agents with $\mathfrak{a} = 1$ the expected value of a successful match is 1 independent on the number of candidates. Those agents simply do not match with any candidate below 1.

However, the probability of matching is decreasing with the number of candidates for sufficiently low $\mathfrak{a}$'s.

Lemma 2. *The probability of matching is decreasing with N, for sufficiently low $\mathfrak{a}$.*

Proof. See the Appendix, page **??**.

The total expected payoff depends on both of these factors. For agents with higher $\mathfrak{a}$'s, increasing N leads to higher expected payoff (i.e. there are positive indirect network effects). And for agents with lower $\mathfrak{a}$'s, increasing N leads to decreasing expected payoff. As we have seen above, agents with $\mathfrak{a} < \frac{1}{4}$ have higher expected payoff when $N = 1$ than when $N = 2$, and their expected payoff further decreases as N increases. For agents with $\mathfrak{a}$ between $\frac{1}{4}$ and $\frac{\sqrt{46}-1}{9} \approx 0.64$ there is positive network effect at $N = 1$, but at $N = 2$ the positive network effect is gone: The expected payoff is lower when $N = 3$ than when $N = 2$, and it further decreases as N grows. That is, for the agents with $\mathfrak{a}$ between 0.25 and approximately 0.64, the network effect reaches its limit at $N = 2$.

In Proposition **??**, we show that for every $\mathfrak{a}$, there is a limit to the positive network effect. At first, adding one more candidate on both sides of the market increases agents' expected payoffs (for $\mathfrak{a} > \frac{1}{4}$). But for every $\mathfrak{a}$, there exists a limit $\bar{N}(\mathfrak{a})$ above which adding more candidates on both sides of the market *decreases* agent's $\mathfrak{a}$ expected payoff. There are positive indirect network effects for $\mathfrak{a}$ when $N < \bar{N}(\mathfrak{a})$, but not when N is higher.

Proposition 1. *For every $\mathfrak{a}$, there exists $\bar{N}(\mathfrak{a})$ such that $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N)$ is positive for $N < \bar{N}(\mathfrak{a})$, and negative for $N \geq \bar{N}(\mathfrak{a})$. Moreover, $\bar{N}(\mathfrak{a})$ is non-decreasing with $\mathfrak{a}$.*

Proof. See the Appendix, page **??**.

For any agent $\mathfrak{a}$, the choice effect — reflected in Lemma **??** — declines in strength as N increases. Each additional candidate increases the expected value of a successful match by a smaller amount than the previous one. At the same time, the competition effect — incorporated in Lemma **??** — increases in N. In result, the positive network effect experienced by agent $\mathfrak{a}$ declines in strength as N increases, until it reaches its limit at $\bar{N}(\mathfrak{a})$. Above that level, an increase of N decreases agent's expected payoff: Above $\bar{N}(\mathfrak{a})$ the network effect is negative.

Moreover, Proposition **??** states that for higher $\mathfrak{a}$'s, $\bar{N}(\mathfrak{a})$ is larger. Therefore, agents with lower $\mathfrak{a}$'s prefer markets where there is less choice and less competition. And agents with higher $\mathfrak{a}$'s prefer markets with more choice and more competition. Figure **??** illustrates how this limit to the network effect varies with $\mathfrak{a}$.

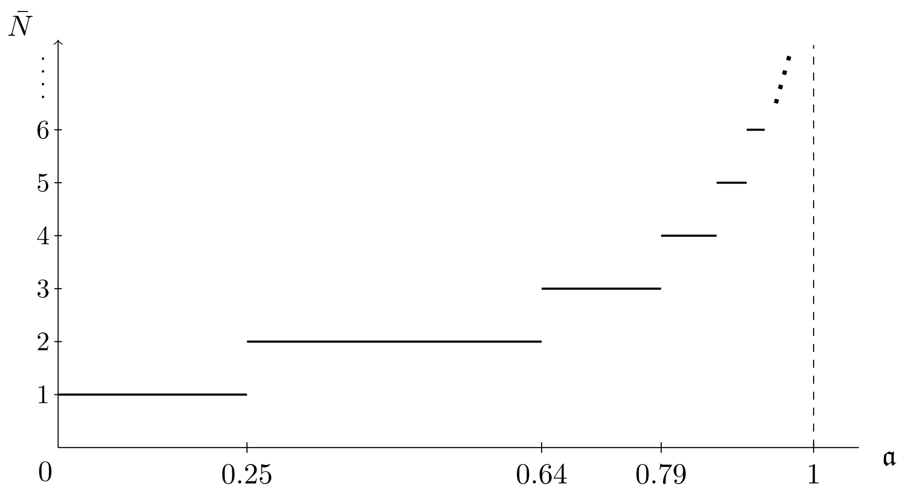

Figure 1: Limit to the network effect as a function of $\mathfrak{a}$.

In this section, we have shown that the strength of network effects depends not only on agent's type, $\mathfrak{a}$, but also on the level of choice and competition, N. This property has not been observed in the existing literature. Many papers on platform competition assume that the number of other agents enters the payoff function linearly (e.g., Rochet and Tirole, 2003). Such formulation makes the limits to network effects irrelevant. Under the linearity assumption every additional agent on the opposite side of the market contributes the same amount to the payoff. If the competitive effect is included, every additional agent on the same side also affects the payoff in the same way. The positive network effect is present when the choice effect outweighs the competition effect. But since the absolute and relative strength of the choice and competition effects is assumed to be constant, if the positive network effect exists, there is no limit to it: The agent would always prefer more choice and competition.

In the environment characterized by that traditional literature, it would not be possible for a platform that restricts choice and competition to attract agents away from a market where more choice and more competition is available.[10] In the next section, we show that under our analysis of network effects, coexistence of such platform and such market is possible, as agents of some types welcome restricted choice and competition.

5 Matching Platform

In the previous section we have investigated the strength of and the limits to positive indirect network effects. We have shown that both vary with the type of agent, as characterized by $\mathfrak{a}$. In this section, we explore strategic opportunities that those properties offer to a matching platform.

In particular, we focus on the fact that a platform may profit when providing *fewer* candidates to its participants than the outside market, as long as it offers fewer candidates to agents on both sides. The *outside market* can be a decentralized market, or an existing matching platform offering a large number of candidates. Every agent who participates in the outside market meets some number Ω of candidates. The fee for participating in the outside market is normalized to 0. In this paper we focus on opportunities for a platform that offers fewer candidates than Ω and charges a positive participation fee (i.e., charges more than the outside market). Providing fewer candidates restricts the choice, but also restricts the competition. Agents for whom the competition effect is large compared to the choice effect are willing to pay a positive fee to participate in such a platform.

In Section **??**, an agent meets candidates with $\mathfrak{a}$ drawn from the whole distribution $U[0, 1]$. In the current section agents decide whether to participate in the platform that restricts the number of candidates, or stay in the outside market. We assume single-homing, i.e., agents cannot participate in both the platform and the outside market. Only agents with low enough $\mathfrak{a}$ prefer to participate in the platform at a given positive fee. Agents with higher $\mathfrak{a}$ prefer to stay in the outside market. Therefore, candidates that can be met in the platform have $\mathfrak{a}$'s distributed according to truncated distribution. This property further influences the rejection probabilities in the platform and in the outside market.

[10] Section **??** discusses literature which analyses coexisting platforms, but under different conditions than ours.

5.1 Example of a Platform Offering Fewer Candidates than Outside Market

We start this section with a careful analysis of an example. In an outside market where everybody meets two candidates ($\Omega = 2$), there is a matching platform that offers only one candidate to its participants ($N = 1$). We denote this platform by M_1. To join the platform, the agent needs to pay fee f. Participating in the outside market is free of charge. Once the agent joined the platform, he no longer participates in the outside market. In the platform, every agent from one side meets a random agent from the other side who also has joined the platform. After they meet, they privately learn their respective Λ's and decide whether to make an offer. Offers are made simultaneously. If both of them make offers to each other, they are matched, and receive their respective Λ's. Otherwise, they remain alone, and receive their respective $\mathfrak{a}$'s.

The expected payoff from participating in the platform is similar to (**??**):

$$EU(\mathfrak{a}|\mathrm{M}_1) = \underbrace{[\mathfrak{a} + (1-\mathfrak{a})\Pr(rej|\mathrm{M}_1)]}_{\text{prob of not matching}} \cdot \mathfrak{a} + (1 - \Pr(rej|\mathrm{M}_1))\frac{1}{2}(1-\mathfrak{a}^2)\,,$$

where $\Pr(rej|\mathrm{M}_1)$ is the probability of being "rejected," i.e. *not* having own offer reciprocated, in platform M_1. Agent $\mathfrak{a}$ matches successfully when the candidate is above $\mathfrak{a}$ and has reciprocated the offer. Otherwise, the agent remains alone.

The probability of being rejected in M_1 depends on the number of candidates, $N = 1$, but also on the types of agents join the platform. This, in turn depends on the fee f. Given that lower $\mathfrak{a}$'s prefer fewer candidates, we can expect[11] that agents with $\mathfrak{a} < \mathfrak{a}^*(f)$ are attracted to the platform; where $\mathfrak{a}^*(f)$ is the threshold value of $\mathfrak{a}$, which depends on f. Agents with higher $\mathfrak{a}$ prefer to stay in the outside market.

To find the probability of rejection in the matching platform, notice that an agent with $\mathfrak{a}$ makes an offer to the candidate with probability $1 - \mathfrak{a}$, and with the remaining probability, $\mathfrak{a}$, the agent does not make an offer to the candidate (i.e. rejects the candidate). Thus, the probability of being rejected in M_1 is

$$\Pr(rej|\mathrm{M}_1) = \frac{\int_0^{\mathfrak{a}^*} \mathfrak{a}\, d\mathfrak{a}}{\int_0^{\mathfrak{a}^*} d\mathfrak{a}} = \frac{1}{2}\mathfrak{a}^*\,.$$

The expected payoff for agent $\mathfrak{a}$ from joining the platform is then

$$EU(\mathfrak{a}|\mathrm{M}_1) = \mathfrak{a}\left[\mathfrak{a} + (1-\mathfrak{a})\frac{1}{2}\mathfrak{a}^*\right] + \left(1 - \frac{1}{2}\mathfrak{a}^*\right)\frac{1}{2}(1-\mathfrak{a}^2)\,.$$

[11] This "threshold" property is formally proven through Lemma **??** in Appendix (page **??**).

Agent $\mathfrak{a}$ who stays in the outside market, where everybody meets two candidates, receives the expected payoff of

$$EU(\mathfrak{a}|\text{OUT}_2) = \mathfrak{a}[\underbrace{\mathfrak{a}^2 + (1-\mathfrak{a}^2)Pr(rej|\text{OUT}_2)}_{\text{prob of not matching}}] + (1 - Pr(rej|\text{OUT}_2))\frac{2}{3}(1-\mathfrak{a}^3)\,.$$

The agent stays unmatched either when none of the candidates were acceptable (which happens with probability $\mathfrak{a}^2$), or when an offer was rejected.

In an environment with two candidates an agent with utility of being alone $\mathfrak{a}$ makes an offer to a particular candidate with probability[12] $\frac{1}{2}(1-\mathfrak{a}^2)$. Since agents with $\mathfrak{a} < \mathfrak{a}^*(f)$ participate in the platform, only agents with $\mathfrak{a} > \mathfrak{a}^*(f)$ stay in the outside market. Thus, the probability of being rejected, for any agent staying in the outside market is[13]

$$\Pr(rej|\text{OUT}_2) = \frac{\int_{\mathfrak{a}^*}^{1}\left(1 - \frac{1}{2}(1-\mathfrak{a}^2)\right)d\mathfrak{a}}{1-\mathfrak{a}^*} = \frac{1}{6}\left[(\mathfrak{a}^*)^2 + \mathfrak{a}^* + 4\right]\,.$$

The expected payoff in the outside market is then

$$EU(\mathfrak{a}|\text{OUT}_2) = \mathfrak{a}\left(\mathfrak{a}^2 + (1-\mathfrak{a}^2)\frac{1}{6}\left[(\mathfrak{a}^*)^2 + \mathfrak{a}^* + 4\right]\right) + \left(1 - \frac{1}{6}\left[(\mathfrak{a}^*)^2 + \mathfrak{a}^* + 4\right]\right)\cdot\frac{2}{3}(1-\mathfrak{a}^3)\,.$$

An agent with the threshold value of being alone, $\mathfrak{a}^*$, is indifferent between joining M_1 at fee f and staying in the outside market. Thus, the threshold $\mathfrak{a}^*$ depends on fee f and is characterized by the following indifference condition

$$EU(\mathfrak{a}^*|\text{M}_1) - EU(\mathfrak{a}^*|\text{OUT}_2) = f\,,$$

which reduces to

$$\underbrace{\frac{1}{36}\left(2(\mathfrak{a}^*)^3 + 6(\mathfrak{a}^*)^2 - 9\mathfrak{a}^* + 10\right)(1-\mathfrak{a}^*)^2}_{f(\mathfrak{a}^*)} = f\,.$$

There is no closed form solution for $\mathfrak{a}^*(f)$. However, we can still learn about some of its properties by investigating its inverse function, $f(\mathfrak{a}^*)$. Function $f(\mathfrak{a}^*)$, shown in Figure **??**, is a continuous function, positive and monotonically decreasing on the interval $[0,1]$. For $\mathfrak{a}^* = 1$ and $\mathfrak{a}^* = 0$, the function takes values $f(1) = 0$ and $f(0) = \frac{10}{36}$. Thus for any fee $f \in [0, 10/36]$ there

[12]With probability $\mathfrak{a}^2$ neither of the candidates is acceptable, and no offer is made. Otherwise, each of the two candidates has an equal probability of getting an offer.

[13]Notice that $\Pr(rej|\text{OUT}_2) > \Pr(rej|\text{M}_1)$ for any level of $\mathfrak{a}^* \in [0,1]$.

exists a threshold $\mathfrak{a}^*(f) \in [0,1]$. For a fee above $\frac{10}{36}$, no agent wants to participate in the platform.

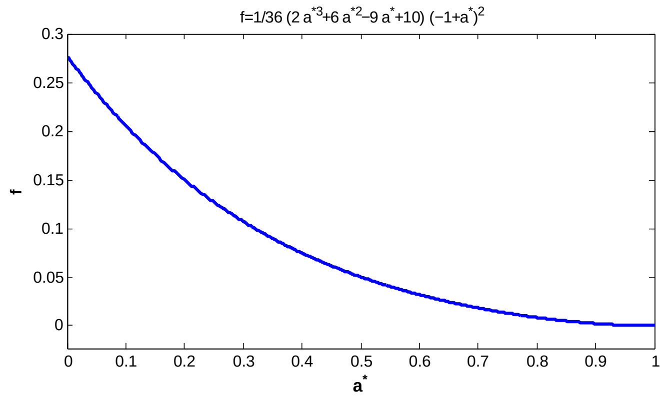

Figure 2: Fee f as a function of threshold $\mathfrak{a}^*$, $f(\mathfrak{a}^*)$.

The platform chooses the fee with the objective to maximize profit. The fee is chosen and announced before agents decide whether to join the platform or not. We assume that the platform has no marginal cost. Then the platform's profit, shown in Figure **??**, is

$$\Pi_{\mathrm{M}_1} = f \cdot \mathfrak{a}^*(f) = f(\mathfrak{a}^*) \cdot \mathfrak{a}^* = \frac{1}{36}\mathfrak{a}^* \left(2(\mathfrak{a}^*)^3 + 6(\mathfrak{a}^*)^2 - 9\mathfrak{a}^* + 10\right)(1-\mathfrak{a}^*)^2 .$$

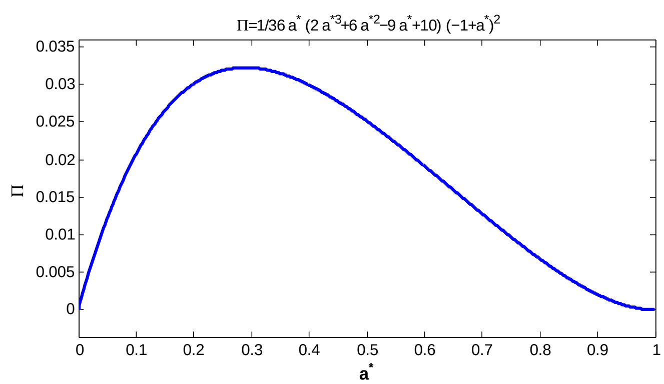

Figure 3: Profit of the matching platform as a function of threshold $\mathfrak{a}^*$.

The profit first increases with $\mathfrak{a}^*$, as more agents participate and pay the fee. However, for large levels of participation, the profit decreases with $\mathfrak{a}^*$. This is because to attract a large number of agents, the platform needs to set a lower fee. Larger number of participants does not compensate for the lower fee at large levels of $\mathfrak{a}^*$. The profit has a unique maximum.

The maximal profit can be found directly by using the first order condition

$$\frac{\partial \Pi_{MM1}}{\partial \mathfrak{a}^*} = \frac{1}{18}(\mathfrak{a}^* - 1)\left(6(\mathfrak{a}^*)^4 + 11(\mathfrak{a}^*)^3 - 27(\mathfrak{a}^*)^2 + 24\mathfrak{a}^* - 5\right) = 0\,.$$

There exists a unique maximum on the interval $\mathfrak{a}^* \in [0, 1]$. This maximum is reached at $\mathfrak{a}^* \approx 0.29$. Profit related to this threshold is $\Pi(\mathfrak{a}_0) = 0.0323$, and the fee — $f = 0.1112$. That is, in this particular example where in an outside market everyone meets two candidates, a profit-maximizing fee for a platform offering one candidate is $f = 0.1112$. At this fee agents with $\mathfrak{a} < 0.29$ prefer to pay the fee and join the platform. And agents with $\mathfrak{a} > 0.29$ prefer to stay in the outside market.

Observation 2. Notice that if fee $f = 0$, then $\mathfrak{a}^*(f = 0) = 1$. That is, if participating in the platform is free, then *all agents* prefer to join the platform. Mathematically, it follows from the fact that $f(\mathfrak{a}^*)$ is positive on the whole interval $\mathfrak{a}^* \in [0, 1)$. However, this global preference to join the platform with *fewer* candidates seems puzzling, especially for the agents with high $\mathfrak{a}$'s. In Section **??**, we have seen that those agents prefer markets where there are more candidates. Yet, presented with the choice between the platform with one candidate and outside market with two, they would prefer to join the platform with fewer candidates. This is an effect of selection of lower $\mathfrak{a}$'s away from the outside market. Agents with lower $\mathfrak{a}$'s participate in the platform because they prefer the market with restricted number of candidates. Only people with higher $\mathfrak{a}$ stay outside. The probability of being rejected is higher in the outside market than in the platform because of the larger number of candidates. But then the rejection probability further increases, because agents with higher $\mathfrak{a}$'a are more likely to reject a candidate. We do not see this reinforcement in Section **??**. It is present only in an environment where agents are allowed to self-select into the platform and outside market. Higher fee for participating in the platform deters higher $\mathfrak{a}$'s from joining. However, at price 0, the increase of rejection probability outweighs the benefits of meeting more candidates, for all agents.

5.2 General Case of Platform Restricting Number of Candidates

Having illustrated our point with a specific example, we now present the results for a general case. Consider a market with a matching platform offering N candidates (M_N), where in the outside market agents meet $\Omega > N$ candidates. In this section, we characterize equilibria in such a market. It is easy to verify that a situation where no agents join the platform is always an equilibrium. However, there also always exist equilibria where some agents participate in the platform. We focus our investigation on the equilibria where the platform is active (i.e., there are some agents who participate in the platform). Especially, we show that for $N < \Omega$, there always exists exactly one equilibrium where some agents pay a positive price to participate in the platform.

In such equilibrium, for a given Ω and $N < \Omega$, there exists a threshold $\mathfrak{a}^*$ such that agents with $\mathfrak{a} < \mathfrak{a}^*$ decide to joint the platform, and agents with $\mathfrak{a} > \mathfrak{a}^*$ stay outside. The threshold is closely related to fee f that the platform charges for participation. The platform sets the fee to maximize its profit.

In the main text, we focus on the rationale behind the results. All the formal proofs are deferred to the appendix.

We start by investigating the decision of each agent whether to join the platform. From the perspective of an individual agent, the rejection probabilities in the platform and in the outside market are given and constant. The agent then makes the decision whether to join the platform or not based on those probabilities, his own utility of being alone $\mathfrak{a}$, and the fee f.

The rejection probabilities and the fee are the same for all agents. However, the decision depends also on $\mathfrak{a}$, which varies among the agents. An agent with $\mathfrak{a}$ prefers to join the platform if the benefit of joining outweighs the fee, i.e., when $EU(\mathfrak{a}|\text{M}_\text{N}) - EU(\mathfrak{a}|\text{OUT}_\Omega) > f$. For $N < \Omega$, $EU(\mathfrak{a}|\text{M}_\text{N}) - EU(\mathfrak{a}|\text{OUT}_\Omega)$ is strictly decreasing in $\mathfrak{a}$.[14] Thus, $EU(\mathfrak{a} = 0|\text{M}_\text{N}) - EU(\mathfrak{a} = 0|\text{OUT}_\Omega)$ is the highest fee that could be paid in the market (and only agent with $\mathfrak{a} = 0$ would pay it). For any fee between $EU(\mathfrak{a} = 0|\text{M}_\text{N}) - EU(\mathfrak{a} = 0|\text{OUT}_\Omega)$ and 0, there is agent with $\mathfrak{a} = \mathfrak{a}^*$ who is indifferent between joining the platform at this fee or not. Since the difference $EU(\mathfrak{a}|\text{M}_\text{N}) - EU(\mathfrak{a}|\text{OUT}_\Omega)$ is decreasing in $\mathfrak{a}$, all agents with $\mathfrak{a}$ lower than $\mathfrak{a}^*$ find it worthwhile to join, while all agents with $\mathfrak{a}$ higher than $\mathfrak{a}^*$ prefer to stay outside.

This threshold property holds for any exogenously given rejection probabilities. In the equilibrium, however, the rejection probabilities in the platform and in the outside market must be

[14]See Lemma **??** in Appendix (page **??**).

consistent with the participation threshold, i.e.:

$$\Pr(rej|\text{M}_\text{N}) = \frac{\int_{\mathfrak{a}<\mathfrak{a}^*} \left(1 - \frac{1}{N}(1-\mathfrak{a}^N)\right) d\mathfrak{a}}{\int_{\mathfrak{a}<\mathfrak{a}^*} d\mathfrak{a}},$$

$$\Pr(rej|\text{OUT}_\Omega) = \frac{\int_{\mathfrak{a}>\mathfrak{a}^*} \left(1 - \frac{1}{\Omega}(1-\mathfrak{a}^\Omega)\right) d\mathfrak{a}}{\int_{\mathfrak{a}>\mathfrak{a}^*} d\mathfrak{a}}.$$

For given Ω and $N < \Omega$ the threshold $\mathfrak{a}^*$ depends on the fee f. The agent with $\mathfrak{a} = \mathfrak{a}^*$ is indifferent between joining the platform or not, i.e., $EU(\mathfrak{a}^*|\text{M}_\text{N}) - EU(\mathfrak{a}^*|\text{OUT}_\Omega) = f$. Because in the equilibrium the probabilities depend only on N, Ω, and $\mathfrak{a}^*$, the expected payoffs of agent with $\mathfrak{a}^*$ depend only on those parameters. Through this indifference condition, we find the relationship between $\mathfrak{a}^*$ and f in the equilibrium. We show that $f(\mathfrak{a}^*)$ is positive and strictly decreasing in the relevant interval.[15]

If f is high, only few agents find it worthwhile to participate in the platform and $\mathfrak{a}^*$ is low. As f decreases, $\mathfrak{a}^*$ increases, since more agents prefer to join at a lower fee. The platform can set up such a high price that no agent joins (for such a high f, $\mathfrak{a}^*(f) = 0$). Conversely, if f decreases to 0, all agents join the platform ($\mathfrak{a}^*(f=0) = 1$). Notice that in both of those extreme cases the platform's revenue is 0.

Corollary. *When fee $f = 0$, then $\mathfrak{a}^*(f{=}0) = 1$. That is, all agents prefer to join the platform if the fee is the same as for participating on the outside market.*

Proof. Observation **??** stated a similar result in the example with M_1 and OUT_2. But the same rationale holds for any $N < \Omega$. Mathematically, it follows from the fact that $EU(\mathfrak{a}^*|\text{M}_\text{N}) - EU(\mathfrak{a}^*|\text{OUT}_\Omega)$ is positive on the interval $[0, 1)$. Agents prefer to join the platform with fewer candidates because they prefer candidates with lower $\mathfrak{a}$ (cf. Observation **??** on page **??**). At fee $f = 0$ the benefit of candidates with lower $\mathfrak{a}$ offsets the benefit of having more choice. □

The platform sets the fee f with the objective to maximize its profit. We assume that all the costs for the platform are fixed costs, and the marginal cost is 0. Thus, the profit maximization is equivalent to revenue maximization. For a given fee f only agents with $\mathfrak{a}$ below the threshold $\mathfrak{a}^*(f)$ participate in the platform and pay the fee; then the revenue is $f \cdot \mathfrak{a}^*(f)$. When the platform chooses a fee that results in an interior $\mathfrak{a}^* \in (0, 1)$, the revenues are positive. There is a unique fee that maximizes the platform's profits. Therefore, for any Ω and $N < \Omega$, there is exactly one

[15] It follows from Lemma **??** in Appendix (page **??**). See also the proof of Proposition **??** in Appendix (page **??**).

equilibrium with active platform. In this equilibrium the platform sets a profit maximizing fee, which depends on N and Ω. The threshold that depends on the fee $\mathfrak{a}^*(f)$ is strictly in the interior of the $[0, 1]$ interval. That means that there are agents who participate in the platform, and agents who stay outside.

Proposition 2. *Suppose that in the outside market agents meet Ω candidates, and that there is a platform offering $N < \Omega$ candidates. For any Ω and $N < \Omega$, there exists an equilibrium where the platform charges a positive fee $f > 0$ and there is a threshold $\mathfrak{a}^* \in (0, 1)$ such that agents with $\mathfrak{a} \in [0, \mathfrak{a}^*)$ participate in the platform, agents with $\mathfrak{a} \in (\mathfrak{a}^*, 1]$ stay in the outside market, and agents with $\mathfrak{a} = \mathfrak{a}^*$ are indifferent. This is the only equilibrium with active platform for $N < \Omega$.*

Proof. See the Appendix, page **??**.

The equilibrium where no agent participates in the platform always exists. Proposition **??** states that for $N < \Omega$ there also always exists a unique equilibrium where some agents participate in the platform, and some not. In this equilibrium, the rejection probability in the platform is lower than the rejection probability in the outside market. This is because agents face less competition in the platform (due to $N < \Omega$), and also because agents in the platform are more likely to make and accept an offer, since they have lower utility of being alone. The outside market offers more candidates. Larger number of candidates increases the expected value of a match if matching is successful. For agents with higher utility of being alone, the positive choice effect outweighs the negative competition effect. Those agents prefer the outside market, which offers more choice and more competition. Agents with lower utility of being alone, however, prefer to join the platform, where they have less competition, but also less choice.

The result in Proposition **??** bears some resemblance to the result in Damiano and Li (2007). They show how different types of agents self-select into different "meeting places," where they meet similar agents. The tool of separation between the meeting places is the price: Only some types find it worthwhile to pay higher price. In both their and our papers, meeting similar agents increases the efficiency of matching. The model in Damiano and Li (2007) differs in many assumptions from our model (see the next section for discussion). Most importantly, they do not investigate the network effects. In every meeting place every agent meets exactly one candidate. In our result there are two effects. One — the self-selection — is the same as described by Damiano and Li (2007), but the other — preferences over the number of candidates — is not captured by their model.

6 Discussion

This section focuses on two major assumptions of the model, and discusses the significance of these assumptions for the result.

Heterogenous value of being alone, $\mathfrak{a}$. Many papers in matching literature (e.g. Damiano and Li, 2007, 2008) assume that agents receive 0 if they remain unmatched. Sometimes this assumption is relaxed by allowing agents to receive some other value when unmatched, but this value is usually assumed the same for all agents. However, in many markets (including dating or labor markets) agents differ by the payoff they obtain when unmatched. It is not a trivial assumption, since equilibria in the market change when we allow agents to differ in their utility of being alone.

Suppose that in our model the value of being alone is 0 for all agents. Then every agent prefers the market with as little competition as possible, and therefore as few candidates as possible. A market with more candidates and more competition increases the probability of being rejected and staying alone. With the payoff of being alone 0, the increase in the expected value of the best candidate does not offset the increased probability of being rejected.

The assumption of the utility of being alone equal to 0 is an extreme assumption. Suppose that the value of being alone is some $\tilde{\mathfrak{a}}$ from the interval $(0, 1)$, but that it is the same for all agents. Since agents are all the same when they make a decision whether to join the platform, they all make the same decision. For some values of parameters Ω, N and $\tilde{\mathfrak{a}}$ there exists an equilibrium with active matching platform. In this equilibrium all agents join the matching platform. There always exists an equilibrium where no agent joins the platform. There are no other equilibria. Specifically, there does not exists an equilibrium in which part of agents strictly prefers to participate in the platform and other agents prefer to stay in the outside market.

Subjective value of a candidate, Λ. In most of the papers on matching (e.g., Damiano and Li, 2007, 2008; McAfee, 2002) agents are endowed with an attribute or attributes that are similarly desired by all potential partners. Usually such an attribute is objective "quality." However, in the labor market employers may value differently different characteristics of employees. Even more so in the dating market, where the taste for the partners is idiosyncratic.

In our model we adopt a completely opposite assumption: the values of Λ are independent. That means that when two men meet a woman, the extent to which one man likes the woman is independent of how much the other man likes her. Such an assumption, applied to the whole market in its pure form, is also not a realistic assumption. However, it gives us opportunity to study the other extreme of the market.

Moreover, such an assumption seems more realistic if applied to a selection from a market. Suppose that agents in the market have objective and subjective characteristics. Once they are separated according to their objective characteristics (e.g., education), the agents are different only according to subjective characteristics. Then the assumption of our model applies.

This interpretation illustrates that most markets involve a mixture of the two extremes. While there is a lot of research investigating one extreme, our paper investigates the other extreme. A natural extension of this research would be investigation of intermediate case, where agents' valuations are correlated, but not identical.

The assumption of subjective valuations is important in models that want to relate to reality, because whether we assume independent or perfectly correlated Λ's changes prediction of the model: For comparison, assume in our model that Λ is an objective quality of an agent, and it is how much utility his partner gets if matched — no matter who the partner is. It turns out that in such a case, all agents prefer to meet more candidates than fewer: The impact of competition is much smaller than the benefit of having more choice. This is because when meeting candidates, an agent knows not only how much he values them, but also how other agents value the same candidates, and how the candidates value his own quality. In such a case, every agent makes an offer to the candidate as close to his own quality as possible. If he makes an offer to a candidate of a higher quality, he significantly decreases the probability of having the offer reciprocated. If he makes an offer to a candidate of a lower quality, he decreases the payoff from matching. The expected payoff from making an offer is larger, the closer the candidate is to agent's own quality. Meeting larger number of candidates increases chances of meeting a candidate closer to the agent's own quality. Thus, all agents prefer a market with more candidates to a market with fewer candidates.

7 Conclusions

Our paper is motivated by an apparent disconnect between existing theory and empirical reality. Theoretical literature on network effects suggests that a matching platform should offer unrestricted access to its participants, as this allows agents to find better matches. However, in practice, we observe numerous markets where platforms that restrict choice exist and prosper alongside platforms that offer more choice. Examples of such platforms come from markets as different as the on-line dating market, labor markets, and the real-estate market. Furthermore, platforms restricting choice are often able to charge higher prices. We propose a model that explains these empirical regularities. The two types of platforms coexist because they compete using different business models. In markets with heterogeneous agents, different business models attract different types of agents. Accounting for such heterogeneity allows us to explain not only why two platforms can coexist without the market tipping, but also why an industry may experience entry of a firm with a seemingly inferior product. In fact, we show that a firm that offers such a product (restricted choice) may charge higher prices and be more profitable than its competitors who offer unrestricted choice.

While our paper captures the stylized facts in a number of important industries, it also delivers a series of additional empirical predictions. Our model predicts that the demand for platform services in non-monotonic in the number of candidates that the platform offers. Moreover, restricted-choice platforms should have a higher probability of transaction occurring (or lower expected time to transaction occurring). An important feature of our model is that when agents choose a platform, they self-select based on their characteristics. Platforms that restrict choice will appeal primarily to agents who are impatient or who have poor outside options. Agents with more patience or better outside option will use platforms that maximize choice. The higher the difference in fees charged, the greater will be the differences between participants of the different platforms. This suggests that empirical methodology used in settings with competing platforms should be robust to such realized heterogeneity.

Finally, our analysis has implications for managers seeking to enter into or compete in industries with strong network effects. While prevailing wisdom suggests that they should always offer the largest possible choice on their platforms, our model shows that this intuition does not always hold. We discuss two important dimensions to consider before deciding how to compete. Specifically, the received wisdom holds in markets where people do not differ much in their utility of being unmatched, or where preferences are fairly homogeneous across agents. However, when people differ dramatically in their outside options, and when preferences are highly subjective, managers have more flexibility in how to compete and may want to enter the market as a restricted-choice platform.

Appendix

A Tentative Offers

The main model assumes that agents can make only a single offer. The goal of this assumption is to reflect the fact that people are able to pursue only limited number of possible relationships. Obviously, such an extreme assumption is not a realistic one. However, this section shows that the qualitative results of the model hold for some other, more realistic, offer-making procedures.

In this section, we analyze a two-step offer-making procedure. After the agents meet their candidates and observe how much they like them, they proceed to making offers. In the first stage they can send a fixed number of *tentative offers*. Simultaneously, other agents send their tentative offers. Every agent observes the tentative offers he or she has received, before sending up to a one final offer in the second stage. The final offers are also sent simultaneously. As before, only if the final offer is reciprocated, the relationship is formed. Otherwise, both agents remain unmatched.

We assume that if agents are indifferent between sending an offer (tentative or final) or not, they do not send it. This eliminates a possible situation when agents send tentative offers to candidates that they like less than being alone, but are sure to be rejected by.

We show here that even with the two-step offer-making procedure there are limits to network effects. Adding tentative offer to the procedure increases the overall probability of getting matched. However, when the number of the tentative offers allowed is constant, but the number of candidates increases, agents with lower utility of being alone prefer markets with fewer candidates, while agents with higher utility of being alone prefer markets with more candidates. A fixed number of tentative offers reflects in a more realistic way the limitations to how many potential relations people can pursue.[16] This section illustrates this point through an example of the market with two tentative offers allowed. However, the same holds for any fixed number of tentative offers.

Consider an equilibrium where every agent makes the tentative offers to his two best candidates, provided that at least two candidates are above the reservation threshold. Otherwise, the agent makes a tentative offer to the best candidate — if the best candidate is above the reservation threshold — or to no candidates, if no candidates are above the threshold. If an agent got a tentative offer from his best candidate, he makes the final offer to this candidate. If an agent did not get a tentative offer from the best candidate, but got one from the second-best candidate, then

[16]In a labor market, it reflects the fact that an agent can go to only a limited number of interviews. In the case of an auction site, an agent can follow only a limited number of ongoing auctions.

the agent makes the final offer to the second-best candidate. If the agent did not get a tentative offer either from the best or the second-best candidate, he does not make a final offer and remains unmatched.[17]

For the purpose of the comparative statics we are looking for, we need to find the expected payoff of agent $\mathfrak{a}$ when everyone meets $N \geq 2$ candidates. An agent gets a tentative offer from a particular candidate when he is either the first or the second choice of this candidate, and he is above the candidate's reservation value. An agent is the first choice of a candidate (and above the reservation value) with a probability

$$Pr(best|N) = \frac{1}{N+1}\,.$$

An agent is the second choice of a candidate (and above the reservation value) with a probability

$$Pr(2nd|N) = \int_0^1 \int_{\mathfrak{a}}^1 (N-1)(1-\Lambda)\Lambda^{N-2} d\Lambda d\mathfrak{a} = \frac{N-1}{N(N+1)}\,.$$

Thus, the probability that the agent gets a tentative offer from a particular candidate is

$$Pr(tentative|N) = Pr(best|N) + Pr(2nd|N) = \frac{2N-1}{N(N+1)}\,.$$

An agent makes the final offer to the best candidate when he got a tentative offer from this candidate, and he likes the candidate more than being alone. However, it may be that the agent has not got a tentative offer from the best candidate, but he got one from the second-best candidate. If this is the case, and the second-best candidate is above the reservation threshold, the agent makes the final offer to the second-best candidate.

The agent gets the final offer when he is the most preferred candidate, or when he is the second-best candidate, but the best candidate did not make a tentative offer. Moreover, the agent gets the final offer from a candidate only if both he and the candidate made tentative offers to each other. The probability that the candidate makes tentative offer is already incorporated in the probability of getting the final offer. But we need to remember that the agent makes a tentative offer to the

[17] There are also other equilibria possible. All have the following structure: Let Λ^{MAX} be the Λ of the best candidate. If agent $\mathfrak{a}$ got a tentative offer from a candidate whose Λ is at least $x(\Lambda^{MAX})$, he makes the final offer to the best of such candidates, even if he did not make a tentative offer to this candidate. If the agent did not get a tentative offer from any of the candidates above $x(\Lambda^{MAX})$, he makes the final offer to his best candidate, even though he did not received a tentative offer from this candidate. The additional probability of successfully matching in such equilibrium is very small and decreasing with the number of candidates. Therefore, it does not change the qualitative results of this section.

best or the second-best candidate only if the candidate is above the reservation value $\mathfrak{a}$. That is, the probability of getting both the tentative and the final offers is

$$\Big[Pr(best|N) + Pr(2nd|N)\cdot\big(1 - Pr(tentative|N)\big)\Big]\cdot Pr(\text{candidate above } \mathfrak{a}) =$$
$$= \Big[\underbrace{Pr(tentative|N)\cdot\big(1 - Pr(2nd|N)\big)}_{Pr(final|N)}\Big]\cdot Pr(\text{candidate above } \mathfrak{a}) =$$
$$= \frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\cdot Pr(\text{candidate above } \mathfrak{a})\,.$$

For the future reference, it is useful to define $Pr(final|N) = Pr(tentative|N)\cdot\big(1 - Pr(2nd|N)\big)$.

The agent matches with the best candidate when he received a tentative and the final offers from that candidate and the candidate was better than being alone. The probability that the best candidate out of N is above $\mathfrak{a}$ is $1-\mathfrak{a}^N$. Therefore, the agent matches with the best candidate with probability

$$Pr(match\ best|N,\mathfrak{a}) = \frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\left(1-\mathfrak{a}^N\right)\,.$$

The agent matches with the second best candidate when he received a tentative and final offer from that candidate, the second-best candidate was better than being alone, and he did not receive a tentative offer from the best candidate. The probability that the second-best candidate is above $\mathfrak{a}$ is

$$N(N-1)\int_{\mathfrak{a}}^{1}\Lambda^{N-2}(1-\Lambda)d\Lambda = 1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\,.$$

Thus, the agent matches with the second-best candidate with probability

$$Pr(match\ 2nd|N,\mathfrak{a}) = \big(1-Pr(tentative|N)\big)\cdot Pr(final|N)\cdot\big(1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big) =$$
$$= \left(1-\frac{2N-1}{N(N+1)}\right)\frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\cdot\big(1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\,.$$

With the remaining probability of

$$1 - Pr(match\ best|N,\mathfrak{a}) - Pr(match\ 2nd|N,\mathfrak{a}) =$$
$$= 1 - Pr(final|N)\Big(1-\mathfrak{a}^N + \big(1-Pr(tentative|N)\big)\big(1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\Big) =$$
$$1-\frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\left(1-\mathfrak{a}^N + \left(1-\frac{2N-1}{N(N+1)}\right)\big(1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\right)$$

the agent remains unmatched and receives the payoff of $\mathfrak{a}$.

The expected payoff from matching with the best candidate out of N is

$$EU(match\ best|\mathfrak{a}, N) = N \int_{\mathfrak{a}}^{1} \Lambda^N dN = \frac{N}{N+1}\left(1-\mathfrak{a}^{N+1}\right) .$$

Notice that this formula already accounts for probability that the best candidate is above $\mathfrak{a}$.

The expected payoff from matching with the second-best candidate out of N is

$$EU(match\ 2nd|\mathfrak{a}, N) = N(N-1) \int_{\mathfrak{a}}^{1} \Lambda^{N-1}(1-\Lambda)d\Lambda = \frac{N-1}{N+1} - (N-1)\mathfrak{a}^N \left(1 - \frac{N}{N+1}\mathfrak{a}\right) .$$

Therefore, the expected payoff for agent $\mathfrak{a}$ in a market where two tentative offers are allowed and there are N candidates is

$$\begin{aligned}
&EU(\mathfrak{a}|N) = \\
&= Pr(final|N) \cdot EU(match\ best|\mathfrak{a}, N) + \big(1 - Pr(tentative|N)\big) Pr(final|N) \cdot EU(match\ 2nd|\mathfrak{a}, N) + \\
&\quad + \mathfrak{a} \cdot \Big[1 - Pr(final|N)\big(1 - \mathfrak{a}^N + \big(1 - Pr(tentative|N)\big)\big(1 - \mathfrak{a}^N - N\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\Big] = \\
&= \frac{2N-1}{N(N+1)} \cdot \frac{N^2+1}{N(N+1)} \frac{N}{N+1}\left(1 - \mathfrak{a}^{N+1}\right) + \\
&\quad + \left(1 - \frac{2N-1}{N(N+1)}\right) \frac{2N-1}{N(N+1)} \cdot \frac{N^2+1}{N(N+1)} \cdot \left(\frac{N-1}{N+1} - (N-1)\mathfrak{a}^N\left(1 - \frac{N}{N+1}\mathfrak{a}\right)\right) + \\
&\quad + \mathfrak{a}\left[1 - \frac{2N-1}{N(N+1)} \cdot \frac{N^2+1}{N(N+1)}\left(1 - \mathfrak{a}^N + \left(1 - \frac{2N-1}{N(N+1)}\right)\left(1 - \mathfrak{a}^N - N\mathfrak{a}^{N-1}(1-\mathfrak{a})\right)\right)\right] .
\end{aligned}$$

By graphing this formula for $N \geq 2$, we see that all agents prefer $N = 3$ to $N = 2$. But the agents are divided whether they prefer 3 or 4 candidates. Agents with $\mathfrak{a} < 0.1379$ (approximately) prefer 3 candidates and agents above that thresholds prefer 4. Similarly, agents with $\mathfrak{a} < 0.3739$ prefer 4 candidates and agents above that prefer 5. In a similar way as in the basic model, it can be shown that the optimal number of candidates is weakly increasing with the utility of being alone.

Interestingly, if there is no limit on tentative offers (i.e., one can always make tentative offers to all candidates above the reservation value, as the number of candidates increases), then the probability of matching with someone above the reservation value increases with the number of candidates. There is no trade-off, and all agents always prefer to meet more candidates.

A matching platform in the market with two tentative offers. We extend the analysis of this environment to illustrate that the results for a market with a platform also apply to the case of tentative offers: For sufficiently large number of candidates in the outside market, a platform offering fewer candidates attracts agents with $\mathfrak{a}$ lower than some threshold $\mathfrak{a}^*$. Agents above the threshold stay in the outside market.

Suppose that in the environment where agents can make up to two tentative offers, there is a matching platform. We assume that agents can make the same number of tentative offers in the platform and in the outside market, but the platform differs from the outside market in the number of candidates it offers. In the outside market the agents meet Ω candidates. The platform offers fewer candidates, $N < \Omega$, and charges a positive fee f.

Agents decide whether to participate in the platform at fee f or to stay outside by comparing their expected payoff. The derivations of the expected payoffs are similar to those above. However, the presence of the platform in the market significantly changes the probabilities of matching with the best and the second-best candidate, $\Pr(best)$ and $\Pr(2nd)$. This affects the probabilities of getting a tentative offer, $\Pr(tentative)$, as well as $\Pr(final)$. Those values are affected because they depend not only on then number of candidates, but also on the types of the candidates. As different types of agents self-select to participate in the platform or stay outside, this affects the probabilities $\Pr(best)$ and $\Pr(2nd)$. Conversely, the expected payoffs of matching with the best and second-best candidate, $EU(match\ best)$ and $EU(match\ 2nd)$, as well as the probabilities that the best and second-best candidates are above $\mathfrak{a}$, are affected only by the number of candidates. Those values do not depend on the presence of multiple platforms in the market.

Given that agents with lower $\mathfrak{a}$'s prefer fewer candidates, we expect that only agents with $\mathfrak{a}$'s below some threshold $\mathfrak{a}^*$ decide to join the platform. The probabilities of getting a tentative or final offer depend on the distribution of $\mathfrak{a}$ among the participants of the platform. The probability of being the best choice of a candidate is

$$\Pr(best|\mathrm{M_N}) = \frac{1}{N}\left(1 - \frac{1}{N+1}(\mathfrak{a}^*)^N\right).$$

The probability of being the 2nd-best choice of a candidate is

$$\Pr(2nd|\mathrm{M_N}) = \frac{\int_0^{\mathfrak{a}^*}\int_{\mathfrak{a}}^1 (N-1)(1-\Lambda)\Lambda^{N-2}d\Lambda d\mathfrak{a}}{\int_0^{\mathfrak{a}^*} d\mathfrak{a}} = \frac{1}{N}\left(1 - (\mathfrak{a}^*)^{N-1} + \frac{N-1}{N+1}(\mathfrak{a}^*)^N\right).$$

Similarly, in the outside market, the probability of being the best choice of a candidate is

$$\Pr(best|\text{OUT}_\Omega) = \frac{\int_{\mathfrak{a}^*}^1 \int_{\mathfrak{a}}^1 \Lambda^{\Omega-1} d\Lambda d\mathfrak{a}}{\int_{\mathfrak{a}^*}^1 d\mathfrak{a}} = \frac{1}{\Omega}\left(1 - \frac{1}{\Omega+1}\cdot\frac{1-(\mathfrak{a}^*)^{\Omega+1}}{1-\mathfrak{a}^*}\right).$$

And the probability of being the 2nd-best choice of a candidate is

$$\Pr(2nd|\text{OUT}_\Omega) = \frac{\int_{\mathfrak{a}^*}^1 \int_{\mathfrak{a}}^1 (\Omega-1)(1-\Lambda)\Lambda^{\Omega-2} d\Lambda d\mathfrak{a}}{\int_{\mathfrak{a}^*}^1 d\mathfrak{a}} = \frac{\frac{\Omega-1}{\Omega(\Omega+1)}\left(1-(\mathfrak{a}^*)^{\Omega+1}\right) - \frac{1}{\Omega}\mathfrak{a}^*\left(1-(\mathfrak{a}^*)^{\Omega-1}\right)}{1-\mathfrak{a}^*}.$$

The expected payoff of agent with $\mathfrak{a}$ is calculated based on the same formula:

$$\begin{aligned} EU(\mathfrak{a}|\,X) = {} & \Pr(final)\cdot EU(match\ best|\mathfrak{a},X) + \\ & + \big(1-\Pr(tentative)\big)\Pr(final)\cdot EU(match\ 2nd|\mathfrak{a},X) + \\ & + \mathfrak{a}\cdot\left[1-\Pr(final)\Big(1-\mathfrak{a}^X + \big(1-\Pr(tentative)\big)\big(1-\mathfrak{a}^X - X\mathfrak{a}^{X-1}(1-\mathfrak{a})\big)\Big)\right]. \end{aligned}$$

The expected payoff for $\mathfrak{a}$ in the platform is obtained by substituting $N = X$ and the appropriate probabilities. Similarly, the expected payoff for $\mathfrak{a}$ in the outside market is obtained by substituting $\Omega = X$ and the appropriate probabilities. In this way, we obtain $EU(\mathfrak{a}|\text{M}_\text{N})$ and $EU(\mathfrak{a}|\text{OUT}_\Omega)$ as functions of $\mathfrak{a}$, N and Ω. It is worth noting that some of the formulas are obtained assuming $N \geq 2$, and should not be applied to $N = 1$.

Given the threshold, every agent compares these two expected payoffs, and decides whether to join the platform at fee f or not. In the equilibrium, the agent with $\mathfrak{a} = \mathfrak{a}^*$ is indifferent between joining at fee f or staying outside:

$$EU(\mathfrak{a}^*|\text{M}_\text{N}) - f = EU(\mathfrak{a}^*|\text{OUT}_\Omega).$$

For any values of N and Ω, this equation provides a relation between the fee f and the threshold $\mathfrak{a}^*$. In the case when f as a function of $\mathfrak{a}^*$ is strictly monotonic for $\mathfrak{a}^* \in [0,1]$, there is exactly one possible threshold $\mathfrak{a}^*$ for each level of f. Examining the examples, we observe that for $N = 2$ or when $N > \Omega$, the monotonicity sometimes does not hold. However, if $N > 2$ and $\Omega > N$, f is strictly decreasing in $\mathfrak{a}^*$ on the interval $[0,1]$.

When $f(\mathfrak{a}^*)$ is strictly decreasing, for every fee within a range, all agents with $\mathfrak{a} < \mathfrak{a}^*$ prefer to join the platform and pay the fee. And all agents with $\mathfrak{a} > \mathfrak{a}^*$ prefer to stay outside. Moreover, given the unique response of the market to each fee, the platform chooses a fee (indirectly choosing the threshold $\mathfrak{a}^*$) to maximize the profit.

Therefore, for markets with two tentative offers, it is also true that a platform offering fewer candidates than the outside market is attractive for some parts of the market, and achieves positive profits. And the result holds for any number of tentative offers, as long as the number of tentative offers is fixed and lower than the number of candidates available.

B Proofs

Statement and Proof of Lemma ??

Many of the proofs make use of the following Lemma:

Lemma 3. *In a market with N candidates:*

(1) Every man receives an offer from any woman he meets with probability $\frac{1}{N+1}$. With the remaining probability $\frac{N}{N+1}$ he does not receive an offer from her (i.e. the probability of having an offer reciprocated is $\frac{1}{N+1}$).

(2) An agent $\mathfrak{a}$ matches successfully with probability

$$\frac{1}{N+1}\left(1-\mathfrak{a}^N\right) .$$

(3) For an agent $\mathfrak{a}$ the expected value of a match, conditional upon successfully matching is

$$\frac{N}{N+1}\,\frac{1-\mathfrak{a}^{N+1}}{1-\mathfrak{a}^N} .$$

(4) The total expected payoff for agent $\mathfrak{a}$ in the market with N candidates is

$$EU(\mathfrak{a}|N) = \underbrace{\frac{N+\mathfrak{a}^N}{N+1}}_{\text{prob of not matching}} \cdot\mathfrak{a} + \underbrace{\frac{1-\mathfrak{a}^N}{N+1}}_{\text{prob of matching}} \cdot \underbrace{\frac{N}{N+1}\,\frac{1-\mathfrak{a}^{N+1}}{1-\mathfrak{a}^N}}_{\text{exp payoff if matched}} = $$
$$= \frac{1}{(N+1)^2}\mathfrak{a}^{N+1} + \frac{N}{N+1}\mathfrak{a} + \frac{N}{(N+1)^2} .$$

Proof.

(1) With N candidates, a woman that the man meets has $N+1$ possible actions: make an offer to one of the N candidates and make no offer at all (when $\mathfrak{a}^w$ is larger than any of the relevant Λ's). All Λ's

and $\mathfrak{a}^w$ are drawn independently from the same distribution $U \sim [0,1]$. Therefore, without knowing $\mathfrak{a}^w$, each of the actions is equally likely: $\frac{1}{N+1}$.

(2) With probability $(1-\mathfrak{a}^N)$ the best Λ is above $\mathfrak{a}$. The agent makes an offer to the best Λ. Independently, the best Λ makes an offer to agent $\mathfrak{a}$ with probability $\frac{1}{N+1}$ (from point (1) of this Lemma).

(3) Unconditional value of matching is $(1 - Pr(rej)) \cdot E(\max \Lambda | \max \Lambda > \mathfrak{a})$. And $1 - Pr(rej) = \frac{1}{N+1}$. The probability of matching is $\frac{1}{N+1}(1-\mathfrak{a}^N)$. Thus, the value of matching, conditional on matching is $\frac{E(\max \Lambda | \max \Lambda > \mathfrak{a})}{1-\mathfrak{a}^N}$.

To find the conditional expected value of $E(\max \Lambda | \max \Lambda > \mathfrak{a})$, we first characterize the distribution function of $\max \Lambda$ under N candidates. Notice that the cdf of $\max \Lambda$ is $Pr(\max \Lambda < x) = x^N$. Thus, the pdf is $\frac{\partial x^N}{\partial x} = N x^{N-1}$. Using the probability density, we calculate the expected value of $\max \Lambda$, given that $\max \Lambda > \mathfrak{a}$:

$$\int_{\mathfrak{a}}^{1} (N x^{N-1}) \cdot x dx = N \int_{\mathfrak{a}}^{1} x^N dx = N \left[\frac{1}{N+1} x^{N+1} \right]_{\mathfrak{a}}^{1} = \frac{N}{N+1} \left(1 - \mathfrak{a}^{N+1}\right) .$$

(4) Follows directly from parts (2) and (3) of the Lemma.

This completes the proof of Lemma **??**. □

Proof of Lemma ?? (page ??)

Proof. By Lemma **??**(3), for an agent $\mathfrak{a} < 1$, the expected value of a match, conditional upon successfully matching is (this follows from Lemma **??**(3)):

$$EV_{succ}(\mathfrak{a}, N) = \frac{N}{N+1} \frac{1-\mathfrak{a}^{N+1}}{1-\mathfrak{a}^N} .$$

Then

$$EV_{succ}(\mathfrak{a}, N+1) - EV_{succ}(\mathfrak{a}, N) =$$
$$= \frac{N+1}{N+2} \frac{1-\mathfrak{a}^{N+2}}{1-\mathfrak{a}^{N+1}} - \frac{N}{N+1} \frac{1-\mathfrak{a}^{N+1}}{1-\mathfrak{a}^N} = \frac{(N+1)^2(1-\mathfrak{a}^{N+2})(1-\mathfrak{a}^N) - N(N+2)(1-\mathfrak{a}^{N+1})^2}{(N+1)(N+2)(1-\mathfrak{a}^{N+1})(1-\mathfrak{a}^N)} > 0$$

The inequality is obtained by direct algebraical manipulation. This completes the proof of Lemma **??**. □

Proof of Lemma ?? (page ??)

Proof. By Lemma **??**(2), an agent $\mathfrak{a}$ matches successfully with probability

$$MP(\mathfrak{a}, N) = \frac{1}{N+1}(1 - \mathfrak{a}^N).$$

Let $\Delta MP(\mathfrak{a}, N) = MP(\mathfrak{a}, N) - MP(\mathfrak{a}, N+1)$. Suppose that there exists $\mathfrak{a}^* < 1$, such that $\Delta MP(\mathfrak{a}^*, N) = 0$, i.e. agent $\mathfrak{a}^*$ is has the same probability of matching under N candidates as under $N+1$. Then for all $\mathfrak{a} < \mathfrak{a}^*$, $\Delta MP(\mathfrak{a}, N) > 0$, i.e., they are more likely to match successfully under N candidates than under $N+1$; and for all $\mathfrak{a} > \mathfrak{a}^*$, $\Delta MP(\mathfrak{a}, N) < 0$, i.e., they are more likely to match successfully under $N+1$ candidates than under N. To see that, notice that

$$\Delta MP(\mathfrak{a}, N) = \Delta MP(\mathfrak{a}, N) - \Delta MP(\mathfrak{a}^*, N) = \frac{1}{N+2}\left[\frac{1}{N+1}((\mathfrak{a}^*)^N - \mathfrak{a}^N) + ((\mathfrak{a}^*)^N - \mathfrak{a}^N) - ((\mathfrak{a}^*)^{N+1} - \mathfrak{a}^{N+1})\right]$$

For $\mathfrak{a} < \mathfrak{a}^*$, $(\mathfrak{a}^*)^N > \mathfrak{a}^N$ and $(\mathfrak{a}^*)^{N+1} - \mathfrak{a}^{N+1} < (\mathfrak{a}^*)^N - \mathfrak{a}^N$, so $\Delta MP(\mathfrak{a}, N) > 0$. Conversely, for $\mathfrak{a} > \mathfrak{a}^*$, $(\mathfrak{a}^*)^N < \mathfrak{a}^N$ and $(\mathfrak{a}^*)^{N+1} - \mathfrak{a}^{N+1} > (\mathfrak{a}^*)^N - \mathfrak{a}^N$, so $\Delta MP(\mathfrak{a}, N) < 0$. (Note: $\left|(\mathfrak{a}^*)^{N+1} - \mathfrak{a}^{N+1}\right| < \left|(\mathfrak{a}^*)^N - \mathfrak{a}^N\right|$ for all $\mathfrak{a}, \mathfrak{a}^* < 1$.)

Moreover, for this $\mathfrak{a}^*$ and $N+1$, $\Delta MP(\mathfrak{a}^*, N+1) > 0$, i.e., the same $\mathfrak{a}^*$ that was as likely to match under N as under $N+1$, is less likely to match successfully under $N+2$. It follows that all $\mathfrak{a} < \mathfrak{a}^*$ are less likely to match under $N+2$ then under $N+1$. Therefore, they are more likely to match under N candidates then under any larger number of candidates. To see that, suppose that for $\mathfrak{a}^*$, $\Delta MP(\mathfrak{a}^*, N) = \frac{1}{N+2}\left[\frac{1}{N+1}(1 - (\mathfrak{a}^*)^N) - (1 - \mathfrak{a}^*)(\mathfrak{a}^*)^N\right] = 0$, i.e., $\frac{1}{N+1}(1 - (\mathfrak{a}^*)^N) = (1 - \mathfrak{a}^*)(\mathfrak{a}^*)^N$. Then for $N+1$,

$$\frac{1}{N+2}(1 - (\mathfrak{a}^*)^{N+1}) - (1 - \mathfrak{a}^*)(\mathfrak{a}^*)^{N+1} = \frac{1}{N+2}(1 - (\mathfrak{a}^*)^{N+1}) - \mathfrak{a}^* \cdot \frac{1}{N+1}(1 - (\mathfrak{a}^*)^N) =$$

$$= \frac{1 - \mathfrak{a}^*}{(N+1)(N+2)}\left[(N+1) - \mathfrak{a}^* \sum_{i=0}^{N-1} (\mathfrak{a}^*)^i\right] > 0\,.$$

The inequality holds because $\mathfrak{a}^* \sum_{i=0}^{N-1} (\mathfrak{a}^*)^i < N$ for $\mathfrak{a}^* < 1$. Therefore, for $\mathfrak{a} < \mathfrak{a}^*$ the probability of matching decreases with N. This completes the proof of Lemma **??**. □

Proof of Proposition ?? (page ??)

Proof. This proof is based on the following formula $\Delta EU(\mathfrak{a}|N) = EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N)$, which becomes

$$\Delta EU(\mathfrak{a}|N) = \underbrace{(\mathfrak{a}-1)^2 \frac{1}{(N+2)^2(N+1)^2}}_{\geq 0} \underbrace{\sum_{i=0}^{N} \left[(N+1)^2 - i(2N+3)\right] \mathfrak{a}^{N-i}}_{G(N,\mathfrak{a})}$$

Value $\mathfrak{a}^*(N)$ which is indifferent between a market with N and a market with $N+1$ candidates is the solution to $\Delta EU(\mathfrak{a}^*|N) = 0$. Of course, $\mathfrak{a} = 1$ satisfies this condition for any N. Any $\mathfrak{a} < 1$ that satisfies $\Delta EU(\mathfrak{a}^*|N) = 0$ must also satisfy $G(N, \mathfrak{a}^*) = 0$.

We can show that

(1) There is exactly one solution $\mathfrak{a}^*(N) \neq 1$ that satisfies $G(N, \mathfrak{a}^*) = 0$. Moreover, for $\mathfrak{a} < \mathfrak{a}^*(N), G(N, \mathfrak{a}) < 0$ (i.e., they prefer N candidates) and for $\mathfrak{a} > \mathfrak{a}^*(N), G(N, \mathfrak{a}) > 0$ (i.e., they prefer $N+1$ candidates).

(2) $N' > N \implies \mathfrak{a}^*(N') > \mathfrak{a}^*(N)$.

With (1) and (2) satisfied, it must be that the most-preferred number of candidates is non-decreasing with $\mathfrak{a}$.

On point (1). We show that $G(N, \mathfrak{a})$ has exactly one solution on the interval $[0, 1]$ in three steps:

(i) For $\mathfrak{a} = 0$, $G(N, 0)$ is always strictly negative: $G(N, 0) = -N(N+1) + 1 \ < \ 0$ for $N \geq 1$.

(ii) For $\mathfrak{a} = 1$, $G(N, 1)$ is always strictly positive: $G(N, 1) = (N+1)(\frac{1}{2}N + 1) > 0$.

(iii) $\exists!\, \mathfrak{a}^*$ s.t. $G(N, \mathfrak{a}) < 0$ for all $\mathfrak{a} < \mathfrak{a}^*$ and $G(N, \mathfrak{a}) > 0$ for all $\mathfrak{a} > \mathfrak{a}^*$.
Suppose, to the contrary, that $\exists\, \mathfrak{a}', \mathfrak{a}''$ s.t. $\mathfrak{a}' < \mathfrak{a}''$ but $G(N, \mathfrak{a}') > 0 > G(N, \mathfrak{a}'')$. Since $G(N, \mathfrak{a})$ is a continuous function, it must be then that for some $\mathfrak{a} \in (\mathfrak{a}', \mathfrak{a}'')$, $G(N, \mathfrak{a}) > 0$ and $\partial G(N, \mathfrak{a})/\partial \mathfrak{a} < 0$. But this is not possible, because it can be shown that[18] $\frac{\partial G(N,\mathfrak{a})}{\partial \mathfrak{a}} > G(N, \mathfrak{a})$ for any $\mathfrak{a} \in [0, 1]$.

On point (2). To show $\mathfrak{a}^*(N') > \mathfrak{a}^*(N)$ when $N' > N$, it is sufficient to show that when for some $\mathfrak{a}^* < 1$, $\Delta EU(\mathfrak{a}^*|N) = 0$, then for all $N' > N$, $\Delta EU(\mathfrak{a}^*|N') < 0$. And since for all $\mathfrak{a} < 1$, $(1-\mathfrak{a})^2 \frac{1}{(N+1)^2(N+2)^2} < 0$, then it is enough to show that the property holds for $G(\mathfrak{a}^*, N)$.

Fix an N, and assume that for $\mathfrak{a}^*$, $G(\mathfrak{a}^*, N) = 0$. Since $G(\mathfrak{a}^*, N) = 0$, then $\mathfrak{a}^* \cdot G(\mathfrak{a}^*, N) = 0$

$$G(\mathfrak{a}^*, N) \cdot \mathfrak{a}^* = \sum_{i=0}^{N} \left[(N+1)^2 - i(2N+3)\right] (\mathfrak{a}^*)^{N-i+1}$$

For $N' = N+1$, $G(\mathfrak{a}^*, N') = \sum_{i=0}^{N'} \left[(N'+1)^2 - i(2N'+3)\right] (\mathfrak{a}^*)^{N'-i}$. Then, by algebraical manipulation

$$G(\mathfrak{a}^*, N+1) = \sum_{i=0}^{N+1} \left[(N+2)^2 - i(2N+5)\right] (\mathfrak{a}^*)^{N-i+1} - \frac{(N+2)^2}{(N+1)^2} \underbrace{\sum_{i=0}^{N} \left[(N+1)^2 - i(2N+3)\right] (\mathfrak{a}^*)^{N-i+1}}_{=0} =$$

$$= \left[(N+2)^2 - (N+1)(2N+5)\right] (\mathfrak{a}^*)^0 + \sum_{i=0}^{N} i \left[\frac{(N+2)^2}{(N+1)^2}(2N+3) - (2N+5)\right] (\mathfrak{a}^*)^{N-i+1} \geq \frac{-(N+2)}{2(N+1)} \ < \ 0$$

Therefore, $\Delta EU(\mathfrak{a}^*|N+1) < 0$, and it can be shown for any $N' > N$. That means that for $\mathfrak{a}^*$, $EU(\mathfrak{a}^*|N) = EU(\mathfrak{a}^*|N+1) > EU(\mathfrak{a}^*|N'+1)$ for any $N' > N$. Since we know that $\Delta EU(\mathfrak{a}|N) < 0$ for all

[18] Detailed proof available from the authors upon request.

$\mathfrak{a} < \mathfrak{a}^*$, then for all $\mathfrak{a} < \mathfrak{a}^*$, $EU(\mathfrak{a}|N') < EU(\mathfrak{a}|N)$ for any $N' > N$. That is, for those $\mathfrak{a}$'s, more candidates than N always bring lower expected payoff. Therefore, the most-preferred N is non-decreasing with $\mathfrak{a}$.

This completes the proof of Proposition **??**. □

Statement and Proof of Lemma ??

Lemma **??** and Lemma **??** are used in the proof of Proposition **??**.

Lemma 4. $\frac{\mathfrak{a}^N - \mathfrak{a}^\Omega}{1-\mathfrak{a}^N}$ *is strictly monotonic for* $\mathfrak{a} \in (0,1)$.

Proof. Let $\Omega = N - n$. The derivative of $\frac{\mathfrak{a}^N - \mathfrak{a}^\Omega}{1-\mathfrak{a}^N}$ with respect to $\mathfrak{a}$ is

$$\frac{\left(N\mathfrak{a}^{N-1} - (N+n)\mathfrak{a}^{N+n-1}\right)\left(1-\mathfrak{a}^N\right) + N\mathfrak{a}^{N-1}(\mathfrak{a}^N - \mathfrak{a}^{N+n})}{(1-\mathfrak{a}^N)^2}$$

The denominator is always positive. The enumerator is equivalent to $N\mathfrak{a}^{N-1}\left(1-\mathfrak{a}^n\right) + n\mathfrak{a}^{N+n-1}\left(1-\mathfrak{a}^N\right)$, which is strictly positive for all $\mathfrak{a} \in (0,1)$ when $n > 0$, and strictly negative when $n < 0$. □

Statement and Proof of Lemma ??

Lemma 5. *For any* Ω *and* $N < \Omega$, *if the difference* $\Delta EU(\mathfrak{a}) = EU(\mathfrak{a}|\text{M}_\text{N}) - EU(\mathfrak{a}|\text{OUT}_\Omega)$ *is positive, it is decreasing in* $\mathfrak{a}$.

Proof. The expected payoffs for agent $\mathfrak{a}$ from participating in M_N or from staying in the outside market are, respectively,

$$\begin{aligned} EU(\mathfrak{a}|\text{M}_\text{N}) =& \mathfrak{a}(\mathfrak{a}^N + (1-\mathfrak{a}^N)\Pr(rej|\text{M}_\text{N})) + (1-\Pr(rej|\text{M}_\text{N}))\frac{N}{N+1}\left(1-\mathfrak{a}^{N+1}\right), \\ EU(\mathfrak{a}|\text{OUT}_\Omega) =& \mathfrak{a}(\mathfrak{a}^\Omega + (1-\mathfrak{a}^\Omega)\Pr(rej|\text{OUT}_\Omega)) + (1-\Pr(rej|\text{OUT}_\Omega))\frac{\Omega}{\Omega+1}\left(1-\mathfrak{a}^{\Omega+1}\right). \end{aligned}$$

In a given market, from a point of view of individual agent, the rejection probabilities are constant: $\Pr(rej|\text{M}_\text{N})$ and $\Pr(rej|\text{OUT}_\Omega)$. The platform charges a positive fee f. Only those agents prefer to join the platform, for whom $EU(\mathfrak{a}|\text{M}_\text{N}) - EU(\mathfrak{a}|\text{OUT}_\Omega) \geq f > 0$.

Let $\Delta\Pr = \Pr(rej|\text{M}_\text{N}) - \Pr(rej|\text{OUT}_\Omega)$, and let $\Delta EU(\mathfrak{a}) = EU(\mathfrak{a}|\text{M}_\text{N}) - EU(\mathfrak{a}|\text{OUT}_\Omega)$. Then

$$\frac{\partial \Delta EU(\mathfrak{a})}{\partial \mathfrak{a}} = \underbrace{\left(1-\Pr(rej|\text{OUT}_\Omega)\right)}_{+}\underbrace{\left(\mathfrak{a}^N - \mathfrak{a}^\Omega\right)}_{?} + \underbrace{\Delta\Pr}_{?}\underbrace{\left(1-\mathfrak{a}^N\right)}_{+}.$$

Note that for $\mathfrak{a} = 1$, the derivative $\frac{\partial \Delta EU(\mathfrak{a})}{\partial \mathfrak{a}} = 0$. For $\mathfrak{a} = 0$, the derivative equals $\Delta \Pr$, so it has the same sign as $\Delta \Pr$: either positive or negative. The derivative can change its sign on the interval $\mathfrak{a} \in (0, 1)$. However, it can change the sign at most once on this interval. This property follows from the fact that $\frac{\mathfrak{a}^N - \mathfrak{a}^\Omega}{1-\mathfrak{a}^N}$ is strictly monotonic for $\mathfrak{a} \in (0, 1)$, by Lemma ??.[19] For future reference, it is also worth noting that the ratio $\frac{\mathfrak{a}^N - \mathfrak{a}^\Omega}{1-\mathfrak{a}^N}$ takes values from 0 for $\mathfrak{a} = 0$ to $\frac{\Omega - N}{N}$ for $\mathfrak{a} = 1$.

Therefore, the difference $\Delta EU(\mathfrak{a})$ is either single-peaked or monotonic on $\mathfrak{a} \in [0, 1]$. Based on the shape of $\Delta EU(\mathfrak{a})$, we can distinguish following cases (represented in Figure ??):

case (1): $\Delta EU(\mathfrak{a} = 0) < 0$, $\frac{\partial \Delta EU(\mathfrak{a})}{\partial \mathfrak{a}} > 0$ for all $\mathfrak{a} \in [0, 1)$. This case occurs when
$N < \Omega$ and $\Delta \Pr > 0$, as well as
$N > \Omega$ and $\frac{-\Delta \Pr}{1-\Pr(rej|\text{OUT}_\Omega)} < -\frac{N-\Omega}{N}$ (which implies $\Delta \Pr > 0$).

case (2): $\Delta EU(\mathfrak{a} = 0) > 0$, $\frac{\partial \Delta EU(\mathfrak{a})}{\partial \mathfrak{a}} < 0$ for all $\mathfrak{a} \in [0, 1)$. This case occurs when
$N < \Omega$ and $\frac{-\Delta \Pr}{1-\Pr(rej|\text{OUT}_\Omega)} > \frac{\Omega - N}{N}$ (which implies $\Delta \Pr < 0$), as well as
$N > \Omega$ and $\Delta \Pr < 0$.

case (3a): $\Delta EU(\mathfrak{a} = 0) < 0$, there exists $\mathfrak{a} \in (0, 1)$ such that $\frac{\partial \Delta EU(\mathfrak{a})}{\mathfrak{a}} = 0$ and $\Delta \Pr < 0$ (i.e., the difference initially decreases, and then increases). This case occurs when
$N < \Omega$ and $\frac{-\Delta \Pr}{1-\Pr(rej|\text{OUT}_\Omega)} \in \left[0, \frac{\Omega - N\left(\frac{1-\Pr(rej|\text{M}_\text{N})}{1-\Pr(rej|\text{OUT}_\Omega)}\right)}{N\Omega}\right)$.

case (3b): $\Delta EU(\mathfrak{a} = 0) > 0$, there exists $\mathfrak{a} \in (0, 1)$ such that $\frac{\partial \Delta EU(\mathfrak{a})}{\mathfrak{a}} = 0$ and $\Delta \Pr < 0$ (i.e., the difference initially decreases, and then increases). This case occurs when
$N < \Omega$ and $\frac{-\Delta \Pr}{1-\Pr(rej|\text{OUT}_\Omega)} \in \left(\frac{\Omega - N\left(\frac{1-\Pr(rej|\text{M}_\text{N})}{1-\Pr(rej|\text{OUT}_\Omega)}\right)}{N\Omega}, \frac{\Omega - N}{N}\right]$.

case (4a): $\Delta EU(\mathfrak{a} = 0) < 0$, there exists $\mathfrak{a} \in (0, 1)$ such that $\frac{\partial \Delta EU(\mathfrak{a})}{\mathfrak{a}} = 0$ and $\Delta \Pr > 0$ (i.e., the difference initially increases, and then decreases). This case occurs when
$N > \Omega$ and $\frac{-\Delta \Pr}{1-\Pr(rej|\text{OUT}_\Omega)} \in \left[-\frac{N-\Omega}{N}, \frac{\Omega - N\left(\frac{1-\Pr(rej|\text{M}_\text{N})}{1-\Pr(rej|\text{OUT}_\Omega)}\right)}{N\Omega}\right)$.

case (4b): $\Delta EU(\mathfrak{a} = 0) > 0$, there exists $\mathfrak{a} \in (0, 1)$ such that $\frac{\partial \Delta EU(\mathfrak{a})}{\mathfrak{a}} = 0$ and $\Delta \Pr > 0$ (i.e., the difference initially increases, and then decreases). This case occurs when
$N > \Omega$ and $\frac{-\Delta \Pr}{1-\Pr(rej|\text{OUT}_\Omega)} \in \left(\frac{\Omega - N\left(\frac{1-\Pr(rej|\text{M}_\text{N})}{1-\Pr(rej|\text{OUT}_\Omega)}\right)}{N\Omega}, 0\right]$.

For $N < \Omega$ only cases (1), (2), (3a) and (3b) may occur.[20] In those cases, whenever $\Delta EU(\mathfrak{a}) > 0$, it is strictly decreasing. □

[19] The derivative $\frac{\partial \Delta EU(\mathfrak{a})}{\partial \mathfrak{a}} = 0$ if and only if

$$\underbrace{\frac{\mathfrak{a}^N - \mathfrak{a}^\Omega}{1 - \mathfrak{a}^N}}_{\text{strictly monotonic}} = \underbrace{-\frac{\Delta \Pr}{1 - \Pr(rej|\text{OUT}_\Omega)}}_{\text{constant}}.$$

[20] In an equilibrium the probabilities are exogenous. Therefore, in an equilibrium case (3a) could never occur. Since the difference $\Delta EU(\mathfrak{a})$ is negative, no agent joins the platform. Thus, $\Pr(rej|\text{M}_\text{N}) = 1$ and $\Pr(rej|\text{OUT}_\Omega) = \frac{\Omega}{\Omega+1} < 1$. Thus, $\Delta \Pr > 0$, which violates a condition for case (3a).

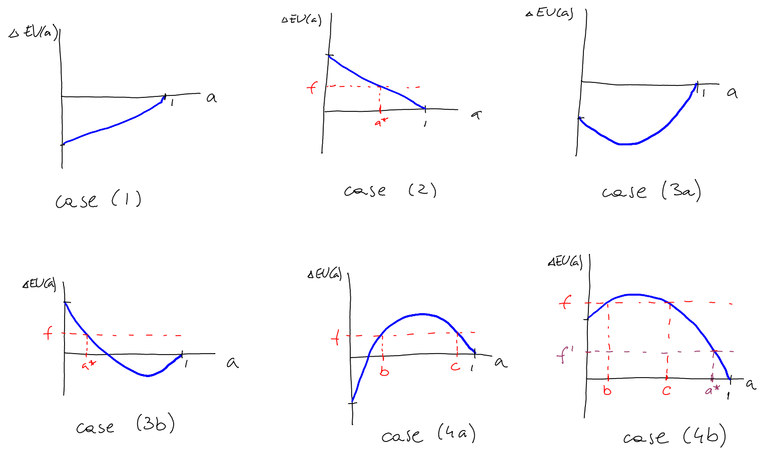

Figure 4: All possible shapes of the difference $\Delta EU(\mathfrak{a})$.

Proof of Proposition ?? (page ??)

Proof. For arbitrary Ω and $N < \Omega$, suppose that the rejection probabilities are given by

$$\Pr(rej|\text{M}_\text{N}) = 1 - \frac{1}{N} + \frac{1}{N(N+1)}(\mathfrak{a}')^N \quad \text{and} \quad \Pr(rej|\text{OUT}_\Omega) = 1 - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)}\,\frac{1-(\mathfrak{a}')^{\Omega+1}}{1-\mathfrak{a}'}\,, \tag{4}$$

for some $\mathfrak{a}' \in (0,1)$.

First, we show that for any $\mathfrak{a}' \in (0,1)$, the difference $\Delta EU(\mathfrak{a})$ — with rejection probabilities given by (??) — satisfies case (2) or (3b) in Lemma ??, i.e. the difference is positive and strictly decreasing for $\mathfrak{a} \in [0, \bar{\mathfrak{a}}]$ for some $\bar{\mathfrak{a}} \in (0,1]$. The difference satisfies case (2) or (3b) under $N < \Omega$ if and only if

$$\frac{\Pr(rej|\text{OUT}_\Omega) - \Pr(rej|\text{M}_\text{N})}{1 - \Pr(rej|\text{OUT}_\Omega)} > \frac{\Omega - N\left(\frac{1-\Pr(rej|\text{M}_\text{N})}{1-\Pr(rej|\text{OUT}_\Omega)}\right)}{N\Omega} > 0 \iff$$

$$\iff (\Omega+1)\left(1 - \frac{(\mathfrak{a}')^N}{N+1}\right) - (N+1)\left(1 - \frac{1}{\Omega+1}\,\frac{1-(\mathfrak{a}')^{\Omega+1}}{1-\mathfrak{a}'}\right) > 0\,.$$

Notice that $\frac{(\mathfrak{a}')^N}{N+1}$ is increasing in $\mathfrak{a}'$, and its highest value is $\frac{1}{N+1}$, for $\mathfrak{a}' \to 1$. Moreover, $\frac{1-(\mathfrak{a}')^{\Omega+1}}{1-\mathfrak{a}'}$ is also increasing in $\mathfrak{a}'$, and its lowest value is 1, for $\mathfrak{a}' \to 0$. Therefore

$$(\Omega+1)\left(1 - \frac{1}{N+1}(\mathfrak{a}')^N\right) - (N+1)\left(1 - \frac{1}{\Omega+1}\,\frac{1-(\mathfrak{a}')^{\Omega+1}}{1-\mathfrak{a}'}\right) >$$

$$> (\Omega+1)\left(1 - \frac{1}{N+1}\right) - (N+1)\left(1\frac{1}{\Omega+1}\right) = \underbrace{\frac{1}{(\Omega+1)(N+1)}}_{>0}\underbrace{(\Omega-N)}_{>0}\underbrace{(N\Omega-1)}_{>0} > 0\,.$$

Thus, for any $\mathfrak{a}'$, with the rejection probabilities given in (??), the difference $\Delta EU(\mathfrak{a})$ satisfies case (2) or (3b).

Since $\Delta EU(\mathfrak{a})$ satisfies case (2) or (3b) for any $\mathfrak{a}' \in (0,1)$, it satisfies the cases for $\mathfrak{a}' = \mathfrak{a}^* \in (0, \bar{\mathfrak{a}})$. Therefore, for any $\mathfrak{a}^* \in (0, \bar{\mathfrak{a}})$, with rejection probabilities

$$\Pr(rej|\text{M}_\text{N}) = 1 - \frac{1}{N} + \frac{1}{N(N+1)}(\mathfrak{a}^*)^N \quad \text{and} \quad \Pr(rej|\text{OUT}_\Omega) = 1 - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1-(\mathfrak{a}^*)^{\Omega+1}}{1-\mathfrak{a}^*}, \tag{5}$$

$\Delta EU(\mathfrak{a}^*) = f > 0$ (i.e., at those probabilities agent with $\mathfrak{a}^*$ is indifferent between paying f for participating in the platform or staying in the outside market). Moreover, for all $\mathfrak{a} < \mathfrak{a}^*$, $\Delta EU(\mathfrak{a}) > f$ (i.e. agents prefer to pay f and participate in the platform), and for all $\mathfrak{a} > \mathfrak{a}^*$, $\Delta EU(\mathfrak{a}) < f$ (i.e. agents prefer to say in the outside market). Given the participation threshold induced by $f = \Delta EU(\mathfrak{a}^*)$, the rejection probabilities are consistent with (??).

In the equilibrium, the platform chooses such a fee that maximizes its profit. The difference $\Delta EU(\mathfrak{a}^*)$ satisfies case (2) or (3b) in Lemma ??, and therefore it is positive and strictly decreasing for $\mathfrak{a}^* \in (0, \bar{\mathfrak{a}})$; and for $\mathfrak{a}^*$ in this interval, the difference takes values from $\bar{f} = \lim_{\mathfrak{a}^* \to 0} \Delta EU(\mathfrak{a}^*)$ to 0. Thus, for any fee $f \in (0, \bar{f})$ that the platform sets, there is a unique corresponding threshold $\mathfrak{a}^*(f)$ s.t. $\Delta EU(\mathfrak{a}^*) = f$. Since $f > 0$ and $\mathfrak{a}^* > 0$, the platform's profit, $\mathfrak{a}^* \cdot f$ is positive. For any other non-negative fee, the profit is 0. Since the profit function is positive and single-peaked — first increasing and then decreasing — on the relevant interval, there exists a unique fee that maximizes the platform's profit.

Therefore, for every Ω and $N < \Omega$, there always exists a unique equilibrium with active platform. □

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