**Momentum, Motion Picture Profit, and the Curse of the Superstar***

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# Momentum, Motion Picture Profit, and the Curse of the Superstar

*Motion picture profit follows a stable Paretian distribution with finite mean and infinite variance. The Paretian tails of the distribution are evidence of bandwagon effects. The infinite variance and long tails account for the "nobody knows" principle and the skewed shape accounts for the curse of the superstar (if superstars are paid their expected contribution to profit, their movies almost surely will lose money). These results are used to explain the nature of risk and the elusiveness of profit in Hollywood as well as features of contracts involving A list artists.*

## 1 Introduction

Motion picture box office revenues are at an all-time high, yet the rate of return in the industry is a meager 0 to 4 percent. Half the theater screens in the United States are in bankruptcy and only 18% of movies earn a profit. Yet, in spite of this gloomy picture, the fees stars command have continued to rise and have reached extraordinary levels. Top directors earn from $6 to $10 million per picture and a handful of A+ actors earn $20 million or more per picture. And the number of superstars continues to grow. How can an industry that is so risky earn such a low rate of return? And why does it pay such a high proportion of its profit to a few superstars?

Robert Frank and Philip Cook (1996) suggest that high professional salaries are the result of the spread of markets in which the value of production depends primarily on a handful of top players when millions have a small interest in the winner's performance. Sherwin Rosen (1981) attributes the pay of stars to small differences in talent and a distribution system that acts as a gateway to talent. Both explanations have a "winner take all" property that relies on a process similar to a ranking tournament that magnifies small differences into large ones. Studio executives have a similar explanation; they argue that superstars can make a movie "open". In the clamor to gain attention in a crowded market, they argue, a movie has to open big and a star can attract the media attention and the large number of opening screens needed to gain a high rank in the box office tournament. A big opening and an early lead at the box office, the industry seems to believe, will attract attention and may start a bandwagon that will translate that lead into more demand. The industry believes in momentum.

Momentum is a dynamical process through which small advantages eventually become large ones. If people choose movies, entertainers, or products in part by observing the choices of others, then an early lead in the rank tournament would eventually be leveraged into a very large advantage later.[1] Momentum has the property that the growth in demand depends on the level of demand already attained so that leaders become more dominant. When demand has the momentum property then small initial differences become large ones and the prize distribution will be concentrated on the top ranked competitors.

A dynamical explanation of superstars should possess these three features: the momentum characteristic, a skewed distribution of outcomes (the "winner-take-all" principle), and the distribution should be stable in form to changes in initial conditions. The stable Paretian distribution meets these requirements.

The stable Paretian distribution is known to be in the basin of attraction of dynamical processes that possess the "momentum" characteristic in which initial differences are magnified over time.[2] This means that any dynamical process with the momentum characteristic eventually converges to a distribution of outcomes with a Paretian tail. With its characteristic high peak, long upper tail and skew, the

[1] The leveraging of an early choices into a large market share later goes under different names in the economics literature: network effects, bandwagons, path-dependence, momentum, and information cascades all have a momentum characteristic in which the growth in demand depends on demand already revealed.

[2] Ijiri and Simon (1977) investigated dynamic processes that possessed the momentum characteristic where growth depends on size. They showed that these processes converged on distributions with Paretian tails and that this result is not sensitive to changes in initial conditions. A more general proof is by Hill, 1974 where it is shown that a Bose-Einstein process has the momentum property and converges to a stable Paretian distribution. De Vany and Walls, 1997 showed that motion picture revenue dynamics follow the Bose-Einstein process. De Vany and Lee show that an information cascade that is a mixture of "follow the leader" and "word of mouth" information is a Bose-Einstein process and generates Paretian tails of box office revenue.

Pareto distribution captures the winner-take-all nature of outcomes which Frank and Cook, and Rosen allude to and which is found in professional salaries, motion picture revenues (De Vany and Walls, 1996, 1997), and patents (Scherer, 2000). The Paretian distribution also is stable in form to changes in initial conditions and so has the robust character required to capture a phenomenon that is universal. Thus, momentum processes and skew distributions with Paretian tails seem to be at the heart of the superstar phenomenon.

But, if bandwagon or momentum effects are to explain the superstar phenomenon then several important questions need to be answered. First, is there evidence of stronger momentum for products involving superstars than for other products? If momentum occurs unpredictably and cannot be tied to the participation of superstars, then momentum fails to be an explanation for the superstar phenomenon. We show that the upper tail of the superstar movie profit distribution shows strong evidence of momentum. Second, superstar compensation is more often tied to outcomes than is compensation of lesser stars. For example, only "A-list" artists receive "back end" participation in motion picture profit.[3] What explains the tying of superstar compensation to outcomes? We show that the "nobody knows" principle (Goldman, Caves, De Vany and Walls,) is involved and hinges on the long upper tail of the probability distribution of profit. Third, while momentum processes may affect revenue, little is known about how they affect profit. Are superstar products more profitable than others? We show that if superstars receive their expected contribution to profit, then their movies almost surely will lose money—this is the curse of the superstar.[4]

This paper shows that the "nobody knows" principle, the superstar curse, and the form of artist contracts all follow from our finding that the correct statistical model of motion picture profit is the $\alpha$-stable Paretian distribution with a finite mean and an infinite variance. We also show that the "nobody knows" principle and

[3] See Acheson and Maule (1995) for a discussion of aspects of the A-list in the context of uncertainty regarding film success.

[4] In De Vany and Walls (2001) we show that there is a similar illusion of expectation in the ranking distributions of G-, PG-, PG13-, and R-rated movies.

an investor's form of the superstar curse can reconcile the motion picture industry's low rate of return in the face of its extreme risks.

## 2 Motion Picture Profit

In spite of the high profit that individual movies earn, most movies are unprofitable. Seventy-eight percent of movies lose money and only twenty-two percent are profitable. Profit is unevenly distributed among those movies that are profitable: just 35% of *profitable* movies earn 80% of total profit.[5] Losses are more evenly distributed. Among *unprofitable* movies, a more moderate 50% accounted for 80% of all losses.[6] The most dramatic statistic is this: just 6.3% of *all* movies earned 80% of the total profit earned by Hollywood over the past decade.[7] By these measures, the movies is clearly a "winner-take-all" business.

Table 1 reports attributes and quantiles of the distribution of movie profit.[8] The sample mean profit is negative for all movies and for movies with and without stars. The mean loss is slightly greater for non-star movies than for star movies ($-3.595$ million versus $-2.083$ million). The standard deviation is large, but is over twice as large in star movies than in non-star movies; this reflects the heavy tails of star movies. The percentiles reveal that a movie must land above the 75th percentile to reach a positive profit. More interesting are the differences in the percentiles in the three groups of movies. Until somewhere between the 50th and 75th percentile, star movies show larger loses at each percentile than non-star movies. From the 75th percentile and higher, star movies show higher profit at each percentile than non-star movies, more evidence of the importance of extreme outcomes to this segment of movies.

[5] The Gini coefficient of profit, a measure of inequality, is 0.608.

[6] The Gini coefficient of inequality of losses, 0.461, is less than the coefficient of profit.

[7] This is the estimate from our sample of 2015 movies released in the North American theatrical market from 1984 through 1996. This concentration of profit is not unlike stock market returns, where a similar 6% of trading periods earn 80% of returns.

[8] We analyze a sample of 2015 movies exhibited in North America between 1984–1996 inclusive. The data were obtained from AC Nielson EDI. The composition of the sample—ratings, genres, revenues, budgets, star presence, etc.—is given in detail in De Vany and Walls (1999).

The skewness and kurtosis statistics indicate that the profit distribution is asymmetric. The data depart from normality and this is plain in Figure 1 where the empirical density is overlaid with the fitted normal density. In addition to this visual evidence, statistical tests clearly reject the normal distribution as a model of profit.[9]

## 3 The Stable Distribution

De Vany and Walls (1999) showed that the upper and lower tails of motion picture profit are asymptotically Pareto-distributed. The existence of Paretian tails is a clue that warrants further investigation to determine if the central part of the motion picture profit distribution conforms to stable distribution theory. If the stable model is a correct model of the data, then the proof by Ijiri and Simon (1977) that momentum processes converge on distributions with Paretian tails and its generalization by Mandelbrot and others (Lévy, Gnedenko, Feller) that general momentum (subordinated) processes converge on stable distributions can then be used to deduce that motion picture profit results from a momentum process.[10] The theoretical reason to think a stable distribution might apply to motion pictures is because Mandelbrot (1963) showed that a process that is stable under choice, mixture, and aggregation converges in distribution to the stable distribution. The limits of a stable process form a generalized central limit theorem which states that the sums of independent and identically distributed terms is stable.[11]

Empirically it has been found that many observed quantities that are the sums of many small terms—for example, stock returns—exhibit heavy tails and skewness that are inconsistent with the normal distribution. Lévy showed that there is a class of distribution functions which follow the asymptotic form of the law of

[9]Skewness-kurtosis and Shapiro-Francia tests both reject the null hypothesis of normality at significance levels of practically zero.

[10]The early papers by Mandelbrot (1963) and Fama (1965), and later by McCulloch (1996) apply the stable distribution to financial and futures markets. See also the volume edited by Adler et al. (1998) for applications of the stable distribution in a wide variety of physical and social settings.

[11]See Samorodnitsky and Taqqu (1994).

Pareto which Mandelbrot defined as

$$1 - F_X(x) \sim (\frac{x}{k})^{-\alpha} \quad x \to \infty \tag{1}$$

Such distributions are characterized by the fact that $0 < \alpha < 2$ and they have infinite variance. The Lévy is a generalization of the normal distribution when the variance is infinite. Mandelbrot (1963) found that the distribution of cotton price changes is approximated by the Lévy distribution. Fama (1963) described an information process (similar to Bose-Einstein information updating) that could lead to a Lévy stable distribution. Both S&P 500 stock index and NYSE composite index returns are well-fitted by a Lévy distribution (Mantegna and Stanley, 1995, and Soloman and Levy, 1998, respectively).

The stable distribution's ability to explain the empirical regularities found in data and its statistical foundation on the most general form of central limit theorem make it a natural model of motion picture profit.[12] The stable distribution also nests the normal, Cauchy, and Lévy distributions as special cases which permits it to be tested against them as a model of the data.

There are many parameterizations of the stable distribution. We shall use the parameterization: $X \sim \mathbf{S}(\alpha, \beta, \gamma, \delta; 0)$ where the characteristic function of $X$ is given by

$$\mathrm{E}\exp(itX) = \begin{cases} \exp\{-\gamma^\alpha |t|^\alpha [1 + i\beta(\tan\frac{\pi\alpha}{2})(\text{sign } t)((\gamma|t|)^{1-\alpha} - 1)] + i\delta t\} & \text{if } \alpha \neq 1 \\ \exp\{-\gamma|t|[1 + i\beta\frac{2}{\pi}(\text{sign } t)(\ln|t| + \ln\gamma)] + i\delta t\} & \text{if } \alpha = 1 \end{cases} \tag{2}$$

This parameterization is a variation of that given by Zolotarev (1986) and it is convenient in several respects: 1) the interpretation of the parameters is clear; 2) the parameterization is a location and scale family so that if $X \sim \mathbf{S}(\alpha, \beta, \gamma, \delta; 0)$ then for $a \neq 0$ it follows that $aX + b \sim \mathbf{S}(\alpha, (\text{sign } a)\beta, |a|\gamma, a\delta + b; 0)$; and 3) the characteristic functions are jointly continuous in all four parameters.

The characteristic exponent $\alpha$ is a measure of the probability weight in the upper and lower tails of the distribution; it has a range of $0 < \alpha \leq 2$ and the

[12] Other distributions, such as the student-$t$ can be used to model heavy tails, but they are ad hoc in that their use cannot appeal to the generalized central limit theorem.

variance of the stable distribution is infinite when $\alpha < 2$. The skewness coefficient $\beta$ is a measure of the asymmetry of the distribution; it has a range of $-1 \leq \beta \leq 1$, where the sign indicates the direction of skewness. The scale parameter $\gamma$ must be positive. It expands or contracts the distribution in a non-linear way about the location parameter $\delta$ which indicates the center of the distribution.

The $\alpha$-stable distribution is the limiting distribution of all stable processes so that it contains the other well-known stable distributions as special cases. Its tails are Paretian and the variance is infinite when $\alpha < 2$. Its mean need not exist for values of $\alpha < 1$. The normal (Gaussian) distribution is a special case of the $\alpha$-stable distribution when $\alpha = 2$. The $\alpha$-stable distribution becomes the Cauchy distribution when $\alpha = 1$ and $\beta = 0$, and the Lévy distribution when $\alpha = 0.5$ and $\beta = \pm 1$. As the characteristic exponent $\alpha$ approaches 2, the skewness coefficient $\beta$ has less impact on the shape of the distribution and when $\alpha = 2$ the distribution has only two parameters, location and scale, which correspond to the familiar mean and variance of the normal distribution.

# 4 Stable Estimation and Diagnostics

The stable distribution parameters are estimated by the method of maximum likelihood.[13] Estimates of the stable parameters for all the movies in the sample are reported in the upper panel of Table 2. Log-likelihood values for the general stable distribution as well as for the symmetric stable ($\beta = 0$) and the normal ($\alpha = 2, \beta = 0$) distributions are shown. The test clearly rejects the null hypothesis of normality at the 1% marginal significance level in favor of the symmetric stable distribution with a likelihood ratio test statistic of $\chi^2_{df=1} = 1151$. Symmetry is not rejected at the 5% level but it is rejected at the 8% level where the likelihood ratio test statistic is $\chi^2_{df=1} = 0.82$. The distribution is stable and there is evidence of a lack of symmetry.

Diagnostic tests suggested by Nolan (1999) were used to detect a departure

[13] Parameter estimation and computation of the theoretical density function were performed using John Nolan's (1998) STABLE software package.

from stability. First, Figure 1 shows the fitted stable density function corresponding to the maximum likelihood estimates. For comparison, the fitted normal density function where $\alpha$ is restricted to equal 2 and $\beta$ is restricted to equal 0 is also shown in the plot. The empirical density function corresponding to the data is also shown in the figure.[14] It is clear that the stable distribution is a better approximation to the empirical distribution than is the normal distribution. Second, the cumulative distribution functions of the fitted stable are plotted with the data in the probability plot shown in Figure 2. The fit is good, even for the extreme values.[15] Third, the quantile-quantile plots shown in Figure 3 show that the data are consistent with the random variation of a stable distribution and fall with the corresponding 95% confidence bands.[16] The PP-plot, QQ-plot, and comparison of empirical and fitted densities support the $\alpha$-stable distribution over the normal and other stable distributions. Indeed, the fits are remarkable and even the deviations at the extreme values are predicted by the model and fall within its confidence bands.

## 5 Probability under the Stable and Normal Models

If one were only to glance at Figure 1 the data would appear to be nearly normal. As a consequence, many motion picture analysts and studio executives might well use a normal model when forecasting profit. But, the differences between the normal and the stable distributions are profound and the consequences of using the wrong statistical model are large. For example, the probability that a movie is profitable at all is 0.30 according to the stable estimates, but 0.39 according to the normal distribution, a sizable error. The magnitude of this error might be enough to explain the low rate of return in Hollywood if film investors were to forecast returns using a normal model. But, the differences between the models go far deeper

[14] The empirical density function is calculated using the Gaussian kernel with the width parameter equal to $2(\text{inter-quartile range})n^{-1/3}$.

[15] Another diagnostic for detecting variation near the extreme values is the variance-stabilized PP-plot, where a transformation is applied to make the variance in the PP-plot uniform. The variance-stabilized PP-plot is nearly the same as the PP-plot shown in Figure 2 so we have not included it here.

[16] See Nolan (1999) for the formulas of the pointwise confidence bands.

than this.

The low values of the tail parameter $\alpha$ shown in Table 2 indicate that both the symmetric and the general $\alpha$-stable distributions place far more probability mass in the tails than the normal distribution. The tail weight parameter $\alpha$ has values around 1.6 for the stable distributions. This value is consistent with the value found for box office revenue (De Vany and Walls, 1999) and suggests that the upper tail of profit is driven by box office momentum. These values are less than the $\alpha = 2$ value that would hold if profit followed the normal distribution. When $\alpha$ is less 2 profit has infinite variance. The variance one would estimate from a normal distribution is finite. Thus, an investor would have more confidence than is justified in profit forecasts were she to use a normal model instead of the correct $\alpha$-stable model.

The $\alpha$-stable distribution also puts a lot more probability mass than the normal distribution on extreme events. To illustrate this point, consider the probabilities of observing the extreme outcomes in the data. *Home Alone* was the most profitable movie in the sample at nearly \$93 million (1982–4 dollars) and *Waterworld* was the least profitable at $-\$85$ million in real dollars. According to the stable distribution estimates, the chance of observing a hit as profitable or more profitable than *Home Alone* is nearly 0.83% and the probability of observing a bomb as unprofitable or more unprofitable than *Waterworld* is 0.45%. According to the normal distribution one would incorrectly calculate the respective probabilities to be essentially zero ($2.817 \times 10^{-16}$ and $3.413 \times 10^{-12}$). The normal model fails to predict the probabilities of extreme outcomes which account for the overwhelming share of loss and profit.

To see this more clearly, consider the shape of the tails of the normal and $\alpha$-stable distributions. An expanded view of the upper tails of each distribution is shown in Figure 4. For profit in excess of \$30 million, the stable density has many times more probability mass than the normal density. The normal distribution is a poor model of the extreme events that drive profit and loss.

The $\alpha$-stable distribution is also a better model of the central part of the distribution. First, consider the location parameter $\delta$. Table 2 contains the estimates of $\delta$ for the normal distribution and the symmetric and general $\alpha$-stable distributions. The location parameter of the normal distribution is larger than it is for either of the symmetric stable models. In all cases, the distribution is located at a negative value. The central or typical outcome for movies is a loss of about \$4 million. The normal mis-estimates this typical outcome as a \$3.35 million loss.

The probability peak of the $\alpha$-stable distribution is higher and thinner than the peak of the normal distribution. The peak of the distribution corresponds to the most probable event. Thus, the stable distribution shows that a loss is more probable than the normal distribution suggests. The central probability mass of the $\alpha$-stable distribution falls off far more rapidly from the peak than the normal distribution and then shoots into the far upper and lower tails. This is an indication of the fragility of the most probable event in estimating the expected outcome.

The scale parameter of the normal model is more than double that of the $\alpha$-stable model. The scale parameter is some indication of the natural scale or size of events. By constraining the estimate of $\alpha$ to be equal to 2, the estimates of the normal distribution give larger values than the stable distribution of the scale parameter. As a consequence of this positive bias in scale, the normal model implies that the usual size of outcomes is larger than they really are. The positively biased scale suggests there is more mass in the central part of the distribution than there actually is. In the stable model, the variance is infinite although the scale parameter is less than half of the improperly fit normal model.

We can conclude that estimating the profit distribution under the constraint of normality (namely that $\alpha = 2$) leads to significant changes in location and scale parameters and seriously biased estimates of probabilities at all points in the support of the distribution.

## 6 Conditional $\alpha$-Stable Analysis and Superstars

To address the issue of superstars, estimates of the parameters of the normal, symmetric stable, and general stable models were made for movies that featured a superstar from the "A-list" of the 100 top stars in Hollywood and for movies that did not feature one of these stars. These estimates are reported in the two lower panels of Table 2.

Tests reject the hypothesis that the parameters are equal for the both groups of movies. Tests for superstar movies, reject normality in favor of the stable distribution. They also reject symmetric stability in favor of asymmetric stability. Thus, the profit distribution for superstar movies is not normal and it is not symmetric; it is a general, asymmetric $\alpha$-stable distribution.

The profit distribution for movies that do not feature an A-list star is $\alpha$-stable symmetric. Tests reject normality in favor of $\alpha$ stability. The general $\alpha$-stable estimates indicate that the profit distribution for non-star movies is skewed toward losses, but they do not reject symmetric stability in favor of the general stable distribution.[17] We conclude that the profit distribution of non-star movies is stable, symmetric, and has infinite variance. These subtle distinctions between the distributions of star and non-star movies are important.

Figure 6 shows the extreme upper tails of the probability density functions of superstar and other movies. These tails are Paretian (power laws), a confirmation of the momentum principle. If the distribution were normal, the tails would decline at an exponential rate and this would be inconsistent with the momentum principle. Movies with stars have much higher probability mass in the upper Paretian tail. The probability of a movie without a star having a profit greater than $20 million is 0.02 while for a movie with a star the probability is .10. For movies with stars, the probability of earning a positive profit is 0.37. For movies without stars, the probability of earning a positive profit is 0.23.

Table 3 tabulates the upper and lower tail probabilities of profit for superstar

[17] The $\chi^2_{df=1}$ test statistic is 3.04 and the upper 5th percentage point of the $\chi^2$ distribution with one degree of freedom is 3.84.

and other movies in the interval $[-100, 100]$. As is evident from the fitted density functions, stars place much more mass in the upper tail of the profit distribution. The tables reveal what is not clear in the figures, which is that the probability of extreme catastrophes, say losses in excess of \$95 million, is higher for movies *without* stars. Putting a star in a movie places more mass on the upper tail and less on the lower tail.

Expected profit can be calculated from the estimates of the stable probability densities. The results in Table 4 show that expected profit is positive for star movies and negative for non-star movies. These values are consistent with the fact that probability is skewed to the positive tail in superstar movies and to the negative tail for others.

The longer and heavier upper tail of the distribution of superstar movies supports the hypothesis that momentum is stronger in superstar than in other movies. The Paretian form of the tails is strong evidence that momentum drives the extreme events located there. The upper tail of the superstar profit distribution has the same shape as the upper tail of the box office revenue distribution; both distributions share the same value of the tail weight parameter $\alpha$. These facts suggest that motion picture profit outcomes in the upper tail of the distribution are driven there by box office momentum.

## 7 The Curse of the Superstar

The superstar curse originates from the differences between the average, expected, and most probable profit for superstar movies. The difference between average and expected value biases judgment to rely too heavily on recent information. The curse of the superstar has its origin in the skew of the distribution that causes the expected value to be substantially greater than the most likely outcome, hence if a studio pays the superstar the expected profit of her movie, the movie will, with high probability, lose money.

When a stable process generates profit, the sample average and theoretical expected value differ randomly from one another. Average profit is not stationary when profit follows a stable distribution; Figure 7 shows the instability of the sample average profit (the x-axis is ordered by release date so the graph also demonstrates the falling profitability of the industry from 1984 to 1996). The reason average profit does not converge is that it depends on the extreme events that occur in an $\alpha$-stable process, where changes in profit come from a distribution with infinite variance. Consequently, the sample average is (ironically) unstable in an $\alpha$-stable process and is a poor predictor of the expectation. This is another major difference between a normal process and an $\alpha$-stable process. If profit were normally distributed, the sample average would converge to the theoretical mean. Because large events loom large and are not improbable in an $\alpha$-stable process, the sample average is an unreliable estimate of the theoretical mean or expectation. The expected value need not even exist because large events may completely dominate the average.

For superstar movies, the difference between the sample average and the expected value is large; for non-star movies the difference is not great. The tails of superstar movies are so long and skewed that expected profit is a positive $7.684 million while average profit is a negative -$2.083 million.

But, the real reason for the curse of the superstar is the big difference between the most probable event—the small interval of profit in the support that is of maximum likelihood—and the expected value. How can the expected and most likely profit be so different as to mislead a studio executive? If the distribution of profit were normal, these would approximately equal one another. But, the shape of the $\alpha$-stable distribution—skewed, asymmetric, and “heavy tailed”—makes all three of these values, the average, expected, and most likely profit, different from one another.

Table 3 contains the values of the average, expected, and most likely profit for superstar and other movies. Note that the expected profit of non-star movies is negative and less than the average and most probable profit, which are nearly equal and also negative. And note that the relationship is different for superstar movies where the expected profit is positive and far exceeds (by $9 million) the average profit, which is negative. Because of the heavy tails, we must numerically evaluate the superstar distribution well into the tails before the expected profit becomes positive.[18] This shows that much of the expected profit of superstar movies comes from the tail of the distribution above $100 million.[19]

More striking is the very large difference ($15.1 million) between the $7.6 million expected profit and the $-\$7.5$ million most likely profit. This difference arises because the superstar distribution is highly asymmetric and positively skewed. Unlike nonsuperstar movies, where the expected value introduces a note of caution, with superstar movies the opposite is true. The expected value is positive and greatly exceeds both the average and most likely profit.

An executive faced with a decision to "greenlight" a nonsuperstar movie is not apt to be misled by much when comparing the average, expected, and most likely outcomes. The expected profit is negative and less than the average profit, and this difference adds a measure of caution to the executive's evaluation of the sample average as a forecast. Because most movies lose money, if the movie is to be greenlighted, it must be because the studio believes it has something that might carry it into the upper tail, which, being Paretian, contains non-trivial probabilities of extremely high profit. Because the non-star distribution is symmetric, the most likely profit is not too different from the expected profit, so expected profit does not bias judgment relative to the most likely outcome. The symmetry of the non-star profit distribution prevents the decision maker from systematic error, though the harsh reality that most movies lose money is not easily overlooked.

A studio looking at a superstar movie is more apt to be misled by the differences between the average, expected, and most likely profit and this is the source of the

[18] Just using the range from $-\$100$ to $100 million gives a negative estimate of expected profit. One must calculate over the whole range of outcomes. For our numerical integration, we used a range from $-\$5$ billion to $5 billion.

[19] Although, the positive tail eventually dominates, the moderately unprofitable movies in the range of $-\$10$ to $-\$30$ million bring an expected loss of about -$6 million. Movies in this range correspond to the overpopulated category of money losing R-rated movies featuring stars that De Vany and Walls (2001) discovered.

curse of the superstar. The average profit is a loss of $2 million, but that will change in value and never settle down. It is not really relevant to evaluating the prospects of the superstar movie. The expected profit of a superstar movie is positive at $7.684 million, over $9 million greater than the sample average, though in another time period the sample average may differ in another direction by more or less. This large difference between the average and expected values is another indication that the average profit is a poor estimator of the expected profit. The important difference that biases judgment is the difference between the expected and most likely profit. The most likely profit is a loss of $7.5 million while the expected profit is a positive $7.684 million, a difference slightly more than $15 million. This value is economically significant, being nearly eight times the average profit (at its current value). It is the positive skew of the superstar distribution that makes the expected profit so much larger than the most likely profit.

How does the superstar curse spread its spell over a studio? A simple calculation makes the point: if a superstar is paid the expected profit of her movie, she would receive about $7.5 million. The most probable outcome for the movie would be a loss of $7.5 million. So, the studio is looking at a probable loss of $15 million when its expected profit is zero because the superstar has extracted all the expected rent.[20] If the movie earns the sample average superstar loss of $2.0, then paying the star a fee equal to expected profit of $7.5 million would lead to a total loss of $9.5 million.

In order to earn a profit when the star is paid a salary equal to the expected profit, the movie has to hit a profit of $10.0 million or greater, an event whose probability, according to Table 3, is about 0.19. So, the movie has an 81% chance of losing money. This is the superstar curse: if a superstar extracts a fee equal to the expected profit then the movie will almost surely lose money.[21] In this instance, "almost surely" means it has an 80% chance of losing money.

[20]This calculation is biased because the cost of movies in our data already include the star's pay. It serves only to illustrate the point. More detailed data on movie budgets would be required to do the calculation precisely and they simply are not available.

[21]John Brodie and Anita Busch noted the losses studios incurred in superstar "megapix".

The studio that bids closest to the superstar's expected profit is most vulnerable to the curse. Thus the curse need not depend on a poor assessment of the odds, as in the bidder's curse. The curse may befall a bidder even when he has a true assessment of the odds so long as he bids up to the expected value of profit. Just as in the bidder's curse, a rational studio who is bidding for a script or a superstar should shade the bid well below expected profit and shade more as the number of bidders increases. Better yet, the bidder should guard against the curse by offering the superstar some form of contingent compensation in exchange for a reduced fixed fee, a practice that is becoming more common.[22]

## 8 Sharing the Tail of the Distribution: Superstar Contracts

The superstar curse (the almost sure loss principle) is one reason stars are offered profit participation. The studio must avoid the mistake of paying a star her expected rent. Many Hollywood superstar movie contracts provide that they will receive some form of participation in profit. Actors, screenwriters, and directors on the A+ list are the only players whose fees are high enough to warrant profit participation.[23] In a profit participation deal, the artist takes less than her customary upfront fee in the form of a fixed payment which is independent of the success of her movie in exchange for compensation which is contingent on the outcome.[24] If the superstar curse is to be avoided, the superstar's fixed fee must be less than the expected profit or the movie almost surely will lose money. Hence, contingent compensation avoids the superstar curse of an almost-sure-loss by paying a fee less

[22]This is hard to document, because the contracts are private information. In a collection of 22 superstar contracts that we reviewed we find that all of them have profit participation. More evidence comes from the number of superstars who have formed their own production companies to take back end participation through their company in addition to collecting a fee. The growing incidence of stars who take producer credits, and thereby hold a profit interest, is further evidence of the rise of back end participation.

[23]Not even all of the artists on the A+ list will have fees so high as to warrant participation. De Vany and Walls (1999) estimated that only 19 stars on the A+ list had a significant impact on revenue.

[24]Mark Weinstein (1998) explains the origin of profit sharing contracts.

than the expected profit while permitting the star to capture some of the upper tail of the distribution. The positive skew and long tail of the distribution make it clear that this is where the action is.

Contingent compensation takes many forms, but it has two common features. Only outcomes on the positive tail are shared, and the events must be well out on the tail to be "in the money". Thus, a high profit or box office gross must typically be met before the superstar shares profit. The share of profit or gross revenue may rise at higher outcomes.

Even within these schemes there may be considerable variation. For example, a star may receive a fixed fee and then an additional, contingent, payment of a fixed amount if the movie attains a certain gross revenue. A contract that pays the star a share of gross revenue may contain "breakpoints" where the share increases where revenue crosses over the breakpoint. In a complex contract, there may be several breakpoints where the star's percentage share increases. More sophisticated yet is a contract, usually between studios cofinancing a movie, that allocates all or a different portion of revenue to different participants at different revenue points above some level at which each of the participant's cost has been recovered. These contracts allocate portions of the upper tail of the distribution. Thus, after the film has attained a revenue at which both parties have agreed that their costs are covered in a 50/50 split, or a split that approximates each's contribution to cost, they may "take turns" in receiving revenue at different revenue points along the upper tail.[25] One studio might take all the revenue from $100 to $200 million, and the other all the revenue above $200 million. The alternation may continue at higher revenue.

This contract is a set of options that break "into the money" only when the movie's gross the revenue exceeds the strike points in the contract.[26]

[25]Theatrical exhibition licenses share events in the center and the tail of the distribution as well. Events in the tail are high revenue weeks during the run. High revenues trigger an escalation of the rental rate from a minimum percentage that may range during the run from 70% to 40% to the 90% over "house nut" clause. During the high revenue weeks the rental clause allows the exhibitor to retain his (negotiated) cost per week of operation plus 10% and allocate 90% of the amount over the nut to the distributor. See De Vany and Eckert (1991) on the exhibition license.

[26]Valuing these option-like participation contracts is a difficult problem because neither the distribution of revenue nor profit is log-normal which is assumed in the Black-Scholes option pricing

The existence of profit sharing contracts with breakpoints is a recognition of the long and heavy tails of the revenue and profit distributions. Superstars may also take some form of compensation in their selection of films.

## 9 Return, Risk, and Uncertainty

What draws investors to this business in numbers sufficient to bring the rate of return to less than 4% (Vogel, 1990) in spite of the risk? While the risks and returns in other businesses have been analyzed extensively, there is little formal statistical modelling of returns in the movie business. This paper begins to remedy that void and its findings are sobering. Only twenty percent of movies earn positive profit. More sobering still, less than seven percent of movies account for nearly all the industry's profit. The most probable outcome is that a movie will lose money. The usual measure of risk, the variance of outcomes, is infinite (nobody knows).

There are several reasons we can give for the low rate of return and they all follow from the $\alpha$-stable model. One is an investor's form of the superstar curse. If an investor invests an amount close to the expected profit, then he will with high probability lose money. Another explanation for the curse could be given in terms of incorrect probability judgment. If investors base their decisions on a normal approximation to the correct distribution, then they will be misled into a false perception of the odds. The most probable event estimated from a normal distribution is a smaller loss than the loss associated with most probable event under the $\alpha$-stable distribution. In addition, with the normal distribution, the true probabilities of outcomes near the central part of the distribution will be overestimated and the probabilities of extreme outcomes will be greatly underestimated. Together, these would give a false perception of the odds and investors would fail to realize how important the extreme events are to the probabilities of all outcomes—the high probability placed on extreme outcomes pulls probability away from the center. Stability is important if we want to analyze risk correctly.

formula. The pricing of options when the stable distribution holds is an active area of research (see Hugh McCulloch, 1996 and Mittnik et al., 1998.)

## 10 Momentum, Paretian Tails, and the Angel's Nightmare

What are the driving factors in determining profit? Momentum in the dynamics of revenue can account for the positive Paretian tail of the distribution. In the revenue tail of the superstar distribution, the conditional probability that revenue will exceed $x \geq x_0$ is $x^{-1.624}$. Since $\alpha = 1.624 > 1$, the conditional mean exists and equals $x\alpha/(\alpha - 1),\ x \geq x_0$. What happens to the conditional expected value of revenue as revenue rises? It rises too. For example, the expected revenue when a film has grossed $50 million is $130 million; when it has grossed $100 million, its expected revenue is $260 million. Expected revenue, conditional on what has already been earned, rises as revenue grows. This is revenue momentum.

Momentum is also behind the "bombs" in the loss tail of the distribution because it too is Paretian. The budget distribution has a long upper tail whose exponent is $d = -2.64$. Thus, the mean and variance of budget are finite, but the variance is very large. The upper (high) budget tail is long and heavy because a few massively large budgets strongly influence the average budget. This is consistent with the way studios budget movies. But, cost momentum, where a movie's production cost grows in proportion to the costs already expended, is also a force that drives budgets to the far end of the distribution. This follows from the Paretian property.[27]

To show the momentum property of the Pareto distribution consider a movie whose budget stands at $x million. The expected budget for all budgets above $x million is $x\frac{d}{(d-1)} = x1.609$. Now suppose an amount equal to $20 million has already been spent on a movie that was expected to cost only the mean amount of $16 million. What is the expected cost of the movie, conditional on $20 million having been expended? It is $20\frac{d}{(d-1)} = 32$ an amount that exceeds the original expectation.[28] At this point, the movie is already $4 million over budget and yet

[27]Since cost is roughly linear in time, the expected time to completion will, like cost, be proportional to the time already expended in production (on this "self-similarity" property of power laws see Schroeder, 1990).

[28]If the probability distribution of cost were normal rather than Paretian, then the conditional expected cost of finishing the movie would diminish.

the expected cost of finishing it, \$32 million, is twice the original budget! This is cost momentum, it is the "angel's nightmare".

Though they may not know its basis in probability, Hollywood executives are aware of the implications of cost momentum. Cost momentum is the nightmare a backer (the angel) lives through as he watches funds disappear and (rationally) anticipates that the expected cost will continue to grow even as more money and time are spent on the movie.

The angel's nightmare comes from the self-similarity of the probability distribution, a form that is typically found in complex iterative processes (Shroeder, 1990). It haunts movies and other complex, sequential production processes; it is a feature of the processe rather than a hazard of morals (though moral hazard may further complicate matters).

Because the angel's nightmare affects all complex film projects, motion picture investors require a completion guarantee as a remedy. A completion guarantee hands authority to an independent party once the original budget is exhausted (or exceeded by a specified amount) and this party's sole responsibility is to finish the film. The completion guarantee thus brings the potentially infinite process to an end, though the final product that results may not be artistically or financially successful.[29] The stable Paretian model explains the need for and existence of the completion guarantee.[30]

## 11 The Theory of Profit

The $\alpha$-stable model appears to have some general implications for the theory of risk and profit. We leave that to specialists, but it seems useful to close with a few comments on that topic.

[29] Caves, 2000 makes the observation that handing off part of the production sequence to another agent at each step in the production sequence helps to obviate moral hazard as a source of the angel's nightmare. The angel's nightmare may exist when there is no moral hazard so long as the production process has the Paretian self-similarity property typical of complex iterative processes.

[30] Many other iterative, complex production processes are likely to be self-similar and experience this form of cost overrun. Aircraft carriers and bridges come to mind.

Measures of risk aversion that depend on the second moment are not defined since the variance does not exist when $\alpha < 2$. Decision models that depend on expectation may fail since the first moment need not exist when $\alpha < 1$.

The $\alpha$-stable model tells us that risk has some unusual features in the movie business:

- Every movie is unique (there is no central tendency or "typical" movie according to the $\alpha$-stable model).
- No single company is able to produce enough movies to pool variance since the variance is infinite.
- The average return fails to converge and may not exist.
- There need be no correlation structure among a studio's offerings.
- Extreme value securities that pay on events high in the tail of the distribution might be effective financing instruments.
- Extreme value contracts partly remedy the failure of the market for extreme value securities to exist.

These observations suggest that risk must be managed through the stock market in the pricing and ownership of shares in motion picture companies. Merging motion picture companies into media conglomerates may be a risk management tactic. Securities in motion picture portfolios do not exist and, generally, motion picture securities markets do not exist.

On a more general level, one is struck by the way the $\alpha$-stable distribution seems to fall between the customary definitions of risk and uncertainty. If you know the distribution, you know the odds, so you are in a situation involving risk. But, even when they know the odds, "nobody knows anything" because of the properties of the stable Paretian distribution. The stable distribution seems to stand somewhere between knowing the probabilities (risk) and not knowing them (uncertainty).[31] The asymmetry, skew, and possible non-existence of the mean of the $\alpha$-stable distribution raise questions about a definition of increasing risk that depends on mean-preserving spreads.

## 12 Conclusions

The stable Paretian model is essential for understanding many important aspects of the movie business: the "nobody knows" principle, the "curse of the superstar", the large share that superstars command of industry profit, the use of completion guarantees, the sources of the "angel's nightmare", the existence of contracts that condition on extreme values, the surprisingly low rate of return in a business as risky as this, and the "winner-take-all" distribution of profit are features of the business that only the $\alpha$-stable Paretian model can explain. And, momentum is part of it all.

[31] This would not confuse a Bayesian studio executive, but her diffuse prior would not converge with more information because the tails remain heavy. Learning, therefore, is hard and experience (sample information) has little value in choosing winners. Since technological innovations appear to follow an $\alpha$-stable process too (Scherer 2000, and Sornette and Zajdenweber 1999, a technology policy predicated on picking winners cannot succeed.

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Table 1: Statistical Properties of Motion Picture Profit

| Attribute | All Movies | without Stars | with Stars |
|---|---|---|---|
| Mean | $-3.351$ | $-3.595$ | $-2.083$ |
| Std Dev | $11.938$ | 9.601 | 20.062 |
| Skewness | $1.855$ | 2.488 | 0.860 |
| Kurtosis | $15.384$ | 22.744 | 5.599 |
| Percentile | | | |
| 1st | $-30.328$ | $-25.582$ | $-38.865$ |
| 5th | $-19.976$ | $-16.562$ | $-28.715$ |
| 10th | $-13.796$ | $-12.375$ | $-22.643$ |
| 25th | $-8.299$ | $-7.461$ | $-13.117$ |
| 50th | $-3.787$ | $-3.692$ | $-4.983$ |
| 75th | $-0.466$ | $-0.863$ | 7.012 |
| 90th | $7.012$ | 3.655 | 23.661 |
| 95th | 14.546 | 9.988 | 34.487 |
| 99th | 47.558 | 32.922 | 56.693 |
| Observations | 2015 | 1689 | 326 |

Note: All dollar magnitudes are reported in millions of 1982–4 U.S. dollars.

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Table 2: Maximum Likelihood Parameter Estimates

| | $\alpha$ Index | $\delta$ Location | $\gamma$ Scale | $\beta$ Skewness | Log-Likelihood |
|---|---|---|---|---|---|
| ALL MOVIES | | | | | |
| Normal | 2 | $-3.351$ | 8.442 | 0 | $-7855.37$ |
| symmetric $\alpha$-stable | 1.268 | $-4.079$ | 4.032 | 0 | $-7279.87$ |
| $\alpha$-stable | 1.259 | $-4.042$ | 4.020 | $0.043$ | $-7279.46$ |
| MOVIES WITH STARS | | | | | |
| Normal | 2 | $-2.083$ | 14.186 | 0 | $-1439.69$ |
| symmetric $\alpha$-stable | 1.582 | $-4.568$ | 10.555 | 0 | $-1419.16$ |
| $\alpha$-stable | 1.624 | $-6.385$ | 10.805 | 0.768 | $-1410.82$ |
| MOVIES WITHOUT STARS | | | | | |
| Normal | 2 | $-3.595$ | 6.789 | 0 | $-6216.46$ |
| symmetric $\alpha$-stable | 1.358 | $-3.932$ | 3.507 | 0 | $-5739.47$ |
| $\alpha$-stable | 1.335 | $-3.827$ | 3.441 | $-0.122$ | $-5737.95$ |

Table 3: Upper and Lower Tail Probabilities

| | Lower Tail Probilities Prob($\pi \leq -$Profit) | | Upper Tail Probabilities Prob($\pi \geq$ Profit) | |
|---|---|---|---|---|
| Profit | Without Star | With Star | Without Star | With Star |
| 0 | .7693248 | .6288536 | .2306752 | .3711464 |
| 5 | .3875867 | .4990274 | .0910419 | .2668165 |
| 10 | .1285646 | .3582706 | .0498598 | .1906549 |
| 15 | .0596434 | .2300214 | .0328644 | .1380565 |
| 20 | .0362870 | .1339655 | .0239798 | .1024961 |
| 25 | .0253501 | .0746570 | .0186244 | .0783693 |
| 30 | .0191672 | .0431966 | .0150847 | .0616906 |
| 35 | .0152458 | .0275664 | .0125905 | .0498569 |
| 40 | .0125605 | .0194864 | .0107485 | .0412221 |
| 45 | .0106185 | .0148503 | .0093385 | .0347486 |
| 50 | .0091556 | .0118935 | .0082283 | .0297738 |
| 55 | .0080181 | .0098512 | .0073339 | .0258656 |
| 60 | .0071110 | .0083590 | .0065997 | .0227354 |
| 65 | .0063726 | .0072236 | .0059873 | .0201855 |
| 70 | .0057610 | .0063328 | .0054697 | .0180774 |
| 75 | .0052471 | .0056168 | .0050269 | .0163119 |
| 80 | .0048098 | .0050302 | .0046445 | .0148161 |
| 85 | .0044337 | .0045417 | .0043111 | .0135360 |
| 90 | .0041072 | .0041293 | .0040182 | .0124305 |
| 95 | .0038214 | .0037772 | .0037591 | .0114682 |
| 100 | .0035693 | .0034735 | .0035285 | .0106244 |

Note: Profit is in millions of dollars.

Table 4: Expected, Average, and Most Probable Profit

| Measure | Stars | No Stars |
| --- | --- | --- |
| Average | $-2.083$ | $-3.595$ |
| Expected | $7.684$ | $-4.477$ |
| Most Probable | $-7.500$ | $-3.750$ |

Figure 1: Empirical and Fitted Density Functions

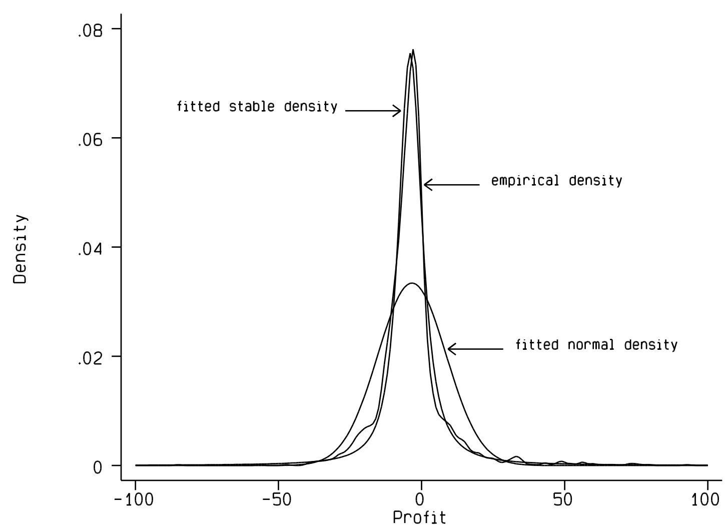


Figure 2: Probability Plot of Stable Fit against Data

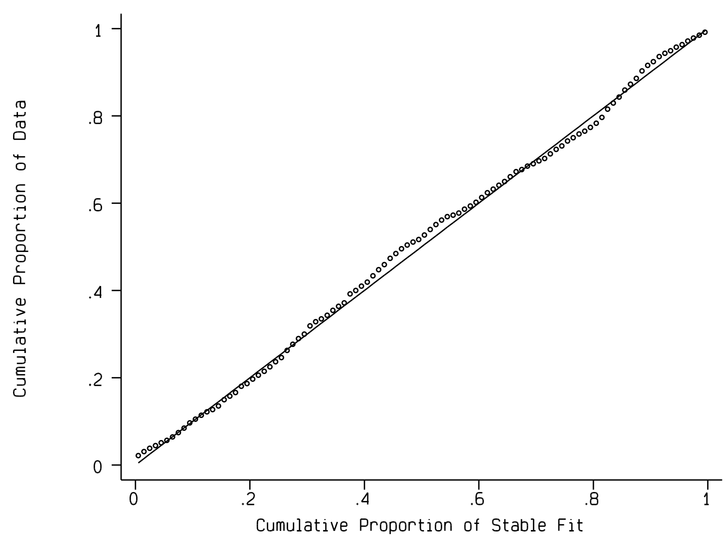


Figure 3: QQ-Plot of Stable Fit against Data

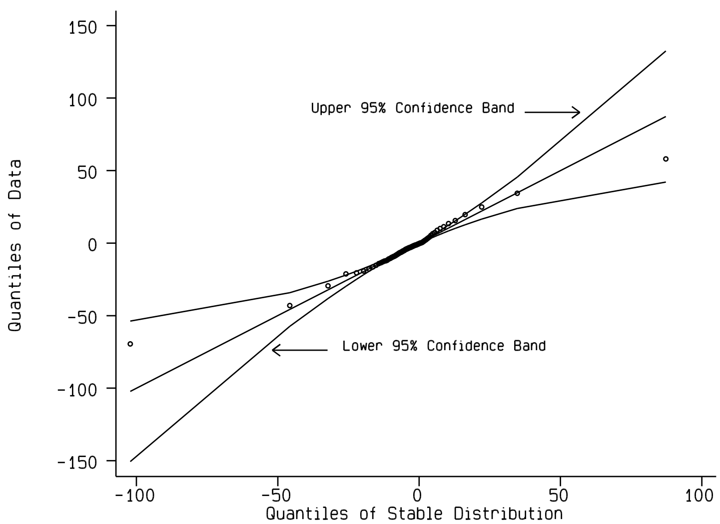
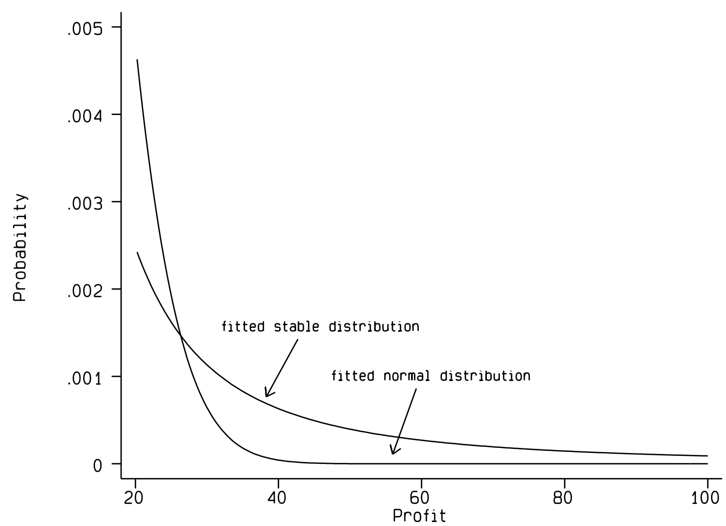


Figure 4: Expanded Upper Tail of Density Functions




Note: Expanded upper tails of density functions shown in Figure 1.

Figure 5: Fitted Stable Density Functions with and without Stars

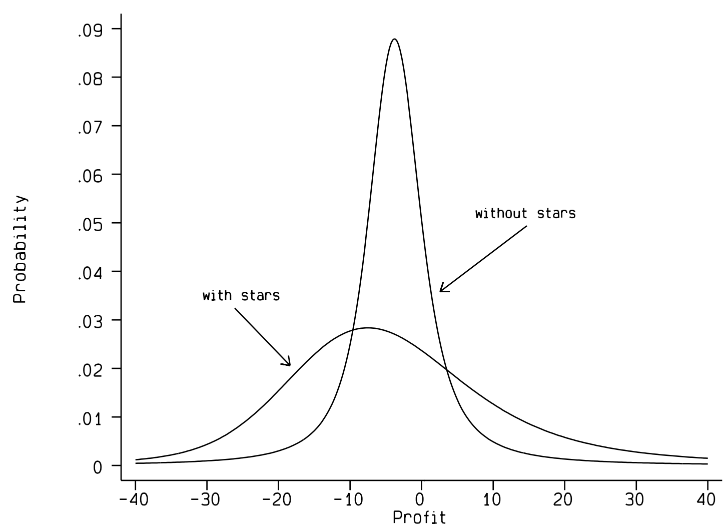
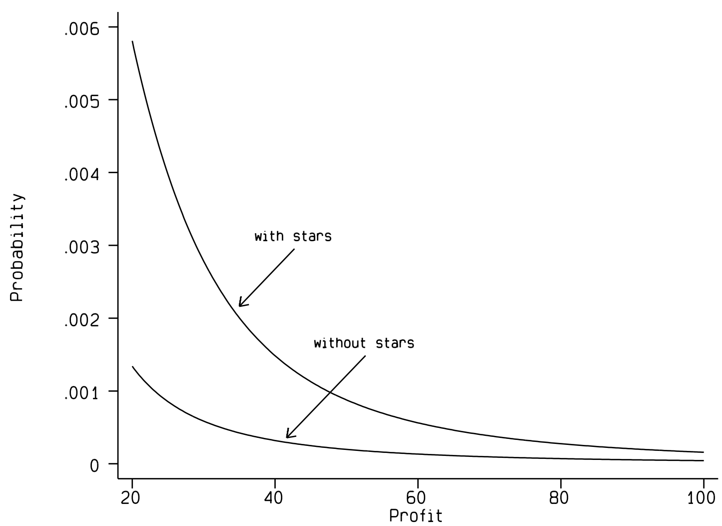


Figure 6: Expanded Upper Tail of Density Functions with and without Stars




Note: Expanded upper tails of density functions shown in Figure 5.

Figure 7: Time Series of Average Cumulative Profit