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Proprietary vs. Open Two-Sided Platforms and Social Efficiency

Andrei Hagiu

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Abstract

This paper identifies a fundamental economic welfare tradeoff between two-sided open platforms and two-sided proprietary (closed) platforms connecting consumers and producers. Proprietary platforms create two-sided deadweight losses through monopoly pricing but at the same time, precisely because they set prices in order to maximize profits, they partially internalize two-sided positive indirect network effects and direct competitive effects on the producer side. We show that this can sometimes make proprietary platforms more socially desirable than open platforms, which runs against the common intuition that open platforms are more efficient. By the same token, inter-platform competition may also turn out to be socially undesirable because it may prevent platforms from sufficiently internalizing indirect externalities and direct intra-platform competitive effects.

Keywords: Two-Sided Markets, Platforms, Indirect Network Effects, Product Variety, Social Efficiency.

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1. Introduction

An increasing number of industries in today's economy are organized around two-sided platforms, which enable consumers to purchase, access and use a variety of products supplied by independent producers: software systems, Internet portals, mobile networks, shopping centers, etc. Policy makers have recently started to devote considerable attention to some of these markets: in particular, the rising popularity of the open source software movement has led many governments around the world to enact policies promoting open source software systems at the expense of proprietary systems[1]. Oftentimes, these policies seem to stem from a presumption (shared by some economists) that open software platforms are *inherently* more efficient than their proprietary counterparts. The social efficiency issues associated with different modes of platform governance (open vs. closed) in the type of markets described above are quite important and they have not yet been addressed by the growing economics literature on two-sided markets, which has up to now been mostly concerned with platform pricing structures[2].

In this context, the key contribution of our paper is to formally reveal a fundamental welfare tradeoff between two-sided *proprietary* (i.e. profit-maximizing) platforms and two-sided *open* platforms, which allow "free entry" on both sides of the market. Using the model of two-sided platforms connecting buyers and suppliers of many varied products first introduced by Hagiu (2004a), we show that on the one hand, a profit-maximizing platform creates two-sided deadweight losses through monopoly pricing, but on the other hand, precisely because it sets prices in order to maximize profits, it internalizes at least partially the positive indirect network externalities between consumers and product suppliers and the direct competitive effects between producers. By contrast, an open platform internalizes

[1] See Hahn et al. (2002).

[2] See for instance Armstrong (2005), Rochet and Tirole (2003), Hagiu (2004b).

neither of these two effects since it essentially sets prices equal to marginal costs (zero) on both sides. Therefore it is by no means obvious which type of platform will create higher product variety, consumer adoption and total social welfare. We also show that the same tradeoff arises when comparing a situation with competing platforms and one with a single monopoly platform: for a certain range of parameters, the latter generates higher product variety and social welfare. This suggests that there is a sense in which platform competition is undesirable because it prevents platforms from sufficiently internalizing indirect network effects and therefore from inducing the appropriate levels of product variety.

This paper is also a generalization of the earlier economics literature on product variety, free entry and social efficiency, in particular the seminal contribution of Mankiw and Whinston (1986). They study the inefficiencies associated with free-entry in product markets and show that the sign of the inefficiency (i.e. whether there is excessive or insufficient entry) depends on the interplay between the competitive (business-stealing) effect and the product-diversity effect. Our analysis can be viewed as an extension of theirs in two important dimensions. First, Mankiw and Whinston's model is "one-sided" in the sense that the number of consumers participating in the market is fixed and only the number of producers is variable. This allows them to focus exclusively on *direct* (negative) competitive effects on the producer side and abstract from the positive *indirect* network effects between the consumer side and the producer side, which are central to our paper. Thus, our two-sided open platforms are similar but more general than the free-entry regimes studied by Mankiw and Whinston (1986), Kiyono and Suzumura (1987), Spence (1976), Dixit and Stiglitz (1977) and Salop (1979) because consumer participation in the market is endogenous in our framework. Second and most important, our two-sided proprietary platforms controlling market access through prices charged to both consumers and product suppliers constitute a novel form of market organization, which has not been analyzed by the literature on product variety.

Finally, our paper is related to the literature on indirect network effects, especially Church and Gandal (1992) and Church Gandal and Krause (2002). Both papers study two-sided technology (or platform) adoption, however in both models, the platform is assumed to be entirely passive, i.e. there is no strategic pricing on either side of the market. This is equivalent to an open platform in our model.

The paper is organized as follows: the next section lays out the basic model and sets up the two-sided mechanisms determining platform adoption by consumers and producers for a monopoly two-sided proprietary platform, a social planner and a monopoly open platform. Section 3 analyzes social efficiency, by first comparing product variety, consumer adoption and social welfare under a proprietary platform and an open platform. It then extends the basic model in two directions. First, it introduces vertical differentiation on the producer side and shows that in this context, contrary to common intuition, a monopoly platform can induce socially excessive product variety. Second, it compares a regime with two competing platforms with one in which only one platform is active and shows that for a range of parameter values, platform competition is not socially desirable. Section 4 concludes.

2. Modelling framework

The modeling framework is derived from that developed in Hagiu (2004a). We are interested in two-sided platforms whose value to consumers (users) is increasing in the number of developers[3] they support and whose value to developers is increasing in the number of

[3] Developers are third-party product suppliers: developers of software applications or games, content providers, etc. For simplicity and ease of interpretation throughout the paper we will use the blanket term "developers" instead of third-party producers and "applications" in order to refer to their products.

users who adopt them. The platform controls the extent of adoption on both sides of the market through prices.

Net surplus for a user indexed by θ from joining a platform which charges her P^U[4] and is supported by n applications is:

$$u(n) - P^U - \theta$$

where $u(n)$ is the surplus obtained from the n applications, *net of the prices charged by application developers* and the parameter θ is the user's intrinsic "distance" in preference space to the system comprised by the platform and the applications[5]. It is distributed over a support $[\theta_L, \theta_H]$ (we allow θ_H to be infinite). The number of users "closer" than θ (i.e. characterized by $\theta' \leq \theta$) is $F(\theta)$, where F is a differentiable and strictly increasing function with continuously differentiable derivative f, mapping $[\theta_L, \theta_H]$ into $[0, +\infty]$ and such that $F(\theta_L) = 0$. We denote by ε_F the elasticity of F, which is to be interpreted as the "elasticity" of user demand for the platform:

$$\varepsilon_F(\theta) = \frac{\theta f(\theta)}{F(\theta)} > 0$$

Similarly, net profits for a developer indexed by ϕ from supporting a platform which charges P^D and is adopted by all users with $\theta \leq \theta^m$ are[6]:

$$\pi(n) F(\theta^m) - P^D - \phi$$

[4] Hagiu (2004b) studies the pricing aspect of the problem into detail. Here we are concerned with social efficiency, therefore we choose the simplest possible pricing game which allows us to derive the main insights.

[5] For example, it can be interpreted as the difference between the fixed (sunk) cost of learning how to use the system and the standalone value of the platform (in case it comes bundled with some applications).

[6] Indeed, given the structure of user preferences assumed above, if user θ adopts the platform given n and P^U then all users $\theta' \leq \theta$ will also adopt.

where $\pi(n)$ is the profit *per platform user net of variable costs* and the parameter ϕ is the fixed cost of writing an application, distributed on $[0, \phi_H]$ (we allow ϕ_H to be infinite). The number of developers with fixed costs less than or equal to ϕ is $H(\phi)$, where H is a differentiable and strictly increasing function with continuously differentiable derivative h, mapping $[0, \phi_H]$ into $[0, +\infty]$ and such that $H(0) = 0$. The elasticity of developer demand for the platform is:

$$\varepsilon_H(\phi) = \frac{\phi h(\phi)}{H(\phi)} > 0$$

As suggested by this formulation we will ignore integer constraints and treat n as a continuous variable throughout the paper. The reason is that in the markets we have in mind there are hundreds or even thousands of applications: each individual developer is then "very small" and ignores the influence of his decision on platform adoption by users and other developers. Continuity also renders the analysis very convenient by allowing us to use demand elasticities.

Let:

$$V(n) = u(n) + n\pi(n)$$

denote the gross surplus created by n applications for each platform user.

We make the following assumption:

Assumption 1 *$u(n)$ is strictly increasing, $\pi(n)$ is strictly decreasing and $V(n)$ is strictly increasing and concave.*

This assumption is quite reasonable: it simply says that net user surplus $u(n)$ is increasing in the number of applications used, that each developer's profits per user are decreasing in n (crowding effect) and that the gross user surplus created by n applications is increasing at a decreasing rate (the 100th application is less valuable than the 10th).

Let us denote by ε_V the elasticity of V:

$$\varepsilon_V(n) = \frac{nV'(n)}{V(n)} \in]0,1[$$

The elasticity ε_V measures the intensity of users' preference for variety. The higher ε_V, the less concave $V(.)$ and therefore the higher the marginal contribution of an additional application to gross surplus per platform user.

Also, it will prove useful to define:

$$\lambda(n) = \frac{\pi(n)}{V'(n)}$$

the ratio between developer profits and the marginal contribution of an additional developer to gross surplus per platform user. Intuitively, when $\lambda(n) > 1$, each developer is gaining more than his marginal contribution, therefore one would expect a bias towards socially excessive entry on the developer side of the market under an open platform (or free entry regime), and viceversa, when $\lambda(n) < 1$, an open platform regime contains a bias towards socially insufficient developer entry (cf. Mankiw and Whinston (1986)). A two-sided proprietary platform may either correct or exacerbate this bias to a certain extent through its prices.

In order to illustrate how the reduced forms $u(n)$, $\pi(n)$ and $V(n)$ are obtained, we provide two specific examples, both of which satisfy assumption 1 and which we will use throughout the paper.

Example 1 Suppose users' gross utility has the Spence-Dixit-Stiglitz form $G\left(\sum_i v(q_i)\right)$, where q_i is the "quantity" of application i *consumed*, $v(0) = 0$, $v'(.) > 0$ and $v''(.) < 0$ and $G'(.) > 0$, $G''(.) < 0$.

User maximization implies that the quantity q_k demanded by each platform user[7] from developer k charging p_k satisfies:

$$p_k = v'(q_k)\, G'\left(\sum_i v(q_i)\right)$$

Each developer takes the market price $G'\left(\sum_i v(q_i)\right)$ as given when setting his price. Consequently, the stage 3 pricing equilibrium among developers is symmetric and defined by:

$$v'(q_n)\, G'(nv(q_n)) = p_n = \arg\max_p \left\{(p-c)\, v'^{-1}\left(\frac{p}{G'(nv(q_n))}\right)\right\}$$

Then: $\pi(n) = (p_n - c)\, q_n$, $u(n) = G(nv(q_n)) - np_nq_n$ and $V(n) = G(nv(q_n)) - ncq_n$. Letting $v(q) = q^\sigma$ and $G(z) = z^{\frac{\alpha}{\sigma}}$, with $0 < \alpha < \sigma < 1$, we obtain $p = \frac{c}{\sigma}$ and:

$$\pi(n) = (1-\sigma)\,\alpha\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}} n^{-\frac{\sigma-\alpha}{\sigma(1-\alpha)}}$$

$$u(n) = (1-\alpha)\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}} n^{\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}}$$

$$V(n) = (1-\sigma\alpha)\left(\frac{\alpha\sigma}{c}\right)^{\frac{\alpha}{1-\alpha}} n^{\frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}}$$

$$\varepsilon_V = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)} \in]0,1[$$

$$\lambda = \frac{\sigma(1-\alpha)}{1-\sigma\alpha} \in]0,1[$$

Example 2 Suppose users have unitary demand for applications (i.e. buy either 0 or one unit of each application) and gross benefits from using n applications are $V(n)$ with $V'(.) > 0$, $V''(.) < 0$. In this case the stage 3 price equilibrium is: $p_n = V'(n)$ leading

[7] This is because all users "agree" on the incremental benefits offered by applications.

to[8]: $\pi(n) = V'(n)$, $u(n) = V(n) - nV'(n) > 0$ and $\lambda = 1$. Letting $V(n) = An^{\beta}$, with $0 < \beta < 1$, we obtain[9]:

$$\pi(n) = \beta A n^{\beta-1}$$

$$u(n) = (1-\beta) A n^{\beta} > 0$$

$$\varepsilon_V = \beta$$

Let us now clearly specify the timing of the pricing game we consider throughout the paper. There are 3 stages:

- Stage 1) The platform sets prices P^U and P^D for consumers and developers simultaneously
- Stage 2) Users and developers make their adoption decision simultaneously
- Stage 3) Developers having adopted the platform set prices for consumers and those consumers who have adopted the platform in the second stage decide which applications to buy.

The slightly odd-sounding assumption that users decide whether or not to buy the platform *before* developers set their prices is made in order to simplify the analysis of the two-sided pricing game. Given that developers are atomistic in our model, it is entirely harmless: developers ignore the effect of their pricing decision on total consumer demand for the platform anyway.

[8] Here we assume developers have 0 marginal costs: many of the real-life platforms we have in mind support digital applications whose marginal costs are virtually 0. In example 1 marginal costs are necessarily positive in order to keep prices and profits finite.

[9] This example is used by Church Gandal and Krause (2002).

2.1. The optimization problem for a two-sided proprietary platform

Given the platform's prices P^U and P^D, it is indeed an (interior) equilibrium for n developers and $F(\theta^m)$ users to adopt the platform in stage 2 only if the following two conditions hold:

$$\pi(n) F(\theta^m) - P^D - H^{-1}(n) = 0 \tag{2.1}$$

$$u(n) - P^U = \theta^m \tag{2.2}$$

The first condition says that in equilibrium all profit opportunities are exhausted for developers (assuming the supply of developers is large enough) and the second condition says that the marginal user θ^m must be indifferent between adopting and not adopting the platform.

Equation (2.1) determines developer demand n as a function $N\left(\theta^m, P^D\right)$ of user demand and the price charged to developers, whereas equation (2.2) determines the marginal user θ^m (and therefore user demand $F(\theta^m)$) as a function $\Theta\left(n, P^U\right)$ of developer demand and the price charged to users. Note that these two-way demand interdependencies or indirect network externalities are positive: $N\left(., P^D\right)$ and $\Theta\left(., P^U\right)$ are both increasing.

Plugging (2.2) into (2.1), we obtain n as an implicit function of the platform's prices P^D and P^U:

$$\pi(n) F\left(u(n) - P^U\right) = H^{-1}(n) + P^D \tag{2.3}$$

This expression makes clear that on the developer side of the market there are both positive *indirect* network effects contained in the term $F\left(u(n) - P^U\right)$ and *negative direct* or competitive effects contained in the term $\pi(n)$.

Throughout the paper we normalize for simplicity and without any loss of substance the platform's marginal costs on both sides to 0. The expression of platform profits is then:

$$\Pi^P = P^U F(\theta^m) + nP^D$$

Using (2.1) and (2.2) we obtain:

$$\Pi^P = (V(n) - \theta^m) F(\theta^m) - nH^{-1}(n) \tag{2.4}$$

which depends only on (θ^m, n). Therefore, rather than maximizing platform profits over $\left(P^U, P^D\right)$ we will do so directly over (θ^m, n)[10].

The first-order conditions determining the optimal $\left(\theta_p^m, n_p\right)$ are:

$$\frac{V(n) - \theta^m}{\theta^m} = \frac{1}{\varepsilon_F(\theta^m)} \tag{2.5}$$

$$V'(n) F(\theta^m) = nH^{-1\prime}(n) + H^{-1}(n) \tag{2.6}$$

Given the profit-maximizing $\left(n_p, \theta_p^m\right)$, the corresponding profit maximizing prices $\left(P_{2sp}^U, P_{2sp}^D\right)$ are then *uniquely* determined by (2.1) and (2.2).

2.2. The optimization problem for the social planner

A benevolent social planner maximizes total welfare, which in our framework is the difference between total surplus from indirect network effects and the costs of entry on the two sides of the market. Its expression when n developers and all $\theta \leq \theta^m$ users are allowed in

[10] A similar "trick" is used by Armstrong (2003) in a linear model. Below we discuss necessary conditions for this transformation to be legitimate in our model.

the market is then:

$$W\left(\theta^m, n\right) = V\left(n\right) F\left(\theta^m\right) - \int_0^{\theta^m} \theta f\left(\theta\right) d\theta - \int_0^{H^{-1}(n)} \phi h\left(\phi\right) d\phi \tag{2.7}$$

which the social planner maximizes over (θ^m, n). This leads to the following first-order conditions:

$$V\left(n\right) - \theta^m = 0 \tag{2.8}$$

$$V'\left(n\right) F\left(\theta^m\right) - H^{-1}\left(n\right) = 0 \tag{2.9}$$

which determine the socially optimal levels of entry on both sides (θ_{so}^m, n_{so}).

2.3. Open platform or two-sided free entry

In our framework, an open platform is characterized simply by free-entry of both users and developers, i.e. the open platform charges prices equal to 0 on both sides of the market $(\left(P_{fe}^U, P_{fe}^D\right) = (0,0))$ and users and developers enter freely until all positive surplus, respectively profit, opportunities are exhausted. Formally, n_{fe} and θ_{fe}^m are determined simultaneously by the following two equations:

$$\pi_m^D\left(n\right) = \pi\left(n\right) F\left(\theta^m\right) - H^{-1}\left(n\right) = 0 \tag{2.10}$$

$$\theta^m = u\left(n\right) = V\left(n\right) - n\pi\left(n\right) \tag{2.11}$$

where $\pi_m^D\left(n\right)$ are net profits of the marginal developer when n developers have entered.

Before proceeding, there are a few issues we need to address in order to be completely rigorous. First, we sidestep the problem of multiple solutions inherent in contexts with indirect network effects by assuming[11]:

Assumption 2 *$\left(\theta_p^m, n_p\right)$, (θ_{so}^m, n_{so}) and $\left(\theta_{fe}^m, n_{fe}\right)$ are well-defined, i.e. (2.5, 2.6), (2.8, 2.9) and (2.10, 2.11) each have a unique interior solution. Moreover $\left(n_p, \theta_p^m\right)$ and (n_{so}, θ_{so}^m) are global maximizers for Π^P and W respectively.*

Second, at a minimum we should also make sure that *given* $\left(P_p^U, P_p^D\right)$, $\left(n_p, \theta_p^m\right)$ arises as a *stable* market configuration and similarly, given zero prices on both sides ($\left(P_{fe}^U, P_{fe}^D\right) = (0,0)$), $\left(n_{fe}, \theta_{fe}^m\right)$ also arises as a stable market configuration. Graphically, stability of configuration (n_x, θ_x^m) *given* $\left(P_x^U, P_x^D\right)$ means that at point (n_x, θ_x^m) the curve $n = N\left(\theta^m, P_x^D\right)$ crosses the curve $\theta^m = \Theta\left(n, P_x^U\right)$ from below in a (n, θ^m) plane[12].

Assumption 3 *The market configurations $\left(n_p, \theta_p^m\right)$ and $\left(n_{fe}, \theta_{fe}^m\right)$ are stable given $\left(P_p^U, P_p^D\right)$,respectively $(0,0)$.*

The following lemma, proven in the appendix, provides a useful example of functional forms, which satisfy assumptions 2 and 3.

Lemma 0 *Assume F, H and V are defined on $[0, +\infty]$ and have constant elasticities, i.e. $F(\theta) = B\theta^{\varepsilon_F}$, $H^{-1}(\phi) = C\phi^{\frac{1}{\varepsilon_H}}$, $V(n) = An^{\varepsilon_V}$, $\pi(n) = \lambda V'(n)$, where $\varepsilon_V = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}$, $\lambda = \frac{\sigma(1-\alpha)}{1-\sigma\alpha}$ in example 1 and $\varepsilon_V = \beta$, $\lambda = 1$ in example 2. Then assumptions 2 and 3 are satisfied if:*

$$\varepsilon_V (1 + \varepsilon_F) \leq 1 + \frac{1}{\varepsilon_H} \tag{2.12}$$

$$\left[\frac{1 - \lambda\varepsilon_V}{1 - \varepsilon_V}\varepsilon_V (1 + \varepsilon_F) - 1\right] \lambda (1 - \varepsilon_V) \leq \frac{1}{1 + \varepsilon_H} \tag{2.13}$$

■

[11] The insights provided by the discussion that follows is unaffected by this assumption and the formal analysis is greatly simplified.

[12] See Hagiu (2004b) for a graphic representation and a dynamic justification.

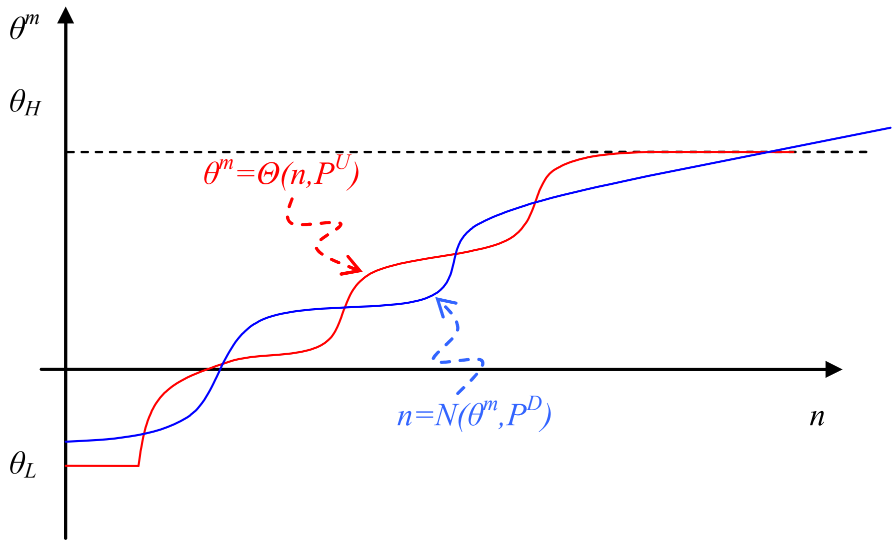

Figure 2.1:

Third and last, for $\left(P_p^U, P_p^D\right)$, (2.1) and (2.2) may have multiple *stable* solutions (θ^m, n) as illustrated in figure 2.1[13].

This is a well-known feature in markets with indirect network effects[14]. Thus, we also make the following assumption.

Assumption 4 *If there are multiple stable market configurations, solutions to (2.1) and (2.2) given* $\left(P_p^U, P_p^D\right) = \left(u\left(n_p\right) - \theta_p^m, \pi\left(n_p\right) F\left(\theta_p^m\right) - H^{-1}\left(n_p\right)\right)$, *then the proprietary platform is able to coordinate users and developers on its most preferred solution, i.e.* $\left(\theta_p^m, n_p\right)$.

This assumption is less restrictive than it might appear at first glance. Even when there are multiple stable equilibria, if developers and users do not coordinate on the equilibrium

[13] Assumption 2 already ensures that given $\left(P_{fe}^U, P_{fe}^D\right) = (0,0)$, (2.1) and (2.2) have a *unique* stable solution $\left(\theta_{fe}^m, n_{fe}\right)$.

[14] See for example Church and Gandal (1992).

desired by the platform, then the latter can "adjust" coordination either by providing some of its own applications or by restricting entry on both sides.

3. Proprietary platforms, open platforms, product diversity and social efficiency

Having set up the equations determining the levels of user adoption and developer entry under a two-sided proprietary platform, a social planner and a two-sided open platform, we can now turn to comparing them, as well as the total levels of social welfare the platforms induce.

3.1. Monopoly proprietary platform vs. monopoly open platform

Although our representation of open platforms as allowing free entry on both sides of the market may be an overly simplified conceptualization of, say, the open source software form of market organization[15], it is sufficient for revealing a fundamental welfare tradeoff relative to proprietary platforms. An open platform does not create two-sided deadweight losses due to monopoly pricing but at the same time leaves uninternalized the positive indirect network effects between users and developers, whereas a proprietary platform has an incentive to internalize them precisely because it sets its prices in order to maximize profits.

Note that in a one-sided market the welfare comparison would be trivial: a firm pricing at marginal cost always entails higher output and higher social welfare than a profit-maximizing monopolist who cannot price-discriminate. By contrast, in a two-sided context, things are more complex: as we show below, a proprietary platform may in fact induce

[15] In particular, "free entry" of users and developers is certainly not a perfect representation of the licensing agreements characteristic of open source software (BSD or GPL).

more developer entry (i.e. product variety), user adoption and higher total social welfare than an open platform and it may even result in *socially excessive* product variety (and user adoption).

To understand precisely where the tradeoffs come from, it is useful to look at the economic mechanisms which drive entry on each side of the market. Consider first the developer side. The derivative of total social welfare with respect to n is:

$$\frac{\partial W}{\partial n} = V'(n) F(\theta^m) - H^{-1}(n) = \pi_m^D(n) + \left(V'(n) - \pi(n)\right) F(\theta^m) \tag{3.1}$$

Thus, if one looks only at the developer side of the market, what drives a wedge between the levels of product diversity under an open platform relative to the socially optimal level is the term $(V'(n) - \pi(n)) F(\theta^m)$. If developer profits per platform user $\pi(n)$ exceed the marginal contribution of an additional developer to social welfare per platform user $V'(n)$ (i.e. $\lambda > 1$), then $\frac{\partial W}{\partial n} < \pi_m^D(n)$ and therefore an open platform tends to induce excessive entry of developers *all other things equal*. And viceversa. This is precisely the insight of Mankiw and Whinston (1986). To see this more clearly, consider example 1:

$$V'(n) - \pi(n) = \underbrace{n\left(G'v' - c\right)\frac{\partial q_n}{\partial n}}_{\text{business-stealing}} + \underbrace{G' \times \left(v - v'q_n\right)}_{\text{product diversity}}$$

Just like in Mankiw and Whinston (1986) the first term represents the business stealing effect and is negative as long as $\frac{\partial q_n}{\partial n} < 0$ and the price $G'v'$ is above marginal cost, whereas the second term is the product diversity effect and is positive since v is concave. The sign of the inefficiency of an open platform on the developer side depends on which of these two effects dominates. In example 2 we have $\pi(n) = V'(n)$, so that the open platform introduces no bias with respect to developer entry *all other things equal*.

But of course, all other things are *not* equal in our model, since developer entry depends on user entry and viceversa. As we show below, the open platform induces too little user entry, which in turn leads to too little developer entry, an indirect effect which does not exist in Mankiw and Whinston (1986).

Consider now the derivative of a proprietary platform's profits with respect to n:

$$\begin{aligned}\frac{\partial \Pi^P}{\partial n} &= V'(n)\,F(\theta^m) - H^{-1}(n) - nH^{-1\prime}(n) \\ &= \pi_m^D(n) + \big(V'(n) - \pi(n)\big)\,F(\theta^m) - nH^{-1\prime}(n) \qquad (3.2)\end{aligned}$$

Comparing (3.2) with (3.1), the proprietary platform introduces no inefficiency through the business stealing and the product diversity effects. This is due to the fact that in our model both users and developers are differentiated only horizontally, so that the platform can fully internalize developer revenues $n\pi(n)$ and user gross surplus $V(n) - n\pi(n)$[16]. What does induce a bias however is the proprietary platform's inability to perfectly price discriminate among developers: it consequently discounts the total social value created by an additional developer by $nH^{-1\prime}(n)$, the revenues lost on existing developers by reducing the price P^D in order to accomodate the additional developer. Since this bias is negative, the proprietary platform tends to induce too little entry on the developer side, keeping everything else constant.

Turning now to the user side of the market, first order condition 2.5 with respect to θ^m

[16] Below we provide an example with vertical developer differentiation.

for the proprietary platform is:

$$u(n) - P_p^U = \theta^m = \frac{\varepsilon_F V(n)}{1+\varepsilon_F} \tag{3.3}$$

Comparing (2.8) to (2.11), the open platform tends to induce too little user adoption all other things equal, because each developer who enters does not take into account the effect of his price on *total* user demand for the platform. Comparing (3.3) to (2.8), the proprietary platform also tends to induce too little user entry: it perceives the benefits of an additional user as the difference between the extra revenues $P_p^U + n\pi(n) = V(n) - \theta^m$, which are exactly equal to the total social value created by the additional user[17], and $\frac{F(\theta^m)}{f(\theta^m)}$, the revenues lost on existing users by reducing the price P_p^U in order to accomodate the additional user.

Comparing (2.11) and (3.3), it is not possible to say in general which of the open platform or the proprietary platform restricts user adoption more. It depends on the sign of P_p^U: all other things equal, the proprietary platform induces less restriction of user entry if and only if it subsidizes users, i.e. sets $P_p^U < 0$. This illustrates the fact that, by being able to balance the interests of the two sides through its pricing structure, a proprietary platform may come closer to the socially optimal level of adoption than a platform simply pricing at marginal cost on both sides.

Thus, given that a proprietary platform induces a bias towards socially insufficient entry on both sides of the market, the combination of the two leads unambiguously to insufficient product diversity and user adoption relative to the socially optimal levels. This of course is not a robust conclusion: it is due to our assumption of horizontal differentiation on both sides. Below we show that introducing vertical developer differentiation is sufficient for

[17] Once again, this is because users are differentiated only horizontally.

overturning this result.

However, even in this simple horizontal differentiation framework, the proprietary platforms may turn out to induce higher levels of product variety and total social welfare. To see this more clearly, we can combine the first-order conditions above in order to obtain:

- the level of product variety n_p induced by a proprietary platform solves:

$$V'(n)\,F\left(\frac{\varepsilon_F V(n)}{1+\varepsilon_F}\right) = nH^{-1\prime}(n) + H^{-1}(n) \tag{3.4}$$

- the level of product variety n_{fe} induced by an open platform solves:

$$\pi(n)\,F(u(n)) = H^{-1}(n) \tag{3.5}$$

- the level of product variety n_{so} chosen by the social planner solves:

$$V'(n)\,F(V(n)) = H^{-1}(n) \tag{3.6}$$

Under sufficient regularity conditions (cf. Assumption 2 and Lemma 0), n_p, n_{fe}, n_{so} are well-defined, i.e. (3.4), (3.5) and (3.6) each have a unique positive solution. Then, since, $\frac{\varepsilon_F}{1+\varepsilon_F} < 1$ and $H^{-1\prime}(n) > 0$, we have $n_p < n_{so}$. However, comparing (3.4) and (3.5), it is not possible to say in general whether $n_p \gtrless n_{fe}$. Figure (**??**) illustrates (3.4), (3.5) and (3.6): in graph a) the positive indirect network effects are outweighed by the negative direct business-stealing effects on the developer side so that the left-hand sides of (3.4), (3.5) and (3.6) are decreasing in n, whereas graph b) depicts the case in which the positive indirect effects are stronger.

The following proposition provides a rigorous illustration of all the previous considera-

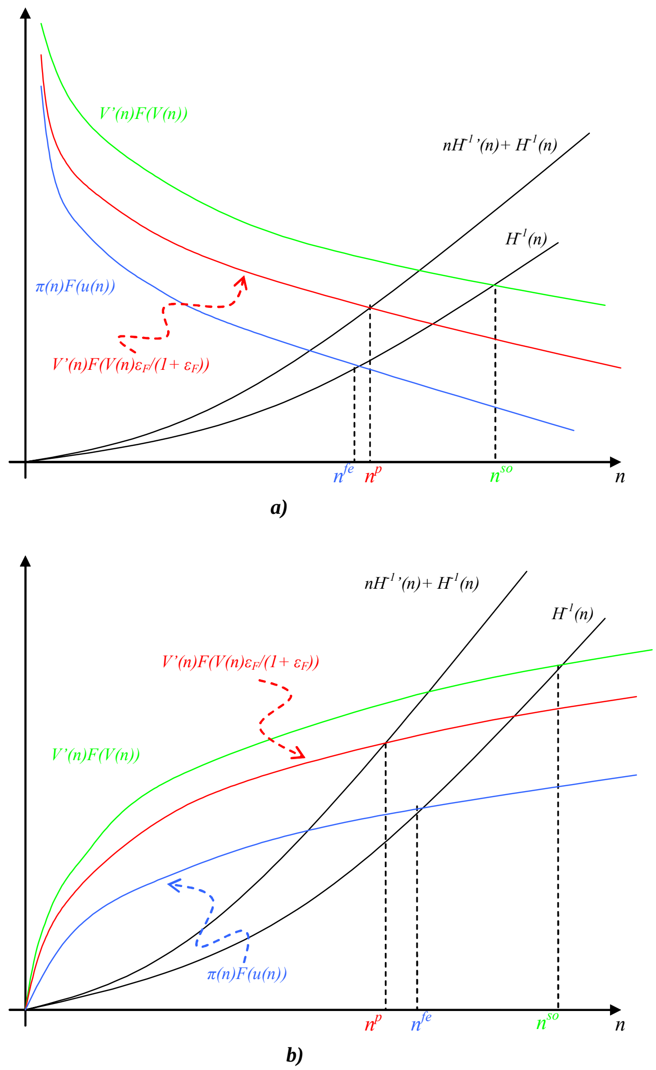
V'(n)F(V(n))
nH-1'(n)+ H-1(n)
H-1(n)
π(n)F(u(n))
V'(n)F(V(n)εF/(1+ εF))
nfe
np
nso
n
a)
nH-1'(n)+ H-1(n)
H-1(n)
V'(n)F(V(n)εF/(1+ εF))
V'(n)F(V(n))
π(n)F(u(n))
np
nfe
nso
n
b)

Figure 3.1:

tions.

Proposition 4 *Assume V, F and H are defined on $[0,+\infty]$ with constant elasticities, $F(\theta) = B\theta^{\varepsilon_F}$, $H^{-1}(n) = Cn^{\frac{1}{\varepsilon_H}}$, $V(n) = An^{\varepsilon_V}$, $\pi(n) = \lambda V'(n)$, $u(n) = (1-\lambda\varepsilon_V)V(n)$ and competition among developers is either as in example 1, with $\varepsilon_V = \frac{\alpha(1-\sigma)}{\sigma(1-\alpha)}$ and $\lambda = \frac{\sigma(1-\alpha)}{1-\sigma\alpha}$, or as in example 2, with $\varepsilon_V = \beta$ and $\lambda = 1$. Also, assume (2.12) and (2.13) hold. Then:*

i) Both the proprietary and the open platforms induce socially insufficient product variety and user adoption: $n_p, n_{fe} < n_{so}$ and $\theta_p^m, \theta_{fe}^m < \theta_{so}^m$

ii) Suppose in addition that $\varepsilon_F = 1$ and that all developers have the same fixed cost ϕ (i.e. $\varepsilon_H = +\infty$, $C = \phi$) and compete as in example 1. Then:

- *$n_p > n_{fe}$ if and only if $(1-\sigma\alpha)^2 > 2\sigma(1-\alpha)^2$.*
- *Total social welfare can be higher with either type of platform: $\frac{W\left(n_p,\theta_p^m\right)}{W\left(n_{fe},\theta_{fe}^m\right)} \to +\infty$ when $\alpha \to 0$, $\sigma \to 0$ and $\frac{\alpha}{\sigma} \to k < 1$; $\frac{W\left(n_p,\theta_p^m\right)}{W\left(n_{fe},\theta_{fe}^m\right)} \to \frac{3}{4}$ when $\sigma \to 1$.*

Proof See appendix.■

The most substantial part of proposition 4 is part ii): it exhibits specific cases in which a proprietary platform dominates an open platform both in terms of product variety and total social welfare.

3.2. Developer vertical differentiation and socially excessive product variety

Despite its tractability, one shortcoming of the two-sided horizontal differentiation model we have used up to now is that it cannot generate cases in which proprietary platforms induce socially excessive levels of product variety[18], as was made clear in the discussion above. This is because when the two sides of the market are differentiated only horizontally, a two-sided platform fully internalizes the indirect network effects between users and developers, as well as the direct competitive effects between developers. The only distortions which arise are the deadweight losses due to monopoly pricing on both sides of the market and they lead to insufficient entry of both users and developers.

In this subsection we wish to make clear that this feature cannot be robust to more general formulations of user and developer demand and that everything hinges crucially on the extent to which a platform internalizes indirect network effects and business-stealing effects. Even though a two-sided platform extracts only a part of *total* user and developer surplus, there is no reason why the *marginal* contribution of an additional developer to platform profits should always be lower than the marginal contribution of that developer to total social surplus so that the platform necessarily restricts entry too much relative to the social optimum. In particular, if developers are sufficiently vertically differentiated by the benefits they offer users (as opposed to being simply heterogeneous in their fixed costs) and if the platform is unable to perfectly price discriminate, then it might *overestimate* the value of the positive indirect network effects relative to the value of negative direct competitive effects and therefore induce socially *excessive* entry.

To formalize this insight, we modify our model by assuming that all developers have the same fixed cost $\phi > 0$ and that they are exogenously differentiated by the quality q of their applications. The quality q is the probability that a given user is interested in a particular application (demand for each application is unitary) and is distributed over a support $[q_L, q_H] \subset [0, 1]$, such that the number of developers with quality lower than q

[18]By contrast, it is easy to construct cases in which the level of product variety generated by an open platform is socially excessive: it suffices to assume inelastic user demand and use our example 1 with the functional forms provided in section 4 of Mankiw and Whinston (1986): $G(z) = \frac{1}{z}$, $v(q) = aq - \left(\frac{b}{2}\right) q^2$.

is $H(q)$, where H is an increasing function with continuous derivative h and satisifying $H(q_L) = 0$ and $H(q_H) < +\infty$.

Although developers are vertically differentiated, we still assume for simplicity that the platform is restricted to charging only fixed access fees on both sides[19]. Given platform prices $\left(P^U, P^D\right)$ and user demand $F(\theta^m)$, only high-quality developers enter, i.e. those with $q \geq q_m$, where q_m is the quality of the marginal developer. Each user buys $\int_{q^m}^{q_H} qh(q)\,dq$ applications so that the equilibrium price of applications is $V'\left(\int_{q^m}^{q_H} qh(q)\,dq\right)$ by straightforward analogy with example 1 above. q_m is then defined by:

$$V'\left(\int_{q^m}^{q_H} qh(q)\,dq\right) q^m F(\theta^m) - P^D - \phi = 0$$

The marginal user θ^m is then given by:

$$V(Q(q^m)) - Q(q^m)\,V'(Q(q^m)) - \theta^m - P^U = 0$$

where:

$$Q(q^m) = \int_{q^m}^{q_H} qh(q)\,dq$$

is the number of applications bought by each platform user. Platform profits are:

$$\begin{aligned}\Pi^P &= P^U F(\theta^m) + P^D (H(q_H) - H(q^m)) \\ &= [V(Q(q^m)) - E(q^m) - \theta^m]\,F(\theta^m) - (H(q_H) - H(q^m))\,\phi\end{aligned}$$

[19] Our insights remain valid when the platform is also allowed to charge variable fees. The only necessary condition is that the platform should be unable to fully extract all developer revenues through its prices. This may be justified for example by the need to provide sufficient innovation incentives to developers, as shown in Hagiu (2004b).

where:

$$E\left(q^{m}\right)=V'\left(Q\left(q^{m}\right)\right)\left(Q\left(q^{m}\right)-\left(H\left(q_{H}\right)-H\left(q^{m}\right)\right)q^{m}\right)>0$$

is the difference between total developer revenues and the portion thereof which is extracted by the platform (per user). In other words, it is the part of developer gross surplus uninternalized by the platform. Note that when all developers have the same quality, $E\left(q^{m}\right)=0$, which brings us back to the horizontal differentiation case, in which the two-sided platform fully internalizes both developer revenues and user surplus.

Assuming the appropriate second order and stability conditions hold, the profit-maximizing marginal product quality q_{p}^{m} and user θ_{p}^{m} are the solutions to the first-order conditions:

$$\left(V\left(Q\right)-E\left(q^{m}\right)-\theta^{m}\right)f\left(\theta^{m}\right)=F\left(\theta^{m}\right) \tag{3.7}$$

$$\left(\frac{dQ}{dq^{m}}V'\left(Q\right)-E'\left(q^{m}\right)\right)F\left(\theta^{m}\right)+h\left(q^{m}\right)\phi=0 \tag{3.8}$$

Social welfare on the other hand has the following expression:

$$W=V\left(Q\right)F\left(\theta^{m}\right)-\int_{0}^{\theta^{m}}\theta f\left(\theta\right)d\theta-\left(H\left(q_{H}\right)-H\left(q^{m}\right)\right)\phi$$

so that the socially optimal marginal product quality q_{so}^{m} and marginal user θ_{so}^{m} are the solutions to:

$$V\left(Q\right)-\theta^{m}=0 \tag{3.9}$$

$$\frac{dQ}{dq_{m}}V'\left(Q\right)F\left(\theta^{m}\right)+h\left(q_{m}\right)\phi=0 \tag{3.10}$$

Comparing (3.9) and (3.10) to (3.7) and (3.8), it is no longer obvious whether the two-sided proprietary platform will induce too little ($q_{p}^{m}>q_{so}^{m}$) or too much ($q_{p}^{m}<q_{so}^{m}$) variety

(and user adoption). Indeed, while the monopoly pricing distortion on the user side still tends to render user adoption sub-optimal[20], on the developer side it all depends on the sign of $E'(q^m)$. Specifically, if $E'(q^m) > 0$, then the left hand side of (3.8) is lower than the left-hand side of (3.10) and consequently, since both expressions are decreasing in q^m (required by our assumption that the maximization problems are well-defined), it might turn out that $q_p^m < q_{so}^m$. In this case there is an excessive variety bias on the developer side, which may exceed the insufficient user adoption bias. The following proposition provides and example in which this happens:

Proposition 5 *Assume there is a mass B of identical users (i.e. user demand for the platform is inelastic), $H(q) = C(q - q_L)$, $V(Q) = V - \frac{A}{Q^\beta}$ and:*

$$\beta q_L > 2q_H$$

Then $\Pi^P(q^m)$ and $W(q^m)$ are concave and the proprietary platform induces socially excessive product diversity, i.e. $q_p^m < q_{so}^m$ or $H(q_H) - H\left(q_p^m\right) > H(q_H) - H(q_{so}^m)$.

Proof In the appendix.■

3.3. Monopoly platform vs. competing platforms

The economic efficiency tradeoff coined above between internalizing two-sided indirect network effects and creating two-sided deadweight loss also has interesting implications regarding the desirability of competition between two-sided platforms. In a static one-sided context, with no fixed set-up costs and no innovation, more competition always increases social welfare as it helps reduce the deadweight losses due to pricing by firms with market

[20] To see this, note that given the same q^m, (3.7) yields a lower θ^m than (3.9).

power. In a two-sided context however, the countervailing force is that platform competition limits the ability of individual platforms to extract surplus from both sides of the market and therefore may generate less product variety and user adoption than those arising under a monopoly platform regime (all other things equal). This negative effect might well outweigh the positive effect of competition (reduction of deadweight loss).

To illustrate this mechanism, consider the following straightforward extension of our model to allow for platform competition (cf. Hagiu (2004b)). The user horizontal differentiation parameter θ is now assumed to be uniformly distributed on a Hotelling segment $[0,1]$; unit transportation costs are t and there is one platform situated at each of the two extremities. Thus, the utility of a user located at $\theta \in [0,1]$ from adopting platform 1 is $u_0 + u(n_1) - t\theta - P_1^U$, whereas that from adopting platform 2 is $u_0 + u(n_2) - t(1-\theta) - P_2^U$, where u_0 is the standalone value of each platform for users. In all that follows we assume u_0 is large enough so that the user market is always entirely covered. For $i = 1, 2$, n_i denotes the number of developers supporting platform i. Then, denoting by D_i^U total user demand for platform i, we have $D_1^U + D_2^U = 1$ and:

$$D_1^U = \frac{1}{2} + \frac{u_1 - u_2}{2t} \tag{3.11}$$

where $u_i = u_0 + u(n_i) - P_i^U$is the utility gross of transportation costs offered by platform i to its users.

Meanwhile we assume there is no differentiation between platforms from the developers' perspective, i.e. a developer with fixed development cost ϕ makes profits $\pi(n_i) D_i^U - P_i^D - \phi$ by joining platform i exclusively and $\pi(n_1) D_1^U + \pi(n_2) D_2^U - P_1^D - P_2^D - 2\phi$ from multihoming[21]. Thus, for each developer, the decision to adopt platform 1 is independent

[21] We also implicitly assume the development cost is platform-independent and there are no economies of

of his decision to adopt platform 2, *given* D_1^U *and* D_2^U[22], so that developer demand n_i for platform $i \in \{1,2\}$ is implicitly defined by:

$$\pi(n_i) D_i^U - P_i^D - H^{-1}(n_i) = 0 \tag{3.12}$$

The assumption that platforms are differentiated from the point of view of users but are perfect substitutes for developers simplifies the analysis and is also quite realistic in most cases. Indeed, at equal platform quality, developers care only about the respective installed bases of users and, compared to the latter, they are relatively less likely to have intrinsic preferences for one platform over the other (i.e. being die-hard MacIntosh or Nintendo fans for example).

Although in principle both users and developers are allowed to multihome, we focus on the symmetric equilibrium in which each platform attracts half the users exclusively, whereas all developers who enter multihome. If the user differentiation parameter t is large enough, this is the only symmetric equilibrium.

Platform 1's profits are then:

$$\begin{aligned}\Pi_1^P &= P_1^U D_1^U + P_1^D n_1 = \left(P_1^U + n_1\pi(n_1)\right)\left(\frac{1}{2} + \frac{u_1 - u_2}{2t}\right) - n_1 H^{-1}(n_1) \\ &= (V(n_1) - u_1)\left(\frac{1}{2} + \frac{u_1 - u_2}{2t}\right) - n_1 H^{-1}(n_1)\end{aligned}$$

In order to find the symmetric equilibrium without explicitly deriving the two-dimensional best-response functions, we use a "trick" developed by Choi (2004). In the symmetric equi-

"platform scale", i.e. the cost of development for an additional platform does not depend on having or not developed for another platform.

[22] This is because developers are atomistic, so that each individual developer does not take into account the effect of his adoption decision on D_1^U and D_2^U through the indirect network effect mechanism.

librium, $u_1 = u_2 = u$ and $D_1^U = D_2^U = \frac{1}{2}$. Consider then varying P_1^D while maintaining $u(n_1) - P_1^U$ fixed equal to u:

$$\Pi_1^P = (V(n_1) - u)\frac{1}{2} - n_1 H^{-1}(n_1)$$

Meanwhile, (3.12) defines a 1-to-1 relationship between n_1 and P_1^D[23]:

$$\pi(n_1)\frac{1}{2} - P_1^D = H^{-1}(n_1)$$

so that we can optimize directly over n_1. We obtain that the number n_c of developers who enter (and multihome) in the symmetric equilibrium with platform competition is defined by[24]:

$$V'(n_c)\frac{1}{2} = n_c H^{-1\prime}(n_c) + H^{-1}(n_c) \tag{3.13}$$

Since we are only concerned with social welfare and prices are simple transfers, we need not worry here about determining the actual equilibrium prices charged by the platforms on the two sides of the market (see Hagiu (2004b) for the complete derivation of the pricing structure): n_c is sufficient for our purposes.

Let us now turn to the case of a monopoly platform situated at one extremity of the Hotelling segment and assume that u_0 is high enough so that the platform covers the entire market for users. It does so by charging $P^U = u_0 + u(n) - t$. Developer demand n is given by:

$$\pi(n) - P^D - H^{-1}(n) = 0$$

[23] This is because π is strictly decreasing and H^{-1} strictly increasing.

[24] If the right-hand side of (3.13) is increasing then n_c is unique. This is the case when H has constant elasticity.

Therefore platform profits are:

$$\Pi^P = P^U + P^D n = V(n) + u_0 - t - nH^{-1}(n)$$

so that the level of product diversity n_p chosen by the monopoly platform is defined by:

$$V'(n_p) = n_p H^{-1\prime}(n_p) + H^{-1}(n_p) \tag{3.14}$$

Comparing (3.14) and (3.13), it is clear that the monopoly platform induces more product diversity than the competing platforms:

$$n_p > n_c$$

The social welfare tradeoff is the following. With a monopoly platform situated at one extremity (say 0) of the Hotelling segment, there is less *platform diversity* so that users situated further than $x = \frac{1}{2}$ incur additional transportation costs ($\frac{t}{2}$ overall). On the other hand however, the monopoly platform offers more *developer product diversity* to its users than any of the two competing platforms because it is able to internalize a larger share of user benefits. Additionaly, there is no duplication of fixed costs for the developers who enter, since they only support one platform rather than two.

Formally, the social welfare gain from having one platform rather than two is:

$$V(n_p) - V(n_c) - \frac{t}{4} + 2\int_0^{H^{-1}(n_c)} \phi h(\phi)\, d\phi - \int_0^{H^{-1}(n_p)} \phi h(\phi)\, d\phi$$

The following proposition establishes rigorously that this expression can be either positive or negative depending on parameter values.

Proposition 6 *Assume u_0 is high enough so that both the competing platforms and a single monopoly platform cover the user market entirely in equilibrium, that all developers have the same fixed development cost ϕ (i.e. developer demand is inelastic, $\varepsilon_H = +\infty$) and that developers for the same platform compete as in example 2 (i.e. $V(n) = An^{\beta}$, $\pi(n) = \beta A n^{\beta-1}$, $u(n) = (1-\beta) A n^{\beta}$). Then there exists a non-empty interval $[t_L, t_H]$ so that total social welfare is higher with a single monopoly platform than with two competing platforms if and only if $t \in [t_L, t_H]$.*

Proof See appendix.■

Thus, proposition 6 confirms that there is a sense in which platform competition may be undesirable because it prevents the competing platforms from sufficiently internalizing positive indirect network effects, so that they do not have enough incentives to induce product variety. A monopoly platform can sometimes be more efficient, even though it creates more deadweight loss. Although here we have focused on the simplest case, in which both the competing platforms and a monopoly platform cover the user market entirely, it should be clear that this insight is valid in more general settings, with partial coverage of the user market and more than two platforms.

4. Conclusion

There is a widely held view among policy makers and economists that open platforms and competition among platforms are intrinsically more desirable for social efficiency than closed, proprietary and monopolistic platforms. This belief seems to rely on an intuition that open and free access to a bottleneck ressource can only improve social welfare. In this paper we have provided a simple model of two-sided platforms which shows clearly how this intuition breaks down in two-sided markets. The key ingredients are positive two-sided indirect network externalities and direct competitive (business-stealing) effects on the producer side of the market, which give rise to a fundamental welfare tradeoff between open and proprietary platforms. Because the latter have market power, they can create two-sided deadweight losses typically by restricting access on both sides of the market more than is socially desirable. At the same time however, precisely because they set prices to maximize profits, they have an inherent interest in internalizing indirect network externalities and direct competitive effects, which are left uninternalized by open platforms. As we have shown formally, this may lead them to induce higher levels of product variety, user adoption and total social welfare than open platforms. We have also shown that proprietary platforms may even induce socially excessive levels of product variety when their profit maximization program leads them to overestimate the value of indirect network externalities. Similarly, platform competition in two-sided markets may turn out to be socially undesirable if it prevents platforms from sufficiently internalizing indirect network externalities and direct competitive effects. The conclusion is that welfare analysis in two-sided markets follows a very different logic from that in one-sided markets and may lead to counterintuitive conclusions.

While here we have focused on two very simple and stark forms of platform governance - open vs. closed -, we believe there are promising perspectives for more in-depth research on subtler aspects of platform governance in two-sided markets (cooperatives, associations, etc.), which should inform both policy-makers and business practitioners.

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5. Appendix

Proof of Lemma 0 With the functional forms assumed, (2.5, 2.6) are equivalent to $\theta_p^m = \frac{\varepsilon_F}{1+\varepsilon_F} A n_p^{\varepsilon_V}$ and:

$$\varepsilon_V A^{1+\varepsilon_F} B \left(\frac{\varepsilon_F}{1+\varepsilon_F}\right)^{\varepsilon_F} n_p^{\varepsilon_V(1+\varepsilon_F)-1} = C\left(1+\frac{1}{\varepsilon_H}\right) n_p^{\frac{1}{\varepsilon_H}}$$

(2.8, 2.9) are equivalent to $\theta^m_{so} = An^{\varepsilon_V}_{so}$ and:

$$BA^{1+\varepsilon_F}\varepsilon_V n_{so}^{\varepsilon_V(1+\varepsilon_F)-1} = Cn_{so}^{\frac{1}{\varepsilon_H}}$$

and (2.11, 2.10) are equivalent to $\theta^m_{fe} = (1-\lambda\varepsilon_V)\, An^{\varepsilon_V}_{so}$ and:

$$BA^{1+\varepsilon_F}\lambda\varepsilon_V\,(1-\lambda\varepsilon_V)^{\varepsilon_F}\, n_{fe}^{\varepsilon_V(1+\varepsilon_F)-1} = Cn_{fe}^{\frac{1}{\varepsilon_H}}$$

Thus, $\left(\theta^m_p, n_p\right)$, (θ^m_{so}, n_{so}) and $\left(\theta^m_{fe}, n_{fe}\right)$ are uniquely defined if $\varepsilon_V\,(1+\varepsilon_F) - 1 < \frac{1}{\varepsilon_H}$.

$\left(\theta^m_p, n_p\right)$ is a global maximum for $\Pi^P\left(\theta^m, n\right)$ if and only if the second order condition holds, i.e. if and only if the Hessian matrix of $\Pi^P\left(\theta^m, n\right)$ evaluated at $\left(\theta^m_p, n_p\right)$ is semi-definite negative. We have:

$$\begin{aligned}
\frac{\partial^2\Pi^P}{\left(\partial\theta^m\right)^2}\left(\theta^m_p, n_p\right) &= \left(V'\left(n_p\right) - \theta^m_p\right) f'\left(\theta^m_p\right) - 2f\left(\theta^m_p\right) \\
&= f\left(\theta^m_p\right)\left(\frac{V\left(n_p\right) - \theta^m_p}{\theta^m_p}\frac{\theta^m_p f'\left(\theta^m_p\right)}{f\left(\theta^m_p\right)} - 2\right) \\
&= -f\left(\theta^m_p\right)\left(1 + \frac{1}{\varepsilon_F}\right) < 0
\end{aligned}$$

$$\frac{\partial^2\Pi^P}{\partial n^2}\left(\theta^m_p, n_p\right) = V''\left(n_p\right) F\left(\theta^m_p\right) - \left(nH^{-1}\left(n\right)\right)''\left(n_p\right) < 0$$

because V is concave and $\left(nH^{-1}\left(n\right)\right)''\left(n_p\right) = C\left(1 + \frac{1}{\varepsilon_H}\right)\frac{1}{\varepsilon_H} n^{\frac{1}{\varepsilon_H}-1} > 0$.

$$\frac{\partial^2\Pi^P}{\partial\theta^m\partial n}\left(\theta^m_p, n_p\right) = V'\left(n_p\right) f\left(\theta^m_p\right) > 0$$

It therefore remains to check that $\left(\frac{\partial^2\Pi^P}{\partial\theta^m\partial n}\left(\theta^m_p, n_p\right)\right)^2 < \frac{\partial^2\Pi^P}{\left(\partial\theta^m\right)^2}\left(\theta^m_p, n_p\right) \times \frac{\partial^2\Pi^P}{\partial n^2}\left(\theta^m_p, n_p\right)$, which, using the expressions above and omitting arguments of some functions in order to avoid clutter, is equivalent to:

$$V'^2 f < \left(1+\frac{1}{\varepsilon_F}\right)\left(-V''F + C\left(1+\frac{1}{\varepsilon_H}\right)\frac{1}{\varepsilon_H} n_p^{\frac{1}{\varepsilon_H}-1}\right)$$

But (2.6) implies:

$$C\left(1+\frac{1}{\varepsilon_H}\right)\frac{1}{\varepsilon_H} n_p^{\frac{1}{\varepsilon_H}-1} = \frac{1}{\varepsilon_H}\frac{V'(n_p)}{n_p} F\left(\theta_p^m\right)$$

so that the inequality above is equivalent to:

$$V'^2 < \left(1+\frac{1}{\varepsilon_F}\right)\left(-\frac{V''\theta_p^m}{\varepsilon_F} + \frac{1}{\varepsilon_H}\frac{V'\theta_p^m}{n_p}\frac{1}{\varepsilon_F}\right)$$

or, using (2.5):

$$\varepsilon_F < \frac{-V''V}{V'^2} + \frac{1}{\varepsilon_H}\frac{V}{n_p V'}$$

Noting that $\frac{V''V}{V'^2} = \frac{\varepsilon_V - 1}{\varepsilon_V}$, this is finally equivalent to (2.12).

Similarly, (n_{so}, θ_{so}^m) is a global maximum for W if and only if the Hessian matrix of $W(n, \theta^m)$ evaluated at this point is semi-definite negative. We have:

$$\frac{\partial^2 W}{(\partial\theta^m)^2}(n_{so}, \theta_{so}^m) = (V(n_{so}) - \theta_{so}^m) f(\theta_{so}^m) - f(\theta_{so}^m) = -f(\theta_{so}^m) < 0$$

$$\frac{\partial^2 W}{\partial n^2}(n_{so}, \theta_{so}^m) = V''(n_{so}) F(\theta_{so}^m) - H^{-1\prime}(n_{so}) < 0$$

$$\frac{\partial^2 W}{\partial\theta^m \partial n}(n_{so}, \theta_{so}^m) = V'(n_{so}) f(\theta_{so}^m)$$

and $\left(\frac{\partial^2 W}{\partial\theta^m\partial n}(n_{so}, \theta_{so}^m)\right)^2 < \frac{\partial^2 W}{(\partial\theta^m)^2}(n_{so}, \theta_{so}^m)\frac{\partial^2 W}{\partial n^2}(n_{so}, \theta_{so}^m)$ is equivalent to:

$$V'(n_{so})^2 f(\theta_{so}^m) < -V''(n_{so}) F(\theta_{so}^m) + H^{-1\prime}(n_{so})$$

or, using $V'(n_{so})\,F(\theta_{so}^m) = H^{-1}(n_{so})$:

$$\frac{-V'(n_{so})^2}{V''(n_{so})\,V(n_{so})}\frac{\theta_{so}^m f(\theta_{so}^m)}{F(\theta_{so}^m)} < 1 - \frac{V'(n_{so})}{V''(n_{so})\,n_{so}\varepsilon_H}$$

or:

$$\frac{\varepsilon_V \varepsilon_F}{1-\varepsilon_V} < 1 + \frac{1}{(1-\varepsilon_V)\,\varepsilon_H}$$

which is in turn equivalent to (2.12).

ii) The necessary and sufficient conditions for stability are:

$$\frac{\partial\Theta}{\partial n}\left(n_x, u(n_x) - \theta_x^m\right) < \frac{1}{\frac{\partial N}{\partial\theta^m}\left(\theta_x^m, \pi(n_x)\,F(\theta_x^m) - H^{-1}(n_x)\right)}$$

for $(n_x, \theta_x^m) = \left(n_p, \theta_p^m\right)$ and $(n_x, \theta_x^m) = \left(n_{fe}, \theta_{fe}^m\right)$. Using the implicit function theorem, the condition corresponding to $\left(n_p, \theta_p^m\right)$ is equivalent to:

$$u'(n_p) < \frac{H^{-1\prime}(n_p) - \pi'(n_p)\,F\left(\theta_p^m\right)}{\pi(n_p)\,f\left(\theta_p^m\right)} \tag{5.1}$$

Using the first order conditions (2.5) and (2.6), (5.1) is equivalent to[25]:

$$(1+\varepsilon_F)\,\frac{n_p u'\pi}{VV'} + \frac{\pi' n_p}{V'} < \frac{1}{1+\varepsilon_H}$$

Since $\pi(n) = \lambda V'(n)$ and $u(n) = V(n) - \lambda n V'(n) = (1-\lambda\varepsilon_V)\,V(n)$, the inequality above is equivalent to:

$$(1+\varepsilon_F)\,\frac{n_p(1-\lambda\varepsilon_V)\,\lambda V'}{V} + \frac{\lambda V'' n_p}{V'} < \frac{1}{1+\varepsilon_H}$$

[25] We omit functional arguments and use the fact that $\frac{nH^{-1\prime}(n)}{H^{-1}(n)} = \frac{1}{\varepsilon_H\left(H^{-1}(n)\right)}$.

or:

$$\left(1+\varepsilon_F\right)\lambda\varepsilon_V\left(1-\lambda\varepsilon_V\right)-\lambda\left(1-\varepsilon_V\right)<\frac{1}{1+\varepsilon_H}$$

which, after factoring $\lambda\left(1-\varepsilon_V\right)$, is exactly the condition (2.13) given in the text.

Similarly, the stability condition corresponding to $\left(n_{fe},\theta_{fe}^m\right)$ is equivalent to:

$$u'\left(n_{fe}\right)\pi\left(n_{fe}\right)f\left(\theta_{fe}^m\right)+\pi'\left(n_{fe}\right)F\left(\theta_{fe}^m\right)\leq H^{-1\prime}\left(n_{fe}\right)$$

which, using (2.10) and (2.11), can be rewritten as:

$$\frac{u'\left(n_{fe}\right)n_{fe}}{u\left(n_{fe}\right)}\frac{f\left(\theta_{fe}^m\right)\theta_{fe}^m}{F\left(u\left(n_{fe}\right)\right)}+\frac{\pi'\left(n_{fe}\right)n_{fe}}{\pi\left(n_{fe}\right)}\leq\frac{1}{\varepsilon_H}$$

or, since $u\left(n\right)=\left(1-\lambda\varepsilon_V\right)V\left(n\right)$, $\pi\left(n\right)=\lambda V'\left(n\right)$:

$$\varepsilon_V\varepsilon_F+\varepsilon_V-1\leq\frac{1}{\varepsilon_H}$$

which is equivalent to (2.12).■

Proof of Proposition 4 i) (3.4), (3.5) and (3.6) are respectively equivalent to:

$$BA^{1+\varepsilon_F}\varepsilon_V\left(\frac{\varepsilon_F}{1+\varepsilon_F}\right)^{\varepsilon_F}n^{\varepsilon_V\left(1+\varepsilon_F\right)-1}=C\left(1+\frac{1}{\varepsilon_H}\right)n^{\frac{1}{\varepsilon_H}} \tag{5.2}$$

$$BA^{1+\varepsilon_F}\lambda\varepsilon_V\left(1-\lambda\varepsilon_V\right)^{\varepsilon_F}n^{\varepsilon_V\left(1+\varepsilon_F\right)-1}=Cn^{\frac{1}{\varepsilon_H}} \tag{5.3}$$

$$BA^{1+\varepsilon_F}\varepsilon_V n^{\varepsilon_V\left(1+\varepsilon_F\right)-1}=Cn^{\frac{1}{\varepsilon_H}} \tag{5.4}$$

As shown in Lemma 0, if (2.12) holds then each of these three equations admits a unique positive solution. Note that we are in the case depicted in graph a) of figure **??** if

$\varepsilon_V(1+\varepsilon_F)<1$ and in the case depicted in graph b) if $\varepsilon_V(1+\varepsilon_F)>1$. Since λ, $\varepsilon_V<1$, (5.2), (5.3) and (5.4) clearly imply that $n_{fe}, n_p < n_{so}$. Then:

$$\theta_{fe}^m = (1-\lambda\varepsilon_V)\,V(n_{fe}) < V(n_{fe}) < V(n_{so}) = \theta_{so}^m$$

$$\theta_p^m = \frac{\varepsilon_F}{1+\varepsilon_F} V(n_p) < V(n_p) < V(n_{so}) = \theta_{so}^m$$

ii) Let $\beta = \frac{\sigma-\alpha}{\sigma(1-\alpha)} = 1-\varepsilon_V$ and recall that $\lambda = \frac{\sigma(1-\alpha)}{1-\sigma\alpha} \in]0,1[$. Then, since $\varepsilon_H = +\infty$, (2.12) is implied by (2.13), which is equivalent to:

$$2(1-\beta) < \frac{\beta}{1-\lambda(1-\beta)} \tag{5.5}$$

Together with $\lambda < 1$, this implies that $1 > \beta > \frac{1}{2}$. (5.2) and (5.3) become:

$$\frac{(1-\beta)}{2} n_p^{1-2\beta} = \frac{C}{BA^2} \tag{5.6}$$

$$\frac{\alpha(1-\sigma)(1-\alpha)}{(1-\sigma\alpha)^2} n_{fe}^{1-2\beta} = \frac{C}{BA^2} \tag{5.7}$$

$$(1-\beta)\,n_{so}^{1-2\beta} = \frac{C}{BA^2} \tag{5.8}$$

We have: $n_p > n_{fe}$ if and only if $\frac{(1-\beta)(1-\sigma\alpha)^2}{2} > \alpha(1-\sigma)(1-\alpha)$, which is equivalent to $(1-\sigma\alpha)^2 > 2\sigma(1-\alpha)^2$. It remains to be verified that this inequality may hold or not, while still satisfying (5.5). If $\alpha \to 0$ then $\beta \to 1$ and $\lambda \to \sigma$, so that (5.5) is satisfied, and in the limit $n_p > n_{fe}$ if and only if $\sigma < \frac{1}{2}$, so that both cases are possible.

Total social welfare has the following expression:

$$W(n,\theta^m) = BV(n)\,\theta^m - \frac{B(\theta^m)^2}{2} - nC$$

Using (5.6), (5.7) and $\theta^m_{fe} = (1 - \lambda(1-\beta)) V(n_{fe})$, $\theta^m_p = \frac{V(n_{2sp})}{2}$ and $V(n) = An^{1-\beta}$ we obtain:

$$\begin{aligned} W\left(n_p, \theta^m_p\right) &= BA^2\left(\frac{1}{2} - \frac{1}{8}\right) n_p^{2-2\beta} - \frac{BA^2(1-\beta)}{2} n_p^{2-2\beta} \\ &= \frac{BA^2}{2}\left(\beta - \frac{1}{4}\right)\left(\frac{BA^2(1-\beta)}{2C}\right)^{\frac{2-2\beta}{2\beta-1}} \end{aligned}$$

and:

$$\begin{aligned} W\left(n_{fe}, \theta^m_{fe}\right) &= BA^2\left[\left(1 - \lambda(1-\beta) - \frac{(1-\lambda(1-\beta))^2}{2}\right) - \frac{\alpha(1-\sigma)(1-\alpha)}{(1-\sigma\alpha)^2}\right] n_{fe}^{2-2\beta} \\ &= \frac{BA^2(1-\alpha)^2}{2(1-\sigma\alpha)^2}\left(\frac{BA^2\alpha(1-\sigma)(1-\alpha)}{C(1-\sigma\alpha)^2}\right)^{\frac{2-2\beta}{2\beta-1}} \end{aligned}$$

Finally:

$$\frac{W\left(n_p, \theta^m_p\right)}{W\left(n_{fe}, \theta^m_{fe}\right)} = \left(\beta - \frac{1}{4}\right)\left(\frac{\sigma}{\lambda}\right)^{\frac{2}{2\beta-1}} \frac{1}{(2\sigma)^{\frac{2-2\beta}{2\beta-1}}}$$

Let $\sigma = x$, $\alpha = kx$ with $0 < k < \frac{1}{3}$ and $x \to 0$. Then $\lambda \to 0$ and $\beta \to 1 - k$ so that (5.5) is satisfied in the limit and at the same time $\frac{\sigma}{\lambda} \to 1$ and therefore $\frac{W_{2sp}}{W_{fe}} \to +\infty$.

Now let $\sigma \to 1$ keeping α fixed: $\lambda, \beta \to 1$ so that (5.5) is satisfied and $\frac{W_{2sp}}{W_{fe}} \to \frac{3}{4}$.$\blacksquare$

Proof of Proposition 5 The expression of platform profits is:

$$\Pi^P(q^m) = B(V(Q(q^m)) - E(q^m)) - C(q_H - q^m)\phi$$

and that of social welfare:

$$W(q^m) = BV(q^m) - C(q_H - q^m)\phi$$

where $Q = \frac{C}{2}\left(q_H^2 - (q^m)^2\right)$ and $E(q^m) = \frac{CV'(Q)}{2}(q_H - q^m)^2$.

q_p^m solves:

$$-BV'(Q(q^m))\,Cq^m - BE'(q^m) + C\phi = 0$$

whereas q_{so}^m is the solution to:

$$-BV'(Q(q^m))\,Cq^m + C\phi = 0$$

We assume that the parameters A, B and C are such that both solutions are interior, so that in order to prove $q_p^m < q_{so}^m$ it is sufficient to prove that $E'(q^m) > 0$ for all $q^m \in [q_L, q_H]$ and the derivatives of the two expressions above are both negative. We have:

$$\begin{aligned}
E'(q^m) &= -\frac{C^2 V''(Q(q^m))\,q^m}{2}(q_H - q^m)^2 - CV'(Q(q^m))(q_H - q^m) \\
&= -\frac{C^2}{2}V''(Q)(q_H - q^m)^2\left[q^m + \frac{V'(Q)}{QV''(Q)}(q_H + q^m)\right] \\
&= -C^2 V''(Q)(q_H - q^m)^2\left(q^m - \frac{q_H + q^m}{\beta + 1}\right)
\end{aligned}$$

so that $E'(q^m) > 0$ is equivalent to:

$$\beta q^m > q_H$$

which is true since $\beta q_L > 2q_H$.

The second derivative of total social welfare with respect to q^m is:

$$BV''(Q(q^m))(Cq^m)^2 - BV'(Q(q^m))\,C$$

and is clearly negative. Therefore, we are done if we show that $E''(q^m) > 0$. We have:

$$\begin{aligned} E''(q^m) &= -\frac{C^2 V''(Q)}{\beta+1}\left[\beta (q_H - q^m)^2 - 2(q_H - q^m)(\beta q^m - q_H)\right] \\ &\quad + \frac{C^3 V'''(Q)}{\beta+1} q^m (q_H - q^m)^2 (\beta q^m - q_H) \end{aligned}$$

Using $\frac{-QV'''(Q)}{V''(Q)} = \beta + 2$, the condition $E''(q^m) > 0$ is equivalent to:

$$\beta (q_H - q^m) - 2(\beta q^m - q_H) + \frac{2(\beta+2)}{q_H + q^m} q^m (\beta q^m - q_H) > 0$$

or:

$$((\beta+2) q_H - 3\beta q^m)(q_H + q^m) + 2(\beta+2) q^m (\beta q^m - q_H) > 0$$

or:

$$(\beta+2) q_H^2 + (2\beta+1) q^m (\beta q^m - 2q_H) > 0$$

which is true when $\beta q_L > 2q_H$.■

Proof of Proposition 6 (3.14) and (3.13) become:

$$n_c = \left(\frac{\beta A}{2\phi}\right)^{\frac{1}{1-\beta}}$$

$$n_p = \left(\frac{\beta A}{\phi}\right)^{\frac{1}{1-\beta}}$$

The expression of the social welfare gain from having a single platform rather than two becomes:

$$A(1-\beta) n_p^\beta - A(1-\beta) n_c^\beta - \frac{t}{4} = A(1-\beta)\left(\frac{\beta A}{2\phi}\right)^{\frac{\beta}{1-\beta}}\left(2^{\frac{\beta}{1-\beta}} - 1\right) - \frac{t}{4}$$

and it is positive if and only if $t \leq 4A(1-\beta)\left(\frac{\beta A}{2\phi}\right)^{\frac{\beta}{1-\beta}}\left(2^{\frac{\beta}{1-\beta}}-1\right) = t_H$.

On the other hand we need to make sure that in the symmetric equilibrium with two competing platforms they make non-negative profits and all users do indeed singlehome, which will yield a lower bound on t. We have:

$$P_c^D = \pi(n_c)\frac{1}{2} - \phi = \frac{1}{2}\beta A n_c^{\beta-1} - \phi = 0$$

$$P_c^U = t - n_c\pi(n_c) + \frac{u'(n_c)\pi(n_c)}{\pi'(n_c)} = t - 2\beta A n_c^{\beta}$$

Hence, the two platforms make non-negative profits and all users singlehome if and only if $P_c^U \geq 0$, i.e.if and only if:

$$t \geq 2\beta A n_c^{\beta} = 2\beta A\left(\frac{\beta A}{2\phi}\right)^{\frac{\beta}{1-\beta}} = t_L$$

Finally, we need to verify that $t_L \leq t_H$, which is equivalent to:

$$\frac{\beta}{1-\beta} \leq 2\left(2^{\frac{\beta}{1-\beta}}-1\right)$$

and this inequality holds for all $\beta \geq 0$.■