Compatibility or Fit?

Pricing and Adoption of Incompatible Technologies Under Network Effects[*]

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Preliminary Draft

February 2004

Abstract

When choosing between incompatible technologies that are characterized by network effects in business or consumer markets, e.g., software (Microsoft vs. Open Source) or "Walkie-Talkie" phones (Direct Connect vs. Push to Talk), customers must decide between compatibility with others and individual fit. Given differing customer preferences, we study the adoption dynamics and pricing strategies of firms selling to a multi-segment market and offering incompatible technologies that are characterized by network effects. When firms enter the market simultaneously, there is less compatibility compared to the social optimum. If one firm enters the market first, it can use a "divide and conquer" strategy to increase its profit compared to the simultaneous entry scenario, even when there are no switching costs. With non-negative switching costs, the first mover can increase its profit or market share, resulting in more compatibility than the social optimum. We study how the threshold switching cost depends on the strength of the network effects. In our model, the early adopters, who are "locked-in" because of switching costs, never regret their decision to adopt, whereas the late adopters, who are not subject to switching costs, are exploited by the incumbent firm. We apply our framework to examine adoption dynamics and pricing strategies when a firm competes with an unsponsored (free) technology. The presence of the free technology increases consumer surplus. If the free technology is available in the market first, increasing switching costs further increases consumer surplus.

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[*] We thank Eric Van den Steen, Thomas Weber, Stefanos Zenios, participants of the Workshop on Information Systems and Economics (Seattle 2003), seminar participants at Stanford University, and especially Tunay Tunca for helpful comments.

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1 Introduction

Many technologies exhibit network effects. Examples include computer software, email, auctions, instant messaging, and on-line dating. When there are competing, incompatible technologies, each user must decide whether it is better to adopt the same technology as others or choose the technology that best fits his own individual needs. For example, in the software market, coordination on a standard platform or application not only facilitates file exchange, but also creates a bigger repository of knowledge for troubleshooting and bug fixes, and increases the availability of complementary software. In cellular telephony, the two-way radio ("Walkie-Talkie") feature is limited to the wireless service provider's own network, e.g., Nextel's Direct Connect vs. Verizon's Push to Talk.[1] Although compatibility on the same technology increases the value of the two-way radio feature, other features of the wireless service, such as the design of the handsets or cellular coverage in different geographical areas, may cause consumers to have different preferences.

We study the adoption dynamics of incompatible technologies with competing, profit-maximizing, multi-segment sponsors (firms). We compare simultaneous entry to the case where a sponsor has the option to offer its product first (sequential entry). When the competing technologies are incompatible, users have to trade off compatibility with others and individual fit. We find that there is less compatibility compared to the social optimum when firms enter the market simultaneously, whereas there can be more compatibility when firms enter the market sequentially, i.e., when there is a first mover. We study how a first mover can leverage its position to increase profit and adjust its pricing strategy as a function of the users' switching costs. We find the threshold switching cost needed to obtain a first-mover advantage and show how it depends on the network effects. In our model, the early adopters, who are "locked-in" because of high switching costs, never regret their decision to adopt, whereas the late adopters, who are not subject to switching costs, are exploited by the incumbent firm.

We also consider the case where a technology sponsored by a profit-maximizing firm competes with a free (unsponsored) technology. An example of this type of market structure is the competition between Microsoft and Open Source software. Microsoft is clearly a dominant player in the software market, however, Open Source has been growing in popu-

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[1] However, technology providers are beginning to create inter-operable platforms (3GPP) under GPRS.

larity. The Apache web server has over 60% of the web server market, three times the share of its closest competitor, Microsoft.[2] Support for the Linux operating system has also been growing. For example, IBM has been porting applications to the Linux operating system since 1998.[3] Although Open Source software is not sponsored by a profit-maximizing firm and does not have a pricing strategy, its presence in the market is a threat to Microsoft.

We apply our framework to analyze the pricing strategies of a firm faced with a competing, incompatible, free technology. We find that if the unsponsored technology is available first and switching costs are strictly positive, then the sponsor of the late entrant will lose market share. We characterize the conditions under which a first-mover advantage exists and derive the threshold switching cost necessary for such an advantage. We find that if a firm generates more total network value than its competitor and consumers value compatibility over fit, then the threshold switching cost is zero.

Network externalities have been studied extensively (see surveys in Katz and Shapiro (1994), Besen and Farrell (1994), Greenstein (1992), Economides (1996), and Hoppe (2002)). Farrell and Saloner (1985) consider the process of switching from an existing technology to a new technology, when players are strategic and uncertain about the preferences of others. They show the equilibrium strategy exhibits a "bandwagon" effect, with players who prefer the new technology leading the adoption. The sequential nature of adoption among heterogeneous consumers is also discussed in Cabral (1990).

Katz and Shapiro (1985) introduce a static model with fulfilled expectations Cournot equilibrium, where consumers make adoption decisions based on expected network sizes and firms set Cournot equilibrium quantities. They show that, with positive network externalities, if consumers expect a firm to have large market share, they will be willing to pay more for the firm's technology, thereby causing the firm to be dominant. Economides and Himmelberg (1995) have applied this equilibrium concept to study the US fax market.

Farrell and Saloner (1986a) study the trade-off between compatibility and fit from a user's perspective, with benefits exogenously given. Farrell and Saloner (1986b) examine the timing of switching technologies when the arrival of a new technology is unexpected. Dewan, Seidman, and Sundaresan (1995) look at the trade-offs between best-of-breed and integrated suite for the adoption of enterprise systems; while their model is different, the trade-off

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[2] Netcraft's (news.netcraft.com) January 2004 web server survey showed Apache with 67% of the global web server market and Microsoft with 21%.

[3] A partial history of the Unix operating system is given in www.opensource.org/sco-vs-ibm.html.

between compatibility and fit is a key feature of that problem. Katz and Shapiro (1992), Matutes and Regibeau (1992), and Regibeau and Rockett (1996) consider compatibility from the firms' product development perspective. Jullien (2001) studies reputation effects in the adoption of incompatible technologies. In our model, we consider consumer strategies as well as market forces that endogenously determine consumer net benefits.

Katz and Shapiro (1986) analyze two competing firms that enter the market simultaneously, but sell to two sequential generations of consumers. They find that there is a "late mover advantage" with the late mover defined as the firm the second generation of consumers prefers. While they focus on the sequence of consumer arrivals, we study the entry sequence of the *firms* and find a result opposite to theirs, namely, that there is a first-mover advantage in the sense that the first firm to enter the market can increase its profit and market share. Katz and Shapiro (1986) do not allow for switching, whereas we show how the first-mover advantage varies as a function of switching cost.

Arthur (1989), David (1985), and Liebowitz and Margolis (1990, 1995) debate whether customers can get "locked-in" to adopting an inferior technology that is early to market. In our model, when "lock-in" occurs, the early adopters never regret their decision to adopt, whereas the late adopters are exploited by the incumbent firm. Farrell and Shapiro's (1988) model examines the adoption dynamics of sequential generations of consumers and firms in price competition. In a very different setting, they find conditions under which an incumbent firm prices to capture the entire market or focuses on its existing customer base and cedes new consumers to the entrant. Klemperer (1987) analyzes the competitiveness of markets with switching costs, but does not consider network effects.

Open Source software is an example of a "free" technology. Programmers who develop Open Source software agree to distribute the compiled program and source code for free, and also to allow others to incorporate the software as a component in a larger software application without paying a royalty or fee.[4] Rosenberg (2000) and Feller, Fitzgerald, and Raymond (2001) describe the evolution and motivation behind Open Source software. The economics of Open Source software is discussed in Lerner and Tirole (2002). Their focus is on the incentives of developers to participate in Open Source and how firms can capitalize on Open Source software by creating complementary products. We focus on the direct competition between a sponsored technology and a free technology.

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[4] For further information on Open Source software, see www.opensource.org.

The rest of the paper is organized as follows. Section 2 presents our model. We present the socially optimal solution in Section 3. We compare the simultaneous and sequential entry games when both technologies are sponsored in Section 4. In Section 5, we analyze the simultaneous and sequential entry games when one technology is sponsored and the other is free. A summary of notation and all proofs are in the Appendix.

2 Model

We consider the adoption dynamics and sponsor pricing strategies for technologies with positive network effects. A "sponsor" (cf. Katz and Shapiro (1986)) is a firm that owns a technology (through patent or copyright) so it can charge above marginal cost, which we assume is zero. Throughout the paper, we refer to customers who adopt the technologies as consumers, although these customers may be firms or individuals. There are two market segments, $i = 1, 2$, of sizes x_1 and x_2, each consisting of a continuum of homogeneous consumers. As we consider one segment, i, we will denote the other segment by j, $j \neq i$. There are two incompatible technologies, A and B, that are available in both segments. If technology $T \in \{A, B\}$ is sponsored, we refer to its sponsor as firm T. The gross benefit that a segment-i consumer receives from adopting either technology is the sum of his standalone benefit and network value. The standalone benefit is the value to the consumer of using the technology if nobody else adopts it. The network value is the benefit a consumer derives from others using the same technology. We assume that the network value increases linearly with network size.

Let $\gamma_i^T \geq 0$ be the standalone benefit for consumers in segment i who adopt technology $T \in \{A, B\}$. We assume that what differentiates the two technologies is their standalone benefits, i.e., $\gamma_i^A \neq \gamma_i^B$. Let $\theta \geq 0$ be the benefit to technology-T adopters of a unit mass of consumers adopting the *same* technology, and $r_i^T \in [0, x_i]$ be the mass of segment-i consumers who adopt technology T. Then, the gross benefit to a segment-i consumer for adopting technology T is $\gamma_i^T + \theta(r_1^T + r_2^T)$. If technology T is sponsored, the net benefit a segment-i consumer receives for adopting technology T is $\gamma_i^T + \theta(r_1^T + r_2^T) - p_i^T$, where p_i^T is the price firm T charges segment i. If a consumer does not adopt either technology, he receives zero benefit.

Segment-i consumers are homogeneous and therefore will always adopt the same technology, T, i.e., $r_i^T \in \{0, x_i\}$, receiving at least gross benefit $\gamma_i^T + \theta x_i \equiv u_i^T$, which we will call the segment standalone benefit. We use the notation TT' to signify that segment 1 adopts technology $T \in \{A, B\}$ and segment 2 adopts technology $T' \in \{A, B\}$. Segment-i's gross and net benefits under adoption pattern TT' are $v_i^{TT'} \equiv u_i^T + \theta x_j \cdot 1_{\{T=T'\}}$ and $w_i^{TT'} \equiv v_i^{TT'} - p_i^T \cdot 1_{\{i=1\}} - p_i^{T'} \cdot 1_{\{i=2\}}$, respectively. We define the total network value and net total network value under adoption pattern TT' to be $V^{TT'} \equiv v_1^{TT'} x_1 + v_2^{TT'} x_2$ and $W^{TT'} \equiv V^{TT'} - p_1^T x_1 - p_2^{T'} x_2$, respectively.

3 Social Optimization

The social planner maximizes total network value. Based on the total network values for the four adoption permutations, V^{AA}, V^{AB}, V^{BA}, and V^{BB}, the socially optimal adoption patterns are summarized in Figure 1. The welfare-maximizing adoption pattern is to coordinate on one of the two technologies unless each segment has a strong preference for a different technology. Let $\Delta_i \equiv u_i^A - u_i^B$ denote segment i's preference for technology A over B. In Figure 1, where $\Delta_i \gg 0$ and $\Delta_j \ll 0$, adopting the technology that best fits each segment's preference results in higher total network value than coordinating to achieve compatibility, thereby resulting in a split market outcome, i.e., AB or BA.

4 Two Strategic Firms

We now consider the scenario where technologies A and B are sponsored by Bertrand competitors, firms A and B. We show that when both firms enter the market simultaneously, there are conditions under which compatibility is lower than what is socially optimal. When one firm is able to enter the market first and switching costs are sufficiently high, the first mover is able to leverage its early entry into the market to increase its profits. We find conditions under which, even at zero switching cost, there is a first-mover advantage.

4.1 Simultaneous Entry

We now examine the adoption dynamics when firms A and B enter the market simultaneously and compete on price in each segment. Farrell and Saloner (1986) derive consumer adoption equilibria for incompatible technologies with network effects, given exogenous benefit functions. They find conditions under which there is more compatibility than the social

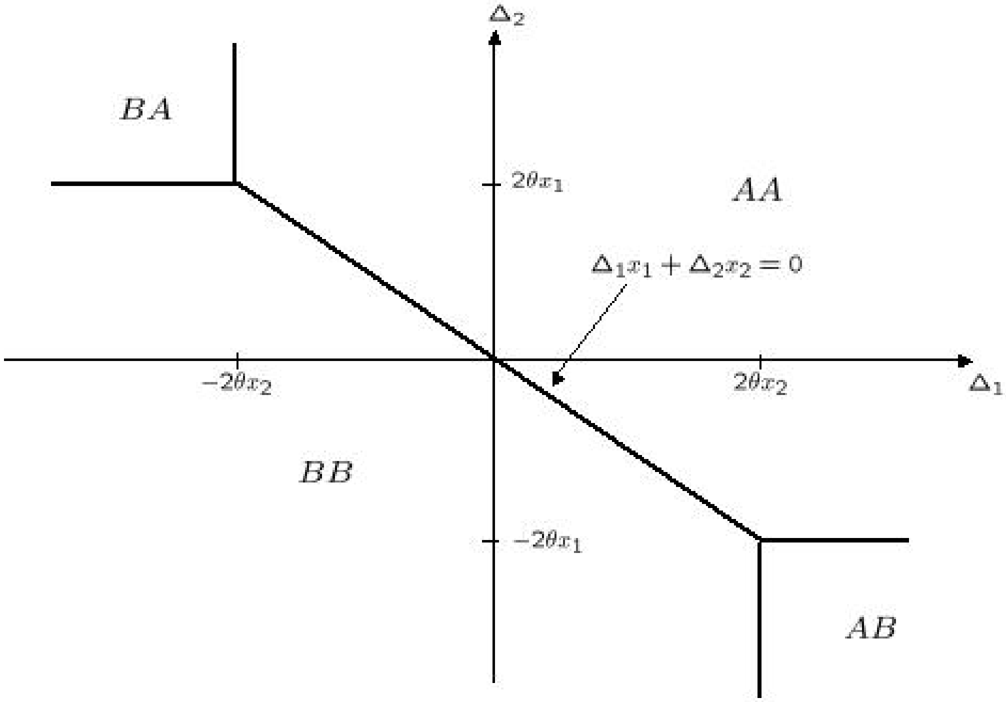

Figure 1: Socially optimal adoption pattern. TT' refers to the region where segment 1 adopts $T \in \{A, B\}$ and segment 2 adopts $T' \in \{A, B\}$.

optimum. In our model, consumer net benefits depend on prices and since firms compete on price, compatibility cannot be an equilibrium if the socially optimal solution is for consumers to adopt their (distinct) preferred technologies.

The timing of the game is as follows:

- Stage 1: Firms A and B set prices p_1^A, p_2^A, p_1^B, p_2^B.
- Stage 2: Consumers decide whether and which technology to adopt.

Since consumers within a segment are homogeneous, we assume that they will act to maxi-

Seg 1 \ Seg 2	Adopt A	Adopt B	Adopt nothing
Adopt A	$u_1^A + \theta x_2 - p_1^A$, $u_2^A + \theta x_1 - p_2^A$	$u_1^A - p_1^A$, $u_2^B - p_2^B$	$u_1^A - p_1^A$, 0
Adopt B	$u_1^B - p_1^B$, $u_2^A - p_2^A$	$u_1^B + \theta x_2 - p_1^B$, $u_2^B + \theta x_1 - p_2^B$	$u_1^B - p_1^B$, 0
Adopt nothing	0, $u_2^A - p_2^A$	0, $u_2^B - p_2^B$	0, 0

Figure 2: Simultaneous entry game, stage-2 consumer payoff matrix. Technologies A and B are both sponsored.

mize the net benefit for the entire segment, which corresponds to the maximum net benefit for each consumer, since network effects are non-negative (cf. Katz and Shapiro (1986)).[5] The stage-2 consumer payoff matrix is shown in Figure 2. Given that A and B are in price competition and u_i^A, $u_i^B \geq 0$, we know that "Adopt nothing" will be weakly dominated by either "Adopt A" or "Adopt B".

The necessary and sufficient conditions for AA to be an equilibrium are

$$\begin{aligned} & (u_1^A - p_1^A) - (u_1^B - p_1^B) \geq -\theta x_2 && (1) \\ \text{and} \quad & (u_2^A - p_2^A) - (u_2^B - p_2^B) \geq -\theta x_1. && (2) \end{aligned}$$

The necessary and sufficient conditions for BB to be an equilibrium are

$$\begin{aligned} & (u_1^A - p_1^A) - (u_1^B - p_1^B) \leq \theta x_2 && (3) \\ \text{and} \quad & (u_2^A - p_2^A) - (u_2^B - p_2^B) \leq \theta x_1. && (4) \end{aligned}$$

AA and BB are both equilibria if

$$\begin{aligned} & -\theta x_2 \leq (u_1^A - p_1^A) - (u_1^B - p_1^B) \leq \theta x_2 && (5) \\ \text{and} \quad & -\theta x_1 \leq (u_2^A - p_2^A) - (u_2^B - p_2^B) \leq \theta x_1. && (6) \end{aligned}$$

If both (5) and (6) hold, we assume consumers choose the equilibrium with the higher net total network value, i.e., they choose AA if

$$(u_1^A - p_1^A)x_1 + (u_2^A - p_2^A)x_2 \geq (u_1^B - p_1^B)x_1 + (u_2^B - p_2^B)x_2, \tag{7}$$

and they choose BB if

$$(u_1^A - p_1^A)x_1 + (u_2^A - p_2^A)x_2 < (u_1^B - p_1^B)x_1 + (u_2^B - p_2^B)x_2. \tag{8}$$

The necessary and sufficient conditions for AB to be an equilibrium are

$$\begin{aligned} & (u_1^A - p_1^A) - (u_1^B - p_1^B) \geq \theta x_2 && (9) \\ \text{and} \quad & (u_2^A - p_2^A) - (u_2^B - p_2^B) \leq -\theta x_1. && (10) \end{aligned}$$

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[5] For example, given segment-1 consumers choose technology T, segment-2 consumers choose T' to maximize $v_2^{TT'} - p_2^{T'}$. Therefore, we eliminate equilibria where, if segment 1 plays A, segment-2 consumers choose B even though $u_2^A + \theta x_1 - p_2^A \geq u_2^B - p_2^B$, because $\gamma_2^A + \theta x_1 - p_2^A < u_2^B - p_2^B$.

The necessary and sufficient conditions for BA to be an equilibrium are

$$(u_1^A - p_1^A) - (u_1^B - p_1^B) \leq -\theta x_2 \tag{11}$$

$$\text{and} \qquad (u_2^A - p_2^A) - (u_2^B - p_2^B) \geq \theta x_1. \tag{12}$$

Given the stage-2 equilibrium strategies of consumers (equations (1)-(12)), we now derive the stage-1 equilibrium pricing strategies of the firms. We assume that when a firm is indifferent between setting a negative price and zero price, it will choose to set a zero price.[6] We define the following sets in (Δ_1, Δ_2) space, $\Delta_1 \in \Re$, $\Delta_2 \in \Re$:

$$\begin{aligned}
D_{AA} &\equiv \{(\Delta_1, \Delta_2) | \{\Delta_1 x_1 + \Delta_2 x_2 \geq 0\} \cap \{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > -\theta x_1\}\}, \\
D_{BB} &\equiv \{(\Delta_1, \Delta_2) | \{\Delta_1 x_1 + \Delta_2 x_2 < 0\} \cap \{\Delta_1 < \theta x_2\} \cap \{\Delta_2 < \theta x_1\}\}, \\
D_{AB} &\equiv \{(\Delta_1, \Delta_2) | \{\Delta_1 \geq \theta x_2\} \cap \{\Delta_2 \leq -\theta x_1\}\}, \\
\text{and } D_{BA} &\equiv \{(\Delta_1, \Delta_2) | \{\Delta_1 \leq -\theta x_2\} \cap \{\Delta_2 \geq \theta x_1\}\}.
\end{aligned}$$

The sets D_{AA}, D_{BB}, D_{AB}, and D_{BA} correspond to the regions in Figure 3 where AA, BB, AB, and BA are the equilibrium adoption patterns, respectively.

Proposition 1 *When both technologies are sponsored and firms enter the market simultaneously, the subgame perfect equilibria are for consumers to follow strategies (1)-(12) and for firms to set the following prices:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}$, *firm A sets* $\{(\tilde{p}_1^A, \tilde{p}_2^A) | \tilde{p}_1^A = \frac{\Delta_1 x_1 + \Delta_2 x_2 - \tilde{p}_2^A x_2}{x_1}$ *and* $\tilde{p}_2^A \in [\Delta_2 - \theta x_1, \Delta_2 + \theta x_1]\}$, *and firm B sets* $\tilde{p}_1^B = \tilde{p}_2^B = 0$, *resulting in adoption pattern AA and consumer net benefits* $w_1^{AA} = u_1^B + u_2^B + \theta x_1 + \theta x_2 - w_2^{AA}$ *and* $w_2^{AA} \in [u_2^B, u_2^B + 2\theta x_1]$,
- *if* $(\Delta_1, \Delta_2) \in D_{BB}$, *firm A sets* $\tilde{p}_1^A = \tilde{p}_2^A = 0$, *and firm B sets* $\{(\tilde{p}_1^B, \tilde{p}_2^B) | \frac{-\Delta_1 x_1 - \Delta_2 x_2 - \tilde{p}_2^B x_2}{x_1}$ *and* $\tilde{p}_2^B \in [-\Delta_2 - \theta x_1, -\Delta_2 + \theta x_1]\}$, *resulting in adoption pattern BB and consumer net benefits* $w_1^{BB} = u_1^A + u_2^A + \theta x_1 + \theta x_2 - w_2^{BB}$ *and* $w_2^{BB} \in [u_2^A, u_2^A + 2\theta x_1]$,
- *if* $(\Delta_1, \Delta_2) \in D_{AB}$, *firm A sets* $\tilde{p}_1^A = \Delta_1 - \theta x_2$, $\tilde{p}_2^A = 0$, *and firm B sets* $\tilde{p}_1^B = 0$, $\tilde{p}_2^B = -\Delta_2 - \theta x_1$, *resulting in adoption pattern AB and consumer net benefits* $w_1^{AB} = u_1^B + \theta x_2$ *and* $w_2^{AB} = u_2^A + \theta x_1$,

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[6] The pricing strategy of the losing firm is then robust to small perturbations in the equilibrium strategy that would result in it making negative profit, i.e., eliminating non-trembling hand perfect equilibria (cf. Felli and Roberts (1999), Osborne and Rubinstein (1994)).

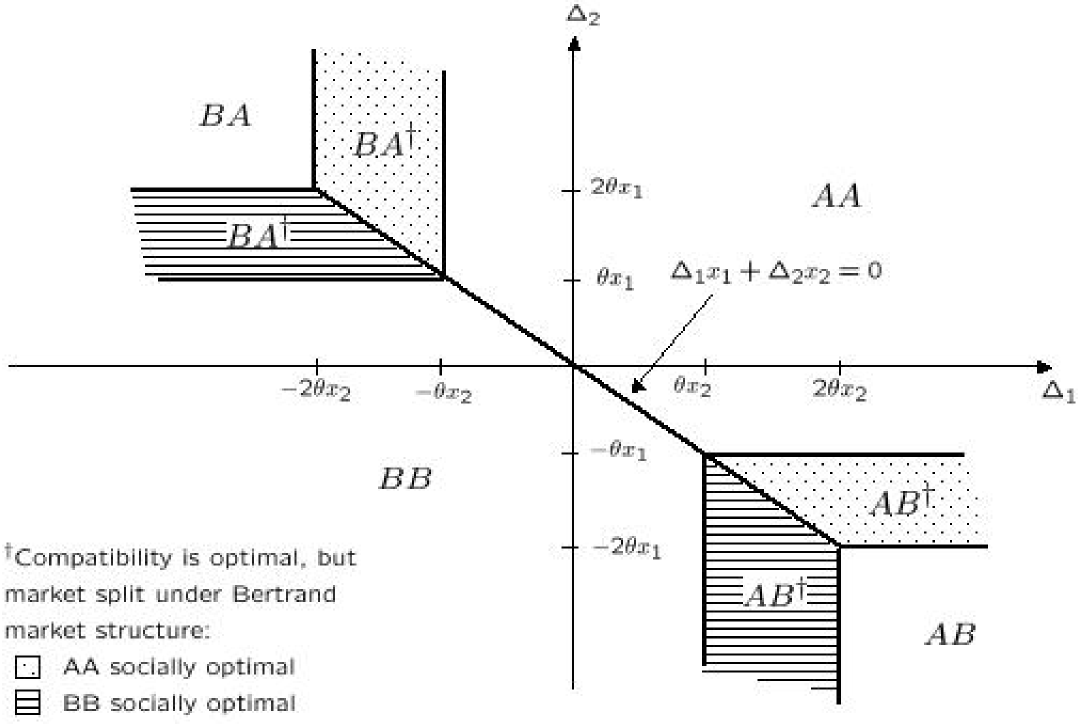

Figure 3: Simultaneous entry game adoption equilibria. TT' refers to the region where segment 1 adopts $T \in \{A, B\}$ and segment 2 adopts $T' \in \{A, B\}$. Technologies A and B are both sponsored.

- *if* $(\Delta_1, \Delta_2) \in D_{BA}$, *firm A sets* $\tilde{p}_1^A = 0$, $\tilde{p}_2^A = \Delta_2 - \theta x_1$, *and firm B sets* $\tilde{p}_1^B = -\Delta_1 - \theta x_2$, $\tilde{p}_2^B = 0$, *resulting in adoption pattern BA and consumer net benefits* $w_1^{BA} = u_1^A + \theta x_2$ *and* $w_2^{BA} = u_2^B + \theta x_1$.

When $(\Delta_1, \Delta_2) \in D_{TT}$, $T \in \{A, B\}$, there exists a range of equilibrium prices which all lead to the same profits for firm T. The intuition is that when firm T wins both segments, it prices to leave consumers' net total benefit equal to the total network value of technology $T' \neq T$, $V^{T'T'}$. However, when there is a split market equilibrium (AB or BA), the equilibrium prices are unique because each firm, T, is maximizing its profit by charging one segment so that the consumers in that segment are indifferent between adopting T and $T' \neq T$. The following proposition compares the simultaneous game adoption equilibrium to the socially optimal outcome.

Proposition 2 *If* $(\Delta_1, \Delta_2) \in D_{AB} \backslash \{\{\Delta_1 > 2\theta x_2\} \cap \{\Delta_2 < -2\theta x_1\}\}$ *or* $(\Delta_1, \Delta_2) \in D_{BA} \backslash \{\{\Delta_1 < -2\theta x_2\} \cap \{\Delta_2 > 2\theta x_1\}\}$, *there is less compatibility than what is socially* *optimal.*

The results of Propositions 1 and 2 are presented in Figure 3. The dotted regions in Figure 3 represent parameter values where AA is the socially-optimal adoption, but price competition between the two firms splits the market. The horizontal lined regions are the analogous areas where BB is socially optimal. In these regions, each segment favors a distinct technology, however, the network effects of one segment swamps the technology preference of the other segment, resulting in a standardized social optimum. Although standardization is optimal, it is too costly for a firm to subsidize the segment that prefers its competitor. Therefore, each firm targets only the segment that prefers its technology, inducing consumers to choose fit over compatibility. As we see from Figure 3, as the network coefficient, θ, increases, so does the region of compatibility.

4.2 Sequential Entry

Suppose now that firm A enters the market before firm B, and consumers have the opportunity to adopt technology A before technology B is available. We define first-mover advantage as firm A's ability to increase its profit by leveraging its early entry into the market.

Katz and Shapiro (1986) analyze a game where consumer adoption opportunities are staggered. In their model, both firms sell in periods 1 and 2, however, only segment 1 can adopt in period 1 and only segment 2 can adopt in period 2. Segments have different preferences and since adoption opportunities are staggered, consumer preferences change over time. Using backwards induction, Katz and Shapiro (1986) show that there is a preference for the technology that is preferred in the second period resulting in a "late-mover advantage," with the late mover defined as the firm that segment 2 prefers (in period 2). In contrast, we focus on the entry sequence of the *firms* and find that the first firm to enter the market can increase its profit and market share.

We allow consumers to switch technologies at a non-negative cost. There are many manifestations of switching costs. Examples of switching costs include learning a new system and converting data to a new format (cf. Shapiro and Varian (1999)). Switching costs have been discussed as a reason for an effect known as "lock-in", namely, that consumers get trapped in an inferior technology. The literature is divided on whether "lock-ins" actually occur. Some (cf. David (1985), Arthur (1989)) claim that consumers can get trapped using an inferior technology that was early to market, whereas others (cf. Liebowitz and Margolis (1995)) claim that market forces will create conditions so that consumers will switch to a superior product that is introduced later on.

In our model, for sufficiently high switching costs, the first-mover can price to induce an equilibrium adoption pattern that is not socially optimal. Since the market is segmented and consumers may have different preferences, the first mover can use a "divide and conquer" strategy to take advantage of the fact that each segment acts to maximize its *own* net benefit. Therefore, the first mover can price to entice one segment to adopt early and then use the network value generated by this adoption to increase its price in the second period. As a result, we find that, compared to their net benefits in the simultaneous entry game, early adopters are equally well off whereas late adopters are weakly worse off. We also find conditions under which the equilibrium adoption pattern is socially optimal when there is a first mover, however, under simultaneous entry, there would be less compatibility than the social optimum.

Let $c \geq 0$ be the per-consumer cost of switching technologies and $p_i^T(t)$ be firm T's segment-i price in period t. The timing of the game is as follows:

- Period 1:
  - · Stage 1: Firm A enters the market and sets prices $p_1^A(1)$, $p_2^A(1)$.
  - · Stage 2: Consumers decide whether to adopt technology A.
- Period 2
  - · Stage 1: Firm A sets prices $p_1^A(2)$, $p_2^A(2)$, and firm B enters and sets prices $p_1^B(2)$, $p_2^B(2)$.
  - · Stage 2: Consumers who have already adopted A decide whether to switch to B at cost c. Consumers who have not yet adopted decide which, if either, technology to adopt.

Once consumers adopt in period 1, their cost is sunk. There are three possible subgames in period 2: no one adopts in period 1, only one segment adopts in period 1, and both segments adopt in period 1. If no one adopts in period 1, firms A and B play the simultaneous entry game (described in Section 4.1) in period 2.

1 \ 2	Adopt A	Adopt B	Adopt nothing
Stay with A	$u_1^A + \theta x_2$, $u_2^A + \theta x_1 - p_2^A(2)$	u_1^A, $u_2^B - p_2^B(2)$	u_1^A, 0
Switch to B	$u_1^B - p_1^B(2) - c$, $u_2^A - p_2^A(2)$	$u_1^B + \theta x_2 - p_1^B(2) - c$, $u_2^B + \theta x_1 - p_2^B(2)$	$u_1^B - p_1^B(2) - c$, 0

Figure 4: Sequential entry game (A is the first mover), period-2, stage-2 consumer payoff matrix when only segment 1 adopts A in period 1. Technologies A and B are both sponsored.

1 \ 2	Stay with A	Switch to B
Stay with A	$u_1^A + \theta x_2$, $u_2^A + \theta x_1$	u_1^A, $u_2^B - p_2^B(2) - c$
Switch	$u_1^B - p_1^B(2) - c$, u_2^A	$u_1^B + \theta x_2 - p_1^B(2) - c$, $u_2^B + \theta x_1 - p_2^B(2) - c$

Figure 5: Sequential entry game (A is the first mover), period-2, stage-2 consumer payoff matrix when both segments adopt A in period 1. Technologies A and B are both sponsored.

The consumer payoff matrix for the period-2 subgame when only segment 1 adopts A in period 1 is shown in Figure 4. Given that A and B are in price competition and $u_i^A, u_i^B \geq 0$, we know that "Adopt nothing" will be weakly dominated by either "Adopt A" or "Adopt B". There is a symmetric case when only segment 2 adopts A in period 1. The consumer payoff matrix for the period-2 subgame when both segments adopt A in period 1 is shown in Figure 5. To solve for the subgame perfect equilibrium, we first solve for the equilibrium prices and adoption patterns in each of the period-2 subgames. The details of these analyses are in Lemmas 1 and 2 in the Appendix.

Consider now the period-1, stage-2 subgame. Consumers must decide whether to adopt A in period 1 or wait until period 2. We say that consumers adopt in sequence (tt') if segment 1 adopts in period t and segment 2 adopts in period t'. Let $w_i^{(tt')}$ be the net benefit to segment-i consumers when the adoption sequence is (tt'). Notice that $w_1^{(22)}$ and $w_2^{(22)}$ are the net benefits of segments 1 and 2 in the simultaneous entry game. The consumer net benefits in the other three adoption scenarios are derived from the period-2 subgame equilibrium and period-1 prices of the firms. For example, $w_1^{(12)}$ is equal to segment-1's equilibrium net benefit in the subgame when only segment-1 adopts A in period 1 *minus* $p_1^A(1)$.

The necessary and sufficient conditions for both segments to adopt in period 1 (sequence

(11)) are:

$$w_1^{(11)} \geq w_1^{(21)} \text{ and } w_2^{(11)} \geq w_2^{(12)}. \tag{13}$$

The necessary and sufficient conditions for both segments to adopt in period 2 (sequence (22)) are:

$$w_1^{(22)} \geq w_2^{(12)} \text{ and } w_2^{(22)} \geq w_2^{(21)}. \tag{14}$$

If (13) and (14) hold, then consumers choose sequence (11) if

$$w_1^{(11)} x_1 + w_2^{(11)} x_2 \geq w_1^{(22)} x_1 + w_2^{(22)} x_2, \tag{15}$$

and they choose sequence (22) if

$$w_1^{(11)} x_1 + w_2^{(11)} x_2 < w_1^{(22)} x_1 + w_2^{(22)} x_2. \tag{16}$$

The necessary and sufficient conditions for segment 1 to adopt in period 1 and segment 2 to adopt in period 2 (sequence (12)) are:

$$w_1^{(12)} \geq w_1^{(22)} \text{ and } w_2^{(12)} \geq w_2^{(11)}. \tag{17}$$

The necessary and sufficient conditions for segment 2 to adopt in period 1 and segment 1 to adopt in period 2 (sequence (21)) are:

$$w_1^{(21)} \geq w_1^{(11)} \text{ and } w_2^{(21)} \geq w_2^{(22)}. \tag{18}$$

When there are multiple equilibria in the simultaneous entry game, the behavior of consumers in the sequential entry game will depend on their beliefs of which equilibrium will be played in the simultaneous entry game. Multiple equilibria are possible in the simultaneous entry game if $(\Delta_1, \Delta_2) \in D_{AA}$ or $(\Delta_1, \Delta_2) \in D_{BB}$. We choose the equilibrium in these regions so that the net benefits of the consumers are in the middle of the range of possible equilibria, i.e., if $(\Delta_1, \Delta_2) \in D_{AA}$, $w_1^{AA} = u_1^B + \theta x_2$ and $w_2^{AA} = u_2^B + \theta x_1$, and if $(\Delta_1, \Delta_2) \in D_{BB}$, $w_1^{BB} = u_1^A + \theta x_2$ and $w_2^{BB} = u_2^A + \theta x_1$.[7]

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[7] The analysis can be done assuming different beliefs. The structure of the solution will remain the same, but the actual results will, of course, be different.

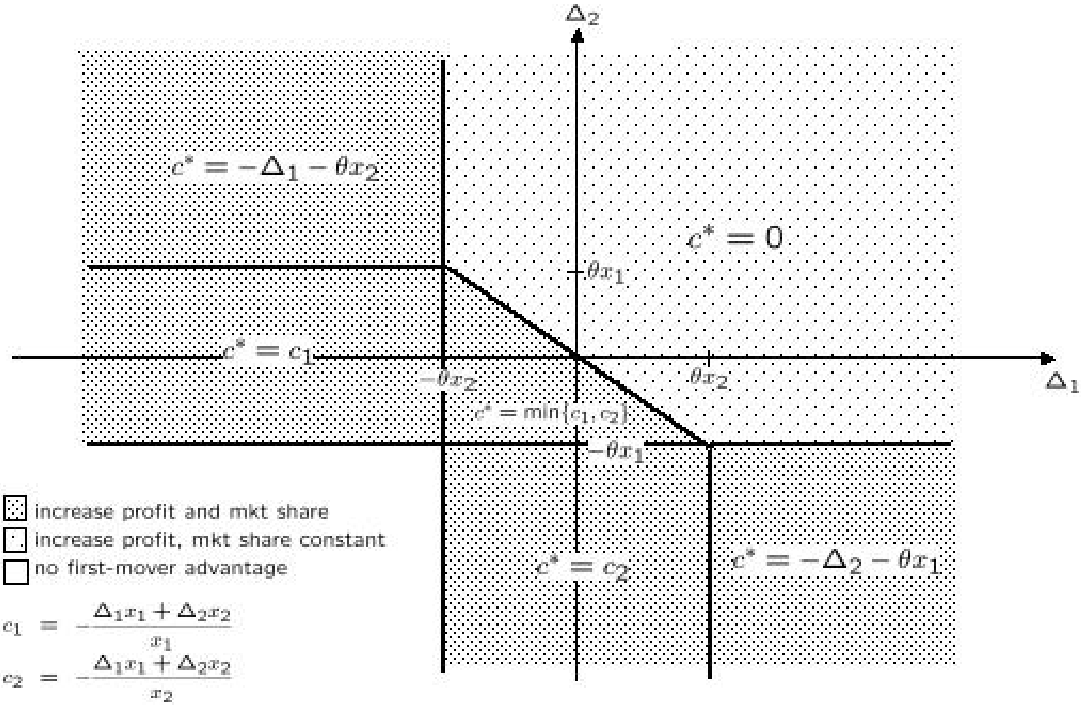

Figure 6: First-mover advantage and threshold switching costs for firm A. Technologies A and B are both sponsored.

We define the threshold switching cost, c^*, as the minimum switching cost required for firm A to have a first-mover advantage. The results of the following proposition are summarized in Figure 6.

Proposition 3 *Firm A has a first-mover advantage if the switching cost, c, is greater than the threshold switching cost, c^*, where c^* is defined as follows:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}$, *then* $c^* = 0$,
- *if* $(\Delta_1, \Delta_2) \in D_{BB} \cap \{\Delta_1 \leq -\theta x_2\} \cap \{-\theta x_1 \leq \Delta_2 < \theta x_1\}$, *then* $c^* = -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_2}$,
- *if* $(\Delta_1, \Delta_2) \in D_{BB} \cap \{\Delta_2 \leq -\theta x_1\} \cap \{-\theta x_2 \leq \Delta_1 < \theta x_2\}$, *then* $c^* = -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}$,
- *if* $(\Delta_1, \Delta_2) \in D_{BB} \cap \{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > -\theta x_1\}$, *then* $c^* = \min\{-\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_2}, -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}\}$,
- *if* $(\Delta_1, \Delta_2) \in D_{AB}$, *then* $c^* = -\Delta_2 - \theta x_1$,
- *if* $(\Delta_1, \Delta_2) \in D_{BA}$, *then* $c^* = -\Delta_1 - \theta x_2$,

- *otherwise, firm A does not have a first-mover advantage, even at infinite switching cost.*

In Figure 6, there is a region (defined by D_{AA}) where the threshold switching cost is $c^* = 0$. Clearly, firm A's first-mover advantage does not depend on being able to "lock-in" consumers with a switching cost. In this region, segments prefer to be compatible on A than be incompatible, even if firm B charges zero price. By moving first, firm A can induce segment i to adopt its technology in period 1. In period 2, segment-j consumers know that segment i will not switch and therefore, will pay a premium (commanded by the network value generated by segment-i consumers) to adopt technology A.

Firm A's general strategy is to either price so that both segments or only one segment adopts A in period 1. It can capture more profit by pricing to induce both segments to adopt in period 1 if, in the simultaneous entry game, there is a split market equilibrium, i.e., if $(\Delta_1, \Delta_2) \in D_{AA}$ or $(\Delta_1, \Delta_2) \in D_{BB}$. Firm A can provide consumers in both segments the same net benefit when they adopt A in period 1 (resulting in a compatible outcome) as they would receive in the simultaneous entry game (resulting in an incompatible outcome). However, because the two segments are compatible on A, firm A can extract the additional network value, $\theta x_1 x_2$, as profit. When firm A prices so that consumer adoption is staggered, it purposely delays the adoption of one segment so that it can build network value in period 1, and extract that value from the late adopters in period 2.

Proposition 4 *The threshold switching cost, c^*, is weakly decreasing in θ.*

When firm A has a first-mover advantage, at least one segment adopts A in period 1. As the network coefficient, θ, increases, consumers value compatibility more and there is an inertial tendency to standardize on A rather than pay the switching cost to be compatible on B. Therefore, the threshold switching cost decreases as the desire for compatibility, θ, increases.

Proposition 5 *When firm A has a first-mover advantage, compared to their net benefit in the simultaneous entry game, consumers who adopt in period 1 are equally well off and consumers who adopt in period 2 are weakly worse off.*

The typical "lock-in" situation is when high switching costs prevent consumers who adopt in period 1 from switching in period 2, and they end up regretting their decision later on. However, in our model, segment-i consumers adopt A in period 1 knowing firm B will enter in period 2. To induce segment-i consumers to adopt in period 1, firm A must price attractively enough to ensure that these consumers are at least as well off as if they waited for the simultaneous entry game. From an expected benefit perspective, segment-i consumers do not regret anything. It is segment-j consumers who bear the burden of the switching cost when firm A raises the price in period 2 as a function of segment i's reluctance to switch, thereby capturing the network value created by its installed base.

5 Competing With Free

We now consider the scenario in which technology A is sponsored and B is not, i.e., the price of technology B is zero. An example of this type of market structure is the competition between Microsoft and Open Source software. Although Open Source software is not sponsored by a profit-maximizing firm and does not have a pricing strategy, its presence in the market is a threat to Microsoft. IBM has been porting applications to the Linux operating system since 1998[8] and a number of large companies are migrating their back-end computing applications to Linux.[9] Another example of a successful Open Source software application is the Apache web server, with 67% market share, vs. 21% market share for Microsoft's web server.[10] In this section, we examine how the presence of a free technology (e.g., Open Source) can affect the adoption equilibrium and profits of a firm (e.g., Microsoft) that sponsors a competing, incompatible technology.

5.1 Simultaneous Entry

We examine the equilibrium adoption when technologies A and B become available in the market at the same time. The timing of the game is as follows:

- Stage 1: Firm A enters the market and sets prices p_1^A and p_2^A. Technology B is available for free.
- Stage 2: Consumers decide whether and which technology to adopt.

The stage-2 consumer payoff matrix is shown in Figure 2, with $p_1^B = p_2^B = 0$.

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[8] A partial history of the Unix operating system is given in http://www.opensource.org/sco-vs-ibm.html.

[9] For example, in September 2003, Ford Motor Company announced its migration plans to Linux. Source: LinuxWorld, September 23, 2003.

[10] Source: Netcraft web survey (news.netcraft.com), January 2004.

Proposition 6 *When technology A is sponsored and B is unsponsored, and both technologies enter the market simultaneously, the subgame perfect equilibria are for consumers to follow strategies (1)-(12) and for firm A to set the following prices:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}$*, firm A sets* $\{(\tilde{p}_1^A, \tilde{p}_2^A) | \tilde{p}_1^A = \frac{\Delta_1 x_1 + \Delta_2 x_2 - \tilde{p}_2^A x_2}{x_1}$ *and* $\tilde{p}_2^A \in [\Delta_2 - \theta x_1, \Delta_2 + \theta x_1]\}$*, resulting in adoption pattern AA and consumer net benefits* $w_1^{AA} = u_1^B + u_2^B + \theta x_1 + \theta x_2 - w_2^{AA}$ *and* $w_2^{AA} \in [u_2^B, u_2^B + 2\theta x_1]$*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BB}$*, firm A sets* $\tilde{p}_1^A = \tilde{p}_2^A = 0$*, resulting in adoption pattern BB and consumer net benefits* $w_1^{BB} = u_1^B + \theta x_2$ *and* $w_2^{BB} = u_2^B + \theta x_1$*,*
- *if* $(\Delta_1, \Delta_2) \in D_{AB}$*, firm A sets* $\tilde{p}_1^A = \Delta_1 - \theta x_2$*,* $\tilde{p}_2^A = 0$*, resulting in adoption pattern AB and consumer net benefits* $w_1^{AB} = u_1^B + \theta x_2$ *and* $w_2^{AB} = u_2^B$*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BA}$*, firm A sets* $\tilde{p}_1^A = 0$*,* $\tilde{p}_2^A = \Delta_2 - \theta x_1$*, resulting in adoption pattern BA and consumer net benefits* $w_1^{BA} = u_1^B$ *and* $w_2^{BA} = u_2^B + \theta x_1$*.*

The interpretation of Proposition 6 is similar to Proposition 1. When firm A wins both segments, there is a range of equilibrium prices which all result in the same profit for A. In a split market equilibrium, the equilibrium prices are unique. If firm A were a monopolist, it could extract the total network value created by charging $p_1^A = u_1^A + \theta x_2$ and $p_2^A = u_2^A + \theta x_1$, leaving consumers with zero net benefit (so that they are indifferent between adopting and not adopting). Comparing with Proposition 6, we see that the presence of technology B increases consumer net benefit.

Firm A's pricing strategy reflects the impact that network effects and the presence of technology B have on firm A's profit. In set D_{AA} (region AA in Figure 3), firm A must lower prices so that its net total network value, W^{AA}, is equal to the total network value of technology B, V^{BB}. In set D_{AB} (region AB in Figure 3), firm A's prices are $p_1^A = \Delta_1 - \theta x_2$ and $p_2^A = 0$. It loses entirely the network value of segment 2 ($u_2^A + \theta x_1$). It also must lower its segment-1 price so that consumers receive the standalone benefit of adopting B (u_1^B) and it must compensate consumers for not being compatible with segment 2 (θx_2).

5.2 Sequential Entry: Sponsored Technology, A, Enters First

In the sequential entry game in which firm A enters first, the timing of the game is the same as when both firms are sponsored (as shown in Section 4.2). The only difference is that the prices for technology B in period 2 are always set to zero. We again have three possible subgames in period 2: no one adopts in period 1, only one segments adopts in period 1, and both segments adopt in period 1. If no one adopts in period 1, the outcome is as described in Proposition 6 (simultaneous entry game when only A is sponsored). It is straightforward to show that the equilibrium adoption and firm A pricing in the other two subgames (when only one segment adopts in period 1 and when both segments adopt in period 2) are the same as described in Section 4.2 (specifically in Lemmas 1 and 2 in the proof of Proposition 3), with $p_1^B(2) = p_2^B(2) = 0$. In the period-1, stage-2 subgame, consumers adopt according to (13)-(18).

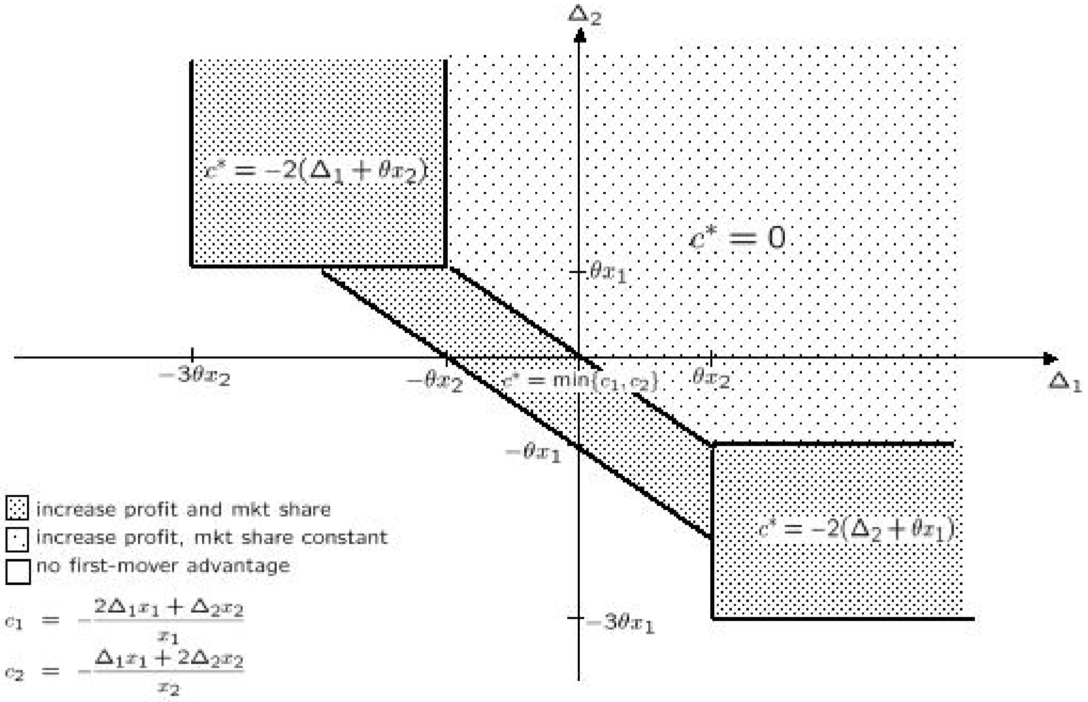


Figure 7: First-mover advantage and threshold switching costs for firm A. Technology A is sponsored and technology B is unsponsored.

Figure 7 summarizes the results of the following proposition.

Proposition 7 *Firm A has a first-mover advantage if the switching cost, c, is greater than the threshold, c^*, where c^* is defined as follows:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}$, $c^* = 0$,
- *if* $(\Delta_1, \Delta_2) \in D_{BB} \cap \{\Delta_1 x_1 + \Delta_2 x_2 \geq -\theta x_1 x_2\}$, $c^* = \min\{-\frac{2\Delta_1 x_1 + \Delta_2 x_2}{x_1}, -\frac{\Delta_1 x_1 + 2\Delta_2 x_2}{x_2}\}$,

$1 \setminus 2$	Adopt A	Adopt B	Adopt nothing
Switch to A	$u_1^A + \theta x_2 - p_1^A(2) - c$, $u_2^A + \theta x_1 - p_2^A(2)$	$u_1^A - p_1^A(2) - c$, u_2^B	$u_1^A - p_1^A(2) - c$, 0
Stay with B	u_1^B, $u_2^A - p_2^A(2)$	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$	u_1^B, 0

Figure 8: Sequential entry game (B is the first mover), period-2, stage-2 consumer payoff matrix when only segment 1 adopts B in period 1. Technology A is sponsored, technology B is unsponsored.

- *if* $(\Delta_1, \Delta_2) \in D_{AB} \cap \{\Delta_2 \geq -3\theta x_1\}$, $c^* = -2(\Delta_2 + \theta x_1)$,
- *if* $(\Delta_1, \Delta_2) \in D_{BA} \cap \{\Delta_1 \geq -3\theta x_2\}$, $c^* = -2(\Delta_1 + \theta x_2)$,
- *otherwise, firm A does not have a first-mover advantage even at infinite switching cost.*

Comparing Figures 6 and 7, we see that firm A's first-mover advantage is greater when B is sponsored than when B is unsponsored. Firm A's first-mover strategy is to price so that consumers who adopt A in period 1 receive at least as much benefit as if they wait for the simultaneous entry game in period 2. When B is unsponsored, consumer surplus is greater in the simultaneous entry game (profits of the sponsor go to the consumers), and therefore, firm A must compensate early adopters more for adopting in period 1.

5.3 Sequential Entry: Unsponsored Technology, B, Enters First

We change the sequential entry game description as follows to reflect that the unsponsored technology is available in the market first:

- Period 1:
  - Stage 1: Technology B is available for free.
  - Stage 2: Consumers decide whether to adopt technology B.
- Period 2
  - Stage 1: Firm A enters the market and sets prices $p_1^A(2)$, $p_2^A(2)$. Technology B remains free.

$1 \setminus 2$	Stay with A	Switch to B
Stay with A	$u_1^A + \theta x_2 - p_1^A(2) - c$, $u_2^A + \theta x_1 - p_2^A(2) - c$	$u_1^A - p_1^A(2) - c$, u_2^B
Switch to B	u_1^B, $u_2^A - p_2^A(2) - c$	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$

Figure 9: Sequential entry game (B is the first mover), period-2, stage-2 consumer payoff matrix when both segments adopt B in period 1. Technology A is sponsored, technology B is unsponsored.

- Stage 2: Consumers who have already adopted B decide whether to switch to A at cost c. Consumers who have not yet adopted decide which, if either, technology to adopt.

If $c = 0$, then the sequential entry game would result in the same outcome as the simultaneous entry game because even if consumers adopt B in period 1, they have paid nothing to adopt B and they can switch to A in period 2 at zero cost. Suppose now that $c > 0$. We have our usual three subgames in period 2. The consumer payoff matrix for the period-2 subgame when only segment 1 adopts B in period 1 is shown in Figure 8. Given that $u_i^A, u_i^B \geq 0$, we know that "Adopt nothing" will be weakly dominated by either "Adopt A" or "Adopt B". There is a symmetric case when only segment 2 adopts B in period 1. The consumer payoff matrix for the period-2 subgame where both segments adopt B in period 1 is shown in Figure 9. To solve for the subgame perfect equilibrium, we first solve for the equilibrium prices and adoption patterns in each of the period-2 subgames. The details of these analyses are in Lemmas 3 and 4 in the Appendix.

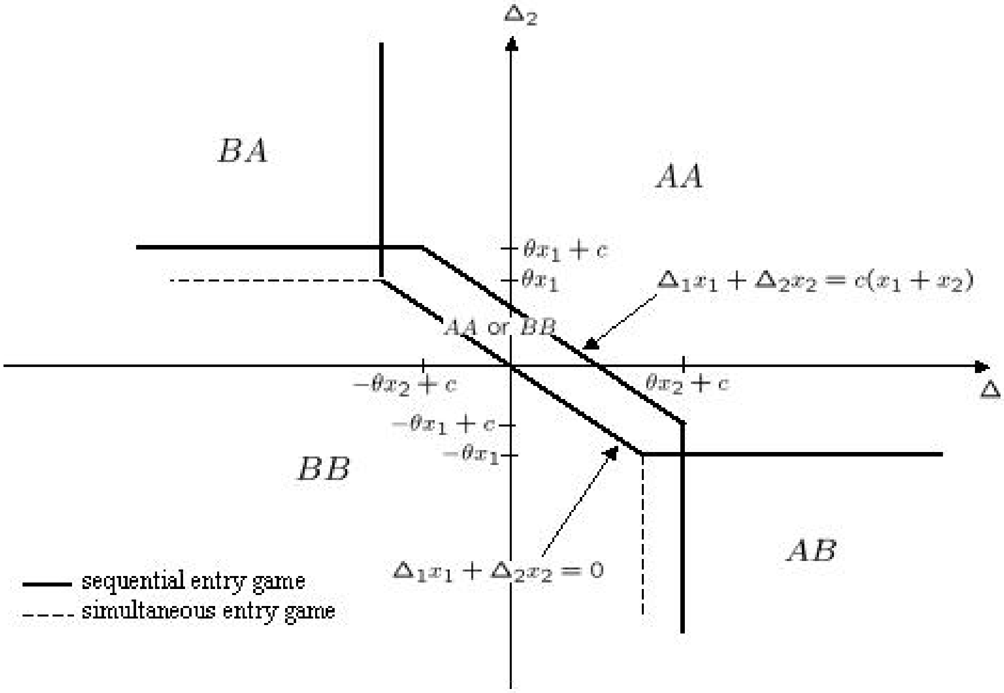


Figure 10: Sequential entry game (B is the first mover) equilibria. TT' refers to the region where segment 1 adopts $T \in \{A, B\}$ and segment 2 adopts $T' \in \{A, B\}$. The switching cost is $0 < c \leq \theta \min\{x_1, x_2\}$. Technology A is sponsored, technology B is unsponsored.

In the period-1, stage-2 subgame, consumers adopt according to (13)-(18). We define the following sets:

$$
\begin{aligned}
E_{AA} &\equiv \{(\Delta_1, \Delta_2) | \{\{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > \theta x_1 + c\}\} \cup \\
&\quad \{\{\Delta_1 > \theta x_2 + c\} \cap \{\Delta_2 > -\theta x_1\}\} \cup \\
&\quad \{\{\Delta_1 x_1 + \Delta_2 x_2 > c(x_1 + x_2)\} \cap \{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > -\theta x_1\}\}\}, \\
E_{BB} &\equiv \{(\Delta_1, \Delta_2) | \{\{\Delta_1 < -\theta x_2\} \cap \{\Delta_2 < \theta x_1 + c\}\} \cup \\
&\quad \{\{\Delta_1 < \theta x_2 + c\} \cap \{\Delta_2 < -\theta x_1\}\} \cup \\
&\quad \{\{\Delta_1 x_1 + \Delta_2 x_2 < 0\} \cap \{\Delta_1 < \theta x_2 + c\} \cap \{\Delta_2 < \theta x_1 + c\}\}\}, \\
E_{AB} &\equiv \{(\Delta_1, \Delta_2) | \{\Delta_1 \geq \theta x_2 + c\} \cap \{\Delta_2 \leq -\theta x_1\}\}, \\
E_{BA} &\equiv \{(\Delta_1, \Delta_2) | \{\Delta_1 \leq -\theta x_2\} \cap \{\Delta_2 \geq \theta x_1 + c\}\}, \\
\text{and } E_{AABB} &\equiv \Delta_1 \times \Delta_2 \backslash \{E_{AA} \cup E_{BB} \cup E_{AB} \cup E_{BA}\}.
\end{aligned}
$$

The sets E_{AA}, E_{BB}, E_{AB}, E_{BA}, and E_{AABB} correspond to the regions in Figure 10 where AA, BB, AB, BA, and (AA or BB) are equilibrium adoption patterns. Figure 10 summarizes the results of the following proposition.

Proposition 8 *In the sequential entry game when the unsponsored technology, B, enters the market first, the equilibrium adoption pattern is* TT' *if* $(\Delta_1, \Delta_2) \in E_{TT'}$, $T, T' \in \{A, B\}$, *and* $\{AA \text{ or } BB\}$ *if* $(\Delta_1, \Delta_2) \in E_{AABB}$.

In the regions $(\Delta_1, \Delta_2) \in D_{AB} \cap \{\Delta_1 < \theta x_2 + c\}$ and $(\Delta_1, \Delta_2) \in D_{BA} \cap \{\Delta_2 < \theta x_1 + c\}$, firm A loses market share and thus, profits, as a result of being the second mover. When competing with an incumbent, free technology under these conditions, firm A has an incentive to reduce switching costs to decrease its likelihood of losing market share. For example, software company A can develop one way converters to port applications developed on B to be compatible with A's technology. When firm A drives the switching cost to zero, technology B cannot gain market share by being the first mover.

When consumer preferences are such that $(\Delta_1, \Delta_2) \in E_{AABB}$, consumers are indifferent between compatibility on A and B. In this case, firm A wants consumers to delay their adoption so that they will adopt its technology. Firm A has an incentive to deploy tactics to spread fear, uncertainty, and doubt about the viability of technology B. By creating this uncertainty, consumers may delay their adoption, thereby giving them the opportunity to adopt A when it becomes available.

The results of Proposition 8 lead to the following corollary.

Corollary 1 *Consumer net benefits weakly increase in the switching cost, c.*

This result arises because, in period 2, if firm A cannot profitably win both segments, it targets only the segment that prefers its technology, thereby splitting the market. The segment that adopts B loses the network value from being compatible with other segment. As switching costs increase, firm A's ability to win even one segment decreases, therefore, consumers are more likely to be compatible on technology B, realizing the full standalone and network value of using the free technology.

Usually, high switching costs are undesirable, as they are associated with trapping consumers in an inferior technology. The QWERTY keyboard is often cited as an example of an inferior technology establishing itself as the standard over the superior Dvorak keyboard because of early adoption by consumers (cf. David (1985), Farrell and Saloner (1986b)). However, we have the opposite effect here. Firm A's own pricing strategy makes the segment that prefers its technology weakly prefer adopting B early over waiting until period 2 for A. As a result, the segment that prefers B increases its net benefit by the network value generated when both segments are compatible on B. Therefore, overall, an increase in switching costs increases the total net consumer benefit.

6 Conclusion

The demand for information and telecommunication technologies is often characterized by network effects. As these industries gain prominence in the economy, it is becoming increasingly important to understand the impact of network effects on technology adoption. The markets for these technologies are often characterized by multiple, heterogeneous segments. In this paper, we have considered the trade-off between compatibility and individual fit when there are competing, incompatible technologies supplied by Bertrand competitors and by a single sponsor competing against a free technology.

We find that when the technologies enter the market simultaneously, there is less compatibility compared to the social optimum. Price competition makes it difficult for either firm to capitalize on network effects, therefore, a split market equilibrium can result when compatibility is socially optimal. When one firm is able to enter the market first (regardless of whether it is competing with another firm or a free technology) it can increase its profit even at zero switching costs, by using a "divide and conquer" strategy to build network value in one period and extract the network value in a later period.

We applied our framework to analyze the pricing strategies of a firm faced with a competing, incompatible, free technology, as in the competition between Microsoft and Open Source software. We find that if the unsponsored technology is available first and switching costs are strictly positive, then the sponsor of the late entrant will lose market share. Under this entry sequence, consumer surplus increases as switching costs increase. When the first mover is the sponsored technology, its first-mover advantage is greater if its competitor is another strategic firm.

7 References

Arthur, B. (1989): "Competing Technologies, Increasing Returns, and Lock-In by Historical Events", *The Economic Journal*, 99 (March 1989), 116-131.

Besen, S.M., and Farrell, J. (1994): "Choosing How to Compete: Strategies and Tactics in Standardization", *Journal of Economic Perspectives*, Vol. 8, No. 2, Spring 1994, pp. 117-131.

David, P. (1985): "Clio and the Economics of QWERTY", *The American Economic Review*, Vol. 75, No. 2, Papers and Proceedings of the Ninety-Seventh Annual Meeting of the American Economic Association, May 1985, pp. 332-337.

Dewan, R., Seidmann, A., and Sundaresan, S. (1995): "Strategic Choices in IS Infrastructure: Corporate Standards versus Best of Breed Systems", *Proceedings of the Sixteenth International Conference on Information Systems (ICIS 95)*, December 1995, pp. 97-107.

Economides, N. (1996): "The Economics of Networks", *International Journal of Industrial Organization*, Vol. 14, No. 6, October 1996, pp. 673-699.

Economides, N. (2000): "Durable goods monopoly with network externalities with application to the PC operating systems market", *Quarterly Journal of Electronic Commerce*, Vol. 1, No. 3, 2000, pp. 193-201.

Farrell, J., and Saloner, G. (1985): "Standardization, compatibility, and innovation", *Rand Journal of Economics*, Vol. 16, No. 1, Spring 1985.

Farrell, J., and Saloner, G. (1986a): "Standardization and Variety", *Economics Letters*, Vol. 20, No. 1, 1986, pp. 71-74.

Farrell, J., and Saloner, G. (1986b): "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation", *The American Economic Review*, Vol. 75, No. 5, December 1986.

Farrell, J., and Shapiro, C. (1988): "Dynamic competition with switching costs ", *Rand Journal of Economics*, Vol. 19, No. 1, Spring 1988, pp. 123-137.

Feller, J., Fitzgerald, B., and Raymond, E. (2001): "Understanding Open Source Software Development", *Addison-Wesley Publishing Company*, 2001.

Felli, L., and Roberts, K.W.S. (1999): "Competition and Hold-Ups", *http://ssrn.com/abstract=171920*, June 1999.

Fudenberg, D., and Tirole, J. (1991): "Game Theory", *The MIT Press*, 1991, Cambridge, MA.

Fudenberg, D., and Tirole, J. (2000): “Pricing a network good to deter entry”, *The Journal of Industrial Economics*, Volume XLVII, No. 4, Dec. 2000.

Greenstein, S. (1992): “Invisible Hands and Visible Advisors: An Economic Interpretation of Standardization”, *Journal of the American Society for Information Science*, 43(8):538-549, 1992.

Hoppe, H.C. (2002): “The Timing of New Technology Adoption: Theoretical Models and Empirical Evidence”, *Manchester School*, Vol. 70, No. 1, pp. 56-76.

Jullien, B. (2001): “Competing in Network Industries: Divide and Conquer”, *IDEI and GREMAQ, University of Toulouse*, Working paper, July 2001.

Katz, M., and Shapiro, C. (1985): “Network Externalities, Competition, and Compatibility”, *The American Economic Review*, Vol. 75, No. 3, June 1985.

Katz, M., and Shapiro, C. (1986): “Technology Adoption in the Presence of Network Externalities”, *Journal of Political Economy*, 1986, Vol. 94, No. 4.

Katz, M., and Shapiro, C. (1992): “Product Introduction with Network Externalities”, *The Journal of Industrial Economics*, Vol. XL, No. 1, March 1992.

Katz, M., and Shapiro, C. (1994): “Systems Competition and Network Effects”, *Journal of Economic Perspectives*, Vol. 8, No. 2, Spring 1994, pp. 93-115.

Klemperer, P. (1987): “The competitiveness of markets with switching costs”, *Rand Journal of Economics*, Vol. 18, No. 1, Spring 1987, pp. 138-150.

Lerner, J., and Tirole, J. (2002): “Some Simple Economics of Open Source”, *Journal of Industrial Economics*, Vol. L, No. 2, pp. 197-234.

Liebowitz, S.J., and Margolis, S.E. (1990): “The Fable of the Keys ”, *Journal of Law and Economics*, vol. XXXIII (April 1990), pp. 1-25.

Liebowitz, S.J., and Margolis, S.E. (1995): “Path Dependence, Lock-In, and History”, *The Journal of Law, Economics, and Organization*, April 1995, v.11, no.1, p.205-226.

Maskin, E., and Tirole, J. (2001): “Markov Perfect Equilibrium: I. Observable Actions”, *Journal of Economic Theory*, 100, pp. 191-219, (2001).

Matutes, C., and Regibeau, P. (1992): “Compatibility and Bundling of Complementary Goods in a Duopoly”, *Journal of Industrial Economics*, Vol. XL, No. 1, pp. 37-54.

Osborne, M.J., and Rubinstein, A. (1994): “A Course in Game Theory”, *The MIT Press*, Cambridge, Massachusetts.

Regibeau, P., and Rockett, K. (1996): “The timing of product introduction and the credibility of compatible decisions”, *International Journal of Industrial Organization*, Vol. 14, No. 6, pp. 801-823.

Rosenberg, D. (2000): “Open Source: The Unauthorized White Papers”, *John Wiley & Sons*, 2000.

Shapiro, C., and Varian, H. (1999): “Information Rules: A Strategic Guide to the Network Economy”, *Harvard Business School Press*, Boston, Massachusetts, 1999.

Witt, U. (1997): ““Lock-in” vs. “critical masses” - industrial change under network externalities”, *International Journal of Industrial Organization*, 15 (1997) 753-773.

Appendix A: Notation

- $T, T' \in \{A, B\}$: technologies / firms
- $i \in \{1, 2\}$, $j \neq i$: segment numbers
- TT': adoption pattern if segments 1 and 2 adopt technologies T and T', respectively
- x_i: size of segment i
- γ_i^T: standalone benefit of segment-i consumer who adopts technology $T \in \{A, B\}$ and no-one else does
- θ: network coefficient, value to a consumer who adopts technology T of one unit mass of other consumers who also adopt technology T
- $u_i^T \equiv \gamma_i^T + \theta x_i$: segment standalone benefit
- $\Delta_i \equiv u_i^A - u_i^B$
- p_i^T: price firm T charges segment i in the simultaneous entry game
- $p_i^T(t)$: price firm T charges segment i in period t of the sequential entry game
- $v_i^{TT'} \equiv u_i^T + \theta x_j \cdot 1_{\{T=T'\}}$: *gross* benefit to a segment-i consumer if the adoption pattern is TT'
- $w_i^{TT'} \equiv v_i^{TT'} - p_i^T \cdot 1_{\{i=1\}} - p_i^{T'} \cdot 1_{\{i=2\}}$: *net* benefit to a segment-i consumer if the adoption pattern is TT'
- $V^{TT'} \equiv v_1^{TT'} x_1 + v_2^{TT'} x_2$: total network value if adoption pattern is TT'
- $W^{TT'} \equiv V^{TT'} - p_1^T x_1 - p_2^{T'} x_2$: *net* total network value if adoption pattern is TT'

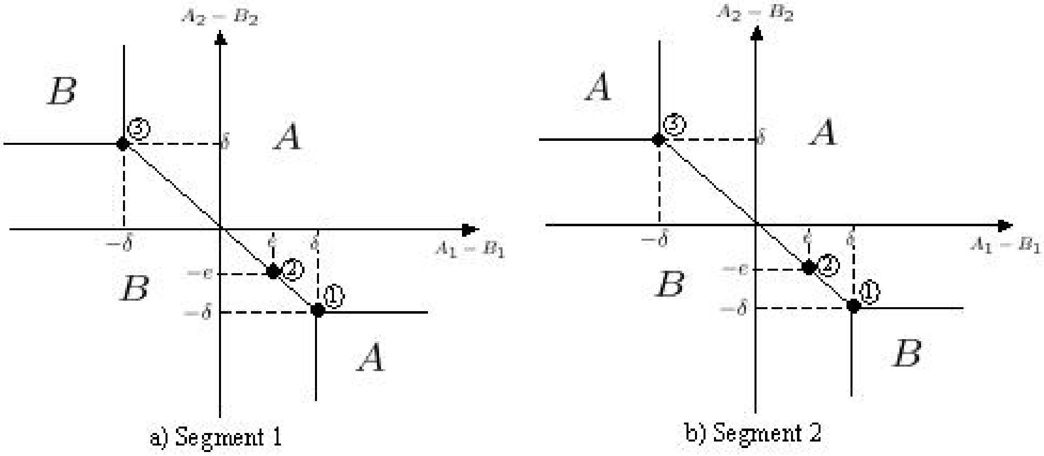
Appendix B: Proofs

We define the following sets in (Δ_1, Δ_2) space, $\Delta_1 \in \Re$, $\Delta_2 \in \Re$, as a function of variables n_1 and n_2:

$$\begin{aligned}
D_{AA}(n_1,n_2) &\equiv \{(\Delta_1,\Delta_2)|\{\Delta_1 x_1 + \Delta_2 x_2 \geq n_1 x_1 + n_2 x_2\} \cap \\
&\quad \{\Delta_1 > -\theta x_2 + n_1\} \cap \{\Delta_2 > -\theta x_1 + n_2\}\}, \\
D_{BB}(n_1,n_2) &\equiv \{(\Delta_1,\Delta_2)|\{\Delta_1 x_1 + \Delta_2 x_2 < n_1 x_1 + n_2 x_2\} \cap \\
&\quad \{\Delta_1 < \theta x_2 + n_1\} \cap \{\Delta_2 < \theta x_1 + n_2\}\}, \\
D_{AB}(n_1,n_2) &\equiv \{(\Delta_1,\Delta_2)|\{\Delta_1 \geq \theta x_2 + n_1\} \cap \{\Delta_2 \leq -\theta x_1 + n_2\}\}, \\
\text{and } D_{BA}(n_1,n_2) &\equiv \{(\Delta_1,\Delta_2)|\{\Delta_1 \leq -\theta x_2 + n_1\} \cap \{\Delta_2 \geq \theta x_1 + n_2\}\}.
\end{aligned}$$

If $n_1 = n_2 = 0$, $D_{TT'}(0,0) \equiv D_{TT'}$, $T, T' \in \{A, B\}$. The sets D_{AA}, D_{BB}, D_{AB}, and D_{BA} correspond to the regions in Figure 3 where AA, BB, AB, and BA are the equilibrium adoption patterns, respectively. If $n_1 \neq 0$ and $n_2 \neq 0$, then $D_{TT'}(n_1, n_2)$, $T, T' \in \{A, B\}$, correspond to the regions in Figure 3, shifted along the Δ_1 and Δ_2 axes by n_1 and n_2, respectively.

B.1 Proof of Proposition 1



a) Segment 1 b) Segment 2

Figure 11: Simultaneous entry game, stage-2 strategies for segments 1 and 2 as a function of $A_1 - B_1$ and $A_2 - B_2$. Technologies A and B are both sponsored.

Let $A_i = (u_i^A - p_i^A)x_i$, $B_i = (u_i^B - p_i^B)x_i$, and $\delta = \theta x_1 x_2$. Making these change in variables in conditions (1)-(12), the stage-2 strategies of segments 1 and 2 are plotted as a function of $A_1 - B_1$ and $A_2 - B_2$ in Figure 11. If AA is the equilibrium outcome in the stage-2 subgame, given p_1^B and p_2^B, firm A's profit is maximized when $A_1 - B_1 + A_2 - B_2 = 0$, i.e., when $A_1 - B_1 = e$, $A_2 - B_2 = -e$, $e \in [-\delta, \delta]$ (point 2 in Figure 11). Firm A's profit when $A_1 - B_1 + A_2 - B_2 = 0$ is $p_1^A x_1 + p_2^A x_2 = u_1^A x_1 + u_2^A x_2 - B_1 - B_2 \equiv \Pi^A_{(2a)}$. Under any other AA outcome, i.e., $A_1 - B_1 + A_2 - B_2 = k > 0$, firm A's profit is $p_1^A x_1 + p_2^A x_2 = u_1^A x_1 + u_2^A x_2 - B_1 - B_2 - k < \Pi^A_{(2a)}$. Firm B's profit is $0 \equiv \Pi^B_{(2a)}$. Likewise, if BB is the equilibrium outcome in the stage-2 subgame, firm B's profit is maximized when $A_1 - B_1 + A_2 - B_2 = 0$. The profits of firms A and B are $0 \equiv \Pi^A_{(2b)}$ and $u_1^B x_1 + u_2^B x_2 - A_1 - A_2 \equiv \Pi^B_{(2b)}$, respectively.

If AB is the equilibrium outcome in the stage-2 subgame, then both firms' profits are maximized when $A_1 - B_1 = \delta$ and $A_2 - B_2 = -\delta$ (point 1 in Figure 11). If $A_1 - B_1 = \delta$ and $A_2 - B_2 = -\delta$, then the profits of firms A and B are $u_1^A x_1 - B_1 - \delta \equiv \Pi^A_{(1)}$ and $u_2^B x_2 - A_2 - \delta \equiv \Pi^B_{(1)}$, respectively. Under any other AB outcome, i.e., $A_1 - B_1 = \delta + k_1$ and $A_2 - B_2 = -\delta - k_2$, $k_1 > 0$ and $k_2 > 0$, the profits of firms A and B are $u_1^A x_1 - B_1 - \delta - k_1 < \Pi^A_{(1)}$ and $u_2^B x_2 - A_2 - \delta - k_2 < \Pi^B_{(1)}$. Similarly, if BA is the equilibrium outcome in the stage-2 subgame, then both firms' profits are maximized when $A_1 - B_1 = -\delta$ and $A_2 - B_2 = \delta$ (point 3 in Figure 11). The profits of firms A and B are $u_2^A x_2 - B_2 - \delta \equiv \Pi^A_{(3)}$ and $u_1^B x_1 - A_1 - \delta \equiv \Pi^B_{(3)}$, respectively.

We now find conditions under which AA is an equilibrium. We assume that when $A_1 - B_1 = e$, $A_2 - B_2 = -e$, $e \in (-\delta, \delta)$, all consumers play A, when $A_1 - B_1 = \delta$ and $A_2 - B_2 = -\delta$ segment-1 (-2) plays A (B), and when $A_1 - B_1 = -\delta$ and $A_2 - B_2 = \delta$ segment-1 (-2) plays B (A). When the adoption pattern is AA, firm B always makes zero profit while and firm A maximizes profit by setting prices to satisfy

$$A_1 - B_1 = e \tag{19}$$

$$\text{and} \qquad A_2 - B_2 = -e. \tag{20}$$

Adding (19) and (20):

$$A_1 + A_2 - B_1 - B_2 = 0$$

$$\Leftrightarrow \qquad \Delta_1 x_1 + \Delta_2 x_2 + p_1^B x_1 + p_2^B x_2 = p_1^A x_1 + p_2^A x_2. \tag{21}$$

However, if $p_1^B x_1 + p_2^B x_2 > 0$, then AA could not be an equilibrium because firm B would have a profitable deviation to lower its prices to capture both segments profitably (condition (8)). Therefore, $p_1^B x_1 + p_2^B x_2 \leq 0$, which implies that 1) $p_1^B = p_2^B = 0$, 2) $p_i^B < 0$ and $p_j^B \leq -p_i^B$, or 3) $p_i^B < 0$ and $p_j^B > 0$. Since firm B makes zero profit under adoption pattern AA, it is indifferent between setting negative price and zero price, and by assumption, we assume that it sets zero price. Therefore,

$$p_1^B = p_2^B = 0. \tag{22}$$

For firm A to be profitable, (21) ≥ 0. Combining (21) and (22), we get

$$\Delta_1 x_1 + \Delta_2 x_2 = p_1^A x_1 + p_2^A x_2 \geq 0. \tag{23}$$

For AA to be an equilibrium, $\Pi^A_{(2a)} \geq \Pi^A_{(1)}$ and $\Pi^A_{(2a)} \geq \Pi^A_{(3)}$ (firm A's best response to firm B's strategy is to price so consumers play AA) and $\Pi^B_{(2a)} \geq \Pi^B_{(1)}$ and $\Pi^B_{(2a)} \geq \Pi^B_{(3)}$ (firm B's best response to firm A's strategy is to price so consumers play AA):

$$\Pi^A_{(2a)} \geq \Pi^A_{(1)} \Leftrightarrow B_2 \leq u_2^A x_2 + \delta \Leftrightarrow p_2^B x_2 \geq -\Delta_2 x_2 - \delta, \tag{24}$$

$$\Pi^A_{(2a)} \geq \Pi^A_{(3)} \Leftrightarrow B_1 \leq u_1^A x_1 + \delta \Leftrightarrow p_1^B x_1 \geq -\Delta_1 x_1 - \delta, \tag{25}$$

$$\Pi^B_{(2a)} \geq \Pi^B_{(1)} \Leftrightarrow A_2 \geq u_2^B x_2 - \delta \Leftrightarrow p_2^A x_2 \leq \Delta_2 x_2 + \delta, \tag{26}$$

$$\text{and} \quad \Pi^B_{(2a)} \geq \Pi^B_{(3)} \Leftrightarrow A_1 \geq u_1^B x_1 - \delta \Leftrightarrow p_1^A x_1 \leq \Delta_1 x_1 + \delta. \tag{27}$$

Equation (22) combined with (24) and (25) give

$$\Delta_2 x_2 \geq -\delta \tag{28}$$

$$\text{and} \quad \Delta_1 x_1 \geq -\delta. \tag{29}$$

Equations (23), (28), and (29) define the set, D_{AA}. Equations (22) are the equilibrium prices for firm B for $(\Delta_1, \Delta_2) \in D_{AA}$. Equations (23), (26), and (27) together define the equilibrium prices for firm A for $(\Delta_1, \Delta_2) \in D_{AA}$. A similar analysis can be done for equilibrium BB by assuming that when $A_1 - B_1 = e$, $A_2 - B_2 = -e$, $e \in (-\delta, \delta)$, all consumers play B, when $A_1 - B_1 = \delta$ and $A_2 - B_2 = -\delta$ segment-1 (-2) plays A (B), and when $A_1 - B_1 = -\delta$ and $A_2 - B_2 = \delta$ segment-1 (-2) plays B (A).

We now find conditions under which AB is an equilibrium. We assume that when $A_1 - B_1 = e$, $A_2 - B_2 = -e$, $e \in (-\delta, \delta)$, all consumers play A, when $A_1 - B_1 = \delta$ and

$A_2 - B_2 = -\delta$ segment-1 (-2) plays A (B), and when $A_1 - B_1 = -\delta$ and $A_2 - B_2 = \delta$ segment-1 (-2) plays B (A). When the adoption pattern is AB, then both firms maximize profits by setting prices to satisfy

$$A_1 - B_1 = \delta \tag{30}$$

$$\text{and} \qquad A_2 - B_2 = -\delta. \tag{31}$$

However, if $A_1 < u_1^B x_1 + \delta$, AB could not be an equilibrium because firm B would have a profitable deviation to charge a positive price and capture segment 1 (condition (3)). Therefore, $A_1 \geq u_1^B x_1 + \delta$, which implies $p_1^A x_1 \leq \Delta_1 x_1 - \delta$. Firm A must be profitable, therefore, we have

$$0 \leq p_1^A x_1 \leq \Delta_1 x_1 - \delta, \tag{32}$$

which implies

$$\Delta_1 x_1 - \delta \geq 0. \tag{33}$$

Equations (30) and (32) together imply $p_1^A x_1 = \Delta_1 x_1 - \delta + p_1^B x_1 \geq 0$ and $p_1^A x_1 \leq \Delta_1 x_1 - \delta$, which in turn imply $-\Delta_1 x_1 + \delta \leq p_1^B x_1 \leq 0$. Since we assume that firms set zero price when they are indifferent between pricing negative and zero, firm B's segment-1 price is

$$p_1^B x_1 = 0. \tag{34}$$

Equations (30) and (34) imply firm A's segment-1 price is

$$p_1^A x_1 = \Delta_1 x_1 - \delta. \tag{35}$$

For AB to be an equilibrium, $\Pi^A_{(1)} \geq \Pi^A_{(2a)}$ (firm A's best response to firm B's strategy is to price so consumers play AB) and $\Pi^B_{(1)} \geq \Pi^B_{(2a)}$ (firm B's best response to firm A's strategy is to price so consumers play AB):

$$\Pi^A_{(1)} \geq \Pi^A_{(2a)} \Leftrightarrow B_2 \geq u_2^A x_2 + \delta \tag{36}$$

$$\text{and} \qquad \Pi^B_{(1)} \geq \Pi^B_{(2a)} \Leftrightarrow A_2 \leq u_2^B x_2 - \delta. \tag{37}$$

By substituting $B_2 = A_2 + \delta$ into (36) and combining with (37), we get

$$u_2^A x_2 \leq A_2 \leq u_2^B x_2 - \delta, \tag{38}$$

which can be expressed as $\Delta_2 x_2 + \delta \leq p_2^A x_2 \leq 0$. Since we assume that firms set zero price when they are indifferent between pricing negative and zero, firm A's segment-2 price is

$$p_2^A x_2 = 0. \tag{39}$$

Equations (31) and (39) imply firm B's segment-2 price is

$$p_2^B x_2 = -\Delta_2 x_2 - \delta. \tag{40}$$

Equation (38) implies

$$\Delta_2 x_2 \leq -\delta. \tag{41}$$

Equations (33) and (41) are sufficient conditions for firms A and B, respectively, to not deviate profitably to BA. Equations (33) and (41) define the set D_{AB}. Equations (35), (39), (34), and (40) define the equilibrium prices for firms A and B for $(\Delta_1, \Delta_2) \in D_{AB}$. The analysis for equilibrium BA is the same with firms A and B switched. Substituting the equilibrium prices into the payoff matrix shown in Figure 2 give the consumer net benefits.

■

B.2 Proof of Proposition 2

Let $D'_{AB} \equiv D_{AB} \backslash \{\{\Delta_1 > 2\theta x_2\} \cap \{\Delta_2 < -2\theta x_1\}\}$ and $D'_{BA} \equiv D_{BA} \backslash \{\{\Delta_1 < -2\theta x_2\} \cap \{\Delta_2 > 2\theta x_1\}\}$. The conditions for AB to be socially optimal are:

$$V^{AB} \geq V^{AA} \Leftrightarrow \Delta_2 \leq -2\theta x_1, \tag{42}$$

$$V^{AB} \geq V^{BB} \Leftrightarrow \Delta_1 \geq 2\theta x_2, \tag{43}$$

$$\text{and} \qquad V^{AB} \geq V^{BA} \Leftrightarrow \Delta_1 x_1 \geq \Delta_2 x_2. \tag{44}$$

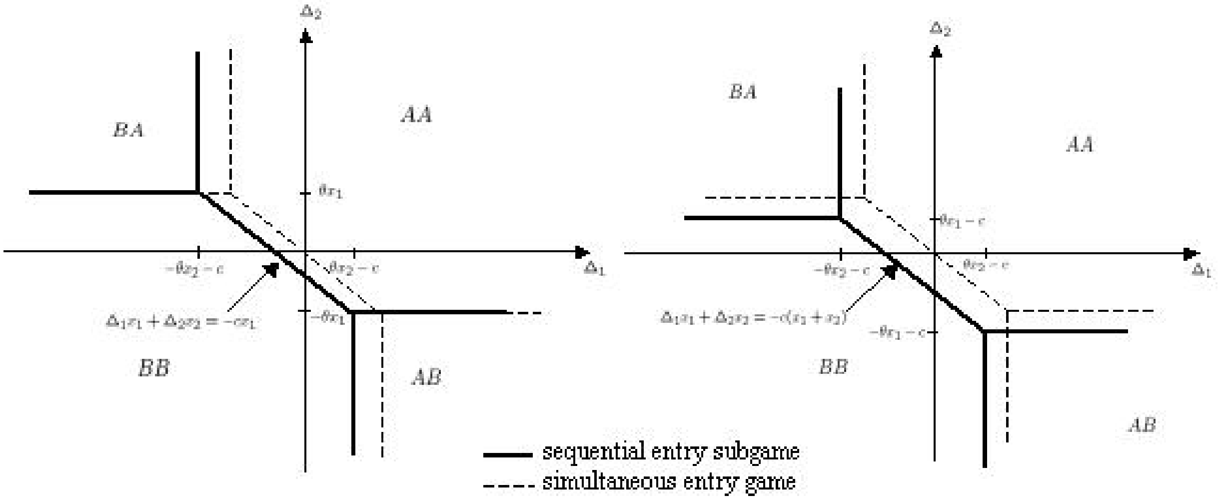
Equations (42) and (43) are binding and together define the set, S_{AB}, in which AB is the socially optimal adoption pattern. Similarly, the set $S_{BA} = \{(\Delta_1, \Delta_2) | \{\Delta_1 \leq -2\theta x_2\} \cap \{\Delta_2 \geq 2\theta x_1\}\}$ defines the set in which BA is socially optimal. In all other regions, either AA

or BB is socially optimal. From Proposition 1, the equilibrium adoption in the simultaneous game for $(\Delta_1, \Delta_2) \in D'_{AB}$ is AB, and for $(\Delta_1, \Delta_2) \in D'_{BA}$ is BA. $D'_{AB} \cap \{S_{AB} \cup S_{BA}\} = \emptyset$ and $D'_{BA} \cap \{S_{AB} \cup S_{BA}\} = \emptyset$.

■

B.3 Proof of Proposition 3

We first solve the period-2 subgames in Lemmas 1 and 2. Figure 12 summarizes the results of Lemmas 1 and 2.



a) Sequential entry subgame when only segment 1 adopts A in period 1.

b) Sequential entry subgame when both segments adopt A in period 1.

Figure 12: Sequential entry (A is the first mover), period-2 subgame adoption equilibria. TT' refers to the region where segment 1 adopts $T \in \{A, B\}$ and segment 2 adopts $T' \in \{A, B\}$. The switching cost is $c \geq 0$. Technologies A and B are both sponsored.

Lemma 1 *In the period-2 subgame when only segment* 1 *adopts* A *in period 1, the equilibrium prices and resulting consumer adoption patterns are:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 > \theta x_2 - c\}$, *firm* A *sets* $\tilde{p}_2^A(2) = \Delta_2 + \theta x_1$, *and firm* B *sets* $\tilde{p}_1^B(2) = \tilde{p}_2^B(2) = 0$, *resulting in adoption pattern* AA,
- *if* $(\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 \leq \theta x_2 - c\}$, *firm* A *sets* $\tilde{p}_2^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1}{x_2}$, *and firm* B *sets* $\tilde{p}_1^B(2) = \tilde{p}_2^B(2) = 0$, *resulting in adoption pattern* AA,

- *if* $(\Delta_1, \Delta_2) \in D_{BB}(-c, 0)$*, firm* A *sets* $\tilde{p}_2^A(2) = 0$*, and firm* B *sets* $\tilde{p}_1^B(2) = -\frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1 - \tilde{p}_2^B(2) x_2}{x_1}$*, resulting in adoption pattern* $\tilde{p}_2^B(2) \in [-\Delta_2 - \theta x_1, -\Delta_2 + \theta x_1]$*,* BB*,*
- *if* $(\Delta_1, \Delta_2) \in D_{AB}(-c, 0)$*, firm* A *sets* $\tilde{p}_2^A(2) = 0$*,* $\tilde{p}_1^B(2) = 0$*, and firm* B *sets* $\tilde{p}_2^B(2) = -\Delta_2 - \theta x_1$*, resulting in adoption pattern* AB*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BA}(-c, 0)$*, firm* A *sets* $\tilde{p}_2^A(2) = \Delta_2 - \theta x_1$*, and firm* B *sets* $\tilde{p}_1^B(2) = -\Delta_1 - \theta x_2 - c$*, resulting in adoption pattern* $\tilde{p}_2^B(2) = 0$*,* BA*.*

Proof : The necessary and sufficient conditions for AA to be an equilibrium are

$$\begin{aligned} & p_1^B(2) \geq -\Delta_1 - \theta x_2 - c && (45) \\ \text{and} \quad & p_2^A(2) - p_2^B(2) \leq \Delta_2 + \theta x_1. && (46) \end{aligned}$$

The necessary and sufficient conditions for BB to be an equilibrium are

$$\begin{aligned} & p_1^B(2) \leq -\Delta_1 + \theta x_2 - c && (47) \\ \text{and} \quad & p_2^A(2) - p_2^B(2) \geq \Delta_2 - \theta x_1. && (48) \end{aligned}$$

AA and BB are both equilibria if

$$\begin{aligned} & -\Delta_1 - \theta x_2 - c \leq p_1^B(2) \leq -\Delta_1 + \theta x_1 - c && (49) \\ \text{and} \quad & \Delta_2 - \theta x_1 \leq p_2^A(2) - p_2^B(2) \leq \Delta_2 + \theta x_1. && (50) \end{aligned}$$

If (49) and (50) hold, then consumers choose the equilibrium with the higher net total network value, i.e., they choose AA if

$$u_1^A x_1 + (u_2^A - p_2^A) x_2 \geq (u_1^B - p_1^B - c) x_1 + (u_2^B - p_2^B) x_2, \tag{51}$$

else they choose BB if

$$u_1^A x_1 + (u_2^A - p_2^A) x_2 \leq (u_1^B - p_1^B - c) x_1 + (u_2^B - p_2^B) x_2. \tag{52}$$

The necessary and sufficient conditions for AB to be an equilibrium are

$$p_1^B(2) \geq -\Delta_1 + \theta x_2 - c \tag{53}$$

$$\text{and} \quad p_2^B(2) \leq -\Delta_2 - \theta x_1 + p_2^A(2). \tag{54}$$

The necessary and sufficient conditions for BA to be an equilibrium are

$$p_1^B(2) \leq -\Delta_1 - \theta x_2 - c \tag{55}$$

$$\text{and} \quad p_2^A(2) \leq \Delta_2 - \theta x_1 + p_2^B(2). \tag{56}$$

We first find conditions under which BB, BA, and AB are equilibria. Since firms are in price competition, firm A will set $p_2^A(2)$ as low as zero in order to win segment 2 (we assume it will not price lower than zero). Given firm A's minimum segment-2 price is $p_2^A(2) = 0$, we find firm B's maximum profit under adoption patterns BB (from (52)), BA (from (55)), and AB (from (54)):

$$\begin{aligned} \Pi_{BB}^B &= -\Delta_1 x_1 - \Delta_2 x_2 - c x_1, \\ \Pi_{BA}^B &= -(\Delta_1 + \theta x_2 + c) x_1, \\ \text{and } \Pi_{AB}^B &= -(\Delta_2 + \theta x_1) x_2. \end{aligned}$$

BB is an equilibrium if and only if $\Pi_{BB}^B \geq 0$, $\Pi_{BB}^B \geq \Pi_{BA}^B$, and $\Pi_{BB}^B \geq \Pi_{AB}^B$:

$$\Delta_1 x_1 + \Delta_2 x_2 \leq -c x_1, \tag{57}$$

$$\Delta_2 \leq \theta x_1, \tag{58}$$

$$\text{and} \quad \Delta_1 \leq \theta x_2 - c x_1. \tag{59}$$

Equations (57), (58), and (59) define set $D_{BB}(-c, 0)$. Equation (52) binds and together with (47) and (48) define the equilibrium prices for firm B. Firm A sets $p_2^A(2) = 0$.

BA is an equilibrium if and only if $\Pi^B_{BA} \geq 0$, $\Pi^B_{BA} \geq \Pi^B_{BB}$, and $\Pi^B_{BA} \geq \Pi^B_{AB}$:

$$\Delta_1 \leq -\theta x_2 - c, \tag{60}$$

$$\Delta_2 \geq \theta x_1, \tag{61}$$

$$\text{and} \quad (\Delta_1 + c)x_1 \leq \Delta_2 x_2. \tag{62}$$

Equations (60) and (61) are binding and together define set $D_{BA}(-c, 0)$. Equation (55) binds and defines firm B's segment-1 equilibrium price. Firm B sets $p^B_2(2) = 0$ since it is indifferent between setting negative price and zero price. Firm A maximizes profit by setting price $p^A_2(2) = \Delta_2 - \theta x_1$.

AB is an equilibrium if and only if $\Pi^B_{AB} \geq 0$, $\Pi^B_{AB} \geq \Pi^B_{BB}$, and $\Pi^B_{AB} \geq \Pi^B_{BA}$:

$$\Delta_2 \leq -\theta x_1, \tag{63}$$

$$\Delta_1 \geq \theta x_2 - c x_1, \tag{64}$$

$$\text{and} \quad (\Delta_1 + c)x_1 \geq \Delta_2 x_2. \tag{65}$$

Equations (63) and (64) are binding and together define set $D_{AB}(-c, 0)$. Equation (54) binds and defines firm B's segment-2 equilibrium price. Firm B sets $p^B_1(2) = 0$ since it is indifferent between setting negative price and zero price. Firm A sets price $p^A_2(2) = 0$.

In the remaining set, $D_{AA}(-c, 0)$, AA is the equilibrium since firm B cannot make non-negative profit by winning one or both segments. Firm B is indifferent between pricing negative and zero, therefore, it sets $p^B_1(2) = p^B_2(2) = 0$. To determine firm A's segment-2 price, we see that (46) is binding (vs. (51)) if and only if $\Delta_1 \geq \theta x_2 - c$, which defines the set $D_{AA}(-c, 0) \cap \{\Delta_1 \geq \theta x_2 - c\}$. Equation (46) defines firm A's segment-2 equilibrium price, i.e., $p^A_2(2) = \Delta_2 + \theta x_1$. Equation (51) is binding (vs. (46)) if and only if $\Delta_1 \leq \theta x_2 - c$, which defines the set $D_{AA}(-c, 0) \cap \{\Delta_1 \leq \theta x_2 - c\}$. Equation (51) defines firm A's segment-2 equilibrium price, i.e., $p^A_2(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1}{x_1}$. ■

There is a symmetric analysis for the period-2 subgame where only segment 2 adopts in period 1, which is not shown in the interest of saving space.

Lemma 2 *In the period-2 subgame when both segments adopt A in period 1, the equilibrium prices and resulting consumer adoption patterns are:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}(-c,-c)$*, firm* B *sets* $\tilde{p}_1^B(2) = \tilde{p}_2^B(2) = 0$*, resulting in adoption pattern* AA*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BB}(-c,-c)$*, firm* B *sets* $\tilde{p}_1^B(2) = -\frac{\Delta_1 x_1 + \Delta_2 x_2 + c(x_1+x_2) - \tilde{p}_2^B(2) x_2}{x_1}$, $\tilde{p}_2^B(2) \in [-\Delta_2 - \theta x_1 - c, -\Delta_2 + \theta x_1 - c]$*, resulting in adoption pattern* BB*,*
- *if* $(\Delta_1, \Delta_2) \in D_{AB}(-c,-c)$*, firm* B *sets* $\tilde{p}_1^B(2) = 0$, $\tilde{p}_2^B(2) = -\Delta_2 - \theta x_1 - c$*, resulting in adoption pattern* AB*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BA}(-c,-c)$*, firm* B *sets* $\tilde{p}_1^B(2) = -\Delta_1 - \theta x_2 - c$, $\tilde{p}_2^B(2) = 0$*, resulting in adoption pattern* BA*.*

Proof : If firm B prices to win both segments (BB), its profit is:

$$p_1^B(2)x_1 + p_2^B(2)x_2 = -\Delta_1 x_1 - \Delta_2 x_2 - c(x_1 + x_2). \tag{66}$$

If firm B prices to win only segment 2 (AB), its profit is:

$$p_2^B(2)x_2 = (-\Delta_2 - \theta x_1 - c)x_2. \tag{67}$$

If firm B prices to win only segment 1 (BA), its profit is:

$$p_1^B(2)x_2 = (-\Delta_1 - \theta x_2 - c)x_1. \tag{68}$$

Firm B maximizes its profit by pricing to win only segment 2 if and only if (67) ≥ 0 and (67) $\geq$ (66), which imply

$$\Delta_2 \leq -\theta x_1 - c \text{ and } \Delta_1 \geq \theta x_2 - c. \tag{69}$$

Equations (69) define the set $D_{AB}(-c,-c)$. Firm B's equilibrium segment-2 price is given by (67) and $\tilde{p}_1^B(2) = 0$.

Firm B maximizes its profit by pricing to win only segment 1 if and only if (68) ≥ 0 and (68) $\geq$ (66), which imply

$$\Delta_1 \leq -\theta x_2 - c \text{ and } \Delta_2 \geq \theta x_1 - c. \tag{70}$$

Equations (70) define the set $D_{BA}(-c,-c)$. Firm B's equilibrium segment-1 price is given by (68) and $\tilde{p}_2^B(2)=0$.

Firm B maximizes its profit by pricing to win both segments if and only if (66) ≥ 0, (66) $\geq$ (67), and (66) $\geq$ (68), which imply

$$\Delta_1 x_1 + \Delta_2 x_2 \leq -c(x_1+x_2) \text{ and } \Delta_1 \leq \theta x_2 - c \text{ and } \Delta_2 \leq \theta x_1 - c. \tag{71}$$

Equations (71) define the set $D_{BB}(-c,-c)$. The necessary conditions for BB to be an equilibrium are

$$p_1^B(2) \leq -\Delta_1 + \theta x_2 - c \text{ and } p_2^B(2) \leq -\Delta_2 + \theta x_1 - c. \tag{72}$$

Equations (66) and (72) define firm B's equilibrium prices. For $(\Delta_1,\Delta_2) \in D_{AA}(-c,-c)$, firm B cannot make positive profit and therefore charges $p_1^B(2)=p_2^B(2)=0$. ■

By substituting the equilibrium prices presented in Lemmas 1 and 2 into the payoff matrices shown in Figures 4 and 5, we can derive the consumer net benefits in the period-2 subgames.

Let $\bar{\Pi}^A$ be firm A's profit in the simultaneous entry game, i.e., when both segments wait until period 2 to adopt. Let $D'_{BB} \equiv D_{BB} \cap \{\Delta_1 \leq -\theta x_2\} \cap \{-\theta x_1 \leq \Delta_2 < \theta x_1\}$, $D''_{BB} \equiv D_{BB} \cap \{\Delta_2 \leq -\theta x_1\} \cap \{-\theta x_2 \leq \Delta_1 < \theta x_2\}$, and $D'''_{BB} \equiv D_{BB} \cap \{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > -\theta x_1\}$. The sets D'_{BB}, D''_{BB}, and D'''_{BB} correspond to the regions in Figure 6 where the threshold switching costs are c_1, c_2, and $\min\{c_1,c_2\}$, respectively.

Clearly, consumers will not pay to adopt A in period 1 if they expect to switch to B in period 2. Therefore, consumers will only adopt A in period 1 if they stay with A in the period 2 subgame.

Consider set D_{AA}. Consumer net benefits in the simultaneous entry game are $w_1^{(22)} = u_1^B + \theta x_2$ and $u_2^B + \theta x_1$, and $\bar{\Pi}^A = \Delta_1 x_1 + \Delta_2 x_2$. Suppose firm A prices so that segment 1 adopts in period 1 and segment 2 adopts in period 2. From Lemma 1, we know that the period-2 subgame results in adoption equilibrium AA, therefore $w_1^{(12)} = u_1^A + \theta x_2 - p_1^A(1)$. To induce segment 1 to adopt in period 1, firm A must set price (from (17a)) so that

$$\begin{aligned} & w_1^{(12)} \geq w_1^{(22)} \Leftrightarrow u_1^A + \theta x_2 - p_1^A(1) \geq u_1^B + \theta x_2 \\ \Rightarrow \quad & p_1^A(1) = \Delta_1. \end{aligned} \tag{73}$$

To deter segment 2 from adopting in period 1, firm A must price so that $w_2^{(11)} \leq w_2^{(12)}$, which it can accomplish by setting $p_2^A(1)$ arbitrarily high. In the period-2 subgame, firm A wins segment 2 by setting price

$$p_2^A(2) \quad = \quad \begin{cases} \Delta_2 + \theta x_1, & (\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 > \theta x_2 - c\} \\ \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1}{x_2}, & (\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 \leq \theta x_2 - c\} \end{cases} . \tag{74}$$

The resulting profit of firm A (from (73) and (74)) is:

$$\Pi^A = \begin{cases} \Delta_1 x_1 + (\Delta_2 + \theta x_1) x_2, & (\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 > \theta x_2 - c\} \\ \Delta_1 x_1 + \Delta_1 x_1 + \Delta_2 x_2 + c x_1, & (\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 \leq \theta x_2 - c\} \end{cases}, \tag{75}$$

which is greater than its profit in the simultaneous entry game, $\bar{\Pi}^A = \Delta_1 x_1 + \Delta_2 x_2$, even if $c = 0 = c^*$. A similar analysis can be done to show that $c^* = 0$ when firm A prices so that only segment 2 adopts in period 1.

Consider set D_{AB}. Consumer net benefits in the simultaneous entry game are $w_1^{(22)} = u_1^B + \theta x_2$ and $w_2^{(22)} = u_2^A + \theta x_1$, and $\bar{\Pi}^A = (\Delta_1 - \theta x_2) x_1$. If firm A prices so that only segment 1 adopts in period 1, the period-2 subgame equilibrium is AB for any switching cost, $c \geq 0$, resulting in $u_1^A - p_1^A(1) = u_1^B + \theta x_2$. Therefore, the most firm A can charge segment 1 in period 1 is $p_1^A(1) = \Delta_1 - \theta x_2$ (from (17a)), resulting in the same profit as it has in the simultaneous entry game.

If firm A prices so that only segment 2 adopts in period 1 and $c \geq -\Delta_2 - \theta x_1$, then AA is the equilibrium in the period-2 subgame. From (18b), we have

$$u_2^A + \theta x_1 - p_2^A(1) = u_2^A + \theta x_1 \Leftrightarrow p_2^A(1) = 0. \tag{76}$$

In period 2, firm A wins segment 1 by charging

$$p_1^A(2) = \min\left\{\frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_2}{x_1}, \Delta_1 + \theta x_2\right\}. \tag{77}$$

From (76) and (77), firm A's profit is $\min\{\Delta_1 x_1 + \Delta_2 x_2 + c x_2, \Delta_1 + \theta x_2\}$, which is greater than its profit in the simultaneous entry game, $\bar{\Pi}^A = (\Delta_1 - \theta x_2) x_1$, for $c \geq -\Delta_2 - \theta x_1$.

If firm A prices so that both segments adopt in period 1 and $c \geq -\Delta_2 - \theta x_1$, then AA is the equilibrium in the period-2 subgame. If firm A prices so that both segments adopt in period 1, but neither segment will adopt in period 1 alone, i.e., (14) holds. Equation (14a) implies:

$$u_1^A - p_1^A(1) \leq u_1^B + \theta x_2. \tag{78}$$

Equation (14b) holds for $p_2^A(1) \geq 0$ because in the period-2 subgame where only segment 2 adopts in period 1, segment-2 consumers switch to B when $(\Delta_1, \Delta_2) \in D_{AB}$. Therefore, segment-2 consumers will never pay to adopt A in period 1. To get both segments to adopt in period 1, (15) must bind:

$$(u_1^A + \theta x_2)x_1 + (u_2^A + \theta x_1)x_2 - p_1^A x_1 - p_2^A x_2 = (u_1^B + \theta x_2)x_1 + (u_2^A + \theta x_1)x_2, \tag{79}$$

resulting in profit $p_1^A x_1 + p_2^A x_2 = \Delta_1$, which is greater than its profit in the simultaneous entry game, $\bar{\Pi}^A = (\Delta_1 - \theta x_2)x_1$. By setting $p_1^A(1) = \Delta_1$, we satisfy (78) and (13a) ($u_1^A + \theta x_2 - (u_1^A - u_1^B) \geq u_1^B + \theta x_2$). By setting $p_2^A(1) = 0$, we satisfy (13b) ($u_2^A + \theta x_1 - 0 \geq u_2^A + \theta x_1$).

If $c \geq -\Delta_2 - \theta x_1 = c^*$, firm A has a first-mover advantage by pricing so that either both segments adopt in period 1 or only segment 2 adopts in period 1. A similar analysis can be done to show that $c^* = -\Delta_1 - \theta x_2$ when $(\Delta_1, \Delta_2) \in D_{BA}$.

Consider set D_{BB}. Consumer net benefits in the simultaneous entry game are $w_1^{(22)} = u_1^A + \theta x_2$ and $w_2^{(22)} = u_2^A + \theta x_1$, and $\bar{\Pi}^A = 0$. Firm A cannot strictly increase its profit by inducing both segments to adopt in period 1 because (15) is binding:[11]

$$\begin{aligned} & (u_1^A + \theta x_2)x_1 + (u_2^A + \theta x_1)x_2 - p_1^A x_1 - p_2^A x_2 = (u_1^A + \theta x_2)x_1 + (u_2^A + \theta x_1)x_2 \\ \Rightarrow \quad & p_1^A x_1 + p_2^A x_2 = 0. \end{aligned}$$

If firm A prices so that only segment 1 adopts in period 1 and $c \geq -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}$, then AA is the equilibrium in the period-2 subgame. To induce segment-1 to adopt in period 1 (from (17a)) we have

$$\begin{aligned} & u_1^A + \theta x_2 - p_1^A(1) = u_1^A + \theta x_2 \\ \Rightarrow \quad & p_1^A(1) = 0 \end{aligned} \tag{80}$$

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[11] However, for switching cost $c \geq -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1 + x_2}$, firm A can always win both segments and make zero profit.

In the period-2 subgame, firm A wins segment 2 by setting price (74). The resulting profit of firm A (from (80) and (74)) is:

$$\Pi^A = \begin{cases} (\Delta_2 + \theta x_1)x_2, & (\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 > \theta x_2 - c\} \\ \Delta_1 x_1 + \Delta_2 x_2 + c x_1, & (\Delta_1, \Delta_2) \in D_{AA}(-c, 0) \cap \{\Delta_1 \leq \theta x_2 - c\} \end{cases}, \tag{81}$$

which is greater than its profit in the simultaneous entry game, $\bar{\Pi}^A = 0$, when $\Delta_2 \geq -\theta x_1$ and $c \geq -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1} \equiv c^*$. $D_{BB} \cap (\Delta_2 \geq -\theta x_1)$ defines set $D'_{BB} \cup D'''_{BB}$. A similar analysis shows that $c^* = -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_2}$ when $(\Delta_1, \Delta_2) \in D''_{BB} \cup D'''_{BB}$. The threshold switching cost when $(\Delta_1, \Delta_2) \in D'''_{BB}$ is therefore $c^* = \min\{-\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}, -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_2}\}$.

B.4 Proof of Proposition 4

Let $\varepsilon > 0$. Let $D'_{BB} \equiv D_{BB} \cap \{\Delta_1 \leq -\theta x_2\} \cap \{-\theta x_1 \leq \Delta_2 < \theta x_1\}$, $D''_{BB} \equiv D_{BB} \cap \{\Delta_2 \leq -\theta x_1\} \cap \{-\theta x_2 \leq \Delta_1 < \theta x_2\}$, and $D'''_{BB} \equiv D_{BB} \cap \{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > -\theta x_1\}$. The sets D'_{BB}, D''_{BB}, and D'''_{BB} correspond to the regions in Figure 6 where the threshold switching costs are c_1, c_2, and $\min\{c_1, c_2\}$, respectively.

Consider $(\Delta_1^*, \Delta_2^*) \in D_{AA}$. The threshold switching cost is $c^* = 0$ and is independent of θ. As θ increases, D_{AA} increases, therefore, $(\Delta_1^*, \Delta_2^*) \in D_{AA}$ when $\theta = \hat{\theta}$ implies $(\Delta_1^*, \Delta_2^*) \in D_{AA}$ when $\theta = \hat{\theta} + \varepsilon$.

Consider $(\Delta_1^*, \Delta_2^*) \in D'_{BB} \cup D''_{BB} \cup D'''_{BB}$. The threshold switching cost is either $-\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_2}$ or $-\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}$, both of which are independent of θ. The set $D'_{BB} \cup D''_{BB} \cup D'''_{BB}$ increases in θ, therefore, $(\Delta_1^*, \Delta_2^*) \in D'_{BB} \cup D''_{BB} \cup D'''_{BB}$ when $\theta = \hat{\theta}$ implies $(\Delta_1^*, \Delta_2^*) \in D'_{BB} \cup D''_{BB} \cup D'''_{BB}$ when $\theta = \hat{\theta} + \varepsilon$.

Consider $(\Delta_1^*, \Delta_2^*) \in D_{BB} \backslash \{D'_{BB} \cup D''_{BB} \cup D'''_{BB}\}$. The threshold switching cost is $c^* = \infty$. The set $D_{BB} \backslash \{D'_{BB} \cup D''_{BB} \cup D'''_{BB}\}$ decreases in θ, therefore, there exists $\hat{\theta}$ such that $(\Delta_1^*, \Delta_2^*) \in D'_{BB}$ or $(\Delta_1^*, \Delta_2^*) \in D''_{BB}$ for $\theta \geq \hat{\theta}$. The threshold switching costs in D'_{BB} and D''_{BB} are finite, therefore, the threshold switching cost decreases as θ increases.

Consider $(\Delta_1^*, \Delta_2^*) \in D_{AB}$. The threshold switching cost is $c^* = -\Delta_2 - \theta x_1$, which is decreasing in θ. The set D_{AB} decreases in θ, therefore, there exists $\hat{\theta}$ such that $(\Delta_1^*, \Delta_2^*) \in D_{AA}$ or $(\Delta_1^*, \Delta_2^*) \in D''_{BB}$ for $\theta \geq \hat{\theta}$. If $(\Delta_1^*, \Delta_2^*) \in D_{AA}$, the switching cost decreases to zero. If $(\Delta_1^*, \Delta_2^*) \in D''_{BB}$, the switching cost decreases to $-\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_2}$. A similar analysis for $(\Delta_1^*, \Delta_2^*) \in D_{BA}$ shows that the threshold switching cost decreases in θ.

■

B.5 Proof of Proposition 5

Let $D'_{BB} \equiv D_{BB} \cap \{\Delta_1 \leq -\theta x_2\} \cap \{-\theta x_1 \leq \Delta_2 < \theta x_1\}$, $D''_{BB} \equiv D_{BB} \cap \{\Delta_2 \leq -\theta x_1\} \cap \{-\theta x_2 \leq \Delta_1 < \theta x_2\}$, and $D'''_{BB} \equiv D_{BB} \cap \{\Delta_1 > -\theta x_2\} \cap \{\Delta_2 > -\theta x_1\}$. The sets D'_{BB}, D''_{BB}, and D'''_{BB} correspond to the regions in Figure 6 where the threshold switching costs are c_1, c_2, and $\min\{c_1, c_2\}$, respectively.

Consider set D_{AA}. The net benefits of the segments in the simultaneous entry game are $w_1^{(22)} = u_1^B + \theta x_2$ and $w_2^{(22)} = u_2^B + \theta x_1$. Firm A prices to induce adoption sequence (12) or (21). If $\Delta_1 \geq \theta x_2 - c$ and $\Delta_2 \geq \theta x_1 - c$, then firm A's profits are $\Delta_1 x_1 + \Delta_2 x_2 + \theta x_1 x_2$ regardless of adoption sequence. Under sequence (12), $w_1^{(12)} = u_1^B + \theta x_2 = w_1^{(22)}$ and $w_2^{(12)} = u_2^A + \theta x_1 - (\Delta_2 + \theta x_1) = u_2^B < w_2^{(22)}$. Under sequence (21), $w_1^{(12)} = u_1^A + \theta x_2 - (\Delta_1 + \theta x_2) = u_1^B < w_1^{(22)}$ and $w_2^{(12)} = u_2^B + \theta x_1 = w_2^{(22)}$.

If $\Delta_1 \geq \theta x_2 - c$ and $\Delta_2 \leq \theta x_1 - c$, then firm A can price to induce sequence (12) by setting $p_1^A(1) = \Delta_1$ and $p_2^A(2) = \Delta_2 + \theta x_1$, resulting in profit

$$\Delta_1 x_1 + (\Delta_2 + \theta x_2) x_2. \tag{82}$$

Firm A can also price to induce sequence (21) by setting $p_1^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_2}{x_1}$ and $p_2^A(1) = \Delta_2)$, resulting in profit

$$\Delta_1 x_1 + 2\Delta_2 x_2 + c x_2. \tag{83}$$

Profit (82) $\geq$ (83) if and only if $\Delta_2 \leq \theta x_1 - c$ (true by assumption). Therefore, firm A prices to induce sequence (12) resulting in $w_1^{(12)} = u_1^B + \theta x_2 = w_1^{(22)}$ and $w_2^{(12)} = u_2^A + \theta x_1 - (\Delta_2 + \theta x_1) < w_2^{(22)}$. A similar analysis shows that for $\Delta_1 \leq \theta x_2 - c$ and $\Delta_2 \geq \theta x_1 - c$, firm A maximizes its profit by pricing to induce sequence (21), resulting in $w_1^{(12)} = u_1^A + \theta x_2 - (\Delta_1 + \theta x_2) < w_1^{(22)}$ and $w_2^{(12)} = u_2^B + \theta x_1 = w_2^{(22)}$.

If $\Delta_1 \leq \theta x_2 - c$ and $\Delta_2 \leq \theta x_1 - c$, firm A can price to induce sequence (12) by setting $p_1^A(1) = \Delta_1$ and $p_2^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1}{x_2}$, resulting in profit

$$2\Delta_1 x_1 + \Delta_2 x_2 + c x_1. \tag{84}$$

Firm A can also price to induce sequence (21) by setting $p_1^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_2}{x_1}$ and $p_2^A(1) = \Delta_2$, resulting in profit (83). Profit (84) $\geq$ (83) if and only if $\Delta_1 x_1 + c x_1 \geq \Delta_2 x_2 + c x_2$.

Consider first the case when $\Delta_1 x_1 + cx_1 \geq \Delta_2 x_2 + cx_2$. Then $w_1^{(12)} = u_1^B + \theta x_2 = w_1^{(22)}$ and

$$\begin{aligned} w_2^{(12)} &= u_2^A + \theta x_1 - \frac{\Delta_1 x_1 + \Delta_2 x_2 + cx_1}{x_2} \\ &= u_2^B + \theta x_1 - \frac{\Delta_1 x_1 + cx_1}{x_2}. \end{aligned} \tag{85}$$

A sufficient condition for (85) $\leq w_2^{(22)}$ is $\Delta_1 \geq 0$. If $\Delta_1 < 0$, that implies that $\Delta_2 \geq 0$ (from the definition of set D_{AA}). Substituting $cx_1 \geq \Delta_2 x_2 - \Delta_1 x_1 + cx_2$ into (85) we get

$$\begin{aligned} w_2^{(12)} &\leq u_2^B + \theta x_1 - \frac{\Delta_1 x_1}{x_2} - \frac{\Delta_2 x_2 - \Delta_1 x_1 + cx_2}{x_2} \\ &= u_2^B + \theta x_1 - \Delta_2 - c \\ &\leq w_2^{(22)}. \end{aligned}$$

A similar analysis can be done for $\Delta_1 x_1 + cx_1 \leq \Delta_2 x_2 + cx_2$ to show that $w_1^{(21)} < w_1^{(22)}$ and $w_2^{(21)} = w_2^{(22)}$.

Consider set D_{AB}. The net benefits of the segments in the simultaneous entry game are $w_1^{(22)} = u_1^B + \theta x_2$ and $w_2^{(22)} = u_2^A + \theta x_1$. Firm A can either price to induce sequence (11) or (21). To induce sequence (11), firm A charges $p_1^A(1) = \Delta_1$ and $p_2^A(1) = 0$, resulting in $w_1^{(11)} = u_1^A + \theta x_2 - \Delta_1 = w_1^{(22)}$ and $w_2^{(11)} = u_2^A + \theta x_1 - 0 = w_2^{(22)}$. To induce sequence (21), firm A charges $p_2^A(1) = 0$ and $p_1^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + cx_2}{x_1}$, where $c \geq -\Delta_2 - \theta x_1$. However, firm A will only price to induce sequence (21) if it can make more profit doing so than under sequence (11), i.e., $c \geq -\Delta_2$. The resulting consumer net benefits are $w_2^{(21)} = u_2^A + \theta x_1 - 0 = w_2^{(22)}$ and

$$\begin{aligned} w_1^{(21)} &= u_1^A + \theta x_2 - \frac{\Delta_1 x_1 + \Delta_2 x_2 + cx_2}{x_1} \\ &= u_1^B + \theta x_2 - \frac{\Delta_2 x_2 + cx_2}{x_1} \\ &\leq u_1^B + \theta x_2 - \frac{\Delta_2 x_2 - \Delta_2 x_2}{x_1} \\ &= w_1^{(22)}. \end{aligned}$$

A similar analysis can be done for set D_{BA}.

Consider set $D'_{BB} \cup D'''_{BB}$. The net benefits of the segments in the simultaneous entry game are $w_1^{(22)} = u_1^A + \theta x_2$ and $w_2^{(22)} = u_2^A + \theta x_1$. Firm A prices to induce sequence (12) by charging $p_1^A(1) = 0$ and $p_2^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 + cx_1}{x_2}$, where $c \geq -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}$. This results in

$w_1^{(12)} = u_1^A + \theta x_2 - 0 = w_1^{(22)}$ and

$$\begin{aligned} w_2^{(12)} &= u_2^A + \theta x_1 - \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1}{x_2} \\ &\leq u_2^A + \theta x_1 - \frac{\Delta_1 x_1 + \Delta_2 x_2 - \Delta_1 x_1 - \Delta_2 x_2}{x_2} \\ &= w_2^{(22)} \end{aligned}$$

A similar analysis can be done for the set $D''_{BB} \cup D'''_{BB}$.

■

B.6 Proof of Proposition 6

The profit of firm A if it prices to win both segments is (from (7)):

$$p_1^A x_1 + p_2^A x_2 = \Delta_1 x_1 + \Delta_2 x_2. \tag{86}$$

The profit of firm A if it prices to win only segment 1 is (from (9)):

$$p_1^A x_1 = (\Delta_1 - \theta x_2) x_1. \tag{87}$$

The profit of firm A if it prices to win only segment 2 is (from (12)):

$$p_2^A x_2 = (\Delta_2 - \theta x_1) x_2. \tag{88}$$

Firm A maximizes its profit by winning both segments (AA) if and only if (86) ≥ 0, (86) $\geq$ (87), and (86) $\geq$ (88). These three inequalities define set D_{AA}. Equations (1), (2), and (7) define firm A's pricing strategy.

Firm A maximizes its profit by winning only segment 1 (AB) if and only if (87) ≥ 0 and (87) $\geq$ (86). These two inequalities define set D_{AB}. Equation (9) binds to maximize profit. Firm A is indifferent between pricing negative and zero when it does not win a segment, therefore, $p_2^A = 0$.

Firm A maximizes its profit by winning only segment 2 (BA) if and only if (88) ≥ 0 and (88) $\geq$ (86). These two inequalities define set D_{BA}. Equation (12) binds to maximize profit. Firm A is indifferent between pricing negative and zero when it does not win a segment, therefore, $p_1^A = 0$.

For $(\Delta_1, \Delta_2) \in D_{BB}$, firm A cannot make non-negative profit by winning either segment, therefore it prices $p_1^A = p_2^A = 0$. Substituting $p_1^B = p_2^B = 0$ and the equilibrium prices of firm A into the payoff matrix shown in Figure 2 gives the consumer net benefits.

■

B.7 Proof of Proposition 7

Let $D'_{BB} \equiv D_{BB} \cap \{\Delta_1 x_1 + \Delta_2 x_2 \geq -\theta x_1 x_2\}$, $D'_{AB} \equiv D_{AB} \cap \{\Delta_2 \geq -3\theta x_1\}$, and $D'_{BA} \equiv D_{BA} \cap \{\Delta_1 \geq -3\theta x_2\}$. These three subsets represent the regions in Figure 7 where the threshold switching costs are $c^* = \min\{c_1, c_2\}$, $c^* = -2(\Delta_2 + \theta x_1)$, and $c^* = -2(\Delta_1 + \theta x_2)$, respectively. Let $\bar{\Pi}^A$ be firm A's profit in the simultaneous entry game, i.e., when both segments wait until period 2 to adopt. The analysis for the threshold switching cost for the D_{AA} is the same as in the proof for Proposition 3.

Consider set D_{AB}. Consumer net benefits in the simultaneous entry game are $w_1^{(22)} = u_1^B + \theta x_2$ and $w_2^{(22)} = u_2^B$, and $\bar{\Pi}^A = (\Delta_1 - \theta x_2)x_1$. If firm A prices to induce sequence (12), the period-2 subgame equilibrium is AB for any switching cost. Therefore, the most firm A can charge segment 1 in period 1 is $p_1^A(1) = \Delta_1 - \theta x_2$ (from (17a)), resulting in the same profit as it has in the simultaneous entry game.

If firm A prices to induce sequence (11) and $c \geq -\Delta_2 - \theta x_1$, then AA is the equilibrium in the period-2 subgame. If firm A prices so that both segments adopt in period 1, but neither segment will adopt in period 1 alone, i.e., (14) holds, then (15) must bind:

$$(u_1^A + \theta x_2)x_1 + (u_2^A + \theta x_1)x_2 - p_1^A x_1 - p_2^A x_2 = (u_1^B + \theta x_2)x_1 + u_2^B x_2, \tag{89}$$

resulting in profit $p_1^A x_1 + p_2^A x_2 = \Delta_1 x_1 + \Delta_2 x_2 - \theta x_1 x_2$, which is greater than its profit in the simultaneous entry game, $\bar{\Pi}^A = \Delta_1 - \theta x_2$, if and only if $\Delta_2 \geq 0$. However, in D_{AB}, $\Delta_2 \leq -\theta x_1$, therefore, firm A does not have a first-mover advantage by pricing to induce sequence (11).

If firm A prices so that only segment 2 adopts in period 1 and $c \geq -\Delta_2 - \theta x_1$, then AA is the equilibrium in the period-2 subgame. From (18b), we have

$$u_2^A + \theta x_1 - p_2^A(1) = u_2^B \Leftrightarrow p_2^A(1) = \Delta_2 + \theta x_1. \tag{90}$$

In period 2, firm A wins segment 1 by charging

$$p_1^A(2) = \begin{cases} \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_2}{x_1}, & c \leq -\Delta_2 + \theta x_1 \\ \Delta_1 + \theta x_2, & c > -\Delta_2 + \theta x_1 \end{cases}. \tag{91}$$

If $c \leq -\Delta_2 + \theta x_1$, firm A's profit is

$$\Delta_1 x_1 + 2\Delta_2 x_2 + c x_2 + \theta x_1 x_2 \geq \bar{\Pi}^A \Leftrightarrow c \geq -2(\Delta_2 + \theta x_1) = c^*.$$

The threshold switching cost satisfies $c^* \leq -\Delta_2 + \theta x_1$ if and only if $\Delta_2 \geq -3\theta x_1$. If $c > -\Delta_2 + \theta x_1$, firm A's profit is

$$\Delta_1 x_1 + \Delta_2 x_2 + 2\theta x_1 x_2 \geq \bar{\Pi}^A \Leftrightarrow \Delta_2 \geq -3\theta x_1.$$

The set D'_{AB} is defined by $D_{AB} \cap \{\Delta_2 \geq -3\theta x_1\}$ and the threshold switching cost is $c^* = -2(\Delta_2 + \theta x_1)$. A similar analysis can be done to show that $c^* = -2(\Delta_1 + \theta x_2)$ for $(\Delta_1, \Delta_2) \in D_{BA} \cap \{\Delta_1 \geq -3\theta x_2\} = D'_{BA}$.

Consider set D_{BB}. Consumer net benefits in the simultaneous entry game are $w_1^{(22)} = u_1^B + \theta x_2$ and $w_2^{(22)} = u_2^B + \theta x_1$, and $\bar{\Pi}^A = 0$. Firm A cannot increase its profit by inducing both segments to adopt in period 1 because (15) is binding:

$$\begin{aligned} & (u_1^A + \theta x_2)x_1 + (u_2^A + \theta x_1)x_2 - p_1^A x_1 - p_2^A x_2 = (u_1^B + \theta x_2)x_1 + (u_2^B + \theta x_1)x_2, \\ \Rightarrow \quad & p_1^A x_1 + p_2^A x_2 = \Delta_1 x_1 + \Delta_2 x_2 \leq 0. \end{aligned}$$

If firm A prices to induce sequence (12) and $c \geq -\frac{\Delta_1 x_1 + \Delta_2 x_2}{x_1}$, the equilibrium in the period-2 subgame is AA. To induce segment 1 to adopt in period 1 (from (17a)) we have

$$\begin{aligned} & u_1^A + \theta x_2 - p_1^A(1) = u_1^B + \theta x_2, \\ \Rightarrow \quad & p_1^A(1) = \Delta_1. \end{aligned} \tag{92}$$

In the period-2 subgame, firm A wins segment 2 by setting price

$$p_2^A(2) = \begin{cases} \frac{\Delta_1 x_1 + \Delta_2 x_2 + c x_1}{x_2}, & c \leq -\Delta_1 + \theta x_2 \\ \Delta_2 + \theta x_1, & c > -\Delta_1 + \theta x_2 \end{cases}. \tag{93}$$

From (92) and (93), firm A's maximum profit (when $c > -\Delta_1 + \theta x_2$) is

$$\Delta_1 x_1 + \Delta_2 x_2 + \theta x_1 x_2 \geq \bar{\Pi}^A \Leftrightarrow \Delta_1 x_1 + \Delta_2 x_2 \geq -\theta x_1 x_2. \tag{94}$$

Otherwise, if $c \leq -\Delta_1 + \theta x_2$, firm A's profit is

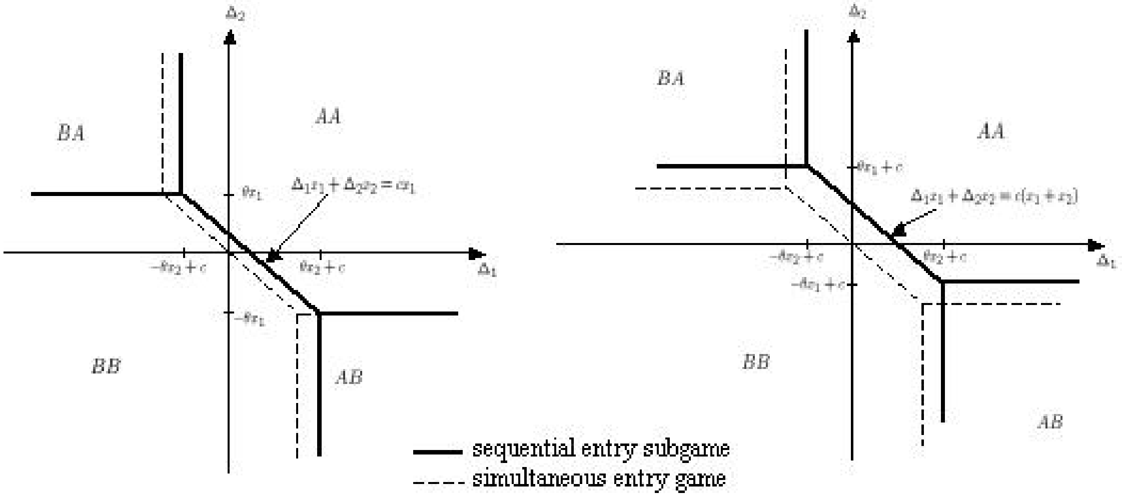
$$2\Delta_1 x_1 + \Delta_2 x_2 + c x_1 \geq \bar{\Pi}^A \Leftrightarrow c \geq -\frac{2\Delta_1 x_1 + \Delta_2 x_2}{x_1} = c^*.$$

The set D'_{BB} is defined by $D_{BB} \cap \{\Delta_1 x_1 + \Delta_2 x_2 \geq -\theta x_1 x_2\}$ (from (94)). A similar analysis can be done to show that if $(\Delta_1, \Delta_2) \in D_{BB} \cap \{\Delta_1 x_1 + \Delta_2 x_2 \geq -\theta x_1 x_2\}$ and $c^* = -\frac{\Delta_1 x_1 + 2\Delta_2 x_2}{x_2}$, firm A has a first-mover advantage by pricing to induce sequence (21).

■

B.8 Proof of Proposition 8

We first solve the period-2 subgames in Lemmas 3 and 4. Figure 13 summarizes the results of Lemmas 3 and 4.



a) Sequential entry subgame when only segment 1 adopts B in period 1.

b) Sequential entry subgame when both segments adopt B in period 1.

Figure 13: Sequential entry (B is the first mover), period-2 subgame adoption equilibria. TT' refers to the region where segment 1 adopts $T \in \{A, B\}$ and segment 2 adopts $T' \in \{A, B\}$. The switching cost is $c > 0$. Technology A is sponsored, technology B is unsponsored.

Lemma 3 *In the period-2 subgame when only segment* 1 *adopts* B *in period 1, the equilibrium prices and resulting consumer adoption patterns are:*

- *if* $(\Delta_1, \Delta_2) \in D_{AA}(c, 0)$*, firm* A *sets* $\tilde{p}_1^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 - c x_1 - \tilde{p}_2^A(2) x_2}{x_1}$, $\tilde{p}_2^A(2) \in [\Delta_2 - \theta x_1, \Delta_2 + \theta x_1]$*, resulting in adoption pattern* AA*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BB}(c, 0)$*, firm* A *sets* $\tilde{p}_1^A(2) = \tilde{p}_2^A(2) = 0$*, resulting in adoption pattern* BB*,*
- *if* $(\Delta_1, \Delta_2) \in D_{AB}(c, 0)$*, firm* A *sets* $\tilde{p}_1^A(2) = \Delta_1 - \theta x_2 - c$, $\tilde{p}_2^A(2) = 0$*, resulting in adoption pattern* AB*,*
- *if* $(\Delta_1, \Delta_2) \in D_{BA}(c, 0)$*, firm* A *sets* $\tilde{p}_1^A(2) = 0$, $\tilde{p}_2^A(2) = \Delta_2 - \theta x_1$*, resulting in adoption pattern* BA*.*

Proof : In period 2, if firm A prices to win both segments (AA), its profit is:

$$p_1^A(2)x_1 + p_2^A(2)x_2 = \Delta_1 x_1 + \Delta_2 x_2 - c x_1. \tag{95}$$

If firm A prices to win only segment 1 (AB), its profit is:

$$p_1^A(2)x_1 = (\Delta_1 - \theta x_2 - c)x_1. \tag{96}$$

If firm A prices to win only segment 2 (BA), its profit is:

$$p_2^A(2)x_2 = (\Delta_2 - \theta x_1)x_2. \tag{97}$$

Firm A maximizes its profit by pricing to win only segment 1 (AB) if and only if (96) ≥ 0 and (96) $\geq$ (95), which imply

$$\Delta_1 \geq \theta x_2 + c \text{ and } \Delta_2 \leq -\theta x_1. \tag{98}$$

Equations (98) define the set $D_{AB}(c, 0)$. Equation (96) gives $\tilde{p}_1^A(2)$. Firm A is indifferent between pricing negative and zero when it does not win a segment, therefore, $\tilde{p}_2^A(2) = 0$.

Firm A maximizes its profit by pricing to win only segment 2 (BA) if and only if (97) ≥ 0 and (97) $\geq$ (95), which imply

$$\Delta_2 \geq \theta x_2 \text{ and } \Delta_1 \leq -\theta x_2 + c. \tag{99}$$

Equations (99) define the set $D_{BA}(c, 0)$. Equation (97) gives $\tilde{p}_2^A(2)$. Firm A is indifferent between pricing negative and zero when it does not win a segment, therefore, $\tilde{p}_1^A(2) = 0$.

Firm A maximizes its profit by pricing to win both segments (AA) if and only if (95) ≥ 0, (95) $\geq$ (96), and (95) $\geq$ (97), which imply

$$\Delta_1 x_1 + \Delta_2 x_2 \geq c x_1, \; \Delta_2 \geq -\theta x_1 \text{ and } \Delta_1 \geq -\theta x_2 + c. \tag{100}$$

Equations (100) define the set $D_{AA}(c, 0)$. The necessary conditions for AA to be an equilibrium are:

$$p_1^A(2) \leq \Delta_1 + \theta x_2 - c \text{ and } p_2^A(2) \leq \Delta_2 + \theta x_1. \tag{101}$$

Equations (95) and (101) define firm A's equilibrium prices. If $(\Delta_1, \Delta_2) \in D_{BB}(c, 0)$, firm A cannot make positive profit and therefore charges $p_1^A(2) = p_2^A(2) = 0$. ■

There is a symmetric analysis for the period-2 subgame where only segment 2 adopts in period 1, which is not shown in the interest of saving space.

Lemma 4 *In the period-2 subgame when both segments adopt B in period 1, the equilibrium prices and resulting consumer adoption patterns are:*

- *if $(\Delta_1, \Delta_2) \in D_{AA}(c, c)$, firm A sets $\tilde{p}_1^A(2) = \frac{\Delta_1 x_1 + \Delta_2 x_2 - c(x_1 + x_2) - \tilde{p}_2^A(2) x_2}{x_1}$, $\tilde{p}_2^A(2) \in [\Delta_1 - \theta x_2 - c, \Delta_1 + \theta x_2 - c]$, resulting in adoption pattern AA,*

- *if $(\Delta_1, \Delta_2) \in D_{BB}(c, c)$, firm A sets $\tilde{p}_1^A(2) = \tilde{p}_2^A(2) = 0$, resulting in adoption pattern BB*

- *if $(\Delta_1, \Delta_2) \in D_{AB}(c, c)$, firm A sets $\tilde{p}_1^A(2) = \Delta_1 - \theta x_2 - c$, $\tilde{p}_2^A(2) = 0$, resulting in adoption pattern AB,*

- *if $(\Delta_1, \Delta_2) \in D_{BA}(c, c)$, firm A sets $\tilde{p}_1^A(2) = 0$, $\tilde{p}_2^A(2) = \Delta_2 - \theta x_1 - c$, resulting in adoption pattern BA.*

Proof : In period 2, if firm A prices to win both segments (AA), its profit is:

$$p_1^A(2)x_1 + p_2^A(2)x_2 = \Delta_1 x_1 + \Delta_2 x_2 - c(x_1 + x_2). \tag{102}$$

If firm A prices to win only segment 1 (AB), its profit is:

$$p_1^A(2)x_1 = (\Delta_1 - \theta x_2 - c)x_1. \tag{103}$$

If firm A prices to win only segment 2 (BA), its profit is:

$$p_2^A(2)x_2 = (\Delta_2 - \theta x_1 - c)x_2. \tag{104}$$

Firm A maximizes its profit by pricing to win only segment 1 (AB) if and only if (103) ≥ 0 and (103) $\geq$ (102), which imply

$$\Delta_1 \geq \theta x_2 + c \text{ and } \Delta_2 \leq -\theta x_1 + c. \tag{105}$$

Equations (105) define the set D_{AB} (c,c). Equation (103) gives $\tilde{p}_1^A(2)$. Firm A is indifferent between pricing negative and zero when it does not when a segment, therefore, $\tilde{p}_2^A(2) = 0$.

Firm A maximizes its profit by pricing to win only segment 2 (BA) if and only if (104) ≥ 0 and (104) $\geq$ (102), which imply

$$\Delta_2 \geq \theta x_1 + c \text{ and } \Delta_1 \leq -\theta x_2 + c. \tag{106}$$

Equations (106) define the set $D_{BA}(c, c)$. Equation (104) gives $\tilde{p}_2^A(2)$. Firm A is indifferent between pricing negative and zero when it does not when a segment, therefore, $\tilde{p}_1^A(2) = 0$.

Firm A maximizes its profit by pricing to win both segments (AA) if and only if (102) $\geq$ 0, (102) $\geq$ (103), and (102) $\geq$ (104), which imply

$$\Delta_1 x_1 + \Delta_2 x_2 \geq c(x_1 + x_2),\ \Delta_1 \geq -\theta x_2 + c \text{ and } \Delta_2 \geq -\theta x_1 + c. \tag{107}$$

Equations (107) define the set $D_{AA}(c, c)$. The necessary conditions for AA to be an equilibrium are

$$p_1^A(2) \leq \Delta_1 + \theta x_2 - c \text{ and } p_2^A(2) \leq \Delta_2 + \theta x_1 - c. \tag{108}$$

Equations (102) and (108) define firm A's equilibrium prices. If $(\Delta_1, \Delta_2) \in D_{BB}(c, c)$, firm A cannot make positive profit and therefore charges $\tilde{p}_1^A(2) = \tilde{p}_2^A(2) = 0$. ■

By substituting the equilibrium prices presented in Lemmas 3 and 4 into the payoff matrices shown in Figures 8 and 9, we can derive the consumer net benefits in the period-2 subgames.

It is a straightforward application of Lemmas 3 and 4 to show that, regardless of adoption sequence, the equilibrium adoption pattern is:

$$\begin{aligned}
& AB \text{ if } (\Delta_1, \Delta_2) \in E_{AB}, \\
& BA \text{ if } (\Delta_1, \Delta_2) \in E_{BA}, \\
& AA \text{ if } (\Delta_1, \Delta_2) \in E_{AA} \cap \{\Delta_1 \geq -\theta x_2 + c\} \cap \{\Delta_2 \geq -\theta x_1 + c\}, \\
\text{and} \quad & BB \text{ if } (\Delta_1, \Delta_2) \in E_{BB} \cap \{\Delta_1 \leq \theta x_2\} \cap \{\Delta_2 \leq \theta x_1\}.
\end{aligned}$$

We define the following subsets (shown in Figure 14):

$$\begin{aligned}
R_1 &\equiv E_{AA} \cap \{\Delta_2 < -\theta x_1 + c\}, \\
R_2 &\equiv E_{BB} \cap \{\Delta_1 > \theta x_2\}, \\
R_3 &\equiv E_{AABB} \cap \{\Delta_2 < -\theta x_1 + c\} \cap \{\Delta_1 > \theta x_2\}, \\
\text{and } R_4 &\equiv E_{AABB} \backslash \{R_3 \cup \{\{\Delta_1 < -\theta x_2 + c\} \cap \{\Delta_2 > \theta x_1\}\}\}.
\end{aligned}$$

By applying Lemmas 3 and 4, we can derive the period-1, stage-2 consumer payoff matrices for sets R_1-R_4, shown in Figures 15-18. The equilibria in Figure 15 are sequences (12) and (22), which lead to AA. The equilibria in Figure 16 are sequences (11) and (12), which lead to BB. The equilibria in Figure 17 are sequences (11), (12), and (22), which lead to AA or BB. The equilibria in Figure 18 are sequences (11), (12), (21), and (22), which lead to AA or BB. This leads to the conclusion of the proposition for $\{\Delta_1 \geq 0\} \cap \{\Delta_2 \leq 0\}$. A similar analysis completes the proof for $\{\Delta_1 \leq 0\} \cap \{\Delta_2 \geq 0\}$.

■

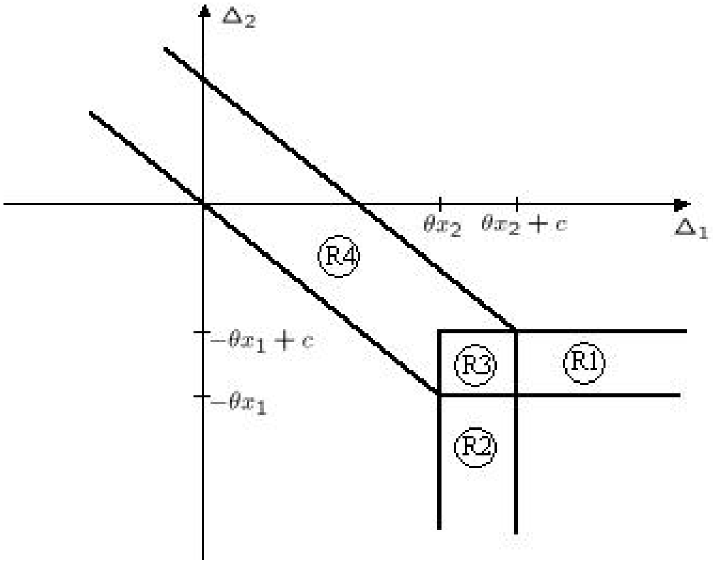


Figure 14: Sets R_1, R_2, R_3, and R_4, as defined in the proof for Proposition 8.

Seg 1 \ Seg 2	Period 1	Period 2
Period 1	$u_1^B + \theta x_2$, u_2^B AB	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ AA
Period 2	$u_1^B + \theta x_2$, u_2^B AB	$u_1^B + \theta x_2$, $u_2^B + \theta x_2$ AA

Figure 15: Sequential entry game (B is the first mover), period-1, stage-2 consumer payoff matrix for R_1.

B.9 Proof of Corollary 1

Let $\varepsilon > 0$. Consider $(\Delta_1^*, \Delta_2^*) \in E_{BB}$. As θ increase, E_{BB} increases, therefore, $(\Delta_1^*, \Delta_2^*) \in E_{BB}$ when $\theta = \hat{\theta}$ implies $(\Delta_1^*, \Delta_2^*) \in E_{BB}$ when $\theta = \hat{\theta} + \epsilon$. Therefore, the net benefit of consumers remains constant as θ increases. The same reasoning applies to $(\Delta_1^*, \Delta_2^*) \in E_{AABB}$.

Consider $(\Delta_1^*, \Delta_2^*) \in E_{AB}$. The set E_{AB} decreases in θ, therefore, there exists $\hat{\theta}$ such that for $\theta \geq \hat{\theta}$, $(\Delta_1^*, \Delta_2^*) \in E_{BB}$. Consumer net benefits weakly increase: $w_1^{BB} = u_1^B + \theta x_2 = w_1^{AB}$ and $w_2^{BB} = u_2^B + \theta x_1 > u_2^B = w_2^{AB}$. A similar analysis applies to $(\Delta_1^*, \Delta_2^*) \in E_{BA}$.

Consider $(\Delta_1^*, \Delta_2^*) \in E_{AA}$. The set E_{AA} decreases in θ, therefore, there exists $\hat{\theta}$ such that for $\theta \geq \hat{\theta}$, $(\Delta_1^*, \Delta_2^*) \in E_{AABB}$. Consumer net benefits remain constant: $w_1^{AA} = u_1^B + \theta x_2 = w_1^{BB}$ and $w_2^{AA} = u_2^B + \theta x_1 = w_2^{BB}$.

■

Seg 1 \ Seg 2	Period 1	Period 2
Period 1	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ BB	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ BB
Period 2	$u_1^B + \theta x_2$, u_2^B AB	$u_1^B + \theta x_2$, u_2^B AB

Figure 16: Sequential entry game (B is the first mover), period-1, stage-2 consumer payoff matrix for R_2.

Seg 1 \ Seg 2	Period 1	Period 2
Period 1	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ BB	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ AA or BB
Period 2	$u_1^B + \theta x_2$, u_2^B AB	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ AA

Figure 17: Sequential entry game (B is the first mover), period-1, stage-2 consumer payoff matrix for R_3.

Seg 1 \ Seg 2	Period 1	Period 2
Period 1	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ BB	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ AA or BB
Period 2	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ AA or BB	$u_1^B + \theta x_2$, $u_2^B + \theta x_1$ AA

Figure 18: Sequential entry game (B is the first mover), period-1, stage-2 consumer payoff matrix for R_4.