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Abstract

We investigate a puzzling phenomenon in which firms make investment decisions that purposefully do not maximize expected profits. Using an extension to the newsvendor model, we focus on a relatively common scenario in which the firm’s investor has imperfect information concerning the quality of the firm’s investment opportunities. We apply Perfect Bayesian equilibrium solution concepts and confirm that over a range of reasonable model parameters the firm’s investment decision does not maximize expected profits. Surprisingly, this includes instances in which a firm with a higher quality investment opportunity finds it attractive to underinvest, thereby behaving as if she faces a lower quality investment opportunity. This is particularly interesting as prior research in the finance literature has shown that firms will overinvest in high quality projects when investors have imperfect information about the quality of the firm’s opportunities. While we conduct our analysis in the context of an inventory stocking decision, our model is generalizable to other types of capacity investment decisions.
1. Introduction

We investigate the effect of short-term objectives and imperfect information on a firm’s\(^1\) operational decisions. In particular, we examine the puzzling phenomenon in which firms make operational decisions that purposefully do not maximize expected profits. Using an extension to the newsvendor model, we focus on a relatively common scenario of a public-traded firm in which the firm’s investor\(^2\) has imperfect information concerning the quality of the firm’s investment opportunities. We apply Perfect Bayesian equilibrium solution concepts and confirm that over a range of reasonable model parameters, the firm makes investment decisions which do not maximize expected profits. More surprisingly, this includes instances in which a firm with a higher quality investment opportunity finds it more attractive to underinvest, thereby behaving as if she faces a lower quality investment opportunity. This finding is particularly interesting as prior research in the finance literature has shown that firms will overinvest in high quality projects when investors have imperfect information about the quality of the firm’s opportunities (Bebchuk and Stole 1993).

In our model we extend the classical newsvendor model to account for two elements that are common in agency relationships. First, there is an external investor in the firm who possesses imperfect information about the firm’s operations. The literature acknowledges that external investors can have information about a firm’s operations that is inferior to the firm’s information (Berle and Means (1932), Stein (1988)). Given this information asymmetry, the short-term valuation of the firm by the investor may differ from the true value of the firm. In our model, firms may either have a high quality investment opportunity or a low quality investment opportunity. Information asymmetry is incorporated into our model by making the quality of the investment opportunity known to the firm but not to the investor. A high quality investment opportunity is represented by a demand distribution for the firm’s product that first order stochastically dominates the demand distribution if the firm instead had a low quality

\(^{1}\) By “firm,” we mean the manager(s) controlling the decisions made within the firm. For ease of expression, we will sometimes refer to this decision maker as “she” or “her.”

\(^{2}\) By “investor,” we mean the marginal equity investor who determines the valuation of the firm and does not have the same information as the firm. For example, this excludes a manager-cum-shareholder who might have insider information, but includes an investor in the firm’s private or public securities.
investment opportunity. The information asymmetry in our model can be generalized beyond product demand to other situations in which the firm’s knowledge of its operations or market prospects is more accurate than that of an external investor, such as the firm having better insight into the effectiveness of an emerging technology, the value of a new supply chain configuration, the potential size of a new market, and so forth.

The second extension we make to the newsvendor model is utilizing a firm objective function that is a linear combination of both her short-term and long-term valuation. In practice, decision makers at a firm are cognizant of and manage the firm’s short-term valuation (Jensen and Meckling (1976), Narayanan (1985)). Managers are motivated to pursue such action for a variety of reasons, including a desire to use the short-term valuation of the firm’s stock as a currency for acquisitions, to raise capital in a secondary offering, to dispose of personal shares in the firm at an attractive price, to prevent takeovers, or to burnish their reputation and careers. There is some empirical evidence supporting the concept that firm inventory policies influence the firm’s valuation in the stock market. Lai (2006) runs a fixed effect regression analysis using lags to account for endogeneity issues and finds that there is an economically significant relationship between inventory levels and firm valuation.

While we utilize the newsvendor model in the context of an inventory stocking decision, our model and our analysis are easily generalized to a wide range of project investment decisions that a firm may encounter. The newsvendor model is merely a framework for making a capacity investment decision in the face of uncertainty. In this respect, our model can be effectively applied regardless of whether the decision is about inventory or some other type of capacity investment, including plant expansions, capital expenditures, and contracting for production inputs.

We review the relevant literature in section 2 and present our model and analytical results in section 3. In section 4 we provide a numerical example of our model. In section 5 we discuss the managerial implications of our model and offer concluding remarks.
2. Literature Review

Our model and analysis make two important contributions to the literature. First, we extend the literature dealing with investment decisions in the presence of short-term managerial objectives and imperfect information by revealing an important variation that has not been previously identified. Second, we add to the relatively sparse research linking the operations management literature on optimal inventory management to the literature on signaling games, which has been more commonly employed in finance and economics.

There is extensive literature on investment decisions in the presence of short-term managerial objectives and imperfect information. This literature can be generalized into two categories – one focused on information asymmetry concerning the amount of the investment made by the firm and the other focused on information asymmetry concerning the quality of the firm’s investment opportunity. In the former category, Stein (1989) shows that firms will underinvest in projects when external investors cannot observe the amount of the firm’s investment. In the latter category, Bebchuk and Stole (1993) demonstrates that firms will overinvest in projects when external investors cannot observe the quality of the firm’s investment opportunity. The overinvestment found in Bebchuk and Stole is due to firms with high quality investment opportunities separating from firms with low quality investment opportunities by using the amount invested to signal the quality of the investment opportunity facing the firm.

Our first contribution is to show the counterintuitive result that over reasonable ranges of the parameters in our model, firms with high quality investment opportunities will underinvest when external investors cannot observe the quality of the firm’s investment opportunity. In this way, firms with high quality investment opportunities choose to mimic the behavior of firms with low quality investment opportunities. This result identifies important exceptions to the findings in Bebchuk and Stole (1993). We also show the less surprising result that for other ranges of the parameters, firms with low quality investment opportunities will overinvest when external investors cannot observe the quality of the firm’s investment opportunity. This finding pertaining to overinvestment also differs from Bebchuk and Stole as their model indicates that firms with a higher type overinvest in order to separate from firms with a lower
Our overinvestment result is due to low type firms overinvesting in order to maintain a pooling equilibrium with high type firms. In this way, firms with low quality investment opportunities choose to mimic the behavior of firms with high quality investment opportunities.

Our second contribution is to provide an important application of signaling game theory to the problem of inventory management in the face of an external investor. While the supply chain coordination literature has employed game theory extensively, there are relatively few applications specifically of signaling games in the operations management literature. Of those papers that do use signaling models, the majority explore how information is shared across the members of a supply chain. Debo and Veeraraghavan (2010) explore how firms may use a combination of prices and congestion to signal quality to consumers. Anand and Goyal (2009) investigate how information is shared within a supply chain that consists of horizontal competitors and a common supplier. Allon and Bassamboo (2011) examine the information that retailers disclose to customers on product availability. There are few instances where signaling models are applied to analyze a firm’s interactions with constituents outside of its supply chain. In one exception, Lai, Debo et al. (2008) show that firms will attempt to signal higher second period demand to investors by channel stuffing in the first period. It should be noted that while these authors also indicate that under-investment in inventory may occur, this finding is due to management trying to optimize expected firm profits by choosing an inventory stocking level that accounts for the cost of channel-stuffing. In our model, on the other hand, management consciously sets an inventory stocking level that they know is sub-optimal for expected profits.

3. The Model

We consider a signaling game with two players – the firm and the investor, and two time periods – period 1 representing the short term and period 2 representing the long term. The firm, who faces uncertain demand, can be one of two types - a high demand type (high type or type $h$) with random demand $\zeta_h$, or a low demand type (low type or type $l$) with random demand $\zeta_l$. The distributions of demand for the high and low types have cumulative distribution functions, $F_h$ and $F_l$, and probability density functions, $f_h$ and
f\(_h\) respectively. All demand distributions have continuous support from zero to infinity. We assume that F\(_h\) first order stochastically dominates F\(_l\), i.e., the high type enjoys a higher quality investment opportunity compared to the low type.

The firm’s investment decision, represented in our model as an inventory stocking decision, is one of two values – a high stocking quantity (q\(_H\)) or a low stocking quantity (q\(_L\)). The values of q\(_H\) and q\(_L\) may be obtained by maximizing the newsvendor’s expected profit function for each demand type in the absence of information asymmetry. However, our analysis holds for arbitrary given values of q\(_H\) and q\(_L\). We restrict our model to two inventory stocking quantities to illustrate the main results. In practice, this discrete inventory stocking choice is representative of a wide variety of possible investment scenarios, including capacity investment, plant additions, research and development projects, new product launches, geographic expansions, or mergers and acquisitions, all of which can often be ‘go or no go’ type decisions.

The sequence of events is as follows. In the short term, the firm receives a private signal about her type and chooses a stocking quantity. The investor observes the firm’s stocking decision but not her type and sets a short-term value for the firm. In the long term, the demand is realized and the firm earns a profit or a loss. Then the firm is dissolved and its proceeds are distributed to the investor. Thus, the firm’s type and true value are revealed to the investor in the long term. Without loss of generality, we assume that the firm and the investor are risk-neutral.

### 3.1. Firm’s utility

We express the firm’s utility as a linear combination of the investor’s valuation of the firm in period 1 and the investor’s expected valuation of the firm in period 2 with weights \(\alpha\) and \(1 - \alpha\), respectively, where \(0 \leq \alpha \leq 1\). A larger value of \(\alpha\) corresponds to a higher emphasis on short-term valuation.

\[
\text{Firm's Utility, } U = \alpha \text{[period 1 valuation]} + (1 - \alpha) \text{[expected period 2 valuation]} \tag{1.1}
\]

The firm pays a cost, \(c\), for every unit of inventory and sells each unit of inventory at a price \(p > c\). If the firm is of type \(\tau \in \{h, l\}\) and makes stocking decision \(q_0 \in \{q_H, q_L\}\), her expected profit function is
\( \pi_{d} = E\left[\text{Profits}(\tau, q_d)\right] = E\left[p \min\{\zeta_1, q_d\} - cq_d\right]. \) Since \( F_d \) first order stochastically dominates \( F_h \), we have \( \pi_{sh} > \pi_{sl} > \pi_{hl} > \pi_{dh}. \)

We assume that the value placed on the firm by the investor in each period is equal to a multiple of the investor’s expectation of the firm’s profits (without loss of generality this multiple is set to 1). In period 1, the investor is uncertain of the firm’s type but does hold probabilistic beliefs about the firm’s type based on the firm’s stocking decision. If the firm decides to stock \( q_L \), let \( \lambda_L \) denote the probability assigned by the investor that the firm is a low type and \( 1 - \lambda_L \) denote the probability assigned by the investor that the firm is a high type. Similarly, if the firm decides to stock \( q_H \), let \( \lambda_H \) denote the probability assigned by the investor that the firm is a low type and \( 1 - \lambda_H \) the corresponding probability that the firm is a high type. As a result, the valuation assigned by the investor in period 1 upon observing stocking decision \( q_D \) is:

\[
\text{period 1 valuation} = \lambda_L E\left[\text{Profits}(l, q_D)\right] + (1 - \lambda_L) E\left[\text{Profits}(h, q_D)\right] = \lambda_L \pi_l + (1 - \lambda_L) \pi_H. \tag{1.2}
\]

In period 2, the demand is realized and the investor updates the valuation. Therefore, the valuation that the firm expects to receive in period 2 is given by:

\[
\text{expected period 2 valuation} = E\left[\text{Profits}(\tau, q_D)\right] = \pi_{dD}. \tag{1.3}
\]

Using (1.1)-(1.3), we obtain the expression for the firm’s utility, \( U(\alpha, \lambda_D, \tau, q_D) \), as a function of the investor’s beliefs \( \lambda_D \), the firm’s emphasis on short-term valuation \( \alpha \), type \( \tau \), and inventory stocking decision \( q_D \):

\[
U(\alpha, \lambda_D, \tau, q_D) = \alpha \left\{ \lambda_D \pi_{lD} + (1 - \lambda_D) \pi_{HD} \right\} + (1 - \alpha) \pi_{dD}. \tag{1.4}
\]

Based on (1.4), the firm’s utility maximization problem can be expressed as:

\[
\max_{q_D} U(\alpha, \lambda_D, \tau, q_D) = \max \left[ \alpha \left\{ \lambda_L \pi_{lD} + (1 - \lambda_L) \pi_{HL} \right\} + (1 - \alpha) \pi_{dD}, \alpha \left\{ \lambda_H \pi_{lD} + (1 - \lambda_H) \pi_{HL} \right\} + (1 - \alpha) \pi_{dH} \right]. \tag{1.5}
\]
3.2. Analysis of a pooling equilibrium on $q_L$

We analyze a signaling game in which the players move sequentially under incomplete information. The literature on signaling games has been developed by several authors and the methodology for our analysis is drawn from Cho and Kreps (1987), Fudenberg and Tirole (1991), Gibbons (1992), and Rasmusen (2007). In general, our model could lead to a pooling equilibrium on $q_L$, a pooling equilibrium on $q_H$, or a separating equilibrium. We focus attention on the pooling equilibrium on $q_L$ since this is the most counterintuitive result from our model. Later we present the other possible equilibria in brief.

The following proposition states the main result of our paper:

**Proposition 1.** There exists a pooling equilibrium on $q_L$ provided the following inequalities hold:

\[
\alpha \lambda_L \pi_{IL} + (1 - \alpha \lambda_L) \pi_{IH} \geq \alpha \lambda_H \pi_{IL} + (1 - \alpha \lambda_H) \pi_{IH} \\
(1 - \alpha + \alpha \lambda_L) \pi_{IL} + (\alpha - \alpha \lambda_L) \pi_{IH} \leq \alpha \pi_{IL} + (1 - \alpha) \pi_{IH} \\
\lambda_H \leq \lambda_L
\]

We prove this result in the rest of this section. For the purpose of brevity and without loss of generality, we assume that the participation constraints of the players are satisfied. In practical situations, firms are compelled to participate in the decisions related to the operation of their businesses.

**Incentive compatibility constraints.** The incentive compatibility constraints that must be met for pooling equilibrium on $q_L$ are:

\[
U(\alpha, \lambda_L, l, q_L) \geq U(\alpha, \lambda_H, l, q_H) \tag{1.6}
\]

\[
U(\alpha, \lambda_L, h, q_L) \geq U(\alpha, \lambda_H, h, q_H). \tag{1.7}
\]

Inequality (1.6) indicates that in order to maintain a pooling equilibrium on $q_L$ the utility derived by a low type firm stocking $q_L$ must be at least as great as her utility from stocking $q_H$. Similarly, inequality (1.7) indicates that the utility for the high type firm stocking $q_L$ must be at least as great as her utility from stocking $q_H$. Inequality (1.6) can be expressed using the terms in (1.5) as:

\[
(1 - \alpha + \alpha \lambda_L) \pi_{IL} + \alpha (1 - \lambda_L) \pi_{IH} \geq (1 - \alpha + \alpha \lambda_H) \pi_{IL} + \alpha (1 - \lambda_H) \pi_{IH}.
\]
By rearranging terms and including \( \pi_{ul} \) and \( \pi_{ul} \) on both sides of the inequality, we get:

\[
\left[ \alpha \lambda_L \pi_{ul} + (1 - \alpha \lambda_L) \pi_{ul} \right] + (1 - \alpha) \pi_{ul} + (1 - \alpha) \pi_{ul} \geq \left[ \alpha \lambda_H \pi_{ul} + (1 - \alpha \lambda_H) \pi_{ul} \right] + (1 - \alpha) \pi_{ul} + (1 - \alpha) \pi_{ul}.
\]

The terms in the square brackets are equivalent to \( U(\alpha, \lambda_L, h, q_L) \) and \( U(\alpha, \lambda_H, h, q_H) \), respectively, so this inequality simplifies to:

\[
\left[ U(\alpha, \lambda_L, h, q_L) \right] + (1 - \alpha) \left( \pi_{ul} + \pi_{ul} \right) \geq \left[ U(\alpha, \lambda_H, h, q_H) \right] + (1 - \alpha) \left( \pi_{ul} + \pi_{ul} \right)
\]

In (1.8), \( (1 - \alpha) \left( \pi_{ul} + \pi_{ul} \right) \geq (1 - \alpha) \left( \pi_{ul} + \pi_{ul} \right) \) is always true. Therefore, if (1.7) holds, then (1.8) shows that (1.6) will hold as well.

Inequality (1.7) can be expressed using the terms in (1.5) as:

\[
\alpha \lambda_L \pi_{ul} + (1 - \alpha \lambda_L) \pi_{ul} \geq \alpha \lambda_H \pi_{ul} + (1 - \alpha \lambda_H) \pi_{ul}
\]

Inequality (1.9) is a necessary condition for a pooling equilibrium on \( q_L \).

**Intuitive criterion refinement.** We next determine if the proposed equilibrium will survive the Intuitive Criterion refinement. This refinement was formalized in Cho and Kreps (1987) as a means to restrict out-of-equilibrium beliefs, thereby eliminating unintuitive equilibria from the solution set. In the current context, the Intuitive Criterion refinement implies that the pooling equilibrium on \( q_L \) can only be maintained if the low type does not strictly prefer the in-equilibrium expected payoff to the payoff from deviating to stocking \( q_H \) and being identified with certainty as a high type (\( \lambda_{ih} = 0 \)). In other words, a low type must be willing to stock \( q_H \) if stocking \( q_H \) results in the investor believing the firm is a high type and therefore awarding a high type valuation. If a low type instead prefers the in-equilibrium expected payoff, the investor will realize that anyone who deviates to stocking \( q_H \) must be a high type. This logic will be replicated by the high type, causing the high type to deviate and the pooling equilibrium on \( q_L \) to break down.

When evaluating a pooling equilibrium on \( q_L \), the Intuitive Criterion refinement can be expressed using the utility function of the low type as \( U(\alpha, \lambda_L, l, q_L) \leq U(\alpha, \lambda_H = 0, l, q_H) \). As a result, the following inequality must hold in order for the pooling equilibrium to survive the Intuitive Criterion refinement:
\[(1 - \alpha + \alpha \lambda_L)\pi_{hl} + (\alpha - \alpha \lambda_L)\pi_{hl} \leq \alpha \pi_{hl} + (1 - \alpha)\pi_{Hl} \quad (1.10)\]

We prove this result in Appendix II. In Appendix III we show that another common refinement, the dominance refinement, does not constrain the solution space beyond inequality (1.10). Inequality (1.10) is a second necessary condition for a pooling equilibrium on \(q_L\).

**Limits on the out-of-equilibrium beliefs: investor-rationality constraint.** It is necessary to specify a plausible out-of-equilibrium belief that can be utilized to test the pooling equilibrium on \(q_L\). When considering a pooling equilibrium on \(q_L\), \(\lambda_L\) is the investor’s prior belief that the firm is a low type and \(\lambda_{Hl}\) is the investor’s out-of-equilibrium belief that the firm is a low type if the firm deviates to stocking quantity \(q_H\). It is not rational that stocking \(q_H\) would send a stronger signal to the investor that the firm is a low type because any deviation to \(q_H\) that is profitable for the low type is also profitable for the high type. Therefore, the firm’s decision to deviate from the pooling equilibrium on \(q_L\) by stocking \(q_H\) can either indicate to the investor that the firm is a high type or it can provide no additional information to the investor regarding the firm’s type. We capture this investor-rationality constraint in our model by requiring that if the firm stocks \(q_H\) the probability assigned by the investor that the firm is a low type is no greater than the probability assigned by the investor that the firm is a low type if the firm stocks \(q_L\):

\[\lambda_{Hl} \leq \lambda_L \quad (1.11)\]

This proves Proposition 1.

**3.3. Pooling equilibrium on \(q_H\)**

It is perhaps less surprising that under certain conditions a pooling equilibrium on \(q_H\) may exist. The following proposition is made without proof.

**Proposition 2.** There exists a pooling equilibrium on \(q_H\) provided the following inequalities hold:

\[\lambda_H \leq \lambda_L\]

The first inequality is the low type’s incentive compatibility constraint. Neither the high type’s incentive compatibility constraint nor the Intuitive Criterion limits the parameter space which supports a
pooling equilibrium on $q_H$. The second inequality is the investor-rationality constraint. For this equilibrium, $\lambda_H$ is the investor’s prior belief that the firm is a low type and $\lambda_L$ is the investor’s out-of-equilibrium belief that the firm is a low type if the firm deviates to stocking quantity $q_L$.

### 3.4. Separating Equilibrium

For certain model parameters a separating equilibrium will exist where both the high type and the low type set inventory levels that are congruent with their types. The following proposition is made without proof.

**Proposition 3.** There exists a separating equilibrium provided the following inequalities hold:

$$
\pi_{hl} + (1 - \alpha + \alpha \lambda_L)(\pi_{hl} - \pi_{ll}) \geq \pi_{hl} + (1 - \alpha + \alpha \lambda_H)(\pi_{hl} - \pi_{hh})
$$

$$
\pi_{hl} + \alpha \lambda_L (\pi_{hl} - \pi_{ll}) \leq \pi_{hl} + \alpha \lambda_H (\pi_{hl} - \pi_{hh})
$$

$$
\lambda_H \leq \lambda_L
$$

The first inequality is the low type’s incentive compatibility constraint. The second inequality is the high type’s incentive compatibility constraint. The third inequality is the investor-rationality constraint. For this equilibrium, $\lambda_H$ is the investor’s belief that the firm is a low type if the firm stocks $q_H$, and $\lambda_L$ is the investor’s belief that the firm is a low type if the firm stocks $q_L$.

### 4. Numerical Example

In this section we demonstrate that each of the three propositions identified above can hold depending on the values of $\lambda_H$, $\lambda_L$, and $\alpha$. In order to satisfy the requirements of first order stochastic dominance of the demand distributions, the expected profit functions for this numerical example are set to $\pi_{hl} = 1.2 \pi_{hl}$, $\pi_{hl} = 1.2 \pi_{hl}$, and $\pi_{ll} = 1.2 \pi_{ll}$. Intuitively, the ranges of parameter values that support different equilibria are influenced by the values of the expected profit functions at $q_H$ and $q_L$. 
4.1. Pooling equilibrium on \( q_L \)

The shaded region in Figure 1 shows the range of parameter values that support a pooling equilibrium on \( q_L \). This equilibrium is maintained when \( \lambda_H \) is close to \( \lambda_L \) and both \( \lambda_L \) and \( \alpha \) are relatively high. The intuition is that when the investor has strong prior beliefs that the firm is a low type (high \( \lambda_L \)) and the investor cannot derive enough information to substantively change this belief when the firm stocks \( q_H \) (\( \lambda_H \) is close to \( \lambda_L \)) and the firm’s utility is heavily influenced by its short-term valuation (high \( \alpha \)) then there is a strong incentive for high type firms to capitulate to the market’s expectations and mimic low types.

The size of the parameter space that supports this equilibrium depends upon the relative magnitude of the expected profit functions. For instance, if \( \pi_{Hh} \) is changed such that the relationship between \( \pi_{Hh} \) and \( \pi_{Lh} \) is \( \pi_{Hh} = 1.1 \pi_{Lh} \) rather than \( \pi_{Hh} = 1.2 \pi_{Lh} \), then the volume of the parameter space that supports a pooling equilibrium on \( q_L \) increases by a factor of approximately 3.

*Figure 1: Feasible region of parameters to support a pooling equilibrium on \( q_L \).*
4.2. Pooling equilibrium on $q_H$

The shaded region in Figure 2 shows the range of parameter values that support a pooling equilibrium on $q_H$. This equilibrium is maintained at lower values of $\lambda_H$ and higher values of $\alpha$. The intuition in this case is that when the investor has strong prior beliefs that any firm which stocks $q_H$ is a high type (low $\lambda_H$) and the firm’s utility is heavily influenced by its short-term valuation (high $\alpha$) then there is a strong incentive for low type firms to mimic high type firms.

Figure 2: Feasible region of parameters to support a pooling equilibrium on $q_H$

4.3. Separating Equilibrium

As show in Figure 3, the necessary and sufficient conditions for a separating equilibrium can be achieved for low values of $\alpha$. The intuition in this case is that when the firm’s utility is not significantly influenced by its short-term valuation (low $\alpha$) then there is a strong incentive for low type firms and high type firms to make stocking decisions that will maximize their respective expected long-term profits. As such, a low type firm will willingly separate from a high type firm by stocking $q_L$ and a high type firm will willingly
separate from a low type firm by stocking \( q_H \). As \( \alpha \) increases, other dynamics come into play as determined by the levels of \( \lambda_H \) and \( \lambda_L \). For instance, in this numerical example there is still a region which supports a separating equilibrium even at very high levels of \( \alpha \).

*Figure 3: Feasible region of parameters to support a separating equilibrium*

4.4. Combined equilibrium regions

Figure 4 shows that when the regions represented in Figures 1 – 3 are combined, all of the available and allowable parameter space on the \( \alpha \), \( \lambda_H \), and \( \lambda_L \) dimensions is covered. This means that the three sets of conditions given by us in Propositions 1-3 fully specify the equilibria that can result from various combinations of out-of-equilibrium and in-equilibrium beliefs. The unallowable region in this parameter space which does not result in any equilibrium solution is represented by the investor-rationality condition that \( \lambda_H \leq \lambda_L \). If this condition is removed, all of the space in Figure 4 will be covered by one of the three equilibria.
Figure 4: Combined regions of parameters to support the three equilibria

Figure 5 shows a cross section of the solution space in the $(\lambda_H, \alpha)$ plane for $\lambda_L = 1$. The region in the top right represents the pooling equilibrium on $q_L$, the region in the top left represents the pooling equilibrium on $q_H$, and the region in the lower half represents the separating equilibrium.
5. Summary of contributions and implications for decision makers

The existing research demonstrates that in the face of imperfect information and short-term objectives, the firm’s investment decision does not maximize expected profits over a range of model parameters. Unlike previous research which has found that firms will always overinvest in projects when investors have imperfect information concerning the quality of the firm’s projects, we find that this will not universally be the case. Critically, firms may knowingly underinvest in projects under such circumstances. We show that the necessary and sufficient conditions for a pooling equilibrium on $q_L$ can be met, specifically as $\lambda_H$ approaches $\lambda_L$ and for higher levels of $\lambda_L$ and $\alpha$. These conditions inform both investors and firms regarding when to expect a pooling equilibrium on $q_L$. The intuition behind these conditions is instructive:

1. **Inability of firms to mitigate information asymmetry** - When $\lambda_H$ approaches $\lambda_L$ it means that if the firm deviates from stocking $q_L$ the investor is not certain of the type of the deviating firm.

   This reflects situations in which the investor believes it is possible for a low type firm to mimic
the investment activities of a high type firm. Put differently, a high type firm finds it difficult to clearly signal its type to the market.

2. **Strong prior belief that firms are low types** - A higher $\lambda_L$ means that there is a higher probability that nature assigns a firm to be a low type (alternatively that there are a higher proportion of low types compared to high types). An example of this is a situation when the majority of the firms in an industry are troubled or if the firm is in a destructively competitive environment.

3. **Emphasis on the short term** - A higher $\alpha$ means that the firm places a greater emphasis on its short-term valuation. This may be captured by circumstances such as the firm seeking to monetize a high stock price in the short term or attempting to stave off takeover or default.

The counter-intuitive investment phenomena captured by our model are present in a variety of real-world situations. French upscale beauty brand Clarins Group provides an example of how one firm avoided the pressures to make sub-optimal investment decisions. This was accomplished when relatives of deceased founder Jacques Courtin-Clarins took the firm private in the summer of 2008. This decision reduced the firm’s emphasis on short-term valuation (reducing $\alpha$) and mitigated information asymmetry with its investors (reducing $\lambda_H$ compared to $\lambda_L$) by moving from public market investors to private and family investors. However, did the choice to go private actually help Clarins avoid making sub-optimal investment decisions? Our model indicates that high-type firms are in jeopardy of imitating the investment decisions of low-type firms only if a third factor is present, namely a general belief among investors that the firm faces dim prospects (a high $\lambda_L$).

This third factor materialized in late 2008 and 2009 as the global recession deepened. Many analysts were generally pessimistic about sales of high-end beauty products that would be discretionary for many consumers and noted that “luring women to invest in high-end skin-care regimens is challenging when shoppers are cutting back” (Byron 2009). Clarins management though – presumably because of private information that would have been difficult to communicate to public market investors – saw
considerable opportunity to sell its high-end products in the U.S. By going private in 2008, the firm was able to invest substantially in “Clarins department-store skin spas,” and product promotions in 2009. According to a Wall Street Journal interview with Chairman Christian Courtin-Clarins, going private allowed Clarins to be “free from short-term shareholder pressure, [and] Clarins is investing in long-term objectives” (Byron 2009). Management’s optimism appears to have been justified by subsequent events and the department store spas have registered rapid growth.

Firms other than Clarins have also argued that in some circumstances they can face “pressures” that prevent them from making investments that are in their long-term interests. For instance, Worldspan and Sabre Holdings, both travel technology providers, have gone private in an attempt “to shrug off short-term shareholder pressure” (Field 2007). Other companies have tried to get around this problem by keeping control in the hands of a few individuals. This is the approach taken by Maersk, the world's biggest container shipping firm. Maersk’s management is focused on two goals: for Maersk to control as many as possible of the companies that influence its containers' movement from country to country, and to pursue scale - both in the size of the firm's main businesses and in its ships. The Financial Times noted, “The company can be single-minded in pursuit of these objectives because it is largely immune from short-term shareholder pressure. The founding Mc-Kinney Møller family holds 55 per cent of the shares and 75 per cent of voting stock, with 49.8 per cent of the shares all held by a single family charitable trust” (Wright 2006).

These examples illustrate the phenomenon we describe in the paper. Companies face pressure to purposefully make suboptimal operations decisions as a result of the collective impact of the three forces in our model – a strong prior belief that firms are low types, an inability for firms to mitigate the information asymmetry regarding their actual type, and an emphasis on short-term valuation.

The crucial insight obtained from our model compared to previous models, particularly Bebchuk and Stole (1993) and Stein (1989), is due to differences in the assumptions on the expected profit function and out-of-equilibrium beliefs. In both of these papers, the investment returns are constrained to be monotonically increasing in the investment decision of the firm; Bebchuk and Stole (1993) restricts the
analysis to situations in which profits are an increasing function of long-term investment, and Stein (1989) restricts the analysis to situations in which future earnings are increasing in current earnings. Therefore, these papers assume separating out-of-equilibrium beliefs, i.e. it is always possible for the high-type firm to make a sufficiently large investment to separate. In contrast, the newsvendor model, like most operational decision models, allows for negative marginal returns on investments beyond an optimum, removing the underlying restrictions of Bebchuk and Stole (1993) and Stein (1989). This feature of the newsvendor model makes it particularly suitable in many practical settings. To accommodate this feature, we consider all possible types of out-of-equilibrium beliefs through the parameters $\lambda_H$ and $\lambda_L$.

While we analyze firm investment decisions in a single decision period, our model has some implications across multiple decision periods. In particular, if there are changes to the parameter space due to internal or external shocks over multiple time periods, firms may oscillate among different equilibria over time, and hence, make different investment decisions even though the firm’s expected profit function does not necessarily change. An internal shock could be modeled as a change in $\alpha$. Such a shock may occur due to changes in compensation structure or firm strategy. An external shock could be modeled as a change in $\lambda_H$ and/or $\lambda_L$. Such a shock may occur due to information revelation about the firm, such as when a large investor takes a substantial stake in a firm. It may also occur due to macroeconomic events such as the dot-com boom and subsequent bust, the onset of a recession, or a competitor filing for bankruptcy. It is interesting to note that the firm’s investment decision will change even if its type remains constant and the expected profits associated with the different investment decisions remain constant. The result will be seemingly erratic firm behavior in which a firm swings between investment levels $q_L$ and $q_H$. The existing research may be extended in order to provide a closer examination of this phenomenon. This fluctuation in firm behavior is generally in line with the findings of Aghion and Stein (2008), who identify a pattern of oscillation in a firm’s choice of business strategy even in the absence of external shocks.
There are two constraints to our model which bear further investigation. First, while the use of two investment levels is applicable to a variety of situations and communicates the main result of the paper, a more generalizable model would allow for a continuum of investment levels by the firm. Second, our model utilizes two firm types, but it may be possible to extend it to account for a continuum of firm types.

References


### Appendix I – Table of Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>Firm utility</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Weight of the short-term valuation on the firm’s utility ($0 \leq \alpha \leq 1$)</td>
</tr>
<tr>
<td>$H, h$</td>
<td>High index (Stocking quantity or firm type)</td>
</tr>
<tr>
<td>$L, l$</td>
<td>Low index (Stocking quantity or firm type)</td>
</tr>
<tr>
<td>$D$</td>
<td>Decision identifier – can take a value of $H$ (High) or $L$ (Low)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Firm type - can take a value of $h$ (high) or $l$ (low)</td>
</tr>
<tr>
<td>$\zeta_\tau$</td>
<td>Random demand for type $\tau$, distributed $F_\tau$</td>
</tr>
<tr>
<td>$F_\tau$</td>
<td>Cumulative distribution function of demand for type $\tau$</td>
</tr>
<tr>
<td>$f_\tau$</td>
<td>Probability distribution function of demand for type $\tau$</td>
</tr>
<tr>
<td>$q_D$</td>
<td>Stocking quantity decision – can take a value of $q_H$ (High) or $q_L$ (Low)</td>
</tr>
<tr>
<td>$\lambda_D$</td>
<td>Investor’s belief that the firm is a low type after seeing $q_D$</td>
</tr>
<tr>
<td>$\pi_{\tau D}$</td>
<td>Expected profit of vendor type $\tau$ making decision $D$</td>
</tr>
<tr>
<td>$p$</td>
<td>Price at which the firm sells each unit of inventory</td>
</tr>
<tr>
<td>$c$</td>
<td>Cost to the firm of each unit of inventory</td>
</tr>
</tbody>
</table>

### Appendix II – Intuitive Criterion Check for Pooling Equilibrium on $q_L$

We apply the Intuitive Criterion refinement, as defined in Cho and Kreps (1987: p.202), to derive inequality (1.10) corresponding to a pooling equilibrium on $q_L$. In Step 1 of the Intuitive Criterion refinement, let $S(q_H)$ be the set of firm types whose expected utility from making the in-equilibrium stocking decision, $q_L$, is strictly greater than their maximum possible utility from making the out-of-equilibrium stocking decision, $q_H$, over the set of best responses available to the investor.

For the high-type firm, the expected utility from making the in-equilibrium stocking decision, $q_L$, is $\alpha \lambda_L \pi_{\tau L} + (1-\alpha \lambda_L) \pi_{\tau L}$ and the maximum possible utility from deviating to $q_H$ is $\pi_{\tau H}$, which is achieved if the investor awards a high type valuation, i.e., at $\lambda_H = 0$. The resulting inequality is never true so the high type newsvendor is never in $S(q_H)$:

$$\alpha \lambda_L \pi_{\tau L} + (1-\alpha \lambda_L) \pi_{\tau L} > \pi_{\tau H} \quad (1.12)$$
For the low-type firm, the expected utility from making the in-equilibrium stocking decision, $q_L$, is 

$$ (1 - \alpha + \alpha \lambda_L) \pi_{hL} + (\alpha - \alpha \lambda_L) \pi_{hl} $$

and the maximum possible utility as a result of deviating to $q_H$ is 

$$ \alpha \pi_{nh} + (1 - \alpha) \pi_{nh}, $$

which is achieved if the investor awards a high type valuation, i.e., at $\lambda_H = 0$. The low type is in $S(q_H)$ if the following inequality holds.

$$ (1 - \alpha + \alpha \lambda_L) \pi_{hL} + (\alpha - \alpha \lambda_L) \pi_{hl} > \alpha \pi_{nh} + (1 - \alpha) \pi_{nh} $$

(1.13)

In Step 2 of the Intuitive Criterion refinement, for any firm type not in $S(q_H)$, test whether their utility from making the in-equilibrium stocking decision, $q_L$, is strictly less than their minimum possible utility from making the out-of-equilibrium stocking decision, $q_H$, over the set of best responses available to the investor given that the best response of the investor is based on excluding any firm type in $S(q_H)$. If this test holds, the equilibrium outcome fails the Intuitive Criterion.

Based on inequality (1.13), there are two possible scenarios that must be considered in Step 2. In the first scenario, inequality (1.13) does not hold, which means $S(q_H)$ is a null set. For this scenario, Step 2 is applied to both the high type and the low type and the minimum best response of the investor must account for the fact that the investor is still uncertain about the type of the deviating firm. For both the low type and the high type, the minimum possible utility from deviating to $q_H$ is achieved if the investor responds by awarding a low type valuation, which occurs at $\lambda_H = 1$. The inequality in Step 2 for the high type is:

$$ \alpha \lambda_L \pi_{hL} + (1 - \alpha \lambda_L) \pi_{hl} < \alpha \pi_{nh} + (1 - \alpha) \pi_{nh}. $$

(1.14)

Inequality (1.14) contradicts inequality (1.9). Since inequality (1.9) must hold for any pooling equilibrium on $q_L$ that the Intuitive Criterion is applied against, inequality (1.14) can never hold.

The inequality in Step 2 for the low type is:

$$ \pi_{hL} < \pi_{hH}. $$

(1.15)

Due to the ordered relationship of the expected profit functions, inequality (1.15) never holds. As neither inequality (1.14) nor inequality (1.15) ever holds, the Intuitive Criterion does not constrain the solution space in the first scenario.
In the second scenario, inequality (1.13) holds, which means \( S(q_{H}) \) contains only the low type. For this scenario, Step 2 of the Intuitive Criterion is applied only to the high type and the minimum best response of the investor must account for the fact that the investor knows that the deviating firm is a high type. The minimum best response of the investor based on this certain knowledge is to award a high valuation, which occurs at \( \lambda_{H} = 0 \). As a result, the inequality in Step 2 for the high type is:

\[
\alpha\lambda_{L}\pi_{L} + \left(1 - \alpha\lambda_{L}\right)\pi_{H} < \pi_{H}
\]

(1.16)

Due to the relationship among the expected profit functions, inequality (1.16) always holds, which means that the pooling equilibrium on \( q_{L} \) always fails the Intuitive Criterion in the second scenario.

Since inequalities (1.14) and (1.15) can never hold when testing a pooling equilibrium on \( q_{L} \), and inequality (1.16) must always hold, a pooling equilibrium on \( q_{L} \) survives the Intuitive Criterion based exclusively on whether or not inequality (1.13) holds. If inequality (1.13) does (does not) hold, then the pooling equilibrium on \( q_{L} \) fails (survives) the Intuitive Criterion. Inequality (1.13) not holding is equivalent to inequality (1.10) holding. This proves that inequality (1.10) is the appropriate constraint for the pooling equilibrium on \( q_{L} \) to survive the Intuitive Criterion.

**Appendix III – Dominance Refinement Check for Pooling Equilibrium on \( q_{L} \)**

Under the Dominance refinement, the investor will assume that the low type will never stock \( q_{H} \) if the worst outcome that the low type can receive by stocking \( q_{L} \) is better than the best outcome that the low type can receive by stocking \( q_{H} \). The following inequality is the basis of the analysis:

\[
\alpha\left\{\lambda_{L}\pi_{L} + \left(1 - \lambda_{L}\right)\pi_{H}\right\} + \left(1 - \alpha\right)\pi_{L} \leq \alpha\left\{\lambda_{H}\pi_{H} + \left(1 - \lambda_{H}\right)\pi_{H}\right\} + \left(1 - \alpha\right)\pi_{H}
\]

(1.17)

The worst outcome that the low type can receive by stocking \( q_{L} \) occurs when the investor believes the firm is a low type, so \( \lambda_{L} = 1 \). The best outcome that the low type can receive by stocking \( q_{H} \) occurs...
when the investor believes the firm is a high type, so $\lambda_H = 0$. As a result, inequality (1.17) is simplified to:

$$\pi_{HL} \leq \alpha \pi_{HH} + (1 - \alpha) \pi_{HH}$$ (1.18)

The right hand side of inequality (1.18) is equivalent to the right hand side of inequality (1.10) and the left hand side of (1.18) must be less than or equal to the left hand side of inequality (1.10) since $\pi_{HL} > \pi_{LL}$. As a result, the Dominance refinement does not impose an additional condition on the value of the parameters that will support a pooling equilibrium on $q_L$ beyond the conditions that are already imposed by inequality (1.10).