

# Resource Allocation and Firm Scope\*

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## Resource Allocation and Firm Scope

**Abstract:** We develop a theory of firm scope based on the benefits and costs of resource allocation within firms. Integrating two firms into one makes it possible to allocate by authority scarce resources that are costly to trade. But to do so efficiently, top management must rely on information that is communicated by self-interested division managers. Two countervailing effects influence the design of optimal incentive contracts: first, intra-firm competition for scarce resources strengthens the incentives of division managers to expend effort on creating profitable projects. Second, a manager who is paid for his own division's performance has an incentive to overstate the quality of his project, which can be corrected only by paying for firm performance as well. We show that the second effect dominates, leading to overall higher costs of providing incentives in an integrated firm, relative to stand-alone firms. Integration thus creates new agency costs that are directly related to its benefits. We derive predictions about when two firms are likely to integrate, and about the optimal structure and incentives contracts in the integrated firm. In particular, we show why it is optimal to separate the tasks of allocating resources and running a division.

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# 1 Introduction

Production in most firms involves the use of scarce firm-specific resources, and allocating them to different uses is one of the most important responsibilities of managers. Managerial authority replaces the price mechanism when contracting over resources is too costly. This simple argument, first expressed by Coase (1937), helps us understand why firms exist. It also helps to explain firms' expansion and merger activities: an often stated reason for two firms to merge is the benefit of pooling capabilities and sharing resources.<sup>1</sup> An implicit assumption here is that it is easier to share resources within an integrated firm than across firm boundaries by contract. Intuitively, it is often preferable to specify who can make decisions about resources than to specify in advance or negotiate how to use resources under different contingencies.

The benefits of integration follow naturally from contracting limitations. The costs of integration, however, are harder to identify, a problem that Williamson (1985, Ch. 6) expressed as the "selective-intervention puzzle". It is widely believed that integration creates new agency costs, but pinning down their precise nature and origin has proven to be difficult.

We develop a theory of firm scope that directly relates the costs and benefits of integration. The benefit of integrating two firms into one is top management's ability to allocate pooled resources to their best use. To do so, top management requires information about opportunities to invest resources. The problem is that this information resides with division managers who may have an incentive to misrepresent it strategically.<sup>2</sup>

We show that establishing truthful upward communication raises the firm's cost of inducing managerial effort. This effect dominates another, positive, effect of managerial competition for resources on effort incentives. The benefit of using resources more efficiently under integration thus necessarily goes along with higher costs of providing incentives for effort. We go on to derive predictions about when two firms are likely to integrate and about the optimal structure of an integrated firm. In particular, we show why it is optimal to separate the tasks of allocating

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<sup>1</sup> Procter and Gamble's acquisition of Gillette in 2005 was aimed at sharing complementary capabilities: "P&G is big in female grooming; Gillette excels in male grooming. Gillette is big in India and Brazil; P&G's strengths lie in China, Japan and places such as Turkey. P&G knows all about molecules; Gillette is a dab hand at gadgets. And P&G's expertise in retail customer development goes hand-in-hand with Gillette's mastery of point of purchase" (Marketing Week 2005).

<sup>2</sup> As Crozier (1965) observed in his field study: "[In making many decisions, higher-level managers] must rely heavily on the information they receive from the section chiefs... The section chiefs, however, ... are running parallel identical units that have to compete for scarce resources. ... Thus they are likely ... to bias the information they give in order to get the maximum of material resources and personal factors with which to run their sections smoothly" (p.45).

resources and running a division.

Resources in our theory have two key characteristics: first, they are firm-specific and not available in an external market, or only available as inferior substitutes. Second, they can be allocated to different uses, but doing so by contract is difficult. Resources with these characteristics fall between the extremes of easily tradeable commodities on one hand, and unique capabilities or other custom-made inputs with only a single use on the other.<sup>3</sup>

Specialized human resources are the best examples that fit these criteria. Many firms' capabilities do not reside in the skills of particular individuals, but instead consist of the "know-how" held collectively by individuals in the firm (see Sutton, 2005). Of interest for us are those resources and capabilities that can be allocated to different possible uses, which means that the resources themselves must be sufficiently well-defined. On the other hand, how resources are to be used is often prohibitively costly to specify in a contract, for instance, because of uncertainty about the duration of demand for them, or for fear of leakage of information to competitors in the case of trade across firm boundaries. The best option then often is to allocate resources by managerial authority.<sup>4</sup>

We develop our theory in a model consisting of two production units that can be either run as stand-alone firms or as divisions of an integrated firm. Each unit is endowed with a fixed amount of resources. To capture the above characteristics in a simplifying way, we assume that the use of resources is not contractible, while *authority* over resources is. We thus follow a literature in which authority over actions, but not the actions themselves are contractible (Hart and Holmström, 2002, Aghion et al., 2004, and the survey of Dewatripont, 2001.)<sup>5</sup>

Each unit is run by a manager whose job it is to create profitable projects. A project's payoff depends on its quality and the resources invested in it. Project quality, in turn, depends on the manager's effort; to induce effort, firms can offer wage contracts based on the projects' payoffs.

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<sup>3</sup> Our notion of resources is not widely used in the economics literature, but is similar to both Chandler's (1990) notion of "organizational capabilities" and to the concept of "unique resources and capabilities" in the strategic management literature, see e.g. Penrose (1959), Wernerfelt (1984) and Barney (1991). The latter, in particular, emphasizes the ability to use unique resources and capabilities in different businesses as the main justification for firms' expansion of scope (see Teece, 1982). Examples of work in economics that builds on the notion of organizational capabilities includes Matsusaka (2001) and Sutton (2005).

<sup>4</sup> In the aviation industry, there is a chronic shortage of engineers who have expertise with composite materials. In response, the headquarters of Airbus Industries, for instance, deploys teams of specialized aviation engineers to different units within the firm, based on an audit process designed to ascertain where the engineers are needed most.

<sup>5</sup> A resource for which this assumption typically does not hold is capital (money). Our theory is therefore strictly speaking not one about internal capital markets; see our discussion in Section 2.

Managers are risk-neutral but protected by limited liability.

If run as an independent firm, each unit uses its endowment of resources, and cannot obtain more resources elsewhere. When the firms merge, they become divisions of one firm; their resources are pooled and placed under the authority of a CEO who can shift the firm's resources to divisions with good projects. We refer to this as a *CEO structure*. For most of the paper, we define "integration" as tantamount to creating such a CEO hierarchy. This rules out the uninteresting case in which the two units are placed under common ownership or management but continue to operate independently with their given resources.

The quality of each division's project is the division manager's private information. Realizing the benefit of integration therefore requires that division managers communicate their information truthfully. But since a project's payoff is higher the more resources it receives, each manager will have a motive to overstate the quality of his project if his wage is based on his division's performance. Pay for individual performance thus gives rise to an endogenous "empire-building" motive, even though managers have no intrinsic preferences for resources.

We show that it is possible to induce truthful communication by paying managers not only for their own division's performance, but for that of the other division as well, or equivalently, for firm performance. Doing so ensures that a manager with a bad project will refrain from claiming to have a good one, since he can gain from the allocation of the firm's resources to a better project. However, providing "team-based" incentives of this sort makes it more costly to induce managerial effort than under non-integration, an effect we refer to as the *information-rent effect* of integration.

There is also a second, positive, *competition effect* of integration on effort incentives: allocating resources based on project quality raises the marginal benefit of effort to create good projects, both because of a complementarity between project quality and resources, and because of competition between the division managers for scarce resources.

We show that even under the most general contracting assumptions, the information-rent effect weakly dominates the competition effect. When contracts contingent on the division managers' messages to the CEO can be written, the two effects cancel each other. In this case, provided that the costs of employing a CEO are low enough, integration is always optimal because it leads to a better resource allocation without any increase in agency costs. In contrast, when communication is cheap talk and the managers' wages can be based only on the divisions' performance (as we assumed in our discussion above), the information-rent effect strictly dominates. There is then a tradeoff between allocating resources optimally and providing effort incentives, and whether integration or non-integration is optimal depends on the relative

importance of these objectives.

We strengthen our result by showing that the CEO structure considered so far in fact minimizes agency costs compared with alternative organizational structures that do not require a CEO. One alternative is a *skewed structure*, in which one of the division managers has authority over all resources. This structure suffers from a conflict of interest, and is strictly dominated by the CEO structure in terms of dealing with incentive problems. Another alternative structure is decentralized *horizontal exchange*, where each manager has authority over part of the firm's resources but can voluntarily lend resources to the other division. We show that using a CEO also dominates horizontal exchange, unless resources are highly complementary, in which case horizontal exchange can work equally well.

Our paper makes three contributions. First, we provide a new and simple theory of the benefit and cost of integration. In a nutshell, realizing the benefits of allocating resources by managerial authority requires aggregating dispersed information, but establishing truthful communication comes at the cost of muted incentives.<sup>6</sup> Like most incomplete-contracting theories, we take limitations of the market as given; in this sense the benefit of integration is assumed rather than explained. What we explain is how integration creates new costs that are directly related to the benefit of integration. In so doing we make minimal assumptions about the underlying agency problem; our model does not rely on conflicting preferences over decisions, influence activities or exogenous empire-building incentives. All we assume is that managers must be given incentives to create investment opportunities, and have private information about them.

Second, we provide an incentive-based rationale for the commonly observed organization of firms as hierarchies in which the “producing and distributing units ... are monitored and coordinated by middle managers [, while] top managers ... [take] the place of the market in allocating resources for future production and distribution” Chandler (1977, p.7). In our theory, a hierarchy with a top manager as pure coordinator avoids the conflict of interest that one or both division managers would have if given the double task of running a division and allocating resources. The endogeneity of the CEO structure also suggests that dispersed information, and not the centralization of authority, is the ultimate source of the costs of integration, in contrast to other theories of the firm (see Section 2).

Third, our model generates several empirically testable predictions: (i) A natural measure of the two units' “relatedness” is the complementarity of their resources. It follows that integration

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<sup>6</sup> At the same time, we provide a micro-foundation for the use of firm-based (“team”) incentives in multidivisional firms. Firm-based incentives are often attributed to the existence of positive externalities without specifying their nature. In our model, the positive externality is the gain from using pooled resources efficiently. It can be realized only if managers communicate truthfully to the CEO, and team incentives help to accomplish this goal.

is more likely the more closely related the two units are, consistent with intuition and evidence. (ii) Integration is more likely the more variable the production units' profits are. This follows not from a desire to reduce risk, but from the benefit of putting more resources into the most profitable projects. Moreover, conditional on integration, greater variability of division profits is associated with lower overall wages and a lower relative weight on firm-based incentives. (iii) Integration is more likely the better firms can hold division managers accountable for their claims about investment opportunities (in our model, through the use of message-contingent contracts). Also, conditional on integration, accountability for claims about projects is associated with lower overall wages and a lower weight on firm-based incentives. (iv) Horizontal exchange of resources between divisions is more likely to occur the more related the divisions are, and the more variable their profits are. An example where this prediction is borne out is BP's organization of units into "peer groups" according to their similarity, see Roberts (2004, p.187) and Section 6.

The next section relates our paper to different strands of the literature, which helps clarify its contribution. Section 3 sets up the model. The analysis of the benefits and costs of integration then proceeds in several steps in Section 4. In Section 5 we study how the optimal firm scope depends on our model's parameters, and derive a number of predictions. In Section 6, we compare different structures of an integrated firms, and show in what way a CEO structure is optimal. Section 7 concludes.

## 2 Related Literature

**Theories of the firm:** A large literature is concerned with integration decisions of firms. One main difference between our paper and this literature is our emphasis on the communication of dispersed information.

An important literature has identified influence activities as a constraint to integration (Milgrom and Roberts, 1988, 1990; Meyer, Milgrom and Roberts, 1992; Scharfstein and Stein, 2000). While division managers in in our model also try to influence the CEO, the information they communicate is a necessary input into efficient resource allocation; in particular, the CEO cannot simply commit himself not to listen to the managers. We thus spell out the channel through which influence activities affect a rational and unbiased CEO's decisions, which allows us to link the costs of integration directly to its benefits.

Our theory is also distinct from incentive-system theories of the firm (see Gibbons, 2005, for a discussion). Holmström and Milgrom (1991, 1994), Holmström and Tirole (1991) and Holmström (1999) argue that firms can solve multitask problems with systems of organizational rules, asset ownership and incentive schemes. The managers' communication in our model could be seen as a

second task. But the incentive-system approach takes as given discrepancies between output(s) and measured performance. No such discrepancies exist in our model; realized output is perfectly measurable. Rather, the organization must solve a team production problem in which managers strategically communicate private information. With cheap-talk communication, there is also no interaction between productive effort and communication in the managers' utility functions, in contrast to some of the papers mentioned.

Our contracting assumptions are borrowed from property-rights theory (Grossman and Hart, 1986) and the more recent literature on authority in organizations. Like Hart and Holmström (2002) and Hart and Moore (2005), for instance, we assume that coordination (here, of the use of resources) requires integration under a central authority because decisions are non-contractible. But in our theory, authority and incentives interact quite differently. In Aghion and Tirole, (1997), Mailath et al. (2005), Dessein et al. (2006), for instance, shifting authority over some decision from A to B reduces A's opportunities (e.g. to implement his preferred project) and consequently his incentives (e.g. to discover projects). In our model there is no divergence of preferences between division managers, CEO, and owner, except that the managers' effort is privately costly. If the CEO had perfect information, his resource allocation decisions would *improve* the managers' incentives. Only when the managers have private information does a new agency problem emerge, which leads to the tradeoff between information aggregation and incentives for effort. Our theory thus explains why "selective intervention" has its costs without relying on a divergence of preferences between CEO and the managers.

**Organizational economics:** Several papers investigate the interactions between coordination, incentives and communication, taking the boundaries of the firm as given. Here, we argue that these interactions and tradeoffs are *determinants* of firm boundaries.

Athey and Roberts (2001) show that individual performance contracts can lead to distortions in decisions that have externalities on others. This problem can be alleviated by introducing a top manager as a pure coordinator, as in our model. But in their model the top manager can obtain all relevant information through monitoring at an exogenous cost. In our model, the CEO must obtain information from strategically communicating managers. As a result, the costs of integration are endogenous, allowing us to make predictions about integration decisions.

In Levitt and Snyder (1997), a principal can get an agent to communicate bad news by paying a reward for the termination of a project, but this undermines the agent's incentives to come up with a good project in the first place.<sup>7</sup> In our model, contracts cannot be based on resource allocations or messages. This leaves team-based performance incentives as the only

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<sup>7</sup>The same tradeoff was also identified by Povel (1999) and Aghion, Bolton and Fries (1999).

way to establish truthful communication. The multi-agent structure of our integrated firm thus plays an important role for our result.

Dessein, Garicano and Gertner (2006) consider a firm that can either adapt products to market conditions or standardize them to reduce costs. The optimal allocation of authority over product design is partly driven by the conflict between communication and effort provision, and partly by the players' divergent preferences, see our discussion of the literature on authority above. In Dessein et al. the firm's three-agent structure (one upstream and two downstream units) is fixed. Our theory endogenizes both firm scope and structure: integration of the two production units may or may not be optimal, and it may or may not be optimal to bring in a third player as coordinator.

Alonso, Dessein and Matouschek (2006) and Rantakari (2006) investigate the tension between adaptation of two divisions' actions to their environment and coordination between the divisions, and ask when decisions should be centralized or decentralized. These papers do not consider monetary incentives and instead assume that managers are biased towards their own division. In our model, the managers' have no exogenous bias; their motive to communicate strategically is driven entirely by the presence of incentive contracts. By endogenizing the managers' incentive contracts and thus the extent of the managers' bias we can derive testable predictions about optimal firm scope and structure that depend only on the firm's production technology.

**Internal capital markets:** There are intersections between our theory and the literature on internal capital markets. A fundamental difference, however, is that capital is typically contractible, while the resources in our model are not. Thus, like in other theories of the firm, integration in our model has its roots in contracting limitations. Without that assumption, integration would be either unnecessary or always optimal, leaving little room for a theory of firm scope. Indeed, most work on internal capital markets takes multidivisional firms as given and hence does not run into this problem.

Moreover, we do not assume that managers are "empire builders" who derive utility directly from the size of their budget or their division as most of the literature does (an exception is Inderst and Klein, 2006). We rely instead on the assumptions of standard incentive theory that agents like money and dislike effort. An empire-building motive emerges endogenously; it is *caused* by incentive contracts that place a large weight on individual performance.

Stein (1997) formalized Williamson's (1975) conjecture that internal capital markets create value by channeling capital to the most productive divisions. We adopt Stein's production technology, but otherwise our assumptions are very different. Stein does not consider effort provision or incentive contracts. Most importantly, he assumes that information is generated

through monitoring (see Gertner, Scharfstein and Stein, 1994). In our model information is communicated strategically by division managers; strategic communication is the source of the costs of integration.

More recent work focuses on agency problems at the division level. In Scharfstein and Stein (2000), inefficiencies in an internal capital market result from division managers' influence activities. Stein (2002) and Inderst and Laux (2005) extend the analysis of Stein (1997) by assuming that project qualities are determined by the agents' effort. In both papers, headquarters' winner-picking leads to a positive competition effect on managers' effort, as in our model. But as we show, if managers communicate strategically and can lie about their projects, the associated information rent effect always cancels, and often dominates, the competition effect. Papers that explicitly model division managers' private information include Ozbas (2004), Wulf (2005), and Inderst and Klein (2006).<sup>8</sup> Like most of the literature, these papers are concerned with the efficiency of investment decisions rather than with firm scope.

### 3 Model

#### 3.1 Setting

**Production units and resources:** There are two ex ante identical production units, 1 and 2. They are either independent firms, or divisions of a single integrated firm. We determine which organizational form will maximize shareholder value and will thus emerge in the market for ownership of the units.

Each production unit is endowed with one unit of resources  $K = 1$ . The resources are specific to the unit and cannot be obtained in an external market. However, each unit may also benefit from using the other unit's resources, as we explain below.<sup>9</sup>

We follow a large literature on authority in distinguishing between the contractibility of decisions themselves and authority over decisions. Specifically, we assume that it is impossible to write contracts about the *use* of resources, whereas *authority* over the allocation of resources is contractible. This distinction is meaningful as long as the designation of authority is a longer-

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<sup>8</sup> Private information also plays a prominent role in the related capital-budgeting literature, see Harris and Raviv (1996, 1998) and Bernardo, Cai and Luo (2001, 2004). This literature is generally concerned with the efficiency of investment decisions, but not with divisional competition for scarce resources or with firm scope.

<sup>9</sup> The logic of our theory does not depend on the amount of resources each unit is endowed with. We would expect to obtain very similar results if, say, unit 1 had two units of resources and unit 2 none. This variation would correspond to the case of an expansion of one firm into a new line of business based on excess supply of a unique resource, as envisioned by Teece (1982).

term decision and cannot be changed on the fly. We thus assume that specifying who has authority over resources is part of the organizational design and precedes decisions over how resources are used.

The noncontractibility of the use of resources rules out bilateral spot contracting between the units. If the units are run as independent firms, therefore, each must make do with its endowment of resources. If the units are divisions of an integrated firm, bilateral trade between the divisions is still impossible. The advantage of integration instead is that the units' pooled resources can be placed under the authority of a single agent, who can shift resources between the divisions.

**Firm scope (integration):** We assume that both with independent firms and an integrated firm, ownership and control are separated: the units' managers have no wealth of their own to finance the necessary assets, and investors lack the necessary managerial skills (cf. Stein 1997). This assumption facilitates comparison of the two cases but is not essential.

For most of this paper, we define an integrated firm as a *CEO structure*, namely an entity in which the two units are divisions headed by a third agent, a CEO, who has authority over the units' pooled resources. We assume that the cost of employing the CEO is small but positive, which rules out a solution in which the CEO merely mimics the resource allocation of independent firms.<sup>10</sup>

This definition of integration obviously goes beyond common ownership of the units and thus differs from that of property-rights theory. It is motivated by the separation of ownership and control: if owners are not in control, then a change in ownership alone is immaterial to how units of a large firm are run. In common use, the terms "integration" or "merger" imply the presence of a common *management*, and we want to know when such a common management is beneficial for the owners. It is important, though, that the CEO structure is not the only conceivable organization that enables the sharing of resources. We investigate alternative organizational solutions in Section 6.

**Managerial effort, projects, payoffs:** Each production unit is run by a manager, whose job is to create profitable investment opportunities, or projects. Once a project has been created, it requires resources to be carried out; the payoff depends on the quality of the project and the resources invested.

Specifically, unit  $i$ 's manager chooses between high ( $e_i = 1$ ) or low effort ( $e_i = 0$ ), his choice

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<sup>10</sup> If the CEO of an integrated firm were also its owner, nothing would change in our model. If independent firms were run by owner-managers, the analysis would be slightly different but the results still largely the same; see Footnote 20 below.

is unobservable. The manager’s cost of low effort is zero; the cost of high effort is  $c > 0$ . High effort generates a good project (type  $\theta_i = G$ ) with probability  $p$ , and a bad one (type  $\theta_i = B$ ) with probability  $1 - p$ . Low effort generates a good project with probability  $q$ , and a bad one with probability  $1 - q$ , with  $q < p$ . Let  $\boldsymbol{\theta} = (\theta_1, \theta_2)$ .

Resources and projects translate into expected payoffs as in Stein (1997), except that we introduce some noise. The resource investment in any project can be either 1 or 2; a zero investment has a zero return. If an amount of  $k_i \in \{1, 2\}$  is invested in a bad project in unit  $i$ , the resulting *expected* payoff  $z_i$  is  $y_{k_i}$  with  $y_2 > y_1 > 0$ . A good project has an expected payoff of  $\varphi y_{k_i}$  for  $\varphi > 1$ :

$$z_i(k_i, \theta_i) = \begin{cases} \varphi y_{k_i} & \text{if } \theta_i = \text{“G”} \\ y_{k_i} & \text{if } \theta_i = \text{“B”} \end{cases} \quad \text{for } k_i = 1, 2 \quad (1)$$

Departing from Stein, we assume that the production process is noisy; a “full support” property ensures that no direct inferences can be made about  $k_i$  or  $\theta_i$  from the observed  $\tilde{z}_i$ . The *actual* payoff for each unit, denoted  $\tilde{z}_i$ , is either  $\mu$  or 0; let  $\tilde{\mathbf{z}} = (\tilde{z}_1, \tilde{z}_2)$ . The probability of the event  $\tilde{z}_i = \mu$  is given by  $z_i(k_i, \theta_i)/\mu$ , where  $\mu$  is assumed to be large enough so that all  $z_i/\mu$  are less than 1. It follows that  $z_i(k_i, \theta_i)$  is indeed the expected payoff.

We make two assumptions:

**Assumption 1**  $1 < y_1 < y_2 < 2$ .

**Assumption 2**  $\varphi y_2 > (1 + \varphi)y_1$ .

These assumptions imply that it is optimal to invest one unit of resources in a bad project, and two in a good one; this follows because Assumption 2 and  $y_1 > 1$  imply  $\varphi y_2 - 2 > \varphi y_1 - 1$ . In particular, both types of projects have a positive NPV. Further, Assumption 1 implies that for each project type there are decreasing returns to resources invested (since  $2y_1 > y_2$ ). Hence, given two equally good or bad projects, it is better to invest 1 in each project instead of 2 in one of them.<sup>11</sup> Assumption 2, in turn, implies that project payoffs are supermodular in project quality and resources: if there are both a good and a bad project to invest in, it is optimal to invest 2 units in the good project rather than 1 in each, which is essential for any benefit from integration to exist.

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<sup>11</sup> For the same reason, if the projects are unknown but have the same expected quality, then it is best to invest the resources equally instead of putting all in one of the production units. This means that a potential gain from integrating the two production units exists only if whoever allocates resources is informed about the project qualities; there is no gain from allocating resources randomly.

Summarizing, the assumptions imply that the efficient way to allocate two units of resources is given by

$$k^*(\theta) = \begin{cases} k_1 = k_2 = 1 & \text{if } \theta_1 = \theta_2 = \text{“G” or if } \theta_1 = \theta_2 = \text{“B”} \\ k_1 = 2, k_2 = 0 & \text{if } \theta_1 = \text{“G” and } \theta_2 = \text{“B”} \\ k_1 = 0, k_2 = 2 & \text{if } \theta_1 = \text{“B” and } \theta_2 = \text{“G”} \end{cases} \quad (2)$$

**Managers’ preferences:** Unit  $i$ ’s manager is risk-neutral but protected by limited liability.

The manager’s utility is given by  $U_i(w_i, e_i) = w_i - ce_i$ , where  $w_i$  is the monetary wage and  $ce_i \in \{0, c\}$  is the disutility of effort. Managers are not empire builders; they do not derive utility directly from their resource allocation or payoff. We assume the managers’ reservation wages are low enough such that in equilibrium, their participation constraints are not binding.

**Contracts:** For simplicity, we restrict the analysis to contracts that are symmetric for both managers. Managers’ wages can be contingent on both production units’ realized payoffs  $\tilde{z}_1$  and  $\tilde{z}_2$ . There is nothing else wages can be based on: resources are not contractible. Moreover, following Crawford and Sobel (1982) and Dessein (2002), we assume in the bulk of the paper that the managers’ communication is cheap talk. That is, messages do not cost a manager anything, nor can any contracts be conditioned on them. To understand the role of this assumption, however, we also examine message-contingent contracts as a benchmark case in Section 4.3.

With two possible payoffs for each unit, there are four possible outcomes overall. General performance-based contracts can therefore be described by a quadruple of wages. We have analyzed this general case; its results are presented in Appendix B and summarized in the main text below. However, the main results are the same and the exposition much simpler if we restrict attention to wage contracts that are separable in the divisions’ payoffs, i.e. contracts of the form:

$$\tilde{w}_i(\tilde{z}_1, \tilde{z}_2) = \alpha + \beta\tilde{z}_i + \gamma\tilde{z}_j \text{ for } i = 1, 2 \text{ and } j \neq i.$$

Expected wages are then given by  $w_i(z_1, z_2) = \alpha + \beta z_i + \gamma z_j$ .<sup>12</sup> With a non-binding participation constraint, the limited-liability constraint must be binding when contracts are optimal. Since the  $\tilde{z}_i$  are either zero or positive,  $\alpha$  should be set to zero, which implies that  $\beta$  and  $\gamma$  must be nonnegative. Optimal contracts are then completely characterized by the parameters  $\beta$  and  $\gamma$ . Most of the results discussed in the text are derived for such contracts that are additively separable in the divisions’ payoffs.

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<sup>12</sup>We refer to  $\beta$  and  $\gamma$  as “bonuses” since the  $\tilde{z}_i$  are binary, but technically  $\beta$  and  $\gamma$  are *shares* of the units’ payoffs, akin to piece rates.

### 3.2 Independent Firms

When the two units are independent firms, each firm's resource investment  $k_i$  is constrained by  $k_i \leq 1$ . The timing of events is as follows:

1. The owner of firm  $i$  offers her manager a wage contract, which he accepts or rejects.
2. Manager  $i$  exerts effort  $e_i \in \{0, 1\}$ . The manager thereupon learns the profitability of his project  $\theta_i \in \{G, B\}$ , which is his private information.
3. Manager  $i$  invests  $k_i = 1$  in his project.
4. The payoff  $\tilde{z}_i$  is realized, and the manager is paid according to the contract.

It is optimal for firm  $i$ 's owner to give the manager authority over resources, since investing at least one unit of resources is always optimal and the manager will do so if given nonnegative incentives based on  $\tilde{z}_i$ . To induce low effort, the owner can simply pay the manager his reservation wage. To induce high effort, the owner can reward the manager for high output. In principle, firm  $i$  could base its manager's wage on  $\tilde{z}_j$  by offering  $\gamma > 0$ , but here there is no reason to do so as  $\tilde{z}_j$  contains no information about manager  $i$ 's effort. It therefore suffices to pay manager  $i$  a bonus  $\beta \geq 0$  (expressed as share of the payoff) if  $\tilde{z}_i = \mu$ .<sup>13</sup> For each firm  $i$ , the optimal contract that induces its manager to exert high effort solves the problem

$$\begin{aligned}
 & \max_{\beta} \quad (1 - \beta)E_{\theta_i}[z_i(1, \theta_i)|e_i = 1] \quad \text{s.t.} \\
 \text{(IC-}e_i\text{)} \quad & \beta E_{\theta_i}[z_i(1, \theta_i)|e_i = 1] - c \geq \beta E_{\theta_i}[z_i(1, \theta_i)|e_i = 0], \\
 \text{(LL)} \quad & \beta \geq 0.
 \end{aligned} \tag{3}$$

### 3.3 Integrated Firm

In the integrated firm, the units' resources are pooled, and there is only a joint resource constraint  $k_1 + k_2 \leq 2$ ,  $k_i \in \{0, 1, 2\}$  for investment in the two divisions. Since contracting on resources is infeasible, someone must have the authority to allocate the firm's resources.

For now we assume that this is a manager at the top, the CEO. We abstract from any agency problems that may arise at the CEO level; it therefore makes no difference whether the CEO is the owner herself or a hired agent. The purpose of this simplification is to understand what the limits to integration are even if top management pursues value-maximizing actions.

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<sup>13</sup> Once again, the manager's actual wage is  $\beta\tilde{z}_i \in \{0, \beta\mu\}$ ; the expected wage then is  $\beta z_i$ .

We assume that the cost of hiring a CEO is  $w_C > 0$  with  $w_C$  close to zero.<sup>14</sup> A positive cost implies that hiring a CEO is optimal only if he plays an active role in allocating resources. That is, we rule out solutions where the CEO simply allocates one unit of resources to each division, which would be the same as under non-integration. On the other hand, as there is no interesting interaction between  $w_C$  and the tradeoff between coordination and incentives that is our main concern, there is no loss from assuming that  $w_C$  is close to zero and hence need not be considered explicitly in the analysis.

Under integration, stage 3 of the timing is as follows:

- 3a. The division managers simultaneously send costless and unverifiable messages  $\hat{\theta}_i$  about their projects to the CEO. Let  $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ .
- 3b. The CEO allocates resources to the two divisions, subject to the constraint  $k_1 + k_2 \leq 2$  and  $k_i \in \{0, 1, 2\}$ .

Thus, the division managers have private information about their projects, which the CEO needs to learn to allocate resources efficiently. Aside from the managers' possible incentive to lie, a further complication is that the CEO cannot commit himself in advance to any allocation rule, since resources are not contractible. Instead, he allocates resources ex post so as to maximize the firm's profit net of wages. This means that the CEO's response to the managers' messages becomes part of the contracting problem.

If the owner wants the managers to exert low effort, she can simply pay them their reservation wage — just as under non-integration. This also induces truth-telling because the managers then have no reason to misrepresent their projects. However, if the firm wants to implement high effort, several incentive constraints must be satisfied to ensure both effort and truth-telling, i.e., for a separating equilibrium to exist.<sup>15</sup> Below, we state the owner's optimization problem for an unspecified set of feasible contracts  $C$ . In subsequent sections, we solve this problem for different assumptions about  $C$ . For any contract  $\zeta \in C$ , denote by  $\bar{w}_i(\theta, \hat{\theta}, \zeta)$  manager  $i$ 's expected wage at stage 3a of the game if his project is of type  $\theta$  and he reports type  $\hat{\theta}$ , under the assumptions that manager  $j$  exerts high effort and reports his type truthfully. Wherever no

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<sup>14</sup> For instance, suppose a hired CEO must exert effort to evaluate information obtained from division managers and to allocate resources, and that this effort is observable. The owner can then simply pay the CEO a wage  $w_C$  for his effort, plus a very small share of total profits to give him an incentive to allocate resources optimally.

<sup>15</sup> The utility functions of managers with good and with bad projects are identical, which would make separation infeasible in most cheap-talk games. Here, separation is possible because the managers differ in their ability to generate profits for the firm. See Fingleton and Raith (2005) for a model of delegated bargaining that exhibits the same properties.

confusion can arise, we will suppress the argument  $\zeta$ . The firm's optimal contract inducing high effort, truthtelling, and an ex-post optimal allocation of resources by the CEO, then solves the following problem:

$$\begin{aligned}
& \max_{\zeta, e_1, e_2, \hat{\theta}_1, \hat{\theta}_2, k_1, k_2} E_{\theta} \{ [z_1(k_1, \theta_1) + z_2(k_2, \theta_2)] - \bar{w}_1(\theta_1, \theta_1, \zeta) - \bar{w}_2(\theta_2, \theta_2, \zeta) \mid e_1 = e_2 = 1 \} \quad \text{s.t.} \\
\text{(IC-e)} \quad & p\bar{w}_i(G, G, \zeta) + (1-p)\bar{w}_i(B, B, \zeta) - c \geq q\bar{w}_i(G, G, \zeta) + (1-q)\bar{w}_i(B, B, \zeta) \quad \text{for } i = 1, 2 \\
\text{(IC-G)} \quad & \bar{w}_i(G, G, \zeta) \geq \bar{w}_i(G, B, \zeta) \quad \text{for } i = 1, 2 \\
\text{(IC-B)} \quad & \bar{w}_i(B, B, \zeta) \geq \bar{w}_i(B, G, \zeta) \quad \text{for } i = 1, 2 \\
\text{(RA)} \quad & \mathbf{k} = \arg \max_{k'_1, k'_2} [z_1(k'_1, \theta_1) + z_2(k'_2, \theta_2) - E[\tilde{w}_1(\tilde{\mathbf{z}}|k'_1, \theta_1), \zeta] - E[\tilde{w}_2(\tilde{\mathbf{z}}|k'_2, \theta_2), \zeta] \\
& \text{s.t. } k_1 + k_2 \leq 2, k_i \in \{0, 1, 2\} \\
\text{(LL)} \quad & \tilde{w}_i(\tilde{\mathbf{z}}|k_i, \theta_i, \zeta) \geq 0 \quad \text{for } i = 1, 2. \tag{4}
\end{aligned}$$

Condition (IC-e) is the standard effort incentive constraint of manager  $i$ . The ancillary assumptions that agent  $j \neq i$  also exerts high effort, that both managers report truthfully, and that (in equilibrium) resources are allocated according to  $k^*$ , are embodied in the definition of  $\bar{w}_i$  above. Conditions (IC-G) and (IC-B) ensure that manager  $i$  reports his project type truthfully, depending on his project type. Finally, (RA) states that the CEO allocates resources to maximize the firm's net profit. As we will see, with separable incentive contracts the constraint (RA) automatically leads to the implementation of the efficient allocation rule  $k^*$ .

## 4 Benefits and Costs of Integration

We solve the model stepwise. After determining the optimal bonus for an independent firm (Section 4.1), we identify in 4.2 the competition effect in an integrated firm in which there is no information asymmetry about the quality of projects. We then consider the general case of private information about projects in 4.3, and finally our main case in which this information is communicated through cheap talk in 4.4.

### 4.1 Independent Firms

Determining each firm's optimal incentive contract for its manager is straightforward:

**Lemma 1** *With independent firms, the optimal contract for each division manager that leads to high effort is given by*

$$\beta^{ni} = \frac{c}{(p-q)(\varphi-1)y_1} \quad \text{and} \quad \gamma^{ni} = 0. \tag{5}$$

For all proofs, see Appendix A.

We already discussed the optimality of  $\gamma^{ni} = 0$  in the previous Section. The optimal bonus for own output,  $\beta^{ni}$ , is increasing in the cost of effort  $c$ , and decreasing in the marginal effectiveness of effort in generating a good project ( $p - q$ ) and the difference in the marginal profitabilities between a good and a bad project ( $(\varphi - 1)y_1$ ). Separability of the wage function is no restriction under non-integration; the optimal contract is the same in the more general case.

As usual, for any interesting agency problem to exist, high effort must be optimal under first-best conditions. For the comparison between integration and non-integration to be interesting, however, high effort must also be optimal under *second-best* conditions in independent firms. Otherwise, an integrated firm can induce low effort *and truth-telling* for the same flat wage as an independent firm, and attain a higher profit because of a better resource allocation. Integration is then always optimal provided that its advantages are not outweighed by the wage cost of the CEO, which we assume. For a zero reservation wage of the division managers, the relevant condition is stated in the next result:

**Lemma 2** *With a zero reservation wage, the contract of Lemma 1 is optimal if*

$$\frac{(p - q)^2(\varphi - 1)^2 y_1}{p(\varphi - 1) + 1} > c. \quad (6)$$

## 4.2 The Competition Effect of Integration on Incentives

Suppose that the CEO can perfectly observe the project types  $\theta_i$ , while effort remains unobservable. We then obtain the following result:

**Proposition 1** *In an integrated firm in which the CEO has perfect information about  $\theta$ , the optimal separable contract for each division manager is given by*

$$\beta^{pi} = \frac{c}{(p - q)\{(1 - p)[\varphi(y_2 - y_1) - y_1] + \varphi y_1\}} \quad \text{and} \quad \gamma^{pi} = 0,$$

where  $\beta^{pi} < \beta^{ni}$ .

Similar results are obtained by Stein (2002), Inderst and Laux (2005) and Marino and Zabolnik (2004). The most important implication of Proposition 1 is that integration with perfect information leads to *better* incentives for effort, in the sense that effort is less costly to induce. This follows from  $\beta^{pi} < \beta^{ni}$  and  $\gamma^{pi} = \gamma^{ni} = 0$ , and is driven by two effects that go in the same direction.

The first effect results from the supermodularity of resources and project quality in the production function: while an independent firm always invests one unit of resources, under

integration the expected amount of resources is higher for a good project and lower for a bad one. The difference in resources invested leads to higher gap between the payoffs generated, and hence a manager’s wage, from good and bad projects. Creating a good project hence becomes more valuable to the manager in an integrated firm. This effect would be present even if the same manager ran both divisions.

The second, more subtle, effect is caused by having two managers compete for resources. More effort exerted by manager  $i$  increases the quality of  $i$ ’s project, which reduces the amount of resources project  $j$  can expect; manager  $i$ ’s effort hence has a negative effect on division  $j$ ’s payoff. This means that running two divisions is a case of “conflicting tasks” in the sense of Dewatripont and Tirole (1999), and the firm benefits from separating the tasks into two jobs. In the following we refer to both effects jointly as the *competition* effect of centralized resource allocation on managerial incentives.<sup>16</sup> Because of the negative effect of  $e_i$  on  $z_i$ , the owner would also want  $\gamma^{pi}$  to be negative rather than positive, but limited liability constrains it to be zero.

### 4.3 Competition vs. Information-Rent Effects: a General Result

When the CEO must rely on the division managers’ communication when allocating resources, the competition effect just discussed is counterbalanced by an *information-rent effect* that raises wage costs: getting a manager to reveal that his project is bad requires rewarding him in some way. Doing so, however, reduces the spread in the manager’s payoff between having a good and a bad project, and thus undermines the incentives for effort (see Levitt and Snyder, 1997).

Because of the countervailing competition and information-rent effects, it is a priori unclear whether centralized resource allocation with privately informed division managers leads to higher or lower wage costs for the firm. Our first main result shows that even under the most general contracting assumptions, the information-rent dominates at least weakly:

**Proposition 2** *Any contract that induces truth-telling must lead to a wage bill at least as high as that under non-integration. The wage bill is strictly higher than under non-integration whenever  $\varphi \bar{w}_1(B, G) > \bar{w}_1(G, G)$ .*

This result is driven by the incentive for a manager with a bad project to claim that his project is good. Such a manager would receive  $\bar{w}_1(B, G)$ , which is always at least  $1/\varphi$  times the

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<sup>16</sup>More precisely, one can compute the optimal bonus  $\beta^{sm}$  for a single manager running both divisions (and exerting effort twice). One can then show that  $\beta^{sm} < \beta^{ni}$ , which captures the pure effect of a better resource allocation on incentives for effort. Furthermore, we have  $\beta^{pi} < \beta^{sm}$ , which captures the additional gain for the firm from having two competing managers. Thanks to Ricardo Alonso for his insights here.

expected wage of a manager who indeed has a good project, i.e.  $\bar{w}_1(G, G)$ . This relationship holds because both types of managers receive the same amount of resources, which means that their expected payoffs differ by the factor  $\varphi$ . It follows that the managers' expected wages differ by the same factor to the extent they are based on realized payoffs. Otherwise they are the same; hence the weak inequality in the relation between the wages. The incentive constraints for truth-telling and effort provision then jointly imply that the firm must pay the managers at least as much under integration with truthful upward communication as they receive in non-integrated firms.

Proposition 2 thus sheds light on the interaction of two opposite effects identified in the literature: we know from Stein (2002) and others that competition for resources can have a positive effect on incentives, and we know from Levitt and Snyder (1997) and Dessein et al. (2006) that inducing truthful communication can have a negative effect on incentives. Both effects are present in our framework, and the latter always weakly dominates the former.

According to Proposition 2, whenever  $\varphi\bar{w}_1(B, G)$  strictly exceeds  $\bar{w}_1(G, G)$ , total wages in the integrated firm strictly exceed those under non-integration. We will see that is the case when communication is cheap talk. In contrast, if either messages or the use of resources are contractible, the benefits of integration can be attained without any additional cost:

**Proposition 3** *Let*

$$w_0 = \frac{y_2 - y_1 + p(2y_1 - y_2)}{p(p - q)(\varphi - 1)[py_1 + (1 - p)y_2]} \quad \text{and} \quad \beta = \frac{c}{(p - q)(\varphi - 1)[py_1 + (1 - p)y_2]}.$$

(a) *If the managers' messages are contractible and  $c$  is not too large, a contract that pays  $w_0$  to a manager who reports  $B$  while the other reports  $G$ , and pays a share  $\beta$  of realized division payoff otherwise, is optimal.*

(b) *If resources are contractible and  $c$  is not too large, a contract that pays  $w_0$  to a manager who does not receive any resources, and pays a share  $\beta$  of realized division payoff otherwise, is optimal.*

*In both cases, the resulting expected total wages are the same as under non-integration.*

Part (a) implies that when messages are contractible, integration is always optimal as the benefits of integration can be realized without any increase in wage costs. In this case, the agency costs of aggregating dispersed information are fully canceled by the agency benefit of creating competition for resources between the agents.

Part (b) implies that when the use of resources is contractible, integration is *unnecessary* because the contract of Proposition 3 can be implemented even under non-integration. Specifically, the owners can first have their managers report project qualities, then trade resources

if doing so is efficient, and then pay the managers according to the final resource allocation and realized outcomes. It is because of this result that capital is not the kind of resource that motivates integration decisions under our informational assumptions, as we already argued in Section 2.

The constraint on  $c$  in the proposition is necessary for high effort, truthtelling and an efficient resource allocation to be feasible at all. For if  $c$  is “too large”,  $\beta$  as stated is so high that the CEO, concerned with maximizing profit *net* of wages, would deliberately misallocate resources to save on wages. Integration with efficient resource allocation would then be feasible only with low managerial effort.

#### 4.4 Integration with Strategic Information Transmission

When the managers’ communication is cheap talk, only performance-based contracts are feasible. The managers’ bonuses for each division’s payoff must then be structured in a way to ensure that they report their projects truthfully:

**Lemma 3** *For any contract  $(\beta, \gamma)$ , and assuming the CEO believes that the division managers’ reports are truthful and allocates the firm’s resources according to  $k^*$ , a manager with a bad project has an incentive to report his type truthfully if and only if*

$$\frac{\gamma}{\beta} \geq \frac{p(2y_1 - y_2) + y_2 - y_1}{y_1 + p[\varphi(y_2 - y_1) - y_1]}, \quad (7)$$

*and a manager with a good project has an incentive to report his type truthfully if and only if*

$$\frac{\gamma}{\beta} \leq \frac{\varphi[y_2 - y_1 + p(2y_1 - y_2)]}{(1 - p)y_1 + p\varphi(y_2 - y_1)}. \quad (8)$$

*The right-hand side of (7) is between 0 and 1, and the right-hand side of (8) is greater than 1.*

Condition (7) implies that individual incentives alone (i.e.  $\beta > 0$  and  $\gamma = 0$ ) can never elicit truthful reports, as a manager with a bad project always has an incentive to claim that his project is good, in order to receive resources. In this sense, pay for performance endogenously generates “empire-building” behavior, even though the managers do not derive any intrinsic utility from the resources they receive. Combining the truthtelling constraints of Lemma 3 with the managers’ effort incentive constraints leads to the second main result of this section:

**Proposition 4** *In an integrated firm, the optimal contract for each division manager that leads to high effort, truthful reports about projects, and an efficient resource allocation, is given by*

$$\beta^{int} = \frac{c}{(p - q)(\varphi - 1)[(1 - p)y_2 + py_1]} \quad \text{and} \quad (9)$$

$$\gamma^{int} = \frac{c[p(2y_1 - y_2) + y_2 - y_1]}{(p - q)(\varphi - 1)[(1 - p)y_2 + py_1][p\varphi(y_2 - y_1) + (1 - p)y_1]}, \quad (10)$$

where  $\beta^{int} \in (\beta^{pi}, \beta^{ni})$  and  $\gamma^{int} > 0$ . The expected wage per agent is strictly higher than under non-integration.

Although a manager with a bad project stands to benefit from resources invested in his division, he also knows that it may be more efficient to invest them in the other division. The key is then to let the manager participate in this gain. While the firm cannot reward the manager directly based on the report of a bad project or the allocation of resources, it can reward him indirectly in the form of incentive pay based on the other division's output (or equivalently, the firm's profit).

The downside is that rewarding manager  $i$  for division  $j$ 's good performance raises the wage costs for the firm. Worse, since manager  $i$ 's effort has a negative effect on  $z_j$  (see Section 4.2), rewarding him for  $j$ 's good performance *reduces* his incentives to exert effort. Thus, as  $\gamma$  is raised from zero to satisfy the manager's truth-telling constraint,  $\beta$  must be raised (starting from  $\beta^{pi}$ ) as well to maintain the manager's incentive to exert high effort. Although the resulting optimal  $\beta^{int}$  is still lower than the bonus  $\beta^{ni}$  required under non-integration, having to pay  $\gamma^{int} > 0$  for the other division's good performance leads to an expected wage bill for the firm that is strictly higher than under non-integration. In other words, the competition effect of centralized coordination on the managers' incentives is outweighed by the information-rent effect.

Propositions 1 and 4 and Lemma 3 are illustrated in Figure 1. Lemma 3 characterizes a cone (shaded in light gray) in which  $\gamma/\beta$  must lie to induce truthful reports about project types. The effort-incentive constraint (IC-e) in (4) defines feasible combinations of  $\beta$  and  $\gamma$  that induce high effort (shaded in medium gray), conditional on truth-telling by both managers. The dashed line represents one of the firm's isoprofit curves, which have a slope of  $-1$  (assuming high effort and truth-telling at each point); the lower the curve, the higher the profit. The isocurves for expected wages look the same but are ordered in the opposite direction.

Under non-integration, the optimal contract is given by  $(\beta^{ni}, \gamma^{ni})$ ; the incentive constraint for effort (not depicted) is a vertical line through that point. Under integration with perfect information, the effort incentive constraint changes to the line IC-e depicted, as a result of the competition effect. The profit-maximizing contract in this case is  $(\beta^{pi}, \gamma^{pi})$ . With strategic communication, the contract must lie in the dark-shaded area in order to satisfy both effort and truth-telling constraints. The profit-maximizing point in that area is  $(\beta^{int}, \gamma^{int})$ , which is associated with a higher wage bill than the contract  $(\beta^{ni}, \gamma^{ni})$  under non-integration.

**Results with general contracts.** General, non-separable contracts can be characterized by  $(\beta, \gamma, \delta)$ , where manager  $i$  gets paid  $\beta$  (as before, as a share of  $\mu$ ) if only  $\tilde{z}_i = \mu$ ,  $\gamma$  if only  $\tilde{z}_j = \mu$ , and  $\delta$  if both units have a high payoff (as before, it is optimal to pay zero if neither

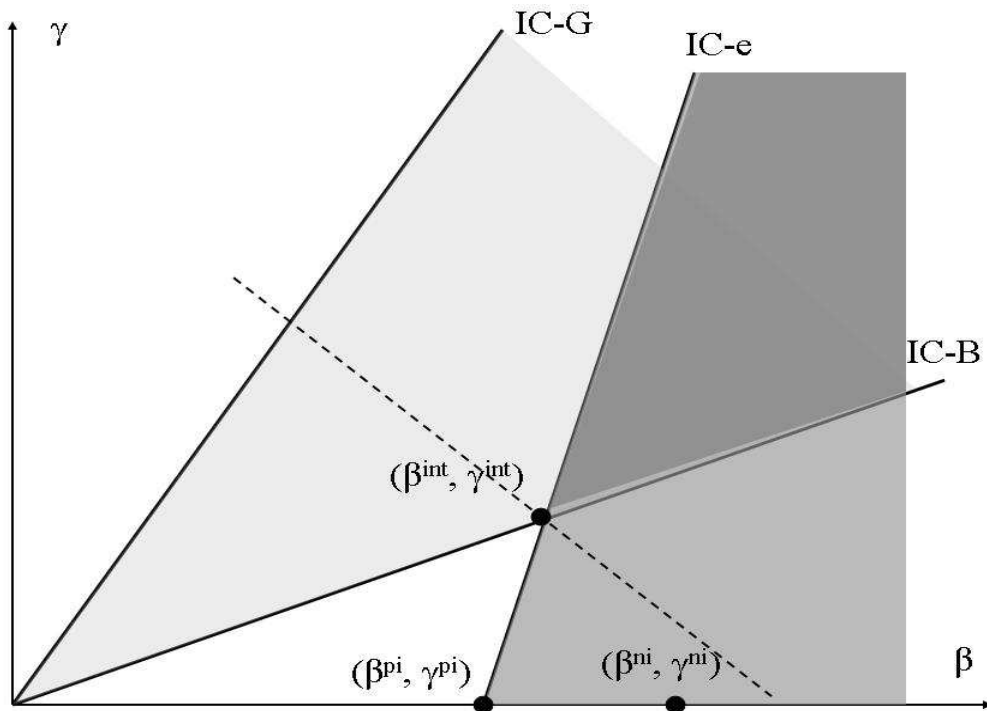


Figure 1: *Truth-telling constraints (IC-B) and (IC-G) and effort incentive constraints (IC-e) as functions of  $\beta$  and  $\gamma$ .*

does). What is new about the general case is that closer attention must be paid to the CEO's incentives to allocate resources. With separable contracts, the firm's net profit is  $1 - \beta - \gamma$  times expected total payoff, and so the CEO always has an incentive to allocate resources efficiently irrespective of  $\beta$  and  $\gamma$ . With general contracts that is no longer the case. For instance, if  $\delta$  is too large, the CEO might be tempted to allocate all resources to one division simply to prevent an outcome in which both units have high payoff. Likewise, if  $\beta$  is large but  $\delta$  small, the CEO might be tempted to allocate the resources equally in order to influence the outcome towards one where both divisions have a high payoff rather than only one.

As we explain in greater detail in Appendix B, there are two main cases to distinguish. If  $p < 1/(1 + \varphi)$ , then  $\delta$  has a positive effect on the truth-telling constraint of a manager with a bad project. It is then possible to induce truth-telling by paying a large enough  $\delta$  while setting  $\gamma = 0$ , and to ensure high effort with  $\beta > 0$ . This solution is optimal as long as it does not lead to a misallocation of resources by the CEO, which means that  $c$  must be sufficiently small. With  $\gamma = 0$ , it is not necessary to pay a manager whose unit does not produce. Consequently, the managers' information can be elicited without additional cost, implying that integration is

always optimal, cf. the discussion in Section 4.3.

If  $p > 1/(1 + \varphi)$ , in contrast, then  $\delta$  has a negative effect on the truth-telling constraint of a manager with a bad project. In this case, inducing truth-telling requires setting  $\gamma > 0$ , and the resulting wage bill is strictly higher than under non-integration, like in Proposition 4. Similarly, the wage bill is strictly higher than under non-integration if  $p$  is small but  $c$  is too large.

To conclude, inducing high effort and truth-telling in general requires higher wages than are needed in independent firms, which may or may not be worth paying from the perspective of the firm's owner. That is, even when incentives for high effort are optimal in independent firms, a constant wage that induces low effort may be optimal in an integrated firm. In a model with continuous instead of binary effort, a higher cost of inducing effort would result in weaker incentives, i.e. a lower reward for a high payoff.<sup>17</sup>

## 5 Optimal Firm Scope

We assume that the organizational form most likely to emerge as the equilibrium in a market for ownership of the production units is the one that maximizes total firm value. In our model, in which there is no debt, this is simply shareholder value. With separate ownership and control, the profit of the owner of an independent firm is the expected payoff minus the manager's wage; if the owner ran the firm herself, the relevant profit would be the total surplus created. For the integrated firm, the separation of ownership and control does not matter for the calculation of profit because we assumed that the CEO's compensation is negligible. Thus, the firm's profit is the expected total payoff minus the division managers' wages.<sup>18</sup>

Recall from Section 4.1 that non-integration with low effort and constant wages is strictly dominated by integration with low effort. This leaves three solutions that can be optimal:

1. Integration with high effort and truth-telling according to Proposition 4,

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<sup>17</sup> We have studied a variant of our model in which effort is continuous. For the model to be tractable, we then need to assume that the project qualities are exogenous rather than determined by effort, and that payoffs depend *additively* on project quality and effort, similar to Dessein et al. (2005). In that modified model, our conjecture above indeed holds; the reward for a high payoff is always lower under integration than under non-integration. But although this result is appealing, overall the modified model leads to less rich implications than our main model because with exogenous project qualities there is no competition effect.

<sup>18</sup> The rents that the managers receive are not to be included in the firm value. If they were — amounting to some kind of *stakeholder* value calculation —, integration would always be optimal since total surplus can only increase when resource allocation is improved while effort is held at a high level. But that would also amount to assuming away agency problems, since the tradeoff between efficiency and rent extraction is the essence of any agency problem when agents have limited liability.

2. Integration with low effort ( $\beta = \gamma = 0$ ) and truth-telling,
3. Non-integration with high effort according to Lemma 1.

The following result states how the optimal solution depends on the model's parameters:

**Proposition 5** (a) *For any given  $y_1, y_2, \varphi, q$ , and  $\mu$ , there exists  $p_0 < 1$  such that non-integration with high effort dominates integration with high effort for all  $p > p_0$ . Moreover, for any given  $y_1, y_2, \varphi, q, \mu$ , and large enough  $\varphi$ , non-integration with high effort dominates integration with low effort.*

(b) *For any given  $y_1, y_2, p, q$ , and  $\mu$ , there exists  $\varphi_0 > 1$  such that integration with high effort is optimal for all  $\varphi > \varphi_0$ .*

(c) *For any given  $y_1, \varphi, p, q$ , and  $\mu$ , if integration with high or low effort dominates non-integration for some  $y_2 \in (y_1, 2)$ , then the same is true for all  $y'_2 > y_2$ .*

Proposition 5 is illustrated in Figure 2.<sup>19 20</sup> The dashed line is the set of  $(p, \varphi)$  pairs for which condition (6) holds with equality, that is, where high and low effort lead to equal profits under non-integration. It is downward-sloping because high effort is better the larger the resulting probability of having a good project ( $p$ ), or its value ( $\varphi$ ).

Below the dashed line, low effort is optimal under non-integration; as we discussed, it follows that integration with low effort is strictly optimal. By continuity, the same holds over some range of  $p$  and  $\varphi$  above the dashed line, where *high* effort is optimal under non-integration.

As  $p$  increases, eventually non-integration with high effort becomes optimal, see part (a) of Proposition 5 and the light-gray area in Figure 2. This is intuitive in comparison with integration with low effort, since a higher  $p$  means a greater value of providing incentives for effort. But

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<sup>19</sup> The figure is based on plot obtained by fixing  $q = 0.2, y_1 = 1.01, y_2 = 1.9$ , and  $c = 0.2$ , and letting  $p$  vary between .5 and 1, and  $\varphi$  between 1.5 and 3.

<sup>20</sup> The separation of ownership and control is not an important part of our story. Suppose that under non-integration both units are run by owner-managers. Then integration is optimal if the resulting profit exceeds the owner-managers' total profits, which equals expected output minus the cost of effort. If upon integration, the owner-managers become the division managers as employees of the new owner, then, somewhat paradoxically, integration is always optimal. This is because the owner-managers can be paid off with the rents they receive as agents of the integrated firm (if necessary, by adding a salary to the total wage). The owner then has no agency costs of integration to bear, and can pocket the full difference in total surplus between integration and non-integration as profit. If in contrast the owner of the integrated firm hires new managers, she must pay the new managers' wages on top of paying off the previous owner-managers. Integration is then less likely to be profitable, but can still be optimal as long as the benefit from centralized resource allocation is large enough. Replicating the analysis above for this case leads to qualitatively the same results as are depicted in Figure 2.

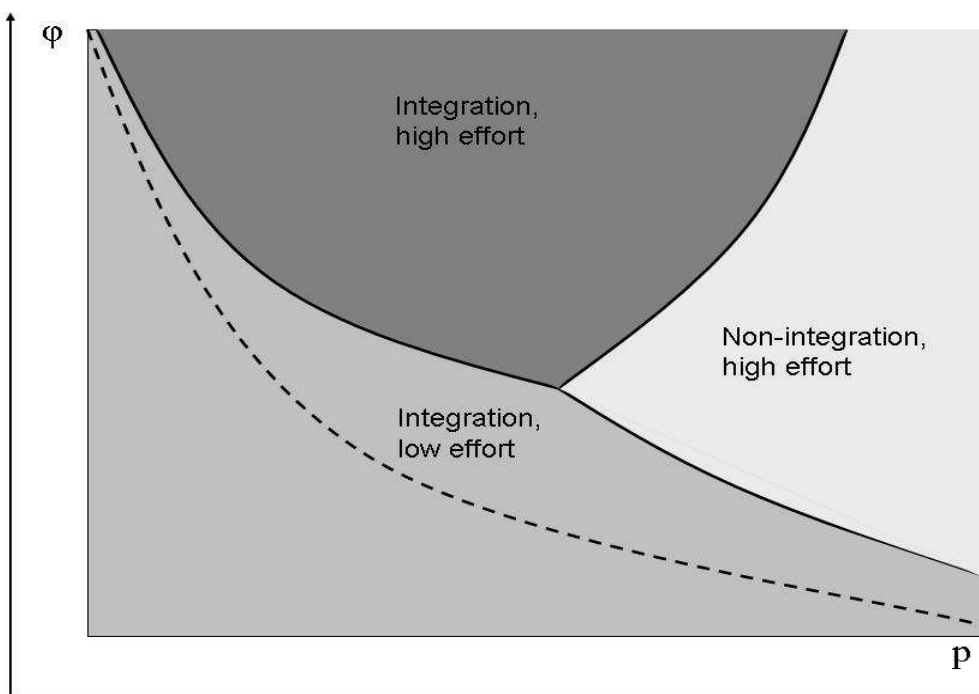


Figure 2: *Optimal organizational form as function of  $p$  and  $\varphi$ . Light gray: non-integration; medium gray: integration with low effort; dark gray: integration with high effort*

compared with integration and high effort, too, non-integration must eventually dominate for  $p$  sufficiently large: the higher  $p$  (approaching 1), the more likely it becomes that both managers have a good project, in which case there is no need to shift all resources to one division. Thus, as  $p$  grows, the benefit of integration shrinks to zero. The costs of integration, meanwhile, remain because  $\gamma/\beta$  must be strictly positive in order to induce truth-telling, and the bonus  $\gamma$  must be paid whenever the “other” division has a high payoff.<sup>21</sup>

As  $\varphi$  increases, eventually integration with high effort becomes optimal, as stated in part (b) of Proposition 5 and depicted by the dark-gray area in Figure 2. Intuitively, integration with high effort eventually dominates integration with low effort, because a greater value of having a good project raises the value of providing incentives to create good projects. Also, integration with high effort eventually dominates non-integration (with high effort). To see why, observe

<sup>21</sup> Figure 2 suggests stronger predictions than are expressed in Proposition 5: whether non-integration or integration is optimal depends *monotonically* on  $p$  and  $\varphi$ , whereas parts (a) and (b) of the proposition only express limit results. The stronger predictions are consistent with all of our numerical simulations, but we have been unable to prove them generally.

that the integrated firm’s ability to move all resources to one division is relevant only when that division has a good project and the other a bad one, in which case the benefit of moving resources is proportional to  $\varphi y_2 - (1 + \varphi)y_1$ , which is positive according to Assumption 2, and increasing in  $\varphi$ .<sup>22</sup>

Not depicted in Figure 2 is part (c) of Proposition 5: integration is more likely the larger  $y_2$ . This is intuitive since  $y_2$  is the expected payoff of investing all resources in one project, which is possible only under integration.

**Predictions:** Our results lend themselves to several testable predictions: First, in our setting in which resources are partly specific to one production unit but can also be used in the other unit within the same firm,  $y_2$  is a measure of “relatedness” of the two divisions, since it measures the value of using one division’s resources in the other division. Proposition 5 part (c) then leads to the intuitive prediction that integration is more likely the more related the two production units are. This prediction is consistent with a large literature, beginning with Montgomery and Wernerfelt (1988), that documents that more widely diversified firms tend to be less highly valued.<sup>23</sup>

Second,  $\varphi$  tells us how much good and bad projects differ in profitability and therefore is a measure of the variability of the divisions’ payoffs. Proposition 5 then leads to the prediction that the more variable profits in two related production units are, the more likely they are to be integrated, because a greater variability raises the benefit of shifting resources to the most profitable projects. Moreover, it is straightforward to establish, using (9), that  $\beta^{int}$ ,  $\gamma^{int}$  and  $\gamma^{int}/\beta^{int}$  are all decreasing in  $\varphi$ . We thus obtain the prediction that conditional on integration, both total wages and the relative weight on firm-based incentives are decreasing in the variability of division profits.<sup>24</sup>

Third, recall from Propositions 2 and 4 that integration with high effort entails additional agency costs (reflected in a larger wage bill) if communication is cheap talk, whereas there is no additional cost if message-contingent contracts are feasible. This comparison suggests the pre-

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<sup>22</sup> With continuous instead of binary effort, the high- and low-effort regions for integration would merge into one region, in which incentives are always weaker than under non-integration but still vary with  $p$  and  $\varphi$  for the same reasons as described above.

<sup>23</sup> Stein (1997) discusses why it would be unsatisfactory to measure the relatedness of the divisions simply by the correlation of project qualities. Instead, he assumes that in a more focused firm headquarters is better able to monitor the divisions’ project qualities.

<sup>24</sup> More precisely, if manager  $i$  is paid based on individual and *total* firm profit according to  $w = \beta' \tilde{z}_i + \gamma' (\tilde{z}_i + \tilde{z}_j)$ , then according to (9) it is optimal to set  $\gamma' = \gamma$  and  $\beta' = \beta - \gamma$ , and the relative weight on firm-based incentives is  $\gamma'/\beta' = 1/(\beta/\gamma - 1)$ , which is decreasing in  $\varphi$  whenever  $\gamma/\beta$  is.

diction that integration is more likely the easier it is to implement contracts and procedures that enable firms to hold managers accountable for their claims about their investment opportunities. The actual use of message-contingent contracts may be rare, but such contracts are approximated when managers whose claims about investment opportunities are not substantiated by results suffer a reputation loss — in other words, when talk is not cheap.

## 6 Optimal Structure of the Integrated Firm

We have assumed that the integrated firm is a pyramidal hierarchy in which a CEO has authority over resources. But why would such a structure be optimal? Is a CEO — a third player — really needed? We now compare the CEO hierarchy with two other structures that do not require a CEO: one is a *skewed hierarchy* in which one of the division managers allocates the firm’s resources in addition to the task of running a division. The other is a structure with *horizontal exchange* where each manager retains authority over his division’s resources, but may lend his resources to the other division.<sup>25</sup>

We will continue to focus on the managers’ incentive constraints rather than on participation constraints. It is rather clear how the latter would affect the owner’s choice of organizational structure: If the division managers in the CEO hierarchy earn substantial rents, then the owner might be able to leave the resource allocation to them (and do without CEO) without paying them any more than before. If the managers’ rents are very small, the owner might have to pay one or both of them more under a decentralized structure to compensate them for any increase in their workload. This could, however, be done with a salary component; there would hence be no interesting interaction with incentives.

**Skewed hierarchy:** We allow for the managers’ contracts to be different since their jobs are now different. The modified timing of events is as follows:

1. The firm’s owner offers each manager  $i$  a contract  $(\beta_i, \gamma_i)$ , which he accepts or rejects.
2. The managers simultaneously exert effort  $e_i \in \{0, 1\}$ . Each manager then learns the profitability of his project  $\theta_i \in \{G, B\}$ , which is his private information.
3. Manager 2 sends a costless and unverifiable message  $\hat{\theta}_2$  about  $\theta_2$  to manager 1.
4. Manager 1 allocates resources subject to the constraint  $k_1 + k_2 \leq 2$  and  $k_i \in \{0, 1, 2\}$ .
5. The payoffs  $\tilde{z}_1$  and  $\tilde{z}_2$  are realized, and the managers are compensated.

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<sup>25</sup> We would like to thank Niko Matouschek for suggesting this second solution.

As before, we look at contracts that lead to high effort and an efficient resource allocation. For manager 2, nothing changes: provided that manager 1 exerts high effort and that resources are allocated efficiently, his effort and truth-telling constraints are the same as before; hence the optimal wage contract for manager 2 is given by Proposition 4. Manager 1’s effort incentive constraint remains unchanged, too, provided that manager 2 exerts high effort and reports truthfully, and that manager 1 himself allocates resources efficiently at stage 4 of the game.

The only difference is that manager 1’s truth-telling constraint is replaced with incentive constraints that induce him to allocate resources efficiently. Two of these constraints may bind in equilibrium: one ensures that manager 1 distributes the resources equally if both divisions’ projects are equally good or bad, instead of allocating all of it to his own division. The other condition ensures that he allocates all resources to division 2 if it has a good project and division 1 a bad one. Both conditions lead to lower bounds on  $\gamma/\beta$ . We can then show the following.

**Proposition 6** *Assume that the owner of an integrated firm wants the managers to exert high effort, and wants resources to be allocated efficiently. Then the incentive constraints in the skewed hierarchy are unambiguously more restrictive than those in the CEO hierarchy.*

The result is the same with general contracts; see Appendix B. The intuition rests on an equivalence between lying to the CEO in one structure and misallocating resources in the other. To make this clear, suppose that in the skewed hierarchy, manager 1 has a bad project and learns that manager 2 has a good project. Manager 1 will allocate resources efficiently if he is better off giving all resources to division 2 rather than dividing the resources equally. The equivalent situation in the CEO hierarchy is a manager 1 with a bad project who — contrary to our assumptions — happens to know that manager 2 has a good project. Manager 1 will then report his type truthfully if he is better off admitting to a bad project than claiming to have a good project, anticipating that the CEO allocates all resources to division 2 in the first case and divides them equally in the second.

Since the outcomes in each structure are the same, the relevant constraints for manager 1 are the same too; the same is true when manager 2 has a bad project. The important difference is that in the CEO hierarchy, manager 1 reports his type *without* knowing manager 2’s, which means his truth-telling constraint is a weighted average of the constraints for each type of manager 2. In contrast, in the skewed hierarchy, manager 1 allocates resources *after* learning 2’s project type, which means that each constraint must be satisfied.<sup>26</sup>

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<sup>26</sup>We believe that Proposition 6 is not specific to our binary model setup but quite general. First, the arguments above should apply to more general type spaces, since what drives the result is the difference in the timing between

The logic of Proposition 6 is reminiscent of Dewatripont and Tirole (1999). In our model the tasks of running a division and of allocating the firm's resources are not directly opposed (unlike, for instance, prosecution and defense), but they are sufficiently misaligned to warrant separation into two jobs. This result is driven by the interaction with a third agent (manager 2), an effect that is absent in Dewatripont and Tirole.

**Horizontal exchange:** Here, each manager retains authority of his division's resources. The timing in this case differs from the previous one only in stages 3 and 4:

3. Each manager  $i$  sends a costless and unverifiable message  $\hat{\theta}_i$  about  $\theta_i$  to manager  $j \neq i$ .
4. Each manager  $i$  can either use division  $i$ 's resources in his own division, or lend them to division  $j \neq i$ .

Recall that the use of resources is not contractible, which rules out bilateral trade involving direct transfer payments. It does not, however, rule out the option for a manager to voluntarily provide the resources under his authority to the other division.

Horizontal exchange of resources requires wage contracts based on both divisions' performance, and thus some form of profit sharing between the two production units. Specifically, it is efficient for (say) manager 1 to lend his resources to division 2 if and only if division 1 has a bad project and division 2 a good one. Getting manager 1 to do so requires a wage contract that allows him to participate in division 2's performance. Wage contracts of this sort may or may not constitute a case of integration. Some firms may be able to share revenues and use team-based incentives while remaining independent, akin to forming a joint venture. In other cases, formal integration may be necessary.

The conditions for inducing high effort and an efficient resource allocation are very similar to those of the CEO hierarchy; the next result states when they differ:

**Proposition 7** *Assume the integrated firm's owner wants to induce high effort and an efficient allocation of resources under horizontal exchange. If  $\varphi \geq y_1^2/(y_2 - y_1)^2$ , then the optimal contract*

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the two organizational structures rather than the binary project types. Second, the binary nature of effort matters insofar as Proposition 6 assumes that the owner wants both managers to exert high effort. In a model with continuous effort (see also Footnote 17), we would expect the owner to offer weaker incentives to manager 1 than to manager 2, precisely because his resource allocation incentive constraints are more difficult to satisfy than the corresponding truth-telling constraint in the firm with CEO (which is the same constraint that determines manager 2's optimal contract). But the latter comparison is what matters for Proposition 6; the precise characteristics of the resulting optimal contract for manager 1 is secondary.

for each manager is given by Proposition 4. If  $\varphi < y_1^2/(y_2 - y_1)^2$ , then the incentive constraints are unambiguously more restrictive than those in the CEO hierarchy.<sup>27 28</sup>

Proposition 7 states that for large enough  $\varphi$ , the incentive-related costs at the division level are the same as in the CEO hierarchy. Given our assumption that the cost of hiring a CEO is small but positive, horizontal exchange then dominates. For smaller  $\varphi$ , the CEO hierarchy is strictly optimal in terms of the incentive constraints involved.

The intuition for Proposition 7 is very simple, since horizontal exchange differs from the CEO hierarchy only in the constraints that lead to an efficient resource allocation. That is, assuming that resources are eventually used efficiently (according to  $k^*$ ), both managers' effort and truth-telling incentive constraints are the same as before. Since with separable contracts the resource allocation constraint (RA) is never binding in the CEO hierarchy, it follows that the wage contract of Proposition 4 is also the optimal contract to induce high effort and truth-telling under horizontal exchange.

But achieving an efficient resource allocation under horizontal exchange also requires that a manager with a bad project is willing to lend his resources to the other division if that division has a good project. The relevant constraint is similar but not identical to the truth-telling constraint (IC-B) already imposed, and may be more or less restrictive than (IC-B) depending on the magnitude of  $\varphi$ . This leads to the case distinction stated in the proposition. Specifically, the larger  $\varphi$ , the *easier* it is to get a manager to lend his resources to the other division (since the manager stands to gain from a larger  $\varphi$ ), and for  $\varphi \geq y_1^2/(y_2 - y_1)^2$  the corresponding incentive constraint is no longer binding given that (IC-B) must already hold.

Together, Propositions 6 and 7 shed light on why — as emphasized by Chandler — firms are often structured as pyramidal hierarchies, with a top management that is in charge of coordinating a firm's activities but is not directly involved in production. We already know part of the story from Athey and Roberts (2001): incentive contracts that provide good incentives for effort tend to provide bad incentives for decisions relevant to other units. These decisions are then perhaps best left to an unbiased decision maker. But how does the decision maker obtain the information needed to make good decisions? Athey and Roberts assume this can be done through monitoring, at a fixed cost. In our model, the information must be communicated by division managers, which introduces a new agency problem. Nevertheless, as we show, the

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<sup>27</sup>Note that Assumption 2 implies a lower bound of  $y_1/(y_2 - y_1)$  on  $\varphi$ , which given Assumption 1 is less than  $y_1^2/(y_2 - y_1)^2$ .

<sup>28</sup>As far as we can tell, Proposition 7 extends to the more general non-separable contracts, but we have not been able to prove this for all possible cases.

associated costs can be less than those of decentralized decision making.

Proposition 7 is related to a result by Alonso et al. (2006) that decentralized decision-making may be preferred to centralization (involving a CEO) if coordination between two divisions is very important. Whether that is the case in their model depends on the managers' bias parameter. Here, in contrast, the condition in Proposition 7 depends only on parameters of the production technology.

**Prediction:** The theoretical result implies that decentralized horizontal exchange of resources is more likely the more closely related the divisions are (the larger  $y_2$ ), and the more variable the division payoffs are (measured by  $\varphi$ ). This prediction is consistent with broad changes in the way modern firms organize themselves, as described by Roberts (2003, p.2). One is a trend towards less diversification and an increased focus on a firm's core strengths; another is a trend among firms to facilitate horizontal communication and coordination between different units, that is, a decrease in reliance on vertical communication. A more specific example is the case of *BP*, which in the course of extensive restructuring efforts in the 1990s introduced " 'peer groups' that linked assets [e.g. oil fields] facing similar technical and commercial challenges to provide mutual support" (Roberts 2004, p.26 and p.187).

## 7 Concluding Remarks

We have investigated the benefits and costs of integration by focusing on one of the key tasks of managers, the allocation of resources. We have shown that this task is associated with incentive conflicts that do not exist in non-integrated firms. More generally, our focus reflects a belief that has its origins in the work of Coase, Barnard, Simon, Williamson and others: understanding hierarchies, and the role of managers in them as people who coordinate others' actions or resolve conflicts through the use of authority, is key to understanding firms' boundaries.

Methodologically, we borrow from both the incentive-system and the property-rights theories of the firm. But while each theory emphasizes either incentive contracts or control rights, in ours the two are inseparable, and are part of the same organizational design problem. We add an element that plays a prominent role in much of recent organizational economics but has been missing in the theory of the firm: the dispersion of information in a firm, the need to communicate critical information to decision makers, and the resulting incentive problems involving the agents who are the sources of information.

It may be considered a limitation of our analysis that we do not consider agency problems between shareholders and top management. Bolton and Scharfstein (1998) discuss how to integrate these problems into the theory of the firm. It seems intuitive, however, that there are limits

to organization even in the absence of shareholder-manager conflicts. While the separation of ownership and control creates its own agency problems, “managerial diseconomies of scale” — a subject of much discussion in economics since at least the 1930s — most likely also exist in firms that are run by their owners. As we have seen, focusing the spotlight on agency problems at lower levels in a firm, while assuming that top management is benevolent, leads to many new insights into this old problem.

## Appendix A: Proofs

**Proof of Lemma 1:**  $E_{\theta_i}[z_i(1, \theta_i)|e_i = 1]$  in (3) is given by  $p\varphi y_1 + (1-p)y_1$ , while  $E_{\theta_i}[z_i(1, \theta_i)|e_i = 0]$  equals  $q\varphi y_1 + (1-q)y_1$ . The manager’s incentive constraint can thus be rephrased as  $\beta(p - q)(\varphi - 1)y_1 \geq c$ . Under an optimal contract, this condition must be binding, which leads to the bonus  $\beta$  stated in the Lemma. ■

**Proof of Lemma 2:** If owner  $i$  wants to implement high effort, her profit is  $(1 - \beta^{n_i})[p\varphi + 1 - p]y_1$ , with  $\beta^{n_i}$  as given by Lemma 1. If the owner wants to implement low effort, she can pay a zero wage (provided that the manager’s reservation wage is zero) and the resulting profit is  $[q\varphi + 1 - q]y_1$ . It is straightforward to show that the difference between these two expressions is positive if and only if (6) holds. ■

**Proof of Proposition 1:** Under perfect information, if manager 1 has a good project, then with probability  $p$  manager 2 has a good project as well, and they receive one unit of resources each. Manager 1 then earns  $\beta\mu$  if division 1 has a high payoff and  $\gamma\mu$  if division 2 does; the probability of each event is  $\varphi y_1/\mu$ . With probability  $1 - p$ , manager 2 has a bad project. All resources are then allocated to division 1, and manager 1 earns  $\beta\mu$  with probability  $\varphi y_2/\mu$ . Manager 1’s expected wage from having a good project is thus

$$W(G) = \varphi[p(\beta + \gamma)y_1 + (1 - p)\beta y_2]. \quad (11)$$

Similarly, if manager 1 has a bad project, then with probability  $p$ , manager 2 has a good one, and all resources go to division 2; whereas with probability  $1 - p$ , manager 2 has a bad project as well. Manager 1’s expected wage from having a bad project then is

$$W(B) = p\gamma\varphi y_2 + (1 - p)(\beta + \gamma)y_1. \quad (12)$$

These expressions are the same for manager 2, and so each manager will exert high effort if  $pW(G) + (1 - p)W(B) - c \geq qW(G) + (1 - q)W(B)$ , or equivalently,

$$(p - q)[(1 - p)(\varphi y_2 - y_1) + p\varphi y_1]\beta - [p\varphi(y_2 - y_1) + (1 - p)y_1]\gamma \geq c. \quad (13)$$

The coefficient of  $\gamma$  in (13) is positive; hence,  $\gamma$  has a negative effect on effort incentives.

Next, let us determine the firm's net profit. With separable contracts and for any division  $i$ , the firm pays  $\beta$  (expressed as share of  $\mu$ ) to manager  $i$  and  $\gamma$  to manager  $j \neq i$  if division  $i$  has a high payoff. The firm's net profit therefore is given by  $1 - \beta - \gamma$  times the expected total payoff in the two divisions, where the latter is given by:

$$Z^{int} = 2p^2\varphi y_1 + 2p(1-p)\varphi y_2 + 2(1-p)^2 y_1. \quad (14)$$

The first (last) term in (14) represents the case where both divisions have a good (bad) project and receive one unit of resources each. The middle term represents the case (with probability  $2p(1-p)$ ) where one division has a good project and the other a bad one, and all resources are allocated to the good project. The firm's expected net profit therefore is

$$2(1-\beta-\gamma)[p^2\varphi y_1 + p(1-p)\varphi y_2 + (1-p)^2 y_1], \quad (15)$$

which is decreasing in  $\beta$  and  $\gamma$ . Since  $\gamma$  has a negative effect on profit *and* on effort incentives, it is optimal to set  $\gamma = 0$ . The optimal  $\beta$  is then obtained by solving (13) as equality for  $\beta$ , setting  $\gamma = 0$ . The result is  $\beta^{pi}$  as stated in the Proposition. To show that  $\beta^{pi} < \beta^{ni}$ , we need to compare  $(1-p)[\varphi(y_2 - y_1) - y_1] + \varphi y_1$  in the denominator of  $\beta^{pi}$  with  $(\varphi - 1)y_1$  in the denominator of  $\beta^{ni}$  in (5). The difference between these two expressions equals  $py_1 + (1-p)\varphi(y_2 - y_1)$ , which is positive, proving our claim.  $\blacksquare$

**Proof of Proposition 2:** The result is derived from three conditions. First, if manager 1 exerts high effort and communicates truthfully, his expected wage is

$$p\bar{w}_1(G, G) + (1-p)\bar{w}_1(B, B) = p[\bar{w}_1(G, G) - \bar{w}_1(B, B)] + \bar{w}_1(B, B), \quad (16)$$

while his effort incentive constraint can be written as

$$\bar{w}_1(G, G) - \bar{w}_1(B, B) \geq \frac{c}{p-q}. \quad (17)$$

Since (17) is binding under non-integration, the difference  $\bar{w}_1(G, G) - \bar{w}_1(B, B)$  in (16) cannot be lower under integration, and thus  $\bar{w}_1(G, G)$  and hence the firm's expected wage payment to manager 1 must increase whenever  $\bar{w}_1(B, B)$  is higher than a bad manager's expected wage under non-integration, which is given by  $\beta^{ni}y_1 = c/[(p-q)(\varphi-1)]$ , cf. (5).

Second, the truthtelling constraint for a manager 1 with a bad project is given by

$$\bar{w}_1(B, B) \geq \bar{w}_1(B, G). \quad (18)$$

The third condition is

$$\varphi\bar{w}_1(B, G) \geq \bar{w}_1(G, G) \quad \text{or} \quad \frac{\bar{w}_1(B, G)}{\bar{w}_1(G, G)} \geq \frac{1}{\varphi}, \quad (19)$$

which is explained in the text. Then, (19), (18) and (17), applied in that order, imply that

$$(\varphi-1)\bar{w}_1(B, B) \geq (\varphi-1)\bar{w}_1(B, G) \geq \varphi\bar{w}_1(B, G) - \bar{w}_1(B, B) \geq \bar{w}_1(G, G) - \bar{w}_1(B, B) \geq \frac{c}{p-q}, \quad (20)$$

and hence  $\bar{w}_1(B, B) \geq c/[(p-q)(\varphi-1)]$ . We have thus shown that  $\bar{w}_1(B, B)$  must be weakly greater than a bad manager's expected wage under non-integration, which proves the main result. It is also clear from (20) that whenever  $\varphi\bar{w}_1(B, G) > \bar{w}_1(G, G)$ , then wage bill under integration must strictly exceed the wage bill under non-integration. ■

**Proof of Proposition 3:** Our proof covers both cases (a) and (b). Consider first manager 1's incentive to choose high effort, assuming that manager 2 does too. Manager 1 gets at least one unit of resources if either his project is good or if both managers have a bad project. In each case, he receives a share  $\beta$  of his division payoff. If manager 1 has a bad project and manager 2 a good one, he receives  $w_0$ . Manager 1 then chooses high effort if (cf. 13)

$$(p-q)[p\varphi y_1\beta + (1-p)\varphi y_2\beta - pw_0 - (1-p)y_1\beta] \geq c. \quad (21)$$

Next, consider manager 1's truthtelling incentives. If his project is bad and he tells the truth and manager 2's project is good, he is paid  $w_0$ . Otherwise, he receives a share  $\beta$  of his allocated resources. Manager 1 then reports truthfully if

$$pw_0 + (1-p)y_1\beta \geq py_1\beta + (1-p)y_2\beta. \quad (22)$$

Similarly, if manager 1 has a good project, he reports his type truthfully if

$$p\varphi y_1\beta + (1-p)\varphi y_2\beta \geq pw_0 + (1-p)\varphi y_1\beta. \quad (23)$$

The expressions for  $w^0$  and  $\beta$  stated in the proposition are the unique solution of (21) and (22) as equalities. It is easy to verify that this contract also satisfies (23). The upper bound on  $c$  is required because for large  $c$ ,  $\beta$  as stated becomes so large that the CEO would prefer to misallocate resources in order to save on wage costs.

Under non-integration, the expected wage bill per firm is  $\beta^{ni}$  times the expected payoff  $[p\varphi + (1-p)]y_1$ , which simplifies to

$$\frac{c[p\varphi + (1-p)]}{(p-q)(\varphi-1)}. \quad (24)$$

The expected wage bill per manager in the integrated firm is given by

$$\frac{1}{2} \left[ p^2 2\varphi y_1\beta + (1-p)^2 2y_1\beta + 2p(1-p)(\varphi y_2\beta + w_0) \right]$$

which upon substituting the expressions in the proposition simplifies to (24), i.e. the same as under non-integration. It follows from Proposition 2 that the contract  $(w_0, \beta)$  must be optimal. ■

**Proof of Lemma 3:** For a manager 1 with a *good* project, the expected payoff from reporting truthfully (under the assumption that manager 2 reports truthfully too), is given by  $\bar{w}_1(G, G) = W(G)$  as given by (11). Suppose manager 1 reports “B” instead (while manager 2 reports truthfully, and the CEO assumes truthtelling on part of both managers). Then all resources go to division 2 if it has a good project, and are allocated equally if it has a bad project. Weighted by the probabilities of these two cases, the expected wage for manager 1 is

$$\bar{w}_1(G, B) = p\gamma\varphi y_2 + (1-p)(\beta\varphi y_1 + \gamma y_1). \quad (25)$$

The truthtelling constraint (IC-G) given by  $\bar{w}_1(G, G) \geq \bar{w}_1(G, B)$  can therefore be expressed as

$$\varphi[y_2 - y_1 + p(2y_1 - y_2)]\beta \geq [(1-p)y_1 + p\varphi(y_2 - y_1)]\gamma, \quad (26)$$

which is equivalent to (8). The difference between the numerator and denominator on the right-hand side of (8) simplifies to  $(1-p)[\varphi(y_2 - y_1) - y_1] + p\varphi(2y_1 - y_2) > 0$ , which means the fraction is greater than 1.

For a manager 1 with a *bad* project, the expected payoff from reporting truthfully (under the same ancillary assumptions as above), is  $\bar{w}_1(B, B) = W(B)$  as given by (12). Suppose manager 1 reports “G” instead. Then with probability  $p$ , manager 2 has a good project too, in which case each division gets one unit of resources. With probability  $1-p$ , manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is

$$\bar{w}_1(B, G) = p(\beta + \varphi\gamma)y_1 + (1-p)\beta y_2. \quad (27)$$

The truthtelling constraint (IC-B) given by  $\bar{w}_1(B, B) \geq \bar{w}_1(B, G)$  can therefore be expressed as

$$\{p[\varphi(y_2 - y_1) - y_1] + y_1\}\gamma \geq [p(2y_1 - y_2) + y_2 - y_1]\beta, \quad (28)$$

which is equivalent to (7). The difference between the numerator and denominator on the right-hand side of (7) simplifies to  $-(1-p)(2y_1 - y_2) - p[\varphi(y_2 - y_1) - y_1] < 0$ , which means the fraction is smaller than 1 (but positive, since both numerator and denominator are). ■

**Proof of Proposition 4:** The optimal separable contract maximizes (15) with respect to  $\beta$  and  $\gamma$ , subject to (i) the effort incentive constraint (13), (ii) the truthtelling constraints (8) and (7), (iii) the constraint (RA) that the CEO allocates resources in an ex-post optimal way, assuming high effort and truthtelling on part of both managers, and (iv) nonnegativity

constraints  $\beta, \gamma, \delta \geq 0$ . This optimization problem is a linear program, and therefore the optimal solution must be a corner point of the parameter set defined by the constraints.

First of all, assuming the other constraints are satisfied, the constraint (RA) leads to the implementation of the efficient resource allocation  $k^*$ . This follows immediately from the fact that the firm's profit is  $1 - \beta - \gamma$  times expected total payoff, which by definition is maximized if resources are allocated efficiently, cf. the discussion in Section 3.1 on the role of our parameter constraints. Thus, for  $\beta + \gamma < 1$ , ex-post profit maximization on part of the CEO leads to an efficient resource allocation, whereas  $\beta + \gamma > 1$  would never be chosen since the resulting profit would be negative.

The relevant constraints are (IC-e) and (IC-B), whereas (IC-G) is redundant. To see why, observe first that the effort constraint (13) must be binding, for otherwise truth-telling would be optimally achieved by setting  $\beta = \gamma = 0$ , when (13) is clearly violated. Second, we already know from Proposition 1 the solution to the relaxed problem in which (IC-G) and (IC-B) are not imposed, and from Lemma 3 we know that it satisfies (IC-G) but not (IC-B). Hence (IC-B) must be binding. Finally, given Lemma 3, any  $(\beta, \gamma)$  that satisfies (IC-B) also satisfies (IC-G). The optimal contract is therefore given by solving (7) and (13) as equalities for  $\beta$  and  $\gamma$ ; the solution is stated in the Proposition.

Since both  $(\beta^{pi}, 0)$  and  $(\beta^{int}, \gamma^{int})$  solve (13) with equality and since (13) is increasing in  $\beta$  and decreasing in  $\gamma$ , it follows that  $\beta^{int} > \beta^{pi}$ , see Figure 2. That  $\beta^{int} < \beta^{ni}$  follows from comparing the expressions for both variables, where  $py_1 + (1-p)y_2$  in the denominator of  $\beta^{int}$  is greater than  $y_1$  in the denominator of  $\beta^{ni}$ , and the expressions are otherwise the same.

The integrated firm's total wage bill is given by  $\beta + \gamma$  times total expected output  $Z^{int}$ , while the total wage bill for two independent firms is given by  $\beta$  times  $2[p\varphi + (1-p)]y_1$ . Upon substituting  $(\beta^{int}, \gamma^{int})$  into the wage bill of the integrated firm and  $\beta^{ni}$  into that of the non-integrated firms, the difference between the two simplifies to

$$2 \frac{c p \varphi y_1}{(p-q)(\varphi-1)} \frac{y_2 - y_1 + p(2y_1 - y_2)}{[p y_1 + (1-p)y_2][p(\varphi(y_2 - y_1) - y_1) + y_1]},$$

which under Assumptions 1 and 2 is strictly positive. ■

**Proof of Proposition 5:** (a) For  $p = 1$ , the expected total payoff of both the nonintegrated firms (given by  $2(p\varphi + 1 - p)y_1$ ) and the integrated firm with high effort (given by  $Z^{int}$  in (14)) equal  $2\varphi y_1$ . Moreover, both  $\beta^{ni}$  and  $\beta^{int}$  equal  $c/[(1-q)(\varphi-1)y_1]$ , whereas  $\gamma^{int}$  is strictly positive. Thus, for  $p = 1$  the total profit of the non-integrated firms strictly exceeds the integrated firm's profit. By continuity, the same holds for some interval of  $p$  close enough to 1, which establishes our claim.

Comparing non-integration with integration with *low* effort, observe that the non-integration profit is increasing in  $p$  while the integration profit does not depend on  $p$ . Moreover, for  $p = 1$  the difference between these profits is given by

$$2\varphi \left( 1 - \frac{c}{(1-q)(\varphi-1)y_1} \right) - 2[(1-q)^2y_1 + q^2\varphi y_1 + q(1-q)\varphi y_2].$$

Because of Assumption 1, this difference is increasing in  $\varphi$  and grows without bound as  $\varphi$  does, which means it must be positive for large enough  $\varphi$ . A lower bound to  $\varphi$  is necessary because for  $\varphi \rightarrow 1$ , Condition (6) is violated, in which case we know integration with low effort dominates non-integration.

(b) First, compare integration with high vs. low effort. The difference between the expected payoff  $Z^{int}$  minus the analogous expression in (14) (with  $q$  instead of  $p$ ) is positive and increases without bound with  $\varphi$ , which can be seen by inspecting the derivative  $2(p-q)[y_2 - (p+q)(y_2 - y_1)]$ , which because of Assumption 1 and  $p+q < 2$  must be positive. Moreover, the managers' total wages,  $2(\beta^{int} + \gamma^{int})$  times  $Z^{int}$ , converge to  $4cp/(p-q)$  as  $\varphi$  approaches infinity. It follows that the net profits under integration with high effort must be larger than with low effort for  $\varphi$  sufficiently large.

Comparing integration with high effort with non-integration, observe that  $\beta^{ni}$ ,  $\beta^{int}$  and  $\gamma^{int}$  also converge to zero as  $\varphi$  approaches infinity; the owners' share of the payoff thus converges to 1 in both cases. The integration payoff eventually exceeds the total no-integration payoff, which can be seen by comparing  $\lim_{\varphi \rightarrow \infty} \frac{1}{\varphi} Z^{int} = 2p[py_1 + (1-p)y_2]$  and  $\lim_{\varphi \rightarrow \infty} \frac{1}{\varphi} 2[p\varphi + (1-p)]y_1 = py_1$ . Thus, since integration with high effort dominates non-integration for  $\varphi \rightarrow \infty$ , the same holds by continuity for an interval of  $\varphi$  above some threshold value.

(c) The non-integration profits do not depend on  $y_2$ . All that remains to show is that an integrated firm's profit is increasing in  $y_2$ . First, under integration with low effort, the firm pays the managers a constant and keeps the rest of the expected payoff  $2q^2\varphi y_1 + 2q(1-q)\varphi y_2 + 2(1-q)^2y_1$ , which is increasing in  $y_2$ . The same is true for the expected payoff  $Z^{int}$  under high effort. Here, the firm keeps only the share  $(1 - \beta^{int} - \gamma^{int})$  of the payoff, but this share is increasing in  $y_2$  too. To see this, consider the sum  $\beta^{int} + \gamma^{int}$ , which can be expressed as

$$\frac{c[y_2 + p(\varphi-1)(y_2 - y_1)]}{(p-q)(\varphi-1)[py_1 + (1-p)y_2][y_1 + p(\varphi(y_2 - y_1) - y_1)]}.$$

Its derivative with respect to  $y_2$  has the same sign as

$$- \left[ p(1-p)(\varphi-1)(y_2 - y_1)^2 - (1-p)(y_2^2 - y_1^2) - py_1^2 \right],$$

which is negative, implying that the owner's payoff share  $1 - \beta - \gamma$  is increasing in  $y_2$ . ■

**Proof of Proposition 6:** As explained in the text, manager 2's effort and truthtelling constraints and manager 1's effort constraint are the same as for the CEO hierarchy. In place of manager 1's truthtelling constraint, there are now two incentive constraints to ensure that manager 1 allocates resources efficiently. Suppose that manager 1 has a bad project. Then, if manager 2's project is bad as well, allocating the resources equally (the efficient choice) leads to an expected wage of  $(\beta_1 + \gamma_1)y_1$  for manager 1, while allocating all resources to division 1 leads to an expected wage of  $\beta_1 y_2$ . Manager 1 therefore allocates resources efficiently if

$$(\beta_1 + \gamma_1)y_1 - \beta_1 y_2 \geq 0 \quad (29)$$

or equivalently  $\gamma_1/\beta_1 \geq (y_2 - y_1)/y_1$ . If manager 2's project is good, then allocating all resources to division 2 (the efficient choice) leads to a payoff of  $\gamma\varphi y_2$  for manager 1, whereas allocating the resources equally instead leads to a wage of  $(\beta + \varphi\gamma)y_1$ . Manager 1 therefore allocates resources efficiently if

$$\gamma_1\varphi y_2 - (\beta_1 + \gamma_1)y_1 \geq 0 \quad (30)$$

or equivalently  $\gamma_1/\beta_1 \geq y_1/[\varphi(y_2 - y_1)]$ . There are more constraints, but all of them are equivalent to or less restrictive than (29) or (30), and therefore need not be considered.

To prove the proposition, we show that (29) and (30) are jointly more restrictive than the truthtelling constraint (28) in the CEO structure. This follows simply from the fact that if we write (28) as follows,

$$\{p[\varphi(y_2 - y_1) - y_1] + y_1\}\gamma - [p(2y_1 - y_2) + y_2 - y_1]\beta \geq 0,$$

the left-hand side is equal to  $1 - p$  times the left-hand side of (29) plus  $p$  times the left-hand side of (30), as is easy to verify. Thus, in the skewed hierarchy, the more restrictive condition of (29) and (30) must hold, whereas in the CEO structure only a weighted average of the two constraints must hold, which is a strictly weaker constraint. ■

**Proof of Proposition 7:** As explained in the text, the effort and truthtelling incentive constraints (IC-e), (IC-B) and (IC-G) are the same under horizontal exchange as in the CEO structure. In addition, an efficient allocation of resource requires that a manager with a bad project lend his resources to the other division if the latter has a good project. If he does, his payoff is  $\gamma\varphi y_2$ , whereas if he keeps his resource his payoff is  $(\beta + \gamma\varphi)y_1$ . It follows that he will lend his resources if  $\gamma/\beta \geq y_1/[\varphi(y_2 - y_1)]$ . It is straightforward to show that this lower bound on  $\gamma/\beta$  is lower than the lower bound stated in Lemma 3 if and only if  $\varphi < y_1^2/(y_2 - y_1)^2$ . When that is this case, the new constraint  $\gamma/\beta \geq y_1/[\varphi(y_2 - y_1)]$  is not binding given that (IC-B) is already imposed; and the contract of Proposition 4 remains optimal. Otherwise, the new

constraint is more restrictive than (IC-B), leading to an optimal wage contract that implies a higher wage bill for the firm than under the CEO structure. ■

## Appendix B: General, Non-separable Contracts

General non-separable contracts specify a wage for each possible realization of  $(\tilde{z}_1, \tilde{z}_2)$ . For a non-integrated firm, the contract given by Lemma 1 remains optimal when non-linear contracts are allowed, since there is no reason to condition wage payments on the other firm's payoff, and since in each firm realized payoff is only  $\mu$  or zero, requiring only one non-zero wage variable.

In the integrated firm, limited liability and a non-binding participation constraint implies that it is optimal to pay each manager zero if both divisions have a zero payoff. The managers' (symmetric) contracts can then be characterized by the triple  $(\beta, \gamma, \delta)$ , where manager  $i$  is paid  $\beta$  (as a share of  $\mu$ ) if only  $\tilde{z}_i = \mu$ ,  $\gamma$  if only  $\tilde{z}_j = \mu$ , and  $\delta$  if both units have a high payoff. Separable incentive contracts are then a special case corresponding to the restriction  $\delta = \beta + \gamma$ .

As explained in the text, with separable contracts the CEO's ex-post optimal resource allocation automatically leads to the efficient allocation  $k^*$ , while with general contracts that is no longer the case. Consequently, in the latter case, solving the firm's program (4) assuming (but not imposing as constraint) that the CEO implements  $k^*$  may lead to wages for which the CEO would rather misallocate resources in order to save on wage costs. However, the only way in which integration can possibly improve over nonintegration is through the ability to shift all resources to one division if its project is good and the other's bad, while any other deviation from  $k_1 = k_2 = 1$  cannot create any benefit. Our conjecture hence is that any optimal solution to (4) is also a solution to the same program with the added constraint that the CEO implements  $k^*$  in allocating resources. Propositions 8-10 below are therefore stated with this additional constraint imposed.

The firm's expected net profit is  $\mu$  times

$$\left[ p^2 \varphi^2 + (1-p)^2 \right] \frac{y_1^2}{\mu^2} (2-2\delta) + 2(1-\beta-\gamma) \left[ p^2 \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) + (1-p)^2 \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) + p(1-p) \varphi \frac{y_2}{\mu} \right]. \quad (31)$$

This expression is obtained as follows. If both divisions have a high payoff, the firm's profit is  $(2-2\delta)\mu$ . This can occur only if both projects are good or both bad, which leads to the first term in (31). Otherwise, if only one division has high payoff, the firm's profit is  $(1-\beta-\gamma)\mu$ ; cf. the coefficient of the second term in (31). This occurs if each division gets one unit of resources but only one has a high payoff (the first two terms in []-brackets in (31)), or if only one division has a good project and gets all resources (the last term in []-brackets in (31)).

Our first result generalizes Proposition 1:

**Proposition 8** *In an integrated firm in which the CEO has perfect information about  $\theta$  and allocates resources efficiently, the optimal contract for each division manager entails  $\gamma = 0$ , and the expected total wage bill is lower under integration than under non-integration.*

*Proof:* Under perfect information, if manager 1 has a good project, then with probability  $p$  manager 2 has a good project as well and each is allocated one unit of resources. Manager 1 can then earn either  $\delta\mu$ ,  $\beta\mu$  or  $\gamma\mu$  (with appropriate probabilities), depending on which of the two divisions has a high payoff. With probability  $1 - p$ , manager 2 has a bad project, all resources are allocated to division 1, and manager 1 earns  $\beta\mu$  with probability  $\varphi y_2/\mu$ . Overall, manager 1's expected wage from having a good project is

$$W(G) = \left\{ p \left[ \frac{\varphi^2 y_1^2}{\mu^2} \delta + \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) (\beta + \gamma) \right] + (1 - p) \frac{\varphi y_2}{\mu} \beta \right\} \mu. \quad (32)$$

If manager 1 has a bad project, then with probability  $p$ , manager 2 has a good one, and all resources go to division 2; whereas with probability  $1 - p$ , manager 2 has a bad project as well. Manager 1's expected wage from having a bad project then is

$$\left\{ p \frac{\varphi y_2}{\mu} \gamma + (1 - p) \left[ \frac{y_1^2}{\mu^2} \delta + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta + \gamma) \right] \right\} \mu. \quad (33)$$

By symmetry, these expressions are the same for manager 2, and so each manager will exert high effort if  $pW(G) + (1 - p)W(B) - c \geq qW(G) + (1 - q)W(B)$ , or equivalently,

$$(p - q) \left\{ \left( p\varphi^2 - 1 + p \right) \frac{y_1^2}{\mu} \delta + \left[ (1 - p) \left( \varphi y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + p\varphi y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right] \beta \right. \\ \left. - \left[ p\varphi \left( y_2 - y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right) + (1 - p)y_1 \left( 1 - \frac{y_1}{\mu} \right) \right] \gamma \right\} \geq c. \quad (34)$$

As in the separable case, the left-hand side of (34) is decreasing in  $\gamma$ , meaning that  $\gamma$  has a negative effect on effort incentives. It is also clear that the firm's profit is decreasing in  $\gamma$ . It is therefore optimal to set  $\gamma = 0$ . Moreover, since the owner cannot do worse with general than with separable contracts, it follows from Proposition 1 that the wage bill must lower under integration than under non-integration.  $\blacksquare$

It is possible but not helpful to derive the precise expressions for  $\beta$  and  $\delta$  for the optimal contract. Note that given  $\gamma = 0$ , separability of the contract imposes the restriction  $\delta = \beta$ , whereas in the general case there is no such restriction. With two variables to choose, only one relevant constraint (IC-e), and a linear program, the optimum is typically a corner solution with one of  $\beta$  or  $\delta$  set to zero and the other positive. The main conclusion from Proposition

1, however, remains intact in the more general case: with perfect information, the *competition effect* of centralized resource allocation *improves* effort incentives relative to non-integration, as reflected in a lower wage bill.

Next, Proposition 2 already covers the general case; there is nothing further to show: the information-rent always dominates the competition effect in the sense that integration with high effort always leads to a wage bill at least as high as under non-integration.

Let us now turn to the case where project types are communicated strategically by the managers. As in Section 4.4, additional constraints come into play. In the following, we first derive these constraints formally, and then generalize Proposition 4.

First, it must be optimal for each manager to report his type truthfully. For a manager 1 with a *good* project, the expected payoff from reporting truthfully, and under the assumption that manager 2 reports truthfully too, is given by  $\bar{w}_1(G, G) = W(G)$  as given by (32). Suppose manager 1 reports “B” instead. Then with probability  $p$ , manager 2 has a good project, in which case all resources go to division 2 and manager 1 earns  $\gamma$  if division 2 has high payoff. With probability  $1 - p$ , manager 2 has a bad project, each division is allocated one unit of resources, and the manager can earn  $\delta$ ,  $\beta$  or  $\gamma$  times  $\mu$ , depending on both divisions’ payoffs. The resulting expected wage for manager 1 is

$$\bar{w}_1(G, B) = \left\{ p \frac{\varphi y_2}{\mu} \gamma + (1 - p) \left[ \varphi \frac{y_1^2}{\mu^2} \delta + \frac{\varphi y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) \beta + \left( 1 - \frac{\varphi y_1}{\mu} \right) \frac{y_1}{\mu} \gamma \right] \right\} \mu. \quad (35)$$

The truthtelling constraint (IC-G) given by  $\bar{w}_1(G, G) \geq \bar{w}_1(G, B)$  can therefore be expressed as

$$\begin{aligned} \varphi \frac{y_1^2}{\mu} (p\varphi - 1 + p) \delta + \varphi \left[ (1 - p) y_2 - (1 - p) y_1 \left( 1 - \frac{y_1}{\mu} \right) + p y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right] \beta \\ + \left[ (p\varphi - 1 + p) y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) - p\varphi y_2 \right] \gamma \geq 0. \end{aligned} \quad (36)$$

For a manager 1 with a *bad* project, the expected payoff from reporting truthfully is  $\bar{w}_1(B, B) = W(B)$  as given by (33). Suppose manager 1 reports “G” instead. Then with probability  $p$ , manager 2 has a good project too, in which case each division gets one unit of resources. With probability  $1 - p$ , manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is

$$\bar{w}_1(B, G) = \left\{ p \left[ \frac{\varphi y_1^2}{\mu^2} \delta + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta \right] + (1 - p) \frac{y_2}{\mu} \beta \right\} \mu. \quad (37)$$

The truthtelling constraint (IC-B) given by  $\bar{w}_1(B, B) \geq \bar{w}_1(B, G)$  can then be expressed as

$$-(p\varphi - 1 + p) \frac{y_1^2}{\mu} \delta - \left[ (1 - p) \left( y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + p y_1 \left( 1 - \frac{\varphi y_1}{\mu} \right) \right] \beta$$

$$+ \left[ p\varphi \left( y_2 - y_1 \left( 1 - \frac{y_1}{\mu} \right) \right) + (1-p)y_1 \left( 1 - \frac{y_1}{\mu} \right) \right] \gamma \geq 0. \quad (38)$$

Second, based on our arguments at the beginning of this section, we will look at contracts that induce the CEO to allocate resources efficiently if he assumes that project types are reported truthfully. Suppose first that both projects are good. If the CEO allocates one unit of resources to each division (the efficient allocation), the expected profit for the firm is

$$2(1-\delta)\frac{\varphi^2 y_1^2}{\mu^2} + 2(1-\beta-\gamma)\frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \quad (39)$$

times  $\mu$  (in the following equations, all profit expressions are stated as shares of  $\mu$ ). If instead the CEO were to allocate all resources to one division, then the expected profit would be

$$(1-\beta-\gamma)\frac{\varphi y_2}{\mu}. \quad (40)$$

For the CEO to choose the efficient allocation requires that (39) be at least as large as (40), or

$$2(\beta+\gamma-\delta)\varphi\frac{y_1^2}{\mu} + (1-\beta-\gamma)(2y_1-y_2) \geq 0. \quad (41)$$

By similar reasoning, it can be shown that the condition for the CEO to allocate resources efficiently if both projects are bad is given by

$$2(\beta+\gamma-\delta)\frac{y_1^2}{\mu} + (1-\beta-\gamma)(2y_1-y_2) \geq 0. \quad (42)$$

Finally, suppose that division 1's project is good and division 2's bad. If the CEO allocates all resources division to 1 (the efficient allocation), the firm's expected profit is

$$(1-\beta-\gamma)\frac{\varphi y_2}{\mu} \quad (43)$$

If instead the CEO were to allocate the resources equally, then the expected profit would be

$$2(1-\delta)\frac{\varphi y_1^2}{\mu^2} + (1-\beta-\gamma) \left[ \frac{\varphi y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) + \left( 1 - \frac{\varphi y_1}{\mu} \right) \frac{y_1}{\mu} \right]. \quad (44)$$

For the CEO to choose the efficient allocation requires that (43) be at least as large as (44), or equivalently

$$(1-\beta-\gamma)[\varphi(y_2-y_1)-y_1] - 2(\beta+\gamma-\delta)\frac{\varphi y_1^2}{\mu} \geq 0. \quad (45)$$

Of these three constraints, (42) is redundant. To see why, notice that since both (41) and (45) must hold, the sum of their left-hand sides, which yields  $\mu(\varphi-1)(y_2-y_1)(1-\beta-\gamma)$ , must be positive. This in turn requires that  $\beta+\gamma < 1$ . Next, given the last result, both (41) and (42) can be binding only if  $\delta > \beta+\gamma$ ; but in that case (40) is the more restrictive condition. We can therefore ignore (42).

The problem we are concerned with, therefore, is that of maximizing (31) with respect to  $\beta$ ,  $\gamma$  and  $\delta$ , subject to the effort incentive constraint (34), the truth-telling constraints (36) and (38), the resource allocation constraints (41) and (45), and the nonnegativity constraints  $\beta, \gamma, \delta \geq 0$ .

**Proposition 9** *In an integrated firm with cheap-talk communication, the optimal non-separable contract for each division manager that leads to high effort, truthful reports about investment projects, and an efficient resource allocation, is given by*

$$\begin{aligned}\beta &= c \frac{1 - p(1 + \varphi)}{(1 - p)(p - q)(\varphi - 1)[(1 - p)y_2 - p(\varphi - 1)y_1]} \\ \gamma &= 0, \\ \delta &= c \frac{(\mu - \varphi y_1)[p(2y_1 - y_2) + y_2 - y_1] + (1 - p)y_1[\varphi(y_2 - y_1) + y_1]}{(1 - p)(p - q)(\varphi - 1)y_1^2[(1 - p)y_2 - p(\varphi - 1)y_1]}.\end{aligned}\quad (46)$$

if  $p \leq 1/(1 + \varphi)$  and  $c$  sufficiently small. In this case, the resulting expected wage per agent is the same as under non-integration. Otherwise, the resulting expected wage per agent is strictly higher than under non-integration. In particular, if  $p > 1/(1 + \varphi)$ , the optimal contract entails  $\gamma > 0$ .<sup>29</sup>

*Proof:* The contract (46) is the unique solution for which  $\gamma = 0$  and both (34) and (38) are binding. Feasibility of this solution requires  $\beta, \delta \geq 0$ . Since the numerator of  $\delta$  in (46) is positive, we need  $(1 - p)y_2 > p(\varphi - 1)y_1$  for the denominator of  $\delta$  to be positive. Since the same term appears in the denominator of  $\beta$ , we need  $p < 1/(1 + \varphi)$  for  $\beta$  to be positive as well. Conversely, if  $p < 1/(1 + \varphi)$  or  $1 - p > p\varphi$ , then it follows that  $(1 - p)y_2 > p\varphi y_1 > p(\varphi - 1)y_1$ , i.e. the same condition we started with. We can conclude that  $p < 1/(1 + \varphi)$  is necessary for the stated solution to be feasible.

However, the resource allocation constraints (41) and (45) need to be satisfied too. It can be shown that  $\delta > \beta$  for the contract (46). It follows that (45) is always satisfied, but (41) may not be. As  $c$  decreases to zero, so do  $\beta$  and  $\delta$  in (46), in which case (41) reduces to  $2y_1 - y_2 \geq 0$ , which means that (46) is overall feasible. However, for larger  $c$  condition (41) is easily violated.

The total expected wage bill for the integrated firm is  $\mu$  times

$$2\delta \left[ p^2 \varphi^2 + (1 - p)^2 \right] \frac{y_1^2}{\mu^2} + 2(\beta + \gamma) \left[ p^2 \frac{\varphi y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) + p(1 - p) \frac{\varphi y_2}{\mu} + (1 - p)^2 \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) \right], \quad (47)$$

cf. the expression for the firm's net profit in (31). Substituting the contract (46) into (47) and simplifying leads to  $2c(p\varphi - 1 + p)/[(p - q)(\varphi - 1)]$ , which is the same as the total wage bill for both firms under non-integration, cf. (24). Optimality of the solution (46) then follows from Proposition 2(a).

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<sup>29</sup> With three variables to specify and eight linear constraints, there are as many as  $8!/(3! 5!) = 56$  different corner points as possible candidates for an optimal solution in the case  $p > 1/(1 + \varphi)$ . It is possible to narrow down the set of possible solutions down to only six; however, there is little to gain from a more complete characterization of the solution.

The contract (46) is not feasible if either  $p > 1/(1 + \varphi)$  or if  $p \leq 1/(1 + \varphi)$  but  $c$  is too large. In the latter case, we just saw that a contract that satisfies (34) and (38) with equality leads to the lowest possible wage cost. If  $c$  is too large, then (41) is violated, and the optimal contract that satisfies (41) while still satisfying (34) and (38) must lead to a higher wage bill.

If  $p > 1/(1 + \varphi)$ , the truth-telling constraint (38) is decreasing in  $\delta$ . Since it is also decreasing in  $\beta$ , the only way to satisfy (38) is to set  $\gamma > 0$ . Evaluating the difference  $\varphi\bar{w}(B, G) - \bar{w}(G, G)$ , using the expressions in (32) and (37), simplifies to  $p\varphi(\varphi - 1)y_1\gamma$ . This means that if  $\gamma > 0$ , then  $\varphi\bar{w}(B, G)$  strictly exceeds  $\bar{w}(G, G)$ . In this case, it follows from (20) in the proof of Proposition 2 that the wage bill under integration is *strictly* higher than under non-integration. ■

Proposition 9 states that unless both  $p$  and  $c$  are small, the conclusions of Proposition 4 carry over to the non-separable case: any *feasible* solution leads to a wage bill higher than under non-integration, typically involving a contract with  $\gamma > 0$ .

Only if  $p \leq 1/(1 + \varphi)$  and  $c$  is small, the managers' information can be elicited without additional cost relative to the case of non-integration, similar to the message-contingent contract of Proposition 2. What makes this possible is that if  $p \leq 1/(1 + \varphi)$ , the truth-telling constraint (38) is increasing in  $\delta$ .<sup>30</sup> It is thus in principle possible to establish truth-telling without requiring  $\gamma > 0$ , by setting  $\delta$  high enough. The problem is that the required  $\delta$  may be too high to satisfy (41).

Our last result generalizes Proposition 6. Like in Section 6, we allow for asymmetric contracts for the managers. The managers' wages for the different possible payoff outcomes can therefore be described by  $\delta_1, \beta_1, \gamma_1$  and  $\delta_2, \gamma_2, \beta_2$ , respectively.

**Proposition 10** *Assume that the owner of an integrated firm wants the managers to exert high effort, and wants resources to be allocated efficiently. Then the incentive constraints in the CEO hierarchy are unambiguously less restrictive than those in the hybrid organization.*

*Proof:* As in the separable case, in the hybrid organization all incentive constraints for manager 2, as well as the effort incentive constraint for manager 1, are the same as in the CEO hierarchy, cf. the proof of Proposition 6. It remains to show how the resource allocation

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<sup>30</sup> To understand the sign of this derivative, observe first that a manager can earn  $\delta$  only if both divisions have high payoffs, which requires that the CEO allocates the resources equally between the divisions (if one division has no resources, it cannot attain a high payoff). If manager 1 has a bad project and reports truthfully, he earns  $\delta$  if manager 2 also has a bad project (the probability of which is  $1 - p$ ), and both have high payoff, which occurs with probability  $y_1^2/\mu^2$ . In contrast, if manager 1 claims to have a good project, he earns  $\delta$  if manager 2 has a good project (the probability of which is  $p$ ), and both have high payoff, which occurs with probability  $\varphi y_1^2/\mu^2$ . Thus, the effect of  $\delta$  to report truthfully is given by  $(1 - p - p\varphi)y_1^2/\mu$ , which is positive if and only if  $p \leq 1/(1 + \varphi)$ .

constraints for a manager 1 with a bad project compare to his truth-telling constraint in the hierarchy with CEO. Suppose manager 1 has a bad project. If manager 2's project is bad too, and manager 1 allocates the resources equally as would be efficient, his expected wage is

$$\left[ \frac{y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta_1 + \gamma_1) \right] \mu.$$

If instead he allocates all resources to himself, his expected wage is  $\beta_1 y_2$ . Manager 1 will therefore allocate resources efficiently if

$$\left[ \frac{y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{y_1}{\mu} \right) (\beta_1 + \gamma_1) \right] \mu - \beta_1 y_2 \geq 0. \quad (48)$$

If manager 2's project is good and manager 1 allocates all resources to division 2 as would be efficient, his expected wage is  $\gamma_1 \varphi y_2$ . If instead he allocates the resources equally, his expected wage is

$$\left[ \frac{\varphi y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta_1 + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma_1 \right] \mu.$$

Manager 1 will therefore allocate resources efficiently if

$$\gamma_1 \varphi y_2 - \left[ \frac{\varphi y_1^2}{\mu^2} \delta_1 + \frac{y_1}{\mu} \left( 1 - \frac{\varphi y_1}{\mu} \right) \beta_1 + \left( 1 - \frac{y_1}{\mu} \right) \frac{\varphi y_1}{\mu} \gamma_1 \right] \mu \geq 0. \quad (49)$$

It can then be shown that left-hand side of (38) is equal to  $(1-p)$  times the left-hand side of (48) plus  $p$  times the left-hand side of (49), which completes the proof (see the proof of Proposition 6 for further details). ■

## References

- Aghion, Philippe, Patrick Bolton, and Steven Fries (1999): Optimal Design of Bank Bailouts: The Case of Transition Economies. *Journal of Institutional and Theoretical Economics* 155(1), 51–70.
- Aghion, Philippe, Mathias Dewatripont and Patrick Rey (2004): Transferable Control. *Journal of the European Economic Association* 2(1), 115–38.
- Aghion, Philippe, and Jean Tirole (1997): Formal and Real Authority in Organizations. *Journal of Political Economy* 105(1), 1–29.
- Alonso, Ricardo, Wouter Dessein, and Niko Matouschek (2006): When Does Coordination Require Centralization? *American Economic Review*, forthcoming
- Athey, Susan; and John Roberts (2001): Organizational Design: Decision Rights and Incentive Contracts. *American Economic Review Papers and Proceedings* 91(2), 200–5.
- Auriol, Emmanuelle, Guido Friebel, and Lambros Pechlivanos (2002): Career Concerns in Teams. *Journal of Labor Economics* 20, 289–307.

- Barney, Jay (1991) Resources and Sustained Competitive Advantage. *Journal of Management* 17(1), 99-120.
- Bernardo, Antonio E., Hongbin Cai, and Jiang Luo (2001): Capital Budgeting and Compensation with Asymmetric Information and Moral Hazard. *Journal of Financial Economics* 61(3), 311-44.
- Bernardo, Antonio E., Hongbin Cai, and Jiang Luo (2004): Capital Budgeting in Multidivision Firms: Information, Agency, and Incentives. *Review of Financial Studies* 17(3), 739-67.
- Bolton, Patrick, and David Scharfstein (1998): Corporate Finance, the Theory of the Firm, and Organizations. *Journal of Economic Perspectives* 12(4), 95-114.
- Chandler, Alfred (1977): *The Visible Hand: the Managerial Revolution in American Business* Belknap Press.
- Chandler, Alfred (1990): *Scale and Scope: The Dynamics of Industrial Capitalism* Belknap Press.
- Coase, Ronald (1937): The Nature of the Firm. *Economica* 4(16), 386-405.
- Crawford, Vincent, and Joel Sobel (1982): Strategic Information Transmission. *Econometrica* 50(6), 1431-51.
- Crozier, Michel (1964): *The Bureaucratic Phenomenon*. University of Chicago Press.
- Dessein, Wouter (2002): Authority and Communication in Organizations. *Review of Economic Studies* 69(4), 811-38.
- Dessein, Wouter, Luis Garicano, and Robert Gertner (2006): Organizing for Synergies. Working paper, University of Chicago.
- Dewatripont, Mathias (2001): Authority. Walras-Bowley Lecture presented at the 2001 North-American Meeting of the Econometric Society.
- Dewatripont, Mathias and Jean Tirole (1999): Advocates. *Journal of Political Economy* 107(1), 1-39.
- Fingleton, John and Michael Raith (2005): Career Concerns of Bargainers. *Journal of Law, Economics and Organization* 21(1), 179-204.
- Gertner, Robert, David Scharfstein, and Jeremy Stein (1994): Internal versus External Capital Markets. *The Quarterly Journal of Economics* 109(4), 1211-30.
- Gibbons, Robert (2005): Four Forma(lizable) Theories of the Firm? *Journal of Economic Behavior and Organization* 58(2), 200-45.
- Grossman, Sanford and Oliver Hart (1986): The Costs and Benefits of Ownership: Vertical and Lateral Ownership. *Journal of Political Economy* 94, 691-719.
- Harris, Milton, and Artur Raviv (1996): The Capital Budgeting Process: Incentives and Information. *Journal of Finance* 51(4), 1139-74.
- Harris, Milton, and Artur Raviv (1998): Capital Budgeting and Delegation. *Journal of Financial Economics* 50(3), 259-89.

- Hart, Oliver, and Bengt and Holmström (2002): A Theory of Firm Scope. Working paper, MIT.
- Hart, Oliver, and John Moore (2005): On the Design of Hierarchies: Coordination versus Specialization. *Journal of Political Economy* 113, 675-702.
- Holmström, Bengt (1999): The Firm as a Subeconomy. *Journal of Law, Economics, and Organization* 15, 74-102.
- Holmström, Bengt and Paul Milgrom (1991): Multi-task Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design. *Journal of Law, Economics and Organization* 7(0), 24-52.
- Holmström, Bengt and Paul Milgrom (1994): The Firm as an Incentive System. *American Economic Review* 84(4), 972-91.
- Holmström, Bengt, and Jean Tirole (1991): Transfer Pricing and Organizational Form. *Journal of Law, Economics, and Organization* 7, 201-28.
- Inderst, Roman, and Christian Laux (2005): Incentives in Internal Capital Markets: Capital Constraints, Competition, and Investment Opportunities. *RAND Journal of Economics* 36(1), 215-28.
- Inderst, Roman, and Manuel Klein (2006): Innovation, Endogenous Overinvestment, and Incentive Pay. Working paper, LSE.
- Levitt, Steven, and Christopher Snyder (1997): Is No News Bad News? Information Transmission and the Role of "Early Warning" in the Principal-Agent Model. *RAND Journal of Economics* 28(4), 641-61.
- Mailath, George J., Volker Nocke and Andrew Postlewaite (2004): Business Strategy, Human Capital, and Managerial Incentives. *Journal of Economics and Management Strategy* 13(4), 617-33.
- Marino, Anthony M. and Jan Zabojsnik (2004): Internal Competition for Corporate Resources and Incentives in Teams. *RAND Journal of Economics* 35(4), 710-27.
- Marketing Week (2005) P&G Scales Up Global Ambitions. London, Feb. 3, 2005
- Matsusaka, John G. (2001): Corporate Diversification, Value Maximization, and Organizational Capabilities. *Journal of Business* 74(3), 409-31.
- Meyer, Margaret, Paul Milgrom, and John Roberts (1992): Organizational Prospects, Influence Costs, and Ownership Changes. *Journal of Economics and Management Strategy* 1(1), 9-35.
- Milgrom, Paul, and John Roberts (1988): An Economic Approach to Influence Activities in Organizations. *American Journal of Sociology* 94 (Supplement), S154-79.
- Milgrom, Paul, and John Roberts (1990): Bargaining Costs, Influence Costs and the Organization of Economic Activity. in: James E. Alt and Kenneth A. Shepsle (eds.), *Perspectives on Positive Political Economy* Cambridge: Cambridge University Press, 57-89.
- Montgomery, Cynthia and Birger Wernerfelt (1988): Diversification, Ricardian Rents, and Tobin's q. *RAND Journal of Economics* 19(4), 623-32.

- Ozbas, Oguzhan (2005): Integration, organizational processes, and allocation of resources. *Journal of Financial Economics* 75(1), 201-42.
- Penrose, Edith (1959): *The Theory of the Growth of the Firm* Oxford: Oxford University Press.
- Povel, Paul (1999): Optimal "Soft" or "Tough" Bankruptcy Procedures. *Journal of Law, Economics and Organization* 15(3), 659-84.
- Rantakari, Heikki (2006): Governing Adaptation. Working paper, MIT
- Roberts, John (2004): *The Modern Firm*. Oxford University Press.
- Scharfstein, David, and Jeremy Stein (2000): The Dark Side of Internal Capital Markets: Divisional Rent-Seeking and Inefficient Investment. *Journal of Finance* 55(6) 2537-64.
- Stein, Jeremy C. (1997): Internal Capital Markets and the Competition for Corporate Resources. *Journal of Finance* 52(1), 111-33.
- Stein, Jeremy (2002): Information Production and Capital Allocation: Decentralized versus Hierarchical Firms, *Journal of Finance* 57(5), 1891-921.
- Sutton, John (2005): Competing in Capabilities: An Informal Overview. Working paper, London School of Economics
- Teece, David (1982): Towards an Economic Theory of the Multiproduct Firm. *Journal of Economic Behavior and Organization* 3, 39-63.
- Wernerfelt, Birger (1984): A Resource-Based View of the Firm. *Strategic Management Journal* 5(2), 171-80.
- Williamson, Oliver (1975): *Markets and Hierarchies: Analysis and Antitrust Implications*. New York: The Free Press.
- Williamson, Oliver (1985): *The Economic Institutions of Capitalism: Firms, Markets, Relational Contracting*. New York: The Free Press.
- Wulf, Julie (2005): Influence and Inefficiency in the Internal Capital Market. Working paper, University of Pennsylvania.