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Abstract

We note some problems in Alesina and Angeletos (2005) and suggest a way to maintain the key insight of that paper, which is that a demand for fairness could lead to different economic systems such as those observed in France versus the US (multiple equilibria).

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1 Introduction

In an important paper Alesina and Angeletos (2005), henceforth AA, argued that a preference for fairness could lead two identical societies to choose different economic systems. In particular, two equilibria might arise: one with low taxes and a belief that the income-generating process is "fair" because effort is important (an "American" equilibrium) and another with high taxes and the belief that the process is "unfair" because luck prevails. Piketty (1995) had shown that a similar pattern could arise from standard preferences if initial beliefs about the relative importance of effort and luck in generating income was different in the two societies, while Benabou and Tirole (2006) study this issue using more realistic preferences (Buera et al, 2010, provides a discussion of the evolution of beliefs about economic systems). A key contribution of AA is to obtain these two equilibria from identical societies assuming

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agents prefer outcomes that are fair, an important modification because fairness considerations seem central in the demand for redistribution and because in some settings (as in some ultimatum games) such preferences for fairness can lead to large (material) inefficiencies.

In this note we discuss some aspects of AA. Our main point is that the way AA specify "fair" preferences (equation 3 in page 965 of AA) has unappealing implications. First, in some situations there is multiplicity and demand for redistribution even if luck plays no role (and the prevailing level of income is "fair"). In other words, there is multiplicity even if the equilibrium tax rate is independent of the signal to noise ratio (a quantity that expresses how important is effort, relative to luck, in the determination of income). This conflicts with the notion that the signal to noise ratio plays a central role in generating multiplicity with AA preferences for fairness. Second, if one defines "fairness" in a way that is consistent with their verbal description, the AA model no longer has multiplicity.[1]

Our final contribution is to show that the central idea in AA, namely that a demand for fairness can lead similar societies to have different equilibrium tax rates, can indeed be obtained in at least one specification of fair preferences. In particular, we study agents for whom a desire for fairness is reduced to a desire to respond like with like, as in the reciprocal altruism preferences of Levine (1998) and Rotemberg (2003) (see their papers for a discussion on how the empirical fit of these preferences improves upon alternative theories). In this alternative model, we obtain multiplicity: two identical societies can have different equilibrium tax rates.

2 The AA Model.

The economy is populated by a measure 1 continuum of individuals $i \in [0,1]$, who live for two periods: in the first period the individuals accumulate capital; in the middle of their lives the taxes are set; in the second period, individuals exert effort (work). Total pre-tax lifetime income

$$y_i = A_i \left[\alpha k_i + (1-\alpha)\, e_i\right] + \eta_i \tag{1}$$

[1] There are minor technical changes that need to be made in AA to obtain the main result, even if one keeps their definition of fairness. In particular, in the appendix we show that by simply adding the assumption of symmetric shocks, one can then prove that the tax rate preferred by the median voter is a Condorcet winner (i.e., it beats any other tax rate in pairwise voting; this approach is less elegant but is required because the median voter does not apply as preferences are not single peaked, even with symmetric shocks). However, some of the appeal of AA is lost, since then the model cannot capture the Meltzer-Richard effect that the authors discuss.

is where A is talent, k is the capital accumulated during the first period, e is effort during the second period, η is "noise" or "luck", and $\alpha \in (0,1)$ is a technological constant. The government imposes a flat tax rate τ on income and redistributes the proceeds in a lump sum fashion, so that the individual's consumption is, for government transfer $G = \tau \int_i y_i$,

$$c_i = (1-\tau)\, y_i + G.$$

Individual preferences are, for $u_i = V_i(c_i, k_i, e_i) = c_i - \frac{1}{2\beta_i}\left[\alpha k_i^2 + (1-\alpha)\, e_i^2\right]$,

$$U_i \equiv u_i - \gamma\Omega \equiv c_i - \frac{1}{2\beta_i}\left[\alpha k_i^2 + (1-\alpha)\, e_i^2\right] - \gamma\Omega$$

where u_i is private utility from own consumption, investment and effort, β_i is an impatience parameter, γ is "distaste for unfair outcomes" and Ω is a measure of the social injustice in the economy. AA assume that A, η and β are iid across agents, and that for $\delta = A^2\beta$, $Cov(\delta, \eta) = 0$. We let $\overline{\delta}$ be the mean of δ, and δ_m its median; AA also assume $\Delta = \overline{\delta} - \delta_m \geq 0$ and normalize $\delta_m = 2$. Similarly, $\overline{\eta}$ is the mean of η and η_m its median. We assume in addition that δ and η are independent (the joint probability distribution is the product of the probability distributions of δ and η), and that their distributions are symmetric (see the appendix).

AA define social injustice as

$$\Omega = \int_i (u_i - \widehat{u}_i)^2$$

where u_i is the actual level of private utility, and $\widehat{u}_i$ is a measure of the "fair" level of utility the individual should have (deserves) on the basis of his talent and effort. They define $\widehat{u}_i = V_i(\widehat{c}_i, k_i, e_i)$ for

$$\widehat{c}_i = \widehat{y}_i = A_i\left[\alpha k_i + (1-\alpha)\, e_i\right]. \tag{2}$$

The individual chooses k when taxes haven't been set, so anticipating a tax rate of τ_e (which will be equal to the actual τ in equilibrium) he maximizes

$$u_i = (1-\tau_e)\, A_i\left[\alpha k_i + (1-\alpha)\, e_i\right] + (1-\tau_e)\,\eta_i + G - \frac{1}{2\beta_i}\left[\alpha k_i^2 + (1-\alpha)\, e_i^2\right] \tag{3}$$

with respect to k, and using the actual tax rate in equation (3) maximizes with respect to e to obtain

$$k_i = (1-\tau_e)\, A_i\beta_i \quad \text{and} \quad e_i = (1-\tau)\, A_i\beta_i. \tag{4}$$

Then, $U_i = u_i - \gamma\Omega$ implies

$$\begin{aligned} U_i &= \frac{\delta_i}{2}\left(1 - \alpha\tau_e^2 - (1-\alpha)\,\tau^2\right) + \eta_i + \tau\left(\overline{\eta} - \eta_i\right) + \tau\left(\overline{\delta} - \delta_i\right)\left[1 - \alpha\tau_e - \tau(1-\alpha)\right] - \gamma\Omega \\ &= \frac{\delta_i}{2}\left(1 - \alpha\tau_e^2 - (1-\alpha)\,\tau^2\right) + \eta_i + \tau\left(\overline{\eta} - \eta_i\right) + \tau\left(\overline{\delta} - \delta_i\right)\left[1 - \alpha\tau_e - \tau(1-\alpha)\right] \\ &\quad -\gamma\left((1-\tau)^2\sigma_\eta^2 + \tau^2\left[1 - \alpha\tau_e - (1-\alpha)\,\tau\right]^2\sigma_\delta^2\right) \end{aligned} \tag{5}$$

3 Fairness in AA

AA define unfairness in the economy as sum of the differences between what people get, c_i, and what people "deserve", $\widehat{c}_i$. In the words of AA, "Following the evidence ... that people share a common conviction that one should get what one deserves, and deserve what one gets, we define our measure of social injustice as" $\Omega = Var\left(c_i - \widehat{c}_i\right)$. That is, in going from what one gets, c_i, to what one deserves, $\widehat{c}_i$, AA change *two* things: the fact that there is a government, and the fact that there is an unfair luck shock in the economy.[2] With this definition of fairness, they find that there are two equilibria, the American with low taxes, and the European, with high taxes. They claim that this is the result of the demand for fairness, and:

"if the demand for fairness is sufficiently high, the complementarity between the optimal level of taxation and the equilibrium signal-to-noise ratio in the income distribution can sustain multiple equilibria."

"The critical features of the model that generate equilibrium multiplicity are (a) that the optimal tax rate is decreasing in the signal-to-noise ratio and (b) that the equilibrium signal-to-noise ratio is in turn decreasing in the tax rate."

To show that multiplicity in AA arises, *not* because of luck and unfairness, but because of the elimination of Government from the definition of fairness, we now show that:

1. In an economy with no unfairness, with luck shocks identically 0, the AA definition of unfairness (which excludes government and luck from fair consumption) gives rise to multiplicity, so it is not unfairness, or the demand for fairness, that gives rise to multiplicity. In fact, in order to illustrate the main point of this section, that the AA definition of fairness is inconsistent with "true" fairness, we show that this quote (p. 970) is problematic: "the US and EU-type equilibria coexist as long as γ is sufficiently high and σ_η is neither too large nor too small relative to σ_δ. Instead ... only the low-tax regime survives if there is ... little noise to correct (low σ_η)." Moreover, they claim in

[2] Di Tella, Dubra and MacCulloch (2010) mention this difficulty and propose an alternative model based on the definition of fairness we introduce in this section, while Alesina et al. (2010) study the dynamic implications of using the preferences of AA and those of Di Tella et al. (2010). Alesina et al. clarify that the definition of fairness in AA is indeed not only about fairness, but about other things as well, since it reflects the fact "that individuals tolerate inequality coming from innate ability and effort, but are averse to inequality arising from everything else, luck and redistribution".

p. 970, "The critical features of the model that generate equilibrium multiplicity are (a) that the optimal tax rate is decreasing in the signal-to-noise ratio and (b) that the equilibrium signal-to-noise ratio is in turn decreasing in the tax rate." The following example has a constant noise-to-signal ratio (at 0), but there is still multiplicity: hence it is not the variation in signal to noise ratio that generates multiplicity, but the exclusion of government from the definition of fair consumption.

2. If fair consumption is defined as the consumption individuals would have in the presence of a government, but luck shocks are identically 0 so that income is fair, $c_i^g = (1-\tau)\,\widehat{y}_i + G$, and unfairness in society is accordingly defined as

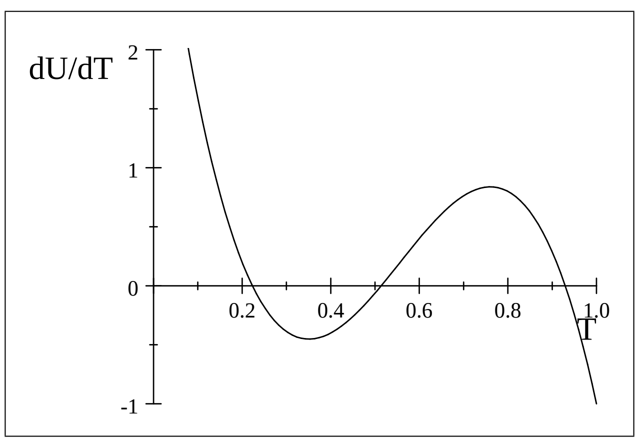
$$\Omega^g = \int \left(c_i - c_i^g\right)^2,$$

then, there is no multiplicity of equilibria (in two identical countries 1 and 2, if the tax rate are $\tau_1, \tau_2 \in (0,1)$, then $\tau_1 = \tau_2$)

With $\delta_m = \overline{\delta} = 2$, and $\overline{\eta} = \eta_m = 0$ and $\gamma = 2$, plus $\sigma_\eta^2 = 1$ and $\sigma_\delta = \frac{5}{2}$, the economy has multiple equilibria, if fair, deserved, consumption is defined as $\widehat{c}_i$ (no government, no luck shocks), as in AA. We have that for U_i, from equation (5), for the individual with the median shocks,

$$\frac{dU}{d\tau} = -\frac{25}{2}\tau^3 - \frac{75}{4}\tau^2\tau_e + \frac{75}{2}\tau^2 - \frac{25}{4}\tau\tau_e^2 + 25\tau\tau_e - 30\tau + 4$$

When we set $\tau_e = \tau$, the equation has 3 roots:



However, imagine that in this same economy, luck shocks were identically 0, so that $\sigma_\eta^2 = 0$. In this case, there is still multiplicity, but it is not a consequence of the "demand for fairness", since this economy is fair. In an economy with no unfairness, we still get multiplicity, if we use the AA definition of fairness, with $\sigma_\eta^2 = 0$:

$$\frac{dU}{d\tau} = -\frac{1}{4}\tau \left(50\tau^2 + 75\tau\tau_e - 150\tau + 25\tau_e^2 - 100\tau_e + 104\right),$$

so setting the same parameter values as before, we see that there are 3 equilibrium tax rates: $\frac{13}{15}, \frac{4}{5}, 0$. Just to repeat ourselves, in this economy, the noise to signal ratio is $Var\left(y_i - \widehat{y}_i\right)/Var\left(\widehat{y}_i\right) = 0$ (independently of the tax rate) but there are three equilibria. Hence, even with no unfairness, there is multiplicity.

Finally, if in any of the two economies (with $\sigma_\eta^2 = 1$ or $\sigma_\eta^2 = 0$) we define fair consumption as that which would arise with government but with no shocks, that is c_i^g, what we argue is a reasonable definition of fairness, there is uniqueness of equilibria (this is true generally, and not only for these two economies). For a proof of the following result, see the appendix.

Theorem 1 *There is a unique equilibrium tax rate when unfairness is defined as* $\Omega^g = Var\left(c_i - c_i^g\right)$.

Proof. See the Appendix. ■

This proves that it is not demand for fairness that generates multiplicity (defining fair consumption as that which would arise if income had no luck shocks yields uniqueness) but the fact that "fair" consumption does not include taxes and transfers (as shown by the fact that in an economy with no luck shocks there is still multiplicity).

One could try to "fix" the model in AA by changing the definition of fairness Ω to some measure of the distance between c_i and c_i^g, some measure different from $\left(c_i - c_i^g\right)^2$ that is, in the population. Although some alternative definitions of Ω do work, they are quite complicated, which may be interpreted as indicating that the basic model is not robust to slight (or reasonable) changes in the definition of Ω.

4 A model of fairness

There is a continuum mass $m < 1$ of firms that behave competitively (they take prices as given). In period 0, firms observe a cost realization $\mu \sim U[0,1]$, and they have three actions available to them: play out, play g (produce a single unit of a good quality product), or play b (produce a single unit of a bad quality product). Quality is observable. The cost of producing the bad quality is $c_b + \mu$, while that of the good quality is $c_g + \mu$ for $c_g > c_b$. The timing of when they observe their cost is irrelevant: they can observe it before or after entry. We have assumed that they observe it before entry.

There is a continuum mass 1 of consumers, and in period 1, after firms have entered, entrepreneurs (the owners of the firms) and consumers must vote for a tax rate τ on sales, that is distributed lump sum to consumers (assuming that it is distributed to all does not change the qualitative features of the model).

In period 2 consumers learn their valuations for the goods, and decide whether to buy (at the prices which are given) one single unit of the good (among the available options, which can be both goods, only g or only b) or none. If a consumer buys good i, for $i = b, g$, with price p_i, his utility increases, for some $\theta \sim U[0,1]$, by $i\theta - p_i$. Whereas several well-known models in economics analyze fairness, including Rabin (1993), Fehr and Schmidt (1999), Falk and Fischbacher (2006), *inter alia*, we follow Levine (1998) and Rotemberg (2008) and assume an individual's kindness towards others depends on their estimation of how kind others have been in relationships with them. In particular, we assume that our consumers have Rotemberg preferences and sometimes reject the hypothesis that the firm was "minimally altruistic" towards them (and chosen a good quality product). In that case, the utility of the firm enters negatively in the consumers' utility (see Rotemberg and Levine, who explain that these preferences better account for the patterns in the data emerging from ultimatum experiments).

Consumer utility is for government transfers G, and profits P of the bad quality firm,

	not buy	buy good i
angry	$G - a_h P$	$i\theta - p_i + G - a_h P$
not angry	$G - a_l P$	$i\theta - p_i + G - a_l P$

Standard preferences are with $a_h = a_l = 0$. We will assume for the parametrizations that $a_h > 0 = a_l$.

A strategy for firm j is a function σ_j from type to $\{Out, b, g\}$, and a voting rule v_j at each node (each configuration of entry of firms) that maps type into $[0,1]$, specifying the tax rate voted by the entrepreneur who owns firm j.

A strategy for consumer i is function from nodes to $[0,1]$, specifying the tax rate he votes in each node, and a purchase decision that depends on his preference parameter θ, the chosen tax t, and the prices of the two goods in that node.

An equilibrium in this economy is a strategy for each firm, a strategy for each consumer, such that:

Given an expected tax rate t^e, and expected prices p_b and p_g, the entry decisions by firms are optimal; each consumer and entrepreneur votes for his/her optimal tax rate knowing how taxes affect the equilibrium price; the tax rate t voted by the majority is $t = t^e$; at prices p_i^e, $i = b, g$, supply equals demand; given the actual tax rate t, and prices p_b and p_g, purchase decisions by consumers are optimal.

Theorem 2 *There exists an open set of parameters for which there are two equilibria in pure strategies with the following features. In one equilibrium, firms expect a high tax rate t_b, those that enter, do so with the bad quality product, and in equilibrium, consumers vote for the tax rate t_b. In the other equilibrium, firms expect a low tax rate $t_g < t_b$, and those that enter do so with a product of good quality, and in equilibrium consumers vote for a low tax rate t_g.*

Proof. We construct the equilibrium starting from the last stage, when firms have entered, and the tax rate has been set. Suppose only firms of type i have entered the market (this will be part of the equilibrium). Demand for good i, $i = b, g$ is then

$$D_i = \begin{cases} 1 - \frac{p_i}{i} & 0 \le p_i < i \\ 0 & i \le p_i \end{cases} .$$

If the price is p_i, and only i firms are in, each firm supplies 1 unit of the good, if and only if $(1-t)\,p_i - c_i \ge \mu$; supply is m times the individual supply of each firm, which is

$$s_i = \begin{cases} 0 & p_i \le \frac{c_i}{1-t} \\ (1-t)\,p_i - c_i & \frac{c_i}{1-t} \le p_i \le \frac{1+c_i}{1-t} \\ 1 & \frac{1+c_i}{1-t} < p_i \end{cases} \Rightarrow S_i = m s_i = \begin{cases} 0 & p_i \le \frac{c_i}{1-t} \\ m\left((1-t)\,p_i - c_i\right) & \frac{c_i}{1-t} \le p_i \le \frac{1+c_i}{1-t} \\ m & \frac{1+c_i}{1-t} < p_i \end{cases}$$

In equilibrium we must have

$$p_i = \begin{cases} i\,(1-m) & t < \frac{i(1-i)-c_i-1}{i(1-m)} \equiv \underline{t}_i \\ \frac{i(mc_i+1)}{1+im(1-t)} & \underline{t}_i \le t \le \frac{i-c_i}{i} \equiv \bar{t}_i \\ i & \bar{t}_i < t \end{cases} . \tag{6}$$

Given this equilibrium price as a function of the tax, consumers and entrepreneurs must vote for their preferred tax rate. We concentrate on the voting behavior of the consumers, since that decides the chosen tax rate, given that there are more consumers than entrepreneurs (because $1 > m$) and all consumers have the same preferences at the voting stage.

For $a = a_h, a_l$, consumer utility for type θ, when firms are of type $i = b, g$, is given by

$$u = \max\{i\theta - p_i, 0\} + G - aP.$$

So for $\theta_i = \frac{p_i}{i}$, and using $\theta \sim U[0,1]$, we obtain

$$\begin{aligned} u &= \left(1-\frac{p_i}{i}\right)\left(iE\left(\theta \mid \theta \geq \theta_i\right) - p_i\right) + G - aP \\ &= \left(1-\frac{p_i}{i}\right)\left(i\frac{1+\theta_i}{2} - p_i\right) + G - aP = \frac{\left(i-p_i\right)^2}{2i} + G - aP. \end{aligned}$$

(a) (This case is irrelevant since for all parameter values we have $\underline{t}_i < 0$). For $t < \underline{t}_i$, $p_i = i(1-m)$ is the equilibrium price, and

$$\begin{aligned} u &= \frac{\left(i-p_i\right)^2}{2i} + tp_i - a\int_0^1 \left((1-t)p_i - \mu - c_i\right)d\mu = \frac{\left(i-p_i\right)^2}{2i} + tp_i - a\left((1-t)p_i - c_i - \frac{1}{2}\right) \\ &= \frac{im^2}{2} + (1+a)i(1-m)t - a\left(i(1-m) - c_i - \frac{1}{2}\right) \end{aligned}$$

which is increasing in t, so the optimal t for consumers will never be $t < \underline{t}_i$.

(b) For $\underline{t}_i \leq t \leq \bar{t}_i$, $p_i = \frac{i(mc_i+1)}{1+im(1-t)}$ and profits are, for $\overline{\mu}_i \equiv \frac{(1-t)i(mc_i+1)}{1+im(1-t)} - c_i$ the largest cost for which the firm produces,

$$P_i = \int_0^{\overline{\mu}_i}\left(\frac{(1-t)i(mc_i+1)}{1+im(1-t)} - \mu - c_i\right)d\mu = \frac{1}{2}\frac{\left(c_i - i(1-t)\right)^2}{\left(1+im(1-t)\right)^2}$$

Government transfers are then

$$G_i = m\frac{it(mc_i+1)\left(i(1-t)-c_i\right)}{\left(im(1-t)+1\right)^2}$$

Consumer utility is therefore

$$\begin{aligned} u &= \frac{1}{2}im^2\frac{\left(c_i - i(1-t)\right)^2}{\left(im(1-t)+1\right)^2} + m\frac{it(mc_i+1)\left(i(1-t)-c_i\right)}{\left(im(1-t)+1\right)^2} - a\frac{1}{2}\frac{\left(c_i-i(1-t)\right)^2}{\left(1+im(1-t)\right)^2} \\ &= m\frac{it(mc_i+1)\left(i(1-t)-c_i\right)}{\left(im(1-t)+1\right)^2} + \frac{\left(im^2-a\right)\left(c_i-i(1-t)\right)^2}{2\left(1+im(1-t)\right)^2} \end{aligned}$$

In this interval, the maximum utility may be either interior, or at $t = \underline{t}_i$, but never at $\bar{t}_i$, since

$$\frac{du}{dt} = \frac{i(mc_i+1)}{\left(im(1-t)+1\right)^3}\left((m+a)(i-c_i) - i\left(c_i m^2 + 2m + a\right)t\right)$$

which is positive for small t, 0 for

$$0 < t_i = \frac{(m+a)(i-c_i)}{ai + 2im + im^2 c_i} < \bar{t}_i \tag{7}$$

and then negative.

(c) For $t > \bar{t}_i = \frac{i-c_i}{i}$, $p_i = i$, $G = P_i = 0$, and utility is 0.

A parameter configuration: $b = 2, g = 4, c_b = 0, c_g = \frac{2}{3}, m = \frac{2}{3}, a_h = 1, a_l = 0$. Suppose firms expect a tax rate of $t_b = \frac{5}{7}$, and prices of $p_b = \frac{42}{29}$ and $p_g = \frac{41}{11}$. Then, a firm prefers to enter with b rather than g if and only if

$$(1 - t_b) p_b - \mu - c_b \geq (1 - t_b) p_g - \mu - c_g \Leftrightarrow p_b + \frac{c_g - c_b}{(1 - t_b)} \geq p_g$$

which in this case is satisfied (with strict inequality). All firms with

$$\mu \leq (1 - t_b) p_b - c_b = \frac{12}{29}$$

enter the market, generating a supply of m times $\frac{12}{29}$, which in this case is $\frac{8}{29}$.

Given prices, consumers with $\theta \geq \frac{p_b}{b} = \frac{21}{29}$ are better off buying than not buying b, and consumers prefer b to g if and only if

$$b\theta - p_b \geq g\theta - p_g \Leftrightarrow p_g - p_b \geq (g - b) \theta$$

which is satisfied (with strict inequality) for all θ. Demand is then $1 - \frac{p_b}{b} = \frac{8}{29}$, which equals supply.

Finally, given that t_b is determined by equation (7), we see that the proposed profile of strategies is an equilibrium. To see that this is an equilibrium for an open set of parameters, note that the incentive constraints are satisfied with strict inequalities. So setting p_b with equation (6), $p_g = \frac{41}{11}$ and t_b with equation (7), we continue to obtain an equilibrium.

Consider now the equilibrium with low taxes. Suppose firms expect a tax rate of $t_g = \frac{15}{44}$, and prices of $p_b = \frac{55}{51}$ and $p_g = \frac{44}{21}$. A firm prefers to enter with g rather than b if and only if

$$(1 - t_g) p_g - \mu - c_g \geq (1 - t_g) p_b - \mu - c_b \Leftrightarrow p_g - \frac{c_g - c_b}{1 - t_g} \geq p_b^d$$

which in this case is satisfied with strict inequality. All firms with

$$\mu \leq (1 - t_g) p_g - c_g = \frac{5}{7}$$

enter the market, generating a supply of m times $\frac{5}{7}$, which in this case is $\frac{10}{21}$.

Given prices, consumers with $\theta \geq \underline{\theta} \equiv \frac{p_g}{g} = \frac{11}{21}$ are better off buying than not buying g; since this consumer prefers g to b, because $0 > b\underline{\theta} - p_b$, all types $\theta' > \underline{\theta}$ also do. In addition, types $\theta < \underline{\theta}$ are such that $0 > b\underline{\theta} - p_b > b\theta - p_b$ so they don't buy b either.

Demand is then $1 - \frac{p_g}{g} = 1 - \frac{11}{21} = \frac{10}{21}$, which equals supply.

As in the other equilibrium, given that t_g is determined by equation (7), we see that the proposed profile of strategies is an equilibrium. To see that this is an equilibrium for an open set of parameters, note that the incentive constraints are satisfied with strict inequalities. So setting p_g with equation (6), $p_b = \frac{55}{51}$ and t_g with equation (7), we continue to obtain an equilibrium. ■

It is easy to show that in the current model it is the possibility of firms behaving in non-altruistic ways (i.e. having the choice between a good product and a bad product), combined with the feature that consumers have reciprocal altruism, that gives rise to multiplicity. If either feature is not present, there is a unique equilibrium in undominated strategies.

5 Conclusion

AA have argued that different economic systems (for example Europe vs America) can be explained as two different equilibria of a model where the demand for redistribution is linked to fairness. In this note we show that there is multiple equilibria in AA even when there is no luck (i.e., with no unfairness). Indeed, multiplicity in AA arises not because of the demand for fairness but because of the exclusion of the government from the definition of fair consumption. There is no easy fix: if fair consumption is defined in the natural way (to include taxes and transfers) then multiplicity disappears. Given that showing how fairness can lead to multiplicity is an important objective of the literature on economic systems that started with Piketty (1995) -using standard preferences- and which Benabou and Tirole (2006) expanded using more realistic preferences, we provide a stylized model of fairness based on "reciprocal altruism" (a la Levine-Rotemberg), in which multiplicity arises. In other words, we show that AA's key intuition (that fairness leads to multiplicity) can be demonstrated, albeit after assuming a somewhat different specification of fair preferences.

6 Appendix: Voting in AA.

AA identify as the median voter the individual with the median values of the shocks. In general he is not. Hence, the tax rate identified by AA may be defeated in voting. Therefore, we identify conditions under which the tax rate identified by AA is the one chosen by the median voter (it is a condition on the symmetry of the shocks). Since the median voter theorem does not apply (preferences are not single peaked), we now show that the tax rate preferred by the median voter wins in pairwise majority voting against any other tax rate (i.e. conditions under which it is a Condorcet winner). This gives conditions under which the AA result is plausible politically.

A tax rate τ_c is a **Condorcet winner** if for every other τ, $\Pr\{(\delta_i, \eta_i) : U_i(\tau_c) \geq U_i(\tau)\} \geq \frac{1}{2}$. That is, τ_c beats every other τ on pairwise majority voting. Formally, let $f(\cdot)$ be the density of δ and $g(\cdot)$ that of η. Recall that δ_m is the median of δ, and that the mean and median of η are 0. We assume that for all x, the densities of δ and η are symmetric: $f(\delta_m - x) = f(\delta_m + x)$, and that $g(-x) = g(x)$.

Theorem 3 *Suppose the densities of δ and η are symmetric, that the draws are independent and let m be the individual with the median values of δ and η. Then, for every fixed τ_e, any tax rate $\tau_m(\tau_e)$ that maximizes the utility of m is a Condorcet winner. If $\tau_m(\tau_e)$ is unique, then it is the unique Condorcet winner.*

Proof. Fix any τ_e, and let $\tau_m(\tau_e)$ (from now on, we suppress the dependence on τ_e, since it will be fixed throughout) be maximal for $m : U_m(\tau_m) - U_m(\tau) \geq 0$ for all τ. All we need to show is that for every τ, $\Pr\{(\delta_i, \eta_i) : U_i(\tau_m) \geq U_i(\tau)\} \geq \frac{1}{2}$, and we will do so by showing that $\Pr\{(\delta_i, \eta_i) : U_i(\tau_m) - U_i(\tau) \geq U_m(\tau_m) - U_m(\tau)\} \geq \frac{1}{2}$.

For all i and j, and all s and t, we have that

$$\begin{aligned} U_i(s) - U_i(t) &\geq U_j(s) - U_j(t) \Leftrightarrow \\ u_i(s) - \gamma\Omega(s) - u_i(t) + \gamma\Omega(t) &\geq u_j(s) - \gamma\Omega(s) - u_j(t) + \gamma\Omega(t) \end{aligned} \tag{8}$$

so that equation (8) holds if and only if $u_i(s) - u_i(t) \geq u_j(s) - u_j(t)$. But then

$$u_i(s) - u_i(t) = (s-t)\left\{\left(\overline{\delta} - \delta_i\right)(1 - \alpha\tau_e) - \frac{1}{2}(1-\alpha)\left(2\overline{\delta} - \delta_i\right)(s+t) - \eta_i\right\}$$

(and similarly for j) imply that equation (8) holds if and only if

$$(s-t)\left\{(\delta_j - \delta_i)\left[1 - \alpha\tau_e - \frac{1}{2}(1-\alpha)(s+t)\right] + \left(\eta_j - \eta_i\right)\right\} \geq 0.$$

Suppose, without loss of generality, that $s > t$, and let $j = m$. Since

$$\begin{aligned} S &\equiv \{(\delta_i, \eta_i) : U_i(\tau_m) - U_i(\tau) \geq U_m(\tau_m) - U_m(\tau)\} \\ &= \left\{(\delta_i, \eta_i) : (\delta_m - \delta_i)\left[1 - \alpha\tau_e - \frac{1}{2}(1-\alpha)(s+t)\right] \geq \eta_i\right\} \end{aligned} \tag{9}$$

we only need to show that $\Pr\left\{(\delta_i, \eta_i) : (\delta_m - \delta_i)\left[1 - \alpha\tau_e - \frac{1}{2}(1-\alpha)(s+t)\right] \geq \eta_i\right\} \geq \frac{1}{2}$.

To understand the following argument, note that $\eta_i = (\delta_m - \delta_i)\left[1 - \alpha\tau_e - \frac{1}{2}(1-\alpha)(s+t)\right]$ is a line through (δ_m, η_m) on the (δ_i, η_i) plane, and that the set in the right hand side of equation (9) is the half space lying "south" of the line. In particular, given $s > t$, if for agent i $\delta_i = \delta_m$, it means that i will prefer s to t (that is, will prefer higher taxes) if he was poorer than the median agent (received a lower η_i than m, namely $0 \geq \eta_i$). We have that for $c \equiv 1 - \alpha\tau_e - \frac{1}{2}(1-\alpha)(s+t)$

$$\Pr\{S\} = \int_{-\infty}^{\infty} \int_{-\infty}^{(\delta_m - \delta_i)c} g(\eta)\, d\eta f(\delta)\, d\delta = \int_{-\infty}^{\delta_m} \int_{-\infty}^{(\delta_m - \delta_i)c} g(\eta)\, d\eta f(\delta)\, d\delta + \int_{\delta_m}^{\infty} \int_{-\infty}^{(\delta_m - \delta_i)c} g(\eta)\, d\eta f(\delta)\, d\delta \tag{10}$$

The symmetry assumption on δ implies that for $z(\delta) = \delta - \delta_m$, the density h of z is such that $h(z) = f(z + \delta_m)$, so that $h(z) = f(z + \delta_m) = f(\delta_m - z) = h(-z)$.

Then, equation (10) and the change of variable $z(\delta) = \delta - \delta_m$ imply that

$$\Pr\{S\} = \int_{-\infty}^{0} \left[\int_{-\infty}^{-zc} g(\eta)\, d\eta \right] h(z)\, dz + \int_{0}^{\infty} \left[\int_{-\infty}^{-zc} g(\eta)\, d\eta \right] h(z)\, dz \tag{11}$$

but symmetry of g implies that

$$\int_{-\infty}^{-zc} g(\eta)\, d\eta = \int_{zc}^{\infty} g(\eta)\, d\eta$$

so that equation (11) becomes

$$\Pr\{S\} = \int_{-\infty}^{0} \left[\int_{-\infty}^{-zc} g(\eta)\, d\eta \right] h(z)\, dz + \int_{0}^{\infty} \left[\int_{zc}^{\infty} g(\eta)\, d\eta \right] h(z)\, dz$$

Since g is symmetric, the pdf of g, G, is such that for all x, $G(-x) = 1 - G(x)$. Therefore

$$\Pr\{S\} = \int_{-\infty}^{0} G(-zc)\, h(z)\, dz + \int_{0}^{\infty} [1 - G(zc)]\, h(z)\, dz = \int_{-\infty}^{0} [1 - G(zc)]\, h(z)\, dz + \int_{0}^{\infty} [1 - G(zc)]\, h(z)\, dz$$

so for $w = -z$, using $h(-w) = h(w)$ and $1 - G(-wc) = G(wc)$ we obtain

$$\begin{aligned} \Pr\{S\} &= \int_{-\infty}^{0} [1 - G(zc)]\, h(z)\, dz + \int_{0}^{\infty} [1 - G(zc)]\, h(z)\, dz = \int_{0}^{\infty} [1 - G(-wc)]\, h(w)\, dw + \int_{0}^{\infty} [1 - G(zc)] \\ &= \int_{0}^{\infty} G(wc)\, h(w)\, dw + \int_{0}^{\infty} [1 - G(zc)]\, h(z)\, dz = \int_{0}^{\infty} h(z)\, dz = \frac{1}{2}. \end{aligned}$$

This completes the proof. ■

As argued in footnote 1, assuming that shocks are symmetric ensures that $\Delta = \overline{\delta} - \delta_m = 0$, which reduces the empirical appeal of the model, since the Meltzer Richard effect then disappears.

Proof of Theorem 1. From $U^g = u_i - \gamma\Omega^g$, equation (5) and

$$\begin{aligned}
\Omega^g &= Var\left(c_i - c_i^g\right) = Var\left((1-\tau)\, y_i - (1-\tau)\, A_i\left[\alpha k_i + (1-\alpha)\, e_i\right]\right) \\
&= Var\left((1-\tau)\left(y_i - A_i\left[\alpha k_i + (1-\alpha)\, e_i\right]\right)\right) = Var\left((1-\tau)\,\eta_i\right) = (1-\tau)^2\,\sigma_\eta^2
\end{aligned}$$

we obtain

$$U^g = \frac{\delta_i}{2}\left(1 - \alpha\tau_e^2 - (1-\alpha)\,\tau^2\right) + \eta_i + \tau\left(\overline{\eta} - \eta_i\right) + \tau\left(\overline{\delta} - \delta_i\right)\left[1 - \alpha\tau_e - \tau\,(1-\alpha)\right] - \gamma\,(1-\tau)^2\,\sigma_\eta^2$$

The best response tax is 0 if

$$\tau\left(\tau_e\right) = \frac{-2\gamma\sigma_\eta^2 - (\delta - 2)\left(1 - \alpha\tau_e\right)}{2\,(1-\alpha)\,(1-\delta) - 2\gamma\sigma_\eta^2}$$

is negative and 1 if $\tau\left(\tau_e\right) > 1$. Hence, if the slope of $\tau\left(\tau_e\right)$ is less than 1, which happens if and only if $\overline{\delta} \geq \frac{1-\alpha-\gamma\sigma_\eta^2}{1-\alpha}$, there is a unique equilibrium. This is the only relevant case since AA assume $\Delta \geq 0$, which implies $\overline{\delta} \geq \delta_m = 2 > 1 \geq \frac{1-\alpha-\gamma\sigma_\eta^2}{1-\alpha}$. If $\Delta \geq 0$ hadn't been assumed, and if $\overline{\delta} < \frac{1-\alpha-\gamma\sigma_\eta^2}{1-\alpha} < 1 < 2 = \delta_m$, there could be 3 equilibria, but the unique interior equilibrium would be unstable.[3] ■

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[3] In any case, the case of 3 equilibria is unappealing for three reasons: (1) because it involves tax rates of 0 and 1, a patently absurd situation; (2) the interior equilibrium is unstable; (3) it arises only if $\overline{\delta} < 1 < 2 = \delta_m$, a situation inconsistent with the observation that mean income is larger than median income, which is ensured by $\Delta \geq 0$.

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