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Incentives in Discrete-time MDP Processes with Flexible Step-size

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1. INTRODUCTION

The problems of resource allocation in economies with public goods has recently been studied from two points of view. On the one hand, procedures have been derived to compute Pareto optima as limit points of a dynamic process, assuming that correct knowledge of the data of the system is available. On the other hand, a possibility of misrepresentation of privately held information, a serious obstacle in such systems, has been overcome by the design of incentive compatible mechanisms in static contexts. These two strains of the literature have been united in the work of Malinvaud, Drèze and de la Vallée Pouissin (MDP) on the assumption that consumers will play maximin strategies at each instant. Because this behaviour cannot be guaranteed, stronger incentives properties are desired. Roberts (1977) showed that if the MDP rules were used but a Nash equilibrium was attained at each instant, the favourable convergence properties of the MDP process were preserved.

Individual strategies in the MDP procedure consist in announcing their marginal rates of substitution (MRS) at each instant. In Roberts' model the strategy space consists of real numbers which play the role of MRS in the dynamic process. Consumers choose these numbers as a function of their true MRS's and of the other agents' announcements, to maximize the time derivative of their utility.

There were two disturbing features of the Roberts model that required further study. First, the strategies in Robert's mechanism were not restricted to be non-negative. The game is well-defined and has Nash equilibria at each instant, but these cannot be viewed as truthful MRS's of agents with monotone preferences. Henry (1977) re-examined the Roberts' process with this non-negativity restriction imposed. He proved that Nash equilibria exist at each date and that their favourable convergence and optimality properties are maintained, provided that one further assumption holds: when indifferent among several possible responses including the truth, the agents will always employ the latter strategy.

The second troublesome aspect of using a Nash concept in Roberts' model is of a more conceptual nature. Indeed it applies to any dynamic decentralized model in continuous time that requires an equilibrium to be achieved at each instant. To attain an equilibrium one usually either proves that it is the limit of an adjustment process or simply asserts that it will be found eventually. In any case, there is in principle an infinite amount of time required to reach the equilibrium exactly, and truncating the process at any point would involve a genuine disequilibrium analysis in which the nature of agents' myopic optimizations would be radically altered. Therefore, assuming that Nash equilibria are reached continually amounts to compressing a double-infinity of time into the adjustment process.

The favourable efficiency properties being achieved only asymptotically, the relevance of such results is called into question.

To overcome this, Schoumaker (1976) considers a discrete time system analogous to the MDP process. The advantage of discrete time processes is that they serve as a more realistic representation of dynamic games played sequentially. The strategy space is assumed to be pairs of real numbers. The first is the willingness to pay of each consumer for a given size of *increment* in the level of the public good production; the second is the compensation the consumer would have if the level of the public good were *decreased* by the same amount. The step size is decreased whenever convergence is attained. Convergence to Pareto optimality is assured subject to the same assumption of correct revelation in matters of indifference as used by Henry.

To avoid this " truthtelling under indifference " postulate, and to provide a process in which the step size is endogenously determined, we have developed a discrete-time adjustment process in which the strategy spaces are monotone concave functions instead of real numbers or pairs of real numbers. The interpretation of strategies in this model is that they represent consumers' willingnesses to pay for changes in the level of production of the public good around the current plan. One interesting feature of our model is that the Nash equilibria attained at each step do not involve " truthtelling ", in that the true demand-price schedules are not the equilibrium strategies. Nevertheless convergence of these Nash equilibria to a Pareto optimum is established.

Because continuous-time procedures are subject to the criticism mentioned above that there is not enough time to find a Nash equilibrium at each of the continuum of dates, one might think that introducing a more complex strategy space would replace this difficulty with another one. The computation of a maximum in a discrete-time model with a flexible step-size would suffer from the same drawback. If, for example, the planner were to use a gradient method at each date, again a " double-infinity " would be created. It turns out, however, that the optimal strategy is always to announce a linear function, so that the Central Planning Board can perform the optimization trivially, given a knowledge of the marginal cost of producing the public good.

In the next section the basic model and notation are set out. Section 3 gives an analysis of the best-replay strategies at each step. Section 4 provides a proof of the existence of Nash equilibria for the game played at each step. Section 5 analyses the properties of these equilibria. The main results of the paper, dealing with convergence towards Pareto optimality of the sequence of these equilibria, are proven there. A brief conclusion follows.

2. NOTATION

Consider a planning process at which a Nash equilibrium is found at each of a countably infinite sequence of dates. We denote a typical point in this sequence by an integer t.

There are I consumers indexed $i = 1, ..., I$, each of which consumes an amount x_i of the private good. The planned output of the public good is denoted y. Consumers' preferences are represented by strictly monotonic quasi-concave differentiable utility functions: $U^i(x_i, y)$, defined for all non-negative values of the arguments. The initial endowment of each consumer consists only in w_i of the private good.

The public good is produced according to a technology with non-increasing returns, described by a production function: $y = f(z)$ where $z \in R$ is the input of private good. We assume that $f''(z) < 0$ and $\lim_{z \to \infty} f' \neq 0$. Let $z = g(y)$ denote the cost function for public good production.

An allocation $(x_1, ..., x_I, y)$ is feasible if

$$\sum_i x_i + g(y) = \sum_i w_i \quad \text{for all } i, \quad x_i \geqq 0.$$

At time $t-1$ the process results in a feasible allocation of public and private goods, denoted $(x_1(t), ..., x_I(t), y(t))$ which forms the data for the process at time t.

The cost function g is known by all agents.   We denote by $q(y, y(t))$ the change in the cost if the planned level of public good were to vary from $y(t)$ to y.   The function q is monotonic increasing, strictly convex, and sign preserving in $[y-y(t)]$.

Consumers are required to announce functions $m_i(y, t)$ at time t.   The spirit of the process is that $m_i(y, t)$ is consumer i's willingness-to-pay function for changes in the level of public good given $x_i(t)$ and $y(t)$.   However, the utility function is not known to the planner and private incentives determine the functions announced.   Nevertheless the planning process is designed so that the allowable announcements $m_i(y, t)$ must be compatible with some monotone quasi-concave differentiable utility function.   This means that $m_i(y, t)$ is monotonic, non-decreasing, sign preserving in $[y-y(t)]$, concave and differentiable in y.   The set of all such functions forms the strategy space of each consumer at time t given the state of the whole system.

The allocation chosen at t, $(x_1(t+1), ..., x_I(t+1), y(t+1))$ is given by

$$y(t+1) \text{ maximizes } \sum_i m_i(y, t) - q(y, y(t))$$

and

$$x_i(t+1) = x_i(t) - m_i(y(t+1), t) + \delta_i \left[\sum_j m_j(y(t+1), t) - q(y(t+1), y(t)) \right]$$

where

$$0 < \delta_i < 1 \quad \text{and} \quad \sum_i \delta_i = 1.$$

First observe that the process is well defined for each collection of strategies $m_1, ..., m_I$ by virtue of our assumptions on the production function which ensures that

$$\max_y \sum_i m_i(y, t) - q(y, y(t))$$

exists and is unique.

Note that

$$\sum_i m_i(y(t+1), t) - q(y(t+1), y(t)) \geqq \sum_i m_i(y(t), t) - q(y(t), y(t)) = 0$$

because $m_i(y, t)$ and $q(y, y(t))$ are sign preserving in $[y-y(t)]$.

Notice also that since each agent i can choose the function $m_i(y, t)$ defined implicitly by

$$U^i(x_i(t) - m_i(y, t), y) = U^i(x_i(t), y(t))$$

(which corresponds to a true revelation of i's willingness to pay for changes in the level of public good from $y(t)$), and since U^i is defined for non-negative values of its first argument, he can insure that:

$$x_i(t+1) \geqq 0$$

and moreover that $U^i(x_i(t+1), y(t+1)) \geqq U^i(x_i(t), y(t))$.

It follows from these observations that starting from any feasible state, the procedure will maintain feasibility as long as consumers play strategies associated with non-negative amounts of private good.

In the tradition of the literature on dynamic planning we are assuming that the agents behave myopically at each t, choosing their strategies to maximize $U^i(x_i(t+1), y(t+1))$ given the information available to them.   Specifically we assume that each agent knows the strategies followed by the others and that a Nash equilibrium in their strategies is achieved at each stage.   By virtue of these conditions the individual's best replay strategy will always be such as to guarantee non-negative consumption of the private good.

3. BEST REPLAY STRATEGY

Given the strategies chosen by the other consumers we can define:

$$\phi_i(y) = \sum_{j \neq i} m_j(y, t) - q(y, y(t)).$$

Note that ϕ_i is strictly concave and differentiable.   Let $\bar{y}_i$ maximize ϕ_i.

If consumer i announces $m_i(y, t)$ then $y(t+1)$ will maximize

$$m_i(y, t) + \phi_i(y).$$

Because $m_i(y, t)$ is required to be non-decreasing,

$$y(t+1) \geqq \bar{y}_i$$

We compute the best replay strategy of consumer i in two steps: first for each $\tilde{y} \geqq \bar{y}_i$ we find the strategy which maximizes $U^i(x_i(t+1), y(t+1))$ subject to $y(t+1) = \tilde{y}$; this obviously amounts to maximizing $x_i(t+1)$ given $\tilde{y}$ and ϕ_i. Then the optimal level of $\tilde{y}$ consistent with $\tilde{y} \geqq \bar{y}_i$ is selected by consumer i and the corresponding strategy is played by i.

Let us denote

$$\frac{\partial m_i(y, t)}{\partial y} = m_i'(y, t).$$

If the maximum of $m_i(y, t) + \phi_i(y)$ occurs at $\tilde{y}$, then

$$m_i'(\tilde{y}, t) = -\phi_i'(\tilde{y}) \qquad \qquad \text{...(1)}$$

We will show that among all monotone concave differentiable and sign preserving functions of $[y - y(t)]$ satisfying (1), the one which maximizes $x_i(t+1)$ is the linear function

$$m_i(y, t) = -\phi_i'(\tilde{y})[y - y(t)] \qquad \qquad \text{...(2)}$$

By definition of $x_i(t+1)$

$$x_i(t+1) = x_i(t) - m_i(\tilde{y}, t) + \delta_i \left(\sum_j m_j(\tilde{y}, t) - q(\tilde{y}, y(t)) \right)$$

$$= x_i(t) - (1 - \delta_i) m_i(\tilde{y}, t) + \delta_i \phi_i(\tilde{y}).$$

Since $0 < \delta_i < 1$, $x_i(t+1)$ will be maximized when $m_i(\tilde{y}, t)$ is minimized.

Note that

$$m_i(\tilde{y}, t) = \int_{y(t)}^{\tilde{y}} m_i'(\xi, t) d\xi, \qquad \qquad \text{...(3)}$$

since $m_i(y(t), t) = 0$.

There are two cases according to the sign of $[\tilde{y} - y(t)]$.

(i) if $\tilde{y} \geqq y(t)$, concavity of m_i requires

$$m_i'(\xi, t) \geqq m_i'(\tilde{y}, t) \quad \text{for all } \xi \leqq \tilde{y} \qquad \qquad \text{...(4)}$$

therefore (3) is minimized when (4) is satisfied with equality.

(ii) if $y(t) \geqq \tilde{y}$, concavity of m_i requires

$$m_i'(\xi, t) \leqq m_i'(\tilde{y}, t) \quad \text{for all } \xi \geqq \tilde{y} \qquad \qquad \text{...(5)}$$

the minimization of (3) implies in this case also that (5) holds with equality throughout the range of integration.

Therefore in both cases the optimal strategy requires that (2) holds for all y between $y(t)$ and $\tilde{y}$. Outside this interval the function $m_i(y, t)$ can be defined in any way compatible with the restrictions on the strategy space. We focus attention on the particular element of this optimal set in which (2) holds for all y.

When the strategy defined by (2) is played, consumer i's utility is

$$U^i(x_i(t) - m_i(\tilde{y}, t) + \delta_i(\sum_j m_j(\tilde{y}, t) - q(\tilde{y}, y(t)), \tilde{y})$$

$$= U^i(x_i(t) + (1 - \delta_i)\phi_i'(\tilde{y})[\tilde{y} - y(t)] + \delta_i \phi_i(\tilde{y}), \tilde{y}). \quad \text{...(6)}$$

The second stage of the optimization is to maximize (6) with respect to $\tilde{y}$ subject to the constraint $\tilde{y} \geqq \bar{y}_i$. As mentioned above this constraint embodies the restriction of m_i to monotone non-decreasing functions.

Let y_i^* satisfy the first order condition for this maximum.

$$U_x^i[(1-\delta_i)(\phi_i''(y_i^*))[y_i^*-y(t)]+(1-\delta_i)\phi_i'(y_i^*)+\delta_i\phi_i'(y_i^*)]+U_y^i = 0$$

or

$$U_x^i[(1-\delta_i)\phi_i''(y_i^*)[y_i^*-y(t)]+\phi_i'(y_i^*)]+U_y^i = 0. \qquad ...(7)$$

It is important to remember that the partial derivatives U_x^i and U_y^i are themselves functions of y and x and are evaluated here at y_i^* and $x_i(t+1)$.

The value of $\tilde{y}$ that maximizes (6) subject to $\tilde{y} \geqq \bar{y}_i$ is clearly:

$$\hat{y}_i = \max(y_i^*, \bar{y}_i)$$

and the optimal strategy of consumer i is

$$m_i(y, t) = -\phi_i'(\hat{y}_i)[y-y(t)], \qquad ...(8)$$

because

$$\phi_i'(\hat{y}_i) = \begin{cases} \phi_i'(y_i^*), & \text{if } y_i^* > \bar{y} \\ 0 = \phi_i'(\bar{y}_i), & \text{if } y_i^* \leqq \bar{y} \end{cases}$$

4. NASH EQUILIBRIA

A collection of response functions $m_1^*(y, t), ..., m_I^*(y, t)$ such that $m_i^*(y, t)$ is an optimal strategy for every consumer i given that agents $j \neq i$ are playing m_j^*, is a Nash equilibrium.

We will show that a Nash equilibrium exists for a system in $(x_1(t), ..., x_I(t), y(t))$. In particular we will demonstrate that there is a Nash equilibrium in which the response functions are all linear:

$$m_i(y, t) = s_i[y-y(t)], \quad s_i \in R_+ \qquad ...(9)$$

We therefore consider the strategy space for each agent as R_+ and his strategy is denoted s_i. A fixed-point argument will be used in which the optimal strategy s_i^* is a function of the strategies of the other consumers s_j, $j \neq i$. We first show that s_i^* can be bounded independently of $\{s_j\}_{j \neq i}$ under the following assumption.

Assumption 1. U_x^i/U_y^i is uniformly bounded over the commodity space.

Let B_1 be the bound whose existence is asserted in this assumption. Let $B_2 = q'(y(t))$.

Lemma 1. $B = \max(B_1, B_2) \geqq s_i$, $\forall s_j, j \neq i, s_j \geqq 0$.

Proof. If $s_i > B_2$ then $y_i^* > y(t)$ since y_i^* will satisfy

$$q'(y_i^*) = \sum_j s_j = s_i + \sum_{j \neq i} s_j \geqq s_i \geqq q'(y(t))$$

and $s_j \geqq 0$ and $q'' > 0$.

y_i^* is defined by (7) which in the case of linear strategies can be written:

$$U_x^i[(1-\delta_i)(-q''(y_i^*)[y_i^*-y(t)])-s_i]+U_y^i = 0$$

or

$$s_i = \frac{U_x^i}{U_y^i} -(1-\delta_i)q''(y_i^*)[y_i^*-y(t)] < B,$$

by the argument above. ‖

Since only non-negative strategies are allowed we know that the optimal strategy of consumer i will lie in $[0, B]$ independently of the strategy choices of others. In order to use a fixed-point argument on the product of these strategy spaces, to assert the existence

of Nash equilibria, we must show that y_i^* defined by (7) is continuous in the variable $s = \sum_{j \neq i} s_j$. (7) can be written as

$$U_x \bigg|_{x = x_i(t+1), \, y = (y_1^*)} \frac{dx_i(t+1)}{dy^*} + U_y \bigg|_{x = x_i(t+1), \, y = y_1^*} = 0. \qquad \ldots(10)$$

In order for there to be a unique and continuously-varying (with respect to s) solution y_i^*, it suffices that $dx_i(t+1)/dy^*$ be monotone decreasing, or in other words that $x_i(t+1)$ be concave. Recalling the definition of $x_i(t+1)$ and using the linear form of m_i:

$$x_i(t+1) = x_i(t) - (1-\delta_i)[q'(y_i^*) - \sum_{j \neq i} s_j](y_i^* - y(t))$$

$$+ \delta_i \left[\sum_{j \neq i} s_j [y^* - y(t)] - q(y^*) \right], \quad \ldots(11)$$

twice differentiating (11) we find:

$$-(1-\delta_i)[q'''(y_i^* - y(t)) + q''] - q''. \qquad \ldots(12)$$

The negativity of (12) cannot be assured without an assumption on the technology.

Assumption 2.

$$| q'''(y) | < \frac{q''(y)}{y} \quad \text{for all } y.$$

This assumption means that in some sense, locally, the degree of diminishing returns to scale is not too large. For example cost functions of the form:

$$z = q(y) = y^\alpha - y(t)^\alpha$$

satisfy Assumption 2 and strict convexity whenever $1 < \alpha < 3$.

Under Assumptions (1) and (2) the argument above implies that y_i^* will be a continuous function of $s_j, j \neq i$. Since y_i^* maximizes

$$s_i[y - y(t)] + \sum_{j \neq i} s_j[y - y(t)] - q(y)$$

s_i will be continuous in $s_j, j \neq i$. The existence of Nash equilibria is guaranteed by Brouwer's fixed-point theorem.

5. NASH EQUILIBRIA AND PARETO OPTIMALITY

In this section we demonstrate the two main results of this paper: Pareto optima are stationary points of the dynamic adjustment process (Theorem 1) and non-optimal allocations lead to Pareto-superior adjustments at every Nash equilibrium (Theorem 2). By virtue of these results we conclude that the sequence of utilities attained is monotone increasing and hence has a limit point. To insure that this limit is a Pareto optimum, it is necessary to know that the correspondence

$$s = F(x(t), y(t))$$

defining the set of Nash equilibrium strategies given the initial point $(x(t), y(t))$ is upper hemi-continuous in $(x(t), y(t))$. Given upper hemi-continuity of F, if $(x(t), y(t)) \to (x^*, y^*)$ and $s_t \in F(x(t), y(t))$ for each t, with $s_t \to s^*$, then s^* is a Nash equilibrium and therefore (by virtue of Theorem 1) (x^*, y^*) is Pareto optimal. To prove the continuity of F in $(x(t), y(t))$ one must show that $\bar{y}_i$ and y_i^* are continuous in the current state. But note that $\bar{y}_i$ depends on the current state only through $y(t)$ which is an argument of q, and the latter is continuous by continuity of the production function. And y_i^* defines a regular maximum of (6); under Assumption (2) the solution to (7) is unique and continuous in the initial state.

Theorem 1. *Let $(x_1(t), \ldots, x_I(t), y(t))$ be Pareto optimal then*

$$s_i = \frac{U_y^i}{U_x^i} \quad i = 1, \ldots, I$$

is a Nash equilibrium and the corresponding allocation is

$$x_i(t+1) = x_i(t) \quad i = 1, \ldots, I$$
$$y(t+1) = y(t).$$

Moreover there are no other Nash equilibria.

Proof. Consider $s_j = U_y^j/U_x^j$, for all $j \neq i$. We will show that the optimal strategy for agent i is to announce

$$s_i = \frac{U_y^i}{U_x^i}.$$

By virtue of the observation in Section 2, the strategy s_j cannot result in a loss in utility for consumer j, independently of the strategies played by the others. Therefore for any $s_i' \neq s_i$ we would have

$$y(t+1) \neq y(t)$$

and consumer i's utility would be decreased.

Hence s_i is the optimal strategy. And by the condition of Pareto optimality:

$$\sum_i \frac{U_y^i}{U_x^i} - q' = 0$$

the specification of the mechanism implies that the allocation remains unchanged when these strategies are played.

To show that the Nash equilibrium is unique, consider the strategies s_j', $j = 1, \ldots, I$, such that

$$s_i' \neq s_i \quad \text{for some } i$$

and assume that these form a Nash equilibrium. First observe that this Nash equilibrium must also result in stationarity because otherwise the utility of at least one agent i would decrease from t to $t+1$, and he would therefore play the strategy s_i. Stationarity implies

$$\sum_i s_i - q' = 0. \qquad \ldots(13)$$

Differentiating the utility attained at $t+1$ with respect to s_i':

$$\frac{d}{ds_i'} U^i(x_i(t) - s_i'[y(t+1) - y(t)] + \delta^i [\sum_j s_j'[y(t+1) - y(t)] - q(y(t+1), y(t))], y(t+1))$$
$$= U_y^i + U_x^i(-s_i'). \qquad \ldots(14)$$

Using (13) and $dy/ds_i' = 1/q''$ (14) is zero only if

$$s_i' = \frac{U_y^i}{U_x^i}$$

and therefore any individual for whom this is not satisfied has an incentive to modify his message slightly in the direction of the true marginal rate of substitution at the stationary point. Hence the s_j' were not equilibrium strategies. ‖

Theorem 2. *Let $(x_1(t), \ldots, x_I(t), y(t))$ be a non-optimal allocation. Then*

$$x_i(t+1) = x_i(t), \quad i = 1, \ldots, I$$
$$y(t+1) = y(t)$$

is not an allocation associated with any Nash equilibrium.

Remark. Because we cannot get " stuck " at a non-optimum, and because the process is utility-monotone at each step, Theorem 2 implies that there is a monotone adjustment from any initial position to a full Pareto optimum.

Proof. Assume that the allocation in $t+1$ given by the statement of the theorem is associated with a Nash equilibrium $s_1, \dots, s_I$. Stationarity implies

$$\sum_i s_i = q'(y(t)). \qquad \dots(15)$$

For each individual define y_i^* by equation (7). By the analysis of Section 3, the optimal strategy of consumer i (see equation (8)) is given by:

$$s_i = \max\left[-\phi_i'(y_i^*),\, 0\right].$$

Let $N = \{i \mid s_i = -\phi_i'(y_i^*)\}$.

For $i \in N$ the stationarity of the Nash equilibrium implies $y_i^* = y(t)$. Substituting into (7) we have

$$U_x^i \phi_i'(y(t)) + U_y^i = 0$$

$$\frac{U_y^i}{U_x^i} = -\phi_i'(y(t)) = s_i. \qquad \dots(16)$$

If $N = \{1, \dots, I\}$ equations (15) and (16) imply the optimality of the stationary allocation, contradicting our hypothesis.

Therefore consider $i \notin N$, $s_i = 0$ and (15) implies

$$\sum_{j \neq i} s_j = q'(y(t)) \qquad \dots(17)$$

further $i \notin N$ implies

$$\phi_i'(y_i^*) > 0 \qquad \dots(18)$$

which together with (17) implies

$$y_i^* < y(t).$$

From equation (7) and $\phi''(y_i^*) < 0$ we have:

$$0 = U_x^i((1-\delta_i)\phi''(y_i^*)(y_i^* - y(t)) + \phi'(y_i^*)) + U_y^i > U_x^i((1-\delta_i)\phi''(y_i^*)(y(t) - y(t))$$
$$+ \phi'(y_i^*)) + U_y^i$$

or

$$> U_x^i \phi'(y_i^*) + U_y^i. \qquad \dots(19)$$

(19) and (18) imply $U_y^i < 0$ contradicting the monotonicity of preferences. Therefore $N = \{1, \dots, I\}$ and (16) implies the optimality of the stationary Nash equilibrium.

6. CONCLUSION

A planning process has been studied in which myopic play at each iteration led to a sequence of Nash equilibria. The allocations associated with these equilibria converge to a Pareto optimum. The procedure is monotone in utility for each agent. This property, together with its discrete-time character, make it an attractive process, when myopic play can be assured. Some restrictions on production and preferences were needed to insure the existence of Nash equilibria.

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