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*The Review of Economic Studies*, Vol. 44, No. 3. (Oct., 1977), pp. 451-463.

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# The Non-existence of Informational Equilibria

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## 1. INTRODUCTION

In [5], we introduced the concept of a long-run equilibrium in which exogenous information received by some, but not all, economic agents led to prices which are used as proxies by the others. It was shown that equilibria of this system are generally inefficient in the sense that the set of individuals who choose to become informed, at a cost, is not optimal given that information can only be transmitted through the price system to the group that does not receive it directly.

Several problems were left open in [5]. Among them was the question of the existence of an equilibrium price system and a corresponding consistent system of expectations relating prices to subjective beliefs in a model with a fixed set of individuals who are not informed directly. This is central to the inefficiency result because a full equilibrium of the system involves individuals deciding whether or not to become informed based on the value this information would have in an equilibrium with fixed sets of informed and uninformed individuals. Were they not to be able to make this calculation because of the absence of an equilibrium with a fixed set of informed agents, the entire concept of a long-run equilibrium with self-fulfilling expectations would be in doubt.

In this paper, a counter-example to the existence of such an equilibrium is presented. It is possible to construct such an example by giving discontinuous density functions to the distributions of the random influences underlying this system. Recent works by Jordan [11] and Kreps [12] explore the problems inherent in such systems. The example given in this paper, however, has all of the regularity properties one would usually expect to be sufficient to guarantee the existence of a general equilibrium.

We then discuss the meaning and implications of such instances of non-existence. It seems to be due to the fact that the space of prices is not “large” enough, in a certain sense, to transmit all of the relevant data. A more detailed discussion must await the presentation of the model and the example, to be given in Section 2. In Section 3 we discuss whether such an example is robust and present some of its further implications.

In this model, as in most general equilibrium systems, prices are the only variables that individuals need to observe in order to calculate their optimal actions. However, in cases in which no equilibria of this type exist, it may be possible to have one in situations where some quantity variables are observable by individuals and their expectations are allowed to depend on these data as well as on prices. Such price-quantity equilibria have interesting implications for the efficiency and informational properties of the model. They are also related, in a sense, to quantity-constrained equilibria studied by Drèze [3], Grandmont-Laroque [4], Younes [14], Benassy [1] and Hahn [8]. This is discussed in Section 3.

Finally, the potential non-existence of equilibria when quantities are not observable provides a further impetus for the use of specific functional forms with properties strong enough to insure the existence of equilibria. For example, the partial equilibrium models studied by Grossman [6] and Grossman-Stiglitz [7] utilize normality assumptions that imply a monotone likelihood-ratio property which in turn suffices to prove the existence of

an equilibrium. This and related conditions to guarantee existence are discussed in Section 4.

The remainder of this section reviews the model and presents the notation. First we present the model in the "noiseless case". The "noisy case", on which the remainder of the paper will centre, is discussed below.

### 1.1. THE NOISELESS CASE

There are  $n$  states of nature. The set  $S$  is the set of all such states.

$$S = \{1, \dots, n\}.$$

There are two periods, the present, in which  $l$  commodities are currently consumable, and the future, in which more commodities will become tradeable. On the markets for presently available goods, the agents can exchange the  $l$  goods of the current period and state-contingent income claims for each of the  $n$  states of nature.

Let

$$x \in R_+^l$$

$$y \in R_+^n$$

be the vectors of present consumption and holdings of state-contingent claims for a typical individual.

The vectors  $\bar{x}$  and  $\bar{y}$  denote the corresponding initial endowments.

Prices will be denoted

$$p = (p^0, p^1, \dots, p^n) \in \Delta^{l+n},$$

where

$$p^0 \in R_+^l$$

$$(p^1, \dots, p^n) \in R_+^n$$

and

$\Delta^{l+n}$  represents the  $l+n$ -dimensional simplex.

Individuals are divided into two groups, the *informed* and *uninformed* described below.

Before the market convenes, informed individuals receive a signal,  $\beta$ , from among a set of all possible signals,  $B$ , indicating the objective relative frequencies of the  $n$  possible states. Thus, for  $\beta \in B \subseteq \Delta^n$ , a typical informed individual,  $a$ , maximizes

$$U_\beta^a(x, y) = \sum_{k=1}^n \beta_k u_k^a(x, y_k)$$

subject to

$$p \cdot (x - \bar{x}, y - \bar{y}) \leq 0.$$

This gives rise to their *demand functions*

$$\xi^a: \Delta^n \times \Delta^{l+n} \rightarrow R_+^{l+n}.$$

The signals are themselves distributed over time such that signal  $\beta$  has probability  $\mu_\beta$ . Uninformed individuals do not receive any signals directly. Instead, they translate the prices they observe into estimates of the relative frequencies of the various states conditional on those prices. Their expectations are given by an *expectation function*:

$$\gamma: \Delta^{l+n} \rightarrow \Delta^n.$$

A typical uninformed individual,  $a'$ , with expectations  $\gamma$  maximizes

$$U_\gamma^{a'}(x, y) = \sum_{k=1}^n \gamma_k(p) u_k^{a'}(x, y_k)$$

subject to

$$p \cdot (x - \bar{x}, y - \bar{y}) < 0.$$

Thus his demand function is given by

$$\xi_{\gamma}^{a'}: \Delta^{l+n} \rightarrow R_+^{l+n}.$$

An *equilibrium* of this system is a pair of functions

$$\pi^*: B \rightarrow \Delta^{l+n}$$

$$\gamma^*: \Delta^{l+n} \rightarrow R^{l+n}$$

such that

$$(i) \quad \Sigma \xi^a(\beta, \pi^*(\beta)) + \Sigma \xi_{\gamma^*}^{a'}(\pi^*(\beta)) = 0 \quad \text{for all } \beta$$

the sums being taken over the sets of informed and uninformed agents respectively

$$(ii) \quad \gamma^*(p) = E(\beta | p) \quad \text{for all } p \text{ such that } \pi^{-1}(p) \neq \phi.$$

The meaning of (i) is just that supply equals demand, for all signals. The right-hand side of (ii) is the conditional distribution of states of nature given that prices are  $p$  and that  $p = \pi(\beta)$  describes the way in which prices are associated with signals in the equilibrium. Thus (2) expresses the fact that expectations are always statistically correct in the sense that beliefs about the distribution of states given prices are equal to the empirical distributions of these states.

### 1.2. THE NOISY CASE

We now introduce the possibility of additional exogenous randomness that is not observable before the signal and is statistically independent of the signal. Suppose that the state of the economy (preferences and endowments) after the random influence is given by  $\omega \in \Omega$ . More specifically, we can suppose that each individual can observe only his own random preferences and endowments,  $\omega_i$ , and that  $\omega = (\omega_i)$ ,  $\omega_i \in \Omega_i$ , where  $i$  indexes all agents. The conditional joint distribution of  $(\omega_j, j \neq i)$  conditioned on  $\omega_i = \bar{\omega}_i$ , will be denoted  $\nu_i(\bar{\omega}_i)$ . Then the demand function for each informed agent is

$$\xi^a: \Delta^n \times \Delta^{l+n} \times \Omega_a \rightarrow R_+^{l+n}.$$

Uninformed agents' expectations now can depend on both prices and their observations on their own preferences and endowments

$$\gamma^{a'}: \Delta^{l+n} \times \Omega_{a'} \rightarrow \Delta^n.$$

Equilibria are defined by

$$\pi^*: \Delta^n \times \Omega \rightarrow \Delta^{l+n}$$

$$\gamma^{a'*}: \Delta^{l+n} \times \Omega_{a'} \rightarrow \Delta^n \quad \text{for each } a'$$

such that

$$(i) \quad \Sigma \xi^a(\beta, \pi^*(\beta, \omega), \omega_a) + \Sigma \xi_{\gamma^{a'*}}(\pi(\beta, \omega), \omega_{a'}) = 0 \quad \text{for all } (\beta, \omega).$$

$$(ii) \quad \gamma^{a'*}(p, \omega_{a'}) = E(\beta | (p, \omega_{a'})).$$

As above, such an equilibrium represents a situation in which, at market clearing prices, every uninformed agent's expectations relating the data he can observe to the ensuing states of nature are statistically confirmed.

Based on the equilibrium attained in this model, each individual decides whether to be a member of the informed group, at a cost, or to use the price system as a proxy. A full equilibrium of the system is attained if no member of the informed group would rather save this expenditure and be a member of the uninformed group, and likewise no uninformed individual would find it advantageous to switch categories. In [5], the inefficiency of such equilibria was explored. In choosing their informational status, individuals do not take into account the fact that their decision will affect the equilibrium prices and hence the information available to all the remaining uninformed agents. There is no clear direction

to the bias in the number of informed agents. On one hand, more informed agents generally means a better correlation between equilibrium prices and the state of nature, and hence a fuller dissemination of the available information. But this is offset by two forces in the opposite direction: Becoming informed involves a cost in real resources, and a price system that responds strongly to the state of nature gives rise to an increased variability of consumption across states of nature. With risk-averse individuals, this means that too much information can be socially harmful unless it is useful in the production process or in intertemporal decision-making—a fact first pointed out by Hirshleifer [10].

In order to study full equilibria of this type, it is necessary to prove the existence of equilibria for an arbitrary fixed set of informed individuals. In the absence of this, the requisite computations regarding the desirability of an individual's classification cannot be performed.

In the noiseless case it is easy to show that equilibria will generally exist. This is because, with a finite number of signals, the equilibrium price system associated with each signal and with expectations that are the same for the informed and uninformed groups will almost surely be distinct for different signals. Thus this perfectly informed expectation pattern will be consistent with such a price system.

Radner [13] pointed out some time ago that there is a natural discontinuity in this type of system. Consider a sequence of price functions all of which give distinct prices to different signals but which converges to a function at which two signals are mapped into the same price. The associated sequence of fulfilled expectations will be discontinuous at this point, and so, therefore, will demand.

The introduction of noise in the system, especially that describable by smooth distributions of the underlying variables, restores the potential for equilibria to exist in which the information obtained by the uninformed group is strictly worse than that received by the informed group. The noisy case of the model therefore may provide theoretical possibilities not available in the simpler case, in addition to being a better characterization of reality. Unfortunately, the example of this paper shows that it is impossible to assume the existence of an equilibrium, even in cases in which continuity conditions are satisfied.

The intuitive relationship between this kind of non-existence result and the discontinuity problem of the noiseless case is as follows. When noise is added we lose the basic feature of the noiseless case that allows us to find perfectly informative equilibrium price distributions in most cases. This is the one-to-one relationship between signals and prices in equilibrium. If the noise takes the form of a continuous random variable, as might be required to insure continuity of demands, it will generally be the case that several signal-random variable pairs are associated with the same price system. In a sense that will be made more precise below, the presence of this noise may require more variation in the price system than the space of equilibrium prices can admit.

## 2. THE COUNTER-EXAMPLE

In this section we shall show that the model presented above may have no equilibria. We will construct an example with a special structure and demonstrate the potential for non-existence using geometric argument.

We assume that there are only two states of nature

$$S = (1, 2)$$

and no presently available commodities.

We assume that the initial endowments of the state-contingent commodities is such that one group of agents hold only state 1-contingent claims, and the remainder of the individuals hold only state 2-contingent claims. The set of informed individuals is assumed to coincide with the first group; and we will refer to these groups as the informed and uninformed, respectively. By virtue of Walras' Law we can concentrate on the contingent futures market for one of these states, say state 1.

It will be assumed that the only random influence on the system is the endowments of the informed. Preferences will be assumed to remain constant. Since the endowments of the uninformed are non-stochastic, their excess demands depend only on prices and their expectations. There are two possible signals,  $\beta_1$  and  $\beta_2$ ;  $\beta_2$  assigns a higher likelihood to state 1 than does  $\beta_1$ .

The demands of the two groups are given as follows:

The demand of the informed group for state 1 claims is derived from a logarithmic utility function. Thus, their demand will depend only on the signal they receive and on the realization of the aggregate (random) endowment of these claims that they possess, but it is independent of price.

Let  $w$  be their random endowment. The functions  $q_1^s(w)$  and  $q_2^s(w)$  are the supply functions for state 1 claims by the informed group, given that the signal is  $\beta_1$  and  $\beta_2$ , respectively, and  $w$  is the endowment.

We denote the density of  $w$  by  $v(\cdot)$ . This induces a pair of distributions of supplies of state 1 claims in the event of the two signals, as shown below. It is essential for the demonstration of this counter-example that the supports of these two distributions overlap, and we denote the region of overlap by  $Q$ .

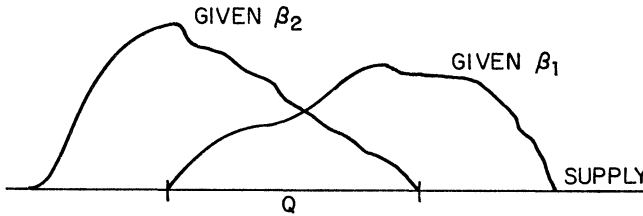


FIGURE 1

Note that supply is higher when  $\beta_1$  is the signal, because  $\beta_1$  is associated with a lower probability of this state than  $\beta_2$ . To preserve continuity, note that we have made  $v(\cdot)$  a continuous function. Discontinuities will not be the source of non-existence.

The uninformed group is assumed to be endowed with a sure level of state 2-contingent claims. Their demand for state 1 claims depends on their expectations and on relative prices. For the purpose of the counter-example we will need only the following properties of their demand.

If expectations were given by  $\beta_1$ , then the demand is given by the inverse demand function

$$p_1 = -q + a \quad \text{for } q \in Q, \tag{2.1}$$

where  $a$  is a constant; and if  $\beta_2$  were the beliefs about the two states, then

$$p_2 = -q + a + c \quad \text{for } q \in Q, \tag{2.2}$$

where  $c > 0$  is another constant.

Nothing is assumed concerning the demand prices for quantities outside of  $Q$ , or for expectations other than  $\beta_1$  or  $\beta_2$ . Of course, by the continuity of demand in expectations and price, we could write quantity demanded as a function of these variables which would coincide with (2.1) and (2.2) at  $\beta_1$  and  $\beta_2$ . But for expectations between these values the demand function might not be invertible, or it might be invertible but non-linear.<sup>1</sup>

In equilibrium the expectations of the uninformed group will be given by  $\gamma(p)$  and a unique quantity demanded will be associated with every price.

For each point  $q \in Q$ , there are two ways in which it could arise as an equilibrium quantity traded: The signal could be  $\beta_1$  and endowments  $w_1(q)$ , or the signal could be  $\beta_2$  and endowments  $w_2(q)$ , where  $w_1$  and  $w_2$  are the inverse functions of  $q_1^s$  and  $q_2^s$  respectively.

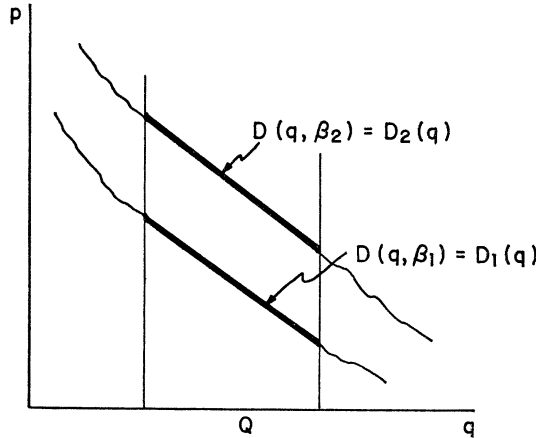


FIGURE 2

For each such  $q$ , there are two possibilities, either the equilibrium price function assigns the same price to each of them, or not. In symbols,

$$\pi(\beta_1, w_1(q)) = \pi(\beta_2, w_2(q)) \quad \dots(2.3)$$

or

$$\pi(\beta_1, w_1(q)) \neq \pi(\beta_2, w_2(q)). \quad \dots(2.4)$$

Whatever these prices are, we know from the remark above that they cannot be equilibrium prices for any  $(\beta, w)$  pair corresponding to a different value of  $q$ , for then the same price would have to give rise to two different levels of demand. Thus, if (2.4) holds, then we know that

$$\gamma(\pi(\beta_1, w_1(q))) = \beta_1 \quad \dots(2.5)$$

and

$$\gamma(\pi(\beta_2, w_1(q))) = \beta_2 \quad \dots(2.6)$$

because observing these prices can only be associated with these  $(\beta, w)$  values.

Recall our assumption that suppliers have logarithmic von Neumann–Morgenstern utility functions. This means that the functions  $q_1^s$  and  $q_2^s$  are linear in  $w$ . Therefore  $w_1(\cdot)$  and  $w_2(\cdot)$  are linear in  $q$ . Denoting their slopes by  $w_1'$  and  $w_2'$  respectively, we can consider the following function mapping quantities into expectations

$$g(q) = \frac{\beta_1 \mu_{\beta_1} w_1' v(w_1(q)) + \beta_2 \mu_{\beta_2} w_2' v(w_1(q))}{\mu_{\beta_1} w_1' v(w_2(q)) + \mu_{\beta_2} w_2' v(w_2(q))} \quad \dots(2.7)$$

The expectation pattern given  $g$  defines an inverse demand function as follows: For each  $q$  in the union of the supports of aggregate supply distributions let  $d(q)$  be the value of  $p$  such that

$$D(q, g(q)) = d(q). \quad \dots(2.8)$$

Given the structure of the demand functions for different expectations, this is well-defined. We note that the possible fluctuations in  $g(q)$  might result in a  $d(\cdot)$  that is not one-to-one. This means that if equilibrium prices are always constant over the two  $(\beta, w)$  pairs that give rise to the same aggregate supply, then the same price might be required to equilibrate the market at different levels of supply. But because the aggregate demand of the uninformed individuals is a unique quantity for each price, equilibria in which prices are constrained in this way will be impossible whenever  $d(\cdot)$  is not one-to-one. The possibility still remains that equilibria exist in which  $\pi(\beta_1, w_1(q)) \neq \pi(\beta_2, w_2(q))$  for some  $q$ . We obtain a counter-

example to this below. The key to this construction is that  $g(\cdot)$  can be made to vary arbitrarily between  $\beta_1$  and  $\beta_2$  by choosing  $v(\cdot)$  appropriately.

Assume that an Equilibrium  $(\pi^*, \gamma^*)$  existed. Let the demand function of the uninformed with the equilibrium expectation function be  $\xi_{\gamma^*}(p)$ . By the analysis above, since  $\gamma^*(p) = g(q(p))$  for almost every  $p$  such that (2.3) holds, and  $\gamma^*(p) = \beta_1$  or  $\beta_2$  for almost every  $p$  such that (2.4) holds,

$$\xi_{\gamma^*}^{-1}(q) = \begin{cases} \{d(q)\} & \text{if (2.3) holds} \\ \{D_1(q), D_2(q)\} & \text{if (2.4) holds.} \end{cases} \quad \dots(2.9)$$

Let

$$\begin{aligned} Q_J &= \{q \in Q \mid \xi_{\gamma^*}^{-1}(q) = \{d(q)\}\} \\ Q_S &= \{q \in Q \mid \xi_{\gamma^*}^{-1}(q) = \{D_1(q), D_2(q)\}\}. \end{aligned} \quad \dots(2.10)$$

(The letters  $J$  and  $S$  are chosen to represent “ joint ” and “ separated ” values of  $q$ , meaning that at  $q$ , the two ways in which the level of supply could have been generated lead to the same or separate systems of prices.)

As indicated in Figure 3, let

$$P = \{p \mid p = D_i(q) \text{ for some } i = 1, 2 \text{ and } q \in Q\}. \quad \dots(2.11)$$

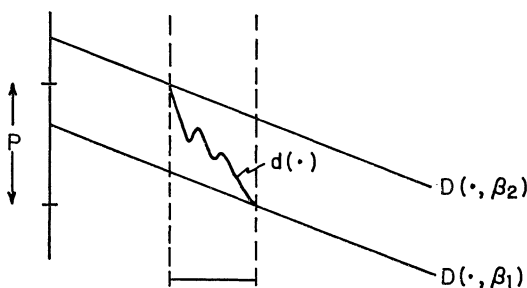


FIGURE 3

If  $\lambda$  denotes Lebesgue measure on the real line, note that

$$\lambda(P) > \lambda(Q) \quad \dots(2.12)$$

since  $D_1$  and  $D_2$  have slope  $-1$  by assumption.

By making demands by the uninformed sufficiently inelastic to changes in expectations, we can make the  $D_1$  and  $D_2$  curves arbitrarily close together and hence

$$\lambda(P) = (1 + \epsilon)\lambda(Q) \quad \dots(2.13)$$

can be satisfied for any  $\epsilon > 0$ .

Let  $A$  be an arbitrary interval in the interior of  $Q$  which will be held fixed for the remainder of the analysis. Let  $\lambda(A) = \delta\lambda(Q)$ . If  $D_1(\cdot)$  and  $D_2(\cdot)$  are as specified we will show that  $d(\cdot)$  can be chosen in such a way that it will:

- (i) not be one-to-one for almost every  $q \in A$ ; that is,  $p = d(q)$ ,  $q \in A$  implies  $p \in d(q')$  for some  $q' \in A$ ,  $q' \neq q$  for almost every  $q \in A$ .
- (ii) be piecewise linear with the absolute value of the slope greater than one throughout  $Q$ .
- (iii) have a constant absolute value of the slope throughout  $A$ .



Before giving the details of the construction of such a  $d(\cdot)$  function, as indicated in Figure 4, we prove that the existence of such a  $d(\cdot)$  for every  $\varepsilon > 0$  implies that no equilibrium will exist for the economy described above when  $\varepsilon$  becomes sufficiently small.

Let

$$\begin{aligned} P_S &= \{p \mid \xi_{y^*}(p) \in Q_S\} \\ P_J &= \{p \mid \xi_{y^*}(p) \in Q_J\}. \end{aligned} \tag{2.14}$$

The set  $P_S$  can be further partitioned into  $P_S^1$  and  $P_S^2$  according to whether  $p \in P_S$  satisfies

$$\begin{aligned} D_1(q) \\ p = \xi_{y^*}^{-1}(q) = \text{or} \\ D_2(q) \end{aligned}$$

for  $q \in Q_S$ .

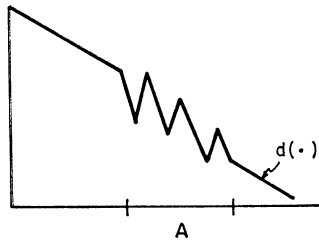


FIGURE 4

The sets  $P_S$  and  $P_J$  are clearly disjoint, for if not, the same  $p$  would correspond to two quantities under the equilibrium demand function, in contradiction to the uniqueness of the quantity demanded at any price.

Because the slope of  $D_1$  and  $D_2$  is  $-1$ , the sets  $P_S^1$  and  $P_S^2$  are essentially copies of  $Q_S$  in the price space. Therefore

$$\lambda(Q_S) = \lambda(P_S^1) = \lambda(P_S^2) = \frac{1}{2}\lambda(P_S). \tag{2.15}$$

The absolute value of the slope of  $d(\cdot)$  being greater than one (property (ii)) implies

$$\lambda(Q_J) < \lambda(P_J). \tag{2.16}$$

Therefore

$$(1 + \varepsilon)\lambda(Q) = \lambda(P) = \lambda(P_S) + \lambda(P_J) > 2\lambda(Q_S) + \lambda(Q_J) = \lambda(Q_S) + \lambda(Q)$$

or

$$\varepsilon\lambda(Q) > \lambda(Q_S). \tag{2.17}$$

This places an upper bound on the measure of  $Q_S$ , which can be made arbitrarily small.

Assuming, temporarily, the existence of a function  $d$ , for each  $\varepsilon$ , having the properties (i), (ii) and (iii) above, we will show that

$$\lambda(Q_J \cap A) \leq \frac{1}{2}\lambda(A). \tag{2.18}$$

Since  $\lambda(A) = \delta\lambda(Q)$  we would have

$$\lambda(Q_J) \leq (1 - \delta)\lambda(Q) + \lambda(Q_J \cap A). \tag{2.19}$$

Hence (2.18) would imply

$$\lambda(Q_J) \leq (1 - \delta)\lambda(Q) + \frac{1}{2}\lambda(A) = \left(1 - \frac{\delta}{2}\right)\lambda(Q). \tag{2.20}$$

From (2.20) it would then follow that

$$\lambda(Q_S) \geq \frac{\delta}{2} \lambda(Q), \tag{2.21}$$

which will contradict (2.17) whenever  $\varepsilon < \delta/2$ . Thus the counter-example will have been shown to exist if (2.18) is proven.

Take  $a \in Q_J \cap A$ . By definition of  $Q_J$  and property (i) there exists  $a' \in A$  such that  $d(a) = d(a')$ . Since the same price cannot equilibrate the market for when the quantity supplied is  $a$  or  $a'$ , it must be that  $a' \notin Q_J$ . By property (iii), we know that  $A \setminus Q_J$  contains a set which is just a translation of  $Q_J \cap A$ . Hence  $\lambda(Q_J \cap A) \leq \frac{1}{2} \lambda(A)$ , which was to be proven, and a counter-example to the existence of an equilibrium arises whenever the function  $d$  can be chosen so that properties (i)–(iii) are satisfied and  $\varepsilon < \delta/2$ .

*Proof that  $d$  can be chosen to have properties stated above, (i), (ii) and (iii) for any  $\varepsilon > 0$ :*

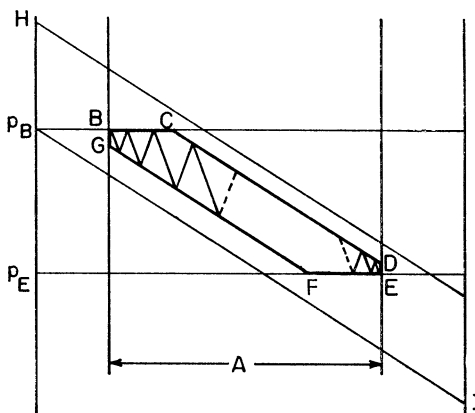


FIGURE 5

*Step 1*

Choose prices  $P_B$  and  $P_E$  such that

$$P_B - P_E < \mu(A).$$

The prices  $P_B$  and  $P_E$  will be the  $d^{-1}(q)$  maximal and minimal value of  $q \in A$ .

*Step 2*

Draw lines CD and GF, with slope  $-1$ , to form the six-sided figure BCDEFG.

*Step 3*

Take  $k > 1$  and, beginning from point B, draw the graph of  $d(\cdot)$  by starting with a line of slope  $-k$  and continuing until the boundary of BCDEFG is reached. At this point, continue with a slope of  $k$ , and so on, reversing the sign of the slope every time the boundary is reached. Terminate when the line reaches segment DE.

Observe

- (i) for  $k$  sufficiently large, every price between  $P_B$  and  $P_E$  will be associated with more than one quantity;
- (ii) the terminal point of the graph constructed within the segment DE is a continuous function of  $k$ , fluctuating between D and E as  $k$  increases.

Therefore  $k$  can be chosen so that the terminal point is E and that the function is never 1:1 within  $A$ , except perhaps on a set of measure zero.

The graph of  $d(\cdot)$  can then be completed by connecting H to B and E to I with straight lines. The resulting function clearly has all the desired properties.

### 3. DISCUSSION

In this section we make several remarks related to the example presented above. First, the robustness of the example is explored. Next we try to present alternative equilibrium concepts for this model, inquiring as to whether a general positive existence result is possible for any of them. Finally we study the role of potential quantity observations for restoring the existence of equilibria. Some conjectures are offered in this regard which will be taken up in future work.

#### 3.1. ROBUSTNESS

This example is based crucially on the fact that by choosing  $v(\cdot)$  appropriately we can cause the ratio  $v(w_1(q))/v(w_2(q))$  to vary arbitrarily rapidly with respect to  $q \in Q$ . This can be done while still preserving the continuity of  $v(\cdot)$ . However, rapid oscillations in the ratio above do require large fluctuations in  $v'$ .

In discussing robustness of the example, it is therefore necessary to distinguish what type of perturbations in  $v'$  are allowed. There is always a sequence of measures  $v^j(\cdot)$  converging in distribution to  $v(\cdot)$  and such that the associated economies along the sequence all have informational equilibria.<sup>2</sup>

However, this does not necessarily mean that the economies without equilibria are "rare". Indeed those for which existence obtains may be rare. To settle this we would have to know more about the (weak-) topological structure of the set of equilibria. This is, at present, an open question.

For stronger topologies, the following robustness property may be obtained. Take  $\phi$  to be "3 or more to 1" on  $A$ .

Assume that the  $v^j(\cdot)$  are differentiable and converge to  $v(\cdot)$  in the  $C^1$ -topology, (i.e.  $v^j$  and  $v'^j$  converge pointwise to  $v$  and  $v'$  respectively), then following exactly the argument used in Section 2, a contradiction between (2.17) and (2.18) is derived for  $j$  sufficiently large.

This type of robustness result is not that attractive however, since, as Debreu [2] and Hildenbrand [9, pp. 123-147] have shown, it is the weak topology which corresponds to the continuity of economic equilibria and the "closeness" of economic behaviour. It is therefore necessary to settle the open question mentioned above before a definite answer to the robustness question can be given.

#### 3.2. ALTERNATIVE EQUILIBRIUM CONCEPTS

We now turn to the possibility of modifying the requirements for an informational equilibrium. One such restriction comes to mind immediately; unfortunately it is a strengthening rather than a weakening of the requirements for an equilibrium.

In the example given in Section 2, if  $(\beta_1, w_1)$  and  $(\beta_2, w_2)$  give rise to a supply of  $q$  contracts for state 1-contingent delivery, we might think that the market should arrive at the same equilibrium in either of these eventualities. Thus one might add the further restriction on  $\pi$  that

$$\pi(\beta_1, w_1(q)) = \pi(\beta_2, w_2(q)) \quad \text{for all } q.$$

This would mean that

$$v(p(q)) \equiv g(q),$$

where  $p(q)$  is the common value of  $\pi$  given above.

Such equilibria will fail to exist whenever  $d(\cdot)$  is not one-to-one. No complicated

construction such as that required for the counter-example to the weaker equilibrium concept of Section 2 is necessary.

In justification of the concept of equilibrium used in Section 2, we might believe that, although the total supply is the same at  $(\beta_1, w_1)$  and  $(\beta_2, w_2)$ , the way in which the economy finds its equilibrium depends on which of these data specify the state of the system. For instance, if the equilibria are limits of differential equation systems, the initial conditions might differ between the two cases so that different prices would be approximated by the solutions. As long as the price-adjustment process is non-stochastic, the same equilibrium price will be achieved (abstracting from stability issues entirely every time a particular  $(\beta, w)$  pair arises. This is an argument for the concept studied.

It is apparent at this point that more general concepts of equilibrium are possible. We can require that the price associated with  $(\beta, z)$  is a random variable which clears the market with probability one. Thus,  $\pi(\beta_1, w_1(q))$  may or may not be equal to  $\pi(\beta_2, w_2(q))$  all the time. Both are random variables whose support is the set of equilibria for the economy with this level of supply.

This concept can be weakened further by requiring only approximate equilibria, that is

$$\text{prob} (\| \xi(\pi(\beta, w)) \| > \varepsilon) < \delta,$$

where  $\varepsilon$  and  $\delta$  can be fixed positive constants, or can be allowed to depend on  $\beta$  and  $w$ , and  $\| \xi \|$  is the norm of the aggregate excess demand.

I have not investigated any of these weakened equilibrium concepts to date, primarily because their justification relies crucially on the fact that they can be attained through a stochastic price-adjustment process, about which no firm theoretical structure exists. They may prove very useful in future work, however, especially if general existence theorems are to be had in this direction.

### 3.3. QUANTITY OBSERVATIONS

The non-existence problem raised above is due to the fact that simultaneous variations in the signal and random supplies cannot be disentangled by individuals who observe only prices. If, however, individuals could observe the aggregate quantity of state 1-contingent claims traded, and could base their expectations about the state of nature on this as well as on price, an equilibrium with self-fulfilling expectations would exist. An equilibrium expectation function would be given by

$$\gamma^*(p, q) = \begin{cases} \beta_1 & \text{if } p = D_1(q) \\ \beta_2 & \text{if } p = D_2(q) \end{cases}$$

and the price function would be

$$\pi^*(\beta_i, w) = D_i(q_i(w)) \quad \text{for } i = 1, 2.$$

This equilibrium has the property that uninformed individuals derive the signal indirectly through their observations of both quantity and price. A central open question remains: If the number of observables is equal to or exceeds the dimensionality of the unobservable signal it would appear that uninformed individuals can deduce the value of the signal without error in equilibrium. A conjecture that we hope to investigate in future work concerns whether equilibria can exist in general in models where complete pass-through of the information is impossible.

This conjecture would, if true, bring us to the rather negative position of being able to guarantee the existence of an equilibrium only in those cases in which all information would be fully disseminated in the resulting equilibrium. Such considerations motivate the next section which is a discussion of one special case in which we can guarantee the existence of an equilibrium with statistically correct expectations that do not disseminate all the information to the uninformed group.

4. CONDITIONS FOR THE EXISTENCE OF EQUILIBRIA

In recent work Grossman [6] and Grossman–Stiglitz [7] have studied the implications of a related, partial equilibrium model in which supplies were normally distributed with an unknown mean. In such a system it can be shown that informational equilibria of the type defined in Section 2 always exist. Moreover, there is an equilibrium with prices dependent only on the quantity supplied, which is discussed in Section 3.

These existence results are a product of the monotone-likelihood ratio property of the normal distribution. That is, if  $\beta_1$  and  $\beta_2$  are two signals giving rise to normal distributions of supplies with means  $\mu_1$  and  $\mu_2$ , with  $\mu_1 > \mu_2$  then  $f_{\mu_1}(q)/f_{\mu_2}(q)$  is monotone increasing in  $q$ , where  $f_{\mu}$  is the normal density function with mean  $\mu$  and fixed variance. This implies the monotonicity of  $g(q)$  as defined in Section 2, and from this monotonicity of  $d(\cdot)$  is obvious.

In this case, therefore, we can take

$$Q_J = Q$$

and

$$\gamma(p) = g(d^{-1}(p))$$

for every  $p$  in the range of  $d(q)$ ,  $q \in Q$ , will be a self-fulfilling expectation pattern.

More generally, in a model with many states of nature and many signals, we have that if

$$g(q) = E(\beta | q)$$

is a one-to-one function from the set vectors of contingent contracts supplied to the convex hull of the set of all signals, then

$$\gamma(p) = g(d^{-1}(p))$$

is an equilibrium expectation pattern. Moreover, this result can be extended to cases in which supplies are not inelastic and the endowments of some uninformed individuals are random. At the present time, however, no general conditions for the existence other than monotone likelihood-ratio are known. This problem will be explored further in future work.

*First version received April 1975; final version accepted October 1976 (Eds.).*

I am grateful to Frank Hahn for several discussions and to Truman Bewley for correcting an earlier error. This research was supported under National Science Foundation Grant SOC 71-03803 to Harvard University.

NOTES

1. To see that this can be done, one can proceed in two steps. First we show that  $D_1$  and  $D_2$  can be constructed so that

$$D_1^{-1}(p) = D_2^{-1}(p) - K$$

over an interval of prices  $[p, \bar{p}]$  for an arbitrarily chosen  $K > 0$ . Then by renormalizing prices appropriately we can make  $D_1^{-1}$  and  $D_2^{-1}$  linear.

This equation requires that

$$\beta_1 u'(x) - p(1 - \beta_1)u'(1 - x) = 0$$

implies

$$\beta_2 u'(x + K) - p(1 - \beta_2)u'(1 - x - K) = 0.$$

Let  $\beta_1$  and  $\beta_2$  be large and let  $u'(x)$  be constant for  $x$  corresponding to consumption levels chosen for state 2.

Thus,  $D_1^{-1}(p)$  must satisfy

$$(i) \quad u'(D_1^{-1}(p)) = A(1 - \beta_1)p/\beta_1$$

and

$$(ii) \quad u'(D_1^{-1}(p) + K) = A(1 - \beta_2)p/\beta_2.$$

From (i) we find the value of  $u'(D_1^{-1}(\bar{p}))$  and by (ii) this gives us the value of  $u'(D_1^{-1}(\bar{p}) + K)$ .

$$u'(D_1^{-1}(\bar{p}) + K) = u'(D_1^{-1}(\bar{p})) \cdot \frac{1 - \beta_2}{1 - \beta_1} \cdot \frac{\beta_1}{\beta_2}.$$

This will also be the marginal utility of consumption in state 1 at the price  $p$  given by

$$u'(D_1^{-1}(\bar{p}) + K) \cdot \frac{\beta_2}{(1-\beta_2)} \cdot \frac{1}{A} = p^1.$$

For  $p \in (p^1, \bar{p})$  we can choose  $u'(D_1^{-1}(p))$  between  $u'(D_1^{-1}(\bar{p}) + K)$  and  $u'(D_1^{-1}(\bar{p}))$ . From these values of  $u'$  we then construct those for  $u'(x)$  for  $x \in [D_1^{-1}(\bar{p}) + K, D_1^{-1}(\bar{p}) + 2K]$ .

Continuing in this way we construct  $u'$  in each interval of length  $K$ . Each time generating a value of  $p$ , denoted  $p^m$ , according to

$$u'(D_1^{-1}(\bar{p}) + mK) \frac{\beta_2}{1-\beta_2} \cdot \frac{1}{A} = p^m.$$

The  $p^m$  will approach zero because

$$u'(D_1^{-1}(\bar{p}) + mK) = \left( \frac{1-\beta_2}{1-\beta_1} \cdot \frac{\beta_1}{\beta_2} \right)^m u'(D_1^{-1}(\bar{p}))$$

and  $\beta_2 > \beta_1$  implies that the parenthetic expression is decreasing to zero. For some  $m$  finite,  $p^m < p$  and the construction can be terminated at that point.

2. I am indebted to a referee for settling this question. The sequence can be constructed by choosing a countably dense collection of points  $W$  in the space of quantities supplied such that

$$\{p = D_1(q_1^s(w)); w \in W\} \cap \{p = D_2(q_2^s(w)); w \in W\} = \phi.$$

Then if endowments are almost always in  $W$ , equilibria will exist. Indeed, one can show that fully informed equilibria will exist. The distribution of endowments can be chosen to converge to an arbitrary distribution, while it remains concentrated on this set all along the sequence.

## REFERENCES

- [1] Benassy, J. P. "Disequilibrium Theory" (Working paper IP-185, Center for Research in Management Science, Berkeley, June 1973).
- [2] Debreu, G. "Neighboring Economic Agents", in *La Decision* (Editions du Centre National de la Recherche Scientifique, pp. 85-90, Paris, 1969).
- [3] Drèze, J. "Existence of an Exchange Equilibrium under Price Rigidities", *International Economic Review*, 16 (2) (June 1975), 301-320.
- [4] Grandmont, J.-M. and Laroque, G. "On Temporary Keynesian Equilibria", *Review of Economic Studies*, 43 (1) (February 1976), 53-68.
- [5] Green, J. "Information, Efficiency and Equilibrium" (Harvard Institute of Economic Research, discussion paper 284, March 1973).
- [6] Grossman, S. "Existence of Futures Markets, Noisy Rational Expectations and Informational Externalities" (Technical report no. 182, Institute for Mathematical Studies in the Social Sciences, September 1975).
- [7] Grossman, S. and Stiglitz, J. "Information in a Competitive Asset Market" (presented to conference on the economics of information, Stanford, March 1975).
- [8] Hahn, F. H. "Neoclassical Economics and its Critics" (Lectures at Harvard University, Winter 1975).
- [9] Hildenbrand, W. *Core and Equilibria of a Large Economy* (Princeton University Press, 1974).
- [10] Hirshleifer, J. "Private and Social Value of Information and the Reward to Inventive Activity", *American Economic Review*, 61 (4) (1971), pp. 561-574.
- [11] Jordan, J. S. "Expectations Equilibrium and Informational Efficiency for Stochastic Environments" (University of Pennsylvania, discussion paper, April 1976).
- [12] Kreps, D. M. "A Note on 'Fulfilled Expectations' Equilibria" (Institute for Mathematical Studies in the Social Sciences, technical report No. 178, Stanford University, September 1975).
- [13] Radner, R. "Equilibre des marchés à terme et au comptant en cas d'incertitude", *Cahiers d'Econometrie*, no. 9 (1967).
- [14] Younes, Y. "On the Role of Money in the Process of Exchange and the Existence of a Non-Walrasian Equilibrium", *Review of Economic Studies*, 42 (4) (October 1975), 489-502.

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