

THE QUESTION OF COLLECTIVE RATIONALITY IN PROFESSOR GALE'S MODEL OF TRADE IMBALANCE *

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1. Introduction

In Gale (1971) a model of international trade is studied in which there are two equilibria, one of which is a state of permanent trade imbalance. In such an equilibrium one country exports commodities to the other in every period. The object of this note is to explore this equilibrium further and to ask whether such a seemingly irrational action by the net exporter might be avoidable by the use of certain policy measures. We shall derive conditions under which it is for two particular policies and discuss how these results depend on the particular structure of Professor Gale's system.

2. The model

There are two countries which we shall later distinguish by superscripts 1 and 2. Each of these is described as follows: Time is measured in discrete intervals $t, t+1, \dots$. Every individual lives two periods and a new generation of identical size replaces the people who have just died.

There is only one good, c , used for consumption. It is produced using β/γ units of labor, l , and $1/\gamma$ units of capital, k . Output of c appears concurrently with the application of inputs. Capital is produced from

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labor alone, but with a one period lag. (The unit of capital is chosen so that 1 unit of labor produces one unit of capital.) Neither capital nor consumption goods are durable. They disappear completely the period after they are made, whether or not they are used or consumed.

Everyone's utility function is $u(c_1, c_2) = c_1^{1-\sigma} c_2^\sigma$, where c_1 and c_2 are consumption the first and second periods of life. Prices are: w_t – current labor services, p_t – current capital services, p_{t+1} – present-value of capital service next period, q_t – current consumption. All are reckoned in units of account, and are market prices at date t .

That is, at each date t , contracts can be traded for the performance of labor services, for delivery of (spot) capital goods or consumption goods, or for the future delivery of capital goods. All payments are made at t .

The coexistence of the two equilibria shown by Professor Gale is a consequence of the structure of trade implicit in his analysis. This is discussed in his 'additional remarks'. Each period young individuals are paid w_t in units of account. They spend $(1-\sigma)w_t$ on consumption and save σw_t for their old age. This saving is accomplished by simply not spending. It is carried forward to the next period by being recorded by a bookkeeper, by the holding of (durable) money, or by some other 'contrivance'. In this way the budget equation is insured.

The question now arises as to who builds capital goods. Since saving can be accomplished by not spending, the return to capital cannot be different from the return to holding money (balances with the bookkeeper, etc.). But what is the return to capital? Capital goods can be built with the intention of selling them next period (when they appear). Thus, presumably, the individual who builds a capital good must know, or forecast, the price of this good in the equilibrium established next period (as well as the price of consumption next period).

However, an alternative is to sell a futures contract at t for delivery at $t+1$ of this capital good at price p_{t+1} , thereby eliminating the uncertainty as to the income received. It is assumed that this is, in fact, what people will do. If they are risk averters and if their expectations about next period's price had mean equal to p_{t+1} then this assumption would follow as a consequence of their maximizing behavior. In general, the decision about how much capital to build at date t depends on expectations about the price of capital at $t+1$. However, the utility function used by Professor Gale has the property that savings are independent of the rate of return (i.e., the price that occurs at the next market date). For this reason Professor Gale did not need to introduce

a separate symbol for this price, and we shall not do so either. Other utility functions would not have had this property, and it would have been necessary to confront the problem of expectations explicitly. It is useful to note the simplification that this choice of utility function induces as well as the problems that would arise from generalization.

Returning to the model at hand, Professor Gale shows the existence of price sequences such that demand for consumption goods equals output of these goods. The first of these equilibria, called balanced equilibrium, has the property that the value of wages of the young people equals the value of consumption of both generations combined. It is therefore called the " $w=c$ equilibrium".

The other equilibrium has the property that the value of savings is equal to the value of the steady state level of capital stock. This equality does not, in general, hold in the $w=c$ case. This equilibrium is unbalanced in the sense that one country exports consumption goods to the other country each period. It is called the " $s=k$ equilibrium".

In the $w=c$ equilibrium some of the capital may be financed by the bookkeeper (government) or some savings may be held in units of account without having been used to build anything. But in the unbalanced $s=k$ equilibrium equality holds between the value of savings and the value of capital stock. As Professor Gale notes, this coincidence allows us to assume a different market structure for which the *only* equilibrium will be the unbalanced one. In this market structure, capital is purchased directly and prices are such that demand and supply for both capital goods futures and for consumption goods are in balance. But, of course, instituting such a market structure (i.e., forcing $s=k$ as well as clearing the consumption goods market) would destroy the $w=c$ equilibrium, and a very interesting stability analysis by Professor Gale depends on the coexistence of these.¹ However, since the rest of my comment is directed at studying properties of $s=k$ equilibria, I shall assume that the market for capital goods futures is cleared (i.e., a bookkeeper or government that could effect the financing necessary in $w=c$ is not present).

To be specific about what markets exist and the resulting equilibrium conditions, we state them explicitly below. (We retain all of Professor Gale's notation and conventions.)

¹ Professor Gale shows that one of the two equilibria is stable and the other unstable. Which is shown to depend on the parameters of the system.

2.1. Factor markets:

Prices for l^i and k^i are determined competitively in each period according to

$$w_t^1 = p_{t+1}^1 \quad (1)$$

$$\beta^1 w_t^1 + p_t^1 = \gamma^1 q_t^1 \quad (2)$$

$$w_t^2 = p_{t+1}^2 \quad (3)$$

$$\beta^2 w_t^2 + p_t^2 = \gamma^2 q_t^2. \quad (4)$$

2.2. Bond markets and capital goods futures markets:

In each country there is a market for delivery of capital goods next period. The steady state level of capital in country i is $1/(1+\beta^i)$ as a consequence of the full employment of labor condition, $k+\beta k=1$ (i.e., labor used in production of capital plus labor used in production of consumption goods equals total labor supply). It is also possible for young people in the country with excess autarchic savings (country 1 under our condition $1/(1+\beta^1) < \sigma < 1/(1+\beta^2)$) to buy bonds (payable in units of account, at interest factor R), from the young people in country 2. Another way of saying this is that they contract with people in country 2 to build them some capital (cf. Gale's remarks).

2.3. Consumption goods market:

Since consumption goods are internationally tradeable we have $q_t^1 = q_t^2$ for all t . q_{t+1}^1 is the present-value price of a contract to deliver consumption goods next period.

Wage income of young people in country 1 (in units of the consumption good) is w_t^1/q_t^1 . They choose to save $\sigma w_t^1/q_t^1$ and there are two alternatives available. They can buy capital goods futures, obtaining $\sigma w_t^1/p_{t+1}^1$ units of them for their savings, which would yield $\sigma w_t^1/q_{t+1}^1$ units of consumption good as their rental next period, or, they can buy bonds at interest factor R , yielding $(\sigma(w_t^1/q_t^1))R$ units of consumption good next period. Clearly, in equilibrium, we must have $R = q_t^1/q_{t+1}^1$, or these markets will not be cleared.

Similarly, young people in country 2 want to save $\sigma(w_t^2/q_t^2)$ and their alternatives are capital yielding $\sigma(q_t^2/q_{t+1}^2)$ or bonds yielding $(\sigma(w_t^2/q_t^2))R$. Thus $R = q_t^2/q_{t+1}^2$.

But the demand for capital goods futures must equal their supply world-wide. This means that, in units of the consumption good,

$$k^1 p_{t+1}^1 + k^2 p_{t+1}^2 = \sigma(w_t^1 + w_t^2).$$

In the bond market, we have equilibrium when the rate of interest is such that the value of exports from the young people in country 2 in period t is equal to the present value of their imports (when they are old) in period $t+1$. Exports of young people in country 1 equals savings minus value of domestic capital: $\sigma\gamma k^1 - k^1\gamma^1 k^1$. Imports next period is the difference between savings and capital in country 2: $-(\sigma - k^2)\gamma^2 k^2$. Thus

$$(\sigma - k^1)\gamma^1 k^1 q_t = -(\sigma - k^2)\gamma^2 k^2 q_{t+1}.$$

2.4. *The system:*

Thus the equilibrium equations for the steady-state reduce to

$$w_t^1 = p_{t+1}^1 \tag{1}$$

$$w_t^2 = p_{t+1}^2 \tag{2}$$

$$\beta^1 w_t^1 + p_t^1 = q_t^1 \gamma^1 \tag{3}$$

$$\beta^2 w_t^2 + p_t^2 = q_t^2 \gamma^2 \tag{4}$$

$$\sigma(w_t^1 + w_t^2) = k^1 p_{t+1}^1 + k^2 p_{t+1}^2 \tag{5}$$

$$p_{t+1}^1 R = p_t^1 \tag{6}$$

$$p_{t+1}^2 R = p_t^2 \tag{7}$$

$$q_t^1 = q_t^2 \tag{8}$$

$$(\sigma - k^1)(\gamma^1 k^1) q_t = -(\sigma - k^2)(\gamma^2 k^2) q_{t+1} \tag{9}$$

$$q_{t+1}^1 R = q_t^1 \tag{10}$$

$$q_{t+1}^2 R = q_t^2 \tag{11}$$

which is a system of eleven equations in the eleven unknowns – 4 prices of capital, 2 wage rates, 4 prices of the consumption good, and the rate of interest. One equation can be derived from the others, but one of the of the prices is also arbitrary.

3. Permanent disequilibrium discussed

As Professor Gale shows, in the ($s=k$) equilibrium, country 1 will continually receive a net inflow of goods if $R > 1$ and will continually export if $R < 1$. We take up, in this section, a reexamination of this phenomenon. First we discuss how this permanent imbalance comes about, with separate reference to different generations in the same country. Second, we discuss the ‘rationality’ of continually exporting, both from an individual’s viewpoint and from the social viewpoint of the exporting country.

We call country 1 the lending country (lenders) and country 2 the borrowing country (borrowers) to reflect the behavior of representative individuals in their younger years. Let us derive the lending and borrowing behavior as a function of R for individuals in these countries.

Dropping the superscripts temporarily, we have $w_t = p_{t+1}$, $\beta w_t + p_t = \gamma q_t$, and $p_{t+1}R = p_t$, which, given R is a system of 3 equations in the four unknowns w_t , p_t , p_{t+1} , q_t . But we can obtain a solution dividing by q_t to yield $W_t = \pi_{t+1}$, $\beta W_t + \pi_t = \gamma$, and $\pi_{t+1}R = \pi_t$, where W_t is the real wage, π_t is the real price of currently available capital good, and π_{t+1} is the price in consumption goods now of a capital good available next period. Thus, $\pi_t = \gamma/(\beta/R + 1)$, and $\pi_{t+1} = W_t = \gamma/(\beta + R)$. The real quantity of savings, σW_t , falls as R rises. The price of next year’s capital stock also falls (it is $k\pi_{t+1}$) at the same rate; thus the real excess of savings over steady-state domestic capital is (replacing superscripts) $(\sigma - k^i)(\gamma^i/(\beta^i + R))$, which is the net real demand for bonds by country i (i.e., this is how much consumption goods is put on the bond market by young people for return R when all other markets have been cleared). The following graph depicts this situation.

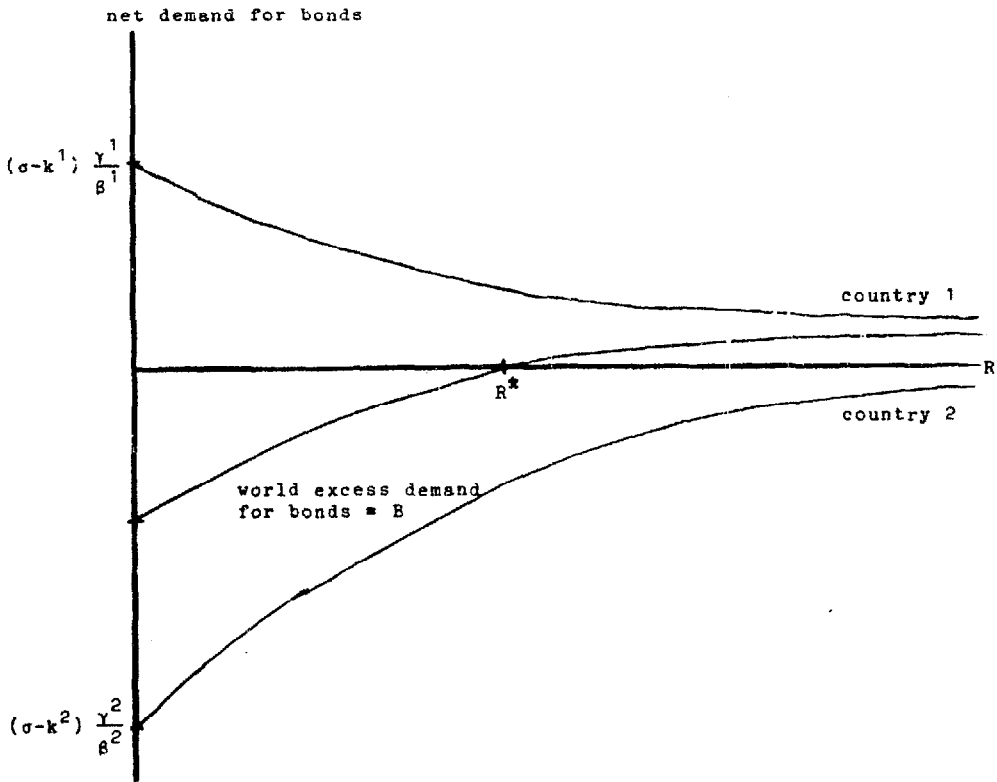


Fig. 1

R^* is the solution of

$$(\sigma - k^1)\gamma^1 \frac{1}{(\beta^1 + R)} + (\sigma - k^2)\gamma^2 \frac{1}{(\beta^2 + R)} = 0$$

or

$$[(\sigma - k^1)\gamma^1 \beta^2 + (\sigma - k^2)\gamma^2 \beta^1] + R[(\sigma - k^1)\gamma^1 + (\sigma - k^2)\gamma^2] = 0.$$

It clearly is unique if it exists, and for it to be economically meaningful, it must be positive, which requires that the two bracketed terms have opposite signs. Now $k^1 < \sigma < k^2$ implies $\beta^1 > \beta^2$, so if the second expression is negative the first one must be as well. Thus we must have the first negative and the second positive, or,

$$\frac{-(\sigma - k^2)}{(\sigma - k^1)} < \frac{\gamma^1}{\gamma^2} < \frac{-\beta^1(\sigma - k^2)}{\beta^2(\sigma - k^1)}.$$

This is precisely equivalent to Professor Gale's expression (2.3).² If

² It looks different because we have defined $R = 1+r$, but his $R = -(\sigma - k_1)\gamma^1 \beta^1 / (\sigma - k_2)\gamma^2 \beta^2$ is not the same thing, though both take the value unity simultaneously.

this condition is violated, it means that one country is so productive relative to the other that no interest rate, no matter how extreme, could equilibrate the bond market. In the above diagram this means that, for instance, the demand curve of country 2 is so low that the world demand curve is asymptotic to the horizontal axis from below but never reaches it. In such situations the world is no better off than the larger country in isolation in terms of the possibility of transferring consumption between generations without using direct intermediation. In all that follows, we assume that the above inequalities hold as well as $k^1 < \sigma < k^2$. It is useful to note that any non-negative value of R is a possible equilibrium value under these constraints. We shall study the properties of equilibria under various world interest rates below.

For R sufficiently small, the sign of B , the world excess demand for bonds, depends on the relationship between

$$(\sigma - k^1)\gamma^1\beta^2 \quad \text{and} \quad -(\sigma - k^2)\gamma^2\beta^1 .$$

By the second of the two inequalities above we can see that the slope of the aggregate demand for bonds is positive in all situations in which equilibrium exists.

If $R^* < 1$, individuals in the lending country receive a smaller amount of consumption goods in return than they loaned out. Because the capital market is also in equilibrium, those individuals who bought capital goods futures find that their real rental is less than real cost of the futures was last period. Thus, if we were to observe a steady-state equilibrium in this case, we would see young people in country 1 loaning out more than their elders receive. Thus, country 1 as a whole is a net exporter every period — but this should properly be viewed as a composition of two phenomena.

We now ask whether or not this is rational for individuals in country 1. But clearly it is, everyone is faced with a given wage rate and rate of interest and is maximizing utility given his options in the various markets. Professor Gale suggests that the behavior of country 1 is collectively irrational because, if they divided their steady-state output of consumption goods $k^1\gamma^1$ by giving $\sigma k^1\gamma^1$ to the old people and $(1-\sigma)k^1\gamma^1$ to the young people every period, they would be better off. Clearly they would. But from this he concludes that the competitive steady-state allocation is not in the core since it can be blocked by the coalition of all country 1 people. However, I contend that we must be more precise in our definition of the core in this Samuelsonian

setting. Usually, the core is the set of allocations that cannot be blocked by any coalition. However, we must define the set of all coalitions. The specification of this set should depend on the model at hand. In this case, we should allow only those coalitions consisting of individuals who are living concurrently, for non-coincidence of lifetime should preclude their cooperation due to informational problems involved in communicating with the dead or the as-yet-unborn. With this stipulation, it is clear that the competitive allocation is unblocked. In this sense, the competitive allocation is not collectively irrational for the net exporting country.

It is clear that perfect intermediation, in the form of a perfectly elastic supply and demand of bonds by the government at price 1, can be a substitute for this cooperative action involving an infinite sequence of individuals. In section 4, we will discuss the possibility that a country can improve its situation (in the Pareto sense) by the creation of such a source of (outlet for) bonds when it is in the midst of a permanent trade imbalance situation as above.

We now proceed to offer another look at trade imbalance from a point of view which will then be related to this discussion. In terms of the diagram above, the perfect intermediary would look like the diagram on the following page. Perfect, that is, because an intermediary that can make the equilibrium rate of interest zero will give the country the highest possible per capita utility. One way of viewing the role of country 2 is that it is a less-than-perfect intermediary for country 1.

Thus, each country acts as an intermediary of some sort for the other, by supplying it with bonds that would not be available to it in isolation. The farther away the isolation demand is from equilibrium the more intermediation would be required for optimality (i.e., higher levels of net credit or debt would be necessary). As can be seen from the diagrams, the country in which the required amount of intermediation is smaller is the one to which the goods will flow, on balance.

The flow of goods can then be interpreted as a payment from the way-out-of-line country to the one whose autarchic situation is closer to equilibrium for the latter over-extending itself to provide so much intermediation that the world is in equilibrium. It is clear that the country with the net outflow of goods is worse off than in the perfectly intermediated situation, but the welfare of the other country is less clear.

On one hand, the steady-state level of aggregate consumption is higher, but the distribution of consumption between old and young

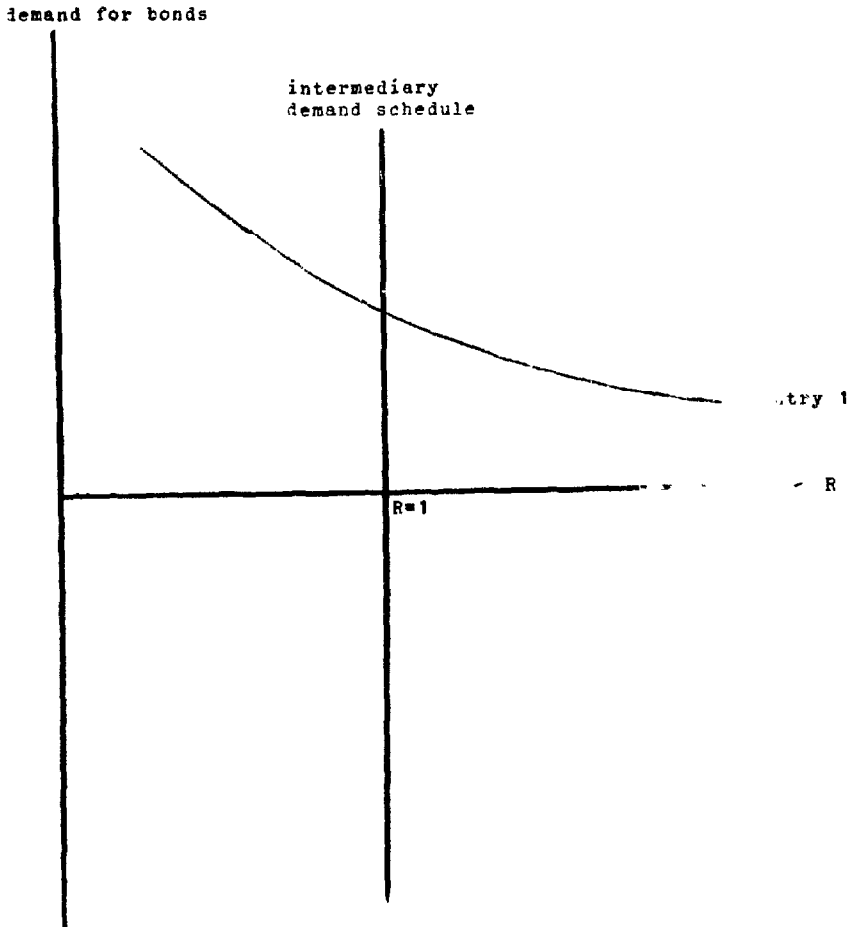


Fig. 2

might be far from optimal – far enough to outweigh the increase in real consumption. We now proceed to analyze this possibility in detail.

We treat the case $R < 1$ and consider, therefore, the utility of a representative individual in country 2. We have the real wage at any interest factor R is: $W^2 = \gamma^2 / (\beta^2 + R)$. The individual will save σ of this, with a return of $\sigma R \gamma^2 / (\beta^2 + R)$; thus his utility will be

$$u = \left[(1-\sigma) \left(\frac{\gamma^2}{\beta^2 + R} \right) \right]^{1-\sigma} \left[\sigma R \left(\frac{\gamma^2}{\beta^2 + R} \right) \right]^\sigma .$$

Differentiating this, we find that

$$\frac{\partial u}{\partial R} = A^2 R^{\sigma-1} \left(\frac{1}{\beta^2 + R} \right) \left(\sigma - \frac{R}{\beta^2 + R} \right) ,$$

where $A^2 = (1-\sigma)^{1-\sigma}(\sigma)^\sigma \gamma^2$. Thus the utility of the representative man is maximized at $R_{2-opt} = \beta^2 \sigma / (1-\sigma)$. Using the fact that $1/(1+\beta^2) > \sigma$ we obtain that $R_{2-opt} < 1$. A similar analysis leads to

$$R_{1-opt} = \frac{\beta^1 \sigma}{(1-\sigma)} > 1.$$

We display this as follows:

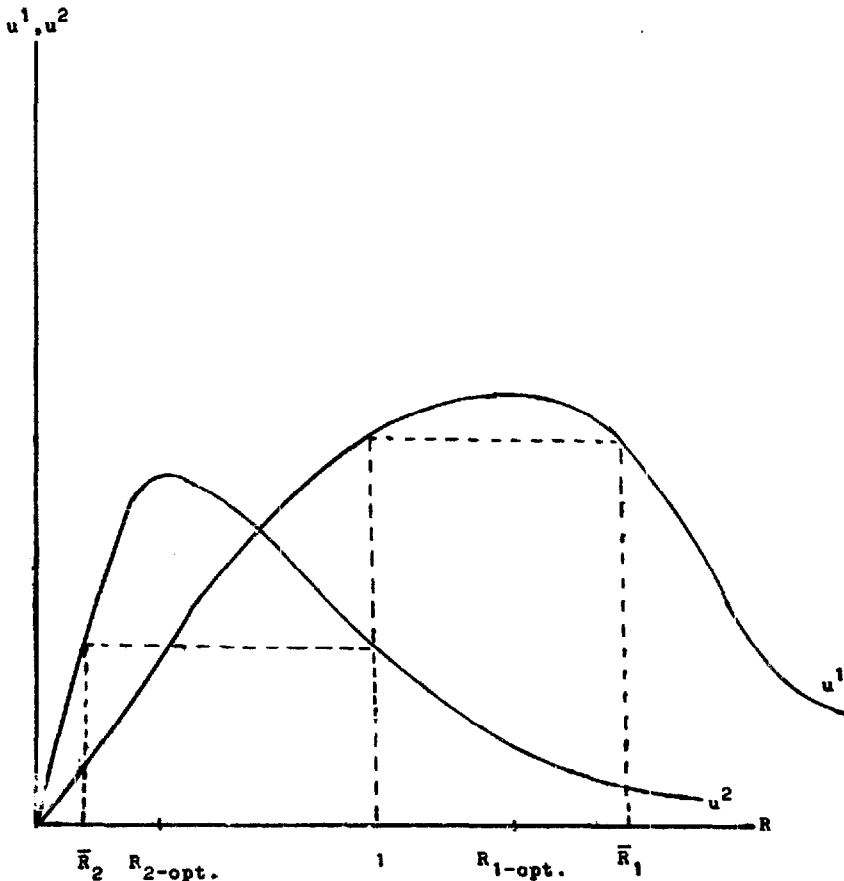


Fig. 3

This means that, for country 2, utility increases as R falls past 1 because this leads to a trade imbalance situation in country 2's favor. However, the distributive distortions set up by the rate of interest overtake this below R_{2-opt} and thus, even though total consumption in the country is rising as R falls, it is made worse off. \bar{R}_2 is defined so that below it country 2 would be better off in the $w=c$ equilibrium, ($R=1$).³ This has the interesting implication that if one country could

³ At the $w=c$ equilibrium, the allocation of consumption goods is the same as if $R=1$ in the $s=k$.

organize itself so that it became a price setter in the international bond market,⁴ it would not choose to force the other country into an extreme interest rate situation (i.e., either $R = 0$ if the home country is a borrower or $R = \infty$ if it is a lender). The reason for this as stated is that the interest rate set leads to redistributive effects in the home country because the factor markets are linked to the bond market as a result of the one period lag in the production of capital goods.

As can be seen from fig. 1 the quantity of bonds demanded decreases in absolute value with R . However, it is easy to see that the extent of trade imbalance increases, for the real (in consumption goods) value of bonds purchased by country 1 is

$$(\sigma - k^1) \gamma^1 \frac{1}{\beta^1 + R},$$

and the repayments in real terms is R times this, so the net inflow is

$$(\sigma - k^1) \gamma^1 \frac{r}{\beta^1 + 1 + r}$$

where $R = 1 + r$. This is of course identical to the expression obtained by Professor Gale for net imports, but leads to some additional insight in the decomposed form of bond purchases of the young plus receipts of the old.

4. The possibility of improving a country's position

As mentioned in sect. 1 we are interested in the possibility that a country that finds itself in a steady-state trade imbalance equilibrium can better its position by cooperative collective action. Let us say that this action is to take place at time t . There will be old people at t who were born in $t-1$ and who have either debt obligations or claims vis-à-vis individuals of the same generation in the other country. We shall assume that any outstanding obligations will be honored by the govern-

⁴ But not a monopolist in any other markets, so that all rates of return adjust to the one set.

ment policy. These old people also own the capital goods in the country. (They bought the futures when they were young.) But the value of these futures may not be what it would have been if free competition had persisted – it can be affected by the collective actions contemplated.

By “can better its position”, we shall mean betterment in the Pareto sense – the current old generation, young generation and all future generations must be better off than they would have been in the prevailing equilibrium.

We shall consider two methods for this improvement. One is the possibility of issuing (or accepting) bonds at interest rate zero. This is the method of intermediation as in Cass and Yaari (1966), and Samuelson (1958). The other possibility we shall call fiat. That is, the consumption and production quantities are collectively decided and must be carried out irrespective of whether such action is compatible with a price system for the existing markets.

4.1. *The case of fiat solutions*

Goods available at time t in country i are output at t , $\gamma^i k^i$, plus debt repayments due from the other country equal to $(\sigma^i - k^i)[R\gamma^i/(\beta^i + R)]$. Of course for country 2 this will be negative so that less than its full output is available.

To sustain the utility level of individuals of generation $t-1$ who are old at t they must be given $R\sigma[\gamma/(\beta+R)]$ where R is the interest factor in the prevailing equilibrium (i.e., before the fiat policy). Thus the amount remaining for the young people at t is

$$\gamma k + (\sigma - k) \frac{R\gamma}{\beta + R} - R\sigma \frac{\gamma}{\beta + R} = \gamma k \left(\frac{\beta}{\beta + R} \right).$$

Next period the output available will be γk and the old people will have to receive at least enough to keep them on the steady-state indifference locus. This amount, call it γx , is the solution to

$$\left[(1 - \sigma) \left(\frac{\gamma}{\beta + R} \right) \right]^{1 - \sigma} \left[\sigma R \frac{\gamma}{\beta + R} \right]^\sigma = \left[\gamma k \left(\frac{\beta}{\beta + R} \right) \right]^{1 - \sigma} [\gamma x]^\sigma. \quad (13)$$

If the amount of consumption goods left over, $\gamma(k - x)$, for the young people of generation $t+1$ is greater than that received by t , $\gamma k[\beta/(\beta+R)]$, then fiat is clearly successful. We shall show that this can never happen. However, fiat will also succeed whenever the marginal rate of substitution at the point $(\gamma k[\beta/(\beta+R)], \gamma x)$ which is the consumption stream of

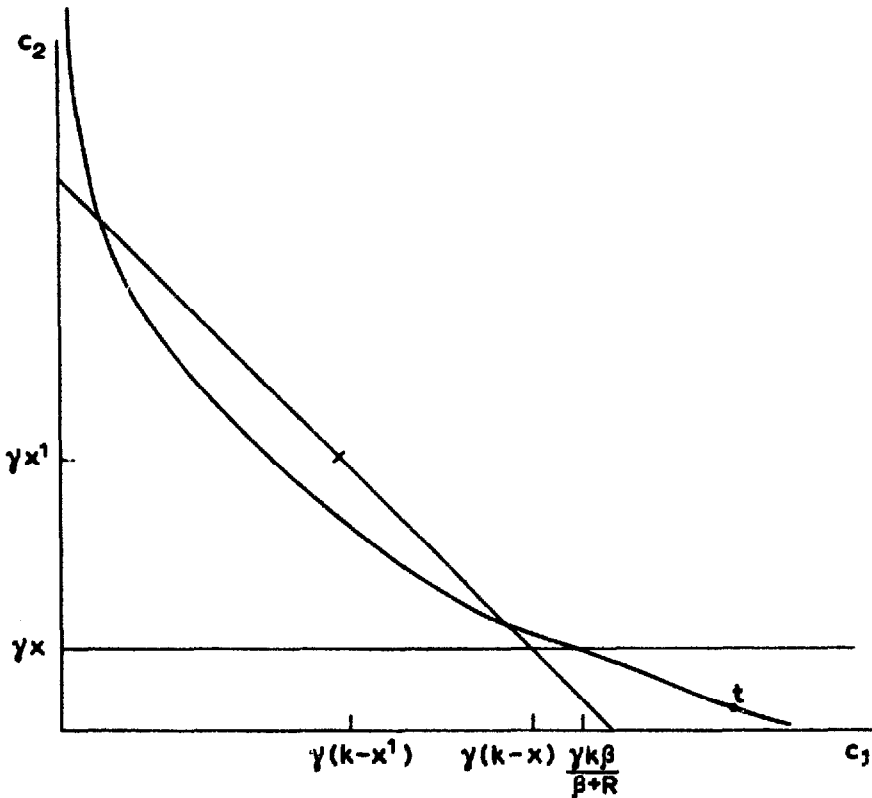


Fig. 4.

generation t individuals is less than one, provided that the steady-state utility level is autarchically attainable. This can be seen in the following diagram. The indifference curve drawn is that corresponding to the steady-state level of utility and $(\gamma k\beta/(\beta+R), \gamma x)$ is on this by construction. The sloping line represents the set of all possible divisions of autarchic output. After giving γx to the old people at $t+1$, $\gamma(k-x)$ is available for the young. $\gamma(k-x)$ is clearly greater than the lower right intersection of the indifference curve and output line and $\gamma k\beta/(\beta+R)$ is to the right of this. Thus the following policy is possible: Give the old people at $t+1$ γx^1 and thereafter divide output $(\gamma(k-x^1), \gamma x^1)$. This is clearly Pareto superior and thus fiat will work. It is easy to see that fiat will fail if $\gamma k\beta/(\beta+R)$ is less than the c_1 value at the upper left hand intersection. This is because the old generation requires so much c_2 in the first period to maintain their autarchic level of utility that any decrease in the c_1 level of the young people will make it impossible to give them enough c_2 to compensate for this. Since we will show that $\gamma k\beta/(\beta+R) + \gamma x > \gamma k$, this is the only other possibility (i.e., the point t is never below the line).

In order to show the latter fact, we solve (13) for x obtaining

$$x = (1-\sigma)^{(1-\sigma)/\sigma} \sigma R \left(\frac{1}{\beta+R} \right) \beta^{1-1/\sigma} (1+\beta)^{1/\sigma-1} . \quad (14)$$

If $\gamma k \beta / (\beta + R) < \gamma(k - x)$, then $\gamma x < \gamma k R / (\beta + R)$. Thus, substituting for x in (14) and manipulating the expression resulting, we arrive at

$$(1-\sigma)^{1-\sigma} \sigma^\sigma < \frac{1}{1+\beta} \beta^{1-\sigma} . \quad (15)$$

To see that this is impossible, note that the right hand side is maximized for fixed σ when $\beta = (1-\sigma)/\sigma$. But at this point its value is $(1-\sigma)^{1-\sigma} \sigma^\sigma$. Thus for every σ , $(1-\sigma)^{1-\sigma} \sigma^\sigma \geq [1/(1+\beta)] \beta^{1-\sigma}$, proving that the point t in the diagram must be above the output line.

Thus fiat works if and only if the following two conditions hold: (1) the steady-state utility level is attainable by dividing γk between the generations; (2) $(\partial u(c_1, c_2)/\partial c_2)/(\partial u(c_1, c_2)/\partial c_1) > 1$, where these derivatives are evaluated at $(\gamma k \beta / (\beta + R), \gamma x)$. The first of these is equivalent to having R outside the interval between 1 and \bar{R}_i for the country, i , in question (\bar{R}_i is defined to be the R at which utility is the same as at $R = 1$), since this is the interval in which the country is doing better than its autarchic steady-state optimum.

The second condition is:

$$\frac{\sigma}{1-\sigma} \frac{c_1}{c_2} > 1 .$$

Substituting $\gamma k \beta / (\beta + R)$ for c_1 and using (14) to find x yields

$$R < \frac{\beta^i}{1+\beta^i} \cdot \frac{1}{1-\sigma}^{1/\sigma} \equiv \tilde{R}_i . \quad (16)$$

Now for country 2 the right hand side of this is less than 1; therefore the interval of R 's in which fiat works is $[0, \min(\bar{R}_2, \tilde{R}_2)]$. The analysis below will demonstrate that $\bar{R}_2 \leq \tilde{R}_2$. For country 1 $\bar{R}_1 > 1$; thus fiat always works for country 1 when $R < 1$ in the steady state. But we may also have that $\tilde{R}_1 > \bar{R}_1$, in which case there will be a second, higher, interval of R 's in which fiat will succeed. In this case fiat will succeed even though country 1 is a steady-state net importer in every period (the interest rate is greater than one).

To see whether such a second interval will exist, that is whether $\tilde{R}_1 > \bar{R}_1$, we need only determine whether $u(\tilde{R}_1) < u(\bar{R}_1) \equiv u(1)$ where we are writing $u(\cdot)$ as the steady-state utility level as a function of R ,

$$u(\tilde{R}_1) = \gamma(1-\sigma)^{1-\sigma} \sigma^\sigma \left(\frac{\beta^1}{1+\beta^1} \cdot \frac{1}{1-\sigma} \right) \left(\frac{1}{\beta^1 + (\beta^1/(1+\beta^1)) \cdot 1/(1-\sigma)} \right)^{1/\sigma}$$

$$u(\bar{R}_1) = u(1) = \gamma(1-\sigma)^{1-\sigma} \sigma^\sigma (1/(1+\beta^1)).$$

Thus $u(\tilde{R}_1)/u(1) < 1$ if and only if

$$\sigma < \left(\frac{1}{1+\beta} \right)^{1/\sigma} \cdot \left(\frac{1-\sigma}{\beta} \right)^{1-1/\sigma} \quad (17)$$

We shall now show that this is impossible. Differentiating the right hand side with respect to β yields that it has a critical point at $\beta = (1-\sigma)/\sigma$. Further, the second derivative is negative. Substituting $\beta = (1-\sigma)/\sigma$ into the right hand side we have that it is equal to σ . Thus

$$\sigma \geq \left(\frac{1}{1+\beta} \right)^{1/\sigma} \left(\frac{1-\sigma}{\beta} \right)^{1-1/\sigma},$$

contradicting (17). Thus fiat works in country 1 only in the interval $0 < R < 1$. This analysis also implies $u(\tilde{R}_2) \geq u(\bar{R}_2)$, or, $\tilde{R}_2 \geq \bar{R}_2$ and therefore $(0, \bar{R}_2)$ is the successful interval for country 2.

4.2. Bond flotation solutions:

In this solution, the government offers to buy or sell bonds at a zero interest rate to citizens of its own country. Thus the government will have a perpetual net debt or credit. This takes the place of the international bond market. All other markets remain competitive and the debts or assets of the older generation in the year in which the policy is instituted accrue to them exactly as before this policy.

The first observation is that no policy of this type in which the rate of return on capital decreases can be Pareto superior because it would decrease the consumption level of the old people who own the capital. Thus bond flotation policies fail if $R > 1$ since

$$\pi_t = \frac{1}{\beta/R + 1}.$$

Country 1 (lenders when young): $R < 1$.

Bond flotation works because steady-state utility is increased and capital income goes up. The country is no longer a steady-state exporter.

Country 2 (borrowers when young): $R < \bar{R}_2$.

Bond flotation works for the same reason. Notice that here it is the steady-state net *importer* that can pursue a better policy.

$$: \bar{R}_2 < R < 1$$

Bond flotation fails because steady-state utility is not attainable in isolation.

We thus see that the bond flotation policy will succeed in exactly the same cases as the fiat policy. Thus all policies that could supplant a current unbalance equilibrium are sustainable by the competitive mechanism. This is a consequence of fixed coefficients in production. In a more general model, fiat would be a more powerful policy in the sense that it could successfully be used in situations in which a market-oriented intermediation fails. The above result, however, summarizes the issue of collective rationality in the model at hand.

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