Abstract

Compensation not only provides incentives to an existing manager but affects the type of manager attracted to the firm. This paper examines the dual incentive and sorting effects of performance pay, in a simple contracting model of endogenous participation. The main result is that sorting dampens optimal pay-performance sensitivity (PPS). This occurs because PPS beyond a nominal amount transfers unnecessary (information) rent from the firm to the manager. The result helps explain why empirical estimates of PPS are much lower than predictions from models of moral hazard alone. Finally, the model delivers a number of comparative statics that can be tested against data, predicting a web of relationships between PPS, the quality of the manager, the variation between types of managers, and the manager’s risk aversion and outside option.
1 Introduction

Managerial compensation serves many functions. It provides incentives, attracts talent, ensures retention, provides feedback, and communicates the goals and objectives of the firm. And yet, the vast majority of the theoretical and empirical work on executive pay only considers its incentive effects. Executive contracts not only provide incentives to the existing manager, but also attract new types of managers to the firm, either from internal or external labor markets. Thus, performance pay has a sorting effect, in that it sorts the potential pool of managers tomorrow in addition to providing incentives to the incumbents today. The objective here is to understand the sorting effects of performance pay, namely, how the firm will solve the dual problem of providing incentives ex-post and sorting new types of managers ex-ante.

The main result is that sorting dampens optimal pay-performance sensitivity (PPS). While a small amount of performance pay is necessary to attract a high quality manager to the firm, excessive performance pay beyond this amount transfers unnecessary rent from the firm to the manager. Excessive performance pay is costly to the firm, and thus, sorting exerts a downward pressure on PPS. Because the firm must also provide incentives to a manager once he is hired, the incentive effect exerts an upward pressure on PPS. The optimal PPS balances these twin competing effects. The downward pressure on PPS from sorting brings the theoretical predictions on PPS closer to empirical estimates. Existing estimates of PPS (example 0.325%, according to Jensen-Murphy, 1990) are much lower than prediction from the canonical model of moral hazard alone, even when factoring in risk aversion.

I adopt a simple contracting framework that permits a solution to the dual sorting and incentive problems. A risk-neutral manager has private information on his ability, and the firm’s contracts are incomplete, since they cannot easily extract this private information through a complex menu of contracts.\(^1\) The firm proposes a contract, which consists of a salary and bonus, representing the fixed and variable components of compensation (in practice, this takes the form of cash and stock or stock options). Based on this contract, the manager decides whether to join the firm. If so, he exerts productive effort. After nature resolves production uncertainty, the output is realized and the firm pays the manager based

\(^1\)This amounts to a restriction of communication between the firm and the agent through the contracts. The firm cannot tailor its contracts to the full complexity of the manager’s private information. Baker and Jorgensen (2003); Lazear (2004); Melumad, Mookherjee, and Reichelstein (1997); and Ray (2007b) make a similar assumption. Bushman, Indjejikian, and Penn (2000) also work in a world of private pre-decision information.
upon the negotiated contract.

The crux of the analysis rests on the firm’s joint choice on salary and bonus, where the bonus measures the PPS of the manager’s compensation. I start with a benchmark model of sorting alone (without incentives) and find that PPS, beyond a nominal amount, only transfers unnecessary rent from the firm to the manager. This downward pressure on PPS from the sorting effect is a robust phenomenon, and is a key component of the more general model, which combines sorting and incentives. There, compensation induces participation ex-ante as well as determines effort (and therefore profits) ex-post. The firm selects the salary and bonus jointly to equalize the marginal rates of substitution between these ex-ante and ex-post effects. In equilibrium, the firm trades off salary and bonus at the same rate for the dual purpose of securing participation (sorting) and inducing effort (incentives).

Next, I extend the model further by making the manager risk-averse, which deepens these tensions between salary and bonus. When the manager’s risk aversion is small, salary and bonus are substitutes, just as with a risk-neutral manager. If the firm raises the bonus, more types of managers are attracted to the firm. Therefore, the firm must lower the bonus in order to keep participation unchanged. Thus, salary and bonus are equivalent instruments in achieving sorting. But if the manager is sufficiently risk-tolerant, salary and bonus become complements. Now, raising the bonus loads risk onto the manager, who requires a larger cash payment to compensate for this increased risk. In this case, the firm will adjust the salary and bonus in the same direction in order to induce participation.

My paper is closest in spirit to Dutta (2008) and Baker and Jorgensen (2003). Both operate in a LEN (linear contract, exponential utility, normal errors) framework, and consider an agent whose ability affects output; for Dutta (2008), effort and ability are substitutes \((e + \theta)\), while for Baker and Jorgensen (2003), effort and ability are complements \((\theta e)\), as in my model. Dutta (2008) allows communication, so the firm offers a menu of contracts to the manager; my model shares the assumption of no communication, as in Baker and Jorgensen (2003). Both papers find that the optimal PPS falls in the variance on output, but may rise in the variance of the agent’s ability distribution (Dutta (2008) calls this information risk, while Baker and Jorgensen call this volatility).\(^2\)

My model shares more assumptions with Baker and Jorgensen (2003) but follows the approach of Dutta (2008). Like Baker and Jorgensen (2003), I work in a world of pre-decision

\(^2\)Moreover, the assumption of communication and the fact that effort and ability are perfect substitutes allows Dutta (2008) to characterize the optimal contract, whereas Baker and Jorgensen (2003) derived comparative statics without solving for the optimal contract in closed form.
information, assume ability and effort are complements, and disallow communication between
the principal and agent. Like Dutta (2008), I am able to characterize the optimal contract
(though implicitly) and make comparisons with the benchmark moral hazard model without
information. My primary difference is that I make participation endogenous. Both papers
assume the principal will hire even the worst type of manager, whereas I show there are some
types of managers who are not profitable for the firm. As such, the contract in my model
must solve the dual problem of participation versus incentives.

Endogenous participation provides a new analytical lens that extends prior work with
new results and implications. Dutta (2008) shows robustly that when a risk-averse manager’s
ability is firm-specific (his outside options are invariant to his ability), then adverse selection
considerations mute PPS. Here, sorting dampens PPS only when the complementarity be-
tween the firm and manager is large; in such cases, the manager is a highly productive match
with the firm, and he collects payoffs stemming from this greater productivity, eliminating
the need for the firm to pay expensive bonuses to attract him. But if this complementarity
is low, the manager can no longer collect this payoff, and hence requires a higher bonus for
compensation, and so sorting inflates optimal PPS. Dutta (2008) also finds that if a manager
has firm-specific human capital, optimal PPS falls in the variance in managerial ability. I
show the reverse: sorting has features of an option contract, and when variance (on ability)
increases, so does the value of the option, so the firm raises PPS to attract the now larger
upper tail of the ability distribution.

Finally, unlike Dutta (2008), Baker and Jorgensen (2003), and the canonical agency
model, I find the standard risk-incentives tradeoff between PPS and output risk does not
always hold. When output risk increases, this creates a disutility for a risk-averse manager,
and it is even more important for the firm to hire the best manager whose output will
overcome this disutility, and hence the firm raises PPS to attract these high types. This result
is consistent with empirical studies that document that the negative relationship between
risk and incentives does not always hold (Prendergast 2002). Ultimately, the core conceptual
difference is that sorting considers how an incentive contract attracts managerial talent in
an outside marketplace, whereas the adverse selection models (for example, Dutta (2008))
rely on how contracts induce different forms of communication.

Models of adverse selection and moral hazard each enjoy voluminous theoretical litera-
tures (see Baiman, 1991 and Hart and Holmstrom, 1987 for surveys). But there have been
only limited attempts to combine both in a single model. The fusion has proven notoriously
difficult and researchers have made simplifying assumptions in order to make the analysis

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tractable. In general, the literature remains largely separate even though real life contracts must solve both problems simultaneously.

2 The Basic Model

To fix ideas, consider the basic model with sorting, but no incentives. A firm (the principal) employs a single manager (the agent). The manager is risk neutral. The manager has a type $\theta$, which he knows, but the firm does not. Hereafter, “type” refers to the type $\theta$ of the manager; while there is a single manager, there is a continuum of types. The firm’s uncertainty on $\theta$ is represented by the density $f$ with cumulative distribution function $F$, over support $\Theta = [0, \infty)$, with mean $\mu_\theta$ and variance $\sigma^2_\theta$. The firm’s output with a (manager of) type $\theta$ is

$$x = \gamma \theta + \epsilon$$ (1)

where $\epsilon$ is distributed symmetrically with 0 and variance $\sigma^2$. The parameter $\gamma > 0$ represents the complementarity between the firm and the manager. High $\gamma$ firms produce more output with high $\theta$ types than with low $\theta$ types. The manager enjoys an outside option $\bar{u} > 0$, which represents his outside opportunities. The firm bears a fixed cost $k - m\theta$ to employ a manager of type $\theta$, where $k, m > 0$. This reflects the non-negative fixed cost $(k - m\theta)^+ \equiv \max\{0, k - m\theta\}$ of hiring and training the worker, as well as a variable cost $m\theta$, where a better manager is less costly to employ. A better manager is less likely to make mistakes or bad decisions. The parameter $k$ can also track, for example, the level of general versus firm-specific human capital: firms that require more specialized skills (finance or technology) may bear a larger cost of installing and training the manager, compared to agency models. However, this assumption is without loss, since the results here will still hold under outside options that are increasing and linear in the manager’s type. Details are available from the author on request.

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4Armstrong, Larcker, and Su (2010) solves the joint problem numerically, simulating the optimal CEO contract under realistic assumptions on the agent’s risk aversion and actual executive contracts.

5The manager’s outside options $\bar{u}$ are fixed and do not vary with $\theta$. This is the standard assumption in agency models.
firms that require more general skills (retail, commodities). The parameter $m$ captures the return to a better manager; when $m$ is large, the cost saving of a better manager is large.

### 2.1 First Best

A social planner maximizes total surplus, which is output from the manager less his cost of employment. Expected surplus for each $\theta$ is $E[TS|\theta] = \gamma \theta - (k - m\theta)^+$. Ex-post efficiency will require that total surplus be positive for each $\theta$. This occurs when

$$\theta > \frac{k}{m + \gamma} \equiv \theta^{FB} \tag{2}$$

Thus, ex-post efficiency establishes a marginal type $\theta^{FB}$ above which a manager of type $\theta$ generates positive surplus. Observe that this first best cutoff $\theta^{FB}$ rises in $k$ and falls in $m$ and $\gamma$. So it is efficient for the firm to hire better types when employment is expensive (to compensate for the high fixed cost of hiring), when the quality of the match with the firm is low (to compensate for the lower productivity from a poor match), and when the returns to a better manager are large (because the incremental variable cost savings from better managers are large).

The formula for the first best cutoff provides insight into when sorting matters. The cost of hiring the manager has both a fixed component ($k$) and a variable component ($m$), where $m$ varies not per unit produced, but rather for an incremental change in the manager’s ability. Firms with high fixed components (high $k$) are those were it is costly to install a manager. For example, these can be firms in technical industries that require a high level of industry-specific or firm-specific human capital (biotechnology, financial services). For such firms, it is important to obtain a high quality manager to compensate for these high fixed costs; as such, the efficient cutoff $\theta^{FB}$ will be high. Firms that require more general human capital (consumer products, retailing) may have lower fixed costs, and therefore lower needs for able managers.

The variable component of the cost function, $m$, tracks how much an incremental increase in quality decreases the cost to the firm. Firms with high variable components are those that markedly benefit from managerial ability. In such companies, the need for sorting is lower because it is built into the cost function. Such companies are very sensitive to ability, since they markedly decrease the firm’s cost function. High ability managers, for sure, are

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6See Corollary 2 in Section 3 for a discussion of empirical proxies for $k$ and implications for cross-sectional variation.
productive at such firms, but so are even low ability managers, because of the sensitivity of
the cost function (its steep slope). In contrast, firms with a low variable component (low \(m\))
need sorting the most, as only highly able managers will be able to produce value for the
firm. Under a low \(m\), lower ability managers are worth less to the firm, hence they must be
screened out through a high \(\theta^{FB}\) hurdle.\(^7\)

The firm cannot observe the type of the manager, and hence, must induce his employment
through a compensation contract. A contract is a salary \(s \geq 0\) and a bonus \(b \geq 0\). For
tractability, I restrict attention to linear contracts of the form

\[
w = s + bx.
\]  

(3)

This reflects the main feature of most compensation schemes, which have a fixed salary
and a bonus that depends on some performance measure.

Contracts are incomplete in that the firm cannot offer a menu of contracts to the manager,
which depends on an announcement of the manager’s type. This occurs because communi-
cation between the manager and the firm is costly, and the type \(\theta\) is sufficiently complex
that it cannot be embedded within a contract. For example, \(\theta\) represents the ability of the
manager, which is a complex mix of skills and attributes, such as vision, leadership abil-
ity, efficiency of decision making, aptitude with building relations (within and outside the
firm), time management skills, and so on. This form of “soft” information and “soft” skills
are not contractable, yet are nonetheless important for productivity.\(^8\) Finally, observe that
most CEO pay contracts do not condition on messages sent between the candidate manager
and the firm. I assume this message game does not take place, because a legal employment
contract cannot condition on the soft information that characterizes the manager’s type \(\theta\).
This is the reason contracts are incomplete.

The timing of the game, displayed in Figure 1, runs as follows: Nature reveals \(\theta\) to
the manager; the firm selects a contract \((s, b)\); each type \(\theta\) decides whether to join the
firm; Nature reveals production uncertainty \(\epsilon\); and the firm pays the manager based on the
realization of output.

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\(^7\)While this may seem counterintuitive, think formally that total surplus rises in \(k\) and falls in \(m\). In
particular, \(m\) governs the slope of the surplus function, and \(\theta^{FB}\) is the cutoff where total surplus breaks
even. Firms with high \(m\) have steep surplus functions, and therefore can afford to hire less able managers,
hence they have lower thresholds. Firms with low \(m\) have surplus functions that rise slowly in \(\theta\), so a high
\(\theta\) is necessary to create value.

\(^8\)Baker Jorgensen (2003) impose a similar assumption on restricting communication between the firm and
the agent.
2.2 Manager’s Problem

A manager of type $\theta$ will join the firm if his expected wage exceeds his outside options, i.e. if $E[w|\theta] \geq \bar{u}$. Since the manager’s expected wage $E[w|\theta] = s + b\gamma \theta$ is linear in $\theta$, there exists a marginal type $\theta^*$ who is indifferent between joining or leaving the firm, so $E[w|\theta^*] = \bar{u}$. This $\theta^*$ is

$$\theta^* = \frac{\bar{u} - s}{b\gamma}$$  \hspace{1cm} (4)

The marginal type $\theta^*$ depends on the contract parameters $(s, b)$ and, therefore, is the primary instrument through which the firm sorts types. This sorting takes place if and only if $\theta^* > 0$, which occurs when $s < \bar{u}$ and $b > 0$. If any sorting occurs at all ($\theta^* > 0$), it occurs when the manager’s expected wage strictly increases in his type. Finally, observe that for the marginal type $\theta^*$, his expected wage exactly equals his outside option ($E[w|\theta^*] = \bar{u}$), whereas every $\theta > \theta^*$ enjoys an information rent $E[w|\theta] - \bar{u} > 0$. This information rent accrues because the manager knows his type, and can extract payoffs from the firm through his wage. The firm is forced to pay a wage that increases in $\theta$ to attract the manager away from his best outside option. The manager captures this difference (the information rent).

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9These conditions guarantee that $\theta^*$ is well-defined and positive. If $s \geq 0$, the contract attracts all types to the firm, so $\theta^* = 0$. If $b = 0$, then all types either strictly prefer their outside option (if $s < \bar{u}$), strictly prefer the firm (if $s > \bar{u}$), or are indifferent between the two (if $s = \bar{u}$). Positive sorting occurs when $\theta^* > 0$ and is outside of these corner solutions.

10Negative bonuses are ruled out by assumption, but they would never exist in equilibrium anyway. The marginal type $\theta^* > 0$ will still exist if the manager’s expected wage decreases in his type ($s > \bar{u}$ or $b < 0$). But the firm would never set a negative bonus because this would either attract no one (if $s < \bar{u}$) or would attract only the low types ($s \geq \bar{u}$).
2.3 Firm’s Problem

The firm earns profit from output, pays out wages, and bears the costs of employing the manager. Thus, the ex-post expected profits for each \( \theta \) is

\[
E[\pi|\theta] = \gamma \theta (1 - b) - s - (k - m\theta)^+.
\]

(5)

Thus, the firm’s profits will rise in the manager’s productivity \( \theta \) and the quality of his match with the firm \( \gamma \), but fall in the compensation parameters \( s \) and \( b \). When the firm chooses its contract \( (s, b) \), this will determine the marginal type \( \theta^* \). In particular, this will induce sorting, since only agents with \( \theta > \theta^* \) will choose to work at the firm. The firm therefore selects a salary and bonus to maximize its expected profits for all \( \theta > \theta^* \):

\[
\max_{s,b} \int_{\theta^*(s,b)}^{\infty} E[\pi|\theta] f(\theta) d\theta.
\]

(6)

Write the marginal manager as \( \theta^*(s,b) \) to illustrate his dependence on the contract parameters. Even without a moral hazard problem of the manager, the compensation contract has a role to play as a sorting instrument for the firm. The contract parameters \( s \) and \( b \) will affect the firm’s payoff in two ways. First, they will determine the mix of types attracted to the firm, and second, they will determine the firm’s expected wage payments made to every manager who then joins the firm. This dual effect of the contract (determining participation ex-ante and expected wage payments ex-post) is apparent from (6), and will be a constant theme throughout, even under moral hazard and risk aversion.

Because there is no incentive problem in the basic model, the contract serves only to sort types. Ex-post expected profit \( E[\pi|\theta] \) decreases in salary and bonus, so sorting is costly for the firm. But it is necessary because the marginal type \( \theta^*(s,b) \) decreases in \( s \) and \( b \). Thus, as the firm raises either salary or bonus (decreasing ex-post profits), it can conceivably raise ex-ante profits because it attracts more types to the firm (thereby expanding the area of integration in (6)). The manager’s participation decision depends only on whether his expected profits exceed his outside options. He is effectively indifferent to receiving salary or bonus as long as he earns more at the firm than in the outside market. The firm, however, prefers to sort using salary, rather than bonus. Intuitively, the manager’s information rent \( E[w|\theta] - \bar{u} \) increases in his bonus, since a high bonus boosts the manager’s compensation relative to his outside option. The firm seeks to minimize these information rents, since, like all rents, they come at the cost of the firm’s surplus through higher wages. In fact, the firm will pay as small a bonus as possible and a salary as close to the manager’s outside
option. This is just enough to induce him to accept the job, leading to the first proposition (all proofs are in the appendix).

**Proposition 1** In the pure sorting model, the optimal contract is \( s \approx \bar{u} \) and \( b \approx 0 \).

In the optimal contract, the firm will set a salary arbitrarily close to \( \bar{u} \) and a bonus arbitrarily close to 0. The firm does not set \( s = \bar{u} \) and \( b = 0 \) exactly, because then every \( \theta \) would weakly prefer to work at the firm. But the firm could do strictly better by reducing the salary by an arbitrarily small amount and raising the bonus by an arbitrarily small amount. Doing so will provide just enough steepness to the wage schedule, such that only \( \theta > \theta^{FB} \) prefer to work at the firm. The proof of Proposition 1 shows that such a contract dominates \( s = \bar{u} \) and \( b = 0 \) and achieves efficient sorting.\(^{11}\)

\[\text{Figure 2: Equilibrium in the Benchmark Model}\]

To see this visually, refer to Figure 2. The firm has two contract parameters, salary and bonus, to induce the sole decision of the agent (participation). Therefore, there are many, in fact a continuum of, contracts which induce efficient participation. The heavy

\(^{11}\)At a technical level, \( \theta^* \) is undefined at the point \( s = \bar{u} \) and \( b = 0 \). The firm’s maximization problem in (1), therefore, has a discontinuity at the point \((s, b) = (\bar{u}, 0)\). The firm’s expected payout at this discontinuity is strictly less than the payoff from the contract \( s = \bar{u} - k\gamma\eta/(m + \gamma) \) and \( b = \eta \) for sufficiently small \( \eta > 0 \). Thus, the optimal contract is arbitrarily close, but not exactly equal to \((s, b) = (\bar{u}, 0)\). In practice, take \( \eta \) to be the smallest possible value in the available currency, such as one cent.
The line in Figure 2 represents the efficient set, namely contracts \((s, b)\), which induce efficient participation: \(\{(s, b) : \theta^*(s, b) = \theta^{FB}\}\). The efficient set slopes downward, so salary and bonus are substitutes rather than complements in inducing participation. Raising the bonus alone will attract more (and worse) types, since large incentive payments increase everyone’s wages. The firm can compensate for this by lowering the salary, bringing participation back to its efficient level. In this sense, the mix of compensation is as relevant, if not more relevant, than simply the level of compensation. Figure 2 also graphs the isoprofit lines of the firm. These are the isoquants of the firm’s profit function given in (6).\(^\text{12}\) Simple algebra shows these isoprofit lines must be steeper than the slope of the efficient set. As the fixed level of profit rises, the isoprofit lines shift downward. Thus, the firm will choose the contract whose isoprofit line gives maximum profit and still achieves efficiency, which happens exactly at the corner solution where the two lines intersect.

Why is a small bonus better at sorting than a large bonus? Everyone wants a large bonus, both the high-types and the low-types. But only the high-types will choose a small bonus, because their high ability can outweigh the low pay from the small bonus. Low-types, on the other hand, receive a very low payoff when the bonus is small, and therefore would not choose to join the firm. This is the sense in which a small bonus is more effective at sorting than a large bonus. The small bonus achieves the separation of types, while the large bonus does not.

Proposition 1 shows that performance pay does indeed have a sorting effect, but that this effect operates at very low PPS. All that is needed to induce efficient sorting is a non-zero slope of the wage profile. More than this is unnecessary because it transfers unnecessary information rent to the agent. This is consistent with the vast empirical literature of PPS of executive contracts.\(^\text{13}\) The sorting effect exerts downward pressure on performance pay.

\(^{12}\)Fixing ex-post profits at a given level and rewriting that equation as salary as a function of the bonus delivers the dashed isoprofit lines in Figure 2.

\(^{13}\)Jensen and Murphy (1990) find that CEO wealth changes $3.25 for every $1000 change in shareholder wealth. Morck, Shleifer, and Vishny (1988) argue empirical estimates of PPS are “too low,” since they deviate widely from theoretical predictions. Attempts to justify these low empirical estimates on risk-aversion, such as Haubrich (1994), rely on estimating parameters of the model that are notoriously difficult to measure, such as the manager’s cost of effort parameter.
3 Combining Sorting and Incentives

Now suppose the manager exerts costly and unobservable effort at the firm. This induces a moral hazard problem on the part of the manager, since the firm cannot observe effort perfectly, but must induce it through its compensation contract. At the same time, this same compensation contract is used to attract managers to the firm. Thus, contracts will now serve the dual purpose of attracting workers and providing incentives. Output is now given by

\[ x = \gamma \theta e + \epsilon, \]  

where \( \epsilon \) still has mean 0 and variance \( \sigma^2 \). The manager exerts effort \( e \) at a quadratic cost of effort at \( C(e) = 0.5ce^2 \) with \( c > 0 \), and he continues to enjoy an outside option \( \bar{u} \). Observe that the manager’s type \( \theta \) and effort choice \( e \) are complements, so more able types have higher marginal productivities of labor, and are therefore more productive to the firm. In fact, there are two levels of complementarity: between the ability \( \theta \) and effort \( e \), as well as between ability and the quality of the match between the firm and manager \( \gamma \). In practice, \( \gamma \) is general to the firm, whereas \( \theta \) and \( e \) are specific to each individual manager. Of course, \( e \) is a choice variable, \( \theta \) is a random variable, and \( \gamma \) is an exogenous parameter.

The firm pays the manager \( w = s + bx \). As before, the contract is linear and the firm cannot condition the contract on the manager’s type. The expected output for each manager \( \theta \) is \( E[x|\theta] = \gamma \theta e \), so more effort from the manager produces more revenue for the firm. For each \( \theta \), the average wage is \( E[w|\theta] = s + b\gamma \theta e \). Assume the manager is risk-neutral.\(^{14}\) Figure 3 illustrates the timeline of the game.\(^{15}\)

3.1 First Best

The manager has two decisions: whether to join the firm and how hard to work. As such, the first best benchmark will also have two components, an efficient effort level and an efficient cutoff for participation. Observe that the expected total surplus for each \( \theta \) is

\(^{14}\)The next section considers a risk averse manager.

\(^{15}\)The timeline of the game opens the possibility of renegotiation of the contract after the manager accepts. But because the manager cannot communicate \( \theta \) at any point, a manager who accepts the initial contract would also accept a renegotiated contract, since the information environment is unchanged. The firm’s optimization problem is the same, and therefore would offer the same contract to a manager after acceptance than it would before acceptance. Thus, the optimal contract would be renegotiation-proof. Details are available from the author upon request.
Nature reveals $\theta$ to manager

Firm proposes contract $(s, b)$

Each manager decides whether to join firm

Managers who join exert effort $\epsilon$

Nature resolves $\epsilon$, and thus $x$

Firm pays manager $w = s + bx$

Figure 3: Timeline of the Game with Effort.

$$E[TS|\theta] = \gamma \theta e - C(e) - (k - m\theta)^+.$$  \hspace{1cm} (8)

Total surplus now not only includes expected output and the fixed cost of hiring a manager, but also the manager’s cost of effort. The first best effort level that maximizes this is $e^{FB} = \gamma \theta/c$. Observe that the first best effort level rises in both $\theta$ as well as in $\gamma$. It is efficient for more productive types to exert more effort, where productivity is measured in either a manager-specific sense ($\theta$) or a firm-specific sense ($\gamma$). And naturally, it is efficient for a manager with high cost of effort to exert less effort, since this high cost reduces the manager’s utility and therefore, total surplus as well. Evaluated at $e^{FB}$,

$$E[TS|\theta] = \frac{(\gamma \theta)^2}{2c} - (k - m\theta)^+.$$  \hspace{1cm} (9)

Expected total surplus rises in both $\gamma$ and $\theta$, and falls in $c$ and $k$. This is positive if and only if

$$\theta > \frac{\sqrt{c(m^2c + 2k)} - mc}{\gamma} \equiv \theta^{FB}.$$  \hspace{1cm} (10)

As before, $\theta^{FB}$ denotes an efficient cutoff, namely the minimal managerial type, such that it is efficient for the firm to employ any manager with $\theta > \theta^{FB}$. The cutoff $\theta^{FB}$ rises in $k$, falls in $\gamma$, and rises in $c$. Thus, as technological or market factors lower the cost of supplying effort, it is efficient for the firm to hire more types and have each of them work more.

3.2 Manager’s Problem

Each manager of type $\theta$ maximizes his expected wage less his cost of effort:

$$\max_{\epsilon} E[w|\theta] - C(e)$$  \hspace{1cm} (11)
The first order condition yields the manager’s incentive constraint (IC): \( \hat{e} = b\gamma \theta / c \). Higher bonuses now have a clear incentive effect of inducing more effort. In addition, effort rises in both \( \gamma \) and \( \theta \), so more able types work more, as do types who are a better fit with the firm. This is exactly the sense in which there is complementarity in production: both \( \gamma \) and \( \theta \) are complements with respect to effort.

The participation decision now involves the manager’s effort choice, which she makes conditional on facing a contract \((s, b)\). A manager of type \( \theta \) will join the firm if, in equilibrium, the manager earns more inside the firm than outside the firm, which occurs if \( E[\hat{w}|\theta] - C(\hat{e}) \geq \hat{u} \). There exists a marginal manager \( \theta^* \) who satisfies \( E[\hat{w}|\theta^*] - C(\hat{e}) = \bar{u} \), or

\[
\theta^* = \frac{\sqrt{2c(\bar{u} - s)}}{b\gamma}.
\]  

(12)

As before, the marginal manager \( \theta^* \) falls in both \( s \) and \( b \), reinforcing the intuition that higher wage payments attract more types to the firm. And just as with \( \theta^{FB} \), \( \theta^* \) rises in \( c \).

### 3.3 Impediments to First Best

Can the firm implement first best? Presumably, this may be possible because there is no difference in risk preferences between the firm and the agent. The canonical agency model of a risk neutral principal and agent obtains first best, so we may expect the same here. However, sorting complicates the analysis, which we now show.

Expected profits of the firm are \( E[\pi|\theta] = E[x|\theta] - E[w|\theta] - (k - m\theta)^+ \). Plugging in (IC),

\[
E[\pi|\theta] = (\gamma \theta)^2 b(1 - b)/c - s - (k - m\theta)^+.
\]  

(13)

As before, the firm’s profits rise in \( \gamma \) and \( \theta \) and fall in \( c \) and \( k \). However, the effect of the bonus on profits is more subtle even though paying out salary clearly drains profits. In particular, a small bonus will raise expected profits, but as the bonus becomes very large, it will eventually lower the firm’s profits. This occurs because of the dual sorting and incentive effects.

Comparing the efficient effort \( e^{FB} \) to the manager’s equilibrium effort \( \hat{e} \) reveals that the firm must implement first best by setting \( b = 1 \). Intuitively, the firm grants maximum incentives to the agent in order to induce him to exert the efficient effort level. Because the firm has two instruments (salary and bonus) to solve two problems (effort and participation), it seems plausible that the firm could implement first best. However, observe that if \( b = 1 \), the expected profit of the firm is negative. Thus, to implement first best, the firm goes bankrupt.
Even though the firm has two separate instruments to solve the effort and participation decisions, this is not sufficient to induce efficiency.

3.4 The Firm’s Problem: Second Best Solution

Now that it’s clear the firm cannot achieve first best, it will select a salary and bonus to maximize its expected profits. Thus, the firm will select $s$ and $b$ to maximize

$$
\Pi(s, b) = \int_{\theta^* (s, b)}^{\infty} E[\pi | \theta] f(\theta) d\theta.
$$

(14)

As before, $\theta^*(s, b)$ makes prominent the dependence of the marginal manager on the parameters of the compensation contract that the firm sets. Because of the moral hazard problem, the compensation contract plays a dual role of both sorting types through $\theta^*$, as well as providing incentives through the expected wage $E[w|\theta]$. Solving this program explicitly is difficult because of the interaction between the sorting and incentive effects. In particular, $b$ affects the manager’s incentives to work, as well as the decision on whether to participate at all. Thus, the participation decision $\theta^*(s, b)$ is now endogenous. Nonetheless, the next proposition, proved in the appendix, presents the implicit solution that still has enough structure to provide insight into the dual sorting and incentive effects.

**Proposition 2** A firm contracting with a risk neutral agent will select an optimal contract $(s, b)$:

$$
s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*)}{f(\theta^*)} b \frac{\theta^*^2}{c} \right] - (k - m\theta^*)^+
$$

(15)

$$
b = \left( 2 - \frac{\theta^*^2}{E[\theta^2 | \theta > \theta^*]} \right)^{-1}.
$$

(16)

The sorting effect is immediately apparent in the optimal contract, since the contract now depends on the distribution of manager types. Recall that in the canonical agency problem of a principal contracting with a risk-neutral agent, the principal will set a salary equal to the agent’s outside option, and make the agent the full residual claimant on the firm’s output ($b^* = 1$); this is the “sell-the-firm” contract, in which the agent keeps the entire share of firm output, and the principal takes back those rents in expectation via the (possibly negative) salary. The canonical model has empirical difficulties both because negative salaries are uncommon, and because empirical estimates of PPS are much lower than $b^* = 1$. 

14
Adding sorting to the canonical model however, moves the equilibrium away from the “sell-the-firm” contract. For all \( \theta > \theta^* \), observe \( E[\theta^2|\theta > \theta^*] > \theta^{*2} \), and therefore, \( b < 1 = b^* \). Thus, the presence of sorting once again dampens the optimal bonus. This confirms the intuition from Proposition 1 that while a positive bonus is necessary to attract higher quality managers to the firm, too much weight on this bonus is wasteful because it transfers excessive rent to the agent. Here, the firm must provide incentives to the manager in order to induce him to work, which puts upward pressure on the bonus. The firm must also attract quality managers and yet, this transfers rent to a manager and places downward pressure on his bonus. The optimal bonus will trade off these twin effects, namely the downward pressure from sorting and the upward pressure from incentives.

How exactly will the firm trade off the optimal choice of salary and bonus? In the canonical model of risk-neutral agent without sorting, there is a clean separation between salary and bonus; the bonus provides incentives, while the salary guarantees participation. Here, though the salary does not play a role in incentives, bonuses do affect sorting, since the marginal manager \( \theta^* \) falls in \( b \). Just like salaries, higher bonuses will attract more types to the firm. The proof of Proposition 2 details the first order condition for the firm’s optimization with respect to the optimal salary and bonus, which leads to the equilibrium condition

\[
\frac{\int_{\theta^*}^{\infty} \partial E[\pi|\theta]/\partial s \ dF}{\int_{\theta^*}^{\infty} \partial E[\pi|\theta]/\partial b \ dF} = \frac{\partial \theta^*/\partial s}{\partial \theta^*/\partial b}.
\] (17)

This equilibrium condition is the marginal rate of substitution between salary and bonus. Recall from Section 2 that sorting gives contracts a dual effect, namely determining participation ex-ante and the expected wage payments ex-post. The same is true here, after including a moral hazard problem. Each piece of the compensation contract will have an ex-post (left-hand side of (17)) and an ex-ante effect (right-hand side of (17)). The equilibrium condition above states that the firm will optimally equalize the ratio of these ex-ante and ex-post effects. This is equivalent to equalizing the marginal rate of substitution between salary and bonus. The firm chooses its salary and bonus such that the tradeoff between the costs of wages against the benefits of participation is equal across both contract parameters. At the equilibrium, the firm is indifferent between using salary and bonus because their marginal rates of substitution are equal.

To see this visually, refer to Figure 4. First, observe that the existence of both a sorting and incentive problem implies a multidimensional measure of efficiency: there is now an efficient participation level \( \theta^{FB} \) as well as an efficient effort level \( e^{FB} \). Since the firm has two contract parameters (salary and bonus) to solve the participation and effort problems, the
efficient set now collapses to a single point, rather than a continuum as in the benchmark model. Figure 4 plots the efficient contract \((s, b) = (\bar{u} - k, 1)\).

The expected profit \(\pi(s, b)\) from (14) gives the isoprofit curve pictured as the upside down U-shaped hyperbole in Figure 4. This contour line gives the set of salary and bonus pairs that guarantee a fixed level of profit. As that fixed level increases, the curve moves downward. This is exactly why the firm cannot implement efficiency, since the isoquant that contains the efficient contract would require an upward shift of the isoquant, possible only with a negative profit level. Thus, the efficient contract sits above and outside of the isoprofit line of the profit maximizing contract.

The other curve in Figure 4 is the isoparticipation line, the isoquant for the marginal manager \(\theta^*\). The isoquant graphs the contract parameters that induce a fixed level of participation, and the downward sloping shape of this isoquant confirms that salary and bonus are substitutes. They are different but equivalent instruments to inducing participation, as an increase in the bonus will require a decrease in the salary in order to keep participation fixed. This reinforces the point that the firm must set the choice of salary and bonus jointly, rather than individually, to determine participation. If the firm wants to induce a higher level of participation \(\theta^*\), then the isoquant in Figure 4 will tilt leftward.\(^{16}\)

The isoprofit and isoparticipation lines represent the ex-post and ex-ante effects of the contract, respectfully. A given contract determines not only who participates ex-ante, but also how hard the manager works ex-post. The slope of each isoquant is the marginal rate of substitution between salary and bonus. The equilibrium condition shows that this occurs precisely when the marginal rates of substitution between salary and bonus are equivalent for both the ex-ante and ex-post effects. This occurs precisely when the two isoquants have identical slopes. Since the isoparticipation line is downward sloping, this will necessarily occur on the downward sloping portion of the isoprofit line.

Recall that in the benchmark model, the firm always prefers to substitute salary for incentives to induce sorting. This is precisely because there is no incentive problem in the benchmark model. Without moral hazard issues, the firm need not induce effort, and therefore, paying bonuses simply drains expected profit. Thus, the equilibrium condition above collapses to a corner solution in which the firm pays as small a bonus as possible to guarantee participation and positive sorting. Adding an effort problem to the manager’s decision gives a positive role for the bonus. Now, the firm’s choice of bonus not only draws a

\(^{16}\)In particular, the \(y\)-intercept of the isoquant remains fixed at \(\bar{u}\) while the \(x\)-intercept shrinks, shifting leftward.
manager to the firm, but also provides incentives to the managers once they accept the job. This extra incentive effect of the bonus is what gives a non-trivial solution for the optimal contract.\footnote{If $b_S$ is the bonus under sorting alone, $b_I$ the bonus under incentives alone, and $b_{I,S}$ the bonus under both incentives and sorting, then $0 = b_s < b_{I,S} < b_I = 1$. This gives a strict ordering. Dutta (2008) finds that $b_{I,S} < b_I$, but does not offer a clear ordering between $b_S$ and either $b_{I,S}$ or $b_I$. Disaligned communication gives a fuller picture of the ranking of the bonuses here.}

The other important result here is that we are now operating in a second best world. Before, the firm could achieve first best because it had two instruments (salary and bonus) to satisfy two constraints (participation and solvency). In this section, the firm now has an additional constraint (effort-provision), but still has only two instruments. This prevents the firm from achieving first best, and therefore, it must choose the contract to balance participation against incentives, all while staying solvent. Figure 4 illustrates how the firm balances these effects by equating the marginal rates of substitution of participation against incentives. This theoretical result shows precisely how contracts can solve multiple problems at once, rather than the classical case of moral hazard or adverse selection alone.

Figure 4: Equilibrium in the Combined Incentives and Sorting Model
3.5 Comparative Statics

Since the optimal contract serves a sorting role in addition to incentives, the optimal salary and bonus both depend on the distribution of managerial talent $f$.

**Corollary 1** When cost of effort ($c$) or the outside option ($\bar{u}$) are sufficiently large, the PPS increases in the variation between types of the manager.

This corollary\textsuperscript{18} complements the results of Dutta (2008) and Baker-Jorgenson (2003). Both papers find in their model that their optimal bonus can rise in what Dutta (2008) calls “information risk,” namely, the variance on $\theta$. For Baker-Jorgenson, the firm wants the manager to make use of his private information. Increasing the bonus induces the manager to make use of this information when the value of that information rises when the variance of $\theta$ rises. Dutta (2008) arrives at the same result only when the manager’s outside option increases in $\theta$, but not otherwise. Both papers fundamentally rely on how contracts affect communication, whereas I focus on how contracts change participation. Sorting has features of an option-like contract. By offering a contract in the market place, the firm automatically caps its downside risk and captures its upside potential from the managerial labor pool. This occurs precisely because the firm can now attract all $\theta > \theta^*$. Just as the value of an option increases in the volatility, so the value to the firm increases in the variance on $\theta$. As the tail of the ability distribution becomes fat, the upside potential grows, and the firm increases the bonus to attract these high types. Thus, the intuition behind Corollary 1 fundamentally rests on inducing participation, rather than eliciting communication.

**Corollary 2** As the manager’s cost of employment ($k$) rises, the firm will lower salaries, but leave the bonus unchanged.

This corollary follows directly from inspection of the optimal contract in Proposition 2. The first order condition from the firm’s choice of salary is

$$E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial s} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta$$

This first order condition equates the marginal cost and marginal benefit of adjusting the salary. The left-hand side is the marginal benefit of a salary change. This comes from the extra profit that the firm earns on each new type of manager attracted to the firm because of the salary adjustment. Thus, a small change in salary will change the marginal manager.

\textsuperscript{18}Proofs of all corollaries are available from the author on request.
and this will yield a return to the firm measured by the expected profit of that marginal manager. The right-hand side of (18) captures the marginal cost of an incremental salary change. This is the change in the cost of wages for the new salary. Namely, the change in profits for every type of manager the firm hires. In equilibrium, the marginal cost and marginal benefit of this incremental change in salary are equal.\(^{19}\)

Consider the effect of increasing \(k\). The right-hand side of (18) is unchanged, but increasing \(k\) lowers expected profits. Because the right-hand side of (18) is independent of \(k\), and because the equality must hold in equilibrium, the firm will compensate for this reduction in expected profit by lowering the salary. Thus, the firm exerts a countervailing effect on expected profit to respond to an exogenous rise in the cost of employment. This corollary predicts that in industries with high fixed costs of hiring, firms will employ lower salaries, and yet leave the bonus unchanged. While this may seem counterintuitive, remember that higher fixed costs drain firm profits, and don’t affect participation (\(\theta^*\) is independent of \(k\)). So the firm recovers profit lost by lowering salary.

Empirical proxies of \(k\) can be diverse. One example is a measure of firm or industry-specific human capital. As an example, a firm in the finance or technology sector, that requires deep, specialized knowledge, may have a high \(k\), while a more general consumer products or retail firm, that requires more general business knowledge, may correspond to a low \(k\). One possible way to measure \(k\) may be the recruitment of managers across different industries or sectors. An industry that recruits future managers from within the same industry may be a signal of a high fixed cost of training, rather than an industry that recruits managers from across different industries. That investment banks hire CEOs with long careers in the financial sector may indicate a high requirement for industry-specific knowledge, and therefore a high \(k\). Corollary 2 gives a prediction on the direction of the cross-sectional variation between industries on managerial salaries.

\(^{19}\)Because the salary has no incentive effects, the change in profit from a change in salary is simply \(-1\), so \(\frac{\partial E[\pi|\theta^*]}{\partial s} = 1\). Thus, a $1 increase in salary drains firm profit by exactly $1. Thus, (18) simplifies to

\[
E[\pi|\theta^*]H(\theta^*) \frac{\partial \theta^*}{\partial s} = 1
\]

where \(H(\theta^*) = f(\theta^*)/(1 - F(\theta^*))\) is the hazard rate, familiar from the literature on adverse selection models.
4 Risk Aversion

Now suppose the manager is risk-averse, and dislikes volatility in his income. Without sorting, the canonical agency model with a risk averse agent delivers the standard risk-incentives trade-off where the principal seeks to provide incentives for the agent to work, but such incentives load risk onto the agent, which he dislikes. How does sorting affect this analysis?

To fix ideas, suppose production uncertainty $\epsilon$ is normally distributed, and as before, has a mean of 0 and a variance of $\sigma^2$. Assume the agent has CARA preferences with coefficient of absolute risk aversion $r > 0$. The timing of the game is the same as before, as are the first best effort level $e^{FB}$ and first best participation cutoff $\theta^{FB}$.

4.1 Manager’s Problem

Given output $x = \gamma \theta e + \epsilon$ and a linear compensation contract $w = s + bx$, the certainty equivalent of the $\theta$-manager’s payoff is now

$$CE(\theta) = s + b\gamma \theta e - \frac{r}{2} b^2 \sigma^2 - C(e).$$  \hspace{1cm} (20)

Observe that uncertainty in production $\sigma^2$ affects the wage variance $V[w|\theta] = b^2 \sigma^2$, but does not affect the mean $E[w|\theta] = s + b\gamma \theta e$. Therefore, the manager’s effort choice optimizes his expected wage less his cost of effort. Given a quadratic cost of effort $C(e) = 0.5ce^2$, the incentive constraint (IC) for the manager of type $\theta$ is the same as before: $e = b\gamma \theta / c$. Given a contract $(s, b)$, a manager of type $\theta$ will participate if the certainty equivalent of his contract exceeds his outside option $\bar{u}$, or $CE(\theta) \geq \bar{u}$. As before, there exists a marginal manager $\theta^*$ who is indifferent between accepting and rejecting the compensation contract, and higher types $\theta > \theta^*$ will accept. This $\theta^*$ is

$$\theta^* = \frac{\sqrt{2c(\bar{u} - s) + crb^2 \sigma^2}}{b\gamma}.\hspace{1cm} (21)$$

Observe that a risk-averse manager now bears a disutility from uncertainty captured by the risk premium term $\frac{r}{2} b^2 \sigma^2$. This risk premium is the additional compensation necessary to induce a risk-averse manager to accept risk. Indeed, only a risk-averse manager of higher $\theta$ will be indifferent between accepting and rejecting the contract. Thus, the marginal type $\theta^*$ rises in the measure of the agent’s risk aversion. A risk-averse manager requires an additional
risk premium to equalize his expected utility with that of a risk-neutral manager. This is true in general for each \( \theta \), and in particular is true for \( \theta^* \).

The firm can achieve positive sorting if \( \theta^* > 0 \). This occurs if \( b > 0 \) and \( s < \bar{u} + 0.5rb^2\sigma^2 \). Namely, the bonus must be positive and the salary must not exceed the outside option plus the risk premium. Observe that this is a looser condition on the salary than before, since without risk aversion, \( s < \bar{u} \) was the sufficient condition to guarantee positive sorting. The risk premium thus grants the firm slightly more leeway in setting its salary, since the firm must embed within the salary the manager’s risk premium. Consistent with earlier results, \( \theta^* \) falls in \( s \) and \( b \), and rises in the outside option \( \bar{u} \). And if \( s < \bar{u} \), \( \theta^* \) rises in the cost of effort parameter \( c \), reinforcing the natural intuition that as effort becomes more costly, fewer types choose to work at the firm.

The other new comparative static in this model of risk aversion is how participation changes with output volatility. As \( \sigma^2 \) rises, this loads volatility onto the manager’s compensation, which he dislikes because of his risk aversion. For a given \( \theta \), this lowers the manager’s certainty equivalent of his payoff, and therefore makes him less likely to work at the firm. Said differently, only a manager with a high \( \theta \) will generate enough revenue for the firm, and therefore enough incentive compensation for the additional decrease in utility from the rise in volatility. Thus, high productivity types will select the firm, whereas low productivity types will select their outside option. The marginal manager \( \theta^* \), who is indifferent between his inside and outside options, will therefore increase. Raising output volatility will not only cause fewer types to participate, but also cause better types to participate. Insofar as it is possible to measure production uncertainty \( \sigma^2 \) and managerial type \( \theta \), this predicts that better types work in more uncertain environments.

The presence of the risk premium in the formula for the marginal manager \( \theta^* \) complicates the relationship between \( \theta^* \) and \( b \). In the benchmark model, \( \theta^* \) uniformly decreased in \( b \), but now the bonus appears in both the numerator and denominator in the expression for \( \theta^* \). Simple algebra shows \( \theta^* \) increases in \( b \) if and only if the manager’s salary exceeds her outside option (\( s > \bar{u} \)). Thus, when the firm sets a salary below the manager’s outside option, increasing the manager’s bonus transfers rents to the manager, thereby attracting more types to the firm. But if the firm pays the manager an efficiency wage (where the salary exceeds the outside option), increasing the bonus lifts the risk premium so much that the rise in \( \theta^* \) due to the risk premium outweighs the fall in \( \theta^* \) due to the rent transfer (the numerator effect exceeds the denominator effect in the formula for \( \theta^* \)). Increases in \( \theta^* \) correspond to higher standards, and thus better types come to the firm; likewise, decreases in \( \theta^* \) lead to
worse managers attracted to the firm. Thus, the presence of risk aversion illustrates that the sorting effects of compensation are subtle. How the firm sets salary will affect the sign of $\partial \theta^*/\partial b$, i.e. whether sorting has a positive or negative effect on the quality of the manager attracted to the firm.

To understand how the firm will optimally select its salary and bonus, we now turn to the firm’s problem.

### 4.2 Firm’s Problem

The firm maximizes expected profit $E[\pi|\theta] = E[x|\theta] - E[w|\theta] - k$ for each $\theta > \theta^*$ that the firm hires. Using (IC), the firm’s expected profit, conditional on $\theta$, is

$$E[\pi|\theta] = (\gamma \theta)^2 b(1 - b)/c - s - (k - m\theta)^+. \quad (22)$$

Observe that the form for expected profit is the same as before, namely, in the prior section with a risk-neutral agent. This occurs because risk aversion does not alter the agent’s incentive constraint, but does affect his participation decision. Because the incentive constraint is the same, conditional on hiring a manager $\theta$, the firm’s ex-post profits from that manager are unchanged in the presence of risk aversion. For sure, risk aversion will affect the optimal bonus, which in turn will drive both incentives and participation. But taking this bonus as exogenous, as the firm does prior to its optimization, risk aversion does not change the expected profit function.

The problems with implementing first best are similar in nature, and even more severe than in the prior section with a risk-neutral manager. In order to achieve efficient effort, the firm must make the manager the full residual claimant on firm output. And with $b = 1$, the firm can induce efficient participation by setting a $s = \bar{u} + \frac{\sigma^2}{2} - k$. To guarantee participation, the firm must compensate the manager with a base wage of at least his outside option, less the cost of his employment, plus his risk premium, to induce the risk-averse manager to accept the risky compensation scheme. However, the firm’s expected profit at this contract is $E[\pi|\theta] = -\bar{u} - \frac{\sigma^2}{2} < 0$. Firm profit is even lower than before because of this additional risk premium, which must be a component of the salary. As before, the firm cannot simultaneously achieve efficient participation and avoid bankruptcy.

Thus, the firm will select $s$ and $b$ to maximize profits, which are $E[\pi|\theta]$ for each $\theta > \theta^*$:

$$\Pi(s, b) = \int_{\theta^*(s, b)}^{\infty} E[\pi|\theta] f(\theta) d\theta. \quad (23)$$
The firm will select its salary and bonus to maximize this expression. Just as in the prior model with the risk-neutral agent, optimizing jointly over $s$ and $b$ will lead to an equilibrium condition that equalizes the marginal rates of substitution between salary and bonus. This substitution trades off the ex-ante effect on participation against the ex-post effect on expected profits. The presence of sorting guarantees this equilibrium condition is non-trivial. The salary and bonus will each receive non-zero weight at the optimum, as each instrument has different effects on both participation and on ex-post wages and profits.

The equilibrium condition takes the same form as before, but it does have a few key differences. The main similarity is in the form of the expected profit function, which as argued earlier, is identical to the expression under a risk-neutral agent. The only term that is changed is $\frac{\partial \theta^*}{\partial b}$, which now is

$$\frac{\partial \theta^*}{\partial b} = \frac{rc\sigma^2}{2b\theta^*} - \frac{\theta^*}{b} = \frac{rc\sigma^2}{\gamma^2 b\theta^*} - \frac{\theta^*}{b}.$$  (24)

This expression makes the effect of risk aversion clear. Under risk neutrality, $r = 0$ and this derivative becomes unambiguously negative, just like $\frac{\partial \theta^*}{\partial s}$. Increasing salary and bonus raises the compensation of the agent, making the contract more attractive to outsiders, thus drawing in greater participation from the labor market, and reducing the marginal type $\theta^*$.

But now, $r > 0$ implies that risk aversion enters the participation decision. Raising incentives no longer unambiguously increases participation, since higher bonuses load more risk onto the compensation, which risk averse agents dislike. In particular, if the level of risk aversion is sufficiently low, the effect will be unambiguous as before. The marginal type will fall, and paying bonuses will attract more types. But, if risk aversion is sufficiently high (namely that $r > \frac{\gamma^2 \theta^*}{c\sigma^2}$), then raising incentives will actually repel all types, and cause the marginal manager $\theta^*$ to rise in $b$. This shows that the relationship between performance pay and participation is subtle in the presence of risk aversion. Importantly, risk aversion does not affect the incentive margin as directly as it does the participation margin. This is somewhat surprising given the traditional tradeoff between risk and incentives. The relationship is more nuanced in that risk averse agents dislike risky compensation, and therefore, they will stay away from firms that offer large incentive packages.

To see this visually, consider the isoquants from the participation decision of the profit function. If risk aversion is sufficiently low ($r < \frac{\gamma^2 \theta^*}{c\sigma^2}$), then the isoquants are visually identical to the risk-neutral case and look like those in Figure 4. In that case, low levels of risk-aversion are similar to risk-neutrality, delivering a downward sloping isoparticipation line (salary and bonus are substitutes). But if risk-aversion is sufficiently high ($r > \frac{\gamma^2 \theta^*}{c\sigma^2}$),
the isoprofit line actually slopes upward, shown in Figure 5. As the firm raises bonuses, this loads risk onto the manager, repelling the lower quality types. In order to counteract this effect and keep the participation level fixed, the firm raises salary, thereby attracting those risk-averse types back to the firm. Thus, when a manager is sufficiently risk-averse, salary and bonus are complements, rather than substitutes, and they therefore reinforce one other in obtaining participation.

Because the ex-post profit function is identical under risk-aversion and risk-neutrality, the isoprofit curve will look similar as before. In Figure 5, the slopes of the two isoquants (the isoparticipation curve and the isoprofit curve) reflect the marginal rates of substitution between salary and bonus. The equilibrium condition ensures that the optimal contract occurs where the slopes of the two isoquants are the same. This gives the tangency condition in the figure. Since the isoparticipation curve slopes upward, the optimal contract now occurs on the upward sloping portion of the isoprofit curve.

\[
\theta^* = \sqrt{\frac{2c(u-s)+crb^2\sigma^2}{b\gamma}}
\]

\[
\Pi = \int_{\theta^*}^{\infty} E[\pi|\theta]dF
\]

Figure 5: Equilibrium Under Sufficiently High Risk-Aversion

Solving for the optimal contract in closed form is even more difficult than before, since the risk aversion adds an additional complexity to the participation decision. Now, every manager, and therefore the marginal manager, requires a risk premium to guarantee participation. This risk premium depends on the bonus \( b \), and therefore, so does the participation decision. Because \( \theta^* \) is no longer monotonic in \( b \), the lower bound of integration in expected profit \( \Pi(s, b) \) makes the first order condition from the firm’s problem more complex.
Nonetheless, it is still possible to arrive at an implicit solution that can deliver intuition.

**Proposition 3** A firm contracting with a risk-averse agent will select an optimal contract \((s, b)\) that satisfies

\[
s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*)}{f(\theta^*)} b \right] \frac{b \theta^* \gamma^2}{c} - (k - m \theta)^+.
\]

\[
b = \left( \frac{r \sigma^2}{\gamma^2 E[\theta^2 | \theta > \theta^*]} + 2 - \frac{\theta^*}{E[\theta^2 | \theta > \theta^*]} \right)^{-1}.
\]

The formula for the optimal bonus in Proposition 3 is more complex than in Proposition 2 because of the risk aversion, but the underlying forces at work are the same. It is clear from the formula for the optimal bonus above that risk aversion will dampen PPS, just like in the standard moral hazard model. Indeed, the right side of the denominator of the bonus above is identical to the denominator of the bonus under risk neutrality.

Notice the similarity here with the canonical model of a principal contracting with a risk averse agent under CARA preferences and linear contracts. In the canonical model, the optimal bonus is \(\hat{b} \equiv 1/(1 + r \sigma^2)\). Both there and here, the optimal contract falls in the cost of effort parameter \(c\), the coefficient of absolute risk aversion \(r\), and variance on output \(\sigma^2\).

Ultimately, introducing risk aversion does more than dampen PPS. It also fundamentally changes the nexus of tradeoffs that the firm faces when choosing salary and bonus. Without risk aversion, salary and bonus are always substitutes, and therefore two equivalent ways of attracting the worker. But under sufficiently high risk aversion, they become complements. When risk aversion is high, a high bonus loads a large disutility onto the manager, and this, in fact, repeels the manager from the firm. To rein him back in, the firm must counteract by raising his salary. Thus, risk aversion changes how the firm jointly considers salary and bonus in relation to each other when designing compensation.

### 4.3 Comparative Statics

The relationship between the optimal bonus with sorting \((b^*)\) and without sorting \((\hat{b})\) is more subtle here. Before, sorting uniformly depressed the bonus coefficient relative to the benchmark without sorting. Now the presence of risk aversion introduces the reliance on \(\gamma\), the quality of the match between the firm and the manager.
Corollary 3  If $\gamma$ is sufficiently small, sorting inflates optimal PPS. If $\gamma$ is sufficiently large, sorting dampens optimal PPS if $c$, $r$, or $\sigma^2$ are sufficiently small.

This gives the condition under which the optimal bonus in the canonical model without sorting ($\hat{b}$) exceeds the optimal bonus in this model with sorting ($b^*$). The proof of Corollary 3 gives the threshold level $\gamma^*$ above which $\hat{b} < b^*$. Intuitively, Corollary 3 shows that the sorting effect dampens the optimal bonus when the complementarity between the firm and the manager is large, and the size of the agency problem is small ($cr\sigma^2$ tracks the magnitude of the agency problem, as increases in any of the individual parameters $c$, $r$, or $\sigma^2$ will exacerbate the agency problem between the manager and the firm). Said differently, the sorting effect inflates the optimal bonus, relative to the canonical model, when the complementarity between the firm and the manager is small. This gives a testable prediction on when the firm will reduce incentives because of the sorting effect.

To interpret further, observe that the complementarity parameter itself affects the agent’s behavior. From his incentive constraint, the agent’s effort increases in $\gamma$ since he works harder when his marginal productivity of labor is high. Furthermore, the optimal $\theta^*$ decreases in $\gamma$ since the firm can hire low types when the quality of the match with each manager is high. Therefore, when $\gamma$ is small, the manager reduces effort and the pool of types attracted to the firm shrinks (since $\theta^*$ rises). To compensate for this, the firm raises incentives. When the complementarity between the firm and manager is large, the forces work in the opposite direction, with a slight adjustment. Now, a large $\gamma$ increases effort and expands the pool of types attracted to the firm (since $\theta^*$ sinks), causing the firm to reduce the bonus if the agency problem is small. When $c$, $r$, or $\sigma^2$ is large, leading to a large agency problem, then this will depress both $b^*$ and $\hat{b}$, but reduce $\hat{b}$ by a larger magnitude, so $\hat{b} < b^*$.

This model under risk aversion provides a theoretical link between the complementarity parameter $\gamma$ and the sorting effect of dampening PPS. Firms and industries where the quality to match is high should experience smaller incentive packages because of sorting. This stands in contrast to prior work, such as Dutta (2008), who finds that optimal PPS under symmetric information always exceeds optimal PPS under asymmetric information (when managerial talent is firm-specific, the relevant case here). Here, a low $\gamma$ offers a low payoff to the manager, and is insufficient to lure him, so the firm must compensate by offering a large bonus. This is why sorting can increase optimal PPS, unlike the rest in Dutta (2008).

Corollary 4  Under risk aversion, there is a negative relationship between risk and incentives ($\frac{\partial b}{\partial \sigma} < 0$) if and only if $s < \bar{u}$.
Suppose that the outside option exceeds the salary, so \( s < \bar{u} \). In this case, sorting does not impact the classic tradeoff between risk and incentives. Even though the contract must now solve the dual problem of attracting talent and motivating effort, a risk-averse agent still dislikes volatility in his output, and this interferes with incentive provision. Because of the manager’s risk-aversion, the firm reduces its incentives to prevent the manager from bearing excessive risk. As the level of this risk (\( \sigma^2 \)) increases, the firm reduces incentives even further. In response to this reduction in incentives, the firm raises \( \theta^* \), thereby attracting better types.

On the other hand, if the salary exceeds the outside option (\( s > \bar{u} \)), this reverses the classic tradeoff between risk and incentives. In this intermediate range of \( s (\bar{u} < s < \bar{u} + \text{the risk premium}) \), the firm still achieves positive sorting (\( \theta^* > 0 \)), but the relationship between risk and incentives is now positive, rather than negative. As output risk (\( \sigma^2 \)) increases, the firm raises incentives. In response to these higher incentives, the firm also increases \( \theta^* \), attracting better types. Thus, the relationship between \( \theta^* \) and \( \sigma^2 \) is still positive, as before. The only remaining case is when salary exceeds the outside option and the risk premium (\( s > \bar{u} + \text{RP} \)). But this case is degenerate, since sorting is negative (\( \theta^* < 0 \)), even though the relationship between risk and incentives is positive.

These results on risk and incentives do not complement the results of Dutta (2008) or Baker Jorgensen (2003). Those papers both find a negative relationship between output risk and incentives and a positive relationship between information risk and incentives (output risk is determined by \( \sigma^2 \), whereas information risk is determined by the variance of \( f \)). Here, I expand on the relationship between output risk and incentives, and show that this relationship depends partly on the choice of salary, which itself is determined endogenously to solve the dual sorting and incentives problem. If risk-aversion is low, then the negative relationship holds. But if risk-aversion is high, an increase in output risk loads disutility onto the manager, which repeals him from the firm. To compensate, the firm increases the bonus to attract this highly risk-averse manager back, so the relationship between risk and incentives is positive. This corollary may help explain why the negative relationship between risk and incentives does not hold under all circumstances (Prendergast, 2002).

**Corollary 5** As output risk (\( \sigma^2 \)) increases, the firm hires better types of the manager (\( \theta^* \) rises).

When output risk increases, regardless of what happens to the choice of bonus, the firm will uniformly hire better types. Since a risk averse manager dislikes volatility in his income,
an increase in output risk will raise his risk premium, thereby making the manager worse off. Only the best types with the highest marginal impact on firm output will then accept the job.

5 Conclusion

Combining the sorting and incentive effects of performance pay is possible exactly because both problems are made simple and tractable. Research has tried for dozens of years to solve the full-blown combined moral hazard and adverse selection problems, and this has not yet yielded strong results. The primary goal of this paper is to understand how sorting affects the firm’s design of optimal compensation. Rather than focusing on inducing appropriate communication like prior work, sorting concerns how the contract will affect the type of manager attracted to the firm. This new logic confirms some existing work (such as incentives raising PPS while selection reduces PPS) but also provides new insights (PPS can increase in the variance on ability; sorting can inflate PPS if manager-firm complementarity is small; the risk-incentives tradeoff can be positive). Future work in this area will expand on the sorting and incentive effects of performance pay to include repeated interactions, retention effects, subjective bonuses, and multiple managers.

6 Appendix: Proofs of Propositions

Proof of Proposition 1: Define the participation set for a contract \((s, b)\) to be

\[
P(s, b) = \{\theta \in \Theta | E[w|\theta] \geq \bar{u}\}.
\]

For a given contract \((s, b)\) the expected profits of the firm is

\[
E\pi(s, b) = \int_{P(s, b)} E[\pi|\theta] f(\theta) d\theta,
\]

where \(E[\pi|\theta] = \gamma \theta (1 - b) - s - (k - m\theta)^+\). Now if \(b > 1\), \(E[\pi|\theta] < 0\). Thus the firm will never set \(b > 1\). If \(b \leq 1\), \(E[\pi|\theta]\) increases in \(\theta\), and thus is monotonic in \(\theta\).

Consider a contract \((s, b) = (\bar{u}, 0)\). Observe that \(P(\bar{u}, 0) = \Theta\) Call this contract \((\bar{u}, 0)\) the fallback contract, when it pays only a flat salary, no bonus, and attracts the entire population.
Suppose $b < 0$. Then either $s < \bar{u}$ or $s \geq \bar{u}$. If $s < \bar{u}$, then $P(s, b) = \emptyset \subset \Theta = P(\bar{u}, 0)$. By monotonicity of expected profit, $E\pi(s, b) < E\pi(s, 0)$, and thus the fallback contract generates more profit for the firm. Alternatively if $s \geq \bar{u}$, $P(s, b) = [0, \theta^*] \subset \Theta = P(s, 0)$. By monotonicity of expected profit, $E\pi(s, b) < E\pi(s, 0)$ and the fallback contract again dominates. In either case, the firm will never select $b < 0$, since it can always do better with the fallback contract.

Now suppose $b \geq 0$. If $s > \bar{u}$, then $P(s, b) = \Theta$. But the firm could always do better by lowering the salary to $\bar{u}$, which does not affect participation, but increases profits.

Thus, $s \leq \bar{u}$. Therefore, the feasibility set is $FS \equiv \{(s, b) | s \leq \bar{u}, b \geq 0\}$. Define the efficient set to be all contracts that implement efficient sorting:

$$ES \equiv \{(s, b) | \theta^* = \theta^{FB}, s \leq \bar{u}, b \geq 0\} \subset FS.$$  

Observe that every contract in the efficient set generates expected surplus less expected wage payments. Because every such contract implements $\theta^{FB}$, the contract parameters do not change the variable of integration above, and only lower the expected wage payment. Thus maximizing expected surplus over the efficient set is equivalent to minimizing the expected wage payments:

$$\arg \max_{ES} E\pi(s, b) = \arg \min_{ES} \int_{\theta^{FB}} E[w|\theta] f(\theta) d\theta.$$  

Now for every contract in the efficient set, $s = \bar{u} - \gamma b \theta^{FB}$ and $b \geq 0$. Therefore,

$$E[w|\theta] = s + b \gamma \theta = \bar{u} + b \gamma (\theta - \theta^{FB})$$

is a linear and nondecreasing function in $b$ for all $\theta > \theta^{FB}$. Minimizing this function is equivalent to setting $b$ as low as possible. It remains to show the fallback contract ($b = 0$) is not optimal.

Let $\eta > 0$. Take $b = \eta$ and $s = \bar{u} - k \gamma \eta / (m + \gamma)$. Observe that $\theta^*(s, b) = \theta^{FB}$.

It is straightforward (but tedious) to show that

$$\lim_{\eta \downarrow 0} E\pi(s, b) > E\pi(\bar{u}, 0).$$

Thus, the optimal contract is $s = \bar{u} - k \gamma \eta / (m + \gamma) \longrightarrow \bar{u}$ and $b = \eta \longrightarrow 0$ is optimal, as it is efficient and dominates the fallback contract. Thus $s \approx \bar{u}$ and $b \approx 0$.  

$\blacksquare$
Proof of Proposition 2: The firm chooses salary $s$ and bonus $b$ to solve

$$\max_{s,b} \int_{\theta^*(s,b)}^{\infty} E[\pi|\theta] f(\theta) d\theta$$

Using Leibnitz’s Rule, the first order conditions are

$$E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial s} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta$$

$$E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial b} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta$$

Combining these two equations leads to the equilibrium condition

$$\frac{\partial \theta^*}{\partial b} \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta = \frac{\partial \theta^*}{\partial s} \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta. \quad (25)$$

Now, the ex-post expected profit $E[\pi|\theta] = E[x|\theta] - E[w|\theta] - (k - m\theta)^+ = \gamma \theta e(1 - b) - s - (k - m\theta)^+$. Plugging in the incentive constraint $e = b\gamma\theta/c$,

$$E[\pi|\theta] = \frac{(\gamma\theta)^2}{c} b(1 - b) - s - (k - m\theta)^+. \quad (26)$$

The derivatives of expected profit with respect to salary and bonus are, respectively:

$$\frac{\partial E[\pi|\theta]}{\partial s} = -1 \quad \text{and} \quad \frac{\partial E[\pi|\theta]}{\partial b} = \frac{(\gamma\theta)^2}{c} (1 - 2b) \quad (27)$$

Recall that $\theta^* = \sqrt{2c(\bar{u} - s)/b\gamma}$. The partial derivatives are:

$$\frac{\partial \theta^*}{\partial s} = \frac{-c}{(b\gamma)^2 \theta^*} \quad \text{and} \quad \frac{\partial \theta^*}{\partial b} = -\frac{\theta^*}{b} \quad (28)$$

Combining these gives

$$\frac{\partial \theta^*}{\partial b} = \frac{\partial \theta^*}{\partial s} \cdot \frac{b\theta^2 \gamma^2}{c} \quad (29)$$

Combining (27) with the equilibrium condition (25) gives

$$-\frac{\partial \theta^*}{\partial b} \int_{\theta^*}^{\infty} \frac{(\gamma\theta)^2}{c} (1 - 2b) f(\theta) d\theta. \quad -\frac{\partial \theta^*}{\partial b} \int_{\theta^*}^{\infty} \frac{(\gamma\theta)^2}{c} (1 - 2b) f(\theta) d\theta. \quad (25)$$

Combining with (29) gives

$$\left(-\frac{b\theta^2 \gamma^2}{c}\right) \cdot (1 - F(\theta^*)) = \int_{\theta^*}^{\infty} \frac{(\gamma\theta)^2}{c} (1 - 2b) f(\theta) d\theta.$$
Rearranging and simplifying,

\[ b = \left( 2 - \frac{\theta^*}{E[\theta^2|\theta > \theta^*]} \right)^{-1}. \]

Inserting (27) into the first order condition for \( s \) gives

\[ E[\pi|\theta^*] f(\theta^*) \frac{-\partial \theta^*}{\partial s} = 1 - F(\theta^*). \]

Combining with (26), (28), and simplifying, this yields

\[ s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*) b}{f(\theta^*)} \right] \frac{b \gamma^2 \theta^* c}{c} - (k - m\theta)^+. \]

Proof of Proposition 3: The firm chooses salary \( s \) and bonus \( b \) to solve

\[ \max_{s,b} \int_{\theta^*(s,b)}^{\infty} E[\pi|\theta] f(\theta) d\theta \]

Using Leibnitz’s Rule, the first order conditions are

\[ E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial s} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta \]

\[ E[\pi|\theta^*] f(\theta^*) \frac{\partial \theta^*}{\partial b} = \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta \]

Combining these two equations leads to the equilibrium condition

\[ \frac{\partial \theta^*}{\partial b} \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial s} f(\theta) d\theta = \frac{\partial \theta^*}{\partial s} \int_{\theta^*}^{\infty} \frac{\partial E[\pi|\theta]}{\partial b} f(\theta) d\theta \]  

\[ (30) \]

Now, \( E[\pi|\theta] = E[x|\theta] - E[w|\theta] = \gamma \theta e(1 - b) - s - (k - m\theta)^+ \). Plugging in (IC),

\[ E[\pi|\theta] = \frac{\gamma \theta e e(1 - b) - s - (k - m\theta)^+}{c} \]  

\[ (31) \]

The derivatives of expected profit with respect to salary and bonus are, respectively:

\[ \frac{\partial E[\pi|\theta]}{\partial s} = -1 \text{ and } \frac{\partial E[\pi|\theta]}{\partial b} = \frac{\gamma^2 \theta^2 c}{c} (1 - 2\theta) \]

\[ (32) \]

Let \( X = 2c(\bar{u} - s) + c r b^2 \sigma^2 \). Observe that
\[ \theta^* = \sqrt{X/b \gamma}. \]  

(33)

The partial derivatives of \(X\) are

\[ \frac{\partial X}{\partial s} = -2c \text{ and } \frac{\partial X}{\partial b} = 2crb \sigma^2 = -\frac{\partial X}{\partial s}rb \sigma^2. \]  

(34)

Differentiating (33) gives

\[ \frac{\partial \theta^*}{\partial s} = \frac{1}{2b \gamma \sqrt{X}} \left( \frac{\partial X}{\partial s} \right) \]  

(35)

\[ \frac{\partial \theta^*}{\partial b} = \frac{1}{2b \gamma \sqrt{X}} \left( \frac{\partial X}{\partial b} \right) \frac{\sqrt{X}}{b^2 \gamma}. \]  

(36)

Combining (36) with (34) gives

\[ \frac{\partial \theta^*}{\partial b} = \frac{1}{2b \gamma \sqrt{X}} \left( \frac{\partial X}{\partial s} \right) (-rb \sigma^2) - \frac{\sqrt{X}}{b^2 \gamma}. \]

Substituting in (35) and (33),

\[ \frac{\partial \theta^*}{\partial b} = -r b \sigma^2 \frac{\partial \theta^*}{\partial s} - \frac{\theta^*}{b} \]  

(37)

Now, combining (33), (34), (35) gives

\[ \frac{\partial \theta^*}{\partial s} = -\frac{c}{b \gamma \sqrt{X}} = -\frac{c}{(b \gamma)^2 \theta^*} \]  

(38)

Combining (32) with the equilibrium condition (30) gives

\[ -\frac{\partial \theta^*}{\partial b} (1 - F(\theta^*)) \frac{\partial \theta^*}{\partial s} \int_{\theta^*}^{\infty} \frac{\gamma^2 \theta^2}{c} (1 - 2b) f(\theta) d\theta. \]

Combining this with (37) gives

\[ \left( r \sigma^2 b \frac{\partial \theta^*}{\partial s} + \frac{\theta^*}{b} \right) (1 - F(\theta^*)) = \left( \frac{\partial \theta^*}{\partial s} \right) \int_{\theta^*}^{\infty} \frac{\gamma^2 \theta^2}{c} (1 - 2b) f(\theta) d\theta. \]

Collecting terms,

\[ r b \sigma^2 \frac{\partial \theta^*}{\partial s} + \frac{\theta^*}{b} = \frac{\partial \theta^*}{\partial s} \frac{\gamma^2 (1 - 2b)}{c} E \]

Where \( E = E[\theta^2 | \theta > \theta^*] \). Substituting in (38), rearranging, and simplifying,
\[ b = \left( 2 + \frac{cr\sigma^2 - \theta^*\gamma^2}{\gamma^2 E} \right)^{-1}. \]

Inserting (32) into the first order condition for \( s \) gives

\[ E[\pi|\theta^*]f(\theta^*)\left(\frac{-\partial \theta^*}{\partial s}\right) = 1 - F(\theta^*). \]

Combining with (31) and (38), and solving for \( s \),

\[ s = \left[ \theta^*(1 - b) - \frac{1 - F(\theta^*)}{f(\theta^*)}b \right] \frac{b\theta^*\gamma^2}{c} - (k - m\theta)^+. \]

References


