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A New Framework for Analyzing and Managing Macrofinancial Risks of An Economy

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Abstract

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of national economic and financial systems. This paper proposes a new comprehensive approach to measure, analyze, and manage macroeconomic risk based on the theory and practice of modern contingent claims analysis (CCA). We illustrate how to use the CCA approach to model and measure sectoral and national risk exposures, and analyze policies to offset their potentially harmful effects. This new framework provides economic balance sheets for inter-linked sectors and a risk accounting framework for an economy. CCA provides a natural framework for analysis of mismatches between an entity's assets and liabilities, such as currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability. It also provides a new framework for sovereign capital structure analysis. It is useful for assessing vulnerability, policy analysis, risk management, investment analysis, and design of risk control strategies. Both public and private sector participants can benefit from pursuing ways to facilitate more efficient macro risk accounting, improve price and volatility discovery, and expand international risk intermediation activities.

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Introduction

Vulnerability of a national economy to volatility in the global markets for credit, currencies, commodities, and other assets has become a central concern of policymakers, credit analysts, and investors everywhere. This paper describes a new framework for analyzing a country's exposure to macroeconomic risks based on the theory and practice of contingent claims analysis.[1] In this framework, the sectors of a national economy are viewed as interconnected portfolios of assets, liabilities, and guarantees, that can be analyzed like puts and calls. We measure the sensitivities of the market values of these portfolios to "shocks" in underlying market risk factors, and we illustrate how to use contingent claims analysis to quantify sovereign credit risk. This framework makes it transparent how risks are transferred across sectors, and how they can accumulate in the balance sheet of the public sector and ultimately lead to a default by the government. CCA provides a natural framework for analysis of mismatches between an entity's assets and liabilities, such as currency and maturity mismatches on balance sheets. The framework also facilitates the quantification of these risk relationships and highlights their non-linear character.

Contingent Claims Analysis

A contingent claim is any financial asset whose future payoff depends on the value of another asset. The prototypical contingent claim is an option – the right to buy or sell the underlying asset at a specified exercise price by a certain expiration date. A call is an option to buy; a put is an option to sell. Contingent claims analysis is a generalization of the option pricing theory pioneered by Black-Scholes (1973) and Merton (1973). Since 1973, option pricing methodology has been applied to a wide

[1] See Merton (1974, 1977, 1992, 1998). When applied to debt and equity securities, contingent claims analysis is commonly called the "Merton Model."

variety of contingent claims. In this paper we focus on its application to the analysis of credit risk and guarantees against the risk of default.

The contingent claims approach is based on three principles: (i) the values of liabilities are derived from assets; (ii) liabilities have different priority (i.e. senior and junior claims); and, (iii) assets follow a stochastic process. The liabilities consist of senior claims (such as senior debt), subordinated claims (such as subordinated debt) and the junior claims (equity or the most junior claim). As total assets decline, the value of risky debt declines, and credit spreads on risky debt rise. (See Annex 1 for more details.)

Balance sheet risk is the key to understanding credit risk and crisis probabilities. Default happens when assets cannot service debt payments. Uncertain changes in future asset value, relative to promised payments on debt, is the driver of default risk. Shocks to flows, prices, or liquidity frequently end up being converted into credit risk in a crisis. But macro models do not handle credit risk well. Financial fragility is intimately related to probability of default. Default is hard to handle in traditional macro models in part due to assumptions which usually exclude the possibility of default. In addition, flow-of-funds and accounting balance sheets cannot provide measures of risk exposures which are forward-looking estimates of losses.

Contingent Claim Balance Sheets for Sectors

We view an economy as a set of interrelated balance sheets with three types of aggregate sectors – corporate, financial, and public sector.[2] The same general principles of contingent claims that apply to analysis of a single firm can also be applied to an aggregation of firms. The liabilities of a firm, a portfolio of firms in a sector, or the public sector (combined government and monetary authorities) can be valued as contingent claims on the assets of the respective firm, sector or public sector. The corporate sector refers to an aggregation of all non-financial firms. A more accurate model of a sector would be CCA models for each individual firm or financial institution and then group them into an aggregate portfolio. Treating the sector as one large firm is simpler and capture certain risk characteristics of the sector for the purposes of this

[2] Gray, Merton, Bodie (2002); Draghi, Merton, Giavazzi, (2002); Gray (2002).

analysis, but data permitting, a portfolio of CCA models of individual firms or institutions provides a richer model.

Governments and central banks typically provide explicit or implicit financial support to large financial institutions in the case of serious deposit runs, illiquidity or insolvency. The financial guarantee from the government is a contingent asset, which is modeled as a put option. Interlinked CCA balance sheets for the corporate sector, the financial sector and the public sector are shown in Figure 1.[3]

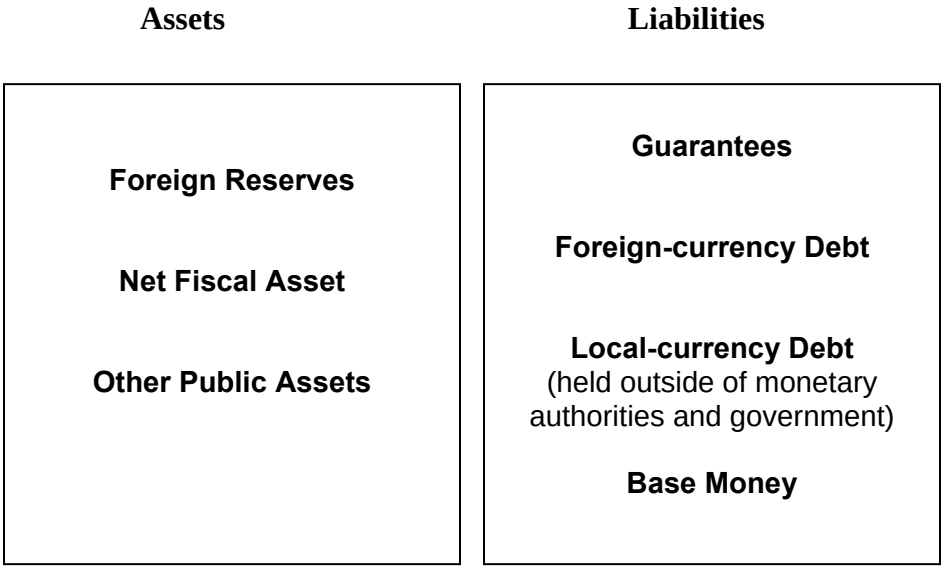
[3] The household sector balance sheet can be added, with household income and assets comprising assets. Household non-discretionary expenditures are the senior liability, debt as a subordinated obligation and discretionary expenditures of households being the junior claim.

Figure 1 – Interlinked CCA Balance Sheets for the Economy

Assets	Liabilities
CORPORATE SECTOR	
Corporate Assets	**Debt** (default-free value minus put option) **Equity** (call option on corporate assets)
FINANCIAL SECTOR	
Loans and other Assets (including loans to corporate sector and public sector) **Financial Guarantees** (modeled as put option)	**Debt / Deposits / Liabilities** (default-free value minus put option) **Equity** (call option on financial sector total assets)
PUBLIC SECTOR (Government and Monetary Authorities)	
TOTAL ASSETS **Foreign Currency** (including contingent foreign reserves) **Net Fiscal Asset and Other Public Assets** **Value of Other Public Sector Assets**	**Financial Guarantees** (modeled as put options related to too-important-to-fail financial and other entities) **Foreign-currency Debt** (default-free value of debt minus put option) **Base Money and Local-currency Debt** Held Outside of the Government and Monetary Authorities (call options on public sector assets)

The public sector balance sheet is an analytical economic balance sheet of the combined government and the monetary authorities.[4] The goal is to construct the liability side of the balance sheet so that the liabilities can be valued and linked to the value of total assets. The combined balance sheet for the public sector is shown in Figure 2. The numeraire can be in local or foreign currency units. The CCA balance sheets for large developed economies with "hard" currencies are measured in units of local currency. The CCA balance sheets of emerging market countries with "soft" currencies are usually measured in a "hard" currency (e.g. US dollar) because it simplifies the analysis and we are interested in valuation and credit risk associated with claims denominated in hard currencies, such as foreign-currency denominated debt.

Figure 2 – Stylized Balance Sheet for the Public Sector



[4] This analytical combined balance sheet includes the monetary authority activities related to foreign currency reserves and "net domestic credit" to government but excludes the direct activities of the

Assets include: foreign currency reserves and contingent foreign currency reserves;[5] net fiscal asset (present value of taxes and revenues, including seignorage, less present value of government expenditures); and other public assets (e.g. equity in public enterprises, value of the public sector's monopoly on the issue of money, and other financial and non-financial assets). Liabilities include: local-currency debt; foreign-currency debt; financial guarantees; and base money. See Appendix 2 for details. It is useful to look into the relationships between items in four categories: fiscal activities, monetary and foreign currency reserve activities, risky debt, and financial guarantees.

Fiscal Assets and Liabilities – In this framework, the items related to fiscal assets and liabilities are taxes, revenues and expenditures. Expenditures can be divided into non-discretionary expenditures which are senior claims, and discretionary expenditures which are junior claims. Non-discretionary expenditures are core expenditures (e.g., defense, education, core infrastructure, welfare, etc.) that will not be given up before giving up on paying the debt. Operationally, discretionary expenditures are ones that are subordinated to the explicit liability claims against the government. Discretionary expenditures may become especially significant in situations of high fiscal revenue such as windfalls from oil or natural resources.

Under stress situations, the government maintains the non-discretionary expenditures and cuts the discretionary expenditures. Under these assumptions, we can subtract the present value of non-discretionary expenditures from the present value of tax capability to obtain the *net fiscal asset*,[6] given that non-discretionary expenditures are clearly senior claims and have the same maturity patterns as taxes and fiscal revenues. The *net fiscal asset* is thus similar to the present value of the primary fiscal surplus over time (the present value of fiscal surplus minus interest payments). This step also simplifies the process of constructing the CCA balance sheet because it is much easier to

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monetary authority with the banking sector, such as credit and liquidity support activities that do not go through the government balance sheet or affect foreign exchange reserves.

[5] The total foreign reserves of the public sector include actual reserves plus *contingent* reserves from international financial institutions, such as the IMF, other governments or contingent credit lines.

[6] The value of assets of an operating firm can be considered as the present value of stochastic future cash flow from income minus net new investment expenditures to create that income. For the public sector, the net fiscal asset is the present value of stochastic future fiscal flows from taxes and revenues minus non-discretionary expenditures equivalent to the present value of the primary fiscal surplus.

obtain market values for the other non-expenditure related liabilities, as will be discussed in more detail later.

Monetary and Foreign Reserve Assets and Liabilities – Base money is a liability of the monetary authorities and thus a liability of the public sector.[7] Base money consists of currency in circulation, bank reserves (required reserves, excess reserves, vault cash). The counterpart of base money liabilities are the assets of the monetary authorities net foreign assets and net domestic assets (including credit to government less government deposits, claims on banks and other items). Changes in base money correspond to changes in net foreign assets and net domestic assets.

Sovereign Local Currency Debt Liabilities – Local-currency debt of the government is a claim on sovereign assets. On the combined balance sheet of the public sector, the local-currency debt is the portion of debt held outside of the monetary authorities and the government.

Sovereign Foreign Currency Debt Liabilities – Foreign currency denominated debt is risky debt includes foreign-currency debt.

Financial Guarantees – As described earlier, implicit or explicit guarantees to "too-important-to-fail" banks and other financial institutions or pension obligations are liability items on the public sector's balance sheet which are modeled as put options.

Base money does not pay a "dividend"; it provides a convenience yield of money for transactions. The quantity of base money can be increased with subsequent consequences for inflation. Sovereign local currency debt is a claim on the public sector balance sheet, paying a "dividend" equal to the promised interest payment. A risk associated with holding sovereign local currency debt is that the government may dilute (or inflate away) part of the value or the debt, or may forcibly restructure some of the debt. The "dilution/inflation risk premium" is an extra premium demanded by the holders of local currency debt.[8]

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[7] Base money is also known as high-powered money or reserve money. As is the common practice, it is the main liability of the monetary authorities (IMF, 2000, Buiter, 1993, Blejer and Schumacher, 2000). Base money is "multiplied" by the banking system; the multipliers relate base money to M1, M2, etc. When a country joins a currency union (i.e. merges with another sovereign or dollarizes) base money is exchanged for foreign currency reserves.

[8] See Gray, Lim and Malone for more details.

Measuring Implied Asset Value and Volatilities Using Market Prices

From the observed prices and volatilities of market-traded securities, one can estimate the implied values and volatilities of the underlying assets.[9, 10] These implied asset values and asset volatilities can be used to calibrate the pricing and risk model of major sectors in the economy.

Domestic equity markets provide pricing and volatility information for the calculation of corporate, bank and non-bank financial assets, and asset volatilities. The Merton model is widely used to estimate implied assets and asset volatility for firms and financial institutions with traded equity. The method uses solves two equations for two unknowns, asset value and asset volatility (details in Annex 1, Merton (1974), KMV (1999), and Crouhy et. al. 2000).[11]

For the sovereign balance sheet, the prices in the international markets (including foreign currency market, debt market, and credit derivatives market), together with information from domestic market prices, provide the market information for the value and volatility of liabilities on the public sector balance sheet.[12] This information can be used to calculate implied asset values, volatilities, and higher moments of implied asset distributions for the sovereign (details are in Annex 2). Applications to a wide range of countries is described in Gray (2001 and 2002), Gapen et. al. (2004 and 2005), and IMF (2006). The key sectors of and economy can be calibrated and linked into an economy-wide CCA balance sheets framework. Subsequently, we can do "forward" simulations to estimate the impact of "shocks" and policy changes on the economic and financial system.[13]

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[9] An implied value refers to an estimate derived from other observed data. Techniques for using implied values are widely practiced in options pricing and financial engineering applications.

[10] See Bodie and Merton (1995).

[11] The CCA approach is used to calibrate balance sheets for listed corporates and banks. For unlisted corporates and banks, information from balance sheets is used along with proxies and comparables from CCA type models of similar firms in the same sector. In the household sector, data permitting, the portfolio of assets is constructed (pension, mutual funds, deposits, PV of labor income and other estimated assets) and the CCA model is used to get implied household net worth and its volatility.

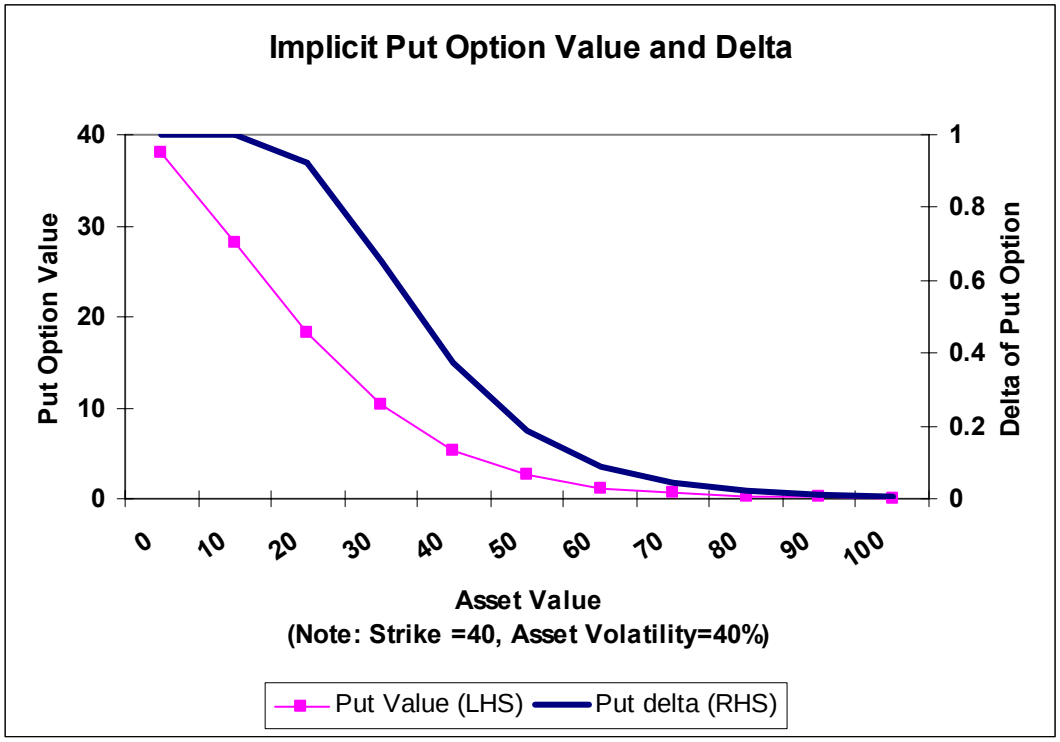
[12] Gray, 2000, 2001.

[13] The MF Risk® models use simulated shocks and policy adjustments to assess impact on risk indicators.

Measuring Risk Exposures

So far, we have discussed how to calculate the value of debt, guarantees, and equity using the CCA approach. We now turn to how to measure the risk exposures. The values of the contingent claims on the CCA balance sheets contain embedded implicit options which can be used to obtain certain risk measures. These include risk exposures in risky debt, probabilities of default, spreads on debt, the sensitivity of the implicit option to the underlying asset (the delta), sensitivity to other parameters, distance to distress, value-at-risk and other measures. The implicit put option changes in a non-linear way as the underlying asset changes. The slope of the put option vs. asset is the sensitivity of the put option value to the underlying asset value which is the delta. The delta measures the change in the value of an option per unit change in the value of the underlying asset. For example, the government's exposure to its guarantee to the banking sector changes as banking assets change. Figure 3 illustrates the value of the implicit put option and the delta for the implicit put option plotted against the banking system asset. This is simply (the absolute value of) the slope of the tangent to the function defining the value of the option at any point. The implicit put option can be the risk exposure that the holders of debt of a sector have, e.g. holders of sovereign debt. The implicit put option can also be a measure of the government's exposure, acquired through implicit or explicit guarantees, if the government provides such guarantees. The implicit put option increases in a non-linear way as the market value of the sector's assets decline.

Figure 3:


Implicit Put Option Value and Delta
Put Option Value
Delta of Put Option
40
30
20
10
0
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
Asset Value
(Note: Strike =40, Asset Volatility=40%)
Put Value (LHS)
Put delta (RHS)

Linkages in a Simple Three-Sector Framework

To show how we can apply the CCA framework, we focus on a simplified model with three aggregate sectors – corporate, financial, and public sectors. In this model, the corporate sector's liabilities include bank loans which are the banking sector's assets. The system's financial stability depends on the government's financial guarantee to the banks.

The debt of the corporate sector can be described as default-free debt combined with a short of a put option on corporate assets. The economic balance sheet of the banking sector has assets consisting of corporate loans (default-free debt minus the value of a put option). The banking sector also includes guarantees from the government as an asset, which is a liability on the government's economic balance sheet (Figure 4).

Figure 4 Balance Sheets for Simple Three-Sector Framework

Corporate Sector Balance Sheet

Assets	Liabilities
Corporate Assets	Debt (=Default-free value of debt minus implicit put option)
	Equity (Implicit call option)

Banking Sector Balance Sheet

Assets	Liabilities
Loans (Debt of Corporate Sector)	Debt
Other Assets	Deposits
Financial Guarantee (Implicit Put Option)	Equity (Implicit call option)

Public Sector Balance Sheet

Assets	Liabilities
Foreign Reserves	Financial Guarantee (Implicit put option)
Net Fiscal Asset and Other Assets	Foreign Debt (Default-free value of debt minus implicit put option)
Value of Monopoly on Issue of Money	Base Money and Local-currency Debt (Implicit call options)

These three economic balance sheets demonstrate the interdependence among sectors; with one sector "long" a certain implicit option and another sector "short" the same implicit option. The sector CCA balance sheets can be integrated together as shown in Figure 5. Each sector contains assets adjusted for guarantees and when the junior claims/equity and risky debt are subtracted the net is zero (columns). When shocks affect the corporate sector it feeds into the financial sector and then could transmit risk to the government. The sum of all positions can be calculated and broken down into the portfolio of the claims of foreigners and other domestic residents. The foreigner's portfolio of claims is the value of what could be viewed as the *present value of the risk adjusted current account.* The framework provides a relative valuation tool for market and credit risk within sectors (e.g. sovereign foreign debt/CDS, local debt, foreign exchange instruments, and interest rates) and across sectors (e.g. sovereign debt, exchange rates, stock market index and banking sector equity or deposits). The economic balance[14] sheets for each of the three sectors with illustrative numbers are shown in Figure 6 and in Annex 3.

[14] The economic balance sheet is the "mark-to-market" balance sheet of the sector's assets and liabilities, including the economic values of other relevant contingent assets and contingent liabilities. This is in contrast to a traditional GAAP accounting balance sheet. For example, the government financial guarantee to the banking system is not a GAAP entry.

Figure 5 Economy-Wide CCA Balance Sheet Matrix

CCA Balance Sheet (billion US$)

	Sovereign	Banking Sector	Non-Bank Financial	Corporate	Horizontal Sum**
Asset without Guarantee	$V_{Sovereign}$	V_{Banks}	$V_{NBank\ Financial}$	$V_{Corporate}$	Σ V
Implicit Guarantee	minus G	+G	+G	+G	0
Asset (including Guarantee)	$A_{Sovereign}$	A_{Banks}	$A_{NBank\ Financial}$	$A_{Corporate}$	Σ A
Jr Claim or Equity	Jr Claim	E_{Banks}	$E_{NBank\ Financial}$	$E_{Corporates}$	Σ (J+E)
Default-free Debt Value	B_S	B_B	B_{NBF}	B_C	Σ B
Expected Loss	EL	EL	EL	EL	Σ EL
Risky Debt (Default-free - EL)	B_S-EL	B_B-EL	B_{NBF}-EL	B_C-EL	Σ Risky Debt
Assets minus Liabilities*	0	0	0	0	0
Risk Indicators					
Distance to Distress	$D2D_S$	Ave $D2D_B$	Ave $D2D_{NB}$	Ave $D2D_C$	
Default Probability	DP_S	Ave DP_B	Ave DP_{NB}	Ave DP_C	
Spread (BPS)	Spr_S	...	...	...	

**Equals Asset + Guarantee - Jr Claim - (Default-free Value of Debt minus EL)*

*** Can be broken down into risk adjusted positions of residents and foreigners*

Integrated Value and Risk Transmission between Sectors

The framework described above is versatile and can be used to understand many types of crises and risk shifting that cannot as easily be analyzed with other techniques. The risk-transmission patterns can be dampened or may be magnified depending on the capital structure and linkages. The framework can help identify situations where volatility gets magnified and negative feedback loops that can trigger severe crises. The patterns of value and default correlation across different asset classes, sectors and sovereign debt values depend on these structures and links, unique to a particular economy. An example of inter-linked sectoral economic balance sheets is shown in Figure 6.

Figure 6 Example Inter-linked Sectoral Balance Sheets (Base Case)

(Units are in Foreign Currency, billion US $)

Corporate Sector Balance Sheet

Assets		Liabilities	
Corporate Assets	120	Loans (Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8)	87.2
		Corporate Equity (call option)	32.8
Total	120		120

Banking Sector Balance Sheet

Assets		Liabilities	
Loans (90 -2.8)	87.2	Deposits	81.3
Financial Guarantee	7.4	Equity	13.3
Total	94.6		94.6

Public Sector Balance Sheet

Assets		Liabilities	
Domestic assets	100	Financial Guarantee to Banks	7.4
Foreign reserves	40	Foreign Debt	82.15
		Local-currency Debt & Other Liabilities	50.45
Total	140		140

Note: The *delta* of the public sector financial guarantee is -0.35 in this base case.

Actual risk transfer in an economy is likely to involve several risk-transmission channels. Below are some examples of risk transmission between sectors. Annex 3 shows example changes to base case balance sheet (Figure 6) for different types of risk transmission.

Risk Transmission from the Corporate Sector to the Banking Sector and to the Government

The corporate sector's financial distress – possibly caused by stock market declines which reduce the value of corporate assets, recession, commodity price drops, or excessive unhedged foreign debt accompanied by currency devaluation – can be transmitted to the financial sector.

Risk Transfer

Corporate Sector → **Banking Sector** → **Government**

We can use the three-sector framework to show how the risk can be transmitted from the corporate sector to the banking sector and to the public sector through implicit and explicit guarantees. The example of a negative shock to the corporate sector is a drop in assets as a result of recession, equity sell-offs, the combination of currency devaluation and unhedged foreign debt, or other negative shocks (shown in Annex 3, Figure A3-2). The value of the assets of the corporate sector declines, so does the value of the debt (and equity) which leads to a decline in bank assets and an increase in the implicit government guarantee. As the corporate assets decline, the government guarantee to the banking sector increases in a non-linear way.

Risk Transmission from the Banking Sector to the Government

The banking sector's financial distress, such as systemic banking crises due to deposit runs, a decline in asset value or mismanagement, can be transmitted to the government through guarantees.

Risk Transfer

Banking Sector → **Government**

The example of the impact of a deposit run on the balance sheets of the three-sector model in Annex 3, Figure A3-3 shows that the banking sector's default barrier

rises, when the banking sector faces a large deposit run resulting in a large increase in the implicit guarantee.

Risk Transmission from the Government to the Banks and Feedback

The public sector's financial distress or default can transmit risk to the financial system. When the banking sector is holding a significant proportion of government securities, and there is a negative shock to the government financial position, it can have a detrimental impact on the banks. The government's implicit guarantee is also likely to increase. This, in turn, makes the government financial position worse, creating a compounding effect, which may result in the government's failure to honor its guarantee obligations and cause a collapse of the banking system.

Risk Transfer

Banking/Financial System ⇄ **Government**

Figure A3-4 in Annex 3 shows the impact of a decline in government assets resulting in lower value of sovereign debt relative to the base case. In Figure A3-5, we assume the same decline in government assets but that the banking sector's assets consist of *half* government securities and *half* loans to the corporate sector (as compared to 100% corporate loans in the base case). The vicious cycle could arise, when the lower value of government securities lowers bank assets, and raises the implicit financial guarantee, which in turn lowers government assets further. This means that the implicit guarantee is higher than what is shown above. In some situations, this vicious cycle can spiral out of control, eventually resulting in the inability of the government to provide sufficient guarantees to banks and leading to a systemic financial crisis.

Risk Transmission from the Pension System to the Government

The financial distress related to pension plans can result in the transmission of risk to the government.

Risk Transfer

Pension System → **Government**

Figure A3-6 shows an example of this type or risk transmission. We assume that the pension system contains one-half of corporate sector equity (in a defined benefit plan which has an implicit government guarantee). A decline in corporate assets would cause the corporate equity value to drop. This, in turn, increases the government guarantee to the pension system and the implicit guarantee to banks.[15]

Risk Transmission from the Public Sector to Holders of Public Sector Debt

Fiscal, banking and other problems can cause distress for the government which can transmit risk to holders of government debt.

Risk Transfer

Public Sector ⟶ Debt Holders (sovereign foreign currency denominated debt or sovereign local currency denominated debt)

Holders of foreign-currency debt have a claim on the value of the debt minus the potential credit loss, which is dependent on the level of assets of the public sector (in foreign currency terms) compared to the foreign-currency default barrier. Thus, we can use the CCA approach to analyze the value of public sector foreign-currency debt by comparing how the volatility of the public sector assets (measured in US dollar terms) changes relative to the foreign-currency default barrier. A large component of the spread on sovereign foreign-currency debt is the credit spread to compensate for the risk of default over the horizon. The credit spread on sovereign foreign-currency debt is a function of: (i) the ratio of sovereign asset, A, to the default barrier, DB_F (associated with default free debt value of foreign debt; (ii) the volatility of sovereign assets, σ_A; and, (iii) horizon and risk-free interest rate.[16] As the term (A/ DB_F) declines and/or σ_A increases, the spread increases in a *non-linear* way and eventually turns sharply higher. The total public sector asset includes foreign currency reserves, the net fiscal asset, and the value

[15] See Bodie 2006..

[16] Spread = $- 1/T \ln[N(d_2) + (A/ (DB_F e^{-rt})) N(-d_1)]$, see Annex 1.

of seigniorage in US dollar terms. Thus a decline in foreign currency reserves, lower fiscal revenues, and/or a rise the foreign debt default barrier will raise spreads.[17]

The value of (risky) local currency debt is influenced by the risk that the government may dilute (or inflate away) part of the value or the debt, or may forcibly restructure some of the debt. The "dilution/inflation risk premium" is an extra premium demanded by the holders of local currency debt.

The volatility of the public sector asset is heavily influenced by exchange rate and fiscal volatilities. In the crisis periods, the fiscal volatility and exchange rate volatility can combine to produce a higher volatility of the sovereign asset. This means that the risk premium on local currency debt is very likely to be higher and lead to an increase sovereign spreads on foreign currency debt. A stylized distress scenario for an emerging market is a decline in the sovereign asset, rolling over local currency debt which becomes more difficult as the holders of the local currency debt demand a higher premium, likely monetization of the deficit leading to higher inflation and depreciation of the exchange rate. This lower foreign currency value of sovereign assets and higher volatility increases spreads on foreign currency debt as default probability can increase. A sovereign can, in principle always issue more money but foreign currency cannot be printed. This is somewhat analogous to a firm that can dilute stock holders, e.g., stock splits, and issue shares but cannot print hard cash needed to service debt.

[17] See IMF GFSR April 2006, Box 3.6 for sovereign CCA and impact of changes in debt structure.

Interrelationship of Macro Financial Contingent Claim Balance Sheets, Risk Exposures and Traditional Macroeconomic Flows

The combined accounts – income/flow, mark-to-market balance sheets, and risk exposure measures – comprise the three important sets of interrelated accounts in the economy which are somewhat similar to those in large modern financial institutions. Risk managers would find it difficult to analyze the risk exposure of their financial institution by relying solely on the income and cash flow statements, and not taking into account (mark-to-market) balance sheets or information on their institution's derivative or option positions. The country risk analysis that relies only on macroeconomic flow-based approach is deficient in a similar way, given that the traditional analysis does not take into account the volatility of assets. Note that when the volatility of assets in the CCA balance sheet equations is set to zero the values of the implicit put and call options go to zero. Something very similar to the traditional macroeconomic flow of funds is the result since the change in assets is equal to changes in cash and book value of debt. Flow of Funds can be seen as a special deterministic case of the CCA balance sheet equations when volatility is set to zero and annual changes are calculated. The risk transmission between sectors is lost.

Controlling and Transferring Risk

The application of CCA to analyze risk exposures in the sectors of an economy offers a rich framework for comparing alternative ways to control and transfer risk. There are several benefits. First, it gives the interrelated values and risk exposure measures across sectors. Understanding of these values and risk exposures can help identify particularly vulnerable situations and potential chain reactions of default. This allows formulation of various alternative ways to control and transfer risk. Second, the framework dovetails with risk-management strategies involving *explicit derivatives and swaps* used by the private and public sectors to control, hedge or transfer risk.[18]

Four broad categories of strategies are: a direct change in the financial structure (the structure of assets and liabilities within the existing institutional structure); managing

[18] One example, in Blejer and Schumacher (2000), includes central bank forward contracts.

guarantees; risk transfer (diversification, hedging and insurance); and, in the longer run, an institutional change to tailor the institutional structure to fulfill financial functions more efficiently within the specific geopolitical environment.

Direct Change in Financial Structure

Increases in assets and declines in default barriers can reduce the vulnerability to distress, reduce spreads on debt, and reduce the value and the deltas of put options (whether they are embedded in risky debt or financial guarantees from the government). CCA, by its nature, shows how the changes in value of assets relate to changes in values of liabilities. Thus, it provides a natural framework for analysis of mismatches, such a currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability.

Management of Guarantees

CCA provides the key to measuring the value and understanding the risk of guarantees. The three basic methods that a guarantor of liabilities has to manage the risks of guarantees are:

Monitoring – The method requires frequent marking-to-market of assets and liabilities of the insured party and collateral that can be seized when the insured party's assets fall below a predetermined target.

Asset Restrictions – This method of controlling costs and managing the cost of the guarantee requires the insured party to (at least partially) hedge its guaranteed liabilities with restrictions on assets in a manner that limits the volatility of net worth.

Risk-based premiums – Under this method, the guarantor charges a fee that is commensurate with the riskiness of the guarantee.[19]

Guarantees on the debt of financial institutions, whether explicit or implicit, should be openly recognized in the government's balance sheet. This framework provides a way to measure the value of the guarantee and the risk exposures associated with the guarantee.

[19] See Merton and Bodie (1992, 1993) and Bodie and Merton (1993).

Risk Transfer

There are three ways to transfer risk, *diversification, hedging and insurance*. Much of the risk described here results from concentration risk and diversification to parties who have a comparative advantage in bearing various risks. If the balance sheets of corporations and financial institutions are weak when the economy is weak – as it is generally the case – then it is precisely when tax revenue is low, and the cost of debt service is high because sovereign risk has increased. In this case, the value of the guarantees will be particularly high. This observation offers a powerful argument for diversification of the government exposure to local shocks (see Box 1).

The financial markets, especially in emerging markets, are often "incomplete", meaning that they provide only limited possibilities to shift risk across various entities and groups. In such situations, diversification through international capital mobility is the obvious alternative. However, the transfer across borders of the ownership of real and financial assets is a rather inflexible way to achieve diversification (as it is costly to reverse); often it also runs against political constraints.

Box 1

Examples of Diversification, Hedging and Risk Mitigation

- *Asset Diversification in Banking Sector* – Asset diversification would suggest that a bank which invests part of its assets in domestic government bonds enhances its exposure to local macro shocks; the value of government bonds will be low precisely when the value of the loan book is low. Therefore, in such economies, banks should hedge the exposure of their loan book by investing in non-domestic assets—such as bonds.
- *Equity Swaps as a Method of Diversifying Internationally* – An equity swap would enable a small country to diversify internationally without violating possible restrictions on investing capital abroad. Suppose that small-country pension funds who already own the domestic equity were to enter into swaps with a global pension intermediary (GPI). In the swap, the total return per dollar on the small country's stock market is exchanged annually for the total return per dollar on a market-value weighted-average of the world stock markets. The swap effectively transfers the risk of the small-country stock market to foreign investors and provides the domestic investors with the risk-return pattern of a well-diversified world portfolio. Since there are no initial payments between parties, there are no initial capital flows in or out of the country. Subsequent payments, which may be either inflows or outflows, involve only the *difference* between the *returns* on the two stock market indices, and no "principal" amount flow.
- *Contingent Reserves or Contingent Sovereign Capital* – Corporations sometimes contract for contingent equity or debt purchases triggered under pre-agreed conditions. Similarly, governments could make arrangements with external public or private sector entities for pre-agreed purchase of government local-currency debt under specific circumstances such as a sudden stop in capital flows or certain revenue losses, commodity price drops or natural disasters. The value of such contingent capital can be compared to the costs of increasing paid-in capital reserves via debt issues. This macrofinance framework could be used to do value-at-risk for the sovereign balance sheet to help determine the appropriate level of foreign currency reserves and contingent reserves or contingent sovereign capital.
- *Sovereign Bonds with Special Features* – GDP-linked bonds or bonds with specific roll-over clauses can help manage risk. Indexed bonds. Commodity linked bonds linked to major exports such as oil or copper. Catastrophe bonds (CAT) and similar instruments.
- *Diversification and Hedging Related to Management of Foreign Reserves* – A sovereign holds foreign currency reserves, in part, to as a cushion against potential losses of the monetary authorities or government. The framework described here can be used to assess the costs of increasing reserves via issue of foreign debt, local currency debt, money or contingent capital contracts against the benefits of having a cushion to mitigate losses (e.g. mitigate the risks posed by the implicit put options associated with government foreign and local currency debt and contingent obligations of the monetary authority). If there are excess reserves, the framework here could be used to assess the investments or strategies that provide the likely optimal hedging, diversification or risk mitigation tailored for the specific risk characteristics of the country's sovereign balance sheet.
- *Others* – Other types of swaps could include assets, equity, or debt of the corporate sector, the financial sector, and the public sector. Weather derivatives. Credit derivatives. Positions taken by various public organizations to lay-off risk in adverse circumstances and/or or to mitigate tax revenue and spending risks.

The macro finance analytical framework could be useful for the design of new risk intermediation and risk transfer products, whereby various risks in one economy could be packaged and sold internationally to improve the efficiency of risk sharing and enhance returns.

Institutional Change

In the longer-term, institutional changes to satisfy certain financial functions more efficiently can reduce risk. In the case of the banking sector, this is particularly important, given the vulnerabilities and costs of crisis in the banking system. Fiscal costs of banking crisis show no sign of declining and can range from costs of 3% to 80% of GDP, not to mention the inefficiencies caused before, during and after the crises. The potential for very costly government guarantees to the banking system, which can arise quickly and can have large associated risk exposures and costs, support the arguments *that it may be best to safely shrink the banking system*. [20] Structural reform, over time, could aim to reduce the size of the banking system and increase the role of institutions that can fulfill the key functions of banks (payments functions and pooling and investment of resources) but do so in an efficient and less risky manner.

The combination of a smaller banking system, improved management of guarantees, equity swaps between the pension system and international counterparties, and direct change in the financial structure would dramatically reduce risk exposures and systemic vulnerability.

[20] Posen, A. 2001, "*A Strategy to Prevent Financial Crises: Safely Shrink the Banking Sector*"

Conclusions

The high cost of international economic and financial crises highlights the need for a comprehensive framework to assess the robustness of countries' economic and financial systems. This paper proposes a new approach to measure, analyze, and manage macroeconomic risk based on the theory and practice of modern contingent claims analysis (CCA). We illustrate how to use CCA to model and measure sectoral and national risk exposures, and we analyze policies to offset their potentially harmful effects. The framework provides economic balance sheets for inter-linked sectors and a risk accounting framework for an economy. It provides a new framework adapting the CCA model to the sovereign balance sheet which can help forecast credit spreads and a framework for relative valuation of credit and market risks for the sovereign and across economic sectors. CCA provides a natural framework for analysis of mismatches between an entity's assets and liabilities, such as currency and maturity mismatches on balance sheets. Policies or actions that reduce these mismatches will help reduce risk and vulnerability. It is useful for assessing vulnerability, policy analysis, risk management, investment analysis, and design of risk control strategies. Both public and private sector participants can benefit from pursuing ways to facilitate more efficient macro risk accounting, improved price and volatility discovery, and expanding international risk intermediation activities.

Annex 1 – Framework for Contingent Claims Analysis, Risk Measures, and Spreads Using Black-Scholes-Merton Formula

Black-Scholes-Merton Equations for Pricing Contingent Claims

CCA defines these fundamental relationships between the value of assets and the value of claims. The total market value of assets, A(t), of an entity financed with debt and junior claims (most junior claim or equity) is equal to the market value of junior claims and market value of risky debt, J(t) + D(t). Assets are stochastic and thus assets, at time t, in the future may decline below the point where debt payments on scheduled dates cannot be made. The junior claim (equity in the case of firms) can be viewed as an implicit call option.

Assets = Junior Claim + Risky Debt
= Junior Claim + Default-Free Debt – Debt Guarantee

The value of the junior claim, the debt guarantee embedded in the value of risky debt, and the financial guarantee can all be formulated in terms of *implicit options* (Merton, 1974). The value of "risky" debt is the default-free value of the debt minus the debt guarantee.

Risky Debt = Default-Free Debt – Debt Guarantee
= Default-Free Debt – Implicit Put Option

Financial Guarantee = Implicit Put Option

The implicit put option is $P = Be^{-rt}(N(-d_2)) - A_0(N(-d_1))$. The value of the risky debt, D, is thus the default-free value minus the expected loss:
$D = Be^{-rt} - P = Be^{-rt} - (Be^{-rt}N(-d_2) - A_0N(-d_1))$. The value of the junior claim, J, is equal to the value of call option, $J = A_0\mathrm{N}(d_1) - Be^{-r_fT}\mathrm{N}(d_2)$

where $d_2 = \dfrac{\ln\left(\dfrac{A_0}{B}\right) + \left(r_f - \dfrac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}$, and $d_1 = d_2 + \sigma_A\sqrt{t}$, r is the risk free rate and t is the horizon period. N(d) = the probability that a random draw from a standard normal distribution will be less than d.

$N(-d_2)$ is the risk-neutral default probability and $N(-d_2^*)$ is the "real world or physical" default probability. $N(-d_2^*) = N(-d_2 - \lambda\sqrt{t})$ where λ is the market price of risk. The credit spread, s, is the premium required to compensate for the expected loss. The get the formula for the spread note that the yield-to-maturity for the risky debt D is y_t, then

$\exp(-y_t t) = \frac{D}{B} = \frac{Be^{-rt} - P}{B}$ This can be rewritten to get the spread, s.

$$s = y_t - r = -\frac{1}{t}\ln\left(1 - \frac{P}{Be^{-rt}}\right) = -\frac{1}{t}\ln\left(N(d_2) + \left(\frac{A}{Be^{-rt}}\right)N(-d_1)\right)$$

The delta of the put option is $N(d_1) - 1$

Example: Assuming that:

Asset value A=$100,
Asset return volatility of σ = 0.40 (40%),
Default-free value of debt = default barrier = DB = $ 75
(derived from short-term debt, $30, plus one-half of long-term debt, $90)
Risk-free rate = 0.05 (5%)
Time horizon = 1, one year

The value of the junior claim/equity is $32.2 and the value of risky debt is $67.8 (equal to the present value of the default barrier minus put option = 75*0.95 – 3.55 = 67.8). The "delta" for the call option is $N(d_1)$, is 0.89 in the above example. The "delta" for the put option (implicit guarantee) is $N(d_1) - 1$, or – 0.11 in the example. Using the spread formula above, the one-year spread for the example is calculated as: 0.0510, or 510 basis points over the risk-free rate. The *probability of default*, using this model, is $N(-d_2)$, or 0.20 (20%) in the example above.

Calculating Implied Assets and Implied Asset Volatility for Firms, Banks, Non-bank Financials with traded equity

In the Merton model for firms, banks and non-bank financials with traded equity the following two equations are used to solve for the two unknowns A, asset value, and σ_A, asset volatility.

$$J = A_0 \mathrm{N}(d_1) - Be^{-r_f T}\mathrm{N}(d_2)$$

$$A\sigma_A = J\sigma_J \frac{\partial J}{\partial A} = J\sigma_J N(d_1)$$ [21]

[21] For a recently published book explaining these concepts applied to credit risk, see Crouhy, Galai, and Mark, 2000.

Annex 2 – Public Sector CCA Balance Sheet and its Calibration Using the Contingent Claims Approach

This Annex describes a framework how the segregated contingent claim balance sheet of the monetary authorities and the government can be combined together and how the implied sovereign assets and asset volatility can be calculated and risk indicators estimated.[22] Useful insights can be obtained when one views relationship between the assets and liabilities of the public sector[23] in a similar way as separate balance sheets of the government and monetary authorities where there are cross-holdings and financial guarantees between these two public sector "partners." Under this structure, the assets of the monetary authority include foreign reserves, credit to the government and other claims. The liabilities of the monetary authority partner are base money and financial guarantees to the government, including guarantees to supply foreign currency to service the sovereign foreign-currency denominated debt. The assets of the government partner include the net fiscal asset, other assets, while the liabilities include credit to the monetary authority (and could include local-currency debt held by the monetary authority) , local-currency debt held outside of the government and monetary authority, financial guarantees and foreign currency denominated debt.

Figure A2-1 shows the structure of this segregated balance sheet structure. This simplified framework is not meant to be a comprehensive catalogue of all the guarantees, the nature of which varies by country and by the detailed structure of the relationship between monetary authorities and the government. There may also be implicit financial support from the monetary authorities to the government via purchase of government local-currency debt under certain circumstances, but this is not shown here. The action of the monetary authority "partner" of buying additional government local-currency debt entails issue of additional base money. There are also "options," that the government and the monetary authorities have to "default" on the obligations to convert local currency into foreign currency. Similarly the government could "forcibly" restructure local-currency debt or to dictate "mandatory" purchases of government bonds by certain public or private institutions or the option to inflate to cover potential shortfalls. Also, in some countries, banks may have deposits with the monetary authorities that receive a higher priority claim on foreign currency reserves than the holders of local currency, which could be junior to claims on foreign currency for payment of external foreign-currency debt.

[22] See Gapen, Gray, Lim, Xiao, 2005.
[23] See Buiter, W. 2000.

Figure A2-1 – Segregated Balance Sheet for the Public Sector

Assets	Liabilities
MONETARY AUTHORITY "PARTNER"	
Foreign Reserves ***Credit to Government*** **Credit to other Sectors**	***Obligation to supply FX to Government to pay FX Debt*** **Base Money**
GOVERNMENT "PARTNER"	
Net Fiscal Asset **Other Public Sector Assets** ***Obligation from Monetary Authority to supply FX to Government to pay FX Debt***	**Guarantees (to too-important-to-fail entities)** **Foreign-currency Debt** **Local-currency Debt Held Outside of the Government & Monetary Authorities** ***Credit from Monetary Authorities***

Note that the cross-holdings of government debt and guarantees from monetary authorities to Government are in italics. Liquidity operations of the monetary authorities are not included.

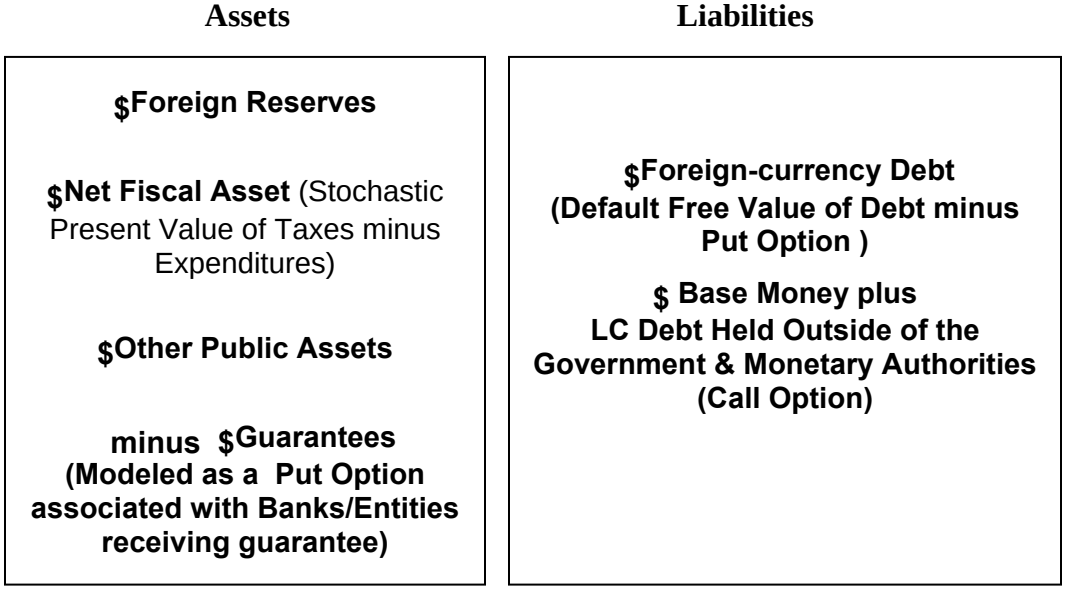
The priority of the claims on the assets certainty can vary from country to country. In many cases, though, we can think of the guarantees to banks or other "too big to fail entities" as senior claims. Also most governments find it easier to inflate or dilute local currency debt in a distress situation before defaulting of foreign currency debt. Thus a case can be made that foreign currency debt is senior to local currency debt. The government may certainly take the view that credit from the monetary authorities is the most junior obligation and many governments may or may not honor that claim. The credit from the monetary authorities is an asset on the side of the monetary authority

partner and a liability of the government partner. Similarly, the financial guarantees to the government partner are an asset on its balance sheet and a liability of the monetary authority partner. When the balance sheets are combined these two items drop out. The segregated balance sheet above reduces to the combined balance sheet in Figure 1. Contingent claims approach can be applied to the segregated or the combined balance sheets, the choice of which depends on the purposes of the analysis.

Calculating Implied Sovereign Assets and Implied Sovereign Asset Volatility Using CCA for the Public Sector Balance Sheet

This section describes how the CCA framework can be used to estimate implied assets and asset volatility, since the market value of sovereign assets cannot be observed directly. The public sector balance sheet in Figure A2-2 has liabilities structured in a way that we can observe the market value of the junior claims and the distress barrier of foreign currency debt so as to be able to adapt the Merton model to the sovereign. One adjustment needed is to subtract the "senior" guarantee to too-big-to-fail entities from both sides the balance sheet as shown in Figure A2-2. On the simplified balance sheet, the local-currency debt of the government, held outside of the monetary authorities, and base money are local currency liabilities which can be modeled as a call option on the public sector assets with the default barrier derived from the foreign-currency debt.

Figure A2-2 Example Public Sector Contingent Claims Balance Sheet with Liabilities Modeled with Options (all items in $ terms)



The units in which the balance sheet is measured can be nominal local currency units, in real terms in local currency units, or in foreign currency. Any numeraire can be used. Since one of the goals of the analysis here is to analyze the value of the CCA

balance sheet in the international context (including effects of changes in FX reserves and the credit risk embedded in foreign debt), the units are in US $.

Local currency debt is a claim on sovereign assets whose value, in principle, can be diluted or inflated. Money is a claim; its issue can be used to increase sovereign assets, via increase in reserves or credit to government. It can in most cases, be exchanged for foreign currency and in the event of dollarization reserves are used to buy out the monetary base. In most cases it is a long-term claim and in that sense junior to debt obligations. Local currency debt and money have some similarities to "shares" and the value of money and local currency debt times the exchange rate can be seen a sort of "market cap" of the sovereign.

This model combines money and local currency debt together to get Local Currency Liabilities (LCL) in a simple two claim CCA framework in order to calibrate the sovereign balance sheet by calculating implied sovereign assets and asset volatility. Value of local currency liabilities in foreign currency terms, $LCL_{\$}$, is a call option of sovereign assets in foreign currency terms, $V_{\$\text{Sov}}$, with strike price tied to the distress barrier for foreign currency denominated debt B_f derived from the promised payments on foreign currency debt and interest payments up to time t.

$$LCL_{\$} = V_{\$\text{Sov}}\text{N}(d_1) - B_f e^{-r_f T}\text{N}(d_2)$$

The formula for the value of local currency liabilities in foreign currency terms is:

$$LCL_{\$} = M + B_{d\$,t=0} = \frac{(M_{LC}e^{r_d T} + B_d)e^{-r_f T}}{X_F}$$

The volatility of the local currency liabilities is:

$$\sigma_{\$LCL} = f\left(M, B_d, r_d, \sigma_M, \sigma_d, \sigma_{X_F}, X_F, \rho_{M,Dd\$}\right)$$

The definition of the variables is shown below.

M_{LC} base money in local currency terms; r_d domestic interest rate; r_f foreign interest rate; Domestic currency denominated debt is B_d (derived from the promised payments on local currency debt and interest payments up to time t); X_F forward exchange rate; σ_{X_F} volatility of forward exchange rate; σ_{D_d} volatility of domestic debt in local currency terms; ρ_{D_d, X_F} correlation of forward exchange rate and vol of domestic debt in local currency terms; $\rho_{M,D_{d\$}}$ correlation of money (in foreign currency terms) and local currency debt (in fc terms); σ_{MLC} Volatility of money (in lc terms); σ_M volatility of money (in fc terms); and, $\sigma_{D_{d\$}}$ volatility of local currency debt (in fc terms).

The two key equations relating assets and local currency liabilities are:

$$LCL_{\$} = V_{\$\mathrm{Sov}}\mathrm{N}(d_1) - B_f e^{-r_f T}\mathrm{N}(d_2)$$

$$V_{\$Sov}\sigma_{\$Sov} = LCL_{\$}\sigma_{\$LCL}\frac{\partial L_{\$LCL}}{\partial A_{\$Sov}} = LCL_{\$}\sigma_{\$LCL}N(d_1)$$

These two equations can be used to calculate the two unknowns, sovereign asset value and sovereign asset volatility. The sovereign default probabilities, spreads and other risk indicators can be calculated.[24] Stochastic interest rates (both domestic and foreign) can be incorporated in the model with frameworks such as Shimko et. al. (1993) which integrate a Vasicek term-structure model into the Merton model.

Breaking Down Sovereign Assets into Key Components

The sovereign asset value can be broken down into its key components, Reserves (R), net fiscal asset or present value of the primary fiscal surplus (PVPS), Implicit Guarantees (G), and Other remainder items.

$$V_{Sov\$,0} = R + PVPS_{\$} - G_{\$} + Other_{\$}$$

The value of the foreign currency reserves can be observed and the guarantee can be estimated from the banking and corporate sector CCA model. Subtracting these from the Implied sovereign asset we can calculate the residual which includes the primary fiscal surplus. If we estimate the expected present value of the primary fiscal surplus (an obvious approximation) the remainder "Other" can be estimated. "Other" may be due to various factors, including contingent financial support from other governments or multilaterals.

$$V_{Sov\$,0} - R + G_{\$} - E[PVPS_{\$}] = Other_{\$}$$

We can use this valuation formula to evaluate the effects of changes in reserves, the primary fiscal balance, and the implicit guarantee on the sovereign asset value. This can be used with changes in the composition of short-term and long-term debt and with money and the exchange rate for sensitivity and stress tests to evaluate changes in sovereign credit spreads and other values and risk indicators.

[24] Xu, D. and Ghezzi, P. 2002 develop a stochastic debt sustainability model and show how it is related to the CCA model described in Gray, Merton, Bodie 2002 and this paper.

Annex 3 – Examples of Risk Transmission Channels between Economic Balance Sheets of Sectors

This Annex describes the CCA equations and inter-linkages among sectors for a simple three sector framework (base case is shown in Figure 6) and numerical examples for a number of different scenarios, Figures A3-2 through A3-6, which demonstrate value and risk transmission changes and calculation of risk exposures.

Figures in Annex 3:

1. Illustrative Equations for Inter-linked Balance Sheets.

(Changes in parentheses in all figures below are all relative to the base case balance sheet shown in Figure 6.)

2. Negative shock to corporate sector assets and the subsequent impact on banking assets and increase in government implicit guarantee.

3. Deposit run and subsequent rise in government implicit guarantee.

4. Negative shock to government assets resulting in lower value of sovereign debt.

5. Negative shock to government assets and thus a decline in the value of government securities held by banks leading to an increase in implicit financial guarantee (and feedback loop).

6. Negative shock to corporate sector assets (as in 2. above) in the case where the pension system contains one-half of corporate sector equity (in a defined benefit plan which also has an implicit government guarantee). The negative shock to corporate sector assets results in lower pension system's asset value with higher implicit guarantee for pension system, in conjunction with higher implicit guarantee to banks.

Annex 3 (cont.) – Figure A3-1

Illustrative Equations for Inter-linked Sectors

MV stands for market value, A is assets, E is equity, J is junior claim, D is debt, FS is financial guarantee from government, C means call option, P means put option. (Subscripts C, B, and G refer to the corporate, banking and financial, and public sector, respectively.)

Corporate Sector

Market value balance sheet equation for the corporate sector(s) using contingent claims.

$MV(A_C) = MV(E_C) + MV(D_C)$ *[Market Value Balance Sheet Equation]*

$MV(D_C) = DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$ *[Equation for $MV(D_C)$]*

$MV(A_C) = C[MV(A_C), DB_C, \sigma_C, T, r] + DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$

Bank and Financial Institutions Sector

Using the market value balance sheet equation, we substitute variables representing the contingent claims:

$MV(A_B) + FS_G = MV(E_B) + MV(D_B)$ *[Market Value Balance Sheet Equation]*

$FS_G = P[MV(A_B), DB_B, \sigma_B, T, r]$ *[Contingent Financial Support, i.e. Guarantee]*

$MV(A_B) + FS_G = C[MV(A_B), DB_B, \sigma_B, T, r] + MV(D_B)$

Banking sector assets contain loans to the corporate sector

$MV(D_C) = DB_C - P[MV(A_C), DB_C, \sigma_C, T, r]$ *[Loans Made to the Corporate Sector]*

Public sector (Government and Monetary Authority) Sector

Using the market value balance sheet equation, we substitute in variables representing the contingent claims:

$MV(A_G) = MV(J_G) + MV(D_G) + FS_G$ *[Market Value Balance Sheet Equation]*

$MV(A_G) = C[MV(A_G), DB_G, \sigma_G, T, r] + DB_G - P[MV(A_G), DB_G, \sigma_G, T, r] + P[MV(A_B), DB_B, \sigma_B, T, r]$

$FS_G = P[MV(A_B), DB_B, \sigma_B, T, r]$ *[Contingent Financial Support to Banks/Financial Institutions]*

$MV(D_G) = DB_G - P[MV(A_G), DB_G, \sigma_G, T, r]$ *[Equation for $MV(D_G)$]*

Annex 3 (cont.) – Figure A3-2

Negative Shock to Corporate Sector Assets, decline of $40 billion (from $120 to $80 billion) as compared to base case.

Corporate Sector Balance Sheet

Assets		Liabilities	
Corporate Assets	80 (- 40)	Loans (Default-free value = 90, minus implicit loan guarantee – or put option of 15.8)	74.1 (- 13.1)
		Corporate Equity	5.9 (- 26.9)
Total	80 (- 40)		80 (- 40)

Banking Sector Balance Sheet

Assets		Liabilities	
Loans (90 -15.8)	74.1 (- 13.1)	Deposits	81.3
Financial Guarantee	13.3 (+ 5.7)	Equity	6.1 (-7.2)
Total	87.4 (- 7.2)		87.4 (- 7.2)

Public Sector Balance Sheet

Assets		Liabilities	
Domestic assets	100	Financial Guarantee to Banks	13.3 (+ 5.7)
Foreign reserves	40	Foreign Debt	80.4 (- 1.75)
		Local Currency Debt & Other Liabilities	46.2 (- 4.25)
Total	140		140

(Units are in Billions of $)

Risk Transmission: Lower corporate assets → lower corporate equity and debt → lower bank assets → higher guarantee from government required → higher cost of government guarantee lowers value of sovereign debt. The value of the assets of the corporate sector declines, so does the value of the debt (and equity) which leads to a decline in bank assets and an increase in the implicit government guarantee. The *delta* of the guarantee is -0.56 in this case, as compared to -0.35 in the base case.

Annex 3 (cont.) – Figure A3-3

Deposit Run – Default barrier rises for banks as $36 billion of long term savings and time deposits become short-term liabilities with deposit run. In this contingent claims framework for banks, the default barrier for banks includes deposits calculated as default-free value of short and long-term deposits which is approximated by demand deposits plus a fraction of time and saving deposits. In a deposit run, a portion of the long-term time and savings deposits shift to the short-term category, thus raising the overall default barrier and raising the size of the implicit guarantee of the government. As $36 billion of long term savings and time deposits become short-term liabilities, the result is a significant increase in implicit financial guarantees from $7.4 billion to $32.6 billion, an increase of $25.2 billion from the base case. The *delta* of the guarantee is -0.83 in this case, more than double the -0.35 delta value in the base case.

Corporate Sector Balance Sheet

Assets		Liabilities	
Corporate Assets	120	Loans (Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8)	87.2
		Corporate Equity	32.8
Total	120		120

Banking Sector Balance Sheet

Assets		Liabilities	
Loans (90 -2.8)	87.2	Deposits	117.3 (+ 36)
Financial Guarantee	32.6 (+ 25.2)	Equity	2.5 (- 10.8)
Total	119.8 (+ 25.2)		119.8 (+ 25.2)

Public Sector Balance Sheet

Assets		Liabilities	
Domestic assets	100	Financial Guarantee to Banks	32.6 (+ 25.2)
Foreign reserves	40	Foreign Debt	73.7 (- 8.4)
		Local-currency Debt & Other Liabilities	33.7 (- 16.8)
Total	140		140

(Units are in Billions of $)

Annex 3 (cont.) Figure A3-4

Negative Shock to Public Sector Assets of $20 billion, result is lower "market value of liabilities" as compared to base case.

Corporate Sector Balance Sheet

Assets		Liabilities	
Corporate Assets	120	Loans (Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8)	87.2
		Corporate Equity	32.8
Total	120		120

Banking Sector Balance Sheet

Assets		Liabilities	
Loans (90 -2.8)	87.2	Deposits	81.3
Financial Guarantee	7.4	Equity	13.3
Total	94.6		94.6

Public Sector Balance Sheet

Assets		Liabilities	
Domestic assets	90 (- 10)	Financial Guarantee to Banks	7.4
Foreign reserves	30 (- 10)	Foreign Debt	79.1 (- 3.1)
		Local Currency Debt & Other Liabilities	33.5 (- 16.9)
Total	120 (- 20)		120 (- 20)

(Units are in Billions of $)

Risk Transmission: Negative shock to government assets → lower value of foreign debt (senior debt in this example) and lower value of local-currency debt and other liabilities.

Annex 3 (cont.) Figure A3-5

Negative Shock to Government Assets and Decline in Value of Government Securities Held by Banks – Banking sector assets consist of half government securities and half loans to corporate sector (as compared to 100% corporate loans in the base case). The market value of government securities (local-currency debt) declines due to decline in government assets of $20 billion relative to the base case. The decline in government assets of $20 billion increases the guarantee to banks by $6.2 billion to $13.6 billion. The vicious cycle could arise, when the lower value of government securities lowers bank assets, and raises the implicit financial guarantee, which in turn lowers government assets further. This means that the implicit guarantee is higher than what is shown above.

Corporate Sector Balance Sheet

Assets		Liabilities	
Corporate Assets	120	Loans (Default-free value = 90, minus implicit loan guarantee – or put option – of 2.8)	87.2 (43.6 loans from banks, and 43.6 to non-banks)
		Corporate Equity	32.8
Total	120		120

Banking Sector Balance Sheet

Assets		Liabilities	
Loans (43.6) and Govt. Securities (30)	73.6 (-16.4)	Deposits	81.3
Financial Guarantee	13.58 (+ 6.18)	Equity	6.1 (-7.2)
Total	94.6		94.6

Public Sector Balance Sheet

Assets		Liabilities	
Domestic assets	90 (- 10)	Financial Guarantee to Banks	13.58 (+ 6.18)
Foreign reserves	30 (- 10)	Foreign Debt	76.8 (- 5.35)
		Local-currency Debt & Other Liabilities	29.9 (- 20.55)
Total	120 (- 20)		120 (- 20)

(Units are in Billions of $)

Annex 3 (cont.) – Figure A3-6

This scenario describes a negative shock to corporate sector assets (as in A5-2. above), in the case where the pension system contains one-half of corporate sector equity (in a defined benefit plan which also has a government guarantee). The results are shown below in Figure A5-6. A decline in corporate assets by $40 billion (from $120 to $80 billion) would cause the corporate equity value to drop by $26.9 billion to $5.9 billion (as compared with the base case). This increases the government guarantee to the pension system by $9 billion and the implicit guarantee to banks by $5.7 billion. In total, the government guarantees to pension system and banking system would increase to $22.3 billion, significantly higher than $7.4 billion in the base case).

Figure A3-6

Corporate Sector Balance Sheet

Assets		Liabilities	
Corporate Assets	80 (- 40)	Loans (Default-free value = 90, minus implicit loan guarantee – or put option of 15.8)	74.1 (- 13.1)
		Corporate Equity	5.9 (- 26.9)
Total	80 (- 40)		80 (- 40)

Banking Sector Balance Sheet

Assets		Liabilities	
Loans (90 -15.8)	74.1 (- 13.1)	Deposits	81.3
Financial Guarantee	13.3 (+ 5.7)	Equity	6.1 (-7.2)
Total	87.4 (- 7.2)		87.4 (- 7.2)

Pension System

Assets		Liabilities	
Corporate Equity (initially 16.4)	3 (- 13.4)	Defined Benefit (Present Value)	12
Financial Guarantee	9 (+ 9)		
Total	12		12

Public Sector Balance Sheet

Assets		Liabilities	
Domestic assets	100	Financial Guarantee to Banks & Financial Guarantee to Pension System	13.3 + 9 =22.3 (+ 14.7)
Foreign reserves	40	Foreign Debt	78.9 (- 3.25)
		Local-currency Debt & Other Liabilities	39 (- 11.45)
Total	140		140

(Units are $ Billions.)

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