

INFORMATION, EFFICIENCY AND EQUILIBRIUM

by

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December 1975

Abstract

When economic agents receive information over time concerning future events it is likely that prices for commodities whose value is influenced by these events will fluctuate in response to changes in the state of knowledge. If such events occur periodically, participants in the market will notice that the prices could be used, to some extent, as a proxy for the relevant information. Learning of this type will take place by those agents who do not receive the information directly. Therefore, under stationary conditions, equilibria in this market will be characterized by the endogenous formation of expectations, dependent on the price system. The purpose of this paper is to define and characterize this type of equilibrium under various conditions and to study its efficiency properties. We will allow the choice of whether to receive information directly to be endogenous. The model will hopefully have useful implications for the theories of speculation and temporary equilibrium.

1. Introduction

In the theory of general equilibrium under uncertainty, the subjective beliefs of economic agents are generally treated as data of the system. Radner(1968) considers a model in which information about the state of nature is revealed sequentially to the agents, and in which the structure of this revelation over time may vary over the agents. Though the main part of his paper is concerned with the case in which the information structure is fixed, Radner extends his model¹ to the case in which agents have a choice over information structures, more informative structures being more costly to obtain. The knowledge possessed by an agent at any date is represented by a partition of the set of states of nature. The interpretation of this is that the agent will know, at any date, the element of the associated partition in which the true state lies. But he cannot distinguish between states in the same element of the partition. In particular, the kind of information that can be treated in this way does not allow for improvements in probabilistic knowledge.² Subjective beliefs are imbedded in the structure of preferences.

In this paper we present a general equilibrium model in which it is possible for agents to receive (at a cost) exogenous signals that are correlated with the true state of nature. These signals do not necessarily narrow down the class of possible states of nature, as an improvement of information in the Radner sense would require. Better information in our model, loosely speaking, will be a higher level of correlation between the signal and the true state.

In a recent paper, Hirshleifer (1971), treated some examples which fit into the framework we will be using. He addressed the question of whether the competitive system results in appropriate incentives for inventive activity, or information gathering. The conclusions of his study were that, in many cases, the private incentive to information gathering exceeds its social value - a result contrary to many of the discussions implying that inventive activity is a public good.

In this paper we introduce an additional effect into the Hirshleifer model. People who do not engage in information gathering still can draw inferences about the true state by observing the equilibrium price system because this is influenced by the signals received by those people who have invested in this activity. In this way the price system disseminates private information to the market. Thus, although there may be no direct interaction between the agents, prices are a natural source of informational externalities. We will be concerned with specifying precisely how they arise and in classifying those cases in which their effects will be positive or negative. Finally we ask whether there is scope for public action to improve the efficiency of the competitive system.

2. Basic Model

We are concerned with an economy in which there are commodities at various dates and in which the environment may evolve along any one of a number of possible paths. Although the model of this section conforms to that of standard general equilibrium theory in all aspects other than the treatment of information, the generalizations we shall discuss in later sections and elsewhere are more appropriately studied in the light of a different institutional structure. We shall suppose, for simplicity, that there are only two periods and that there are markets at the initial date for currently deliverable commodities and for all future deliverable commodities contingent on the state of nature. Further all agents know that, after the state of nature is realized, there will be another meeting of the market for commodities uncontingently deliverable. Of course in the frictionless Arrow-Debreu world such as we assume in this section, such a reconvening of markets is irrelevant since relative prices will not change. However, when transactions costs, random preferences or other imperfections are superimposed on this system, the existence of trading in the second period becomes necessary and exerts a substantial influence on the behavior of the system.³

We assume that there are l commodities in each period and n states of nature. All consumption sets are taken to be the non-negative orthant of the commodity space.

The set of states of nature is written $S = \{s_1, \dots, s_n\}$. Currently deliverable commodities are denoted by $x \in R_+^l$; future commodities contingent on state k are also elements of R_+^l , for each

state. Price systems are vectors in the $l(n + 1)$ dimensional simplex, where the first l coordinates refer to currently deliverable commodities and the rest of the prices are for contingent commodities. The traders know that at the second date, after the true state of nature is known, markets will reconvene, and some price vector, $q \in \Delta^l$, will be an equilibrium. The value of q will depend of course on which state has occurred, among other things.

In such an economy there are two types of uncertainty. One reflects uncertainty as to which state of nature will obtain, the other concerns what the equilibrium price system will be at the second date, given a knowledge of the state. The latter type of uncertainty has been treated elsewhere⁴, and for the present we wish to exclude this phenomenon. It should be possible to synthesize these models to include both forms of uncertainty. However, in this paper we shall concentrate on uncertainty about the state of nature, and on information bearing on these contingencies.

We assume that the future price systems in the various states are perceived identically and without uncertainty by all agents in the present. Thus we can reduce the model to one in which only state contingent income claims need to be traded at the initial date. Thus, let the consumption by any agent be represented as

$$(x, y) = (x, y^1, \dots, y^n)$$

where

$x \in R_+^l$ is the vector of commodities in the present and $y^k \in R_+$, $k=1, \dots, n$ is the income of the individual contingent on the occurrence of state k .

Similarly endowments are denoted

$$(\bar{x}, \bar{y}) = (\bar{x}, \bar{y}^1, \dots, \bar{y}^n)$$

with the same interpretation.

Prices will be denoted

$$p = (p^0, p^1, \dots, p^n) \in \Delta^{\ell+n}$$

where

$p^0 \in R_+^{\ell}$ is the price vector for current consumption and $p^k \in R_+$ is the price of contingent income in state k .

We assume that preferences of the agents may be represented as follows:

$$U(x, y) = \sum_k t_k u(x, y^k, k)$$

where t_k is the subjective probability of state k .

There may be two reasons for the explicit dependence of u on k . First, some commodities may be more useful in some states than in others.⁵

Second, the anticipated future price systems may vary with the state.

Thus, even though the underlying preferences for commodities may be represented by

$$V(x, x^1, \dots, x^n) = \sum_k t_k v(x, x^k)$$

preferences in the reduced space of current commodities and state contingent income claims will depend on k .⁶

The essential feature of this model is that subjective beliefs are not regarded as completely exogenous. In this section we take the following very simple approach to endogenous belief and outside information. It should be regarded only as an introduction to the more general models to follow.

The set of consumers is divided into two disjoint groups, denoted

A and A', which we shall call the informed and uninformed groups respectively. The number of consumers in the entire economy is denoted by I. A typical informed individual, a, receives an exogenous signal before the market meets at the initial date. This signal is denoted

$$\beta \in \Delta^n$$

and is interpreted by the individual to be the correct objective set of frequencies (probabilities) of the various states being realized. The signal affects his actions in the following way:

He maximizes

$$\sum_k \beta_k u^a(x, y^k, k)$$

subject to

$$p \cdot (\bar{x} - x, \bar{y} - y) \geq 0$$

where u^a is the utility function for agent a.⁷ This results in an excess demand function⁸

$$\xi^a: \Delta^n \times \Delta^{n+l} \rightarrow R^{n+l}$$

Thus the signal affects demands because it is used as the subjective probability vector by all agents who know it.

Individuals in the uninformed group, A', do not receive the signal. However, they try to use the market prices they observe as an indication of the relative frequencies. That is, they do not have fixed subjective probabilities in mind, but rather their subjective probabilities are given in general by a mapping

$$\gamma^{a'}: \Delta^{l+n} \rightarrow \Delta^n.$$

Temporarily let us speak of γ as if it were fixed, although later

we shall use a concept of equilibrium in which γ will be determined jointly with the price system in a self-fulfilling way. We may write the excess demand function for an uninformed individual, a' , as

$$\xi_{\gamma}^{a'}: \Delta^{n+l} \rightarrow R^{n+l}$$

where we have written the subscript γ just as a reminder that demands have been calculated using subjective probabilities given by γ . Thus prices affect demands through their induced expectations as well as through the budget equation.

Let us suppose that the signal can be any of a finite number⁹ of vectors and denote the set of all possible signals by $B \subseteq \Delta^n$. For each β , we can find a market clearing price system given expectations $\{\gamma^{a'}\}_{A'}$, defined as a vector $\beta \in \Delta^{l+n}$ such that

$$\sum_A \xi^a(\beta, p) + \sum_{A'} \xi_{\gamma}^{a'}(p) = 0.$$

The conditions under which this can be done are the usual continuity, desirability and convexity assumptions on utility functions, and continuity of expectations. We omit the proof.

Doing this for each $\beta \in B$, we define an equilibrium with expectations $\{\gamma^{a'}\}_{A'}$ as a mapping

$$\pi: B \rightarrow \Delta^{l+n}$$

such that for every $\beta \in B$, $\pi(\beta)$ is market clearing.

In such an equilibrium it generally will not be the case that expectations will be fulfilled. By this we mean that, if such a market is repeated many times, the asymptotic frequencies of the

different events given each signal will not correspond to the expectations associated with the market-clearing price systems for these signals. Thus one should expect that learning, or revision of expectations patterns will take place. In this paper we will not be concerned with modelling the learning process explicitly.¹⁰ The object will be to characterize the equilibria that are achieved after no further learning is possible.

We assume that this economy is repeated many times. The frequencies of occurrence of the various signals are denoted μ_β . Thus the frequencies of the states of nature are just the mean of β with respect to its distribution, since each signal is a statistically correct indicator of the probabilities of the states.

Thus, an expectatious pattern γ will be correct, or fulfilled by the price mapping π if, for each p in the image of B under π ,

$$\gamma(p) = \frac{\sum_{\beta \in \pi^{-1}(p)} \beta \cdot \mu_\beta}{\sum_{\beta \in \pi^{-1}(p)} \mu_\beta}.$$

If $\pi^{-1}(p)$ is a one-element set, then this condition is just $\gamma = \beta$. But if different signals lead to the same price system, then they will be confused by uninformed agents who can observe prices only and not the signal directly. In this case a weighted combination of the relevant signals will be associated with this price vector in a statistical equilibrium of this kind.

We shall say that (π, γ) is an equilibrium if

- 1) γ is fulfilled by the price mapping π
 and 2) π is an equilibrium with expectations $\{\gamma\}$.

We first discuss briefly the question of the existence of an equilibrium in this system. If for each β the resulting market clearing price system were distinct, then we could show that an equilibrium exists as follows:

Take $\beta^1 \in B$ and let $\gamma^1(p) \equiv \beta^1$ for all $p \in \Delta^{l+n}$, for the economy with expectations γ^1 . Then take $\beta^2 \in B$ $\beta^2 \neq \beta^1$ and let

$$\gamma^2(p) = \begin{cases} \beta^2 & \text{for } p \neq p^1 \\ \beta^1 & \text{for } p = p^1. \end{cases}$$

By assumption we can find a market clearing

price system, p^2 , for the economy with expectations γ^2 and p^2 will be different from p^1 . Repeating the process in this way, we arrive at an expectations mapping $\gamma^{|B|}$ and finitely many prices, $p^1, \dots, p^{|B|}$. The equilibrium γ can be taken to be any mapping which has value β^j at price system p^j for all j .

This proof rests crucially on the assumption that if γ^i and γ^j are different constant expectations functions, then the market clearing prices associated with them will be different, and hence that the problem of two signals giving rise to the same price system in an equilibrium will not arise. We would have, in such a case, that for every $\beta \in B$, $\gamma(\pi(\beta)) = \beta$ - as opposed to the more general possibility allowed in the definition of fulfilled expectations. In complete generality, however, it is not possible to guarantee that such coincidences will not arise. A characterization of certain conditions under which they can be avoided, and hence equilibrium is sure to exist, is given by Kihlstrom

and Mirman (1975). There are, however, two other approaches. One is the possibility of showing that such coincidences are "rare", using differential topology and related techniques. The other would be to let B be an infinite set and μ a non-atomic measure on B . In this case, if results on "rareness" were to be had, the corresponding interpretation would be that the condition for expectation fulfillment would be satisfied for almost-every price system.

While both of these extensions would be interesting, they have not been pursued herein. The reason, apart from the difficulty of the project itself, is that the model of genuine economic interest is, I believe, that of the next section. The present section is merely a necessary prelude. There, however, the problems are more severe. After a long search for an existence theorem, the author constructed a non-pathological counter-example. Conditions sufficient to insure existence are known, but they are not of the usual type. That is, they concern the shapes of functions rather than their topological properties or limiting behavior, see Green (1975).

3. Endogenous Information Gathering: the case of random excess demands and noisy information

In this section we extend the model of section 2 to a case in which uninformed individuals do not receive information through the price system that is as accurate as that received directly by the informed group. We suppose that becoming informed is costly in terms of real resources, and that the decision about whether or not to incur such costs is taken by each individual.

In this model there is an additional element of uncertainty. Within each state of nature we suppose that the endowments of agents are random. Although this could formally be eliminated by treating each different endowment distribution as a separate state of nature, it is assumed that, for reasons of transaction costs perhaps, such markets do not exist and some residual variability of excess demand functions is present. More generally, any model with an incomplete set of markets has the potential that direct information concerning the underlying states of nature will be a less noisy signal than this price system. On dimensionality grounds the reasons are clear - direct information gives $(n-1)$ independent relative frequencies whereas with fewer markets than this, many different signals will generally be reflected in the same market-clearing price. Presently we consider the special case in which endowments and exogenous events are both random, but contingent markets exist only for the latter.

Let $(\Omega, \mathcal{F}, \nu)$ be a measure space and let $e: \Omega \rightarrow R^{(k+n)I}$ be a random variable whose value is the endowment distribution for each $\omega \in \Omega$ ¹¹. Thus the excess demands for both the informed and uninformed groups depends on ω , and for the informed group, of course on β . An equilibrium

with a fixed set of informed individuals is a pair (π, γ) , where

$$\pi: \Delta^n \times \Omega \rightarrow \Delta^{l+n}$$

and
$$\gamma: \Delta^{l+n} \rightarrow \Delta^n$$

with the properties that π is an equilibrium with expectations γ for every $(\beta, \omega) \in B \times \Omega$, and that γ is fulfilled at π .¹²

In this setting, the definition of γ being fulfilled is that, for every¹³ p in the image of $\Delta^n \times \Omega$,¹⁴

$$\gamma(p) = \frac{\sum_{\beta \in B} \beta \cdot \mu_{\beta} v(\{\omega | \pi(\beta, \omega) = p\})}{\sum_{\beta \in B} \mu_{\beta} v(\{\omega | \pi(\beta, \omega) = p\})}$$

We now allow that individuals can choose whether or not to become informed. The cost of being informed is given by a vector of currently available commodities $b \in R_+^l$. We assume that the decision to become informed is taken once and for all and that individuals who choose to receive the signal have the vector b subtracted unconditionally from their endowments of currently available commodities for all $\omega \in \Omega$ in every iteration of the economy. If this subtraction makes some component of the endowment vector negative and leads the individual to have negative wealth for some p that occur with positive probability in an equilibrium, then the individual is prohibited from becoming informed.¹⁵

In order to determine what course of action the individual will follow, it is necessary to be explicit about what he knows regarding his situation at present vis à vis what information he will have if he acquires the signal. We assume that the individual knows B and $\{\mu_{\beta}\}_{\beta \in B}$. That is, before becoming informed he knows what the set of potential signals is and what their relative frequencies will be. Using this knowledge he calculates the anticipated average utility

from being informed as follows. Once receiving the signal, he will be maximizing, in each iteration,

$$\sum_k \beta_k u(x, y^k, k)$$

subject to $p. (\bar{x} - x - b, \bar{y} - y) \geq 0$

To calculate the average utility of the solution to this problem, it is necessary for him to know the joint distribution of p and β . Let the measure η on $\Delta^{l+n} \times B$ be this joint distribution.

In order to estimate η he uses his knowledge of three things, the marginal distribution of β , $\{\mu_\beta\}$, the marginal distribution of p , which he observes in the ongoing equilibrium, and the conditional distribution of the states for each p , which he has correctly assessed as $\gamma(p)$. The conditional distribution of the states is also the conditional mean signal, since the signals are statistically correct.

It is clear that if B is a set of linearly independent vectors in Δ^n , then knowledge of the conditional means and the set B is sufficient to imply the joint distribution, η . However if this condition fails, in particular if there are more than n members of B , this cannot be done and the individual is faced with a statistical decision problem. To treat this problem in full generality would require that the individual have a subjective distribution over the set of all η that are consistent with his evidence, and then maximizes his expected average utility with respect to this distribution.¹⁶ This would introduce still another complexity into our problem, and we shall avoid it simply by assuming from now on that B is a set of linearly independent vectors. The basic character of the inefficiency results to follow should not

be changed in dropping this restriction; though at present this remains an open problem.

A full equilibrium is a triple (A, π, γ) where A is a subset of the set of agents, (π, γ) is an equilibrium with a fixed set of informed agents A , no member of A would like to become uninformed and no uninformed agent would rather pay for receiving the signal. It is important to note that, because of the competitive character of the model, each individual fails to take account of the change he would effect in (π, γ) if he were to switch from one group to another.

In a model with a finite set of informed individuals the indivisible nature of agents may cause non-existence of a full equilibrium. As more agents become informed the equilibrium (π, γ) will change and this will be discontinuous due to the finiteness of the number of agents. Although this discontinuity could be removed by introducing a continuum of agents, it would still not be the case that the existence of an equilibrium could be guaranteed. This is explored more fully in Green (1975). The present paper will be confined to efficiency considerations in situations sufficiently non-pathological to admit the existence of equilibria. Section 4 below presents an example, so that the concept is shown to be non-vacuous.

There is an essential difference between this model and that of section 2. There we argued that it would be highly unlikely that two different signals would give rise to the same price system. Uninformed individuals therefore do not have any less information than informed individuals in an equilibrium. But in this

model, for each β there is a distribution of possible prices and for different β they will have overlapping supports. Uninformed people will have inferior information whenever prices turn out to be in this region of intersection. We shall be interested in the implications of this induced confusion for the efficiency of the system, a topic to which we shall turn in Section 5. Before doing so, however, we will present an example of an equilibrium of this type.

4. Example

Let us suppose that there are two possible states of nature and no currently available commodities, so that $l = 0$, $n = 2$. There are three individuals in the economy, one informed and two uninformed. All individuals have the same utility function

$$u(y^1, y^2) = t_1 \log(y^1) + t_2 \log(y^2)$$

where t_k is the subjective probability of state k occurring. There are two possible signals, occurring with equal probability

$$\beta^1 = \left(\frac{1}{4}, \frac{3}{4}\right)$$

$$\beta^2 = \left(\frac{3}{4}, \frac{1}{4}\right)$$

The endowments of the two uninformed individuals are non-random and equal to $(0, \frac{1}{2})$. The informed individual has a random endowment $(\lambda, 0)$, of the two contingent income claims. The distribution of λ is given by the density function

$$g(\lambda) = \frac{3}{8}(3\lambda - 1) \quad \lambda \in \left[\frac{1}{3}, 1\right]$$

$$g(\lambda) = \frac{3}{8}(3 - \lambda) \quad \lambda \in [1, 3]$$

$$g(\lambda) = 0 \quad \text{otherwise}$$

Let p be the relative price of y^2 - that is, let y^1 be numeraire.

Let $\gamma(p)$ be the subjective probability of state 1.

From this data one can compute the excess demand function for the economy as it depends on β , $\gamma(p)$, p and λ .

For $\beta = \beta_1$, excess demand for y^1 is given by

$$\gamma(p) \cdot p - \frac{3\lambda}{4}$$

For $\beta = \beta_2$, excess demand for y^1 is

$$\gamma(p) = p - \frac{\lambda}{4}$$

Excess demands for y^2 can be computed using Walras law.

If expectations patterns for both uninformed individuals are given by

$$\begin{aligned} \gamma(p) &= \frac{3}{4} & p &\leq \frac{1}{3} \\ \gamma(p) &= \frac{1}{4} & p &\geq 3 \\ \gamma(p) &= \frac{1}{1+p} & \frac{1}{3} < p < 3 \end{aligned}$$

then it can be computed that the following $\pi(\cdot, \cdot)$ is an equilibrium with expectations γ :

$$\begin{aligned} \pi(\beta^1, \lambda) &= 3\lambda & 1 &\leq \lambda \leq 3 \\ &= \frac{3\lambda}{4 - 3\lambda} & \frac{1}{3} &\leq \lambda \leq 1 \\ \pi(\beta^2, \lambda) &= \frac{\lambda}{4 - \lambda} & 1 &\leq \lambda \leq 3 \\ &= \frac{\lambda}{3} & \frac{1}{3} &\leq \lambda \leq 1 \end{aligned}$$

To verify that this is infact an equilibrium with a fixed set of informed individuals, first notice that if $p < \frac{1}{3}$ then $\beta = \beta^2$ and if $p > 3$ then $\beta = \beta^1$. Thus we must have $\gamma = \frac{3}{4}$ in the former case and $\gamma = \frac{1}{4}$ in the latter, which is true as γ has been constructed. Consider p between $\frac{1}{3}$ and 3. If p arose from β^1 then we must have that the λ from which it resulted solves $\pi(\beta^1, \lambda) = p$. This can be shown to be $\frac{4}{3} \frac{p}{1+p}$. If the same p occurred after signal β^2 , then the value of λ must have been $4 \frac{p}{1+p}$.

If γ is fulfilled by π we must have that

$$\gamma(p) = \frac{\beta^1 \mu_{\beta^1} g(\lambda^1(p)) + \beta^2 \mu_{\beta^2} g(\lambda^2(p))}{\mu_{\beta^1} g(\lambda^1(p)) + \mu_{\beta^2} g(\lambda^2(p))}$$

where $\lambda^k(p)$ is the value of λ such that $\pi(\beta^k, \lambda^k(p)) = p$.

In our example this may be rewritten as

$$\gamma(p) = \frac{\frac{1}{4} g(\frac{4}{3} \frac{p}{1+p}) + \frac{3}{4} g(4 \frac{p}{1+p})}{g(\frac{4}{3} \frac{p}{1+p}) + g(4 \frac{p}{1+p})}$$

Since $p \in [\frac{1}{3}, 3]$, $\frac{4}{3} \frac{p}{1+p} \in [\frac{1}{3}, 1]$ and $4 \frac{p}{1+p} \in [1, 3]$.

Using the definition of g ,

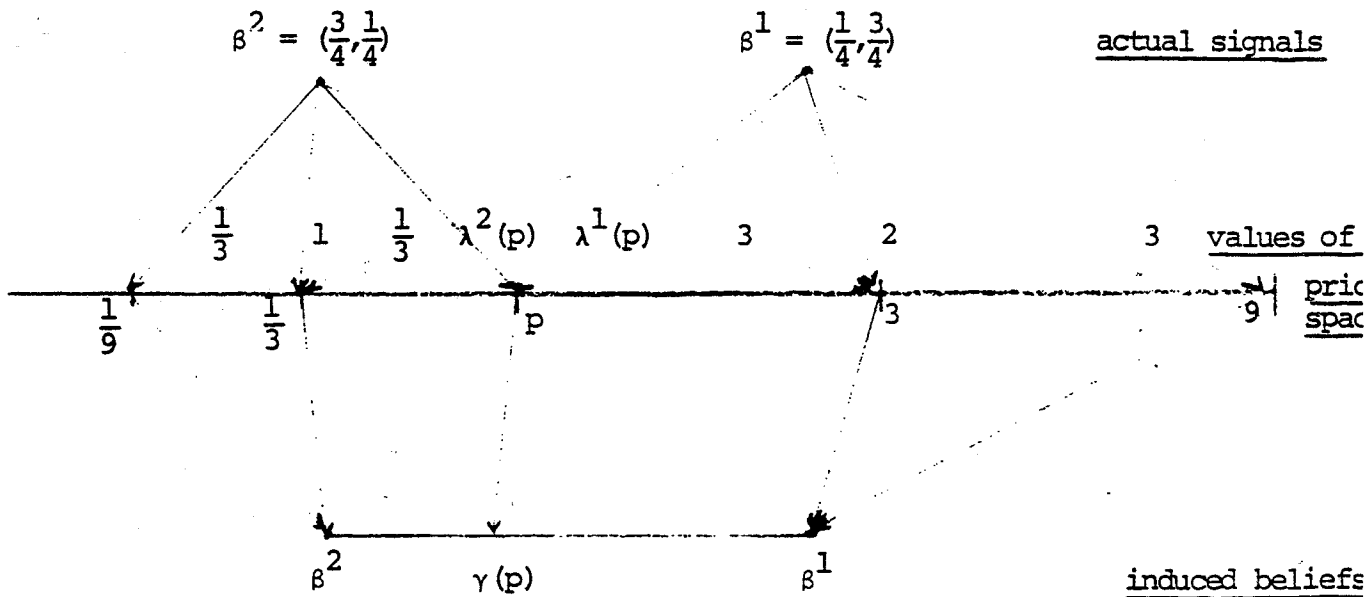
$$g(\frac{4}{3} \frac{p}{1+p}) = \frac{3}{8} \left(\frac{3p-1}{1+p} \right)$$

and

$$g(4 \frac{p}{1+p}) = \frac{3}{8} \left(\frac{3-p}{1+p} \right)$$

Upon substituting these expressions for g and $\gamma = \frac{1}{1+p}$ (which is valid in this range of p) one verifies that γ is fulfilled by π , and hence that (π, γ) is an equilibrium.

We can depict this equilibrium in the following diagram



Let us consider the position of an uninformed individual.

Whenever $p \in (\frac{1}{3}, 3)$, he would have benefited from knowing the signal directly because he would be faced with either the problem

$$\max \frac{1}{4} \log y^1 + \frac{3}{4} \log y^2$$

subject to

$$y^1 + py^2 \leq 1$$

or the problem

$$\max \frac{3}{4} \log y^1 + \frac{1}{4} \log y^2$$

subject to

$$y^1 + py^2 \leq 1$$

with probabilities $\alpha = \frac{3p-1}{2p+2}$ and $1-\alpha$

It can be calculated that the average utility attainable from solving this problem is

$$\frac{1}{1+p} \log p + \frac{1}{4} \log \frac{1}{4} + \frac{3}{4} \log \frac{3}{4}$$

Whereas, in the existing situation he solves the problem

$$\max \frac{1}{1+p} \log(y^1) + \frac{p}{1+p} \log(y^2)$$

subject to

$$y^1 + py^2 \leq 1$$

which results in an average utility of

$$\frac{1}{1+p} \log p + \frac{1}{1+p} \log \frac{1}{1+p} + \frac{p}{1+p} \log \frac{p}{1+p}.$$

It is easy to show that the individual is better off with the more detailed information for all $p \in (\frac{1}{3}, 3)$. Thus the total private benefit from acquiring this information as perceived by this individual is the integral of these utility differences with respect to the equilibrium

distribution of prices $\pi(,)$. However the cost of acquiring the signal might outweigh these benefits. For the informed individual, they may be lower and it would not therefore be to his advantage to cease receiving the signal.

5. Efficiency Considerations

In evaluating the efficiency of equilibria as defined in the previous sections there are several questions which one can ask. First, one may inquire as to how each iteration of the system allocates commodities given the particular signal that has occurred. The answer to this seems rather clear, but it is also not the most relevant type of efficiency, as we shall argue below. It is true, of course, due to the theorems of neo-classical welfare economics,¹ that with expectations fixed at their equilibrium values no reallocation of commodities can give any individual a higher expected utility without decreasing the expected utility of another. However, expectations are not really fixed in this model - they are generated through the price system. Thus it is somewhat misleading to use a price system to find an equilibrium, generating expectations in the process, and then discard this price system in making the alternative distributions of resources. Suppose instead that we consider all expected utilities over individuals that can be attained in competitive equilibria after having redistributed the endowments. In such a regime it seems clear, the original endowment distribution has no special claim to optimality in this subjective sense. It is possible to redistribute endowments in such a way that the price system changes to allow much more information to be received by the uninformed group. Indeed, due to the possibility of multiple equilibria at each (w, β) , it is not even clear that every equilibrium is Pareto optimal in this ex ante sense, since some may be associated with much better information dissemination than others.

Since we have viewed this system in a repetitive context in establishing statistically correct expectations, it is natural to pose questions regarding the efficiency of the competitive equilibria "on average" over time. We will be more precise about this shortly; but we shall first discuss the concepts of efficiency studied by Starr (1973). His model is identical to the above, except that expectations are exogenous to the system and informational considerations are omitted. It is shown that for an ex ante optimum (in the expected utility sense) to be optimal ex post, after the occurrence of state k , it is necessary that expectations be sufficiently similar. In the case of differentiable demand functions, without corner solutions, "sufficiently similar" can be taken to mean identical. Optimality ex post is a condition of "no regrets". That is, allowing reallocations only of current consumptions and state k - contingent consumptions, no superior position could be reached, given that state k is the true state.

In the perfectly frictionless model of section 2, one should note that (as long as one individual is receiving the signal) every uninformed person receives the same information indirectly. Thus, for each signal the result is ex post optimal in this sense.

Let us define an allocation pattern over signals B to be a function

$$f: B \times \Omega \rightarrow R^{I(l+n)}$$

which is feasible for each $\omega \in \Omega$, where there are I individuals in the economy. The vector $f_i(\beta, \omega) \in R^{n+l}$ is the consumption plan of the i^{th} individual given that the signal has been β and that ω

is the underlying point in the probability space. The average utilities resulting from an allocation pattern f

$$U(f) \in R^I$$

are defined by

$$U_i(f) = \sum_B \sum_k \int \beta_k u_{\beta}^i(f_i(\beta, \omega)) d\nu, i=1, \dots, I$$

It is easy to show that if for each $\beta \in B$ every individual has correct subjective probabilities, then the average utilities resulting from the competitive allocation pattern are optimal relative to the set of all allocation patterns over the same set of signals B . Naturally a set of signals that is more detailed or more highly correlated with the state of nature will make a better allocation possible. Thus, due to the fact that information is disseminated perfectly in this frictionless world, the resulting allocation is optimal ex post in Starr's sense.

The situation is somewhat different, however, in the noisy information case of section 3. In general there will be some (β, ω) for which the uninformed people have different expectations than the informed people. This is not to say that their expectations are incorrect - they are statistically verified and as accurate as possible given the identification problem they face. Diversity of beliefs gives rise to a potential misallocation of resources ex post as shown in Starr (op.cit., corollary 6.1). However we should be cautious lest we conclude too quickly that the competitive process is inefficient - though it will be argued below that indeed it is. Although it is true that the resulting allocation would be preferable

if everyone had correct beliefs, information is costly, and the potential of the price system to transmit information places a constraint on the performance of the system. In defining efficiency we should take account of these facts and use a set of alternative allocations that are attainable under the same informational restrictions.

This suggests that any alternative allocation should be attainable as an equilibrium with expectations generated through the price system, but perhaps with some government intervention in the form of taxes or subsidies. Naturally an optimum relative to such a set of allocations is a second-best in a very strong sense, since with internally generated expectations many allocation patterns could not be sustained as competitive equilibria. Hirshleifer has shown that there may be a divergence between the private and social returns to information gathering activity. In his model, individuals who did not engage in such activity held stastically correct expectations that were independent of prices and hence of the signal received by the informed group. The model we have used introduces another effect into this system. In acquiring the signal, the resulting price system will change, and the correlation between the expectations of the uninformed group and the signal received will also be altered. In general one can expect that the distribution of prices will be more highly correlated with the true signal, and relatively less dependent on the random component of excess demand as the informed group grows. This causes beneficial externalities to the uninformed group to the extent that they have preferences that are temporally

dependent or are engaged in intertemporal production activities, which we have not treated but which can easily be included in this model. On the other hand, with separable utility and no productive value to information, the information dissemination effect studied here will reinforce the original Hirshleifer effect. The presence of this type of informational externality, which can only be of relevance in the case of noisy information, is the principle conclusion of this study.

There are, therefore, three relevant considerations for the efficiency of equilibria in this model. The Hirshleifer effect and its reinforcement through the information dissemination effect are externalities of improved information which are effective for every possible set of informed individuals. Additional externalities are generated through the decision to become informed directly. When acquiring superior information in this way, the individual correctly weighs its private benefit to him against the private cost. The induced change in his demand function will affect the equilibrium price system and hence the accuracy with which the information is transmitted to those who remain uninformed. In this way the decision to become informed sets up forces which interact with the informational externalities present in the fixed-category situation.

6. Summary and Possible Extensions

We have tried to introduce the choice of obtaining exogenous information into a model of temporary equilibrium which is repeated over time. The concept of equilibrium we have used is one in which expectations, though not correct in the sense of perfect foresight, are statistically verified. It has been argued that inherent in such a system is the possibility of inefficiency due to externalities of various types.

There are many possible extensions of this model and applications of this framework to other situations. One of the more interesting of these is to treat cases in which the adjustment to equilibrium takes place through quantity variables as well as prices. For a variety of reasons it may be that prices are rigid, or at least imperfectly flexible in the period of adjustment to temporary equilibrium. Thus the actual path of economic activity may involve some unrealized plans - perhaps unintended fluctuations in inventories, unemployment or unfilled orders. Because information about the level of inventories or unfilled orders is usually private, whereas prices are publically disseminated, the framework explored in this paper may prove useful. If there are such frictions in the economy, they may lead to the same types of inefficiency as noisy prices, since both disguise the true state of the system from the view of those individuals who are not directly informed.

Other potential applications are to capital market theory or to the theory of speculation. Some recent work by Grossman (1975a,b) and Grossman and Stiglitz (1975) pursues these issues.

Footnotes

* This research was supported under NSF Grant SOC 71-03803.

1. op.cit. section 9, also Marschak and Radner (1972).
2. In the framework of Marschak and Radner (1972), Ch. 2, such improvements can be treated by expanding the definition of states of nature to include the possible values of the information to be received. In a competitive model without transactions costs it would then be required to have a separate market operating for each of these states. In our framework states will be defined ab initio, the information structure being treated separately. This requires only one market for each state of nature, regardless of the informational environment. The preferability of either method depends on the relevant assumption to be made concerning the timing of market-clearing vis-à-vis the receipt of new information.
3. See e.g. Green, J. (1971) or remarks in Foley (1970).
4. Green, J., op. cit.
5. For example, medicine in a state of ill health.
6. Hence the axioms for choice over lotteries implying that preferences depend on outcomes and not directly on the state will not be valid and it would be incorrect to write

$$U = \sum t_k u(x, y^k).$$
7. We shall follow the practice of indexing individuals with superscripts a or a' .
8. We are neglecting problems of potentially multi-valued demands. As the main purpose of this paper is to introduce some concepts and point out new phenomena this lapse can perhaps be temporarily excused.
9. This is just for expository convenience and can easily be relaxed.
10. On this problem, which can be very serious in imperfectly competitive settings, see Arrow and Green (1973). Rothschild (1972) treats a related, but different aspect of the learning problem. Rothschild (1971) contains an excellent survey of the imperfect information literature.
11. To be formally consistent here we should allow each individual's expectations to depend on his endowment as well as on the prices. This is reasonable since the individual must be able to observe his own endowment, even though those of other agents are unobservable. Thus γ would be defined by

$$\gamma: \Delta^{l+n} \times R^{l+n} \rightarrow \Delta^n$$

and the condition for expectation fulfillment for a typical individual would become

$$\begin{aligned} \gamma^{a'}(p, e_a) &= E(\beta \mid p, e_a) \\ &= \frac{\sum \beta_i \mu_{\beta_i} \text{Prob}(\pi(\beta_i, \omega) = p \text{ and } e_a(\omega) = e_a)}{\text{Prob}(\pi(\beta_i, \omega) = p \text{ and } e_a(\omega) = e_a)} \end{aligned}$$

12. This is an application, to the special case of non-stochastic tastes but stochastic endowments, of the structure of Hildenbrand (1971). The equilibrium concept we shall use, however, is closer to that proposed in Hildenbrand as an alternative, (op. cit., p. 427) and pursued by Bhattacharya and Majumdar (1971).

13. We will not distinguish between statements made everywhere or almost everywhere. Statements about (β, ω) are to be understood relative to the product measure - β and ω are assumed to be independently distributed.

14. In the case in which Ω has infinitely many points and the distribution of $\pi(\beta, \cdot)$ is atomless for every β , this should be written as

$$\gamma(p) = \frac{\sum_{\beta \in B} \beta \mu_{\beta} g(\beta, p)}{\sum_{\beta \in B} \mu_{\beta} g(\beta, p)}$$

for almost-every p in the range of $\pi(\cdot, \cdot)$, where $g(\beta, p)$ is the density function of the distribution of $\pi(\beta, \cdot)$ evaluated at p .

15. Of course this discussion of the bankruptcy issue is not rigorous because we have not been specific about what it means for p to be an equilibrium when someone has negative wealth. The reader is referred to Arrow and Hahn (1971), Grandmont (1970) or Green (1972) for explicit treatments of this issue. Without belaboring the point, it will not detract from the present discussion if we assume that b is so small that bankruptcy does not happen.
16. It would also be possible to specify non-Baysian decision rules; but one would then be forced to use criteria other than the maximization of expected utility.
17. See, e.g., Debreu (1959).

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