



# Search Diversion and Platform Competition Online Appendix

Andrei Hagiu  
Bruno Jullien

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# Search Diversion and Platform Competition

## Online Appendix

Andrei Hagiu\* and Bruno Jullien†

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In this online appendix we provide the complete analysis for the case in which the two platforms compete for exclusive affiliation on both sides of the market (with no fixed fees). The expressions of platforms' profits and of the advertiser's payoff when the latter affiliates exclusively with platform  $A$  are:

$$\Pi_A = X(s_A, r_A) \left( \frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right) \quad \text{and} \quad \Pi_B = F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_A)}{2t} \right)$$

$$\Pi^{adv}(A) = (X(s_A, \pi) - X(s_A, r_A)) \left( \frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right)$$

The key difference with the other two competition scenarios is that here it is unclear whether both platforms wish to compete for the advertiser. This is because the "losing" platform B obtains higher consumer demand, which may compensate for its lower revenues per consumer (no advertising). Thus, given the levels of search diversion set in stage 1, it is possible that B obtains a larger profit without the advertiser than the maximum profit it could expect to achieve if it were to attract the advertiser. When this is the case, platform B prefers not to make an offer to the advertiser in stage 2 and platform A is a de facto monopoly on the advertiser side of the market. Consequently, according to the same logic as before, platform A's optimal

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\*Harvard University (HBS Strategy Unit), ahagiu@hbs.edu

†Toulouse School of Economics (IDEI & GREMAQ), bjullien@cict.fr

choice of search diversion maximizes its joint profits with the advertiser. Let then:

$$s_{VI}(\pi, t) \equiv \arg \max_{s \in [0,1]} \left\{ X(s, \pi) \left( \frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) \right\} \quad (1)$$

be the level of search diversion that maximizes the joint profit of the advertiser and the platform it affiliates with, when facing a platform that offers product 1 (content) only.

Of course, the alternative scenario is that the two platforms actually compete to attract the advertiser. For this case, we define

$$s_T(\pi, t) \equiv \arg \max_{s \in [0,1]} \left\{ X(s, \pi) \left( \frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s)}{2t} \right) \right\} \quad (2)$$

as the level of search diversion that maximizes total industry profit, i.e. the joint profits of the advertiser and both platforms. Since  $V(s)$  is decreasing, we have:

$$s_{VI}(\pi, t) < s_T(\pi, t)$$

Whether or not the two platforms effectively compete for the advertiser depends on  $z_{VI}(\pi, t) \in [0, 1]$ , which solves:

$$X(s_{VI}(\pi, t), \pi) \left( \frac{1}{2} + \frac{V(s_{VI}(\pi, t)) - V(0)}{2t} \right) = F(u) \left( \frac{1}{2} + \frac{V(0) - V(z(\pi, t))}{2t} \right),$$

with the convention  $z_{VI}(\pi, t) \equiv +\infty$  if:

$$X(s_{VI}(\pi, t), \pi) \left( \frac{1}{2} + \frac{V(s_{VI}(\pi, t)) - V(0)}{2t} \right) > F(u) \left( \frac{1}{2} + \frac{V(0) - V(1)}{2t} \right)$$

When  $z_{VI} \in [0, 1]$ , it represents the level of search diversion that the "winning" platform A (who obtains the advertiser) would have to choose in order to render the "losing" platform B indifferent between extracting maximum joint profits with the advertiser and ceding the advertiser to platform A. If  $z_{VI} = +\infty$  then platform B always prefers competing for the advertiser (no matter the level of search diversion chosen by A). Furthermore, if  $z_{VI} \in [0, 1]$

then, by choosing  $s_A > z_{VI}$ , platform A can secure the affiliation of the advertiser (i.e. platform B prefers not to compete for it).

Focusing on the range of  $t$  such that the two platforms are actually competing<sup>1</sup>, we have:

**Lemma 1** *There exists  $\bar{t}_3 > V(0)$  such that, for all  $t \in [0, \bar{t}_3]$ , the maximum level of search diversion that can be sustained in equilibrium is:*

$$s^*(\pi, t) = \begin{cases} s_{VI}(\pi, t) & \text{if } z_{VI}(\pi, t) \leq s_{VI}(\pi, t) \\ z_{VI}(\pi, t) & \text{if } s_{VI}(\pi, t) \leq z_{VI}(\pi, t) \leq s_T(\pi, t) \\ s_T(\pi, t) & \text{if } z_{VI}(\pi, t) \geq s_T(\pi, t) \end{cases}$$

*The maximum level of search diversion  $s^*(\pi, t)$  is continuous in its 2 arguments, (weakly) increasing in  $t$  for  $t \in [0, \bar{t}_3]$  and (weakly) increasing in  $\pi$ .*

**Proof.** We assume that  $X(s, \pi) \left( \frac{1}{2} + \frac{V(s) - V(0)}{2t} \right)$  is quasi-concave and its maximizer  $s_{VI}$  is interior to  $[0, 1]$ . For any  $s$ , denote then by  $z(s) \in [0, 1]$  the solution to

$$X(s, \pi) \left( \frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) = F(u) \left( \frac{1}{2} + \frac{V(0) - V(z)}{2t} \right)$$

with the convention  $z(s) = +\infty$  for all  $s$  such that:

$$X(s, \pi) \left( \frac{1}{2} + \frac{V(s) - V(0)}{2t} \right) > F(u) \left( \frac{1}{2} + \frac{V(0) - V(1)}{2t} \right)$$

Since  $V(z)$  is decreasing, the function  $z(s)$  is inverted U-shaped in  $s$ , with maximal value

$$z_{VI} = z(s_{VI}) = \max_s z(s).$$

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<sup>1</sup>If  $t$  is large enough, then the two platforms behave as local monopolists. Platform A then chooses  $s_A = s_{XV} = s^M$  for this range of  $t$ .

We restrict attention to levels of search diversion  $(s_A, s_B)$  such that:

$$X(s_i, \pi) \left( \frac{1}{2} + \frac{V(s_i) - V(0)}{2t} \right) \geq F(u) \frac{1}{2} \text{ for } i = A, B$$

Fix  $(s_A, s_B)$  chosen in the first stage of the game. Denote by  $\Pi^{adv}(i)$  the payoffs obtained by the advertiser when it affiliates exclusively with platform  $i$  and by  $\Pi^i$  the profits derived by platform  $i$ , where  $i \in \{A, B\}$ . If the advertiser affiliates exclusively with platform  $i \in \{A, B\}$  in the equilibrium of the game beginning in the second stage then the equilibrium fees  $(r_A, r_B)$  chosen in stage 2 must be such that (assuming interior demand<sup>2</sup>) the following conditions hold:

$$\begin{aligned} \Pi^{adv}(i) &= \max \{ \Pi^{adv}(j), 0 \} \\ \Pi^{adv}(j) &= X(s_j, \pi) \left( \frac{1}{2} + \frac{V(s_j) - V(0)}{2t} \right) - F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_i)}{2t} \right) \\ \Pi^i &= X(s_i, \pi) \left( \frac{1}{2} + \frac{V(s_i) - V(0)}{2t} \right) - \Pi^{adv}(i) \\ \Pi^i &\geq F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_j)}{2t} \right) \text{ if } \Pi^{adv}(j) \geq 0 \\ \Pi^i &\geq F(u) \frac{1}{2} \text{ if } \Pi^{adv}(j) < 0 \end{aligned} \tag{3}$$

The first condition ensures that the advertiser weakly prefers to affiliate with platform  $i$  and that platform  $i$  cannot increase its profits by raising  $r_i$ . The second condition requires platform  $j$  to offer the advertiser all the surplus in excess of  $j$ 's outside option that would be created if the advertiser were to affiliate with  $j$  instead of  $i$ . If  $j$  was offering any less in equilibrium, it could then profitably deviate by slightly decreasing  $r_j$  and getting the advertiser to affiliate with it (because of the first condition). The third condition is a simple accounting equality. The fourth and fifth conditions ensure that platform  $i$  prefers the equilibrium to a deviation in which it would let the advertiser affiliate with platform  $j$  or no platform at all (depending on whether platform  $j$  offers the advertiser a positive payoff).

With the notation above, we have  $\Pi^{adv}(j) \geq 0$  if and only if  $s_i \leq z(s_j)$ .

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<sup>2</sup>Choosing  $s_i$  that induces zero demand for platform  $i \in \{A, B\}$  is weakly dominated by a small  $s_i$ , so we rule it out.

Furthermore, if  $\Pi^{adv}(j) \geq 0$  then the conditions above imply:

$$\begin{aligned} & X(s_i, \pi) \left( \frac{1}{2} + \frac{V(s_i) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_i)}{2t} \right) \\ \geq & X(s_j, \pi) \left( \frac{1}{2} + \frac{V(s_j) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_j)}{2t} \right) \end{aligned}$$

Suppose that the equilibrium  $(s_A^*, s_B^*)$  is such that the advertiser affiliates with platform A. There are three possibilities:

**Case I:**  $s_A^* < z(s_B^*)$

In this case, we must have:

$$\begin{aligned} \Pi^A &= X(s_A^*, \pi) \left[ \frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right] \\ &\quad - X(s_B^*, \pi) \left[ \frac{1}{2} + \frac{V(s_B^*) - V(0)}{2t} \right] + F(u) \left[ \frac{1}{2} + \frac{V(0) - V(s_A^*)}{2t} \right] \\ &\geq F(u) \left[ \frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right]. \end{aligned}$$

Local optimality of  $s_A^*$  implies that we must have  $s_A^* = s_T$ . For this to be an equilibrium, we first must be able to find  $s_B^*$  such that  $z(s_B^*) \geq s_T$ , which is possible if and only if  $z_{VI} \geq s_T$ .

Second, neither A nor B can be able to profitably deviate. Consider any deviation  $s_A$  by platform A. For the deviation to be profitable, A must continue to win the advertiser, otherwise it would obtain  $F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right)$ , which is (weakly) dominated by the pre-deviation profits. Since  $s_A^* = s_T$ , we also know that there is no profitable deviation  $s_A \leq z(s_B^*)$ , so the only possibility would be  $s_A \geq z(s_B^*)$ , in which case platform A's deviation profits would be:

$$\Pi^A = X(s_A, \pi) \left( \frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right)$$

But  $s_{VI} < s_T < z(s_B^*)$ , so the expression above is decreasing for  $s_A \geq z(s_B^*)$ . Thus, there is no profitable deviation for A.

Consider now a deviation  $s_B$  by platform B. The only way it can be profitable is if B wins the advertiser and makes profits larger than  $F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_T)}{2t} \right)$ . This requires  $s_B > z(s_T)$  (otherwise

either A wins the advertiser or B wins it but its resulting profits are still  $F(u) \left( \frac{1}{2} + \frac{V(0)-V(s_T)}{2t} \right)$ , which yields deviation profits (recall conditions (3)):

$$X(s_B, \pi) \left( \frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right)$$

But  $s_B > z(s_T)$  is equivalent to:

$$F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) > X(s_T, \pi) \left( \frac{1}{2} + \frac{V(s_T) - V(0)}{2t} \right)$$

and we have:

$$\begin{aligned} & X(s_B, \pi) \left( \frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) \\ \leq & X(s_T, \pi) \left( \frac{1}{2} + \frac{V(s_T) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_T)}{2t} \right) \end{aligned}$$

Combining the last two inequalities, we obtain:

$$X(s_B, \pi) \left( \frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) < F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_T)}{2t} \right),$$

which means B's deviation is not profitable.

We have thus proven that if  $z_{VI} > s_T$  then there exists an equilibrium  $(s_A^*, s_B^*)$  with  $s_A^* = s_T$  and  $s_B^* = s_{VI}$ .

**Case II:**  $s_A^* > z(s_B^*)$

In this case, we must have:

$$\Pi^A = X(s_A^*, \pi) \left( \frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) \geq F(u) \frac{1}{2}$$

Local optimality of  $s_A^*$  implies that we must have  $s_A^* = s_{VI}$ . For this to be an equilibrium, we first must be able to find  $s_B^*$  such that  $z(s_B^*) < s_{VI}$ , which is always possible since  $z(0) = 0$ .

Second, neither A nor B can be able to profitably deviate. Any deviation  $s_A$  by platform A such

that  $s_A \geq z(s_B^*)$  cannot be profitable, so we must have  $s_A \leq z(s_B^*)$ . If the advertiser affiliates with platform A after such a deviation then A's deviation profits are:

$$X(s_A, \pi) \left( \frac{1}{2} + \frac{V(s_A) - V(0)}{2t} \right) - X(s_B^*, \pi) \left( \frac{1}{2} + \frac{V(s_B^*) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_A)}{2t} \right)$$

But  $z(s_B^*) < s_{VI} < s_T$ , so these profits are increasing in  $s_A$  for  $s_A \leq z(s_B^*)$ , which means this deviation is not profitable. The remaining possibility is that  $s_A \leq z(s_B^*)$  and the advertiser affiliates with platform B, which platform A can always induce by setting  $s_A = 0$ . In this case, A's deviation profits are  $F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right)$ . This deviation is not profitable if:

$$X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) \geq F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right),$$

i.e. if  $s_B^* \leq z_{VI}$ . Note that it is always possible to find  $s_B^*$  that satisfies both this condition and  $z(s_B^*) < s_{VI}$  above: for example,  $s_B^* = 0$  works.

Consider now a deviation  $s_B$  by platform B: it can be profitable only if it leads to the advertiser affiliating with B. In such a deviation, platform B's profit would be:

$$X(s_B, \pi) \left( \frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) - \max \left\{ 0, X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) - F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) \right\} \quad (4)$$

Suppose that  $s_{VI} \geq z_{VI}$ , i.e.

$$X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) \leq F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right)$$

Then platform B's best deviation is  $s_B = s_{VI}$ , which yields profits:

$$X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right),$$



less than the pre-deviation profits  $F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right)$ . Thus, B has no profitable deviation.

Now suppose that  $s_{VI} < z_{VI}$ . There are two possibilities:

- If  $z_{VI} \geq s_T$  then B's best deviation (from (4)) is achieved for  $s_B = s_T$ , yielding profits:

$$\begin{aligned} & X(s_T, \pi) \left( \frac{1}{2} + \frac{V(s_T) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_T)}{2t} \right) \\ & - X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) \\ & > F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right) \end{aligned}$$

- If  $s_{VI} < z_{VI} < s_T$  then B's best deviation (from (4)) is achieved for  $s_B = z_{VI}$ , yielding profits:

$$X(z_{VI}, \pi) \left( \frac{1}{2} + \frac{V(z_{VI}) - V(0)}{2t} \right)$$

But  $s_{VI} < z_{VI} < s_T$  implies that:

$$\begin{aligned} & X(z_{VI}, \pi) \left( \frac{1}{2} + \frac{V(z_{VI}) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(z_{VI})}{2t} \right) \\ & > X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right), \end{aligned}$$

which is equivalent to (recall the definition of  $z_{VI}$ ):

$$X(z_{VI}, \pi) \left( \frac{1}{2} + \frac{V(z_{VI}) - V(0)}{2t} \right) > F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right)$$

Thus, in both cases B has a profitable deviation so  $(s_A^* = s_{VI}, s_B^*)$  cannot be an equilibrium for any  $s_B^*$ .

We have thus proven that there exists an equilibrium  $(s_A^* = s_{VI}, s_B^* = 0)$  if and only if  $s_{VI} \geq z_{VI}$ .

**Case III:**  $s_A^* = z(s_B^*)$

In this case, we must have:

$$\Pi^A = X(s_A^*, \pi) \left( \frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) \geq F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B^*)}{2t} \right),$$

i.e.  $s_B^* \leq z(s_A^*)$  (this ensures A does not want to deviate by ceding the advertiser to B). Furthermore, any deviation  $s_A > z(s_B^*)$  such that the advertiser stays with A cannot be profitable, so we must have  $s_A^* = z(s_B^*) \geq s_{VI}$ . Similarly, any deviation  $s_A$  by platform A such that the advertiser stays with A and  $s_A < z(s_B^*)$  cannot be profitable, so we must also have  $s_A^* = z(s_B^*) \leq s_T$ .

Platform B's profit is

$$F(u) \left( \frac{1}{2} + \frac{V(0) - V(z(s_B^*))}{2t} \right) = X(s_B^*, \pi) \left( \frac{1}{2} + \frac{V(s_B^*) - V(0)}{2t} \right).$$

For B to profitably deviate, it must obtain the advertiser. In such a deviation ( $s_B$ ), platform B's profit would be:

$$X(s_B, \pi) \left( \frac{1}{2} + \frac{V(s_B) - V(0)}{2t} \right) - \max \left\{ 0, X(s_A^*, \pi) \left( \frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) - F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_B)}{2t} \right) \right\}$$

There are three possibilities:

a) If  $z(s_A^*) < s_{VI}$  then B's optimal deviation is  $s_B = s_{VI}$ , which yields  $X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right)$ .

This deviation is strictly profitable unless  $s_B^* = s_{VI}$ , which means  $s_A^* = z(s_{VI})$ .

b) If  $z(s_A^*) > s_T$  then B's optimal deviation is  $s_B = s_T$ , which yields:

$$\begin{aligned} & X(s_T, \pi) \left( \frac{1}{2} + \frac{V(s_T) - V(0)}{2t} \right) \\ & - X(s_A^*, \pi) \left( \frac{1}{2} + \frac{V(s_A^*) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_T)}{2t} \right) \\ & \geq F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_A^*)}{2t} \right) \end{aligned}$$

This deviation is strictly profitable unless  $s_A^* = s_T$ , which is possible only if  $z(s_A^*) \geq s_T$ .

c) The remaining case is  $s_{VI} \leq z(s_A^*) \leq s_T$ . Here, B's optimal deviation is  $s_B = z(s_A^*)$ , which yields  $X(z(s_A^*), \pi) \left( \frac{1}{2} + \frac{V(z(s_A^*)) - V(0)}{2t} \right)$ . For this deviation not to be profitable we must have  $z(z(s_A^*)) \leq s_A^*$ .

Using the analysis of the 3 cases above, we can infer the following:

- In all cases, the equilibrium level of search diversion  $s_A^*$  verifies  $s_{VI} \leq s_A^* \leq s_T$
- If  $z_{VI} > s_T$  then the maximum level of search diversion that can be supported in equilibrium is  $s_A^* = s_T$  (case I).
- If  $z_{VI} < s_{VI}$  then the only possible equilibrium level of search diversion is  $s_A^* = s_{VI}$  (case II).
- If  $s_{VI} \leq z_{VI} \leq s_T$  then the case I equilibrium cannot be sustained since  $z(s_B^*) \leq z_{VI} \leq s_T$  for all  $s_B^*$ . Consequently, the maximum level of search diversion that could potentially be sustained in equilibrium is  $s_A^* = z_{VI}$  (case 3). Let us make sure that this equilibrium does indeed work. Let  $s_B^* = s_{VI}$  so that  $s_A^* = z(s_B^*) = z_{VI}$ . To ensure that A has no profitable deviation, we must show that  $s_{VI} \leq z(s_A^*) = z(z_{VI})$ . Recalling the definition of  $s_T$  and  $s_{VI} \leq z_{VI} \leq s_T$ , we have:

$$\begin{aligned} & X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right) \\ \leq & X(z_{VI}, \pi) \left( \frac{1}{2} + \frac{V(z_{VI}) - V(0)}{2t} \right) + F(u) \left( \frac{1}{2} + \frac{V(0) - V(z_{VI})}{2t} \right) \end{aligned}$$

By the definition of  $z_{VI}$  we also have:

$$X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) = F(u) \left( \frac{1}{2} + \frac{V(0) - V(z_{VI})}{2t} \right)$$

We can then derive:

$$F(u) \left( \frac{1}{2} + \frac{V(0) - V(s_{VI})}{2t} \right) \leq X(z_{VI}, \pi) \left( \frac{1}{2} + \frac{V(z_{VI}) - V(0)}{2t} \right),$$

i.e.  $s_{VI} \leq z(z_{VI})$ , as desired. Finally,  $s_{VI} \leq z_{VI} \leq s_T$  also implies  $z(z_{VI}) \leq z_{VI} \leq s_T$  (recall  $z(s)$  is weakly decreasing for  $s \geq s_{VI}$ ). To ensure that B does not have a profitable deviation we must then verify that  $z(z(z_{VI})) \leq z_{VI}$ , which follows directly from  $s_{VI} \leq z(z_{VI})$ . We can then conclude that the maximum level of search diversion that can be sustained is indeed  $s_A^* = z_{VI}$ .

This concludes the proof of the first part of the Lemma.

Let us now turn to the second part.  $s^*(\pi, t)$  is continuous because  $s_{VI}(\pi, t)$ ,  $z_{VI}(\pi, t)$  and  $s_T(\pi, t)$  are continuous in both of their arguments. The fact that  $s_{VI}$  and  $s_T$  are increasing in  $t$  is seen from the respective first-order conditions that determine them:

$$\begin{aligned} X_s(s_{VI}, \pi) (t + V(s_{VI}) - V(0)) + X(s_{VI}, \pi) V'(s_{VI}) &= 0 \\ X_s(s_T, \pi) (t + V(s_T) - V(0)) + (X(s_T, \pi) - F(u)) V'(s_T) &= 0 \end{aligned}$$

Both  $X_s(s_{VI}, \pi)$  and  $X_s(s_T, \pi)$  are positive, so both left-hand sides above are increasing in  $t$ .

The proof that  $s_{VI}$  and  $s_T$  are increasing in  $\pi$  is very similar to the proof that  $s^M$  is increasing in  $\pi$  (cf. proof of Proposition 1 in the main paper), therefore omitted.

Let us now turn to  $z_{VI}(\pi, t)$ . From the definition of  $s_{VI}$  we have:

$$X(s_{VI}, \pi) \left( \frac{1}{2} + \frac{V(s_{VI}) - V(0)}{2t} \right) \geq X(0, \pi) \frac{1}{2} = F(u) \frac{1}{2},$$

which implies  $X(s_{VI}, \pi) > F(u)$ . We can then write the equation determining  $z_{VI}(\pi, t)$  as:

$$\begin{aligned} V(0) - V(z_{VI}) &= \frac{X(s_{VI}, \pi) (t + V(s_{VI}) - V(0))}{F(u)} - t \\ &= \max_s \left\{ \frac{X(s, \pi) (t + V(s) - V(0))}{F(u)} - t \right\} \equiv W(\pi, t), \end{aligned}$$

where the last equality follows from the definition of  $s_{VI}$ . Using the envelope theorem, we have:

$$\begin{aligned} W_t(\pi, t) &= \frac{X(s_{VI}, \pi)}{F(u)} - 1 > 0 \\ W_\pi(\pi, t) &= s_{VI} F \left( \frac{u}{1 + s_{VI}} \right) \frac{t + V(s_{VI}) - V(0)}{F(u)} > 0 \end{aligned}$$

Since  $V(0) - V(z_{VI})$  is increasing in  $z_{VI}$ , we can therefore conclude that  $z_{VI}(\pi, t)$  is increasing in  $t$  and increasing in  $\pi$ , just like  $s_{VI}$  and  $s_T$ .

Note that  $s^*(\pi, t)$  is always equal to the middle value among  $s_{VI}(\pi, t)$ ,  $s_T(\pi, t)$  and  $z_{VI}(\pi, t)$ . As a result,  $s^*(\pi, t)$  inherits the same properties: increasing in  $t$  and decreasing in  $\pi$ .

Furthermore, note that  $s^*(\pi, t)$  is a well-defined equilibrium only if  $t$  is such that:

$$V(s^*) + V(0) > t,$$

i.e. such that the marginal consumer on the Hotelling segment derives positive net utility when the platform that attracts the advertiser chooses  $s^*$  in equilibrium. Since  $s^*$  is increasing in  $t$ , the above inequality holds for all  $t \in [0, \bar{t}_3]$ , where  $\bar{t}_3 > V(0)$ . ■

Note that the equilibrium is not unique when  $z_{VI}(\pi, t) \geq s_{VI}(\pi, t)$  (in that case, there is a continuum of equilibria), which is why we have chosen to focus on the equilibrium that involves the highest level of search diversion in the Lemma above. Fundamentally, equilibrium multiplicity stems from the fact that search diversion has opposite effects on the two sides of the market. It arises when the platform that wins the advertiser is in a situation where raising  $s$  would strongly depress consumer demand, whereas reducing  $s$  would lead the advertiser to jump ship to the other platform.

According to the most basic intuition, the equilibrium level of search diversion ought to maximize joint profits of the "winning" platform and the advertiser, i.e.  $s^* = s_{VI}(\pi, t)$ . Perhaps surprisingly, this is not always the case here. It is important to understand how the other two outcomes may arise.

First, when  $z_{VI} \geq s_T$ , the equilibrium level of search diversion  $s_T$  maximizes joint profits of the advertiser, the winning platform (say, A) and the losing platform (B). The fact that platform B's profit is taken into account (note that it depends on the search diversion level chosen by A) might seem odd at first glance. The interpretation is as follows: A must offer the advertiser a payoff just above the largest payoff that can be offered by B, who in turn can only offer the advertiser the difference between joint profits and B's outside option, i.e. what B gets when the advertiser affiliates with A. This term is *increasing* in the search diversion level chosen by A: a higher  $s_A$  increases B's outside option (more consumers go to B instead of A) and thereby decreases the hurdle that A needs to overcome in order to attract the advertiser. In other words, when it expects platform B to compete for the advertiser, the winning platform A maximizes

total industry profit because it internalizes the fact that yielding more consumer demand to platform B (through more search diversion) reduces the cost of attracting the advertiser, by reducing platform B's willingness to compete.

On the contrary, when  $z_{VI} \leq s_{VI}$ , the winning platform can, with the same level of search diversion, obtain maximum joint profits and secure the affiliation of the advertiser without having to give up additional consumer demand. The equilibrium is then the same as if one platform were vertically integrated with product 2.

The intermediate range  $s_{VI} \leq z_{VI} \leq s_T$  arises because the total industry profit is maximized at a higher level of search diversion ( $s_T$ ) than the vertically integrated profit ( $s_{VI}$ ). In this range, when neither  $s_T$  nor  $s_{VI}$  are sustainable, the winning platform is cornered in an equilibrium where the losing platform is just indifferent between competing and not competing for the advertiser (as discussed above).

Finally, the comparative statics in  $(t, \pi)$  are the same as in the case of competition for consumers and interpreted in the same way. Comparing with the level of search diversion chosen by a monopolist, we obtain (the proof is in the appendix):

**Proposition 1** *Relative to the level of search diversion chosen by a monopoly platform, the maximum equilibrium level of search diversion when platforms compete for both consumers and the advertiser is strictly lower for low  $t$  and strictly higher for large  $t$ . Specifically, there exists  $\bar{t}_2 \in [0, V(0)]$  such that:*

- $s^*(\pi, t) \leq s^M(\pi, t)$  for  $0 \leq t \leq \bar{t}_2$
- $s^*(\pi, t) > s^M(\pi, t)$  for  $\bar{t}_2 < t \leq \bar{t}_3$

**Proof.** From lemma 1,  $s^*$  is continuous and increasing in  $t$  for  $t \in [0, \bar{t}_3]$ . We also know that  $s^M$  is continuous and decreasing in  $t$  for all  $t \geq 0$ . Furthermore,  $\lim_{t \rightarrow 0} s^* = 0 < s_X = \lim_{t \rightarrow 0} s^M$ , whereas if  $t \in (V(0), \bar{t}_3)$  then  $s^M = s_{XV} < s_{VI} < s_T$ , which implies  $s^M < s^*$ . Consequently, there exists  $\bar{t}_2 \in [0, V(0)]$  such that  $s^* \leq s^M$  for  $t \in [0, \bar{t}_2]$  and  $s^* > s^M$  for  $t \in (\bar{t}_2, \bar{t}_3)$ . ■

This result confirms the one obtained under competition for consumers only (despite a significantly more complex analysis): once again, the equilibrium level of search diversion with competing platforms is lower than the one chosen by a monopolist when competition is intense (low  $t$ ) and higher when competition is not too intense (high  $t$ ). The explanation is the same.