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Abstract. Many firms in the business to consumer market sell identical products online using auctions and posted prices at the same time. In this paper, we develop and analyze a model of the key tradeoffs sellers face in such a dual-channel setting that is built around the interplay of three design parameters, the posted price, the auction lot size, and the auction duration. Our results show that a monopolist seller can increase his revenues by offering auctions and a fixed price concurrently, and we identify when either a posted price only or a dual channel strategy is optimal for the seller. We model consumer choice of channels, and thus market segmentation, and find that consumers who value the item for more than its posted price use a threshold policy to choose between the two channels. The threshold defines an upper bound on the remaining time of the auction. We explain how optimizing the design parameters enables the seller to effectively segment the market so that the two channels reinforce each other and cannibalization is mitigated. Our findings also demonstrate that there are two dominant auction design strategies in this setting: one-unit auctions that tend to be short and long multi-unit auctions. Which of these two strategies is optimal for the seller depends on the consumer arrival rate and the disutility of delivery delay incurred by high valuation consumers. In either case, the optimal auction design of the dual channel can significantly outperform a single posted price channel. Our results indicate that the seller's surplus from offering auctions is lower when consumers are more sophisticated in estimating their expected discount from participating in the auction.

1. Introduction

In the business to consumer market many firms are selling the same or almost identical products online using auctions and fixed prices simultaneously. The Internet enables firms to operate the two venues in the same space (the WWW) and time and allows consumers to observe and compare the two selling channels with no additional costs. For example, IBM offers selected products via auctions on eBay while the same products are sold for posted prices on IBM's website; CompUSA conducts auctions for new and refurbished products on a dedicated auction website, while selling identical items for posted prices on its catalog web site; Sam's Club, operates the two selling channels within the same website; airline tickets are sold for fixed prices as well as via reverse and forward auctions.

The practice of operating auctions and fixed prices in parallel, on the Internet, raises many important questions. Clearly the two selling channels cannot be treated independently. Optimizing each channel separately results in a suboptimal global design as the two channels compete in the same market. Though the problem of optimally selecting and designing a single selling mechanism (auction or posted price) has been well addressed in the literature, there is little research on how to operate and design such selling mechanisms in parallel. In addition, the economic benefits and limitations of using both auctions and posted price are not clear. On one hand, an auction creates a venue for selling to consumers who do not value a product as much as the posted price, thereby increasing revenues. On the other hand, the auction channel may attract consumers who otherwise would have bought at the posted price, thereby reducing revenues. Our research focuses on the following issues:

- How do consumers behave when faced with the choice between the two channels?
- What is the optimal choice of the auction parameters i.e., auction length and quantity, when identical items are sold for a posted price by the same outlet?
- What is the optimal posted price, when identical items are being auctioned?
- When does the dual channel outperform the single channel (posted price only)?

We develop a mathematical model that addresses the questions stated above. In our model, a monopolist sells identical items using sealed-bid second-price auctions and a posted price at the same time. Our research contributes to the existing auction literature by introducing the following three changes. First, the auctions are conducted parallel to a posted price and serve the same stream of consumers, so arriving consumers can choose between purchasing the item and bidding. Second, in our model the number of bidders is stochastic, and consumers can arrive at any time during the auction. Thus, different consumers spend different amounts of time in the auction and incur different delay costs, depending upon their arrival time. Finally, the auctioned quantity is an endogenous decision variable, set by the seller in order to maximize total (auction and fixed price) revenue.

Our results rely on a model of consumer behavior that defines how consumers choose between bidding and purchasing the item for its posted price. We prove that under fairly general assumptions consumers who value the item for more than its posted price, the "high valuation consumers", use a threshold policy to choose between the two selling channels. The threshold defines an upper bound on the remaining time of the auction: if the remaining time observed by a high valuation consumer upon his arrival exceeds the threshold, the consumer chooses to purchase the item for its posted price. If the remaining time is less than the threshold, the consumer chooses to participate in the auction. We also show that the optimal bidding strategy in the sealed-bid, second-price auction is no longer truth-revealing. For a consumer who values the item for more than the posted price bidding his true valuation is weakly dominated by placing a bid equal to the posted price.

We formulate a nonlinear optimization problem for choosing the posted price, the auction quantity, and the auction length when the seller's objective is to maximize expected revenue per unit time. Based on numerous numerical experiments, we argue that the optimal posted price in the dual channel is unique and is higher than the optimal posted price in the absence of auctions and that a seller can significantly increase his revenue by adding an auction channel to the posted price channel. Depending on the model parameter values, we find that the optimization results in one of the following two strategies: (1) the seller should conduct one-unit auctions and decrease the auction length as the consumer arrival rate increases, or (2) the seller should conduct long auctions and increase the size of the auction lot as the consumer arrival rate increases. In the first strategy the number of units auctioned per consumer is small, but in the second strategy this ratio is significantly higher.

Our result reflects the main tradeoffs that the seller faces: Setting one-unit auctions or long auctions deters high valuation consumers from bidding. Yet these settings also reduce the number of units sold via auction per unit time, reducing total sales. When the seller offers one-unit lots, he would like to make the auctions short enough to have a significant increase in sales while maintaining a high auction price. Thus, the auction length is decreasing with the consumer arrival rate. When the seller uses long auctions, he should offer larger lots in order to increase sales, but since increasing the quantity offered also decreases the auction price, the auction lot is increasing in the arrival rate.

The first strategy, *one-unit auctions*, deters high valuation consumers from participating in the auction by limiting the size of the auction lot. The second strategy, *long auctions*, limits the number of high valuation consumers who bid due to long waiting times. Both strategies increase the competition (number of bidders per unit) that high valuation consumers face from other bidders, reducing their chances of winning and raising expected auction prices, and thus decrease their incentive to bid. Which of these two strategies is optimal for the seller depends on the consumer arrival rate and the delay cost per unit time incurred by high valuation consumers. Our results confirm that the seller's revenue from the dual channel can be higher than the optimal revenue achieved by using a posted price alone, or managing the two channels independently.

The paper's structure is as follows. In §2, we review the relevant literature. In §3, we consider a monopolist seller and construct a model of selling identical items using auctions and a posted price simultaneously. We develop a detailed model of consumer behavior and show how to calculate the expected auction price of the dual channel for a stochastic number of bidders. We describe the characteristics of an optimal design of the dual channel through numerical examples in §4 and conclude in §5.

2. Literature Review

Auction markets have been of interest to researchers for generations. There exists an extensive literature that examines the optimal design of auctions and the ranking of different auction mechanisms: Milgrom (1987), McAfee and McMillan (1987), and Klemperer (1999) provide excellent surveys. Traditionally, analysis of auction design has focused on the auction mechanism itself. In these analyses, an auction is fully characterized by how bidder valuations are revealed and how the actual goods are allocated. Much of the focus is on showing under which circumstances common auction mechanisms are equivalent and on proving when these mechanisms are truth-revealing. The widespread use of online auctions has brought a new set of managerial problems to the forefront that have yet to be studied. Pinker et al. (2003) survey the current state of research on the specific problems faced in the design of online auctions.

The problem of optimally selecting and designing a single selling mechanism has been well addressed in the literature. Wang (1993) considers the impact of the dispersion in the distribution of buyers' valuations on the choice between a posted price and an auction for a seller selling one unit of a good. Wang demonstrates that an auction is preferred when buyer valuations become more disperse. This model, like many others, ignores buyers' costs that are associated with auctions, such as the cost of delays and the cost of monitoring the auction. Wang does not model how consumers would choose between the two mechanisms, since he does not consider simultaneous use of both. Hence, the set of consumers is identical for both methods (it does not depend on the choice of mechanism). Harstad (1990) uses a model in which the seller's choice of selling method and reserve price does affect the number of bidders attending the auction. Ehrman and Peters (1994) consider a waiting cost for bidders due to the disappearance of outside alternatives. De Vany (1987) considers a seller with one unit of a commodity choosing between three mechanisms: an auction after a fixed time T; an auction after a fixed number of consumers have arrived; and posted price. Consumers incur a cost of waiting when an auction mechanism is chosen.

Some researchers have considered the choice of mechanism when the seller offers multiple homogenous units of the good (Arnold and Lippman (1995), Harris and Raviv (1981), Riley and Zeckhauser (1983), Maskin and Riley (1989)). Others have tried to explain the coexistence of the different selling mechanisms in a market and have examined the equilibrium of mechanisms in a competitive environment (Peters (1999), Epstein and Peters (1999), and Kultti (1997)). Recently, Gallien (2002) has compared the fixed price, dynamic posted price, and online auction mechanisms when selling multiple units to risk-neutral and time-sensitive consumers. Each buyer is characterized by his valuation of the item and his arrival time and a buyer's net value decreases in the interval between his arrival and the time he obtains the item. Yet, though mechanism selection has been well covered, there is little research on how to operate and design such selling mechanisms in parallel. In addition, the economic benefits and limitations for a firm that concurrently employs multiple selling mechanisms are not clear.

Vakrat and Seidmann (1999) study simultaneous sales of identical products using online auctions and a fixed price catalog. Their empirical research shows that the auctions result in an average discount of 25% relative to the catalog prices. They model a one-unit English auction and assume that the number of bidders is deterministic, and that consumers have full information about the catalog price. They find that the expected auction price is a function of the number of bidders and of delay and search costs associated with the auction. Their paper does not model consumers' choice between the two channels, nor does it show what incentives the seller has to conduct such an auction. Van Ryzin and Vulcano (2002) examine the optimal pricing-replenishment policy when the firm sells in two markets, one fixed price market and one auction market, and demand comes from two different and independent streams of customers, so there is no need to model consumers' choice. In their model, the seller decides how to split the inventory between the two markets.

Another use of the two selling mechanisms simultaneously is the "buy now" price offered on many C2C auction sites. On Yahoo Auctions, for example, the auction will automatically close when a bidder meets the specified "buy now" price, and the item is sold to that bidder. Another example is E-bay's "Buy It Now" option, which is only shown on listings until an item receives its first bid, or, when the seller sets a reserve price, until the reserve is met. In this business model the auction is the main selling venue, and the "buy now" price is viewed as the secondary channel. Budish and Takeyama (2001) model an English auction with a buy now price. Their model has only two bidders and two possible types: a high-valuation and a low-valuation consumer. They show that the seller is strictly better off by adding a "buy now" price to the auction only when bidders are risk-averse. This result, however, does not hold in a more general framework with *N* valuations. Hidvegi et al. (2002) model an auction with *N* bidders having continuously distributed private valuations and show that a bidder with a very high valuation compared to the buy price will use the buy price unconditionally, a bidder with a valuation close to the buy price will only use the buy price when the current bid reaches a threshold price, and there is no change in the optimal bidding strategy for a bidder with valuation lower than the buy price. They find that when either party is risk-averse, a buy-price auction is strictly better for the seller. They do not consider delay costs for bidders.

It is important to mention that, with the exception of Gallien (2002), the existing research does not model the facts that different consumers arrive at different stages of the online auction, and that their expected utility from bidding is a function of their arrival time during the auction. Even those papers that associate a delay cost with bidding, assume that all bidders spend the same amount of time in the auction and thus incur the same delay cost. Clearly, such assumptions are not suitable for the modeling of online auctions, which can last for as long as a week or more. Our model addresses this issue.

3. The model

We model an online seller who offers identical items using two selling mechanisms, posted price and auctions, simultaneously. The auctions have a fixed duration and are then repeated. The seller's objective is to maximize his revenue per unit time. The seller chooses the auction duration T, the quantity to auction q, and the posted price p. Without loss of generality, we assume that the marginal cost of each unit is zero (if this is not so, consumers' valuations of the product can be taken net of the marginal cost). The seller's publically declared reserve price is R.[1] We also assume that the seller can satisfy any demand. Consumers visit the web site according to a Poisson process with rate λ and each consumer is interested in purchasing one unit of the good. Consumers differ in their valuations of the good, having independent private values. We assume that each consumer's valuation, V, is independently drawn from a probability distribution with cumulative density function $F(\ )$ with support set $[\underline{v}, \overline{v}]$ where $\underline{v} \geq R$.

Since the two selling channels are being offered simultaneously on the same platform or in the same space (the Internet), we assume that consumers can observe both channels on arrival, with no additional costs. Hence, consumers are fully informed: they observe the item's posted price and they know whether an auction is currently offered, what the auctioned quantity is and what the remaining time of the auction is. They do not know the number of other bidders.

We model the auctions using the sealed bid *(q+1)- price* format with risk-neutral bidders having unit demand and independent private values for the good. In a sealed bid *(q+1)- price* auction the winners are the bidders with the q highest bids (q being the auctioned quantity), and each pays a price equal to the *(q+1)* highest bid (the highest losing bid). We assume a random tie-breaking rule. In such a sealed bid *(q+1)- price* auction, with no posted price offering, the dominant strategy for each bidder is to bid his true valuation of the item (Milgrom, 1987). By doing so, the bidder is setting an upper bound on the price that he is willing to pay: he will accept any price below his reservation price and none above.

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[1] A public reserve price is equivalent to setting a minimum initial bid.

Most online auctions are conducted in the open, ascending bid English auction format, where bidders can observe the lowest bid needed to win in every moment of the auction and, on some web sites, can see how many other bidders there are. This suggests that consumers have more information available to them when choosing between the posted price and auction participation than in the sealed bid auction we model. However, in practice last minute bidding or 'sniping' is very prevalent (Roth and Ockenfels 2002). This suggests that actually very little information is available to the consumers since they do not know how many other bidders are lurking in the background nor do they have any indication of what these other bidders are willing to bid. The result is a de facto sealed-bid auction.

In our setting, the existence of the posted price option adds considerable complexity to the analysis of the auction because it splits the consumers into several subgroups (see Figure 1). The posted price first splits the consumers into those with low valuations (i.e., valuations less than the posted price), and those with high valuations, i.e. valuation greater than or equal to the posted price. In this paper, all low valuation consumers become bidders in the auctions, we explain the rationale for this in §3.1. A fraction β of the high valuation consumers also become bidders while a fraction 1-β purchase at the posted price. The participation probability, β, depends on the design variables (q, T, and p) and much of the analysis of §3 is devoted to how it is determined. Some of the bidders win the auction and some lose. The high valuation bidders who lose will purchase at the posted price at the conclusion of the auction. To explain the choices made by consumers depicted in Figure 1 we next describe our model of consumer behavior. Table 1 summarizes the notation used throughout the paper.

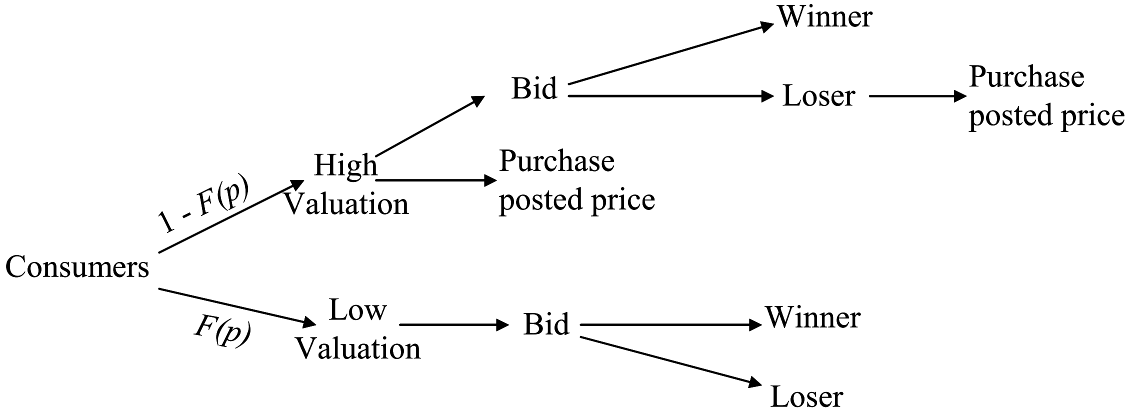

Figure 1: Schematic of customer splitting in the presence of dual channel

Decision variables:	
p	posted price.
q	auctioned quantity (per auction).
T	auction length.
Model parameters:	
λ	arrival rate of consumers to the web site.
$F(v)$	cumulative density function of consumers' valuation distribution with support $[\underline{v}, \bar{v}]$.
w	delay cost incurred by high valuation consumers per unit time.
R	seller's public reserve price.
Other notation:	
$\mathrm{E}[p_a]$	expected auction price.
N^+	random number of bidders who value the item for more than its posted price.
N^-	random number of bidders who value the item for less than its posted price.
$\bar{t}$	maximum time remaining in an auction for which high valuation consumers choose to participate in auction.
$O\{x,y\}$	expected value of the x^{th} order statistic of y draws from the consumer valuation distribution truncated on $[\underline{v}, p]$.
β	probability a high valuation consumer participates in the auction.

Table 1: Summary of notation

3.1 Consumer Behavior

We model the behavior of risk-neutral consumers with bounded rationality, when the seller offers both auctions and a posted price. We now examine how consumers choose between the available channels and define a weakly dominant bidding strategy (and thus a dominant equilibrium) for those who choose to bid. The bounded rationality manifests itself in the use of a heuristic algorithm for choosing between channels.

As noted in the literature review, most auction models examine markets where auctions are the sole selling mechanism and the number of bidders is deterministic. In such markets consumers face a simple choice between bidding and not. In the absence of auction-related costs, the expected value from bidding is always nonnegative so the set of bidders is the same as the set of arrivals: each arriving consumer chooses to participate in the auction rather than stay out of the market.

When a risk-neutral consumer has the option of a posted price channel, he bases his choice on his expectation of a greater surplus. When consumers choose the auction channel, it is because they believe that there is an opportunity to purchase the good at a discount over the posted price. Yet, there are costs to participating in an auction, so in expectation the auction price discount must exceed these costs. There are essentially two auction participation costs, the cost of monitoring and making bids and the cost of deferring the purchase of the good until the end of the auction. There is empirical evidence that these costs influence the behavior of auction participants.

Hann and Terwiesch (2003) study bidder behavior on a German name-your-price service that allows bidders to update their bids after only a few minutes. They find that bidders do not bid very frequently with small increments, as one would expect, but rather bid only a few times (typically less than four) with significant bid increments. They explain this behavior as demonstrating that there is a significant participation cost in these online negotiation settings. Lucking-Reiley (1999) conducts experiments with Internet auctions of collectible trading cards, comparing the profit generation of Dutch auctions with that of first-price auctions which theory predicts are equivalent. He finds, to the contrary, that Dutch auctions had closing prices on average 30 percent higher than first-price auctions. He speculates that one possible reason is the fact that the Dutch auctions were much longer and bidders might have been impatient to complete their purchase. Motivated by Lucking-Reiley's observation, Carare and Rothkopf (2001) develop decision and game theoretic models of slow Dutch auctions to show how including auction transaction costs related to the auction duration alter their outcomes. In their models the value the bidder receives from the auctioned good decreases with the time spent in the auction i.e., there is a delay cost. We use a similar approach in our model.

Some auction mechanisms require more active participation by the bidders and therefore have higher participation costs. The sealed-bid auction we model does not require the bidders to constantly monitor the auction's progress or to analyze the behavior of other bidders. In our setting therefore, it is reasonable to assume that the most significant component of the auction participation cost is the delay cost.

To develop the intuition behind our model we consider the following numerical example.

Example 1: A seller is offering two computer keyboards in an auction while simultaneously selling them online for \$100. Let's say seven consumers $\{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$ with respective valuations $\{5, 10, 80, 90, 101, 110, 120\}$ arrive to the website. Consumers $\{B_1, B_2, B_3, B_4\}$ have no other option but to bid in the auction because the posted price is too high for them. We demonstrate in Lemma 1 that their optimal bidding strategy is to bid their valuations. If only these four consumers bid the closing price of the auction will be \$10 with B_3 and B_4

winning. What about B_5? He can purchase the keyboard at the posted price and have it shipped to him immediately while receiving a surplus of \$1, but he might be able to do better by bidding in the auction. We demonstrate in Lemma 1 that B_5's optimal bidding strategy is to bid the posted price \$100. If he bid this way against consumers $\{B_1, B_2, B_3, B_4\}$ he would win and pay \$80, yielding a surplus of \$21. If B_5 arrived at the auction 3 days before it ended choosing to participate would force him to incur a waiting cost because if he wins he will receive the good three days later than if he had purchased at the posted price. If this cost was \$3 per day his surplus would be reduced to \$12. If consumer B_6 also bid in the auction the closing price would increase to \$90 reducing B_5's surplus further to just \$2. We can see from this example that a high valuation consumer may find it worthwhile to participate in the auction if they anticipate receiving a discount over the posted price larger than the delay cost. Where the discount is determined by the number and types of the other bidders.

3.1.1 Consumer's problem

Low valuation consumers, those with $V < p$, cannot buy the item for its posted price because the value they get from doing so is negative, they thus choose between bidding and staying out of the market. Clearly, these consumers prefer to receive the item earlier rather than later and they may choose not to bid if the remaining time of the auction is significantly long (hence, we later set an upper bound on the feasible auction length when solving the seller's optimization problem in §4). However, since they have no other option for obtaining the item, we assume that the delay cost per unit time perceived by these consumers is significantly lower than the delay cost per unit time perceived by consumers who can obtain the item instantly by paying the posted price. To simplify the problem, we therefore assume that the delay cost per unit time is $w = 0$ for consumers with $V < p$. The optimization problem faced by these consumers is:

$$Max\{U^A, 0\} \text{ where, } U^A(V) = Max_{b\in[0,\infty)} \Pr(win|b)V - E[auction\_payment\,|\,b]. \quad (1)$$

$\Pr(win|b)$ is the probability that the consumer wins the item in the auction by bidding b and $E[auction\_payment|b]$ is the expected auction payment.

High valuation consumers, those with $V > p$, would buy the item for its posted price if auctions were not offered. High valuation consumers choose between buying the item for its posted price and participating in the auction. It is never optimal for these consumers to do nothing because their utility from buying the item for the posted price is nonnegative. We assume that when high valuation consumers purchase the item for its posted price they obtain the item instantly. When they choose to bid, they are choosing to experience a delay in obtaining and using the item because they must wait until the end of the auction. Hence, when choosing to bid, these consumers incur a delay cost that is an increasing function of the time remaining until the end of the auction. We define $U^A(V)$ as the maximum expected value from participating in the auction and $U^B(V)$ as the value from purchasing the item for the fixed price, for a consumer with valuation V. A consumer arriving with t^e time units remaining in the auction solves the following optimization problem:

$$Max_{i \in A,B} U^i(V)$$

where,

$$U^A(V) = Max_{b \in [0,\infty)} E[U(b;V)] = Max_{b \in [0,\infty)} \Pr(win|b)V - E[auction\_payment \,|\, b] + \Pr(lose|b)(V-p) - wt^e \qquad (2)$$

$$U^B(V) = V - p$$

Pr(*win*|*b*) is the probability that the consumer wins the item in the auction by bidding *b* and Pr(*lose*|*b*) = 1- Pr(*win*|*b*). E[*auction_payment*|*b*] is the expected auction payment, and E[$U(b;V)$] is the expected value from the auction for a consumer with valuation V when he bids *b* where the expected value is taken over the bids of all other bidders in the auction. The consumer evaluates the expected payoff from bidding using an optimal bidding strategy and compares it with the payoff from purchasing the item for the posted price. The first two terms of E[$U(b; V)$] give the expected value from bidding *b* when the product is not offered for a posted price. The existence of a posted price offering has two opposing effects on the auction's value for a high valuation consumer:

- $\Pr(lose|b)(V-p)$: If the customer loses the auction, we assume he can and will purchase the item for the same posted price with a payoff of (*V*-*p*). Hence, the existence of a posted price increases the expected value from participating in the auction by reducing the cost of losing the auction. We discuss this assumption further after we have completed analyzing the participation decision in §3.1.2.
- $-wt^e$: Since the consumer could have bought the product for the posted price and obtained the item instantly, he incurs a delay cost when he chooses to bid and wait until the end of the auction to receive the item. We assume that this delay cost is linear in the remaining time of the auction that the consumer observes upon his arrival. It may be that *w,* the delay cost per

unit time*,* is an increasing function of V, that is, a consumer who values the item more also relates a higher cost to a delay in using the item. To simplify the following analysis, we assume that w is positive and independent of V for consumers with $V > p$.

3.1.2 Optimal auction participation

The sealed bid auction is modeled as a static game of incomplete information. The action space for bidder i is the space of possible bids, $B_i=[0,\infty)$, and the type space is $T_i = \left[\underline{v}, \overline{v}\right]$, the support of the consumer valuation distribution. A strategy, $b(V)$, is a mapping from the type space to the action space. Because valuations are independent, player i believes that V_k for every $k\neq i$ is drawn from the CDF $F(\ )$.

Next, we find a bidding strategy, $b_i(V_i)$, such that for any given number of bidders and combination of other bidders' actions $b_{-i} = \left(b_1, b_2, ..., b_{i-1}, b_{i+1}, ... b_{N^+ + N^-}\right)$ and for any other bidding strategy, $b_i'(V_i)$, $U_i(b_i(V_i), b_{-i}, V_i) \geq U_i(b_i'(V_i), b_{-i}, V_i)$ with strict inequality for some b_{-i}. That is, we find a weakly dominant strategy for this static game of incomplete information and such a strategy provides a dominant equilibrium.

Lemma 1:

A weakly dominant bidding strategy for risk-neutral bidders with independent private values in a sealed bid (q+1)- price auction that is conducted parallel to a posted price, p, is the following:

$$b(V) = \begin{cases} V & \text{for } V < p \\ p & \text{for } V \geq p \end{cases} \tag{3}$$

The proof is in Appendix 1.

Clearly, the weakly dominant bidding strategy given in Lemma 1 is also a symmetric Bayes-Nash equilibrium of the game. Note that the existence of an outside option at price p puts an upper bound of p on the bids placed by high valuation consumers. This is in contrast to the optimal bidding strategy in a traditional sealed-bid second-price auction in which it is optimal to bid ones valuation. Losing the auction in our setting is less of a loss because of the infinite posted price supply.

Based on Lemma 1, $U^A(V) = E_{b_{-i}}[U(p;V)]$ for bidders who value the item for more than the posted price, and $U^A(V) = E_{b_{-i}}[U(V;V)]$ for bidders who value the item for less than the posted price. For consumers with $V < p$, low valuation consumers, the value from participating in the auction is non-negative, so all of these consumers choose to bid rather than stay out of the market. Hence, the number of participants in the auction is the same as the number of arrivals. The number of low valuation participants, N^{-}, is a random variable from a Poisson distribution with rate $\lambda_1 = \lambda F(p)$. Consumers with $V \geq p$, high valuation consumers, choose to participate in the auction rather than to buy the item for its posted price if and only if $E[U(p, V)] > U^{B}(V)$, that is, if and only if

$$\Pr(win|p)V - E[auction\_payment \mid p] + \Pr(lose|p)(V - p) - wt^{e} > V - p \, . \tag{4}$$

Proposition 1: *In a dual channel with a sealed bid (q+1)- price auction in which bidders follow the strategy of Lemma 1, high valuation consumers choose to participate in the auction if and only if*

$$\Pr(win|p)p - E[auction\_payment \mid p] - wt^{e} \geq 0 \tag{5}$$

This decision rule can be restated:

High valuation consumers choose to participate in the auction if and only if

$$t^{e} \leq \bar{t} \, . \tag{6}$$

The proof is in Appendix 1. For the definition of $\bar{t}$, see Table 1.

We note that $\Pr(win|p)p - E[auction\_payment \mid p]$ is the expected discount a high valuation consumer gets over the posted price if he participates in an auction. We define $D(N^{+}, N^{-})$ as the expected discount a high valuation consumer gets over the posted price if he participates in an auction with a total of N^{+} high valuation bidders and N^{-} low valuation bidders. Based on Proposition 1, we conclude that under fairly general conditions high valuation consumers use a threshold policy to choose between buying the item for its posted price and bidding in the auction. If the remaining time of the auction observed by a high valuation consumer, upon his arrival, exceeds the threshold, the consumer chooses to purchase the item for its posted price. If the remaining time of the auction is less than the threshold, $\bar{t}$, the consumer chooses to participate in the auction. This result is shown in Figure 2. The numbered dots in Figure 2 depict the arrival times and valuations of bidders. The horizontal axis represents the time remaining in the auction when the consumer arrives, while the vertical axis represents the consumer's valuation of the good being sold. Figure 2 depicts the two types of consumer segmentation occurring in the dual channel. The posted price splits the consumers into low and high valuation groups. The low valuation consumers all bid in the auction regardless of their arrival times (consumers 2, 4 , and 7). The threshold time $\bar{t}$ segments the high valuation consumers into those that bid and those that don't. Consumers 1, 3 and 5 all have arrived with more than $\bar{t}$ time units remaining in the auction and therefore have delay costs that exceed their expected discount from participating in the auction. On the other hand consumers 6 and 8 arrive late enough that it is worthwhile for them to bid in the auction and delay their purchase of the item.

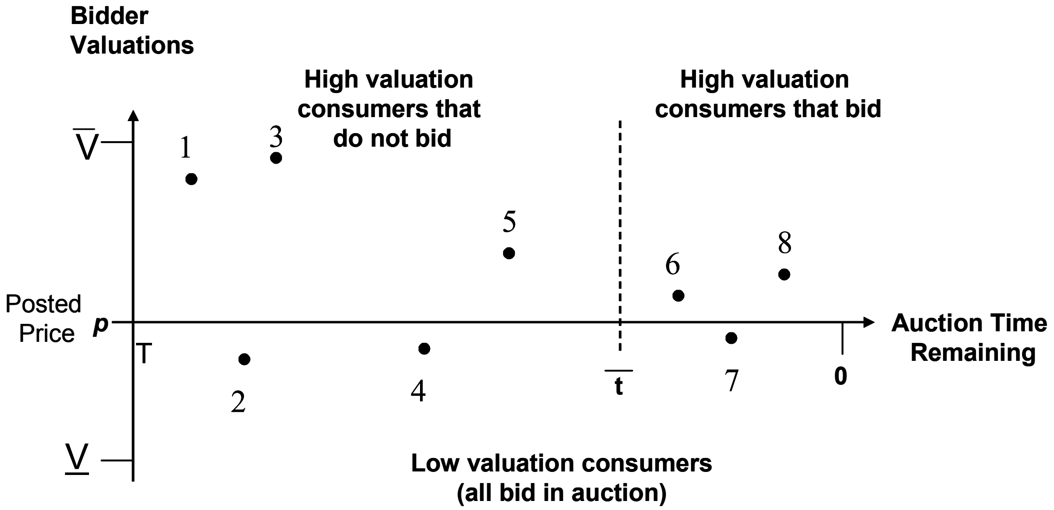

Figure 2: The dynamics of auction participation (p – posted price, T – auction duration, $\bar{t}$ - participation threshold, $[\underline{v},\bar{v}]$ - valuation support)

Since Poisson arrivals are uniformly distributed over a fixed time interval, the fraction of high valuation consumers that participate in the auction is given by $\beta = Min(1, \bar{t}/T)$, where $\bar{t}/T$ is the probability that a consumer arrives in the last $\bar{t}$ units of time of the auction. When $\bar{t} \geq T$ we have the trivial case in which no one purchases the item for its posted price for the entire duration of the auction, and thus it is no longer a dual channel. There is no difference between the cases of $\bar{t} = T$ and $\bar{t} > T$. For the rest of this paper we assume that the design of the dual channel is such that $\bar{t} \leq T$. We conclude that the number of high valuation consumers who bid, N^+, has a Poisson distribution with rate $\lambda_2 = \lambda(\bar{t}/T)(1 - F(p))$ and that the smaller the value of $\bar{t}$, relative to *T*, the more effectively the seller has segmented the low and high valuation consumers.

Proposition 1 also helps provides the rationale for our assumption that high valuation consumers who lose the auction will immediately buy the good at the posted price. An alternative option would be to participate in a subsequent auction. We do not view this option as plausible for the following reasons. First, if high valuation consumers find it optimal to enter a new auction from the beginning it means that their joining threshold $\bar{t}$ exceeds the auction length *T*. In such a scenario no high valuation consumer would purchase at the posted price and here would be no dual channel to discuss. Second, high valuation consumers lose when there are more high valuation consumers than items auctioned. In such a situation even if you win you pay the posted price *p,* and would have been better off skipping the auction altogether. A high valuation consumer who entered one auction and faced a lot of competition from other high valuation consumers has no reason to expect a different outcome in a future auction and thus would expect no discount over the posted price. Finally, in principle, consumers can not be certain there will be another auction.

It is important to note that these results depend on the consumers' assumption that the seller's capacity is unlimited. When the capacity is limited, the probability of being able to purchase the item for the posted price at the end of the auction is less than one because of the positive probability that the seller will run out of stock. The participation threshold would then be dependent upon the consumer's valuation, *V.* Hence, our suggested model holds when consumers believe that the probability that the seller will run out of stock during the auction is zero. This is plausible when the auction's length is relatively short and the seller's capacity is assumed to be large. A different model is needed for items such as airline tickets, end-of-season items, or refurbished goods, for which the probability of being "out of stock" is significant.

Based on Proposition 1, all high valuation consumers solve the same problem so all use the same threshold, $\bar{t}$. Proposition 1 also shows that a high valuation consumer must assess the expected discount the auction will provide over the posted price to determine the optimal threshold for joining an online auction. The larger the expected discount the larger the threshold $\bar{t}$. This assessment is quite complex because the consumer must integrate the effects of several factors such as the posted price, the number of units being auctioned, the duration of the auction, the number of other bidders and their valuations, and the cost they incur from waiting until the end of the auction to purchase the good. The individual effect of any of these factors on the threshold is fairly intuitive. A lower delay cost, *w*, encourages high valuation consumers to participate in the auction earlier. A larger auctioned quantity, q, enables more high valuation consumers to bid, without reducing the individual's probability of winning. We therefore expect $\bar{t}$ to increase with the auctioned quantity and decrease in the delay cost per unit time. All other things being equal, the longer the auction is, the higher the expected auction price is, due to an increase in the number of bidders. Hence, we also expect $\bar{t}$ to be non-increasing in the length of the auction. The combined effect of all of these factors is much harder to anticipate.

In recognition of the complexity of the decision facing the high valuation consumer we assume that he determines the optimal threshold for joining the auction using a heuristic. The high valuation consumer arriving to the auction assumes that the total number of other bidders is deterministic and equal to its expected value. In other words, high valuation consumers use a threshold $\bar{t}$ selected such that if the number of low valuation bidders is $\lambda \Pr(V \le p)T$ and the number of additional high valuation bidders is $\lambda \Pr(V > p)\bar{t}$ then the expected discount from the auction equals $w\bar{t}$. Since auction prices must be calculated using integer numbers of bidders and $\lambda \Pr(V \le p)T$ and $\lambda \Pr(V > p)\bar{t}$ may be non-integer, we further assume that the high valuation consumer interpolates between the nearest integer values for the expected number of low and high valuation bidders to determine the optimal threshold $\bar{t}$. Specifically we have:

Assumption 1: High valuation consumers evaluate the expected auction discount as if the number of high valuation bidders (bidders with $V > p$) arriving in a time period of length t is $\lceil \lambda \Pr(V > p)t \rceil$ with probability ρ and $\lfloor \lambda \Pr(V > p)t \rfloor$[2] with probability (1- ρ), where $\rho = \lambda \Pr(V > p)t - \lfloor \lambda \Pr(V > p)t \rfloor$, and the number of low valuation bidders arriving in a time period of length t is $\lceil \lambda \Pr(V \le p)t \rceil$ with probability γ and $\lfloor \lambda \Pr(V \le p)t \rfloor$ with probability (1- γ), where $\gamma = \lambda \Pr(V \le p)t - \lfloor \lambda \Pr(V \le p)t \rfloor$.

We defined $D(N^+, N^-)$ as the expected discount a high valuation consumer gets over the posted price if he participates in an auction with a total of N^+ high valuation consumers (including himself) and N^- low valuation bidders where N^+ and N^- are deterministic. For general distributions of N^+ and N^- the auction discount is given by

---

[2] $\lceil \ \rceil$ gives the closest larger integer and $\lfloor \ \rfloor$ gives the closest smaller integer.

$$
\begin{aligned}
&p\Pr(win \mid p) - E[auction\_payment \mid p] = p\left(\Pr(N^+ \le q) + \frac{1}{x}\sum_{x=q+1}^{\infty}\Pr(N^+ = x)\right) - \\
&\left(p\frac{1}{x}\sum_{x=q+1}^{\infty}\Pr(N^+ = x) + R\sum_{x=0}^{q}\sum_{y=0}^{q-x}\Pr(N^+ = x)\Pr(N^- = y) + \sum_{x=0}^{q}\sum_{y=q-x+1}^{\infty}\Pr(N^+ = x)\Pr(N^- = y)O\{q-x+1, y\}\right)
\end{aligned}
\tag{7}
$$

Using the above formula for deterministic values of N^+ and N^- we have:

$$
D(N^+, N^-) = \begin{cases} p - O\{q - N^+ + 1, N^-\} & \text{if } N^+ \le q \quad \text{and} \quad N^+ + N^- > q \\ 0 & \text{if } N^+ > q \\ p - R & \text{if } N^+ + N^- \le q \end{cases}
\tag{8}
$$

Proposition 2: *If high valuation consumers make their auction participation decision using Assumption 1 and bid according to the strategy defined in Lemma 1, the* ***unique*** *threshold is given by* $\bar{t} = Min\{T, t_w\}$ *where* t_w *is the solution of the following fixed-point equation*

$$
\begin{aligned}
wt_w = {} & \rho\gamma D\left(\lceil \lambda \Pr(V > p)t_w + 1\rceil, \lceil \lambda \Pr(V \le p)T\rceil\right) - \\
& \rho(1-\gamma)D\left(\lceil \lambda \Pr(V > p)t_w + 1\rceil, \lfloor \lambda \Pr(V \le p)T\rfloor\right) - \\
& (1-\rho)(1-\gamma)D\left(\lfloor \lambda \Pr(V > p)t_w + 1\rfloor, \lfloor \lambda \Pr(V \le p)T\rfloor\right) - \\
& (1-\rho)\gamma D\left(\lfloor \lambda \Pr(V > p)t_w + 1\rfloor, \lceil \lambda \Pr(V \le p)T\rceil\right)
\end{aligned}
\tag{9}
$$

and $D(N^+, N^-)$ *is the expected discount from the auction, for deterministic number of bidders from each type as given in Equation 8. O{x,y} is a function given by the expected value of the* x^{th} *order statistic of y draws from the consumer valuation distribution truncated on* $[\underline{v}, p]$.

The proof is in Appendix 1.

In Appendix 3 we present the results of numerical experiments analyzing performance when consumers do not use a heuristic but instead calculate the expected auction discount exactly, to determine the participation threshold $\bar{t}$. We find that the results are qualitatively similar to those derived using the above heuristic.

3.2 Seller's optimization

Recall that N^- and N^+ are defined, respectively, as the number of low and high valuation consumers who bid. We have shown above that N^- and N^+ are random variables from Poisson distributions with rates λ_1 and λ_2 respectively, where $\lambda_1 = \lambda F(p)$ and $\lambda_2 = \lambda(1-F(p))\beta$. The seller determines these rates by selecting T, q, and p. The seller's decision problem can thus be formulated as follows:

$$\underset{T,\,q,\,p}{Max}\ \frac{1}{\mathrm{T}}\left[E[\pi_a]+\left(\lambda T(1-F(p))-E[N^+]\right)p+p\sum_{x=q+1}^{\infty}(x-q)\frac{e^{-\lambda_2 T}\left(\lambda_2 T\right)^x}{x!}\right] \qquad (10)$$

where

$$E[\pi_a]=p\sum_{x=q+1}^{\infty}\frac{e^{-\lambda_2 T}\left(\lambda_2 T\right)^x}{x!}+\sum_{x=0}^{q}\ \sum_{y=q+1-x}^{\infty}O\{q-x+1,y\}\frac{e^{-\lambda_1 T}\left(\lambda_1 T\right)^y}{y!}\frac{e^{-\lambda_2 T}\left(\lambda_2 T\right)^x}{x!}+$$

$$R\sum_{x=0}^{q}\sum_{y=0}^{q-x}\frac{e^{-(\lambda_2+\lambda_1)T}\left(\lambda_2 T\right)^x\left(\lambda_1 T\right)^y}{x!\,y!}$$

The first term of Equation (10) is the expected revenue from the auction. The second term of Equation (10) is the expected revenue from sales for the posted price during the auction. The number of purchasers for the posted price equals the number of high valuation consumers (consumers with $V > p$) who arrive during the time of the auction less those who choose to bid. The last term in Equation (10) is the expected revenue from sales to high valuation consumers who lost in the auction.

If the number of bidders is less than q the auction price is the seller's reserve price R. Note that, in our model, R is a parameter not a decision variable and only consumers with valuations $V \geq R$ are relevant to the analysis. Our model could be used as an engine to identify the optimal reserve price in situations in which the reserve price is public knowledge. Some online auctions such as those conducted on eBay, SamsClub.com, and Compusaauctions.com allow sellers to post secret reserve prices. A secret reserve price will deter some bidders from participating in the auction as observed by Bajari and Hortacsu (2003). Our model is not designed to capture this effect.

4. Design of the dual channel

To develop our intuition of the seller's perspective let's consider a numerical example modeled on Example 1. **Example 2**: Seven consumers $\{B_1, B_2, B_3, B_4, B_5, B_6, B_7\}$ with respective valuations $\{5, 10, 80, 90, 101, 110, 120\}$) come to a website selling computer keyboards. If there was only a posted price channel with price \$100, the seller would only sell to B_5, B_6, and

B_7 for a revenue of \$100 + \$100 + \$100 = \$300. If he offers the two-unit auction and none of the high valuation bidders participate he will earn an additional \$20 (for a total of \$320) because B_4 and B_3 will win at the price of B_2's bid of \$10. If B_5 and B_6 choose to bid then they will win at the price of \$90 and the seller's total revenue will be \$100+ \$90 + \$90 = \$280, a decrease of \$20 with no additional sales. If however, only B_6 participated in the auction then B_6 and B_4 would win the auction at a closing price of \$80 leading to \$200 of posted price revenue (from B_5 and B_7) and \$160 of auction revenue (from B_6 and B_4) for a total of \$360.

This example illustrates that while keeping high valuation consumers (consumers with $V > p$) out of the auctions can increase sales and revenue, when high valuation consumers do participate in the auction there is the side benefit of an increase in the auction price. Hence, while the design of the auction should aim to discourage high valuation consumers from bidding, which is equivalent to narrowing the time period in which high valuation consumers choose to bid (reducing $\bar{t}$) as depicted in Figure 2, this objective is tempered by the positive effect high valuation bidders have on auction prices. The seller also has two other related goals, namely to increase the number of units sold in the auction per unit time and the prices paid in the auction. To identify auction design strategies, we consider how the choice of q and T affects these three goals. We summarize our findings in Table 2.

Goal:	Limit $\bar{t}$	Increase auction sales	Increase auction price
Strategy 1:	Large T	Large q	Large T
Strategy 2:	Small q	Small T	Small q

Table 2: Potential auction design strategies

Increasing T will increase the auction price because it increases the number of bidders who cannot buy at the posted price i.e., it increases competition for the auctioned items (defined as the number of bidders per unit auctioned). A higher auction price means that $\bar{t}$ will be smaller because high valuation consumers need a smaller delay cost to make the auction worthwhile. On the other hand long auctions decrease the total auction sales per unit time because it takes longer to sell every unit auctioned. Short auctions have the opposite effect on each goal. Decreasing q is another way to increase competition in the auctions. Small lot sizes will increase auction prices and as a result drive away high valuation consumers while at the same time few items are sold via auction, and the auction sales are smaller. Large lot sizes can compensate for the negative aspects of long auctions and short auctions can compensate for the negative aspects of small lot sizes. The two strategies described in Table 2 follow naturally. Considering the seller's needs, limiting high valuation consumers' participation in the auction and increasing the number of units auctioned per unit time while maintaining a high auction price, the seller should choose one of the following two strategies: (1) setting one-unit auctions and decreasing the auctions length as the consumers' arrival rate increases or (2) setting long auctions and increasing the auction lots as the consumers' arrival rate increases. When the arrival rate is low, the seller's main concern is the auction price and cannibalization of sales at the posted price. In this case, the seller may need to set maximum length, one-unit auctions in order to sell auctioned units above marginal cost, or it may even become suboptimal to add the auctions.

4.1 Numerical experiments

In the following numerical experiments we assume that consumer valuations follow a uniform distribution over [\$0, \$100]. Hence, we can use a closed-form expression for the expected value of the order statistics, $O\{x, y\}$, in our model. We vary the arrival rate λ between 0.5 and 60 consumers per day and vary the delay cost, w, between \$0.1 and \$5 per day. For each combination of λ and w we determine the optimal values of q, T and p. We use equation (10) to calculate the seller's expected revenue for given q, T, p, λ, w, and $\bar{t}$. For the threshold used by high valuation consumers, $\bar{t}$, we employ the heuristic approach given in Proposition 2, instead of solving Equation (5). To summarize, given (w, λ) we numerically calculate the exact expected revenue observed by the seller when consumers use the heuristic of Proposition 2 for each triplet (q, T, p) to find the optimal dual channel design.

Recall that in our model consumers with $V < p$ always participate in the auction because they have no other purchase options. Yet it is reasonable to assume that even such bidders will not join an auction if its duration is too long. In other words, we assume that there is a maximum auction duration H, such that consumers ignore the auction until the time remaining in the auction is less than or equal to H. For the base case we assume that $H = 168$ hours (seven days). Experiments with different values of H did not qualitatively change our results (see Appendix 2). To reduce the computational burden we treat T as a discrete variable $T \in \{1 \text{ hour}, 2 \text{ hour}, \ldots, H \text{ hour}\}$. Table 3 summarizes our findings for $H = 7$ days.

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**	**\$5/day**
0.5/day	No auction, P=\$50					1;H; \$56	1; H;\$56
1/day	1; H; \$52	1; H; \$53	1; H; \$53	1; H; \$53	1; H; \$54	1; H; \$54	1; H; \$55
2/day	1; 134; \$52	1; 135; \$52	2; H; \$53	2; H; \$53	2; H; \$54	3; H; \$56	3; H; \$57
5/day	1; 53; \$52	1; 55; \$52	6; H; \$52	7; H; \$54	8; H; \$55	8; H; \$57	9; H; \$58
10/day	1; 27; \$52	1; 27; \$52	13; H; \$52	16; H; \$54	18; H; \$56	19; H; \$58	20; H; \$59
20/day	1; 13; \$52	1; 14; \$52	30; H; \$52	35; H; \$54	38; H; \$56	39; H; \$58	40; H; \$59
30/day	1; 9; \$52	1; 9; \$52	47; H; \$52	54; H; \$54	58; H; \$57	60; H; \$58	62; H; \$60
40/day	1; 7; \$52	1; 7; \$52	1; 7; \$52	75; H; \$55	78; H; \$57	80; H; \$58	83; H; \$60
50/day	1; 5; \$52	1; 5; \$52	1; 5; \$52	94; H; \$55	98; H; \$57	100; H; \$58	104; H; \$60
60/day	1; 4; \$52	1; 4; \$52	1; 4; \$52	113; H; \$55	118; H; \$57	121; H; \$58	125; H; \$60

Table 3: The optimal (q,T [Hr],p[\$]) for various parameter values.

Reinforcing our previous discussion, we see that the optimization results in one of the two cases we predicted, one-unit auctions with the length of the auction decreasing in the arrival rate or long auctions (seven days, the maximum length) with the size of the auction lot increasing in the arrival rate. Which of the two settings, [*one-unit auction*] or [*maximum length auction*], is optimal depends on the delay cost per unit time incurred by high valuation consumers and on the consumer arrival rate. When w is low the only way to deter high valuation consumers from bidding is to reduce the size of the auction lot. A long auction will not work because if w is sufficiently low, high valuation consumers will always choose to bid (regardless of the time they arrive during the auction). For small values of w the seller should therefore, offer one-unit auctions and the length of the auctions should increase as the arrival rate decreases. When the delay cost, w, is high, it deters high valuation consumers from bidding. The seller should set the auction length to the maximum and increase the size of the auction lot as the consumers' arrival rate increases. When arrival rates are low, the main concern is the auction price and cannibalization of the posted price channel, so it becomes optimal to offer one-unit auction with the maximum length (7 days) (which is an extreme case of each of the above two strategies). As the arrival rate decreases, the single channel might outperform the dual channel. As w increases, the dual channel outperforms the single channel even for smaller arrival rates.

λ: w:	**$0.1/day**	**$0.5/day**	**$1/day**	**$2/day**	**$3/day**	**$4/day**	**$5/day**
1/day	.09; .05; .39	.09; .06; .38	.08; .06; .39	.07; .07; .4	.07; .07; .4	.06; .08; .4	.06; .08; .4
2/day	.11; .07; .85	.11; .07; .85	.19; .1; .75	.17; .11; .8	.16; .13; .8	.23; .2; .6	.21; .22; .6
5/day	.29; .17; 2.1	.27; .16; 2.1	.66; .2; 1.7	.71; .29; 1.6	.74; .4; 1.5	.67; .48; 1.5	.68; .61; 1.4
10/day	.56; .33; 4.2	.56; .33; 4.2	1.52; .34; 3.3	1.7; .58; 2.9	1.71; .86; 2.7	1.61; 1.1; 2.6	1.52; 1.33; 2.6
20/day	1.16; .68; 8.4	1.08; .64; 8.5	3.63; .65; 6.0	3.8; 1.2; 5.4	3.66; 1.77; 5.1	3.33; 2.25; 5.1	3.07; 2.65; 5.1
30/day	1.68; .99; 12.7	1.68; .99; 12.7	5.76; .95; 8.6	5.89; 1.82; 7.9	5.55; 2.73; 7.3	5.13; 3.45; 7.5	4.71; 4.15; 7.3
40/day	2.15; 1.28;17.0	2.15; 1.28; 17.0	2.15; 1.28; 17.0	8.15; 2.56; 9.8	7.47; 3.67; 9.7	6.84; 4.59; 10.0	6.31; 5.55; 9.7
50/day	3.; 1.79; 21.0	3.; 1.79; 21.0	3.; 1.79; 21.0	10.23; 3.2; 12.3	9.39; 4.61; 12.1	8.55; 5.73; 12.4	7.91; 6.95; 12.1
60/day	3.75; 2.24;25.0	3.75; 2.24; 25.0	3.75; 2.24; 25.0	12.3; 3.84; 14.7	11.31; 5.55; 14.5	10.35; 6.93; 14.8	9.51; 8.35; 14.5

Table 4: Allocation of items to consumers per 24 hour period (Number of units won by high valuation; Number of units won by low valuation; number bought at posted price)

In Table 4 we display how the units sold are allocated among the different groups of consumers, high valuation auction participants, low valuation auction participants, and high valuation consumers who purchase at the posted price. We can see that in the cases where it is optimal to have short one-unit auctions, the number of units bought by high valuation consumers in the auction is small relative to the number they buy at the posted price. When it is optimal to have long auctions with larger lot sizes, the auctions become responsible for a significant fraction of the sales to high valuation consumers. We also see that more auction sales tend to go to the high valuation consumers than to the low valuation consumers. However, as the delay cost *w* increases (moving horizontally across the table) we see that a higher proportion of auction sales go to the low valuation consumers since participating in the auction becomes too costly in terms of delay for the high valuation consumers. Looking at Table 5 we can see that having more high valuation consumers in the auctions tends to increase the auction price. What is happening is that when *w* gets large enough the seller can offer more units in the auctions without there being a complete collapse of the posted-price channel. The result is that the reduction in posted price sales is more than made up for by the increase in sales in the auction and the increase in auction prices driven by high valuation consumer participation.

λ: w:	$0.1/day	$0.5/day	$1/day	$2/day	$3/day	$4/day	$5/day
1/day	$36.97; 21%	$37.24; 21%	$36.5; 20%	$35.26; 19%	$35.17; 19%	$34.37; 19%	$34.59; 19%
2/day	$42.11; 15%	$41.85; 14%	$43.7; 24%	$38.18; 23%	$34.43; 22%	$47.67; 32%	$43.7; 31%
5/day	$42.05; 15%	$42.26; 14%	$45.98; 30%	$45.81; 35%	$44.38; 38%	$44.75; 38%	$43.1; 40%
10/day	$42.25; 15%	$42.17; 15%	$47.53; 34%	$46.89; 41%	$46.05; 44%	$45.54; 45%	$44.24; 45%
20/day	$41.87; 15%	$42.52; 14%	$48.17; 40%	$47.32; 45%	$46.43; 47%	$46.06; 47%	$45.04; 46%
30/day	$42.21; 15%	$42.21; 15%	$48.43; 42%	$47.46; 46%	$47.41; 48%	$46.04; 48%	$45.69; 48%
40/day	$42.52; 14%	$42.52; 14%	$42.52; 14%	$48.25; 49%	$47.46; 49%	$46.17; 48%	$45.77; 48%
50/day	$41.42; 15%	$41.42; 15%	$41.42; 15%	$48.31; 49%	$47.5; 49%	$46.25; 48%	$45.81; 48%
60/day	$40.99; 16%	$40.99; 16%	$40.99; 16%	$48.36; 49%	$47.52; 49%	$46.21; 48%	$45.84; 48%

Table 5: Expected auction price and fraction of total revenue coming from auction channel (Auction price; auction revenue fraction)

We can also see in Table 3 that the posted price is increasing as *w* increases and as λ increases. When there is no auction, increasing the posted price from the optimal point leads to a loss of sales from those consumers who are priced out that cancels any revenue gains from higher prices. When there is an auction the consumer who was priced out by a price increase will not be completely lost because he may purchase in the auction and his participation increases the auction price. When *w* is large the seller is also less concerned about losing high valuation consumers to the auction, the net effect is that it becomes optimal to raise the posted price as *w* increases. Table 5 shows that as *w* increases the fraction of revenues coming from auctions eventually starts to decrease. When λ increases the posted price increases for a different reason. As the website traffic increases it becomes more attractive to have more sales via auctions. Increasing the posted price means that a larger proportion of the consumers are targeted by the auction channel. In Table 5 we can see that as λ increases the fraction of revenues coming from auctions increases.

Figures 3 and 4 further analyze the changes in the optimal design, based on the numerical results in Table 3. From Figure 3 we see that when *w* is such that it is optimal to set the auction length to the maximum ($T = H$), the optimal lot size increases with the arrival rate at a relatively constant rate, enabling the seller to capture more consumers (sell more units via auction per unit time). When the arrival rate is low, the size of the auction lot decreases to one unit. Figure 4 reveals that for small values of *w*, when it is optimal to have one-unit auctions, the optimal auction length is decreasing with the arrival rate. As the arrival rate increases, the shorter auctions enable the seller to capture more consumers per unit time.

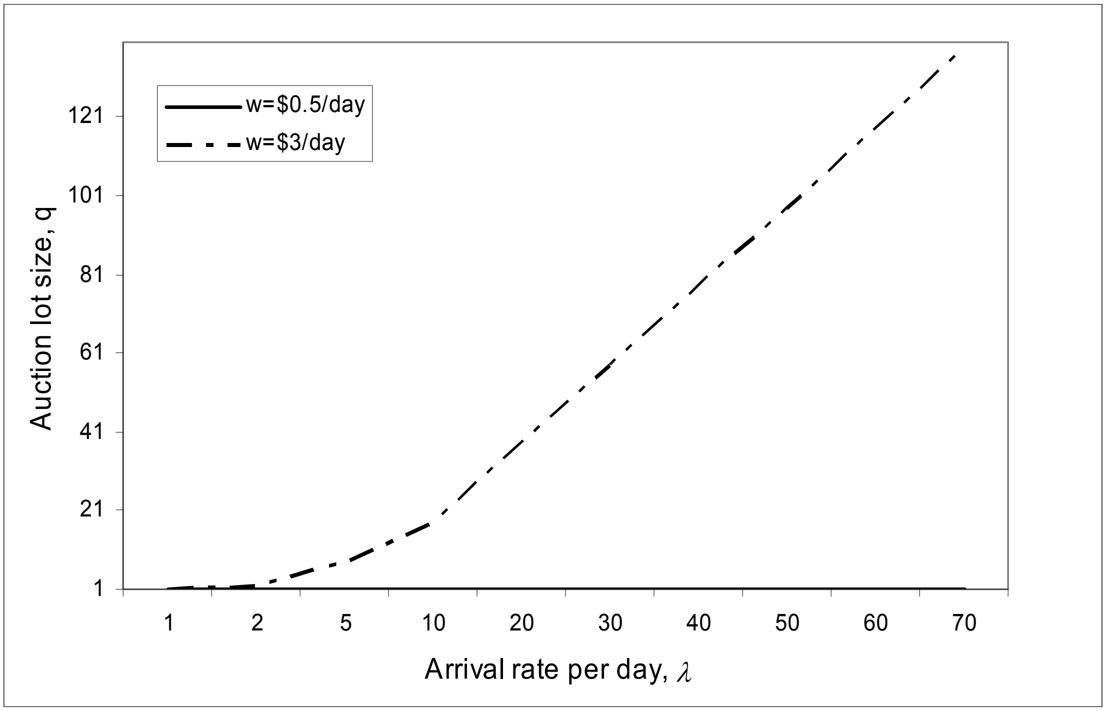

Figure 3: The optimal auction lot size as a function of the arrival rate and delay cost, *w*, when setting auction length $T = H$ is optimal.

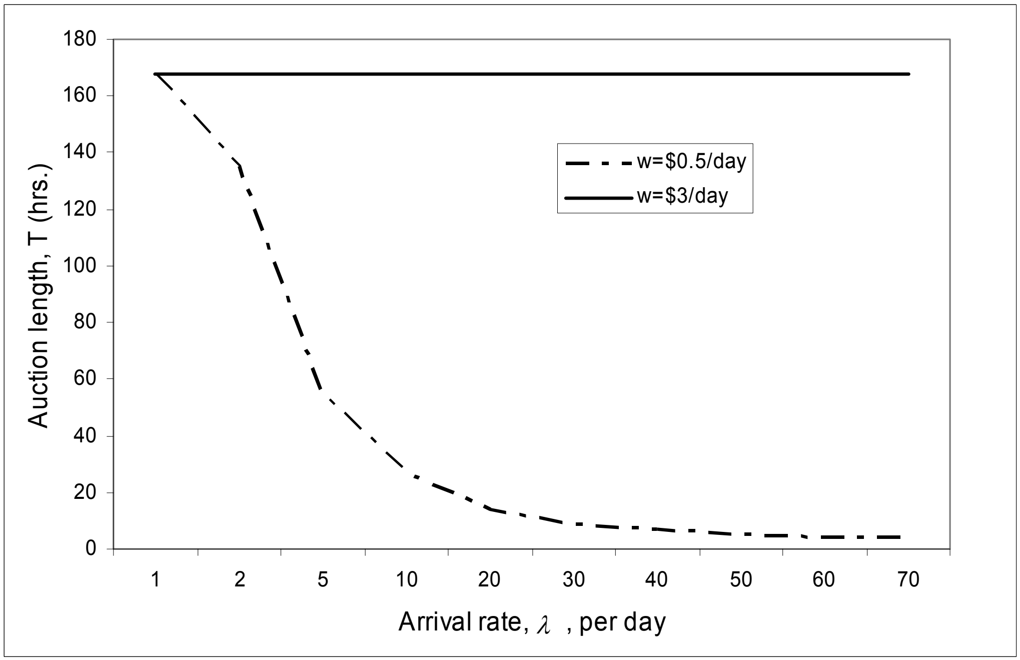

Figure 4: The optimal auction length as a function of the arrival rate and delay cost, *w*.

4.2 Revenue Comparison

In Figure 5 we plot the percentage increase in revenue from using the dual channel over the single channel (only posted price) as a function of the delay cost *w*, for three values of arrival rate , $\lambda = 1$/day, $\lambda = 5$/day and $\lambda = 30$/day. We see that the dual channel's revenue is at least 2.4% greater than the posted price revenue and as *w* increases we have a substantial increase in revenue of 10% or more. It is interesting to see that the revenue increase is not always increasing with the arrival rate.

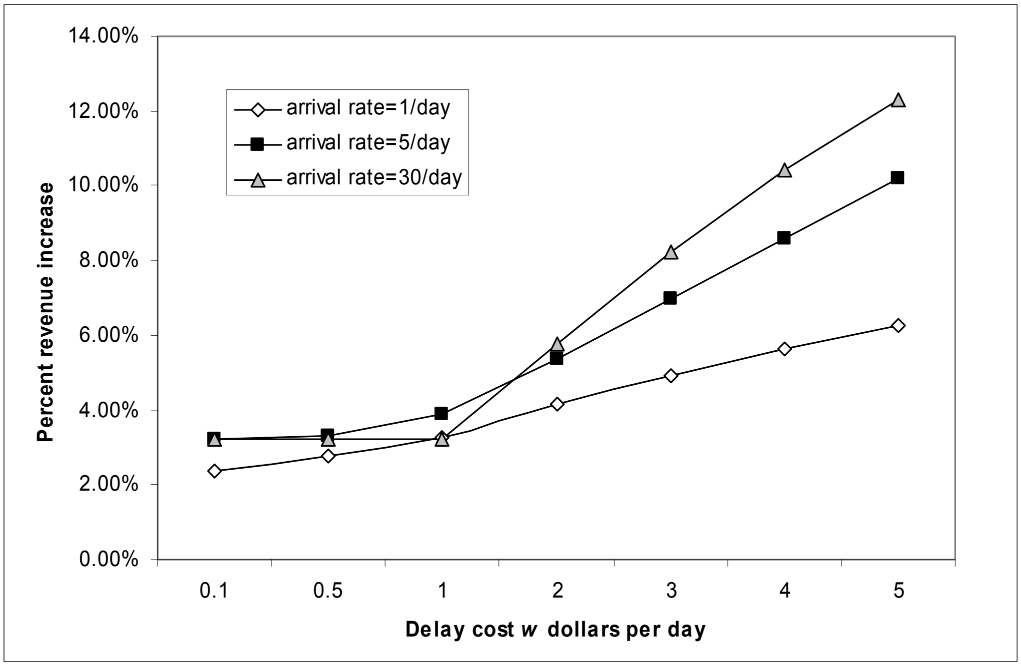

Figure 5: Revenue increase over posted price as a function of delay costs, *w,* and arrival rate, λ.

In Figure 6 we compare the revenue from the optimal design of the dual channel with the revenue when the seller uses the more naïve approach of managing the auction and posted price channels independently. In this naïve approach, the posted price is set as if there were no auction channel, and the auction channel design parameters *T* and *q* are selected as if there were no posted price channel (see Appendix 4 for detailed results). We find that such an approach leads to even lower revenue than having just a posted price channel. We also find that the auction length used in this case is the maximum, *H*, which is often much larger than what would be optimal, and that the auction lot size is much larger than would be optimal if the two channels were managed jointly. When compared to this approach the benefits of the optimal dual channel design are even greater than when compared to just a posted price channel.

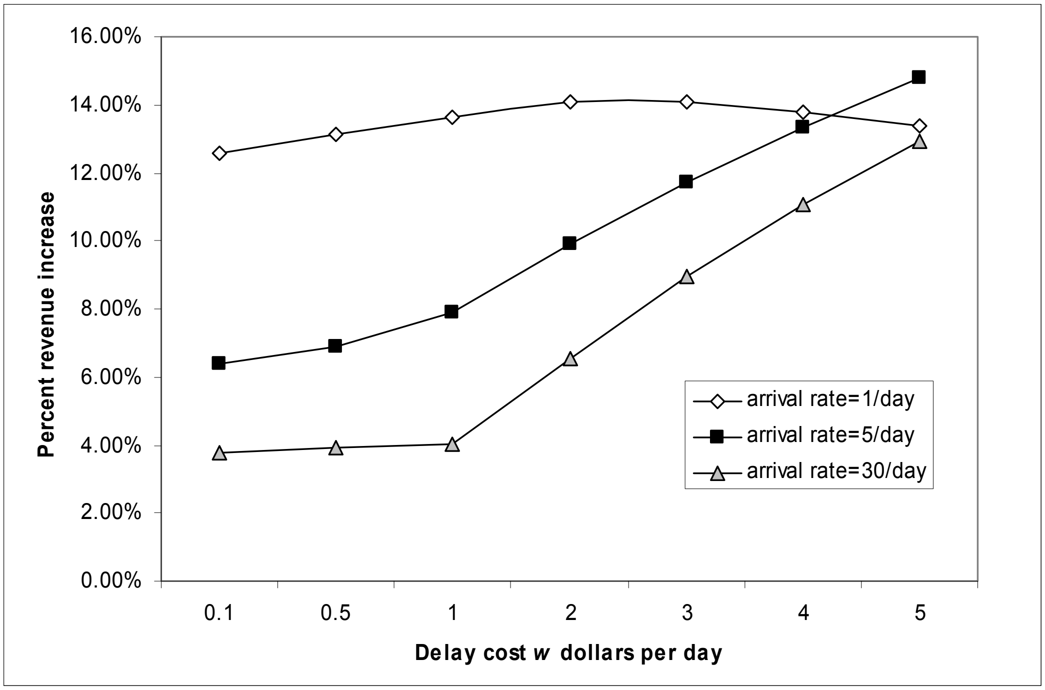

Figure 6: Revenue increase optimal dual channel design gives over independent management of dual channels as a function of delay cost, *w,* and arrival rate, λ.

The seller's incentive to add auctions is greater when the delay cost perceived by consumers, *w*, is high. In such circumstances, more effective segmentation of consumers can be achieved because the seller can sell more via auction without losing too much posted price revenue and it becomes optimal to increase the auction length and lot size. In addition, when *w* is large, the incentive to add auctions increases as the arrival rate of consumers' increases. This is shown in Figure 7. As the arrival rate increases, the expected auction price increases, which deters high valuation consumers from bidding and increases the revenue from the auction at the same time. Figure 8 shows that for lower arrival rates it is most important to jointly optimize both the auction and the posted price channels.

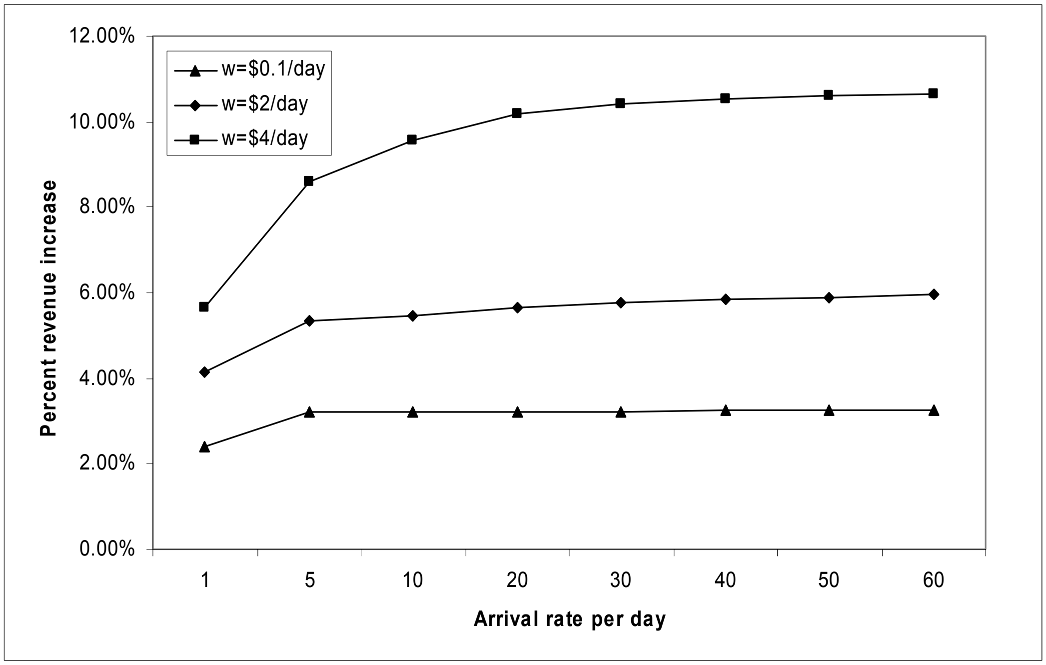

Figure 7: Percent revenue increase over posted price, as a function of arrival rate, λ, and delay cost, w.

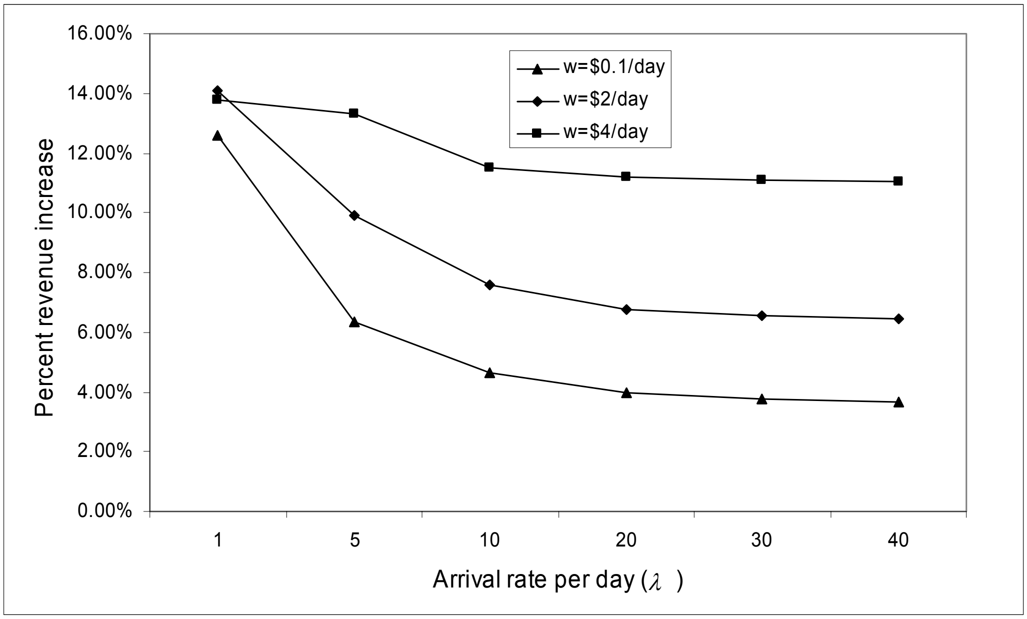

Figure 8: Percent revenue increase optimal dual channel design gives over independent management of dual channels, as a function of arrival rate, λ, and delay cost, w.

5. Concluding Remarks

It is possible to observe many firms selling the same or very similar goods online using posted prices and auctions simultaneously. Our paper explains this phenomenon by showing how posted price, auction length , and auction duration can be used to segment consumers between an online auction channel and posted price channel to increase a seller's revenue. This allows the auction to be used to capture customers who were priced out by the posted price while mitigating the effects of cannibalization of the posted price channel. Numerical simulations show that the dual channel can significantly outperform a lone posted price channel; we see revenue increases ranging from 2.4% to greater than 10% which are considered significant improvements in retail sales. We see even greater benefits over a naïve approach to managing the dual channels by optimizing each independently of the other.

Balancing the need to avoid cannibalizing the posted price channel with the opportunity to exploit the potential of the auction channel may seem daunting to a manager. Interestingly, our analysis shows that managing the dual channel optimally may be simpler than it seems. We find that as long as arrival rates are reasonably high, there are only two dominant strategies for managing the dual channel, and in both the posted price is set higher than when there is no parallel auction. One strategy is to offer successive one-unit auctions parallel to the posted price. This strategy appears to be optimal for a wide range of parameter values and is indeed commonly observed in practice. The second strategy is to offer long auctions and becomes optimal when consumers' delay cost increases. The choice of the optimal strategy depends upon the customer traffic to the website, λ, and the delay cost associated with the product being sold, w.

Our model contains several unique features that are expressed in our model of consumer behavior. We model consumers as making their auction participation decision using an estimate of their expected discount. Their participation decision is directly influenced by the quantity being auctioned relative to expectations about the number of other bidders. I.e., bidders know that if they face greater competition in the auction the discount over the posted price channel will be smaller. The result is a very realistic and rich portrayal of bidder behavior.

There are a number of interesting areas for future research that are related to this paper. By parameterizing our model on R, the seller's reserve price, we could identify the optimal reserve price as well as posted price, lot size and auction length. We suspect that the lot-size and auction length variable already capture most of the effects of the reserve price. Such a result would be interesting in its own right. The heuristic we have proposed in Proposition 2 is optimistic in its assessment of the bidder's ability to estimate the auction discount. Future research could explore alternative heuristics and their impact on seller decisions.

Our model could also be extended to situations in which there is a finite supply that must be allocated between the two channels. Another extension would be to introduce competition between selling channels to address situations in which there are multiple auctioneers and posted price sellers in the market. In all of these possible extensions the underlying interaction between the auction lot size, auction length, and posted price introduced in this paper will play an important role. As demonstrated in Figures 6 and 8, failure to manage their interactions correctly can significantly reduce revenues.

Appendix 1

Proof of Lemma 1:

The auction mechanism awards the items to the bidders with the q highest bids and each pays a price equal to the highest losing bid. We assume a random tie-breaking rule.

For each player i define his type as his valuation V_i, and his action as his bid b_i. A strategy is a function from the type space $[\underline{v}, \bar{v}]$ to the action space, $B = [0,\infty)$[3]. See Gibbons (1992, Chapter 3) for a detailed definition of static Bayesian games.

Define $b_{-i} = (b_1, b_2, \ldots b_{i-1}, b_{i+1}, \ldots b_N)$, as the vector of actions by the N-1 players besides player i. The payoff received by player i, for each combination of actions that could be chosen by the other players, b_{-i}, is given by [4]:

$$U_i(b_i, b_{-i}; V_i) = \begin{cases} \theta \times [V_i - p_a(b_i, b_{-i})] + (1-\theta) \times [V_i - p] & \text{if } V_i \geq p \\ \theta \times [V_i - p_a(b_i, b_{-i})] & \text{else} \end{cases}$$

where $p_a(b_i, b_{-i})$ is the resulting auction price, given by the q+1 highest bid when there are more than q bids, and by the seller's reserve price otherwise, and θ is the probability player i gets an item in the auction.

θ is a deterministic function of b_i, b_{-i}, and q, given by:

---

[3] Or, when the current lowest bid required to win is listed, the action space can be bounded from below by that price (choosing not to bid dominates a bid lower than the current lowest bid required to win).

[4] We do not include any cost of delay because once the consumer decides to bid this cost is sunk. Here we assume the decision to bid has already been made, and the search is for a bidding strategy.

$$\theta = \begin{cases} 1 & if \quad b_i > p_a(b_i, b_{-i}) \\ 0 & \quad b_i < p_a(b_i, b_{-i}) \\ \dfrac{q - C^+(p_a)}{C(p_a)} & \quad b_i = p_a(b_i, b_{-i}) \end{cases}$$

$C(x)$ is the number of bids in b (the vector of all bids $b=(b_{-i};b_i)$) that equal x and $C^+(x)$ is the number of bids in b that are strictly larger than x .

Next we show that when player i values the item for more than its posted price, $V_i \geq p$, he can do no better than bidding p. That is, for any other action $b'_i \neq p$, for any number of bidders N and combination of other bidders actions b_{-i}, $U_i(p, b_{-i}; V_i) \geq U_i(b'_i, b_{-i}; V_i)$, with strict inequality for some b_{-i}.

We divide the space of feasible b_{-i} into three exclusive sets (columns in Table 4): the first set includes instances of b_{-i} that together with $b_i = p$ result in an auction price that is higher than p; the second set includes instances of b_{-i} that together with $b_i = p$ result in an auction price that equals p; and the last set includes instances of b_{-i} that together with $b_i = p$ result in an auction price that is lower than p. Table 4 shows the payoff matrix for a player of type $V_i \geq p$.

	$\boldsymbol{b_{-i} : p_a(p, b_{-i}) > p}$	$\boldsymbol{b_{-i}: p_a(p, b_{-i})=p}$	$\boldsymbol{b_{-i} : p_a(p, b_{-i}) < p}$
$\boldsymbol{b_i = p-\Delta}$	V_i-p	V_i-p	$\theta\times [V_i- p_a(p, b_{-i})]+[1-\theta]\times[V_i-p]$ with $\theta<1$ for some b_{-i}
$\boldsymbol{b_i = p}$	V_i-p	V_i-p	$V_i- p_a(p, b_{-i})$
$\boldsymbol{b_i = p+\Delta}$	$\theta\times [V_i- p_a(p+\Delta, b_{-i})]+[1-\theta]\times[V_i-p]$ with $\theta>0$ for some b_{-i}	$\theta\times [V_i- p_a(p+\Delta, b_{-i})]+[1-\theta]\times[V_i-p]$ with $\theta>0$ and $p_a(p+\Delta, b_{-i})>p$ for some b_{-i}	$V_i- p_a(p, b_{-i})$

Table 4: Payoff matrix for player of type $V>p$

For any $\Delta > 0$ and any b_{-i}, $U_i(p, b_{-i}; V_i) \geq U_i(p+\Delta, b_{-i}; V_i)$ and $U_i(p, b_{-i}; V_i) \geq U_i(p-\Delta, b_{-i}; V_i)$ and there is some b_{-i} for which these inequalities are strict. A detailed analysis follows.

Payoffs when $b_i = p$

- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) > p}$ player i does not win the item in the auction and purchases it for the posted price, with payoff V_i-p.

- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) = p}$ player *i* wins the item in the auction with probability $0 \le \theta(p, b_{-i}) \le 1$. If he wins the item he pays *p* and if he does not win he purchases it for *p*. Either way his payoff is $V_i - p$.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) < p}$ player *i* wins the item in the auction with payoff $V_i - p_a(p, b_{-i}) > V_i - p$.

Payoffs when $b'_i = p+\Delta$

- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) > p}$ player i's payoff as a function of b_{-i} is given by $\theta[V_i - p_a(p+\Delta, b_{-i})]+[1-\theta][V_i - p]$ where $p_a(p+\Delta, b_{-i}) \ge p_a(p, b_{-i}) > p$ and $\theta \in [0,1]$. The payoff thus is never higher than the payoff when he bids *p*. When b_{-i} is such that $p_a(p, b_{-i}) < p+\Delta$ player *i* wins the item in the auction and pays $p_a(p+\Delta, b_{-i}) \ge p_a(p, b_{-i}) > p$ and and so his payoff is less than his payoff from bidding *p*.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) = p}$ player *i*'s payoff as a function of b_{-i} is given by $\theta[V_i - p_a(p+\Delta, b_{-i})]+[1-\theta][V_i - p]$ where $p_a(p+\Delta, b_{-i}) \ge p_a(p, b_{-i}) = p$ and $\theta \in [0,1]$. The payoff is thus never higher than the payoff when he bids *p*. Furthermore, for b_{-i} such that $p+\Delta > p_a(p+\Delta, b_{-i}) > p$ player *i* wins the item in the auction and pays $p_a(p+\Delta, b_{-i}) > p$ so his payoff is less than his payoff from bidding *p*.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) < p}$ player *i* wins the item in the auction with payoff $v_i - p_a(p, b_{-i})$, the same payoff as when he bids *p*.

Notice that $U_i(p, b_{-i}; V_i) \ge U_i(p+\Delta, b_{-i}; V_i)$ for every feasible vector b_{-i}, with strict inequality for some instances of b_{-i}. We conclude that $b'_i = p+\Delta$ is weakly dominated by $b_i = p \ \forall \ \Delta > 0$.

Payoffs when $b'_i = p - \Delta$

- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) > p}$ player *i* does not win the auction and thus purchases the item for the posted price with payoff of $V_i - p$ which is the same as when he bids *p*.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) = p}$ player *i* does not win the auction and thus purchases the item for the posted price with payoff of $V_i - p$ which is the same as when he bids *p*.
- When b_{-i} is such that $\boldsymbol{p_a(p, b_{-i}) < p}$ player i's payoff as a function of b_{-i} is given by $\theta[V_i - p_a(p-\Delta, b_{-i})]+[1-\theta][V_i - p]$ where $\theta > 0$ only if $p_a(p-\Delta, b_{-i}) = p_a(p, b_{-i})$ (if player *i* lowers the auction price by bidding less than *p*, he will be the highest losing bid). The payoff thus is never higher than the payoff from bidding *p* and is strictly lower for b_{-i} such that $\theta < 1$.

We note that $U_i(p, b_{-i}; V_i) \geq U_i(p-\Delta, b_{-i}; V_i)$ with strict inequality for some instances of b_{-i}. We conclude that $b'_i = p-\Delta$ is weakly dominated by $b_i = p$. Since the action $b_i = p$ weakly dominates every other action in B, it is a weakly dominant action for a bidder with $V_i > p$ [Mass-Colell et al, 1995, Chapter 8 pp. 238].

In a similar way we can prove that $b_i = V_i$ is a weakly dominant action when $V_i < p$. Table 5 shows that for any number of bidders N and for any combination of bidders' actions b_{-i}, player i can not do better than bidding V_i by bidding $V_i - \Delta$ or $V_i + \Delta$, for any $\Delta > 0$, and for some instances of b_{-i} he does strictly worse.

	$b_{-i} : p_a(V_i, b_{-i}) > V_i$	$b_{-i} : p_a(V_i, b_{-i}) = V_i$	$b_{-i} : p_a(V_i, b_{-i}) < V_i$
$b_i = V_i - \Delta$	0	0	$\theta[V_i - p_a(V_i, b_{-i})]$ with $\theta<1$ for some b_{-i}
$b_i = V_i$	0	0	$V_i - p_a(V_i, b_{-i})$
$b_i = V_i + \Delta$	$\theta[V_i - p_a(v_i+\Delta, b_{-i})]$ with $\theta>0$ for some b_{-i}	$\theta[V_i - p_a(V_i+\Delta, b_{-i})] \leq 0$	$V_i - p_a(V_i, b_{-i})$

Table 5: Payoff matrix for a player of type $V < p$

Since the strategy $b(V) = \begin{cases} V & \text{for } V < p \\ p & \text{for } V \geq p \end{cases}$ satisfies $U_i(b(V_i), b_{-i}; V_i) \geq U_i(b'(V_i), b_{-i}; V_i)$ for every feasible vector b_{-i}, with strict inequality for some instances of b_{-i}, it is a weakly dominant strategy.

■

Proof of Proposition 1:

Decision rule (5), "participate in auction if $\Pr(win|p)p - E[auction\_payment \mid p] - wt^e \geq 0$," is derived from rearranging the condition that $U^A(V) > U^B(V)$ assuming that bidders bid according to the strategy specified in Lemma 1 so that $U^A(V) = \mathrm{E}U(p, V)$.[5] Then, the key observation is that because $\Pr(win|p)$, the probability of winning with a bid of p, and $E[auction\_payment|p]$, the expected auction payment with a bid of p, are not functions of V, decision rule (5) does not depend on the individual's valuation of the item, V. The only difference between high valuation consumers with respect to the participation decision, is the remaining time of the auction that they observe upon arrival, t^e, where $0 \leq t^e \leq T$.

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[5] Actually, we do not need to assume that bidders bid according to the bidding strategy in Equation (3), but only that they use it to estimate the highest payoff they can obtain from the auction.

Since (5) does not depend on *V*, all high valuation consumers use the same decision rule to choose between purchasing the item for the posted price and participating in the auction. Assume **all** high valuation consumers use the same threshold t^*. A threshold for which $\Pr(win|p)p - E[auction\_payment \mid p] > wt^*$ cannot be optimal, since high valuation consumers who arrive with $t^* + \varepsilon$ time units remaining in the auction would find it optimal to bid (contradicting t^* being the optimal common threshold). Similarly a threshold for which $\Pr(win|p)p - E[auction\_payment \mid p] < wt^*$ can not be optimal as high valuation consumers who arrive with t^*-ε time units remaining in the auction, would not find it optimal to bid. Hence the unique optimal common threshold is given by the solution of the following fixed point equation:

$$\Pr(win|p)p - E[auction\_payment \mid p] = wt^* \tag{11}$$

where both Pr(*win*|*p*) and E[*auction_payment*|*p*] are functions of the commonly used threshold t^*. $\Pr(win|p)$ is non-increasing in the commonly used threshold, t^*, because the expected number of bids that equal *p* (and the rate of the Poisson process generating bids that equal *p*) is increasing with the commonly used threshold so the probability that a high valuation consumer wins (assuming a random tie-breaking rule) is non-increasing with the threshold. Likewise, $E[auction\_payment|p]$ is non-decreasing in the commonly used threshold t^* because holding everything else equal, as t^* increases the expected number of bids that equal *p* (and the rate of the Poisson process generating bids that equal *p*) increases, so the expected auction payment of a high valuation consumer clearly can not decrease. Hence, the LHS of Equation11 is non-increasing in the commonly used threshold t^* and is nonnegative and the RHS of Equation 11 is linearly increasing in t^* so the solution of Equation (11) exists and is unique. We conclude that decision rule (5) can be restated as decision rule (6) where $\bar{t}$ is given by the solution of Equation (11).

■

Proof of Proposition 2:

We need only show that t_w, the solution of the following fixed point equation

$$\begin{aligned} wt_w = {} & \rho\gamma D(\lceil \lambda \Pr(V > p)t_w + 1\rceil, \lceil \lambda \Pr(V < p)T\rceil) - \\ & \rho(1-\gamma)D(\lceil \lambda \Pr(V > p)t_w + 1\rceil, \lfloor \lambda \Pr(V < p)T\rfloor) - \end{aligned} \tag{12}$$

$$(1-\rho)(1-\gamma)D\left(\lfloor \lambda \Pr(V>p)t_w+1\rfloor, \lfloor \lambda \Pr(V<p)T\rfloor\right)-$$
$$(1-\rho)\gamma D\left(\lfloor \lambda \Pr(V>p)t_w+1\rfloor, \lceil \lambda \Pr(V<p)T\rceil\right)$$

is unique. The LHS of Equation (12) equals zero for $t_w = 0$ and is linearly increasing in t_w. As explained above the expected auction discount for a deterministic number of bidders from each type is given by

$$D(N^+, N^-) = \begin{cases} p - O\{q - N^+ + 1, N^-\} & \text{if } N^+ \le q \quad \text{and} \quad N^+ + N^- > q \\ 0 & \text{if } N^+ > q \\ p - R & \text{if } N^+ + N^- \le q \end{cases} . \tag{13}$$

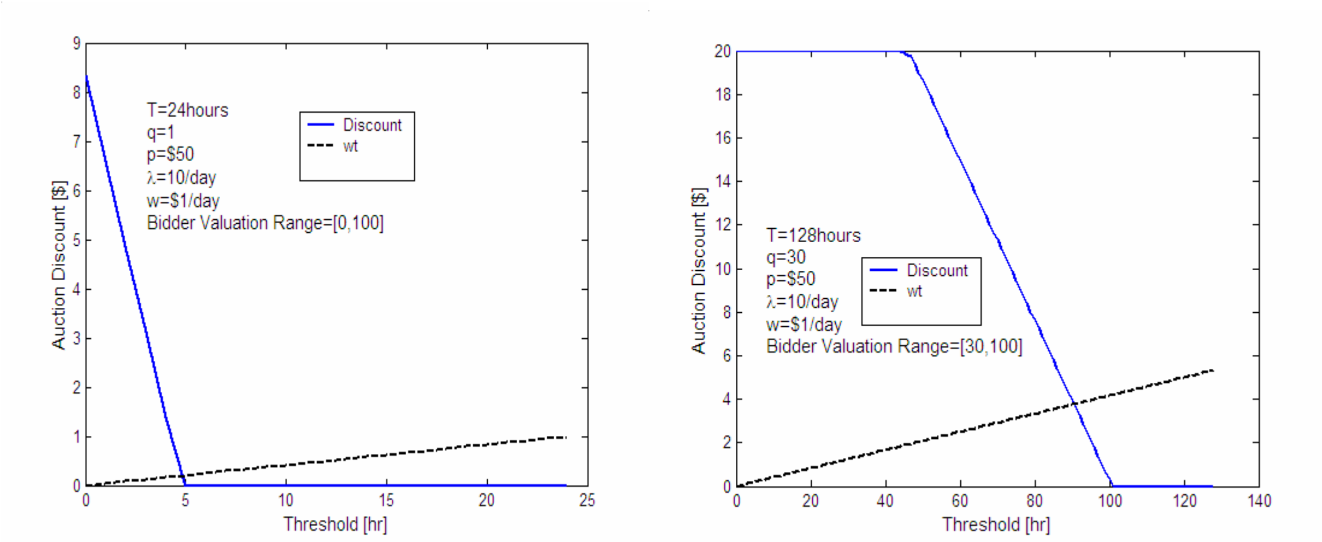
Each of the four possible terms in the RHS of Equation (12) has the following properties:

a) It is non-increasing in t_w . N^+ is non-decreasing in t_w, as $N^+ = \lceil \lambda \Pr(V>p)t_w+1\rceil$ or $N^+ = \lfloor \lambda \Pr(V>p)t_w+1\rfloor$. The expected value of the r^{th} order statistic from n draws is given by $\mu_{r:n} = n\binom{n-1}{n-r}\int_{-\infty}^{\infty} x[P(x)]^{n-r}[1-P(x)]^{r-1}dP(x)$, where $x_{(i)}$ is the i^{th} order statistic and $x_{(1)} \ge x_{(2)} \ge \cdots \ge x_{(N^-)}$ (based on David, 1981, page 33). When $N^+ \le q$ and $N^+ + N^- > q$, $r = q - N^+ + 1$, $n = N^-$ and $P(.)$ is the pdf of the consumer's valuation truncated in $[\underline{v}, p]$. If N^+ increases due to an increase in t_w , r decreases which means that we are looking for the expected value of a higher order statistics so $p - O\{q - N^+ + 1, N^-\}$ is non-increasing in t_w. When $N^+ > q$, an increase in t_w, which might increase N^+, does not change the auction discount (it stays zero). When $N^+ + N^- \le q$ an increase in t_w which might increase N^+ can either decrease the auction discount (if the total number of bidders is higher than the auction quantity) or keep it unchanged. In all cases the auction discount is non-increasing in t_w.

b) It is non-negative for $t_w = 0$. For $t_w = 0$, $N^+ = 1$, and there is one bid that equals p. All other bids are lower than p - drawn from a common distribution with the support $[\underline{v}, p)$. Hence, $O\{q, N^-\}$ is less than p and the auction discount D is non-negative.

c) It is zero for $t_w > \dfrac{q}{\lambda \Pr(V>p)}$ (see Equation 13).

Hence, the sum of the four terms in the RHS of Equation (12) is non-increasing in t_w, non-negative for $t_w = 0$, and zero for $t_w > \dfrac{q}{\lambda \Pr(V > p)}$. It follows that the two curves, the expected auction discount and the delayed-use cost, intersects at a unique value of t_w. Figures 9 and 10 provide illustrative examples of the existence of a unique solution.



Figures 9 and 10: Auction discount and delayed-use cost as function of the participation threshold.

■

Appendix 2 – Additional numerical results

Tables 6 and 7 present the optimal design of the dual channel, when the participation threshold used by high valuation consumers is derived from Proposition 2, for $H = 5$ days and $H = 3$ days (the base case of $H = 7$ days is in §4). We can see that the results are very similar in nature to those in Table 3 except that the auctions are shorter with smaller lot-sizes and the posted prices are lower.

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**	**\$5/day**
0.5/day	No auction, p=\$50						
1/day	1; H; \$52	1; H; \$53	1; H; \$53	1; H; \$54	1; H; \$54	1; H; \$54	1; H; \$55
2/day	1; H; \$52	1; H; \$52	1; H; \$52	1; H; \$52	1; H; \$53	1; H; \$53	2; H; \$55
5/day	1; 53; \$52	1; 55; \$52	3; H; \$52	4; H; \$53	5; H; \$54	5; H; \$55	5; H; \$55
10/day	1; 27; \$52	1; 27; \$52	1; 28; \$52	10; H; \$53	11; H; \$54	12; H; \$55	13; H; \$57
20/day	1; 13; \$52	1; 14; \$52	1; 14; \$52	22; H; \$53	24; H; \$54	26; H; \$56	27; H; \$57
30/day	1; 9; \$52	1; 9; \$52	1; 9; \$52	35; H; \$53	39; H; \$55	40; H; \$56	41; H; \$57
40/day	1; 7; \$52	1; 7; \$52	1; 7; \$52	48; H; \$53	53; H; \$55	54; H; \$56	56; H; \$57
50/day	1; 5; \$52	1; 5; \$52	1; 5; \$52	62; H; \$53	66; H; \$55	68; H; \$56	71; H; \$58
60/day	1; 4; \$52	1; 4; \$52	1; 4; \$52	75; H; \$53	80; H; \$55	82; H; \$56	86; H; \$58

Table 6: The optimal (q, T [hr], p [\$]) for various parameter values for H = 5 days and $\bar{t}$ derived from Proposition 2.

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**	**\$5/day**
0.5/day	No auction, p=\$50						
1/day	No auction, p=\$50						
2/day	1; H; \$52	1; H; \$53	1; H; \$53	1; H; \$53	1; H; \$53	1; H; \$53	1; H; \$54
5/day	1; 53; \$52	1; 55; \$52	2; H; \$52	2; H; \$52	2; H; \$53	2; H; \$53	2; H; \$53
10/day	1; 27; \$52	1; 27; \$52	1; 28; \$52	4; H; \$52	5; H; \$53	5; H; \$53	6; H; \$54
20/day	1; 13; \$52	1; 14; \$52	1; 14; \$52	11; H; \$52	12; H; \$53	13; H; \$53	13; H;\$54
30/day	1; 9; \$52	1; 9; \$52	1; 9; \$52	1; 9; \$52	19; H; \$53	21; H; \$54	21; H;\$54
40/day	1; 7; \$52	1; 7; \$52	1; 7; \$52	1; 7; \$52	27; H; \$53	29; H; \$54	30; H;\$55
50/day	1; 5; \$52	1; 5; \$52	1; 5; \$52	1; 5; \$52	35; H; \$53	37; H; \$54	39; H;\$55
60/day	1; 4; \$51	1; 4; \$52	1; 4; \$52	1; 4; \$52	42; H; \$53	45; H; \$54	47; H;\$55

Table 7: The optimal (q, T [hr], p [\$]) for various parameter values for H = 3 days and $\bar{t}$ derived from Proposition 2.

Appendix 3 – Results with more knowledgeable consumers

We assume that consumers evaluate the expected auction discount as if the total number of bidders of each type is given by the expected value of the relevant Poisson arrival process. We use this assumption to derive the suggested functional form for the threshold used by high valuation consumers. We believe this assumption to be more realistic than to expect that consumers can accurately determine the expected discount from the auction using the Poisson distribution and Equation (5). To examine the effect of this assumption on our results, we look at the optimal design of the dual channel when consumers are so insightful that they evaluate the auction discount using the Poisson distribution of the number of bidders- that is high valuation consumers derive the participation threshold directly from Equation (5) by solving

$\Pr(win|p)p - E[auction\_payment \mid p] = w\bar{t}$ where

$$
\begin{aligned}
&p\Pr(win \mid p) - E[auction\_payment \mid p] = \\
&p\left(\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!} + \frac{1}{x+1}\sum_{x=q}^{\infty}\sum_{y=0}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!} + \sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\right) - \\
&\left(\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}O(q-x,y) + \frac{p}{x+1}\sum_{x=q}^{\infty}\sum_{y=0}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!} + R\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\right) = \\
&\sum_{x=0}^{q-1}\sum_{y=q-x}^{\infty}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}\left(p - \left(\underline{v} + (p-\underline{v})\frac{y-(q-x-1)}{y+1}\right)\right) + (p-R)\sum_{x=0}^{q-1}\sum_{y=0}^{q-x-1}\frac{e^{-\lambda_1 T}(\lambda_1 T)^y}{y!}\frac{e^{-\lambda_2 T}(\lambda_2 T)^x}{x!}
\end{aligned}
$$

Tables 8 to 10 present the optimal design of the dual channel, when the participation threshold used by high valuation consumers is derived directly from Equation (5), for different values of H (the upper limit of the auction length). When consumers are as "knowledgeable" as the seller we see that the optimal auction length remains the longest possible, H time units, even for small w values. For one –unit auctions, the threshold found directly from Equation (5) is significantly larger than the threshold found based on Proposition 2. This means that on average when consumers are "knowledgeable", more high valuation consumers will participate in the auction than when consumers are less knowledgeable. As a result most of the auction sales will be going to the class of buyers that the seller wants buying at the posted price. The only way the seller can increase revenue by adding auctions with small lots is to use long auctions in order to increase the expected auction price and deter the high valuation consumers. For long auctions with large lots, the difference between these two thresholds decreases as w and λ increase. Thus, for large values of w and λ the two thresholds yield the same optimal design. However for small values of w and λ while short one-unit auctions perform very well in the first case (when the participation threshold is given by Proposition 2) they result in decrease in revenue (compared to only posted price) in the second case (when participation threshold is derived from Equation 5). Using the optimal designs listed in Tables 8-10 the seller can still increase his revenue by adding the auctions; but the increase is smaller than when consumers are less "knowledgeable".

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**	**\$5/day**
0.5/day	No auction , p=\$50						
1/day	No auction , p=\$50		1; H; \$52	1; H; \$53	1; H; \$53	1; H; \$54	1; H; \$54
5/day	4; H; \$50	5; H; \$51	6; H; \$52	7; H; \$54	8; H; \$55	9; H; \$57	9; H; \$58
10/day	10; H; \$50	13; H; \$51	15; H; \$52	16; H; \$54	18; H; \$56	19; H; \$58	19; H; \$59
20/day	25; H; \$50	30; H; \$51	32; H; \$52	36; H; \$54	38; H; \$56	39; H; \$58	40; H; \$59
30/day	41; H; \$50	47; H; \$51	49; H; \$52	56; H; \$55	57; H; \$56	59; H; \$58	62; H; \$60
40/day	55; H; \$50	62; H; \$51	68; H; \$52	75; H; \$55	79; H; \$57	79; H; \$58	83; H; \$60
50/day	68; H; \$50	80; H; \$51	87; H; \$52	93; H; \$55	97; H; \$56	100; H; \$58	103; H; \$60
60/day	81; H; \$50	93; H; \$51	101;H;\$52	112;H;\$55	117; H; \$57	119; H; \$58	125; H; \$60

Table 8: The optimal (q, T [hr], p [\$]) for various parameter values for H = 7 days and $\bar{t}$ derived from Equation 5.

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**	**\$5/day**
1/day	No auction, p=\$50					1; H; \$54	1; H; \$54
5/day	2; H; \$50	3; H; \$51	3; H; \$51	4; H; \$52	5; H; \$54	5; H; \$55	5; H; \$55
10/day	6; H; \$50	8; H; \$51	9; H; \$51	10; H; \$53	11; H; \$54	12; H; \$55	13; H; \$57
20/day	16; H; \$50	19; H; \$51	22; H; \$52	23; H; \$53	25; H; \$55	26; H; \$56	27; H; \$57
30/day	26; H; \$50	32; H; \$51	34; H; \$52	37; H; \$53	39; H; \$55	40; H; \$56	41; H; \$57
40/day	37; H; \$50	41; H; \$51	46; H; \$52	49; H; \$53	53; H; \$55	55; H; \$56	56; H; \$57
50/day	45; H; \$50	54; H; \$51	59; H; \$52	63; H; \$53	67; H; \$55	68; H; \$56	69; H; \$57
60/day	58; H; \$50	66; H; \$51	72; H; \$52	76; H; \$53	80; H; \$55	81; H; \$56	85; H; \$58

Table 9: The optimal (q, T [hr], p [\$]) for various parameter values for H = 5 days and $\bar{t}$ derived from Equation 5.

λ : w:	**\$0.1/day**	**\$0.5/day**	**\$1/day**	**\$2/day**	**\$3/day**	**\$4/day**	**\$5/day**
1/day	No auction, p=\$50						
5/day	1; H; \$50	1; H; \$50	1; H; \$51	2; H; \$51	2; H; \$52	2; H; \$52	2; H; \$53
10/day	2; H; \$50	3; H; \$50	4; H; \$51	5; H; \$52	5; H;52	6; H; \$53	6; H; \$54
20/day	8; H; \$50	9; H; \$50	11; H; \$51	12; H; \$52	13; H;53	13; H; \$53	14; H; \$54
30/day	13; H; \$50	15; H; \$50	17; H;51	19; H; \$52	20; H;53	22; H;54	22; H;54
40/day	20; H; \$50	23; H; \$50	24; H; \$51	27; H; \$52	28; H; \$53	29; H; \$54	30; H; \$55
50/day	24; H; \$50	29; H; \$50	32; H;51	34; H;52	36; H;53	38; H;54	39; H;55
60/day	32; H; \$50	35; H; \$50	38; H; \$51	40; H; \$52	43; H; \$53	46; H; \$54	47; H; \$55

Table 10: The optimal (q, T [hr], p [\$]) for various parameter values for H = 3 days and $\bar{t}$ derived from Equation 5.

Appendix 4 - Managing the dual channels independently

In Table 11 we display the suboptimal dual channel designs that would result if the two channels were managed independently. The auction length and lot size are chosen to maximize revenue per unit time when all consumer are low valuation, i.e. there is no posted price option. The result is that the auction design is independent of the waiting costs and lot sizes tend to be larger than in Table 3. The posted price is set as if there is no auction channel and as a result is lower than optimal. The result is that too many high valuation consumers purchase in the auction leading to too few posted-price sales.

λ : *w*:	**$0.1/day**	**$0.5/day**	**$1/day**	**$2/day**	**$3/day**	**$4/day**	**$5/day**
1/day	3; H; $50	3; H; $50	3; H; $50	3; H; $50	3; H; $50	3; H; $50	3; H; $50
2/day	7; H; $50	7; H; $50	7; H; $50	7; H; $50	7; H; $50	7; H; $50	7; H; $50
5/day	17; H; $50	17; H; $50	17; H; $50	17; H; $50	17; H; $50	17; H; $50	17; H; $50
10/day	34; H; $50	34; H; $50	34; H; $50	34; H; $50	34; H; $50	34; H; $50	34; H; $50
20/day	69; H; $50	69; H; $50	69; H; $50	69; H; $50	69; H; $50	69; H; $50	69; H; $50
30/day	104; H; $50	104; H; $50	104; H; $50	104; H; $50	104; H; $50	104; H; $50	104; H; $50
40/day	139; H; $50	139; H; $50	139; H; $50	139; H; $50	139; H; $50	139; H; $50	139; H; $50

Table 11: The optimal (q, T [*hr*], p [*$*]) for various parameter values when the two channels are managed independently

REFERENCES

1. Arnold, M. and S. Lippman 1995. Selecting a selling institution: Auctions versus sequential. *Economic Inquiry* ***33*** (1) pp. 1-23.
2. Bajari, P. and A. Hortacsu. 2003. The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions. *RAND Journal of Economics.* **34 (2)** pp. 329-55.
3. Budish, E. B., and L. N. Takeyama. 2001. Buy prices in online auctions: irrationality on the Internet? *Economics Letters.* **72** pp. 325-333.
4. Carare, O., and M. Rothkopf. 2001. Slow Dutch Auctions. *RUTCOR Research Report* 42-2001.
5. David, H.A. Order Statistics. Wiley Series in Probability and Mathematical Statistics. 1981.
6. De Vany, A. 1987. Institutions for Stochastic Markets. *Journal of Institutional and Theoretical Economics.* **143** pp. 91-103.
7. Ehrman, C., and M. Peters. 1994. Sequential Selling Mechanisms. *Economic Theory.* **4** pp.237-53.
8. Epstein, L. G., and M. Peters. 1999. A Revelation Principle for Competing Mechanisms. *Journal of Economic Theory* **88** pp. 119-160.
9. Gibbons, R. 1992. Game theory for applied economists. Princeton University Press.
10. Hann, I., and C. Terwiesch. 2003. Measuring the Frictional Costs of Online Transactions: The Case of a Name-Your-Own-Price Channel. *Management Science*, **43** (11) pp. 1563-1579.
11. Harris, M., and A. Raviv. 1981. Allocation Mechanisms and the Design of Auctions. *Econometrica* **49** (6) pp. 1477-1499.
12. Harstad, R.M. 1990. Alternative Common-Value Auction Procedures: Revenue Comparisons with Free Entry. *Journal of Political Economy* **98** (2) pp. 421-430.
13. Hidvegi, Z., and Wang, W., A.B. Whinston. 2002. Buy-price English auction. Working paper. Red McComb Graduate School of Business, University of Texas at Austin.
14. Gallien, J. 2002. Dynamic mechanism design for online commerce. Working Paper. MIT Sloan School of Management.
15. Klemperer, P. 1999. Auction theory: A guide to the literature. *Journal of Economic Surveys.* **13** (3) pp. 227-286.
16. Kultti, K. 1997. Equivalence of auctions and posted prices. *Center of Economic Research, Tilbury University Netherlands.* Working Paper.
17. Lu, X., and R. P. McAfee. 1996. The Evolutionary Stability of Auctions Over Bargaining. *Games and Economic Behavior* **15** pp. 228-254.
18. Lucking-Reiley, D. 1999. Using Field Experiments to Test Equivalence Between Auction Formats: Magic on the Internet. *American Economic Review.* **89** (5) pp. 1063-1081.
19. Mas-Colell, A., Whinston, M. D., and J. R. Green. 1995. Microeconomic Theory. Oxford University Press.

20. Maskin, E., and J. Riley. 1989. Optimal Multi-unit Auctions. F. Hahn ed. *The Economics of Missing Markets, Information and Games*. New York: Oxford University. pp. 312-35.
21. McAfee, R., and J. McMillan. 1987a. Auctions and Bidding. *Journal of Economic Literature* **25** pp. 699-738.
22. McAfee, R., and J. McMillan. 1987b. Auctions with a stochastic number of bidders. *Journal of Economic Theory* **43** pp. 1-19.
23. Milgrom, P.R. 1987. Auction Theory. T.F. Bewely ed. *Advances in Economic Theory*, edited by New York: Cambridge University. pp. 1-32.
24. Peters, M. 1999. Competition among mechanism designers in a common value environment. *Review of Economic Design.* **4** pp. 273-292.
25. Pinker, E., Seidmann, A., and Y. Vakrat. 2003. The design of Online Auctions: Business Issues and Current Research. *Management Science* **49** (11) pp. 1457-1484.
26. Riley, J., and W. F. Samuelson. 1981. Optimal Auctions. *The American Economic Review* **71** pp.381-392.
27. Riley, J., and R. Zeckhauser. 1983. Optimal selling strategies: when to haggle, when to hold firm. *The Quarterly Journal of Economics* **98** pp. 267-89.
28. Roth, A. E., and A. Ockenfels. 2002. Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet. *American Economic Review*, **92** (4) pp. 1093-1103.
29. Vakrat, Y., and A. Seidmann. 1999. Can Online Auctions Beat Online Catalogs? *Proceedings of the Twentieth International Conference on Information Systems (ICIS), Charlotte, North Carolina*.
30. van Ryzin, G., and G. Vulcano, G. 2002. Optimal auctioning and ordering in an infinite horizon inventory-pricing system. Working Paper. Graduate School of Business, Columbia University.
31. Wang, R. 1993. Auctions versus Posted-Price selling. *American Economic Review* **83** (4) pp. 838-851.