Long-Run Stockholder Consumption Risk and Asset Returns

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First draft, Feb., 2005
Current draft, Nov., 2006

ABSTRACT

We provide new evidence on the success of long-run risks in asset pricing by focusing on the risks borne by stockholders. Exploiting micro-level household consumption data, we show that long-run stockholder consumption risk better captures cross-sectional variation in average asset returns than aggregate or non-stockholder consumption risk, and provides more plausible economic magnitudes. We find that risk aversion estimates around 10 can match observed risk premia for the wealthiest stockholders across sets of test assets that include the 25 Fama and French size and value portfolios, the market portfolio, bond portfolios, and the entire cross-section of stocks.

*London Business School, University of Chicago and NBER, and Northwestern University and NBER, respectively. We are grateful to David Chapman, John Cochrane, Lauren Cohen, Joshua Coval, Eugene Fama, Francisco Gomes, Lars Hansen, John Heaton, Ravi Jagannathan, Arvind Krishnamurthy, Sydney Ludvigson, Jonathan Parker, Monika Piazzesi, Jacob Sagi, Amir Yaron, Moto Yogo, and seminar participants at the Northwestern Finance lunch, the Board of Governors of the Federal Reserve, the University of Chicago, HBS, LSE, LBS, INSEAD, Copenhagen Business School, Norwegian School of Management, Norwegian School of Economics and Business Administration, University of Amsterdam, University of Copenhagen (CAM), the SED meetings in Budapest, Hungary, the WFA meetings in Portland, OR, and the NBER Summer Institute Asset Pricing meetings for helpful comments and suggestions. We also thank Ken French for providing data. Moskowitz thanks the Center for Research in Security Prices, the initiative on Global Financial Markets at the University of Chicago, and the Neubauer Family Faculty Fellowship for financial support.

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A cornerstone of asset pricing theory, the consumption CAPM, focuses on consumption risk as the key determinant of equilibrium asset prices and expected returns. Recent evidence finds success using long-run aggregate consumption risk to capture cross-sectional and aggregate stock returns (Parker (2001), Bansal and Yaron (2004), Parker and Julliard (2005), Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2005)). The empirical success of long-run consumption risk raises several questions. First, how well does aggregate consumption risk reflect the risks faced by capital market participants who actually own and trade stocks? Second, why does future consumption respond to news in stock returns and what are the underlying economic shocks driving these patterns? This article focuses on the first question.

We show that the long-run consumption risk of households who hold financial assets is particularly relevant for asset pricing. Examining more disaggregated measures of long-run consumption risks across stockholders and non-stockholders, we provide new evidence on the long-run properties of consumption growth and its importance for asset pricing. Our work intersects the recent long-run risk literature with the literature on limited stock market participation and consumption (Mankiw and Zeldes (1991), Parker (2001), Vissing-Jørgensen (2002), Brav, Constantinides, and Geczy (2002), Attanasio, Banks, and Tanner (2002), and Gomes and Michaelides (2006)). Using micro-level household data from the Consumer Expenditure Survey (CEX) for the period 1982 to 2004, we show that the covariance of returns with long-run consumption growth rates of households who own stocks provides a better fit and more plausible risk aversion magnitudes when capturing the cross-sectional variation in average returns across the 25 Fama and French stock portfolios, the market portfolio, eight bond portfolios, and the entire cross-section of individual stocks, than long-run aggregate or non-stockholder consumption growth. In particular, small and value stocks earn low returns and long-term bonds do poorly when the future consumption growth of stockholders is low. The high average returns observed for small and value stocks and long-maturity bonds may therefore reflect the premium stockholders require to bear long-run consumption risk.

The recent empirical success of long-run consumption measures has prompted asset pricing models that feature long-run risks. One set of models are variations of the recursive utility framework of Kreps and Porteus (1978) and Restoy and Weil (1989), which allow for the separation of

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1Prior research finds a weak asset pricing role for contemporaneous consumption risk (Kandel and Stambaugh (1990), Mankiw and Shapiro (1996), Breeden, Gibbons, and Litzenberger (1989), Cochrane (1996), and Lettau and Ludvigson (2001b)). Other studies find some success using conditioning variables in the CCAPM such as the consumption-to-wealth ratio $cay$ of Lettau and Ludvigson (2001a, 2001b) and $cay$ combined with labor income growth $lr$ (Julliard (2005)). However, there is debate over whether these conditioning variables can give rise to adequate dispersion in risks across assets needed to explain observed return premia (Lewellen and Nagel (2006)).
the elasticity of intertemporal substitution (EIS) from risk aversion. Epstein and Zin (1989), Weil (1989), Bansal and Yaron (2004), and Hansen, Heaton, and Li (2005) are examples of such models that feature a role for future consumption. Other preferences such as habit persistence (Sundaresan (1989), Constantinides (1990), Heaton (1995), and Campbell and Cochrane (1999)) or constraints on adjusting consumption as in models of staggered decision making (Lynch (1996) and Gabaix and Laibson (2002)), could also give rise to long-run consumption growth affecting asset prices. We adopt the recursive preference framework, which allows us to structurally interpret our empirical results and to remain consistent with prior work on long-run consumption risk (Bansal and Yaron (2004), and Hansen, Heaton, and Li (2005)).

To implement the recursive preference approach, we use the structural framework of Hansen, Heaton, and Li (2005), focusing mainly on the special case where the EIS equals one. The pricing kernel under this special case simplifies to an expression that only depends on the present value of long horizon consumption growth, substantially simplifying estimation and interpretation of our findings. However, under a different set of structural assumptions, Bansal and Yaron (2004) highlight the importance of long-run risks in conjunction with an EIS > 1 to explain asset price dynamics. For robustness, we also interpret our findings under structures where the EIS differs from one. For the assets we consider (the 25 Fama-French portfolios, the equity premium, eight Treasury bond portfolios, and all individual stocks), altering the EIS parameter makes little difference on the structural point estimate of the risk aversion parameter. We show that the reason the EIS makes little difference in our setting is because we focus on the cross-sectional price of risk, whereas Bansal and Yaron (2004) focus on fitting the low level and low volatility of the riskless rate. To match the riskless rate dynamics, it may very well be that an EIS greater than one is important. However, we do not focus on matching the moments of the risk-free rate.

To match the cross-sectional variation in average returns of the 25 Fama-French portfolios, our structural estimate of the value of risk aversion implied by the cross-sectional reward for long-run

\footnote{From a microeconomic perspective, a framework with habit formation may be less attractive since the support for such preferences is mixed (see Dynan (2000) and Brunnermeier and Nagel (2005) for evidence against habit formation based on consumption and portfolio choice data and Ravina (2005) for a dissenting view based on credit card charges). In contrast, estimates of the EIS in Vissing-Jørgensen (2002) and the results in the present paper on risk aversion suggest that consumption and asset returns may be consistent with a framework where stockholders have an EIS substantially above zero and a risk aversion higher than the reciprocal of the EIS, a structure which is possible within the recursive preference framework.}

\footnote{Attanasio and Vissing-Jørgensen (2003) estimate conditional Euler equations for stockholders in the Consumer Expenditure Survey (CEX) using after-tax returns and find a value for the elasticity of intertemporal substitution around 1.4 when using the after-tax T-bill return as the asset return, and around 0.4 when using the after-tax stock return. These are the same values documented in Vissing-Jørgensen (2002) with the exception that they adjust for the effect of taxes. This evidence suggests that an elasticity of intertemporal substitution of one is not unreasonable.}
consumption risk of stockholders is around 20, and around 10 for the wealthiest third of stockholders with the largest holdings of equity. These implied risk aversion estimates are significantly smaller than those obtained from either aggregate or non-stockholder long-run consumption growth. Long-run stockholder consumption growth is more volatile, more sensitive to aggregate consumption shocks, and more correlated with asset returns than long-run non-stockholder or contemporaneous stockholder consumption growth, and hence requires a lower risk aversion parameter to match the moments of asset returns.

Since CEX data is limited to the period 1982 to 2004, and long-run risks are difficult to measure, we construct factor-mimicking portfolios for long-run stockholder consumption growth to reduce estimation error by focusing on the returns to tradeable assets maximally correlated with consumption growth and generating a longer time-series of data. Reestimating the Euler equations using the longer series of consumption growth factor-mimicking (CGF) portfolio returns, we find even lower risk aversion estimates that are around 6 to 9 for stockholders and 5 to 7 for the wealthiest stockholders. Accounting for time-variation in the relation between consumption growth and asset returns we find risk aversion values that are similar to those estimated from actual consumption growth. In asset pricing tests, we find that the CGF portfolios perform at least as well as the Fama and French (1993) three factor model in explaining the cross-section of returns, even over the out-of-sample period prior to CEX data availability.

To further explore the scope of our findings, we examine the aggregate equity premium, the cross-section of eight Treasury bond portfolios, and the entire cross-section of individual stock returns as additional testing grounds. The magnitudes of the risk aversion parameters required to explain returns for these three separate sets of assets are remarkably similar to those obtained from the 25 Fama-French portfolios. The similarity in values across different sets of test assets increases confidence in the robustness of the results.

Our findings highlight the importance of long-run consumption risk for those who directly bear capital market risk in pricing assets. The fact that estimating the Euler equation for those households who actually own the asset can better match return moments and implies fairly reasonable risk aversion values, is comforting and helps alleviate concerns about the success of long-run consumption risk in pricing assets being spurious.

The rest of the paper is organized as follows. Section I outlines a structural model with recursive preferences linking asset prices to long-run consumption risk, which we use to interpret our findings.
Section II summarizes the data sources and variable construction and provides some preliminary measures of long-run risks. Section III examines the relation between long-run stockholder and non-stockholder consumption risk and the cross-section of returns on the 25 Fama-French portfolios. In Section IV we construct factor-mimicking portfolios for stockholder and non-stockholder consumption growth and employ them in asset pricing tests. Section V examines other assets, including the equity premium, eight Treasury bond portfolios, and the entire cross-section of individual stocks. Section VI concludes.

I. A Structural Asset Pricing Model with Recursive Preferences

Our theoretical setup follows Hansen, Heaton, and Li (2005), who adopt a set of recursive preferences following Kreps and Porteus (1978), Epstein and Zin (1989), Weil (1989), and Bansal and Yaron (2004). The innovation of our study is in the empirical implementation of this setup. We use this model to compute the value of risk aversion implied by our findings, which allows us to evaluate the economic plausibility of our results. Each household has recursive preferences of the form

$$V_t = \left[ (1 - \beta) C_t^{1 - \frac{1}{\sigma}} + \beta \left[ E_t \left( V_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}$$  \hspace{1cm} (1)

where $C_t$ is consumption, $\sigma$ is the elasticity of intertemporal substitution, $\gamma$ is relative risk aversion, and $\beta$ is the discount factor.

A. Euler Equations for an Elasticity of Intertemporal Substitution = 1

We focus first on the special case where the elasticity of intertemporal substitution (for stockholders) equals one. The stochastic discount factor in this case follows Hansen, Heaton, and Li (2005) (henceforth HHL) and simplifies to an expression that only depends on the present value of expectations about future consumption growth rates, making empirical estimation and interpretation of the Euler equation easier. More formally, for $\sigma = 1$, the recursion becomes,

$$V_t = C_t^{1-\beta} \left[ E_t \left( V_{t+1}^{1-\gamma} \right) \right]^{\frac{\beta}{1-\gamma}}$$ \hspace{1cm} (2)
Following HHL, we assume log consumption growth follows a moving-average process\(^4\)

\[
  c_t - c_{t-1} = \mu_c + \alpha(L) w_t = \mu_c + (\sum_{s=0}^{\infty} \alpha_s L^s) w_t = \mu_c + \sum_{s=0}^{\infty} \alpha_s w_{t-s}
\]

where \(\{w_t\}\) is an iid standard normal process. The log of the household’s stochastic discount factor is then given by

\[
  s_{t+1} = \ln \beta - [\mu_c + \alpha(L) w_{t+1}] + (1 - \gamma) \alpha(\beta) w_{t+1} - \frac{1}{2} (1 - \gamma)^2 \alpha(\beta)^2
\]

\[
= \ln \beta - [c_{t+1} - c_t] + (1 - \gamma) (\sum_{s=0}^{\infty} \alpha_s \beta^s) w_{t+1} - \frac{1}{2} (1 - \gamma)^2 (\sum_{s=0}^{\infty} \alpha_s \beta^s)^2
\]

\[
\simeq \ln \beta + (1 - \gamma) [(E_{t+1} - E_t) \sum_{s=0}^{\infty} \beta^s (c_{t+1+s} - c_{t+s})] - \frac{1}{2} (1 - \gamma)^2 (\sum_{s=0}^{\infty} \alpha_s \beta^s)^2.
\]

(See HHL for derivations.) In the last line of the above expression, we follow HHL and drop the term 
\([c_{t+1} - c_t]\). This term does not materially affect the results since one-period consumption growth is known to be poorly correlated with one-period excess returns (this is the failure of the standard CCAPM model). Written in terms of consumption growth rates, the term \((\sum_{s=0}^{\infty} \alpha_s \beta^s) w_{t+1}\) equals \((E_{t+1} - E_t) \sum_{s=0}^{\infty} \beta^s (c_{t+1+s} - c_{t+s})\), representing the innovation in expectations about the present value of consumption growth rates, and the term \((\sum_{s=0}^{\infty} \alpha_s \beta^s)^2\) is the variance of this innovation.

For any valid stochastic discount factor \(S_{t+1}\), the unconditional Euler equation for asset \(i\) states that \(E(S_{t+1} R^i_{t+1}) = 1\). Assuming joint (unconditional) log-normality of the stochastic discount factor, the return on asset \(i\), and the return on the riskless asset, and using the expression for the log stochastic discount factor in (4), the log-linearized unconditional Euler equation for the excess return on asset \(i\) over the riskless rate is

\[
E\left( r^i_{t+1} - r^f_{t+1} \right) + \frac{1}{2} V \left( r^i_{t+1} \right) - \frac{1}{2} V \left( r^f_{t+1} \right) = -cov \left( s_{t+1}, r^i_{t+1} - r^f_{t+1} \right)
\]

\[
\simeq (\gamma - 1) cov \left( E_{t+1} \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}), E_t \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}), r^i_{t+1} - r^f_{t+1} \right)
\]

\[
= (\gamma - 1) cov \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}), E_t \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}), r^i_{t+1} - r^f_{t+1} \right)
\]

\[
- (\gamma - 1) cov \left( E_t \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) \right), E_t \left( r^i_{t+1} - r^f_{t+1} \right) \right).
\]

### A.1. Unconditional covariances

We estimate two versions of the unconditional Euler equation (5). The first approach is to estimate equation (5) using only the unconditional covariance term, \(cov \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}), r^i_{t+1} - r^f_{t+1} \right)\),

\(^4\)Hansen, Heaton, and Li (2005) measure long-run relations between consumption growth and asset returns using this highly structured but interpretable model of long-run growth variation. They also consider an alternative specification for consumption growth with a time trend and find that long-run differential responses to permanent shocks between value and growth portfolios are roughly the same under both specifications.
and ignore the covariance of the conditional expectation of discounted consumption growth rates and the conditional expectation of the excess asset return. This approach leads to a consistent estimate of risk aversion $\gamma$ if expected excess returns are constant over time, or if the covariance between $E_t \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) \right)$ and $E_t \left( r_{t+1}^f - r_{t+1}^i \right)$ is identical across the set of test assets, implying that expected excess returns on all assets move up or down in parallel as the conditional expectation of discounted consumption growth rates changes.\(^5\)

The main advantage of this first approach is that it does not require the estimation of conditional expectations, a difficult task in the CEX data that span only a 20-year time period. An additional advantage is that the resulting Euler equation is a log-linearized version of that used by Parker and Julliard (2005), who use 12-quarter consumption growth rates and a $\beta = 1$ (no discounting), which reduces $\text{cov} \left( r_{t+1}^i, \sum_{s=0}^{\infty} \beta^s (c_{t+1+s} - c_{t+s}) \right)$ to $\text{cov} \left( r_{t+1}^i, \Delta c_{t+1} + ... + \Delta c_{t+12} \right)$. Hence, the recursive framework provides one possible explanation for the success of the risk measure used by Parker and Julliard (2005). We follow HHL and assume a discount factor of 5% per annum, implying $\beta = 0.95^{1/4}$ in quarterly data.\(^6\)

### A.2. Covariance of Conditional Expectations

A second approach is to estimate equation (5) using a VAR to estimate $E_{t+1} \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) - E_t \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j})$. Theoretically, this estimation is more correct since it incorporates conditional expectations and since the VAR calculates sums to the upper limit of infinity. The drawback is that the VAR estimation introduces an additional element of estimation error. The VAR estimated in this second approach takes the form

$$z_{t+1} = Az_t + \varepsilon_{t+1}$$

with $z_t = \left[ c_t - c_{t-1}, r_{t-1}^i - r_t^f, r_{t-1}^f - r_{t-1}^i, ..., r_{t-(K-1)}^f - r_{t-(K-1)}^i \right]'$ and where all variables are demeaned. Following Campbell (1996 p. 311),

$$E_{t+1} \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) - E_t \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) = c1'(I - \beta A)^{-1} \varepsilon_{t+1}$$

\(^5\)In our estimations we include a constant term to ensure consistent estimates of $\gamma$ in this scenario, where the constant estimates the term $-(\gamma - 1) \text{cov} \left( E_t \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) \right), E_t \left( r_{t+1}^i - r_{t+1}^f \right) \right)$. Further, although we cannot continue the sum $\sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j})$ to the upper limit of infinity, it is likely that consumption growth rates very far out in the future will not matter substantially for the resulting estimate of $\text{cov} \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}), r_{t+1}^i - r_{t+1}^f \right)$ since they are unlikely to be correlated with excess asset returns in the current period. We consider several possible upper limits in the sum and show that beyond 16 quarters, additional quarters do not alter the results much.

\(^6\)Discount rates within the range of 0 to 10% ($\beta = 0.9$ to 1) have little effect on our results.
where $e_1$ is a $K+1$ column vector whose first element is one and whose other elements are zero. We estimate one VAR for each asset $i$ to ensure that each VAR contains information only about that asset’s excess return and consumption, making the resulting estimate of the covariance term
\[ \text{cov} \left( E_{t+1} \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_t), r^i_{t+1} - r^f_{t+1} \right) \]
as comparable as possible to the covariance term
\[ \text{cov} \left( \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_t), r^i_{t+1} - r^f_{t+1} \right) \]
used in the first approach that does not employ a VAR. Furthermore, including all asset returns in one large VAR is not feasible due to an unmanageable number of parameters given our sample length.

In addition to estimating the Euler equation using stockholder consumption growth, we also use data for non-stockholders and aggregate US per capita consumption. Since non-stockholders do not hold financial assets and since evidence (e.g., Vissing-Jørgensen (2002)) suggests that non-stockholders likely have an EIS lower than one, the Euler equations will not hold for non-participating households. Hence, we do not treat risk aversion estimates from these estimations as valid risk aversion estimates for non-stockholders, but merely report them for comparison with those obtained for stockholders. Moreover, if all households have an EIS = 1, then the aggregate consumption-to-wealth ratio is constant, which contradicts evidence from aggregate US data that the consumption-wealth ratio varies over time (Lettau and Ludvigson (2001a,b)). However, as long as non-stockholders have an EIS different from one, a general equilibrium model of limited stock market participation also delivers a time-varying aggregate consumption-wealth ratio.

### B. Euler Equations for an Elasticity of Intertemporal Substitution $\neq 1$

When the EIS (of stockholders) differs from one, a host of additional random variables enter the Euler equation. We describe these additional terms and discuss how to estimate them.

Hansen, Heaton, and Li (2005), building on earlier work by Kogan and Uppal (2001), provide the following first-order approximation of the log stochastic discount factor that is valid for values of the EIS not equal to one.

\[ s_{t+1} \simeq s^1_{t+1} + (\sigma - 1) Ds^1_{t+1} \]

where $s^1_{t+1}$ is the log stochastic discount factor for an EIS of one and $Ds^1_{t+1}$ is the derivative of the log stochastic discount factor with respect to the EIS, evaluated at an EIS of one. HHL adopt the following statistical model to estimate $s^1_{t+1}$ and $Ds^1_{t+1}$.

\[ c_{t+1} - c_t = U_c x_t + \alpha_0 w_{t+1} \tag{7} \]
\[ x_{t+1} = G x_t + H w_{t+1} \tag{8} \]
where consumption growth is demeaned.\footnote{This model is consistent with the \textsc{Ma}(\infty) model for consumption growth stated earlier, which can be shown by substituting out \(x_t\) in equation (7) using (8), then substituting out \(x_{t-1}\) in the resulting expression and continuing this process.} They show that, omitting constants that do not affect the subsequent analysis,

\[ s_{t+1} \simeq (1 - \gamma) \alpha(\beta) w_{t+1} + (\sigma - 1) \left( \frac{1}{2} w'_{t+1} \Theta_0 w_{t+1} + w'_{t+1} \Theta_1 x_t + v_1 x_t + v_2 w_{t+1} \right) \quad (9) \]

where \(\Theta_0, \Theta_1, v_1\) and \(v_2\) are functions of the statistical model parameters \(U_c, \alpha_0, G\) and \(H\) and the preference parameters \(\beta\) and \(\gamma\):

\[
\begin{align*}
\Theta_0 &= (\gamma - 1) H' \Upsilon_{dv} H \\
\Theta_1 &= (\gamma - 1) H' \Upsilon_{dv} G \\
v_1 &= U_v \left( G - \frac{1}{\beta} I \right) + (\gamma - 1)^2 \alpha(\beta) H' \Upsilon_{dv} G \\
v_2 &= (1 - \gamma) U_{dv} H + U_v H 
\end{align*}
\]

where

\[
\begin{align*}
\alpha(\beta) &= \alpha_0 + \beta U_c (I - G \beta)^{-1} H \\
\mu_v &= \frac{\beta}{1 - \beta} \frac{1 - \gamma}{2} \alpha(\beta) \alpha(\beta)' \\
U_v &= \beta U_c (I - \beta G)^{-1} \\
\Upsilon_{dv} &= \frac{1 - \beta}{\beta} U_v U_v + \beta G' \Upsilon_{dv} G \\
U_{dv} &= \left( -\frac{1 - \beta}{\beta} \mu_v U_v - \beta (1 - \gamma) \alpha(\beta) H' \Upsilon_{dv} G \right) (I - \beta G)^{-1}.
\end{align*}
\]

In order to calculate the additional term in the log stochastic discount factor in equation (9),

\[(\sigma - 1) \left[ \frac{1}{2} w'_{t+1} \Theta_0 w_{t+1} + w'_{t+1} \Theta_1 x_t + v_1 x_t + v_2 w_{t+1} \right],\]

we need to estimate \(U_c, \alpha_0, G, H\) and \(w_{t+1}\) in addition to making an assumption about the value of \(\sigma\) (the EIS). In order to estimate these terms, therefore, we must estimate the VAR system and cannot adopt the simple approach for the case with an EIS equal to one in which no such additional estimation is necessary.

We implement HHL’s statistical model by assuming 
\[x_t = \left[ r_t^i - r_t^f, r_{t-1}^i - r_{t-1}^f, \ldots, r_{t-(K-1)}^i - r_{t-(K-1)}^f \right]^\prime.\]

Then \(G\) is a \(K \times K\) matrix and \(H\) is a \(K \times (K + 1)\) matrix of the form

\[
G_{K\times K} = \begin{bmatrix} G^*_1 & I_{K-1} \\ 0_{(K-1)\times1} & 0_{K\times1} \end{bmatrix} \quad \text{and} \quad H_{K\times(K+1)} = \begin{bmatrix} H^*_1 & H^*_{1\times K} \\ 0_{(K-1)\times(K+1)} & 0 \end{bmatrix}
\]

and \(w_{t+1} = \begin{bmatrix} w_{t+1}^c & w_{t+1}^2 & 0_{1\times(K-1)} \end{bmatrix}^\prime\) is a \((K + 1) \times 1\) column vector with only the first two elements non-zero (see Hamilton (1994) p. 259 for a similar setting). Equations (7) and (8) are then estimated by equation by equation using OLS.
Denoting the error terms from equation (7) by $\varepsilon_{t+1}^c$ and the error term from (8) by $\varepsilon_{t+1}^x$ we have

$$
\varepsilon_{t+1}^c = \alpha_0 w_{t+1} = \alpha_0^c w_{t+1}^c + \alpha_0^x w_{t+1}^x \quad (10)
$$

$$
\varepsilon_{t+1}^x = H w_{t+1} = H^c w_{t+1}^c + H^x w_{t+1}^x. \quad (11)
$$

Assuming all elements in $w_{t+1}$ have unit variance, the variances and covariance of the error terms are

$$
V (\varepsilon_{t+1}^c) = (\alpha_0^c)^2 + (\alpha_0^x)^2 + 2\alpha_0^c\alpha_0^x \text{cov} (w_{t+1}^c, w_{t+1}^x) \quad (12)
$$

$$
V (\varepsilon_{t+1}^x) = (H^c)^2 + (H^x)^2 + 2H^c H^x \text{cov} (w_{t+1}^c, w_{t+1}^x) \quad (13)
$$

$$
\text{cov} (\varepsilon_{t+1}^c, \varepsilon_{t+1}^x) = \alpha_0^c H^c + \alpha_0^x H^x + (\alpha_0^c H^c + \alpha_0^x H^x) \text{cov} (w_{t+1}^c, w_{t+1}^x). \quad (14)
$$

In order to recover $\alpha_0$, $H$, and $w_{t+1}$ we assume that $\text{cov} (w_{t+1}^c, w_{t+1}^x) = 0$ and that $H^c = 0$ (i.e., that the consumption shocks and return shocks are contemporaneously uncorrelated and that the consumption shock only affects returns with a lag) and use equations (10), (11), (12), (13), and (14) to recover $w_{t+1}^c$, $w_{t+1}^x$, $\alpha_0^c$, $\alpha_0^x$, and $H^x$.

Our implementation of HHL’s statistical model is similar to the VAR model we use in our second approach in the EIS = 1 case. If we stack the equations in HHL’s statistical model then it, too, is of the form $z_{t+1} = Az_t + \varepsilon_{t+1}$ with $z_t$ as defined above. However, HHL’s statistical model imposes the restriction that consumption does not enter the dynamics of the excess return. Furthermore, in the case where the EIS differs from one it is necessary to estimate $\alpha_0$, $H$, and $w_{t+1}$ separately, as opposed to just estimating the residuals $\varepsilon_{t+1}$. For this reason we need the restrictions $\text{cov} (w_{t+1}^c, w_{t+1}^x) = 0$ and $H^c = 0$ (or alternative restrictions that will allow identification). In order to determine if these restrictions are likely to materially affect the results in the EIS $\neq 1$ case, we impose the same restrictions on the VAR estimated for the EIS = 1 case and compare the ones obtained using the unconstrained VAR in (6) (where we do not need the additional restrictions for identification). As we will show, the results are not materially affected by these restrictions in the sense that the risk aversion estimates are very similar.

C. What is the impact of the EIS on risk aversion estimates?

Before proceeding to the data and estimation, we discuss briefly the impact the assumption on the EIS has on our structural estimates. The punchline is that the EIS has little effect on the risk aversion estimate when examining the cross-sectional price of risk.
To understand the impact of the EIS, consider first its role for the level of expected asset returns and the volatility of expected asset returns. The higher the EIS, the lower the level of expected asset returns that is needed in equilibrium to induce households to consume a growing consumption stream (for a given time discount factor $\beta$). Furthermore, the higher the EIS, the smaller the fluctuations in expected asset returns generated by fluctuations in expected consumption growth rates over time. Thus, a higher EIS keeps the level of expected asset returns low and the volatility of expected asset returns low, which helps fit the moments of the riskless rate. As for the equity premium, in the recursive framework the covariance between the excess stock market return and the stochastic discount factor is driven by the sensitivity of each of these quantities to shocks to the long-run component of dividend growth and consumption growth. When the EIS is high, a positive shock to the long-run component leads to a large positive change in the stock market price-dividend ratio and thus a large positive stock return. This result occurs because the shock leads to high dividends but generates only a slight increase in the discount rate when the EIS is high. The sensitivity of the stochastic discount factor to shocks to the long-run component does not depend strongly on the EIS. As a result, the covariance of the excess stock market return and the stochastic discount factor is higher for a higher EIS.

What are the implications of the EIS for the cross-section of returns? If the higher EIS increases covariances of all stock returns with the log stochastic discount factor by a roughly similar amount, then the risk aversion coefficient identified purely from the cross-section of returns will not be affected by the value for the EIS. We confirm this intuition empirically in Section III. This property of the model suggests that using cross-sectional inference to estimate $\gamma$ may be particularly useful when there is lack of agreement about the value of the EIS.

II. Data and Preliminaries

We briefly describe the data sources and variables used in the study and provide some preliminary measures of long-run risks of stockholder and non-stockholder consumption growth.

A. Asset returns

Initial tests focus on the returns of the 25 size and book-to-market equity sorted portfolios of Fama and French (1996) obtained from Kenneth French’s website from July, 1926 to November, 2004. The ending date of November, 2004 is chosen to match the end of the consumption data we use.
as discussed below. We also employ the returns on eight Treasury bond portfolios with average maturities of 3 months, 1 year, 2 years, 5 years, 7 years, 10 years, 20 years, and 30 years, obtained from the Center for Research in Security Prices (CRSP) from July, 1926 to November, 2004. We extract the full cross-section of NYSE, AMEX, and NASDAQ stock returns with beginning of month share prices above $5 from CRSP from July, 1926 to November, 2004. We acquire all available stock-level market capitalization and book equity figures from CRSP and Compustat, and obtain the Fama and French (1993) factors $R_{MRF}$ (excess return on the CRSP value-weighted index), $SMB$ (small minus big portfolio), and $HML$ (high minus low book-to-market equity portfolio) covering July, 1926 to November, 2004, as well as stock-level book values of NYSE firms prior to June, 1962, from Kenneth French’s website. We use the 30-day T-bill rate from CRSP over the period July, 1926 to November, 2004 as the riskless rate of interest.

B. Consumption Growth

We calculate separate quarterly consumption growth rates for stockholders, the wealthiest stockholders, and non-stockholders using data from the Consumer Expenditure Survey (CEX) for the period January, 1982 to November, 2004. We also calculate aggregate per capita consumption growth rates from the National Income and Product Accounts (NIPA) from January, 1959 to November, 2004.

B.1. Household level consumption from the CEX

We describe briefly the disaggregated CEX household-level data and how we compute average growth rates for stockholders, non-stockholders, and top stockholders. CEX data are available from the start of 1980 to the first quarter of 2005. Before 1999 about 4,500 households are interviewed per quarter in the CEX. The sample size increases to about 7,500 households per quarter after 1999. Each household is interviewed five times. The first time is practice and the results are not in the data files. The interviews are three months apart and households are asked to report consumption for the previous three months. While each household is interviewed three months apart, the interviews are spread out over the quarter implying that there will be households interviewed in each month of the sample, enabling us to compute quarterly growth rates at a monthly frequency. Financial information is gathered in the fifth quarter only. Aside from attrition, with about 60 percent of households making it through all five quarters, the sample is representative of the U.S. population.

The consumption definition and sample selection criteria follow Vissing-Jørgensen (2002). The
consumption measure used is nondurables and some services aggregated from the disaggregate CEX consumption categories to match the definitions of nondurables and services in NIPA. We use consumption as reported in the Interview Survey part of the CEX. The service categories excluded are housing expenses (but not costs of household operations), medical care costs, and education costs, since these costs have substantial durable components. Attanasio and Weber (1995) use a similar definition of consumption. In leaving out durables, it is implicitly assumed that utility is separable in durables and nondurables/services.\footnote{Results in the paper are generally similar when using total consumption that adds remaining services and durables to our current measure. We report results only for our nondurable and service measure of consumption for ease of comparison with the existing literature.} Nominal consumption values are deflated by the BLS deflator for nondurables for urban households. To control for consumption changes driven by changes in family size, we regress the change in log consumption on the change in log family size at the household level and use the residual as our quarterly consumption growth measure. Appendix A describes the additional selection criteria imposed on our CEX sample, including the omission of extreme consumption growth outliers.

**B.2. Identifying stockholders**

We consider both a simple definition of stockholders following Vissing-Jørgensen (2002) based on responses to the CEX indicating positive holdings of “stocks, bonds, mutual funds and other such securities” and a more sophisticated definition of stockholders to mitigate response error by supplementing the CEX definition with a probit analysis designed to predict the probability that a household owns stocks. Using the Survey of Consumer Finances (SCF) from 1989, 1992, 1995, 1998, and 2001, which contains the entire wealth decomposition of households (including direct and indirect holdings), we estimate a probit model for whether a household owns stock on a set of observable characteristics that also exist in the CEX (age, education, race, income, holdings in checking and savings accounts, dividend income, and year). The estimated coefficients from the probit model in the SCF data are then used to predict the probability of stock ownership for households in the CEX data who have information on the same observable characteristics and valid responses to CEX checking and savings account questions. The details of this procedure and the probit estimates are described in Appendix A. Under the more sophisticated classification, stockholders are then defined as the intersection of households who own “stocks, bonds, mutual funds, and other such securities” \textit{and} have a predicted probability of owning stock from the probit analysis of greater than 0.50. Non-stockholders are similarly defined as the intersection of those
responding negatively to the CEX question and having a predicted probability of owning stock of less than 0.50. Households whose responses do not match their predicted probabilities are excluded.

These alternative definitions of stock and non-stockholders refine the simple CEX definition to increase confidence that each household actually holds (or does not hold) stocks. Under the simple CEX definition, we classify 77.3% (22.7%) of households as non-stockholders (stockholders). This percentage is too high (low) relative to other sources such as the SCF, probably due to omission of indirect stockholdings in retirement plans by many CEX respondents. Under the alternative definition that includes the probit analysis, we classify 40.3% of households as non-stockholders and 13.7% as stockholders, excluding the 46.0% of households that cannot be confidently classified. While we lose households from the sample when we refine our stockholder definitions, we increase confidence that the remaining households actually do (or do not) participate in capital markets. Consistent with data from other sources (such as the SCF), the fraction of stockholders increases over our sample period.

We also compute consumption separately for the wealthiest third of stockholders based on their beginning of quarter dollar amount of holdings under both stockholder definitions. The cut-offs for being in the top third are defined by year and month. Under the simple stockholder definition, the average cutoff for being in the top third of CEX stockholders is stockholdings of $19,956 (in 1982 dollars) in the first half of our sample (1982 to 1992) and $41,816 in the second half (1993 to 2004). Under the more refined stockholder definition the corresponding average cutoffs are $20,428 for the first half of our sample and $42,208 for the second half.

Since these are minimum values (in 1982 dollars) we are clearly capturing wealthy households. For robustness, we also identify the wealthiest stockholders using a fixed cutoff in real terms, as well as taking the top 5 percent of the population in terms of stockholdings as a cutoff. Results are virtually unchanged across these various definitions.

Appendix A describes the stockholder and non-stockholder classifications in detail. Our final sample contains 206,067 quarterly consumption growth observations across 76,568 households. The median number of consumption observations per month under the first definition is 172 for stockholders and 586 for non-stockholders. Under the alternative definition, these medians are 103 and 314, respectively.

The CEX tends to underweight the super wealthy (see Bosworth, Burtless, and Sabelhaus (1991)). Thus, our results likely understate the importance of focusing on stockholders, particularly the wealthiest stockholders.
B.3. Aggregation of household consumption growth rates

The panel dimension for each household in the CEX allows us to calculate consumption growth rates at the household level. Since households do not appear in the CEX for more than four quarters, however, we cannot calculate a long-run consumption growth rate for a particular household. Instead we construct a time series of average consumption growth for a particular group of households (e.g., stockholders), and average the (log) consumption growth rates for households in that group. Our baseline approach computes the average growth rate from \( t \) to \( t + 1 \) for a group as follows:

\[
\frac{1}{H^g_t} \sum_{h=1}^{H^g_t} \left( c_{h,g,t+1}^{h,g} - c_{h,g,t}^{h,g} \right)
\]

where \( c_{h,g,t}^{h,g} \) is the quarterly log consumption of household \( h \) in group \( g \) for quarter \( t \) and \( H^g_t \) is the number of households in group \( g \) in quarter \( t \). We adjust for seasonal effects by regressing the series on 12 monthly dummies and using the residual from that regression in the subsequent analysis. We then substitute the average growth rate of the group into the asset pricing relations from Section I. For example, for the special case where the EIS = 1, for holders of asset \( i \) (and the riskless asset) the Euler equation to be estimated is

\[
E \left( r_{t+1}^i - r_{t+1}^f \right) + \frac{1}{2} V \left( r_{t+1}^i \right) - \frac{1}{2} V \left( r_{t+1}^f \right) \approx \left( \gamma - 1 \right) \text{cov} \left( \sum_{s=0}^{\infty} \beta^s \frac{1}{H^g_{t+s}} \sum_{h=1}^{H^g_{t+s}} \left( c_{h,g,t+s}^{h,g} - c_{h,g,t+s}^{h,g} \right), r_{t+1}^i - r_{t+1}^f \right).
\]

This equation is the asset pricing relation based on the consumption of a particular household, summed cross-sectionally across households in the group. The cross-sectional summation exploits the fact that the Euler equation should hold for each stockholder at each point in time. It does not assume a representative stockholder, an assumption that would be violated in an incomplete market setting with uninsurable idiosyncratic consumption shocks.

For robustness, we also consider another method for computing average consumption growth rates for a particular group by assuming a representative agent aggregation. This approach, rather than aggregating properly by taking the cross-sectional average of the change in household log consumption growth rates, takes the log change in the cross-sectional average of consumption
growth,

\[
\text{Representative agent } \Rightarrow \log \left( \frac{1}{H^{t+s}} \sum_{h=1}^{H^{t+s}} C_{t+1+s}^{h,g} \right) - \log \left( \frac{1}{H^{t+s}} \sum_{h=1}^{H^{t+s}} C_{t+s}^{h,g} \right)
\]

(16)

where \( C_{t}^{h,g} \) is consumption at quarter \( t \) for household \( h \) in group \( g \). Estimating the Euler equation in (15) using this alternative consumption growth series addresses how sensitive our results are to the representative agent assumption and hence can gauge whether comovements of asset returns with cross-household inequality play a significant role. In addition, this computation reduces the influence of large positive or negative growth rates for some individual households.

One concern with computing long-run growth rates for these groups of households is that the composition and attributes of households may change over time. Consumption at the beginning of the period may pertain to a somewhat different set of households than consumption several quarters later. For instance, increased stock market participation over time suggests households at date \( t \) will be from a higher part of the wealth distribution than households at date \( t + S \). However, results from a consumption-weighted growth measure (unreported) are similar to those from the equal-weighted growth measure, suggesting this concern is not serious.

B.4. Aggregate consumption data

We also compute aggregate consumption growth rates by using seasonally-adjusted monthly aggregate consumption of non-durables from NIPA Table 2.8.3 [line 3] available from January, 1959 to November, 2004. Real per capita growth rates are calculated by subtracting the CPI inflation rate and population growth rates, using monthly population from NIPA Table 2.6 [line 29]. For comparison with the CEX data, we use the monthly aggregate consumption data and calculate quarterly consumption growth rates, available at the monthly frequency.

C. Long-run consumption risks across groups

Before proceeding to the Euler equation estimation, it is useful to examine some direct measures of long-run risk for stockholders and non-stockholders. Appendix B describes our approach to estimating the time-series variance of the cross-sectional mean log consumption growth rate for each group of households (stockholders, non-stockholders, and top stockholders) and each time horizon (we consider \( S = 1, 2, 4, 8, 12, 16, 20 \) and 24 quarter growth rates).

There are several challenges to calculating this variance. First, we observe only the sample average growth rate, not the population mean growth rate. The time series variance of the sample
average consumption growth rate for a given group will therefore be an upward biased estimate of
the time-series variance of the cross-sectional mean consumption growth rate for the group. Fur-
thermore, the number of households in each cross-section differs for stockholders, non-stockholders,
and top stockholders. Since the number of non-stockholders exceeds the number of stockholders
which in turn exceeds the number of top stockholders, the magnitude of the bias will differ for the
three groups. Second, we are interested in quarterly growth rates, but have these available at the
monthly frequency.

We address both issues using a bootstrap methodology described in Appendix B. We address the
first issue by estimating how fast the time-series variance of a group’s sample average consumption
growth rate decreases as the number of households in the cross-section increases and then estimate
the limit as the number of households goes to infinity using a simple linear regression. Figure 1
highlights the idea by plotting the bootstrap estimated variance for a group against the number
of households \((H)\) selected in the cross-section of that group for the bootstrap simulations. The
asymptote is the estimated variance of interest which represents an estimate of the true time-series
variance of the group’s mean consumption growth rate. The second plot in Figure 1 plots the
bootstrapped variances against the reciprocal of the number of households chosen for the bootstrap
simulation \((1/H)\) and shows that the intercept from a linear regression of variances on \(1/H\) identifies
the asymptote.

Table I reports summary statistics for the standard deviations of our baseline consumption
growth measures for stockholder (Panel A), top stockholder (Panel B), non-stockholder (Panel C),
and aggregate (Panel D) consumption across various time horizons. The table highlights the time-
series properties of each group’s consumption growth and their long-run risks. The first row of each
panel reports the “naive” standard deviation of a group’s consumption growth, where no adjustment
is made for the size of the cross-section, and the second row reports the “asymptotic” volatility
based on the procedure in Appendix B. The adjustments matter substantially, as the asymptotic
volatility is much lower than the naive measure across horizons and affects the differences across
groups. However, even with the adjustment, stockholder consumption growth is more volatile than
non-stockholder consumption growth at any horizon and the top stockholders have the highest
consumption volatility across horizons.

The ratio of stockholder to non-stockholder risks is also larger at longer horizons. For \(S = 1\), the
(asymptotic) stockholder standard deviation is 1.4 times higher than the non-stockholder standard
deviation, while for \( S = 20 \), the stockholder volatility is about twice that of non-stockholders. The large volatility of consumption growth for stockholders, especially at longer horizons, suggests that long-run stockholder consumption growth may be better able to match the moments of asset prices with lower levels of risk aversion. Of course, it is the covariance of long-run stockholder consumption with aggregate shocks and returns that matters for asset pricing. The last row of each panel of Table I reports the sensitivity, \( \beta \), of a group’s consumption growth over horizon \( S \) with respect to aggregate consumption over the same horizon. The sensitivity of a group’s consumption to aggregate consumption generally increases with the horizon up to around 16 quarters and then declines slightly. Higher sensitivities with aggregate consumption growth at longer horizons is not surprising since the consumption levels of each group are likely cointegrated with the aggregate consumption level. Across all horizons, sensitivity to aggregate consumption growth is highest for the top stockholders, then stockholders, and lowest for non-stockholders. Most importantly, the differences across groups are relatively small at short horizons and quite large at long horizons.\(^{10}\) Hence, little mileage is gained by using stockholder versus non-stockholder consumption risk at short horizons for explaining asset prices (consistent with the results in Mankiw and Zeldes (1991), Parker (2001), and Vissing-Jørgensen (2002)). It is the combination of stockholder consumption and long-run risks that is crucial.

Focusing on 16-quarter consumption growth rates (which we do for much of the analysis), Table I highlights that stockholder consumption is about 3 times more sensitive to aggregate consumption shocks than non-stockholder consumption growth, while consumption growth of the top stockholders is almost 4 times as sensitive as that of non-stockholders. Non-stockholder consumption growth has a \( \beta \) of about one with aggregate consumption at long horizons, revealing why we obtain very similar results using non-stockholder consumption as we do with aggregate consumption. Figure 2 plots the sensitivities of each group’s consumption growth (the predicted values from linear regressions) with respect to aggregate consumption growth at 16 quarters. The figure illustrates that stockholders bear a disproportionate amount of aggregate consumption risk relative to non-stockholders, exposing why long-run stockholder consumption risk is able to match asset return premia at lower values of risk aversion.\(^{11}\)

\(^{10}\)At long horizons the volatility of consumption growth for non-stockholders approaches that for the U.S. aggregate, but remains slightly higher than the U.S. aggregate. This result could be driven by a type of measurement error in the CEX data which does not cancel out across households. Also, the particular seasonal adjustment procedure used by NIPA could play a role in smoothing the NIPA data.

\(^{11}\)Possible explanations for greater stockholder consumption sensitivity to aggregate consumption risk are (1) that stockholders may have different preferences leading them to accumulate more wealth and take on more capital market

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III. Stockholder Consumption Risk and the 25 Fama and French portfolios

Table II examines the relation between stockholder, top stockholder, non-stockholder, and aggregate consumption risk over various horizons and the cross-section of expected returns on the 25 Fama and French portfolios.

A. Euler equation estimation

We first estimate the unconditional Euler equation (5) for the special case where the EIS = 1, using only the unconditional covariance term. In the following subsections we check the robustness of our results and interpretation to including estimates of the covariance of conditional expectations of consumption growth and excess returns using a VAR as well as the robustness of our structural estimate of risk aversion, $\gamma$, when we allow the EIS to differ from one.

Following equation (15), we run cross-sectional regressions of the average log excess returns on the 25 Fama-French portfolios plus half the excess variance (measured quarterly from July, 1926 to November, 2004) against the covariance of returns with long-run discounted consumption growth of stockholders, the top third of stockholders, and non-stockholders separately. Although CEX consumption data is only available from 1982 to 2004, we employ the entire time-series of returns dating back to July, 1926 to estimate average returns and variances on the 25 Fama-French portfolios, since mean returns are notoriously difficult to estimate. In the next subsection we confirm results are similar when estimating means and variances of returns over the same sample period. Specifically, we run the following cross-sectional regression,

$$
\hat{E} \left[ r_i^{t+1} \right] - r_f + \frac{s^2}{2} = \alpha + (\gamma - 1)\hat{\text{cov}} \left( \sum_{s=0}^{S} \beta^s \left[ \frac{1}{H_t^{s+1}} \sum_{h=1}^{H_t^{s+1}} \left( c_{t+1+s}^{h,g} - c_t^{h,g} \right) \right], r_i^{t+1} \right) + e_i \tag{17}
$$

where $\beta = 0.95^{1/4}$ and $\gamma$ is the implied risk aversion coefficient from the model. We estimate regression (17) via GMM. Appendix C derives the moment conditions and shows that the point estimate we obtain from our GMM framework is equivalent to that obtained from OLS. We compute standard errors under our GMM framework that account for correlation of error terms across assets at a point in time, estimation error in covariances, serial correlation in the consumption growth risk, (2) stockholders may have different labor income or entrepreneurial income, (3) the tax system may transfer aggregate risk from non-stockholders to stockholders, or (4) the bond market may transfer risk to stockholders from non-stockholders, since the latter can only use the bond market to smooth their consumption. Gomes and Michaelides (2006) focus on mechanism (1), while Guvenen (2005) analyzes mechanism (4).
series induced by overlapping consumption data, and the different lengths of the data series used to estimate covariances and average returns.

We can also obtain consistent estimates of \( \gamma \) by estimating the Euler equation in reverse.

\[
\hat{\text{cov}} \left( \hat{r}_{t+1}^i, \sum_{s=0}^{S} \beta^s \left( \frac{1}{H_{t+s}^g} \sum_{h=1}^{H_{t+s}^g} (\hat{c}_{t+1+s}^{h,g} - \hat{c}_{t+s}^{h,g}) \right) \right) = \delta + \frac{1}{\gamma - 1} \left( \hat{E}[r_{t+1}^i] - r_f + \frac{\hat{\sigma}_i^2}{2} \right) + u_i \tag{18}
\]

For brevity, Appendix C omits the proof that GMM (with the appropriately redefined moment conditions) is also equivalent to OLS for this specification. We use the delta method to compute standard errors on \( \gamma \) from the GMM framework. Since equations (17) and (18) both provide consistent estimates of \( \gamma \) and since these estimates could differ substantially in sample, we report results from both specifications.

Our focus is primarily on the structural estimate of risk aversion, \( \gamma \), we obtain from the model. Implied risk aversion estimates provide a direct economic measure of the plausibility of the model. For instance, if the covariance between consumption growth and returns is too small to capture cross-sectional return premia (e.g., Hansen and Singleton (1983), Hansen and Jagannathan (1991), and Lewellen and Nagel (2006)), the regression will produce an implausibly large risk aversion estimate. While we also report cross-sectional \( R^2 \)s and pricing errors, we show that these diagnostics can be misleading. Lewellen, Nagel, and Shanken (2006) demonstrate how cross-sectional \( R^2 \)s and pricing errors can lead to a false impression of the model’s success, particularly when the test assets are highly correlated with each other and contain a strong factor structure as the 25 Fama-French portfolios do (see also Daniel and Titman (2006)). One of their prescriptions for improving asset pricing tests is to impose the theoretical restrictions of the model and to take seriously the parameter value estimates. By backing out the implied risk aversion value from our Euler equation estimation, we evaluate the plausibility of the economic magnitudes of this parameter value as a metric for the success of the model. Another benefit from the structural approach is that the use of covariances, rather than betas, of consumption growth reduces the impact of measurement error in consumption growth since covariances are not scaled by the variance of consumption growth.

B. Results across horizons and groups

Table II reports the results from regressions (17) and (18) for each consumption series: stockholders, the top third of stockholders, non-stockholders, and aggregate consumption growth from quarter \( t \) to \( t + S \) for \( S = 1, 2, 4, 8, 12, 16, 20, \) and 24. We report the estimated intercept, \( \alpha \), and implied risk
aversion coefficient, $\gamma$, from the Euler equations with GMM $t$-statistics as described in Appendix C that account for cross-correlated residuals, covariance estimation error, consumption growth autocorrelation, and different sample lengths used to estimate means and covariances.

Panel A of Table II reports results for stockholder consumption risk. Risk aversion estimates from stockholder consumption growth are unreliable at horizons less than 12 quarters. In contrast, examining growth rates at 12 to 24 quarters, risk aversion estimates are reliably positive and relatively stable. Risk aversion of 18.8, 17.0, 13.9, and 13.9 are obtained from 12, 16, 20, and 24 quarter growth rates, respectively. Defining long-run growth rates from 12 to 24 quarters out makes little difference in terms of the parameter estimates. The cross-sectional $R^2$ is between 0.62 and 0.68 for 16 to 24 quarter growth rates, indicating that long-run stockholder consumption risk captures substantial variation in average returns across the 25 Fama-French portfolios. While cross-sectional $R^2$s alone are not strong tests of the model, a high $R^2$ combined with a relatively modest risk aversion coefficient lends reasonable support for the model. In addition, the constant term is insignificant, indicating that the average pricing errors are insignificantly different from zero.

Results are also reported for the reverse regression (18), which places consumption risk on the left-hand side of the regression equation. The implied risk aversion estimates from the reverse regression are pure noise for less than 12 quarters consumption growth and are between 20 and 26.5 and statistically significant from 16 to 24 quarters out. These values are quite similar in magnitude to those obtained above.

Panel B of Table II reports results for the consumption risk of the top third of stockholders. The implied risk aversion estimates fall to between 7.7 and 11.0 for 16 to 24 quarter growth rates when estimating the regression forward and to between 12.8 and 16.3 when estimating the equation in reverse. Once again, while short-horizon growth rates, even for the wealthiest stockholders, do not pick up variation in average returns, long-run growth rates do. Comparing the results for top stockholders (Panel B) to those for all stockholders (Panel A) illustrates the fallacy of focusing on $R^2$s or pricing errors in judging model success. The $R^2$s are no higher (and sometimes lower) for top stockholder consumption risk, yet the implied risk aversion estimates are more reasonable. Taking the economic magnitudes of the model seriously is a more stringent test of the theory. For roughly the same $R^2$, we obtain lower risk aversion estimates that appear more plausible when using the consumption risk of top stockholders, who own and trade most of the equity in the market.

Panel C of Table II reports results for non-stockholder consumption risk. Since the Euler
equation may not hold for these households, because they do not hold stocks and, moreover, since non-stockholders may have an EIS < 1, the risk aversion parameter $\gamma$ for non-stockholders may not be a valid measure of their true risk aversion. Nevertheless, it is still interesting to compare the results from non-stockholder consumption risk to those from stockholders to emphasize the importance of focusing on stockholders. As Panel C of Table II shows, there is no significant relation between non-stockholder consumption risk and expected returns for horizons less than 20 quarters. For 20 to 24 quarter growth rates, non-stockholder consumption risk captures some of the variation in average returns, which is not surprising since stockholder and non-stockholder consumption is likely cointegrated at long horizons. More importantly, however, the implied risk aversion coefficients from the model are quite large compared to those obtained for stockholders and the top third of stockholders. This result illustrates, again, the fallacy of focusing exclusively on $R^2$s. For 20 to 24 quarter growth rates, non-stockholder consumption risk produces $R^2$s of 65 to 73 percent, which are at least as high as those for stockholders. However, the economic magnitudes are far less plausible. Non-stockholder consumption risk requires risk aversion of 29 to 44 to capture observed risk premia. These point estimates indicate much weaker covariation between returns and the consumption of non-stockholders. Put another way, high cross-sectional $R^2$s, particularly on the 25 Fama-French portfolios, are a relatively low hurdle to clear for a model. Using theory to interpret the economic magnitude of the coefficients is a much tougher test.

Finally, Panel D of Table II reports results for aggregate consumption growth based on covariances for the sample period for which we have CEX micro-level data (1982 to 2004) and for the longer period for which aggregate consumption data is available from NIPA (1959 to 2004). Average returns are estimated from the full 1926 to 2004 sample in both cases. Both sets of covariances yield similar results. Risk aversion required to match aggregate consumption risk with average asset returns ranges from 31.6 to 190 at long horizons and is sensitive to whether the regression is run forward or in reverse. These estimates are substantially higher than those for stockholder and top stockholder consumption growth.

Overall, Table II indicates that the long-run consumption risk of stockholders better captures asset return variation. Since these are the households for which the Euler equation should hold and since the household consumption data we employ comes from a completely separate source than aggregate consumption data, these results help alleviate general concerns about the importance of long-run consumption risk in the literature being spurious.
C. Dispersion in consumption risks

To illustrate what drives the lower risk aversion estimates we obtain by using stockholder and top stockholder consumption growth, Table III reports the dispersion in long-run consumption growth covariances (at 16 quarter horizons) for the 25 Fama-French portfolios along with $t$-statistics from a GMM estimator that accounts for autocorrelation induced by overlapping consumption data.\(^{12}\)

A typical concern with the estimates presented in Table II is that consumption covariances may not differ substantially across the test assets and may be imprecise. However, our results in Table II adjust for covariance estimation error in computing risk aversion estimates to account for the statistical precision of the covariances, and the value of the risk aversion coefficient $\gamma$ is identified off of the economic magnitudes of the differences in covariances across the assets.

Panel A of Table III reports the covariance estimates and $t$-statistics for each of the 25 Fama-French portfolios with long-run stockholder consumption growth. Many of the individual covariance estimates are at least two standard errors from zero, but, more importantly, there is wide economic dispersion in covariances across the portfolios. An $F$-test for the joint equality of the first-stage covariances across the 25 portfolios is rejected at the 5% significance level. More disperse covariances across the test assets implies a smaller risk aversion coefficient is required to match return data. If dispersion in consumption risk were negligible across the test assets, then an infinitely large $\gamma$ would be required to explain returns.

For the top third of stockholders in Panel B of Table III, there is even more dispersion in consumption risk across the 25 assets, which generates the even lower risk aversion estimate we obtain in Table II. The first-stage covariance estimates for non-stockholder and aggregate consumption exhibit little dispersion and are not significantly different from each other across the test assets, requiring very high risk aversion to capture observed return premia.

Figure 2 summarizes the evidence by plotting average log excess returns (plus half the excess variance) on the 25 Fama-French portfolios against their covariances with long-run consumption growth for each group. To highlight the importance of economic magnitudes, the four graphs are plotted on the same scale. The plots make apparent that there is far too little dispersion in non-stockholder or aggregate consumption covariances to be able to match average return dispersion

---

12 Since Table II shows that our results are fairly consistent at long horizons of 12 to 24 quarters, we focus the remainder of the paper on 16-quarter growth rates (except of course for the VAR results) for brevity, emphasizing that there is nothing special per se about the 16-quarter growth rates and that our results are robust to other definitions of long-run consumption.
across the assets with any reasonable risk aversion parameter. On the other hand, stockholder, and particularly top stockholder, consumption covariances are widely dispersed and therefore give much more plausible risk aversion values.

D. Robustness

Results are robust to the aggregation method used to compute each group’s average consumption growth rates, to various definitions of stockholder status, and to different estimation methods.

D.1. Aggregation method

Panel A of Table IV reports results for the representative agent aggregation in equation (16) used to compute the average consumption growth series of each group. Aggregating households under a representative agent assumption improves the results, generating even lower (and still significant) risk aversion coefficients for stockholders. Risk aversion estimates for stockholders decline from 17.0 under the baseline aggregation to 12.1 under the representative agent aggregation, when running the regression forward, and from 26.5 to 19.9 when running the regression in reverse. For the top stockholders, risk aversion declines from 11.0 to 9.7 (16.1 to 15.5) when running the forward (reverse) regression. For non-stockholders, risk aversion is still large at 40 to 62 under the new aggregation. Hence, results are robust to the aggregation method employed.

The baseline, and theoretically correct, aggregation that averages log consumption growth rates not only captures the mean growth rate, but also heterogeneity in consumption growth across households. By comparing the results to those under a representative agent aggregation, which captures only the mean effect, we can gauge whether the pricing relations are driven more by mean effects or heterogeneity across households. The slight improvement in results under the representative agent aggregation suggests that it is the covariances of returns with average consumption growth and not the covariance with the dispersion in consumption growth across households (Mankiw (1986), Constantinides and Duffie (1996)) that is driving the asset pricing relations.\(^{13}\)

\(^{13}\)One reason the representative agent series might deliver slightly better results is that it mitigates the influence of outliers by taking the log of the average rather than the average of the logs across households. Since extreme consumption growth observations are more likely to occur among households with very small consumption levels, we also employed a series that aggregates households correctly but weights a household’s log consumption growth rate by the initial consumption level of the household. Results (available on request) for the consumption-weighted series are also slightly better than our baseline series and similar to the representative agent series, consistent with mitigating the influence of large outliers improving the results.
D.2. Alternative definitions of stockholder status

Panel B of Table IV reports results using the alternative definition of stockholders which combines information from the CEX with a probit analysis from the SCF to classify households as stockholders and non-stockholders. The tighter definition of stockholders and the top third of stockholders improves the results slightly. Risk aversion coefficients are a little lower under the more precise stockholder definition. Non-stockholder consumption growth, however, continues to exhibit a weak relation with returns and requires much larger risk aversion to match returns.\(^\text{14}\)

D.3. Alternative estimation methods

Panel C of Table IV reports results from the Euler equation estimation when we estimate the mean and variance of returns over the same sample period as the consumption growth covariance estimates (1982 to 2004). The disadvantage is that since mean returns are difficult to estimate, by shortening the sample considerably we introduce additional noise into our estimates. As Panel C of Table IV shows, however, the results are quite similar to those obtained in Table II, where the entire time series of returns is used to estimate the mean and variance of returns.\(^\text{15}\)

Finally, Panel D of Table IV reports results from the Euler equation estimation when we force the constant term to be zero, thereby forcing the model to also price the average level of the 25 average excess returns (i.e., approximately, the equity premium). This specification imposes the theoretical restriction of the model that the intercept should be zero (if the Treasury bill rate equals the zero beta rate) and hence provides a more stringent test of the model. As Panel D of Table IV shows, risk aversion estimates are slightly higher under this specification, but still quite similar to those obtained when the intercept is a free parameter. For stockholders, risk aversion estimates are between 24.9 and 26.5, for the top stockholders they are between 14.4 and 16.1, and for non-stockholders they are between 55.9 and 147.9. Hence, results are similar when imposing alternative definitions of stockholder status.

\(^\text{14}\)In unreported results we also examined alternative definitions of the top stockholders that generate a more homogeneous group of top stockholders over time. We use a fixed cutoff in real terms to define the top third of households over the full CEX sample and use the top five percent of the overall population in terms of stockholdings, where the cutoff for the top five percent is by year and month. These alternative definitions provide a more stable group of stockholders in terms of wealth. The results from these alternative definitions are quite similar to those from our baseline approach with risk aversion ranging from 9.9 to 15.9.

\(^\text{15}\)When estimating the regression forward, where mean returns are on the left hand side of the regression equation, risk aversion estimates are slightly smaller than those obtained when using the entire return series to estimate means. However, when estimating the regression in reverse (with mean returns on the right hand side), larger risk aversion estimates are obtained when using the shorter sample period to estimate mean returns. The wider discrepancy between the forward and reverse regression results is testament to the additional noise introduced when estimating mean returns over a much shorter sample period.
this additional theoretical restriction.

E. Alternative structural interpretations

The results in Table II were estimated and interpreted using the unconditional Euler equation (5) under the special case where the EIS = 1 and where we ignored the covariance of the conditional expectation of discounted consumption growth and the conditional expectation of the excess asset returns in our estimation. We chose to avoid estimating the conditional moments due to the short sample of CEX data, rather than introduce substantial additional estimation error into the analysis. In this subsection, we examine the robustness of this interpretation and estimation.

E.1. Covariance of Conditional Expectations

We begin by estimating the Campbell (1996) style VAR in equation (6) to estimate $E_{t+1} \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j}) - E_t \sum_{j=0}^{\infty} \beta^j (c_{t+1+j} - c_{t+j})$. The advantage of this approach is that it incorporates conditional expectations and allows for continuing sums up to infinity, which more closely matches the theoretical framework. The disadvantage is that obtaining reliable conditional moment estimates may be a concern in the short CEX sample.

Panel A of Table V reports the results from using this VAR and an EIS = 1 to estimate the Euler equation. Standard errors used to compute $t$-statistics are calculated using a block bootstrap that resamples the data in 16-quarter blocks 1,000 times. For stockholder consumption growth, the estimated risk aversion needed to match the cross-section of returns is 12.7, which is smaller than the 17.0 point estimate we obtain in Table II under our baseline estimation with 16-quarter consumption growth rates. Similarly, the required risk aversion for top stockholder consumption growth drops from 11.0 to 7.3 under the VAR framework. Hence, estimating the conditional moments and allowing sums up to infinity delivers even lower risk aversion estimates for stockholders. The drawback is that statistical power is weaker due to the additional degrees of freedom expended in the short sample. For non-stockholders, however, we still need large risk aversion parameters to be able to match return moments. The point estimate for non-stockholder consumption is smaller under the VAR than our baseline approach, but still high at 25.9. Our point estimate of risk aversion for aggregate consumption of 45.1 is almost identical under both estimation procedures.

Panel B of Table V reports results from our implementation of Hansen, Heaton, and Li’s (2005) first-order approximation of the Euler equation around an EIS of 1. Our empirical set up of
this model is described in Section I. The same block bootstrap procedure is used to calculate standard errors. For an EIS = 1, the statistical model we employ using the Hansen, Heaton, and Li (2005) framework resembles the Campbell (1996) style VAR with the additional restrictions that consumption does not enter the dynamics of the excess return, that consumption and return shocks are not contemporaneously correlated, and that consumption shocks only affect returns with a lag. As Panel B of Table V indicates, these additional restrictions do not matter for the results, as the point estimates of risk aversion under the Hansen, Heaton, and Li (2005) statistical set up for an EIS = 1 are nearly identical to those obtained under Campbell’s (1996) VAR framework in Panel A of Table V, for each consumption series.

E.2. Interpretation when the EIS ≠ 1

Panel C of Table V reports results from our implementation of Hansen, Heaton, and Li’s (2005) statistical model for an EIS = 1.5. As Section I details, when the EIS differs from one, we have several other terms to estimate in the Euler equation, which will alter our estimate of γ, the risk aversion parameter. However, as discussed in Section I, these terms may not differ significantly across risky assets and hence may have a small effect on estimated risk aversion when pricing the cross-section of returns. Confirming this intuition, Panel C of Table V reports that for an EIS of 1.5 (the value used by Bansal and Yaron (2004)), the point estimates of γ are very similar to those obtained when the EIS = 1 for each consumption series.

Panel D of Table V reports results for EIS = 0.5 for comparison as well. The results are, again, nearly identical to those for an EIS = 1 or 1.5. When estimating the cross-sectional price of risk, the value for the EIS makes little difference for the resulting risk aversion estimates.

IV. Consumption Growth Factor-Mimicking Portfolios

Another way to assess the importance of long-run stockholder consumption growth for asset pricing is to construct factor-mimicking portfolios for long-run stockholder consumption growth, which are tradeable assets designed to be maximally correlated with long-run stockholder consumption growth. These portfolios are not only interesting because they are tradeable (and hence have market prices) but also because they may reduce measurement error in consumption growth by focusing only on the component of long-run consumption growth related to returns. In addition, since the CEX sample is short (1982 to 2004) and long-run risks are difficult to estimate, factor-mimicking
portfolios may improve estimation by allowing a longer time-series of data to be constructed.

To construct these portfolios, we project the present value of consumption growth on a set of instruments (available over a longer period) and use the estimated coefficients to construct a (longer) time-series of stockholder consumption growth. If measurement error in consumption is uncorrelated with the asset returns used to construct the factor-mimicking portfolio, then the factor portfolio may contain less measurement error than actual consumption growth, even in sample.

A. Constant factor loadings

Following Breeden, Gibbons, and Litzenberger (1989) and Lamont (2001), we create the consumption growth factor portfolio, \( CGF \), by estimating the following regression,

\[
\sum_{s=0}^{15} \beta^s \left( \frac{1}{H_{t+s}} \sum_{h=1}^{H_{t+s}} \left( c_{t+1+s}^{h,g} - c_{t+s}^{h,g} \right) \right) = a + b'R_t + \eta_t \tag{19}
\]

where \( c_t^{h,g} \) is the log of consumption of household \( h \) in group \( g \) for quarter \( t \), \( H_t^g \) is the number of households in group \( g \) in quarter \( t \), \( R_t \) are the excess returns over the riskless rate on the base assets (instruments), and \( \beta = 0.95^{1/4} \). We use returns, as opposed to log returns, in this regression so that the coefficients \( b \) are easily interpretable as the weights in a zero-cost portfolio. The return on the portfolio \( CGF \) is,

\[ CGF_t = b'R_t \tag{20} \]

which mimics innovations in long-run consumption growth. The resulting factor portfolio is the minimum variance combination of assets that is maximally correlated with long-run stockholder consumption growth in sample. Moreover, equation (20) is not limited to the CEX sample period if the vector \( b \) is relatively stable over time, an assumption which we discuss below.

We create the \( CGF \) over the entire period for which returns data is available on the base assets. The first set of instruments or base assets we employ are the value-weighted small growth (intersection of the smallest 40% size, lowest 40% BE/ME stocks, based on NYSE breakpoints), large growth (intersection of the largest 40% size, lowest 40% BE/ME stocks), small value (intersection of the smallest 40% size, highest 40% BE/ME stocks), and large value (intersection of the largest 40% size, highest 40% BE/ME stocks) portfolios, whose excess returns are available from July, 1926 to November, 2004.\(^{16}\)

\(^{16}\)Breeden, Gibbons, and Litzenberger (1989) justify the use of covariances with respect to a portfolio that has
Regression (19) is estimated using data from January, 1982 to November, 2004 using the 16-quarter discounted CEX stockholder and non-stockholder consumption growth rates. The results from this first-stage regression, with Newey and West (1987) \( t \)-statistics allowing for autocorrelation up to order 48 (months), are reported in Panel A of Table VI. Consistent with our earlier findings, long-run consumption growth for stockholders is negatively related to small growth and positively related to small value. These relations are even stronger for the top third of stockholders and are negligible for non-stockholder consumption growth.

Using the coefficient estimates from the first-stage regression equation (19) to construct the \( CGF \) in equation (20), we then estimate the Euler equation for the 25 Fama-French portfolios by replacing actual consumption growth with its \( CGF \) returns over the longer period July, 1926 to November, 2004. Standard errors used to compute \( t \)-statistics are calculated via a block bootstrap procedure that resamples consumption growth, base asset return data, and test asset return data using 16-quarter blocks and then recomputes the first-stage coefficients for the \( CGF \) and the risk aversion coefficient from the second-stage Euler equation that uses the bootstrapped \( CGF \). This procedure therefore also accounts for first-stage estimation error in the \( CGF \). The block bootstrap procedure is run using 5,000 replications.

The second-stage Euler equation estimates based on covariances for the CEX-period 1982 to 2004 are shown in the middle of Panel A of Table VI. The estimated risk aversion coefficients are very similar to those obtained using actual consumption growth in Table II. The reliability of the estimates is also similar despite the additional estimation error introduced from the first stage. This additional error may be mitigated by the reduction in measurement error from using returns instead of actual consumption growth.

The second-stage Euler equation estimates based on covariances for the full 1926 to 2004 period at the bottom of Panel A of Table VI show considerably lower implied risk aversion estimates. For the stockholder \( CGF \), implied risk aversion drops from 17.0 using actual consumption growth to 6.3. For the top third of stockholders, implied risk aversion drops from 11.0 using actual consumption growth to 4.7 using the \( CGF \). We also report results for the \( CGF \) of non-stockholder and aggregate consumption growth, obtaining risk aversion estimates of 25.9 and 16.4, respectively.

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maximum correlation with consumption growth in place of covariances with respect to actual consumption growth in a proof where the test assets are the same as the base assets used to construct the portfolio. Technically, therefore, we should construct the \( CGF \) from all 25 Fama-French portfolios or a set of base assets that span the mean-variance frontier. Since, with a limited time-series, we cannot obtain 25 reliable coefficients, and since the four size and value portfolios approximately span the mean-variance frontier of the 25 Fama-French portfolios, we use these four portfolios as the base assets for our \( CGF \) construction.
These estimates are also lower than those obtained with actual consumption growth, but they are still much higher than those obtained for stockholders.

Figure 5 summarizes the results in Panel A of Table VI, and illustrates how $R^2$s can be deceiving, by plotting the average excess returns of the 25 Fama-French portfolios against their covariances with the CGF for stockholder, top stockholder, non-stockholder, and aggregate consumption growth using the same scale. As Figure 5 shows, a high $R^2$ can be obtained with very little dispersion in covariances, causing a large risk aversion estimate needed to explain average returns, as in the case of non-stockholder and aggregate consumption. For stockholders and the top stockholders, however, we obtain both a high $R^2$ and substantial variation in covariances across the assets, and thus require a much more modest risk aversion parameter to explain returns.

Conditional on the assumption of constant factor loadings, the lower risk aversion estimates obtained from using CGF data for the full 1926 to 2004 period indicate that the dispersion of covariances across test assets are larger when estimated over the full sample (remember, that average returns are calculated for the full 1926 to 2004 sample in both second-stage estimates). Naturally, the question arises whether it is reasonable to assume constant factor loadings and if not, how will our conclusions be biased? For instance, increases over time in the fraction of the population who hold stocks could have lowered covariances of stock returns with stockholder consumption growth via increased risk sharing. If so, then the factor-mimicking portfolios computed in equation (20) with constant factor loadings may not be good approximations for actual consumption growth earlier in the sample period when participation rates were quite different. One response to this concern is that if risk sharing was more limited in the pre-CEX sample, then covariances of stockholder consumption with asset returns were likely higher in that period than our estimates suggest. For instance, Parker (2001) finds the covariance between asset returns and aggregate consumption growth to be stronger in the pre-war data. Hence, our CGF may produce upward biased risk aversion estimates.

B. Time-varying factor loadings

In this section we assess empirically the potential bias of assuming the relation between stockholder consumption growth and asset returns is constant through time. Time variation in stock market participation rates and the predictions of limited participation models (e.g., Basak and Cuoco (1998), Guvenen (2005), and Gomes and Michaelides (2006)) suggest this assumption may not be accurate. To account for time variation in the relation between asset returns and stockholder and
non-stockholder consumption growth, we allow for time-varying factor loadings when we estimate the first-stage regression in equation (19). Specifically, we employ an instrument \( z_t \) to capture the time variation in covariances between asset returns and stockholder consumption growth. The first-stage regression is,

\[
\sum_{s=0}^{15} \beta^s \left[ \frac{1}{H^2_{t+s}} \sum_{h=1}^{H^2_{t+s}} \left( c_{t+1+s}^{h,g} - c_{t+1+s}^{h,g} \right) \right] = (a + dz_t) \mathbf{1} + (b + cz_t)' R_t + \eta_t
\]  

(21)

where \( z_t \) is the instrument designed to pick up time variation and \( \mathbf{1} \) is a vector of ones.

Our choice of instrument \( z_t \) is motivated by the limited participation literature. A central theme in models of limited stock market participation (Basak and Cuoco (1998), Guvenen (2005)) is that the consumption (or wealth) share of stockholders is a state variable that should predict excess returns. This prediction holds even if stockholders and non-stockholders have identical preferences, though the effects are stronger if stockholders are less risk averse than non-stockholders. In addition, if non-stockholders have an EIS different from that of stockholders, as suggested by Vissing-Jørgensen (2002), then there may also be a link between stockholder consumption shares and the dynamics of the aggregate consumption-to-wealth ratio. This implication is consistent with micro-level evidence which suggests that wealthy households (e.g., stockholders) have higher savings rates (Dyan, Skinner, and Zeldes (2004), Carroll(2000), Bosworth, Burtless, and Sabelhaus (1991)). Hence, when stockholder wealth increases relative to non-stockholder wealth, the aggregate consumption-to-wealth ratio will decline, providing a link between stockholder wealth (or consumption) shares and the aggregate consumption-to-wealth ratio.

Figure 4 plots the aggregate consumption-to-wealth of Lettau and Ludvigson (2001a) \( cay \) against the ratio of quarterly consumption of stockholders to aggregate quarterly consumption in the CEX (the stockholder consumption share), calculated using CEX survey weights. The figure highlights that the stockholder consumption share varies over time in a manner that mirrors the dynamics of the consumption-to-wealth ratio, generating a correlation of \(-0.44\).\(^{17}\)

We employ Lettau and Ludvigson’s (2001a) \( cay \), linearly interpolated between quarters to produce monthly estimates, as the instrument \( z_t \) that captures time variation in factor loadings. The first stage is estimated over the CEX sample period January, 1982 to November, 2004 and then the coefficients are applied to return and \( cay \) data that is available from June, 1951 to November, 2004 to form a longer time series for the CGF. We then employ this CGF, whose factor loadings on the

\(^{17}\)This evidence suggests that the consumption-to-wealth ratio may be linked to stock market participation, possibly providing an economic story for its empirical success in pricing assets.
size and value portfolios vary over time as determined by the dynamics of \( cay \), in the second-stage Euler equation estimation.

The first column of Panel B of Table VI reports the first-stage estimation results from equation (21) for stockholder consumption growth. The bottom of Panel B of Table VI reports the second-stage Euler equation (forward regression) estimates for the time-varying \( CGF \) for stockholders. The coefficient of risk aversion for the stockholder \( CGF \) is 21.4, which is in line with our earlier estimates using actual consumption data. To assess whether time-variation in the factor loadings or the different sample periods used is contributing to the results, we also computed risk aversion from the constant loading \( CGF \) over the same June, 1951 to November, 2004 sample period to coincide with the availability of \( cay \). The risk aversion estimate we obtain is almost the same as that generated from the time-varying \( CGF \) over the same period. This evidence indicates that accounting for time-variation in factor loadings from 1951 to 2004 does not seem to alter our estimates. Absent an instrument to proxy for time-varying stock market participation prior to 1951, we are unable to answer whether time variation in factor loadings is important for estimating the Euler equation over the earlier period 1926 to 1951 that seemed to produce much lower risk aversion estimates.\(^{18}\)

The remaining columns of Panel B of Table VI report results for the time-varying \( CGF \)s for top stockholder, non-stockholder, and aggregate consumption growth. The findings are similar: For each group, risk aversion estimates are similar in magnitude to actual consumption growth and the constant loading \( CGF \) over the same sample period (not reported).

C. Another instrument

Panel C of Table VI reports results for \( CGF \)s instrumented with other variables besides size and value portfolios. Namely, we employ aggregate consumption itself as the instrument. Risk aversion estimates from the second-stage Euler equation (forward regression) estimation for stockholders, top stockholders, and non-stockholders are 20.8, 14.5, and 53.4, respectively using the \( CGF \) associated with aggregate consumption from 1959 to 2004. These magnitudes are consistent with our previous

\(^{18}\)Moreover, an open question remains as to why the earlier data drives down the risk aversion estimate, which is equivalent to asking why are covariances between asset returns and our \( CGF \)s larger and more disperse across the 25 Fama-French portfolios in the pre-war data? Fama and French (2006) find that the covariance of value (growth) stocks with the market portfolio is much larger (smaller) in the pre-war data and Parker (2001) also finds that asset returns covary more strongly with aggregate consumption in the pre-war period. Investigating why the covariance of asset returns with macroeconomic variables appears larger in the earlier sample period is beyond the scope of this paper, but may be an interesting pursuit to help determine what drives the underlying relations between asset prices and the macroeconomy.
results. As shown in Table I and in the first-stage estimates here, stockholders are more than 2.5 times as sensitive to aggregate consumption growth as non-stockholders and the top third of stockholders are almost 4 times as sensitive to aggregate consumption growth as non-stockholders. This heightened sensitivity to aggregate growth is precisely why we obtain much lower risk aversion estimates based on consumption data or consumption CGFs. This particular set of results also highlights why $R^2$s may not be a good metric to judge model success. In this case, the second-stage $R^2$s will be identical across the groups since the CGFs are constructed based on one variable. Hence, the only comparison to make across groups is the difference in required risk aversion to explain returns.

D. Other applications of the CGFs

A useful feature of the CGFs is that they are tradeable assets. Panel A of Table VII reports that the average excess returns on the constant factor loading stockholder and top stockholder CGFs are 44 and 77 basis points per month, respectively, and are statistically different from zero. The portfolio weights in the CGF ensure maximum covariance with long-run stockholder consumption growth. Hence, it is reassuring that the average returns to these portfolios are reliably positive and large. One application of the CGF is to use them in asset pricing tests and compare their performance against other known factor-mimicking portfolios such as those of Fama and French (1993). Using equations (19) and (20) (or (21)) is a way to form a factor related to size and value that employs consumption growth as an economic guide in determining the weights that should be placed in these asset classes, motivated by the underlying economic theory. In comparison with SMB and HML, the stockholder and top stockholder CGFs place more weight on small value and small growth (with opposite sign) than on large value and large growth, and place more absolute weight on small value than small growth, compared to the Fama-French factors which are an equal dollar long and short in small and large stocks and value and growth stocks, respectively.

In theory, the consumption growth factor should perform at least as well as the Fama and French (1993) factors SMB and HML. An interesting by-product of the CGFs is that this new combination of size and value portfolios, guided by theory, has a much stronger aggregate market component than either SMB or HML. For example, the stockholder CGF exhibits a 0.61 correlation with the market, whereas HML and SMB exhibit only a 0.18 and 0.33 correlation with the market, respectively, as shown in Panel A of Table VII. While Lakonishok, Shleifer, and Vishny (1994) tout the weak cyclicality of HML as evidence in favor of a non-risk based explanation for
the value premium, we emphasize that a slight recombination of size and value portfolios, motivated by the consumption-based model, produces a highly cyclical factor portfolio.

In addition, Panel B of Table VII shows that the \textit{CGF} has a positive alpha with respect to the Fama and French (1996) 4-factor model, even over the out of sample period prior to the existence of CEX consumption data. However, Panel C of Table VII shows that neither \textit{SMB} nor \textit{HML} deliver significant alpha with respect to the \textit{CGF}s. This evidence suggests that the \textit{CGF} may be a better way to form a size and value factor, consistent with the underlying economic theory.

V. The Equity Premium and Other Test Assets

We estimate the Euler equation for stockholder and non-stockholder consumption growth using the equity premium and other test assets instead of the 25 Fama-French portfolios to examine whether our results are sensitive to the specific cross-section of average returns chosen as the testing ground. Using the aggregate market equity return, a cross-section of eight Treasury bond portfolios, and even the entire cross-section of individual stock returns, we find remarkably consistent estimates of the risk aversion parameter across these various testing grounds. Not only do the qualitative results hold, that risk aversion estimates are lowest for wealthiest stockholders and highest for non-stockholders, but, more notably, the quantitative values of risk aversion are quite similar.

A. The equity premium

Table VIII reports implied measures of risk aversion for the equity premium across various measures of long-run consumption growth using stockholder, top stockholder, non-stockholder, and aggregate consumption. Implied risk aversion coefficients are computed from equation (17) with the intercept set to zero and using the (quarterly) excess log return on the CRSP value-weighted index as the single test asset return. We report \textit{t}-statistics for the risk aversion estimate (in parentheses) that are calculated from a block bootstrap procedure that resamples 5,000 times, using 16-quarter blocks.

We report results over three sample periods: the period over which CEX data are available (January, 1982 to November, 2004) for both actual consumption growth and the \textit{CGF}; and the period over which the factor-mimicking portfolios \textit{CGF}s are available (July, 1926 to November, 2004). To minimize the effect of estimation uncertainty in the equity premium, all risk aversion coefficients are calculated using the equity premium estimate from the full July, 1926 to November, 2004 period of quarterly excess log stock returns. Differences in risk aversion estimates across
periods are therefore driven by differences in the covariance of the excess return on stocks with the long-run consumption growth measures. These covariances are also reported in the table.

Panel A of Table VIII reports results for actual consumption growth data over the CEX sample period. For robustness, we report Euler equation estimates and implied risk aversion values for an EIS = 1.5, 1, and 0.5 using Hansen, Heaton, and Li’s (2005) VAR. For an EIS = 1, stockholder consumption growth requires a risk aversion coefficient of 22.4, about the same magnitude as that obtained from the 25 Fama-French portfolios. For an EIS = 1.5, lower risk aversion of 21.7 is required, and for an EIS = 0.5, higher risk aversion of 24.1 is obtained. For comparison, the last row of Panel A of Table VIII reports risk aversion estimates from our baseline approach that does not use a VAR and assumes the EIS = 1. The magnitudes of the risk aversion coefficients are similar.

For top stockholders, risk aversion of 11.4, 12.6, and 15.2 are required for an EIS = 1.5, 1, and 0.5, respectively. For non-stockholders, risk aversion of 64.2, 66.6, and 69.2 are required and for aggregate consumption growth these values are 82.3, 83.9, and 84.7. These risk aversion estimates are similar across EIS values. Hence, while the value of the EIS matters more for the equity premium than it does for the cross-sectional risk premium (as we argued earlier) it still does not have a big effect. Overall, the risk aversion values estimated from the equity premium are in line with those obtained for the 25 Fama-French portfolios.

Given the consistency of risk aversion estimates across EIS values, Panels B and C of Table VIII report estimates for the EIS = 1 case only for brevity. Panel B reports results using covariances based on the constant loading CGFs over the CEX sample period. The results are largely consistent with those in Panel A. Finally, Panel C reports results for the constant loading CGFs over the full sample period from July, 1926 to November, 2004. Required risk aversion of only 8.1 is obtained for the stockholder CGF and only 5.1 for the top stockholder CGF. For non-stockholder and aggregate consumption CGFs, required risk aversion is 26.6 and 24.0, respectively. These risk

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<th>Top Stockholder Risk Aversion</th>
<th>Non-Stockholder Risk Aversion</th>
<th>Aggregate Risk Aversion</th>
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<td>1</td>
<td>22.4</td>
<td>21.7</td>
<td>64.2</td>
<td>82.3</td>
</tr>
<tr>
<td>1.5</td>
<td>21.7</td>
<td>15.2</td>
<td>66.6</td>
<td>83.9</td>
</tr>
<tr>
<td>0.5</td>
<td>24.1</td>
<td>11.4</td>
<td>69.2</td>
<td>84.7</td>
</tr>
</tbody>
</table>

---

19 Bansal and Yaron (2004) find that the EIS matters significantly for the value of risk aversion in their calibration of the equity premium. A key difference between Bansal and Yaron’s (2004) calibration and our estimation is that in their calibration when the EIS changes, both the stochastic discount factor and return process change endogenously. Obviously, when we estimate the risk aversion parameter from the data, we take the returns as given. Hence, when the EIS changes, only the stochastic discount factor changes in our estimation procedure since returns data are exogenous inputs in the estimation. In terms of their model, when the EIS changes, both \( \beta_{m,e} \) and \( \lambda_{m,e} \) change in their equation (7), with most of the action coming via \( \beta_{m,e} \) (we checked this through some calibration exercises). In our estimation using return data, we are essentially only changing \( \lambda_{m,e} \) and therefore conclude that the EIS does not matter much for the risk aversion estimates we obtain from the equity premium. This difference helps reconcile why Bansal and Yaron (2004) reach different conclusions about the importance of the EIS for the risk aversion parameter needed to fit the equity premium.
aversion estimates are similar to those found previously in the cross-section of returns on the 25 Fama-French portfolios. The results suggest that fairly reasonable risk aversion estimates can be obtained using the full sample period and focusing on the long-run consumption risk of stockholders, particularly wealthy stockholders.

B. Cross-section of bond returns

Table IX reports Euler equation estimates using a cross-section of eight Treasury bond portfolios with average maturities of 3 months, 1 year, 2 years, 5 years, 7 years, 10 years, 20 years, and 30 years with returns over the period for which CEX data are available (January 1982 to November 2004). Panel A reports results using actual consumption growth measures from the CEX, and Panel B employs the constant loading CGFs. Despite using two different sample periods and two different measures of consumption risk, both panels deliver very similar results. Consistent with our findings for the 25 Fama-French portfolios and the equity premium, the estimates of risk aversion are similar to those obtained previously from the other tests assets. The success of the CGFs, which are created from size and value portfolios, on the cross-section of bond returns, suggests that there is common variation in these assets related to long-run consumption growth that determines their risk premia.

Figure 6 plots the average excess bond returns against their consumption covariances measured via actual CEX consumption data. The plot highlights the spread in covariance risk across the bond portfolios as well as the high cross-sectional fit.

C. The entire cross-section of individual stocks

As an additional testing ground for the model, we employ the entire cross-section of all individual stock returns in Table X. We estimate the covariance of returns with long-run consumption growth for each individual stock traded on the NYSE, AMEX, and Nasdaq with beginning of month share prices above $5. To reduce noise in individual stock consumption growth covariances, we follow the procedure of Fama and French (1992) and compute portfolio covariance estimates and assign them to each individual stock within the portfolio. In this procedure the individually estimated covariances are used to rank stocks and form portfolios (this is referred to as the pre-ranking step). We then compute return covariances for the constructed portfolios over the full data sample and assign these estimates to each stock in a particular portfolio (these covariances are called post-
ranking covariances).\textsuperscript{20} Fama and French (1992) follow a similar procedure by using size and pre-ranking beta sorted portfolios to form post-ranking betas in testing the unconditional CAPM. In addition, because of the noise in individual stock returns and difficulty in estimating precise covariances for individual stocks, we only employ the CGFs to measure consumption risk since they are available over a much longer sample period than actual CEX consumption data and since the CGFs contain only the component of consumption growth correlated with returns, mitigating the effects of measurement error.

We run Fama and MacBeth (1973) month-by-month cross-sectional regressions of the entire cross-section of log excess stock returns on their covariance with long-run consumption growth. To assess the marginal impact of consumption risk controlling for other known determinants of returns, we also report regression results that include market $\beta$, the log of market capitalization, and the log of BE/ME as regressors. The time-series average of the monthly coefficient estimates and their time-series $t$-statistics are reported in the style of Fama and MacBeth (1973), adjusted for autocorrelation using the Newey and West (1987) procedure.\textsuperscript{21}

Table X indicates that the covariance of returns with long-run consumption growth of stockholders captures significant cross-sectional variation in average returns. The univariate regression is a direct estimate of the Euler equation in regression (17) and yields an implied risk aversion coefficient of 9.3, which is similar to the magnitude obtained from the other testing grounds: 25 Fama-French portfolios, equity premium, and eight bond portfolios. Moreover, controlling for other known determinants of returns (market, size, and BE/ME) does not eliminate the significance of consumption risk. For the top third of stockholders, results are even stronger, requiring a risk aversion of only 5.6, while for non-stockholders risk aversion is 39.2 and for aggregate consumption it is 29.7. Overall, the results and point estimates of risk aversion are surprisingly consistent across the various sets of test assets.

\textsuperscript{20}Specifically, for each individual stock we estimate the covariance of its returns with CGFs using the past 24 to 60 months (as available) of monthly log excess returns before July of year $t$. Stocks are sorted at the end of June into 100 pre-ranking covariance centiles. We then compute the equal-weighted quarterly log excess returns on these 100 portfolios over the next 12 months, from July to June. This procedure is repeated every year, forming a time-series of returns on these 100 portfolios. We then reestimate covariances for the portfolios formed from the pre-ranking sorts using the full sample of returns from July 1926 to 2004 to obtain post-ranking covariances. The post-ranking covariance estimate for a given group is then assigned to each stock in the group, with group assignments updated at the end of June of each year. Even though the post-ranking covariances themselves do not change over time, as an individual stock moves into and out of one of the 100 portfolios due to its pre-ranking covariance changing, that stock will receive a different post-ranking covariance. This procedure reduces estimation error by shrinking individual covariance estimates to a portfolio average and employing the full sample of data.

\textsuperscript{21}A correction for first-stage covariance estimation error via Shanken (1992) has little effect on the standard errors.
VI. Conclusion

We find empirical support for consumption-based asset pricing by focusing on the long-run consumption risk of stockholders. Long-run stockholder consumption risk captures the return premia associated with size and value portfolios, the aggregate stock market, bond portfolios, and the entire cross-section of stocks, requiring a modest risk aversion coefficient of about 10 for the wealthiest stockholders to match return premia. The stronger link between asset prices and the consumption of households that actually own financial assets is comforting and suggests that recent evidence on the success of long-run consumption risk is unlikely to be due to chance.

The fact that stockholders are more sensitive to aggregate consumption movements helps explain why the consumption risk of stockholders delivers lower risk aversion estimates. Understanding further why consumption growth, particularly that of stockholders, responds slowly to news in asset returns will improve our understanding of what is driving these long-run relations. The challenge is to understand how these long-run patterns emerge as an equilibrium outcome. One approach is to examine the potential link between long-run stockholder consumption growth, asset returns, and macroeconomic shocks. Evidence from the macroeconomic growth and real business cycle literature highlights that consumption and production respond slowly to technology shocks, peaking at 3 to 4 year horizons (see Altig, et. al (2005)), which is about the same horizon over which consumption responds fully to asset returns. Theory then needs to determine why stockholders, in equilibrium, end up taking on more of this aggregate risk. As a step in this direction, Gomes and Michaelides (2006) show that a setting with fixed costs of stock market participation combined with preference heterogeneity has potential for generating interesting equilibrium differences in stockholder and non-stockholder consumption patterns.
REFERENCES


Guvenen, Fatih, 2005, Do stockholders share risk more effectively than non-stockholders?, Working paper, University of Rochester.


Appendix A: CEX Sample Choice and Stockholder Definitions

We describe our CEX sample criteria and stockholder definitions.

CEX sample choice

For each household we calculate quarterly consumption growth rates based on reported monthly consumption values. We drop household-quarters in which a household reports non-zero consumption for more than three or less than three months or where consumption is negative.

Extreme outliers are dropped since these may reflect reporting or coding errors. Specifically, we drop observations for which the consumption growth ratio \( \frac{C_{t+1}^h}{C_t^h} \) is less than 0.2 or above 5. In addition, non-urban households (missing for part of the sample) and households residing in student housing are dropped, as are households with incomplete income responses. Furthermore, we drop households who report a change in age of household head between any two interviews different from 0 or 1 year. These exclusions are standard. We also drop all consumption observations for households interviewed in 1980 and 1981, since the CEX food question was changed in 1982 leading to a drop in reported food consumption. The question was changed back to the initial question in 1988, but there is no obvious way to adjust for this change without substantial loss of data. See Battistin (2003) for details on the questions asked. Furthermore, we obtain small sample sizes in the last three months of the sample since all households used in those months must be in their last interview in order for financial information to be available. We therefore drop these three months of data.

Finally, because financial information is reported in interview five, and because we wish to calculate consumption growth values by household, households must be matched across quarters. Therefore, we drop households for which any of interviews two through five are missing. Matching households across interviews creates problems around the beginning of 1986 and the beginning of 1996 since sample design and household identification numbers were changed, with no records being kept of which new household identification numbers correspond to which old ones. We therefore exclude households who did not finish their interviews before the ID change, implying that fewer observations are available for the last 4 months of 1985 and 1995 and the first 9 months of 1986 and 1996 around the ID changes. Furthermore, no households were interviewed in April, 1986 and April, 1996. To avoid a missing value in our time-series (with a resulting longer period of missing long-run consumption growth rates) we set quarterly consumption growth for March 1986 and March 1996 equal to aggregate quarterly real nondurable per capita consumption growth for those months. Results are similar if we simply drop these months.

Stockholder status

The CEX contains information about four categories of financial assets. Households are asked for their holdings of “stocks, bonds, mutual funds and other such securities”, “U.S. savings bonds”, “savings accounts”, and “checking accounts, brokerage accounts and other similar accounts.”

We refer to households with positive responses to the category “stock, bonds, mutual funds and other such securities” as stockholders and those with zero holdings as non-stockholders for our simplest and baseline definition. The Euler equation involving consumption in period \( t \) and \( t + 1 \) should hold for those who hold the asset as of date \( t \). Therefore, holding status must be defined based on holdings at the beginning of period \( t \) (when considering the consumption growth between period \( t \) and \( t + 1 \)). Two additional CEX variables are used for this purpose. The first variable
reports whether the household holds the same amount, more, or less of the asset category compared to a year ago. The second variable reports the dollar difference in the estimated market value of the asset category held by the household last month compared to the value of the asset category held a year prior to last month. We define a household as holding an asset category at the beginning of period $t$ if it 1) reports holding the same amount of the asset as a year ago and holds a positive amount at the time of the interview (the fifth interview) or 2) reports having lower holdings of the asset than a year ago, or 3) reports having had an increase in its holdings of the asset but by a dollar amount less than the reported holdings at the time of the question.\textsuperscript{22}

As discussed in the data section we also define a group consisting of the wealthiest third of stockholders. This is done based on the value of beginning of period $t$ holdings of “stock, bonds, mutual funds and other such securities” calculated using the end of period value minus the change in holdings over the period. Two data issues arise. First, the CEX starts using computers to conduct the interviews (rather than paper) beginning in April, 2003. This change appears to imply that the dollar asset change question (the variable COMPSECX) is not asked if the respondent already replied having the same asset holdings as a year ago (in the data this shows up as “valid blank” responses for the variable COMPSECX). This problem is easily fixed by simply setting the asset change to zero in these cases. Second, for interviews conducted from October, 1990 to March, 1996, about 5 percent of households report holdings of stocks, bonds and mutual funds of $1. We contacted the Bureau of Labor Statistics to determine if this is a coding error, but they were not sure how to interpret the $1 answers. Since all of the households reporting $1 assetholdings answer the question comparing current holdings to holdings a year ago it is likely that they are holding such assets. We therefore include them as stockholders. However, since the $1 households cannot be classified by amount of stockholding, we exclude them in estimations that are based on the wealthiest third of stockholders (by stockholdings).

It is known from the Survey of Consumer Finances (SCF), that many households hold stocks or bonds only in their pension plan. Unfortunately, it is not possible to determine whether households with defined contribution plans report their stockholdings and bondholdings in these plans when answering the CEX questions. The percent of stockholders (documented below) in the CEX is smaller than in other sources. This fact may indicate that some CEX households with stockholdings in pension plans do not report these, leading them to be miscategorized as non-stockholders. Unfortunately, we are unable to address this issue. However, our results may be conservative and would likely strengthen with a cleaner separation of stockholders and non-stockholders.

A complementary issue which can be addressed is that households holding bonds or bond funds exclusively will be misclassified as stockholders when households are categorized based on the definitions above. We therefore also consider a more sophisticated definition of who likely holds stocks using a probit analysis from another data source, the SCF, to predict the probability that a household owns stocks. Using the SCF from 1989, 1992, 1995, 1998, and 2001, which contains the entire wealth decomposition of households (including direct and indirect holdings), we estimate the following probit model for whether a household owns stocks on a set of observable characteristics that also exist in the CEX: age of household, age squared, an indicator for at least 12 but less than 16 years of education for head of household (\textit{highschool}), an indicator for 16 or more years of education, and...\textsuperscript{22}1,834 households in our final sample of 76,568 households report an increase in their holdings of stocks, bonds and mutual funds but do not report their current holdings. Most of these households are likely to have held these assets a year ago and are therefore placed into the stockholder category. 154 households report an increase in their holdings of stocks, bonds or mutual funds larger than the value of the reported end of period holdings. We classify these as non-stockholders.
education (college), an indicator for race not being white/caucasian, year dummies, the log of real total household income before taxes, the log of real dollar amount in checking and savings accounts (set to zero if checking and savings = 0), an indicator for checking and savings accounts = 0, and an indicator for positive dividend income, plus a constant.\footnote{We include checking and savings account holdings and not total financial wealth in the probit because of the suspected underreporting of indirect financial wealth holdings in the CEX discussed above.} Data are averaged across SCF imputations and SCF weights are employed in the probit model to avoid the estimates being unduely influenced by the oversampling of high wealth individuals in the SCF. The estimates of the coefficients from the probit model in the SCF are then used to predict the probability of stock ownership for households in the CEX who have information on the same observable characteristics and valid responses to checking and savings account questions. The estimated probit model coefficients and t-statistics are

$$
Prob(\text{Stockholder}) = \Phi(x'b)
$$

\[
x'b = -7.457 \pm 0.0297age + 0.0004age^2 + 0.3102\text{highschool} + 0.5160\text{college} \\
+ -0.2594\text{nonwhite} + 0.2508y_{1992} + 0.3795y_{1995} + 0.6299y_{1998} + 0.6575y_{2001} \\
+ 0.5513\text{log(income)} + 0.0747\text{log(chk + sav)} + 0.1067(chk + sav = 0) + 1.2438(div > 0)
\]

(7.44) + (9.86) + (14.23) + (7.11) + (10.91) + (18.20) + (19.19) + (10.20) + (1.72) + (36.14)

where the pseudo $R^2$ from the first-stage probit model in the SCF is 0.32.

When calculating a household’s predicted probability of stock ownership in the CEX we use the 1992 dummy coefficient for the years 1990 to 1993, the 1995 dummy coefficient for 1994 to 1996, the 1998 dummy coefficient for 1997 to 1999, and the 2001 dummy coefficient from 2000 onward. We then define stockholders under the more sophisticated definition as the intersection of households who respond positively to holdings of “stocks, bonds, mutual funds and other such securities” and have a predicted probability of owning stocks of at least 0.50. Non-stockholders are similarly defined as those responding negatively to the CEX question and having a predicted probability of owning stocks less than 0.50. These alternative definitions of stock and non-stockholders refine the CEX classification.

Appendix B. Estimating the variance of group level consumption growth rates

This appendix outlines our approach to estimating the time-series variance of the cross-sectional mean log consumption growth rate for each group of households (stockholders, non-stockholders, and top stockholders) and each time horizon ($S = 1, 2, 4, 8, 12, 16, 20$ and $24$ quarters).

There are several challenges to doing this. First, we do not have an infinite number of households in the cross-section, so we observe only the sample average growth rate, not the population mean growth rate. The time-series variance of the sample average consumption growth rate for a given group will be an upward biased estimate of the time-series variance of the cross-sectional mean consumption growth rate for the group. Furthermore, the number of households in each cross-section differs for stockholders, non-stockholders, and top stockholders. Since the number
of non-stockholders exceeds the number of stockholders which in turn exceeds the number of top
stockholders, the magnitude of the bias will differ for the three groups. Second, we are interested
in quarterly growth rates, but have these available at the monthly frequency. While the second
issue is relatively easy to deal with, the first issue is not.

Our approach to deal with the first issue is to estimate, using a bootstrap approach, how fast the
time-series variance of the group’s sample average consumption growth rate decreases as the number
of households in the cross-section increases. The object of interest is the time-series variance for
an infinite number of households in the cross-section and this limit can be estimated based on the
bootstrap results using a simple regression.

Let \( g_{t+s}^h = \ln C_{h,t+s} - \ln C_{h,t} \) denote household \( h \)'s S-quarter log consumption growth rate
(adjusted for family and seasonal effects). Use \( E_h \) to denote a cross-sectional population mean.
We are interested in estimating the time-series variance of \( E_h \) (\( \ln C_{h,t+s} - \ln C_{h,t} \)) , for several different
time horizons \( S \) and for each of the three groups of households. Let \( e_{t+s}^h = (\ln C_{h,t+s} - \ln C_{h,t}) -
E_h \) (\( \ln C_{h,t+s} - \ln C_{h,t} \)) denote the difference between household \( h \)'s growth rate and the group's
mean growth rate (i.e., the idiosyncratic component of household \( h \)'s consumption growth rate).
For households in a given group, assume that the idiosyncratic components \( e_{t+s}^h \) are drawn from the
same distribution. An advantage of our approach (outlined below) is that this distribution need not
be specified (for example, it is not necessary to assume that idiosyncratic shocks are independent
over time). Also, since we are estimating what the time-series variance of the cross-sectional mean
log consumption growth rate will be in the population (i.e., for an infinitely large cross-section),
measurement error does not induce any biases as long as it is purely idiosyncratic across households
(and additive in logs) and thus cancels out in the population mean, \( E_h \) (\( \ln C_{h,t+s} - \ln C_{h,t} \)).

Because we focus on log growth rates,

\[
E_h \left( g_{t,t+s}^h \right) = E_h \left( g_{t,t+1}^h + g_{t+1,t+2}^h + g_{t+s-1,t+s}^h \right). \tag{22}
\]

We do not observe this population mean growth rate. Instead we observe the sample average growth rate

\[
\hat{E}_h( g_{t,t+s} ) = \left( \frac{1}{H_t} \sum_{t=1}^{H_t} g_{t,t+1}^h + \frac{1}{H_{t+1}} \sum_{t=1}^{H_{t+1}} g_{t+1,t+2}^h + \ldots + \frac{1}{H_{t+s-1}} \sum_{t=1}^{H_{t+s-1}} g_{t+s-1,t+s}^h \right) \tag{23}
\]

The two components on the right hand side are by construction uncorrelated, so

\[
V \left( \frac{1}{H_t} \sum_{t=1}^{H_t} g_{t,t+1}^h + \frac{1}{H_{t+1}} \sum_{t=1}^{H_{t+1}} g_{t+1,t+2}^h + \ldots + \frac{1}{H_{t+s-1}} \sum_{t=1}^{H_{t+s-1}} g_{t+s-1,t+s}^h \right) \tag{24}
\]

Assume that the number of households in the cross-section is constant over time and denote this
number by \( H \), which by construction will be the case in our bootstrap analysis outlined below. Then

\[
V \left( \frac{1}{H} \sum_{h=1}^{H} g_{t,t+1}^h + \frac{1}{H} \sum_{h=1}^{H} g_{t+1,t+2}^h + \ldots + \frac{1}{H} \sum_{h=1}^{H} g_{t+s-1,t+s}^h \right) \tag{25}
\]
The structure of the data will determine the last term \( \frac{1}{H^2} V \left( \sum_{h=1}^{H} e_{t+1}^h + \Sigma_{h=1}^{H} e_{t+1, t+2} + \ldots + \Sigma_{h=1}^{H} e_{t+s-1, t+s} \right) \). For a balanced panel, \( \frac{1}{H^2} V \left( \sum_{h=1}^{H} e_{t+1}^h + \Sigma_{h=1}^{H} e_{t+1, t+2} + \ldots + \Sigma_{h=1}^{H} e_{t+s-1, t+s} \right) = \frac{1}{H} V \left( e_{t+1}^h + \ldots + e_{t+s-1, t+s} \right) \), assuming that the idiosyncratic shocks are uncorrelated across households (though they may be correlated over time for a given household). For a series of repeated cross-sections where only one consumption growth rate is available for each household \( \frac{1}{H^2} V \left( \sum_{h=1}^{H} e_{t+1}^h + \Sigma_{h=1}^{H} e_{t+1, t+2} + \ldots + \Sigma_{h=1}^{H} e_{t+s-1, t+s} \right) = \frac{1}{H} V \left( e_{t+1}^h \right) \). For a rotating panel, such as the CEX data, in which 3 consumption growth observations are available for each household and where 1/3 of the households are replaced each quarter24

\[
\frac{1}{H^2} V \left( \sum_{h=1}^{H} e_{t+1}^h + \Sigma_{h=1}^{H} e_{t+1, t+2} + \ldots + \Sigma_{h=1}^{H} e_{t+s-1, t+s} \right) = \frac{1}{H} \left[ V \left( e_{t+1}^h \right) + \ldots + V \left( e_{t+s-1, t+s}^h \right) \right] + \frac{2}{3} \Sigma_{i=0}^{s-1} \text{cov} \left( e_{t+i, t+i+1}^h, e_{t+i+1, t+i+2}^h \right) + \frac{1}{2} \Sigma_{i=0}^{s-2} \text{cov} \left( e_{t+i, t+i+1}^h, e_{t+i+2, t+i+3}^h \right).
\]

The data structure will not matter for the \( V \left( E_{h, t+s} \left( g_{t+s}^h \right) \right) \) estimator we obtain, assuming the bootstrap analysis in Step 1 below is performed properly as described.

The estimation proceeds as follows. We have data series for \( N_{t, t+s} = \left( \frac{1}{H} \sum_{h=1}^{H} e_{t+1}^h + \frac{1}{H} \sum_{h=1}^{H} e_{t+1, t+2} + \ldots + \frac{1}{H} \sum_{h=1}^{H} e_{t+s-1, t+s} \right) \). The following two steps allow us to estimate \( V \left( E_{h, t+s} \left( g_{t+s}^h \right) \right) \) for a given group of households.

1. **Step 1:** Perform a series of bootstrap analyses to estimate \( V \left( N_{t, t+s} \right) \) for several different values of \( H \). For each value of \( H \) the bootstrap procedure proceeds as follows.

   (a) In each iteration of the bootstrap, draw a randomly chosen subset of the original data set for the group. This subset should have \( H \) households in each cross-section and should contain \( T - s \) cross-sections, where \( T \) is the time-dimension of the original data set. The subset should replicate the data structure of the CEX. Thus, of those households drawn at \( t \), 2/3 must also be drawn at \( t + 1 \), and 1/3 must also be drawn at \( t + 2 \).25 The sampling in a given period \( t \) should always be without replacement.

   (b) For each randomly chosen subset, calculate \( N_{t, t+s} \) and estimate \( V \left( N_{t, t+s} \right) \). To estimate \( V \left( N_{t, t+s} \right) \) as precisely as possible, use overlapping data and estimate \( V \left( N_{t, h+s} \right) \) by

\[
\frac{1}{s} \left[ \hat{V}_{s, 1} + \hat{V}_{s, 2} + \ldots + \hat{V}_{s, s} \right],
\]

where \( \hat{V}_{s, 1} \) is the variance of \( N_{t, t+s} \) calculated using observations \( s + 1 \), 2s + 1, 3s + 1, ..., \( \hat{V}_{s, 2} \) is the variance of \( N_{t, t+s} \) calculated using observation \( s + 2 \), 2s + 2, 3s + 2, ..., and so forth. After a pre-chosen number \( M \) of bootstrap iterations (\( M = 100,000 \) in our analysis) use the average of the \( M \) estimates of \( V \left( N_{t, t+s} \right) \) as the final estimate of \( V \left( N_{t, t+s} \right) \). The more bootstrap iterations are performed, the more precise this final estimate will be.

---

24 This formula is only approximate since the fraction of households at time \( t + 1 \) and \( t + 2 \) which were also in the sample at time \( t \) will not always be 2/3 and 1/3, respectively, in the first and last quarters of the data set and around the CEX sample redesigns in 1986 and 1996. However, this does not induce a bias in our estimate of \( V \left( E_{h, t+s} \left( g_{t+s}^h \right) \right) \) since the exact expression for \( \frac{1}{H^2} V \left( \sum_{h=1}^{H} e_{t+1}^h + \Sigma_{h=1}^{H} e_{t+1, t+2} + \ldots + \Sigma_{h=1}^{H} e_{t+s-1, t+s} \right) \) remains a linear function of \( 1/H \).

25 Similarly, in a balanced panel, if a given household is drawn at \( t = 1 \), it must also be drawn at \( t = 2 \), ..., \( t = T - s \). In a series of repeated cross-sections, independent random samples can be drawn in each period of a bootstrap iteration.
(c) Repeat this bootstrap analysis for a series of two or more chosen values of $H$.

2. **Step 2:** Regress the resulting series of variances $V(N_{t,t+s})$ on a constant and the variable $1/H$ to estimate $V\left(E_{h,t+s}\left(g_{t,t+s}^h\right)\right)$. From equations (25) and (26) it follows that the intercept in this regression will estimate $V\left(E_{h,t+s}\left(g_{t,t+s}^h\right)\right)$ (the object of interest), while the regression coefficient on $1/H$ will estimate $\left[\sum_{i=1}^{s} V\left(e_{t,s+i+1}^h e_{t+s+i+1}^h + 2\sum_{i\neq j} \text{cov}\left(e_{t,s+i+1}^h e_{t+s+i+1}^h, e_{t+s+j+1}^h e_{t+s+j+1}^h\right)\right]ight.$.

The above calculations assume quarterly data. If quarterly data are available at the monthly frequency, then simply repeat step 1 three times: First using the quarterly growth rates for months ending 1, 4, 7, and 10, then using the quarterly growth rates for months 2, 5, 8, and 11, and finally using the quarterly growth rates for months 3, 6, 9, and 12. Use the average of the three resulting final estimates of $V(N_{t,t+s})$ to proceed in step 2.

In our analysis we use values of $H = 3, 6, 12, 18,$ and $24$ for stockholders and non-stockholders, and $H = 3$ and $6$ for top stockholders. In a small number of months (7 out of 273 months) the number of top stockholders is only 4 or 5. In these months we add in 1 or 2 observations in order to have 6 observations available. We set the consumption growth for these 1 or 2 observations equal to the average stockholder consumption growth rate. This addition is conservative in that any resulting bias in the estimate of $V\left(E_{h,t+s}\left(g_{t,t+s}^h\right)\right)$ will be negative. We have experimented with the vector of $H$-values chosen and results are not sensitive to the choice.

Finally, in order to ensure that the CEX data in fact do constitute a balanced rotating panel, we need to ensure that each household has exactly three consumption growth observations. Our baseline data set is already restricted to such households. However, 11 percent of observations are for households where one or more observations are dropped because it is a consumption outlier or falls under one of the other drop criteria imposed (see Appendix A). Results for this part of our analysis, the estimation of the variance of group level consumption growth rates, is therefore based on 89 percent of our baseline sample.

### Appendix C: GMM estimation

This appendix first shows the equivalence, in terms of the point estimate for risk aversion, between GMM and a cross-sectional OLS regression in our setting. It then uses the GMM framework to derive standard errors that account for (a) correlation of error terms across assets in a given time period, (b) estimation error in covariances, (c) autocorrelation in our consumption series due to the use of overlapping consumption growth data, and (d) the different length of the data series used to estimate covariances and average returns.

To save space we show the derivations here for the Euler equation written in the form of equation (17). The derivations follow in a similar manner when the Euler equation is written in the form of the reverse regression equation (18) with GMM moment conditions reorganized correspondingly.

#### A. The GMM setup

The conditional Euler equation from the Epstein-Zin setting is given by

$$E_t \left(r_{t+1}^l - r_{t+1}^f + \frac{1}{2} V_t \left(r_{t+1}^l\right) \right) \simeq (\gamma - 1) \text{cov}_t \left(r_{t+1}^l, \Sigma_{s=0}^{\infty} \beta^s (c_{t+1+s} - c_{t+s})\right)$$  \hspace{1cm} (27)
Assume $V_t$ and $cov_t$ do not vary over time. Then the corresponding unconditional Euler equation is

$$E \left( r^i_{t+1} - r^f_{t+1} \right) + \frac{1}{2} V \left( r^i_{t+1} \right) \simeq (\gamma - 1) \, cov \left( r^i_{t+1}, \Sigma_{s=0}^\infty \beta^s (c_{t+1+s} - c_t) \right)$$

(28)

where $\Sigma_{s=0}^\infty \beta^s (c_{t+1+s} - c_t)$.

In moment form, adding the moment for $\varepsilon_{c,t+1}$ to allow estimation of $\mu_{\Sigma_{s=0}^\infty \beta^s (c_{t+1+s} - c_t)}$,

$$\begin{bmatrix} 0_{N \times 1} \\ 0 \end{bmatrix} = E \left[ \begin{bmatrix} r^i_{t+1} - r^f_{t+1} + \frac{1}{2} \sigma^2 - (\gamma - 1) r_{t+1} \varepsilon_{c,t+1} \end{bmatrix} \right] = E \begin{bmatrix} g^i_t \\ g^m_t \end{bmatrix}.$$

$N$ is the number of stocks (25 in our setting using the 25 FF test assets), $g^i_t$ and $g^m_t$ are defined by the last equality, and

$$r_{t+1} = \begin{bmatrix} r^i_{t+1} \\ \ldots \\ r^N_{t+1} \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} V \left( r^1_{t+1} \right) \\ \vdots \\ V \left( r^N_{t+1} \right) \end{bmatrix}.$$

For simplicity we assume $\sigma^2$ is known. Since $\sigma^2$ in practice can be estimated with high accuracy any biases resulting from this should be minimal.

In order to use all available data, we use a different sample length $T$ for the two parts of $E \left( g^i_t \right)$. We use a sample of length $T_1$ for estimating mean excess returns (adjusted for the variance term) $E \left( r_{t+1} - r^f_{t+1} + \frac{1}{2} \sigma^2 \right)$ and a subsample of length $T_2$ for estimating covariances $E \left( r_{t+1} \varepsilon_{c,t+1} \right)$. We also use the subsample of length $T_2$ for estimating $E \left( g^m_t \right)$. When estimating equation (28) we include an intercept term $\alpha$, identical for all stocks, in the first set of moment conditions following Parker and Julliard (2005) to provide estimates of risk aversion that best fit the cross-section of average returns, without the additional constraint of also fitting the level of these returns (and thus the equity premium). We also estimate the equation omitting the intercept term.

The GMM objective function is

$$\gamma \mu_{\Sigma_{s=0}^\infty \beta^s (c_{t+1+s} - c_t)} \alpha g^r W g$$

with

$$g = \begin{bmatrix} g^r \\ g^m \end{bmatrix} = \begin{bmatrix} \frac{1}{T_1} \Sigma_{t=1}^{T_1} \left( r_{t+1} - r^f_{t+1} + \frac{1}{2} \sigma^2 \right) - \alpha 1_N - (\gamma - 1) \frac{1}{T_2} \Sigma_{t=1}^{T_2} r_{t+1} \varepsilon_{c,t+1} \end{bmatrix}.$$

where $1_N$ is a $N \times 1$ vector of ones. The estimation picks three parameters to fit 26 moments as best possible. We use a pre-specified weighting matrix

$$W = \begin{bmatrix} I_{25} & 0 \\ 0 & h \end{bmatrix}$$

rather than efficient GMM to give each of the 25 portfolios equal importance in the estimation as opposed to downweighting portfolios with less precisely estimated average returns. Following Parker and Julliard (2005) we set $h$ to a sufficiently large number that the estimate of $\mu_{\Sigma_{s=0}^\infty \beta^s (c_{t+1+s} - c_t)}$ approximately equals $\frac{1}{T_2} \Sigma_{t=1}^{T_2} \Sigma_{s=0}^\infty \beta^s (c_{t+1+s} - c_t)$ and that further increases in $h$ have only minimal effects on the estimates of $\gamma$ and $\alpha$. 

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B. Equivalence of GMM and OLS

The first order conditions for the GMM minimization are \((\nabla g)' W g = 0\), i.e.

\[
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} & \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial \alpha} \\
I_{25} & 0 & 0 \\
0 & h & I_{25}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} \\
\frac{\partial g}{\partial \mu} \\
\frac{\partial g}{\partial \alpha}
\end{bmatrix}'
\begin{bmatrix}
I_{25} & 0 & g^r \\
0 & h & g^m
\end{bmatrix}
\begin{bmatrix}
I_{25} & 0 & 0 \\
0 & h & I_{25}
\end{bmatrix}'
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} \\
\frac{\partial g}{\partial \mu} \\
\frac{\partial g}{\partial \alpha}
\end{bmatrix}
= 0_{3 \times 1}
\]

\[
\implies
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} & \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial \alpha} \\
I_{25} & 0 & 0 \\
0 & h & I_{25}
\end{bmatrix}'
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} \\
\frac{\partial g}{\partial \mu} \\
\frac{\partial g}{\partial \alpha}
\end{bmatrix}
\begin{bmatrix}
I_{25} & 0 & g^r \\
0 & h & g^m
\end{bmatrix}
\begin{bmatrix}
I_{25} & 0 & 0 \\
0 & h & I_{25}
\end{bmatrix}'
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} \\
\frac{\partial g}{\partial \mu} \\
\frac{\partial g}{\partial \alpha}
\end{bmatrix}
= 0
\]

\[
\implies
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} & \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial \alpha} \\
I_{25} & 0 & 0 \\
0 & h & I_{25}
\end{bmatrix}'
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} \\
\frac{\partial g}{\partial \mu} \\
\frac{\partial g}{\partial \alpha}
\end{bmatrix}
\begin{bmatrix}
I_{25} & 0 & g^r \\
0 & h & g^m
\end{bmatrix}
\begin{bmatrix}
I_{25} & 0 & 0 \\
0 & h & I_{25}
\end{bmatrix}'
\begin{bmatrix}
\frac{\partial g}{\partial \gamma} \\
\frac{\partial g}{\partial \mu} \\
\frac{\partial g}{\partial \alpha}
\end{bmatrix}
= 0
\]

The third row implies

\[
\hat{\alpha} = \frac{1}{N} \sum_{i=1}^{N} \left( r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2 \right) - (\hat{\gamma} - 1) \frac{1}{N} \sum_{i=1}^{N} \left( \frac{1}{T_2} \sum_{t=1}^{T_2} (r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2) \right)
\]

where \( y_i = \frac{1}{T_1} \sum_{t=1}^{T_1} \left( r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2 \right) \) is the average excess return on asset \( i \) (adjusted for the variance term) and \( x_i = \frac{1}{T_2} \sum_{t=1}^{T_2} (r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2) \) is the covariance of asset \( i \)'s return with the long-run consumption growth measure.

The first row implies

\[
\hat{\gamma} - 1 = \frac{\sum_{i=1}^{N} \left( \frac{1}{T_2} \sum_{t=1}^{T_2} (r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2) \right) \sum_{i=1}^{N} \left( \frac{1}{T_1} \sum_{t=1}^{T_1} \left( r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2 - \hat{\alpha} \right) \right)}{\sum_{i=1}^{N} \left( \frac{1}{T_2} \sum_{t=1}^{T_2} (r_{t+1}^{i} - r_{t+1}^{f} + \frac{1}{2} \sigma_i^2 - \hat{\alpha} \right)^2}
\]

where the last equality follows after substituting in the above expression for \( \hat{\alpha} \). Thus, the GMM estimates of \( \alpha \) and \( (\gamma - 1) \) are identical to the OLS estimates obtained from a cross-sectional regression (with \( N \) data points) of average excess returns (adjusted for the variance term) on sample covariances.
C. GMM standard errors

Following Newey and McFadden (1994), Theorem 3.4, the asymptotic distribution of the GMM estimator is (under appropriate regularity conditions) given by

\[
\sqrt{T_1} \left( \begin{bmatrix} \hat{\gamma} \\ \mu \Sigma_{\alpha=0}^{\infty} \Omega(c_{t+1+s} - c_t) \\ \end{bmatrix} - \begin{bmatrix} \gamma \\ \mu \Sigma_{\alpha=0}^{\infty} \Omega(c_{t+1+s} - c_t) \\ \end{bmatrix} \right) \rightarrow^d N \left( 0, [G'WG]^{-1} G'WG [G'WG]^{-1} \right)
\]

where \( G = E \left[ \nabla g \right] \) and \( \sqrt{T_1} \left[ g \right] \rightarrow^d N \left( 0, \Omega \right) \). Since

\[
\nabla g = \begin{bmatrix} \frac{\partial g}{\partial \gamma} & \frac{\partial g}{\partial \mu} & \frac{\partial g}{\partial \alpha} \\ \end{bmatrix} = \begin{bmatrix} \frac{-\frac{1}{T_2} \Sigma_{t=1}^{T_2} \epsilon_{c,t+1}}{0} & \left( \frac{\gamma - 1}{T_2} \right) \frac{1}{T_2} \Sigma_{t=1}^{T_2} r_{t+1} & -1 \\ 0 & -1 & \frac{0}{0} \end{bmatrix},
\]

\( G \) is estimated by

\[
\hat{G} = \begin{bmatrix} \left( -\frac{1}{T_2} \Sigma_{t=1}^{T_2} r_{t+1} \epsilon_{c,t+1} \right) & \left( \frac{\gamma - 1}{T_2} \right) \frac{1}{T_2} \Sigma_{t=1}^{T_2} r_{t+1} & -1 \\ 0 & -1 & \frac{0}{0} \end{bmatrix}.
\]

\( \Omega \) is a 26x26 matrix:

\[
\Omega = E \left[ T_1 gg' \right] = E \left[ T_1 \begin{bmatrix} g^r g^r & g^r g^m & g^m g^m \\ \end{bmatrix} \right] = \begin{bmatrix} \Omega_{rr} & \Omega_{rm} & \Omega_{mm} \end{bmatrix}.
\]

\( \Omega^{mm} \) can be rewritten as follows

\[
\Omega_{mm}^{mm} = E \left( T_1 \frac{1}{T_2} \Sigma_{t=1}^{T_2} \epsilon_{c,t+1} + \frac{1}{T_2} \Sigma_{t=1}^{T_2} \epsilon_{c,t+1} \right)
\]

where \( L \) is the highest order of autocorrelation induced by the use of overlapping consumption growth data (\( L = 48 \) when using 16-quarter discounted consumption growth rates, available at the monthly frequency).

Define \( z_{t+1} = r_{t+1} - r_{t+1}^f + \frac{1}{2} \sigma^2 - \alpha \) and \( w_{t+1} = r_{t+1} \epsilon_{c,t+1} \) (where \( z \) is assumed i.i.d., while \( w \) is autocorrelated). Then we can write \( \Omega^{mm} \) as

\[
\Omega_{mm}^{mm} = E \left( T_1 \left( \frac{1}{T_1} \sum_{t=1}^{T_1} \left( r_{t+1} - r_{t+1}^f + \frac{1}{2} \sigma^2 - \alpha s \right) - (\gamma - 1) \frac{1}{T_2} \sum_{t=1}^{T_2} r_{t+1} \epsilon_{c,t+1} \right) \times \frac{1}{T_2} \sum_{t=1}^{T_2} \epsilon_{c,t+1} \right)
\]

where the second equality exploits the fact that values of \( \epsilon_c \) are uncorrelated with values of \( z \) which are from the part of the long sample (of length \( T_1 \)) which does not overlap with the short sample (of length \( T_2 \)).
Finally,

\[
\Omega_{25x25}^{rr} = E \left( T_1 \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} z_{t+1} - (\gamma - 1) \frac{1}{T_2} \Sigma_{t=1}^{T_2} w_{t+1} \right) \times \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} z_{t+1} - (\gamma - 1) \frac{1}{T_2} \Sigma_{t=1}^{T_2} w_{t+1} \right) \right)
\]

\[
= T_1 E \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} [z_{t+1} - \mu_z] \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} [z_{t+1} - \mu_z]' \right) - T_1 \mu_z \mu_z' \right)
\]

\[
+ T_1 (\gamma - 1)^2 E \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} [w_{t+1} - \mu_w] \left( \frac{1}{T_2} \Sigma_{t=1}^{T_2} [w_{t+1} - \mu_w]' \right) + (\gamma - 1) T_1 \mu_w \mu_w' \right)
\]

\[
- (\gamma - 1) T_1 E \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} [z_{t+1} - \mu_z] \times \frac{1}{T_1} \Sigma_{t=1}^{T_1} [w_{t+1} - \mu_w] \right) + (\gamma - 1) T_1 \mu_w \mu_w'
\]

\[
= E \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} [z_{t+1} - \mu_z] [z_{t+1} - \mu_z]' \right) - T_1 \mu_z \mu_z'
\]

\[
+ \frac{T_1}{T_2} (\gamma - 1)^2 \left( \frac{1}{T_1} \Sigma_{t=1}^{T_1} [w_{t+1} - \mu_w] [w_{t+1} - \mu_w]' \right)
\]

\[
- T_1 (\gamma - 1)^2 \mu_w \mu_w'
\]

\[
- (\gamma - 1) E \left( \frac{1}{T_2} \Sigma_{t=1}^{T_2} [z_{t+1} - \mu_z] [w_{t+1} - \mu_w]' \right)
\]

\[
+ (\gamma - 1) T_1 \mu_z \mu_w'
\]

\[
- (\gamma - 1) E \left( \frac{1}{T_2} \Sigma_{t=1}^{T_2} [w_{t+1} - \mu_w] [z_{t+1} - \mu_z]' \right)
\]

\[
+ (\gamma - 1) T_1 \mu_w \mu_z'
\]

where \( \mu_z \) is the mean of \( z_{t+1} \) and \( \mu_w \) is the mean of \( w_{t+1} \). \( \Omega \) is estimated by removing the \( E \)'s and using the estimated values of \( \varepsilon_{t+1}, \alpha, \gamma, \mu_z, \) and \( \mu_w \). When invertibility problems arise we employ Newey and West (1987) weightings up to lag \( L = 48 \) to ensure invertibility. In cases where invertibility is not a problem we confirm that the Newey and West (1987) weights deliver similar standard errors.
Table I:
Risks of Stockholder, Top Stockholder, Non-stockholder, and Aggregate Consumption Growth Across Horizons

Panel A (B and C) reports summary statistics for the second moments of discounted consumption growth of stockholders (top stockholders and non-stockholders) over various horizons $S = 1, 2, 4, 8, 12, 16, 20, 24$. Since we are estimating a time-series variance of the cross-sectional sample mean of log consumption growth, where the number of stockholders, top stockholders, and non-stockholders in the cross-section differs, we compute the asymptotic standard deviation as outlined in Appendix B to account for measurement error differences across the groups. For comparison, we also report the “naive” standard deviation which simply computes the time-series standard deviation of the cross-sectional sample mean of log consumption growth for each group without accounting for the varying size of cross-sections of each group. From the asymptotic variances, we compute and report the autocorrelations using variance ratio tests. Finally, we report the sensitivity or $\beta$ with respect to aggregate consumption growth over the same horizon for each group. Consumption growth from quarter $t$ to $t + S$ is calculated using data from the Consumer Expenditure Survey over the period January, 1982 to November, 2004. Panel D reports standard deviations and autocorrelations for aggregate consumption from NIPA over the longer period 1959 to 2004 for comparison.

<table>
<thead>
<tr>
<th>$S$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
</table>

**Panel A: Stockholder consumption growth over horizon $S$**

| Standard deviation (quarterly) | Naive | 0.034 | 0.042 | 0.054 | 0.076 | 0.097 | 0.116 | 0.135 | 0.142 |
|                               | Asymptotic | 0.018 | 0.027 | 0.036 | 0.054 | 0.074 | 0.092 | 0.111 | 0.113 |
| Implied autocorrelation      | $\rho_1$ | 0.14  |       |       |       |       |       |       |       |
|                              | $\rho_{2-3}$ | -0.14 |       |       |       |       |       |       |       |
|                              | $\rho_{4-7}$ | 0.11  |       |       |       |       |       |       |       |
|                              | $\rho_{8-11}$ | 0.08  |       |       |       |       |       |       |       |
|                              | $\rho_{12-15}$ | 0.03  |       |       |       |       |       |       |       |
|                              | $\rho_{16-19}$ | 0.10  |       |       |       |       |       |       |       |
|                              | $\rho_{20-24}$ | -0.57 |       |       |       |       |       |       |       |
| Sensitivity to aggregate     | 0.630 | 0.906 | 1.205 | 1.566 | 2.112 | 2.872 | 2.659 | 2.369 |       |

**Panel B: Top Stockholder consumption growth over horizon $S$**

| Standard deviation (quarterly) | Naive | 0.082 | 0.099 | 0.129 | 0.179 | 0.220 | 0.251 | 0.270 | 0.286 |
|                               | Asymptotic | 0.033 | 0.037 | 0.041 | 0.064 | 0.090 | 0.108 | 0.118 | 0.118 |
| Implied autocorrelation      | $\rho_1$ | -0.37 | -0.03 | 0.09  | 0.02  | -0.04 | -0.04 | -0.08 |       |
|                              | $\rho_{2-3}$ | 0.09  | 0.21  | -0.06 | 0.06  | -0.11 | -0.02 |       |       |
|                              | $\rho_{4-7}$ | 0.02  |       |       |       |       |       |       |       |
|                              | $\rho_{8-11}$ | -0.04 | -0.04 | -0.08 |       |       |       |       |       |
|                              | $\rho_{12-15}$ |       |       |       |       |       |       |       |       |
|                              | $\rho_{16-19}$ |       |       |       |       |       |       |       |       |
|                              | $\rho_{20-24}$ |       |       |       |       |       |       |       |       |
| Sensitivity to aggregate     | 0.645 | 1.011 | 1.560 | 2.143 | 2.875 | 3.780 | 3.903 | 3.417 |       |

**Panel C: Non-stockholder consumption growth over horizon $S$**

| Standard deviation (quarterly) | Naive | 0.021 | 0.026 | 0.031 | 0.044 | 0.055 | 0.064 | 0.070 | 0.075 |
|                               | Asymptotic | 0.013 | 0.016 | 0.014 | 0.026 | 0.037 | 0.046 | 0.051 | 0.053 |
| Implied autocorrelation      | $\rho_1$ | -0.27 | -0.19 | 0.21  | -0.06 | 0.06  | -0.11 | -0.02 |       |
|                              | $\rho_{2-3}$ | -0.19 |       |       |       |       |       |       |       |
|                              | $\rho_{4-7}$ | 0.21  |       |       |       |       |       |       |       |
|                              | $\rho_{8-11}$ | -0.06 | -0.06 |       |       |       |       |       |       |
|                              | $\rho_{12-15}$ |       |       |       |       |       |       |       |       |
|                              | $\rho_{16-19}$ |       |       |       |       |       |       |       |       |
|                              | $\rho_{20-24}$ |       |       |       |       |       |       |       |       |
| Sensitivity to aggregate     | 0.496 | 0.406 | 0.591 | 0.835 | 0.951 | 0.962 | 0.944 | 0.781 |       |

**Panel D: Aggregate consumption growth over horizon $S$**

| Standard deviation (quarterly) | 0.007 | 0.011 | 0.017 | 0.024 | 0.028 | 0.030 | 0.032 | 0.033 |
| Implied autocorrelation      | $\rho_1$ | 0.235 | 0.158 | 0.067 | 0.030 | 0.010 | 0.002 | -0.003 |       |
Table II:
Euler Equation Estimation for Stockholder, Top Stockholder, Non-stockholder, and Aggregate Consumption Growth Across Horizons

Panel A reports estimates of the Euler equation based on discounted consumption growth of stockholders over various horizons $S = 1, 2, 4, 8, 12, 16, 20, 24$. Results are presented for regressions of the average log excess returns plus half of the variances of the 25 Fama-French portfolios, which are estimated using quarterly data from July, 1926 to November, 2004, on the covariances between log excess returns on the portfolios and consumption growth over horizon $S$. Results are also reported for the reverse regression that regresses covariances on average log excess returns plus half their variance and a constant term. Covariances between log excess returns and long-run consumption risk are estimated over the period of CEX data availability. Estimation is performed by OLS which is shown to be equivalent to GMM in Appendix C. Reported are the intercepts ($\alpha$) and implied risk aversion coefficients ($\gamma$) with $t$-statistics (in parentheses) computed using the GMM approach in Appendix C that account for the different sample lengths used to estimate means and covariances, cross-correlated residuals, first-stage estimation error in the covariances, and consumption growth autocorrelation. For the reverse regressions, $t$-statistics are computed using the Delta method. $R^2$'s from the cross-sectional regressions are also reported. Panel B reports the same set of regression results using the consumption growth of the top third of stockholders and Panel C reports results using the consumption growth of non-stockholders. Long-run consumption growth from quarter $t$ to $t+S$ is calculated using data from the Consumer Expenditure Survey over the period January, 1982 to November, 2004, assuming a discount rate of 5% per year (quarterly discount factor $\beta = 0.95^{1/4}$). Panel D reports regression results and risk aversion estimates based on aggregate consumption from NIPA over both the CEX period (1982 to 2004) and over the longer period 1959 to 2004 for which NIPA data is available.

<table>
<thead>
<tr>
<th>$S$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.033</td>
<td>0.045</td>
<td>0.065</td>
<td>0.021</td>
<td>0.015</td>
<td>0.011</td>
<td>0.007</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(5.72)</td>
<td>(6.32)</td>
<td>(3.54)</td>
<td>(3.12)</td>
<td>(1.47)</td>
<td>(0.97)</td>
<td>(0.77)</td>
<td>(2.34)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-13.04</td>
<td>-22.34</td>
<td>-44.55</td>
<td>12.21</td>
<td>18.84</td>
<td>17.02</td>
<td>13.90</td>
<td>13.87</td>
</tr>
<tr>
<td></td>
<td>(-0.99)</td>
<td>(-2.04)</td>
<td>(-2.14)</td>
<td>(1.54)</td>
<td>(2.64)</td>
<td>(3.46)</td>
<td>(5.02)</td>
<td>(6.23)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.03</td>
<td>0.03</td>
<td>0.33</td>
<td>0.08</td>
<td>0.46</td>
<td>0.63</td>
<td>0.68</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Panel B: Top Stockholder consumption growth over horizon $S$

| $\alpha$ | 0.020 | 0.070 | 0.100 | 0.060 | 0.010 | 0.000 | 0.020 | 0.010 |
|       | (0.87) | (4.11) | (11.66) | (1.82) | (0.50) | (0.19) | (0.76) | (0.26) |
| $\gamma$ | -529.33 | -726.24 | -136.29 | 135.11 | 20.13 | 26.51 | 20.00 | 21.74 |
|       | (-0.18) | (-0.18) | (-0.72) | (0.40) | (1.54) | (2.40) | (3.64) | (4.47) |
| $R^2$ | 0.03 | 0.33 | 0.03 | 0.19 | 0.19 | 0.19 | 0.20 | 0.44 |

Panel C: Non-stockholder consumption growth over horizon $S$

| $\alpha$ | 0.030 | 0.014 | 0.035 | 0.016 | 0.023 | 0.008 | 0.010 | 0.016 |
|       | (6.10) | (1.59) | (5.37) | (0.92) | (2.30) | (2.83) | (1.16) | (3.56) |
| $\gamma$ | 6.69 | 13.79 | -3.31 | 10.48 | 7.65 | 11.01 | 8.68 | 7.73 |
|       | (1.27) | (2.53) | (-0.85) | (1.53) | (2.07) | (2.85) | (4.20) | (4.71) |
| $R^2$ | 0.03 | 0.08 | 0.03 | 0.19 | 0.31 | 0.66 | 0.65 | 0.44 |

Panel D: Aggregate consumption growth over horizon $S$

| $\alpha$ | -0.001 | 0.110 | 0.140 | 0.090 | -0.020 | 0.020 | 0.010 | 0.020 |
|       | (-0.05) | (3.76) | (1.56) | (2.22) | (-0.41) | (0.53) | (0.21) | (0.36) |
| $\gamma$ | 180.10 | 170.59 | -149.91 | 51.28 | 22.47 | 16.11 | 12.81 | 16.31 |
|       | (0.21) | (0.40) | (-0.19) | (0.50) | (0.96) | (1.91) | (3.04) | (2.94) |
| $R^2$ | 0.03 | 0.08 | 0.03 | 0.19 | 0.31 | 0.66 | 0.65 | 0.44 |
Panel C: Non-Stockholder consumption growth over horizon $S$

Regression:  
$$E[r_{t+1}^s] - r_f + \frac{\sigma^2}{2} = \alpha + (\gamma - 1)\sigma_{i,c} + \epsilon_i$$

<table>
<thead>
<tr>
<th>$S$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.032</td>
<td>0.023</td>
<td>0.020</td>
<td>0.040</td>
<td>0.048</td>
<td>0.004</td>
<td>0.009</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(3.10)</td>
<td>(1.33)</td>
<td>(4.09)</td>
<td>(4.56)</td>
<td>(22.28)</td>
<td>(0.20)</td>
<td>(1.52)</td>
<td>(0.77)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>87.31</td>
<td>72.19</td>
<td>25.05</td>
<td>-20.83</td>
<td>-31.12</td>
<td>48.90</td>
<td>29.12</td>
<td>32.73</td>
</tr>
<tr>
<td></td>
<td>(1.80)</td>
<td>(1.90)</td>
<td>(1.79)</td>
<td>(-1.36)</td>
<td>(-1.79)</td>
<td>(3.38)</td>
<td>(4.22)</td>
<td>(5.84)</td>
</tr>
</tbody>
</table>

Reverses regression:  
$$\sigma_{i,c} = \alpha + \frac{1}{(\gamma - 1)}(E[r_{t+1}^s] - r_f + \frac{\sigma^2}{2}) + u_i$$

| $\alpha$ | -0.010 | 0.000 | 0.030 | 0.050 | 0.070 | 0.030 | 0.010 | -0.010 |
|       | (-1.55) | (-0.63) | (1.93) | (2.87) | (6.02) | (4.28) | (0.51) | (-1.11) |
| $\gamma$ | 671.08 | 224.12 | 300.80 | -392.46 | 174.91 | 44.03 | 32.73 | 44.30 |
|       | (0.37) | (0.93) | (0.50) | (-0.26) | (1.08) | (3.57) | (5.10) | |

$R^2$ | 0.13 | 0.32 | 0.08 | 0.06 | 0.19 | 0.33 | 0.65 | 0.73 |

Panel D: Aggregate consumption growth over horizon $S$

CEX sample period 1982 - 2004

Regression:  
$$E[r_{t+1}^s] - r_f + \frac{\sigma^2}{2} = \alpha + (\gamma - 1)\sigma_{i,c} + \epsilon_i$$

| $\alpha$ | 0.029 | 0.033 | 0.027 | 0.015 | 0.016 | 0.010 | 0.010 | 0.023 |
|       | (7.92) | (8.84) | (7.21) | (0.73) | (0.92) | (0.46) | (2.28) | (1.10) |
| $\gamma$ | 27.55 | -16.98 | 21.98 | 74.99 | 53.61 | 46.34 | 37.97 | 31.85 |
|       | (0.91) | (-0.96) | (1.64) | (2.75) | (6.11) | (3.94) | (5.10) | |

Reverses regression:  
$$\sigma_{i,c} = \alpha + \frac{1}{(\gamma - 1)}(E[r_{t+1}^s] - r_f + \frac{\sigma^2}{2}) + u_i$$

| $\alpha$ | 0.010 | 0.020 | 0.010 | 0.000 | -0.010 | 0.010 | 0.020 | -0.001 |
|       | (0.35) | (0.90) | (0.90) | (0.58) | (0.00) | (-1.53) | (1.18) | (0.91) |
| $\gamma$ | 2073.14 | -822.83 | 461.51 | 147.80 | 89.54 | 124.92 | 70.35 | 70.35 |
|       | (0.12) | (-0.18) | (0.41) | (1.87) | (3.29) | (4.20) | (1.61) | (2.72) |

$R^2$ | 0.01 | 0.02 | 0.05 | 0.50 | 0.68 | 0.51 | 0.14 | 0.44 |

NIPA sample period 1959 - 2004

Regression:  
$$E[r_{t+1}^s] - r_f + \frac{\sigma^2}{2} = \alpha + (\gamma - 1)\sigma_{i,c} + \epsilon_i$$

| $\alpha$ | 0.020 | 0.027 | 0.024 | 0.017 | 0.008 | 0.007 | 0.019 | 0.010 |
|       | (5.91) | (7.16) | (5.60) | (2.63) | (1.51) | (0.47) | (2.56) | (0.98) |
| $\gamma$ | 97.77 | 13.32 | 18.42 | 25.29 | 52.03 | 54.20 | 31.56 | 41.51 |
|       | (2.49) | (0.87) | (1.57) | (2.37) | (4.41) | (3.58) | (1.89) | (2.89) |

Reverses regression:  
$$\sigma_{i,c} = \alpha + \frac{1}{(\gamma - 1)}(E[r_{t+1}^s] - r_f + \frac{\sigma^2}{2}) + u_i$$

| $\alpha$ | 0.010 | 0.020 | 0.030 | 0.040 | 0.010 | 0.020 | 0.040 | 0.030 |
|       | (1.31) | (0.80) | (1.32) | (2.35) | (0.71) | (3.01) | (2.09) | (3.23) |
| $\gamma$ | 538.76 | 1019.33 | 360.52 | 154.87 | 80.00 | 133.52 | 189.96 | 148.40 |
|       | (1.34) | (0.11) | (0.42) | (1.12) | (3.83) | (2.16) | (0.74) | (1.60) |

$R^2$ | 0.18 | 0.01 | 0.05 | 0.16 | 0.65 | 0.40 | 0.11 | 0.27 |
Table III:  
Dispersion in Consumption Growth Covariances  
Across the 25 Fama-French Portfolios

Panel A reports the first-stage covariance estimates and GMM t-statistics allowing for autocorrelation up to 48 lags of each of the 25 Fama-French portfolios with stockholder consumption growth over a 16 quarter horizon. An F-test on the joint equality of the covariances is reported (with p-values in parentheses). Panels B, C, and D report the covariance estimates for top stockholder, non-stockholder, and aggregate consumption growth, respectively.

| First-stage covariance estimates for 16 quarter consumption growth |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
|                     | 1                   | 2                   | 3                   | 4                   | 5                   | avg.                |                     |
|                     | 10                  | 10                  | 10                  | 10                  | 10                  | 10                  |                     |
|                     | 0.60                | 1.20                | 1.64                | 2.04                | 2.02                | 0.60                |                     |
|                     | 0.51                | 1.77                | 1.87                | 3.09                | 2.54                | 0.51                |                     |
|                     | 0.75                | 1.74                | 2.83                | 2.01                | 2.70                | 0.75                |                     |
|                     | 0.12                | 1.76                | 2.47                | 2.01                | 2.92                | 0.12                |                     |
|                     | 1.13                | 2.34                | 2.65                | 2.62                | 3.65                | 1.13                |                     |
|                     | 6.51                | 11.39               | 13.86               | 13.71               | 16.27               | 6.51                |                     |
|                     | 3.39                | (p-value = 0.051)   |                     |                     |                     |                     |                     |

| Panel B: Top Stockholder consumption growth |
|-------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Consumption growth covariance |                      | GMM t-statistics  |                   |                   |                   |                   |
| 1 (small)                     | 16.79              | 22.63             | 24.00             | 27.41             | 34.44             | 25.05             | 0.92               | 1.69               | 1.92               | 2.61               | 2.99               |
| 2                             | 14.44              | 18.94             | 23.56             | 30.26             | 30.99             | 23.64             | 0.93               | 1.88               | 2.60               | 4.29               | 3.94               |
| 3                             | 16.12              | 22.90             | 28.99             | 22.67             | 29.80             | 24.10             | 1.19               | 2.20               | 4.00               | 3.25               | 3.70               |
| 4                             | 8.08               | 17.34             | 24.55             | 17.38             | 21.29             | 17.73             | 0.51               | 2.21               | 3.15               | 2.50               | 3.17               |
| Avg.                          | 13.82              | 20.58             | 24.10             | 23.63             | 28.64             | 13.82             | 0.92               | 1.69               | 1.92               | 2.61               | 2.99               |
| F-stat                        | 3.57               | (p-value = 0.045)  |                   |                   |                   |                   |                     |                     |                     |                     |                     |

| Panel C: Non-Stockholder consumption growth |
|--------------------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Consumption growth covariance              |                   |                   |                   |                   |                   |                   |
| 1 (small)                                  | 5.34               | 5.34              | 5.79              | 5.64              | 7.10              | 5.84              | 1.46               | 2.19               | 2.42               | 2.58               | 2.83               |
| 2                                           | 4.72               | 4.62              | 5.39              | 6.93              | 6.43              | 5.62              | 1.31               | 1.87               | 2.74               | 4.63               | 2.76               |
| 3                                           | 5.81               | 7.11              | 6.43              | 5.07              | 6.57              | 6.20              | 1.62               | 2.99               | 3.48               | 3.11               | 3.79               |
| 4                                           | 4.48               | 4.51              | 6.19              | 5.02              | 4.72              | 4.98              | 1.30               | 2.00               | 3.10               | 2.82               | 2.12               |
| 5 (large)                                   | 3.95               | 5.72              | 5.33              | 5.04              | 5.44              | 4.90              | 1.23               | 2.30               | 2.41               | 2.64               | 4.20               |
| Avg.                                        | 4.86               | 5.46              | 5.83              | 5.34              | 6.05              | 4.86              | 1.23               | 2.30               | 2.41               | 2.64               | 4.20               |
| F-stat                                      | 0.73               | (p-value = 0.493)  |                   |                   |                   |                   |                     |                     |                     |                     |                     |

| Panel D: Aggregate consumption growth       |
|---------------------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Consumption growth covariance              |                   |                   |                   |                   |                   |                   |
| 1 (small)                                  | 5.74               | 4.48              | 5.67              | 4.99              | 6.18              | 5.41              | 1.50               | 1.27               | 1.99               | 1.88               | 2.37               |
| 2                                           | 3.19               | 3.80              | 4.33              | 4.94              | 5.37              | 4.33              | 0.82               | 1.30               | 1.88               | 2.16               | 2.46               |
| 3                                           | 2.43               | 4.17              | 4.74              | 4.36              | 5.12              | 4.16              | 0.72               | 1.70               | 2.25               | 2.19               | 2.51               |
| 4                                           | 2.83               | 3.96              | 4.95              | 5.12              | 4.64              | 4.30              | 0.96               | 1.71               | 2.93               | 3.14               | 2.81               |
| 5 (large)                                   | 3.97               | 3.89              | 3.35              | 4.73              | 4.77              | 4.14              | 1.92               | 1.99               | 2.41               | 3.18               | 3.65               |
| Avg.                                        | 3.63               | 4.06              | 4.60              | 4.83              | 5.22              | 3.63              | 1.23               | 1.99               | 2.41               | 3.18               | 3.65               |
| F-stat                                      | 1.23               | (p-value = 0.310)  |                   |                   |                   |                   |                     |                     |                     |                     |                     |
Table IV: Robustness of Aggregation, Stockholder Definition, and Estimation

Estimates of the Euler equation based on the long-run (discounted) consumption growth of stockholders, the top third of stockholders, and non-stockholders at 16 quarter horizons are reported across different stockholder definitions and estimation methods. Long-run consumption growth from quarter $t$ to $t + 16$ is calculated using data from the Consumer Expenditure Survey over the period January, 1982 to November, 2004, assuming a discount rate of 5% per year (quarterly discount factor $\beta = 0.95^{1/4}$). Results are presented for regressions of average log excess returns plus half their variance on consumption covariances (“Forward” regressions) and for regressions of consumption covariances on average log excess returns plus half their variance (“Reverse” regressions). In the table $\alpha$ denotes the intercept from the regressions and $\gamma$ is the implied risk aversion coefficient calculated from the regression estimates. Estimation is performed by OLS which is shown to be equivalent to GMM in Appendix C with $t$-statistics (in parentheses) computed using the GMM approach in Appendix C that account for the different sample length used to estimate means and covariances, cross-correlated residuals, first-stage estimation error in the covariances, and consumption growth autocorrelation. $R^2$s from the cross-sectional regressions are also reported. Panel A reports estimates using a representative agent aggregation assumption to compute stockholder and non-stockholder consumption growth. Panel B reports results from an alternative definition of stockholders using the predicted probability of being a stockholder from a probit analysis conducted in the SCF. Panel C reports results where the mean and variance of asset returns are estimated over the same time period as the CEX sample. Panel D reports results for regressions that force the intercept to be zero.

<table>
<thead>
<tr>
<th>Regression:</th>
<th>Stockholders</th>
<th>Top stockholders</th>
<th>Non-stockholders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forward</td>
<td>Reverse</td>
<td>Forward</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.013</td>
<td>-0.001</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(1.18)</td>
<td>(-0.03)</td>
<td>(0.73)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>12.14</td>
<td>19.93</td>
<td>9.71</td>
</tr>
<tr>
<td></td>
<td>(3.51)</td>
<td>(2.25)</td>
<td>(3.09)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.59</td>
<td>0.60</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Panel A: Representative agent aggregation

Panel B: Alternative definition of stockholders: CEX and Probit from SCF

Panel C: Mean returns estimated over CEX sample period (1982 – 2004)

Panel D: No intercept
### Table V: Euler Equation Estimation Across Elasticity of Intertemporal Substitution Values

Estimates of the Euler equation based on the long-run (discounted) consumption growth of stockholders, the top third of stockholders, non-stockholders, and aggregate across different values for the elasticity of intertemporal substitution (EIS) are reported. Estimation of both the intercept, $\alpha$, and implied risk aversion parameter, $\gamma$, is done through a series of Vector Autoregressions (VAR). Panel A reports results for EIS = 1 from Campbell’s (1996) VAR that places no restrictions on the coefficient estimates. Panel B reports results for EIS = 1 from Hansen, Heaton, and Li’s (2005) VAR that places restrictions on the coefficient estimates as described in Section I. Panel C reports results for EIS = 1.5 and Panel D for EIS = 0.5 from Hansen, Heaton, and Li’s (2005) VAR. The VARs are estimated for each portfolio individually using up to 16 lags and employing only returns. Standard errors used to compute $t$-statistics (reported in parentheses) are computed using a block bootstrap approach that resamples in 16-quarter blocks 1,000 times.

<table>
<thead>
<tr>
<th>Stockholders</th>
<th>Top stockholders</th>
<th>Non-stockholders</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: EIS = 1 from Campbell’s (1996) VAR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(2.71)</td>
<td>(1.53)</td>
<td>(3.18)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>12.71</td>
<td>7.33</td>
<td>25.91</td>
</tr>
<tr>
<td></td>
<td>(3.10)</td>
<td>(1.78)</td>
<td>(3.12)</td>
</tr>
<tr>
<td><strong>Panel B: EIS = 1 from Hansen, Heaton, Li’s (2005) VAR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(2.62)</td>
<td>(2.11)</td>
<td>(3.31)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>12.68</td>
<td>6.85</td>
<td>30.15</td>
</tr>
<tr>
<td></td>
<td>(2.70)</td>
<td>(1.99)</td>
<td>(2.73)</td>
</tr>
<tr>
<td><strong>Panel C: EIS = 1.5 from Hansen, Heaton, Li’s (2005) VAR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(2.02)</td>
<td>(2.34)</td>
<td>(2.71)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>12.69</td>
<td>6.68</td>
<td>30.51</td>
</tr>
<tr>
<td></td>
<td>(4.46)</td>
<td>(2.72)</td>
<td>(1.70)</td>
</tr>
<tr>
<td><strong>Panel D: EIS = 0.5 from Hansen, Heaton, Li’s (2005) VAR</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>(2.79)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>12.63</td>
<td>6.95</td>
<td>30.72</td>
</tr>
<tr>
<td></td>
<td>(4.61)</td>
<td>(3.18)</td>
<td>(2.64)</td>
</tr>
</tbody>
</table>
Table VI: Consumption Growth Factor-Mimicking Portfolios

The table reports estimates of the Euler equation using the long-run consumption growth of stockholders, the top third of stockholders, non-stockholders, and aggregate consumption using consumption growth factor-mimicking (CGF) portfolios which are constructed by projecting consumption growth on a set of instruments available over a longer sample period and then using the coefficient estimates and the instruments to construct predicted values of consumption growth. Panel A reports results for a CGF formed by regressing 16-quarter (discounted) consumption growth on a constant and the quarterly log excess returns (over the T-bill rate) of small growth (intersection of the smallest 40% size, lowest 40% BE/ME stocks, based on NYSE breakpoints), large growth (largest 40% size, lowest 40% BE/ME stocks), small value (smallest 40% size, highest 40% BE/ME stocks), and large value (largest 40% size, highest 40% BE/ME stocks) portfolios. The regression is estimated over the CEX period (January, 1982 to November, 2004) and the coefficients are then used to project consumption growth over the same period and from July, 1926 to November, 2004. Coefficient estimates of the factor loadings from the first stage are assumed to be constant over time. Panel B reports results allowing for time-variation in factor loadings by interacting the four size and value portfolios with Lettau and Ludvigson’s (2001) consumption-to-wealth ratio \(cay\) (linearly interpolated between quarters to produce monthly estimates). The consumption-to-wealth ratio proxies for the share of stockholder wealth and is available over a longer time period than CEX consumption data. We model factor loadings as having a constant component and a time-varying component which is a function of \(cay\). Panel C reports results from a CGF created by projecting stock and non-stockholder consumption growth on aggregate consumption growth. Each panel reports the first-stage regression parameters that determine the factor loadings in constructing each CGF, including Newey and West (1987) \(t\)-statistics with an adjustment of 48 lags and the \(R^2\) from the regression. Each panel then reports the risk aversion estimate from the Euler equation in the second stage that employs the CGF in place of actual consumption growth. Standard errors used to compute \(t\)-statistics are calculated from a block bootstrap procedure that resamples 16-quarter blocks 1,000 times and repeats both the first-stage CGF construction and second-stage Euler equation estimation.

<table>
<thead>
<tr>
<th>Stockholders</th>
<th>Top stockholders</th>
<th>Non-stockholders</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First-stage estimates of weights for CGF</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small, growth</td>
<td>-0.32</td>
<td>-0.56</td>
<td>-0.14</td>
</tr>
<tr>
<td>((-1.65))</td>
<td>((-1.65))</td>
<td>((-1.59))</td>
<td>((-1.31))</td>
</tr>
<tr>
<td>Large, growth</td>
<td>0.02</td>
<td>0.05</td>
<td>0.11</td>
</tr>
<tr>
<td>((0.08))</td>
<td>((0.11))</td>
<td>((0.83))</td>
<td>((0.22))</td>
</tr>
<tr>
<td>Small, value</td>
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<td>1.07</td>
<td>0.20</td>
</tr>
<tr>
<td>((1.96))</td>
<td>((2.25))</td>
<td>((1.84))</td>
<td>((2.04))</td>
</tr>
<tr>
<td>Large, value</td>
<td>-0.07</td>
<td>-0.24</td>
<td>-0.10</td>
</tr>
<tr>
<td>((-0.24))</td>
<td>((-0.46))</td>
<td>((-0.77))</td>
<td>((-0.73))</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Second-stage Euler equation estimates using CGF</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Regression:</td>
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<td>forward</td>
</tr>
<tr>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>CEX sample period January, 1982 to November, 2004</td>
<td></td>
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</tr>
<tr>
<td>(\alpha)</td>
<td>0.015</td>
<td>-0.008</td>
<td>0.012</td>
</tr>
<tr>
<td>((1.26))</td>
<td>((-0.93))</td>
<td>((1.46))</td>
<td>((-0.66))</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>16.91</td>
<td>25.91</td>
<td>11.09</td>
</tr>
<tr>
<td>((3.16))</td>
<td>((2.19))</td>
<td>((4.54))</td>
<td>((3.24))</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.83</td>
<td>0.75</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>Entire sample period July, 1926 to November, 2004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.007</td>
<td>0.007</td>
<td>0.009</td>
</tr>
<tr>
<td>((1.08))</td>
<td>((1.23))</td>
<td>((1.36))</td>
<td>((0.98))</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>6.25</td>
<td>9.34</td>
<td>4.72</td>
</tr>
<tr>
<td>((6.30))</td>
<td>((5.35))</td>
<td>((8.58))</td>
<td>((6.72))</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.63</td>
<td>0.59</td>
<td>0.63</td>
</tr>
<tr>
<td>Stockholders</td>
<td>Top stockholders</td>
<td>Non-stockholders</td>
<td>Aggregate</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------</td>
<td>------------------</td>
<td>-----------</td>
</tr>
</tbody>
</table>

**Panel B: CGF from size and value portfolios with time-varying factor loadings**

**First-stage estimates of weights for CGF**

<table>
<thead>
<tr>
<th>Subset</th>
<th>Small, growth</th>
<th>Large, growth</th>
<th>Small, value</th>
<th>Large, value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.58</td>
<td>-0.09</td>
<td>0.79</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(-2.96)</td>
<td>(-0.38)</td>
<td>(3.44)</td>
<td>(-0.44)</td>
</tr>
<tr>
<td>SG × cay</td>
<td>-0.17</td>
<td>0.07</td>
<td>0.23</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(-3.92)</td>
<td>(1.44)</td>
<td>(2.23)</td>
<td>(2.14)</td>
</tr>
<tr>
<td>LG × cay</td>
<td>-0.09</td>
<td>-0.08</td>
<td>-0.09</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(-0.38)</td>
<td>(-0.44)</td>
<td>(-0.44)</td>
<td>(0.22)</td>
</tr>
</tbody>
</table>

**Second-stage Euler equation estimates using CGF**

<table>
<thead>
<tr>
<th>Regression:</th>
<th>forward</th>
<th>forward</th>
<th>forward</th>
<th>forward</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.008</td>
<td>0.003</td>
<td>-0.017</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(1.33)</td>
<td>(0.66)</td>
<td>(-1.80)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>γ</td>
<td>21.43</td>
<td>11.53</td>
<td>108.53</td>
<td>56.75</td>
</tr>
<tr>
<td></td>
<td>(6.73)</td>
<td>(6.28)</td>
<td>(5.35)</td>
<td>(5.70)</td>
</tr>
<tr>
<td>R²</td>
<td>0.90</td>
<td>0.74</td>
<td>0.78</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Panel C: CGF from aggregate consumption growth**

**First-stage estimates of weights for CGF**

<table>
<thead>
<tr>
<th>Subset</th>
<th>Aggregate consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.68</td>
</tr>
<tr>
<td></td>
<td>(5.42)</td>
</tr>
<tr>
<td>R²</td>
<td>0.50</td>
</tr>
</tbody>
</table>

**Second-stage Euler equation estimates using CGF**

<table>
<thead>
<tr>
<th>Regression:</th>
<th>forward</th>
<th>forward</th>
<th>forward</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.64)</td>
<td>(0.64)</td>
<td>(0.64)</td>
</tr>
<tr>
<td>γ</td>
<td>20.76</td>
<td>14.47</td>
<td>53.38</td>
</tr>
<tr>
<td></td>
<td>(3.68)</td>
<td>(3.76)</td>
<td>(3.57)</td>
</tr>
<tr>
<td>R²</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Table VII:
Asset Pricing Tests Using Consumption Growth Factor-Mimicking Portfolios

Panel A reports summary statistics on the consumption growth factor (CGF) mimicking portfolios for stockholder and top stockholder long-run consumption growth from Table VI assuming constant factor loadings. The time-series means and t-statistics of the CGF portfolio returns are reported along with their correlation with the Fama and French (1993) factor portfolios RMRF, SMB, and HML and the momentum portfolio UMD. For comparison with existing factors which are available at the monthly frequency, the CGF’s in this table are calculated using the coefficients from the first stage reported in Table VI combined with monthly (as opposed to quarterly) returns on the regressors from that first stage. Panel B reports results from time-series regressions of each CGF on the three-factor model of Fama and French (1993) augmented with a momentum factor. Panel C reports the reverse regression of the Fama-French factor portfolios on each CGF. Results are reported for the full sample of returns from July, 1926 to November, 2004 and from the period prior to the existence of the CEX household consumption series (before January, 1982) used to construct the CGFs.

<table>
<thead>
<tr>
<th></th>
<th>RMRF</th>
<th>SMB</th>
<th>HML</th>
<th>UMD</th>
<th>CGF_stock</th>
<th>CGF_top</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.69</td>
<td>0.24</td>
<td>0.42</td>
<td>0.74</td>
<td>0.44</td>
<td>0.77</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.85</td>
<td>2.12</td>
<td>3.62</td>
<td>4.86</td>
<td>6.95</td>
<td>6.90</td>
</tr>
</tbody>
</table>

Correlations

<table>
<thead>
<tr>
<th></th>
<th>RMRF</th>
<th>SMB</th>
<th>HML</th>
<th>UMD</th>
<th>CGF_stock</th>
<th>CGF_top</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMRF</td>
<td>1.00</td>
<td>0.33</td>
<td>0.18</td>
<td>-0.31</td>
<td>0.61</td>
<td>0.57</td>
</tr>
<tr>
<td>SMB</td>
<td>1.00</td>
<td>0.09</td>
<td>-0.17</td>
<td>0.55</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>HML</td>
<td>1.00</td>
<td>0.09</td>
<td>0.18</td>
<td></td>
<td>0.55</td>
<td>0.59</td>
</tr>
<tr>
<td>UMD</td>
<td></td>
<td>1.00</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>CGF_stock</th>
<th>CGF_top</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample (1926-2004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>CGF_stock</td>
<td>CGF_top</td>
</tr>
<tr>
<td>Out of sample (1926-1982)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Four-factor regressions: $CGF_t = a + bRMRF_t + sSMB_t + hHML_t + mUMD_t + \epsilon_t$

<table>
<thead>
<tr>
<th></th>
<th>CGF_stock</th>
<th>CGF_top</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.15</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(5.20)</td>
<td>(5.03)</td>
</tr>
<tr>
<td>$b$</td>
<td>0.14</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(13.90)</td>
<td>(11.16)</td>
</tr>
<tr>
<td>$s$</td>
<td>0.21</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>(7.13)</td>
<td>(8.14)</td>
</tr>
<tr>
<td>$h$</td>
<td>0.30</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(14.48)</td>
<td>(12.48)</td>
</tr>
<tr>
<td>$m$</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(1.58)</td>
<td>(1.73)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.79</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.81</td>
</tr>
</tbody>
</table>

Panel C: Regressions of Fama-French factors on CGF

<table>
<thead>
<tr>
<th></th>
<th>SMB_t = \alpha + \beta_{CGF}CGF_t + \epsilon_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.19</td>
</tr>
<tr>
<td></td>
<td>(-1.97)</td>
</tr>
<tr>
<td>$\beta_{CGF}$</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(20.05)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.30</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>HML_t = \alpha + \beta_{CGF}CGF_t + \epsilon_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>(-1.02)</td>
</tr>
<tr>
<td>$\beta_{CGF}$</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>(25.14)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.41</td>
</tr>
</tbody>
</table>

60
Table VIII:

The Equity Premium

Implied measures of risk aversion from the equity premium and the covariance of aggregate stock returns with long-run (16-quarter, discounted) consumption growth are reported separately for the consumption of stockholders, the top stockholders, and non-stockholders from the CEX, as well as aggregate nondurable and service consumption from NIPA over the period January, 1959 to November, 2004. Panel A reports results using actual CEX consumption data from January, 1982 to November, 2004. Estimated risk aversion parameters are reported for the Euler equations assuming an EIS = 1.5, 1, and 0.5, and using Hansen, Heaton, and Li’s (2005) VAR. The last row of Panel A also reports estimated risk aversion for an EIS = 1 that does not use a VAR and ignores the covariance between the conditional means of returns and consumption growth (the baseline approach). Panels B and C focus only on the special case where the EIS = 1. Panel B reports results for the consumption growth factor-mimicking portfolios, CGFs, which assume constant loadings and cover the period July, 1926 to November, 2004. Panel C reports results for the constant-loading CGFs covering the full return period July, 1926 to November, 2004. We also report the t-statistic (in parentheses) of the risk aversion estimate γ, calculated using a block bootstrap method that resamples 5,000 times 16 quarters at a time.

<table>
<thead>
<tr>
<th></th>
<th>Stockholder</th>
<th>Top stockholder</th>
<th>Non-stockholder</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance with RMRF</td>
<td>9.34 × 10^{-4}</td>
<td>15.67 × 10^{-4}</td>
<td>4.54 × 10^{-4}</td>
<td>3.14 × 10^{-4}</td>
</tr>
</tbody>
</table>

*Hansen, Heaton, and Li (2005) structural VAR*

<table>
<thead>
<tr>
<th>EIS</th>
<th>γ</th>
<th>v</th>
<th>γ</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>21.69</td>
<td>(3.81)</td>
<td>11.37</td>
<td>(2.32)</td>
</tr>
<tr>
<td></td>
<td>64.22</td>
<td>(5.78)</td>
<td>82.31</td>
<td>(3.52)</td>
</tr>
<tr>
<td>1.0</td>
<td>22.44</td>
<td>(4.35)</td>
<td>12.61</td>
<td>(3.63)</td>
</tr>
<tr>
<td></td>
<td>66.62</td>
<td>(6.34)</td>
<td>83.90</td>
<td>(5.51)</td>
</tr>
<tr>
<td>0.5</td>
<td>24.11</td>
<td>(2.20)</td>
<td>15.21</td>
<td>(2.36)</td>
</tr>
<tr>
<td></td>
<td>69.21</td>
<td>(4.08)</td>
<td>84.65</td>
<td>(2.52)</td>
</tr>
</tbody>
</table>

*Baseline estimate*

<table>
<thead>
<tr>
<th>EIS</th>
<th>γ</th>
<th>v</th>
<th>γ</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>23.91</td>
<td>(4.74)</td>
<td>14.66</td>
<td>(3.84)</td>
</tr>
<tr>
<td></td>
<td>48.172</td>
<td>(6.45)</td>
<td>69.24</td>
<td>(6.11)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Stockholder</th>
<th>Top stockholder</th>
<th>Non-stockholder</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance with RMRF</td>
<td>6.17 × 10^{-4}</td>
<td>11.50 × 10^{-4}</td>
<td>3.67 × 10^{-4}</td>
<td>2.61 × 10^{-4}</td>
</tr>
<tr>
<td>γ</td>
<td>35.68</td>
<td>(2.69)</td>
<td>19.60</td>
<td>(2.84)</td>
</tr>
<tr>
<td></td>
<td>59.29</td>
<td>(2.99)</td>
<td>83.06</td>
<td>(3.04)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Stockholder</th>
<th>Top stockholder</th>
<th>Non-stockholder</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance with RMRF</td>
<td>30.05 × 10^{-4}</td>
<td>51.97 × 10^{-4}</td>
<td>8.36 × 10^{-4}</td>
<td>9.30 × 10^{-4}</td>
</tr>
<tr>
<td>γ</td>
<td>8.12</td>
<td>(3.12)</td>
<td>5.12</td>
<td>(3.26)</td>
</tr>
<tr>
<td></td>
<td>26.59</td>
<td>(3.20)</td>
<td>24.01</td>
<td>(3.12)</td>
</tr>
</tbody>
</table>
**Table IX:**  
**The Cross-Section of Bond Returns**  
Panel A reports estimates of the Euler equation based on discounted consumption growth of stockholders, top stockholders, non-stockholders, and aggregate consumption over $S = 16$ quarters using the cross-section of bond portfolio returns. Results are presented for regressions of the average log excess returns plus half of the variances of 8 Treasury bond portfolios with maturities of 3 months, 1 year, 2 years, 5 years, 7 years, 10 years, 20 years, and 30 years, on the covariances between log excess returns on the portfolios and consumption growth. Estimation is performed by OLS which is shown to be equivalent to GMM in Appendix C. Reported are the intercepts ($\alpha$) and implied risk aversion coefficients ($\gamma$) with $t$-statistics (in parentheses) computed using the GMM approach in Appendix C. $R^2$'s from the cross-sectional regressions are also reported. Panel B reports the same set of regression results using the consumption growth factor-mimicking portfolios from Table VI Panel A for each group’s consumption growth that uses the size and value portfolios and assumes constant factor loadings through time.

<table>
<thead>
<tr>
<th></th>
<th>Stockholders</th>
<th>Top stockholders</th>
<th>Non-stockholders</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Euler equation estimates using Actual CEX consumption growth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(1.10)</td>
<td>(0.46)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>13.30</td>
<td>6.87</td>
<td>31.21</td>
<td>80.56</td>
</tr>
<tr>
<td></td>
<td>(2.38)</td>
<td>(2.48)</td>
<td>(2.00)</td>
<td>(1.52)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.90</td>
<td>0.89</td>
<td>0.95</td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Panel B: Euler equation estimates using CGF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(4.70)</td>
<td>(4.31)</td>
<td>(5.07)</td>
<td>(3.94)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>16.16</td>
<td>11.04</td>
<td>37.87</td>
<td>66.20</td>
</tr>
<tr>
<td></td>
<td>(1.24)</td>
<td>(1.25)</td>
<td>(1.26)</td>
<td>(1.10)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.61</td>
<td>0.59</td>
<td>0.62</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Table X:

The Entire Cross-Section of Individual Stocks

The table reports results from Fama and MacBeth (1973) cross-sectional regressions of the log excess returns plus one half their variance of all NYSE, AMEX, and Nasdaq stocks with share prices above $5 on the consumption growth factor-mimicking portfolio \( CGF \) from Table VI Panel A for stockholder, top stockholder, non-stockholder, and aggregate consumption growth that uses the size and value portfolios and assumes constant factor loadings through time. Covariances are estimated using a procedure similar to Fama and French (1992). For each individual stock the covariance of its returns with the \( CGF \) is estimated using the past 24 to 60 months (as available) of monthly log excess returns before July of year \( t \). Stocks are then sorted in June into 100 pre-ranking covariance centiles. The equal-weighted quarterly log excess returns on these 100 portfolios are then computed over the next 12 months, from July to June for every year. Covariances are then reestimated for the portfolios formed from the pre-ranking sorts using the full sample of returns to obtain post-ranking covariances. The post-ranking covariance estimate for a given portfolio is then assigned to each stock in the portfolio, with portfolio assignments updated each June. Every month the cross-section of stock returns in excess of the 30 day T-bill rate plus half their variance is regressed on a constant (not reported) and the post-ranking covariance estimate with the consumption growth factor \( CGF \) in the style of Fama and MacBeth (1973), where the time-series average of the monthly coefficient estimates and their time-series \( t \)-statistics are reported. The variance of individual stock returns is also estimated over the previous 24 to 60 months (as available). For robustness, regression results are also reported that include the beta with the excess return on the CRSP value-weighted index, \( \beta_{RMRF} \), the log of market capitalization (\( ME \)), and the log of the book-to-market equity ratio (\( BE/ME \)) as additional regressors.

**Fama-MacBeth (1973) cross-sectional regressions using \( CGF \)**

<table>
<thead>
<tr>
<th>Coefficient estimates:</th>
<th>( \gamma )</th>
<th>( b )</th>
<th>( s )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stockholders</td>
<td>9.33</td>
<td>0.016</td>
<td>-0.0005</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td>(3.14)</td>
<td>(0.75)</td>
<td>(-1.75)</td>
<td>(1.67)</td>
</tr>
<tr>
<td>Top stockholders</td>
<td>5.64</td>
<td>0.015</td>
<td>-0.0004</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td>(3.34)</td>
<td>(0.70)</td>
<td>(-1.68)</td>
<td>(1.67)</td>
</tr>
<tr>
<td>Non-stockholders</td>
<td>39.18</td>
<td>0.000</td>
<td>-0.0005</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td>(3.26)</td>
<td>(0.01)</td>
<td>(-1.91)</td>
<td>(1.79)</td>
</tr>
<tr>
<td>Aggregate</td>
<td>29.66</td>
<td>0.061</td>
<td>-0.0004</td>
<td>0.0011</td>
</tr>
<tr>
<td></td>
<td>(3.16)</td>
<td>(0.48)</td>
<td>(-1.57)</td>
<td>(1.80)</td>
</tr>
</tbody>
</table>

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Figure 1. Plots of Time-Series Variance of Average Group Consumption Growth Rates Versus Number of Households in the Group

The first figure plots the bootstrap estimated variance of average consumption growth for a group (stockholders, non-stockholders, and top stockholders) against the number of households ($H$) selected in the cross-section of that group for each bootstrap simulation. Appendix B details the calculation of the bootstrap time-series variance of average consumption growth for a group. Results are based on 100,000 simulations. The second figure plots the bootstrap estimated variance of average consumption growth for a group against the reciprocal of the number of households ($1/H$) selected in the cross-section of that group for each bootstrap simulation.
Figure 2. Plot of the Sensitivity of Stockholder, Top Stockholder, and Non-stockholder Long-Run Consumption Growth to Aggregate Long-Run Consumption Growth

This figure plots the sensitivity of stockholder, top stockholder, and non-stockholder long-run (16-quarter, discounted) consumption growth from the CEX to aggregate long-run consumption growth from NIPA.
Figure 3. Consumption Risk and Expected Returns on the 25 Fama-French Portfolios

The graphs plot the average log excess returns plus half their variance of the 25 Fama-French portfolios against the covariance of returns with long-run (16-quarter, discounted) consumption growth for stockholders, the top third of stockholders, non-stockholders, and aggregate. The entire time-series of returns from July, 1926 to November, 2004 is used to estimate mean returns, while covariances are calculated over the CEX sample (January, 1982, to November, 2004). Also reported are the $R^2$s from the cross-sectional regressions. The 25 Fama-French portfolios are labeled as small, growth = 1,1 ... large, growth = 5,1 ... small, value = 1,5 and large, value = 5,5.
Figure 4. Plots of the Consumption-to-Wealth ratio $cay$ and the Stockholder Consumption Share

The figure plots the consumption-to-wealth ratio of Lettau and Ludvigson (2002) along with the ratio of the quarterly consumption of stockholders (using our baseline stockholder definition) to total quarterly CEX consumption. For readability, each time series is standardized by subtracting its mean and dividing by its standard deviation.

![Consumption-to-Wealth Ratio and Stockholder Consumption Share](image)

Correlation = $-0.44$
Figure 5. Consumption Growth Factor-Mimicking Portfolios

The graphs plot the average log excess returns plus half their variance of the 25 Fama-French portfolios against the covariance of returns with the consumption growth factor mimicking portfolio returns ($CGF$) designed to proxy for the long-run (16-quarter, discounted) consumption growth of stockholders, the top third of stockholders, non-stockholders, and aggregate. The entire time-series of returns from July, 1926 to November, 2004 is used to estimate mean returns and covariances. The construction of the factor-mimicking portfolios for consumption growth are described in Table VI. Also reported are the $R^2$s from the cross-sectional regressions. The 25 Fama-French portfolios are labeled as small, growth = 1,1 ... large, growth = 5,1 ... small, value = 1,5 and large, value = 5,5.
Figure 6. Consumption Risk and Expected Returns on 8 Treasury Bond Portfolios

The graphs plot the average log excess returns plus half their variance of 8 Treasury bond portfolios ranging in maturities from 3 months, 1 year, 2 years, 5 years, 7 years, 10 years, 20 years, and 30 years, against the covariance of returns with long-run (16-quarter, discounted) consumption growth for stockholders, the top third of stockholders, non-stockholders, and aggregate. Also reported are the $R^2$s from the cross-sectional regressions. The Treasury bond portfolios are labeled from 1 to 8 in ascending order according to maturity.