



# Competing Ad Auctions: Multi-homing and Participation Costs

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Working Paper

10-055

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# Competing Ad Auctions: Multi-homing and Participation Costs

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## ABSTRACT

We model competing auctions for online advertising, with attention to the participation costs that limit advertisers' interest in using small ad platforms. When participation costs are large relative to the volume of traffic an ad platform can offer, an advertiser may forego use of an ad platform that the advertiser otherwise finds profitable. Mergers between ad platforms can increase advertiser welfare if the resulting click-through rate and volume of traffic are sufficiently improved relative to the offerings of the ad auctions when separate. When there is an insufficient improvement, such mergers can harm advertisers.

## 1. INTRODUCTION

Online ad auctions sell advertising placements on search engines and elsewhere – providing key funding for a variety of online resources. Advertisers sign up with one or more ad platforms, specify their advertising preferences (including conditions in which they want their ads to be shown, and how much they are willing to pay), and receive clicks from interested users.

In this paper, we explore competition among ad platforms that offer search engine advertising services. Our motivations are several. For one, we want to understand why some advertisers choose to use only certain ad platforms but not others. After all, if an ad network cannot attract a broad selection of advertisers, it will be unable to present ads related to users' requests, and it will also garner substantially lower revenue. In addition, we want to evaluate possible transactions among ad platforms. For example, we want to assess the effects of ad platforms joining their systems so that an advertiser buying from one ad platform automatically receives traffic from another platform also.

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\*Ben Edelman and Hoan Soo Lee serve as consultant and research intern to Microsoft and Microsoft Research, respectively. But they write on their own behalf, not at Microsoft's request or for Microsoft's benefit.

Our analysis is grounded in *participation costs* which each bidder must pay to use each ad platform. Through a model and data, we argue that participation costs *exist* and *matter* – affecting bidders' decision about which ad platforms to use, and changing the welfare consequences of mergers or joins among platforms. To date, the literature reflects no discussion of participation costs – a simplification that is appropriate when modeling a single ad platform, but inadequate for evaluating competition between platforms.

This paper proceeds in four parts. In Section 2, we develop model fundamentals and notation. In Section 3, we model ad auctions with participation costs – offering an initial explanation of why not all advertisers use all ad platforms, and testing that explanation with available data. In Section 4, we analyze competing ad auctions, develop a concept of “joining” two auctions, and identify conditions where joins increase or reduce advertiser welfare. In Section 5, we discuss policy implications.

## 2. AD AUCTIONS - GSP AND VCG

In an *ad auction* there is a set of bidders  $\mathcal{N} = \{1, \dots, N\}$  and  $K$  slots for sale. Each bidder can receive at most one slot. We assume without loss of generality that  $N > K$ . The positions are sold for a single period of time. Each slot  $k$  has an expected *click-through rate*  $\alpha_k > 0$ . The auction has a known capacity  $C > 0$ . Thus, if an advertiser wins slot  $k$ , the advertiser will receive  $C\alpha_k$  clicks in expectation. We assume that  $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_K$ . It will be convenient to define  $\alpha_k = 0$  for every  $k > K$ .

The *value* per click for bidder  $j$  is  $v_j \in [0, 1]$ . Bidders are risk neutral, and the payoff to bidder  $j$  for winning slot  $k$  is  $C\alpha_k v_j$  minus his payments to the auctioneer. We also assume that for each bidder  $j$ ,  $v_j$  is drawn from a commonly known distribution  $F$ .

Each bidder  $j$  submits a bid  $b_j$  which is interpreted as his value per click. We denote by  $b_{(j)}$  the  $j^{\text{th}}$  highest bid. Similarly we denote by  $g(j)$  the identity of the  $j^{\text{th}}$  highest bidder. In case of ties, the order among those bidders is determined randomly.

We consider payments and outcomes under two distinct auction mechanisms. Modern ad platforms generally use the *Generalized Second Price* (GSP) structure, wherein if bidder  $g(j)$  receives position  $k$ , he tenders total payment of  $C\alpha_j b_{(j+1)}$  (and otherwise he pays zero). In the *Vickrey*

Clarke-Groves (VCG) ad auction, each bidder pays his impact on all others' social welfare, assuming bids equal values. Hence, if bidder  $g(j)$  gets position  $k$ , his total payment will be  $p^{V,k} = C \sum_{j=k}^N (\alpha_j - \alpha_{j+1}) v_{(j+1)}$ . Both GSP and VCG ad auctions allocate the first position to the highest bidder, the second position to the second-highest bidder, and so forth.

Bidding truthfully is a dominant strategy for every bidder in the VCG ad auction [10, 3, 7]. [6] and [9] show that there exists an equilibrium in the GSP ad auction under complete information such that its outcome coincides with the outcome of the VCG auction in which each bidder bids truthfully. Furthermore, this equilibrium yields the lowest revenue for the seller, so it is in some sense best for the bidders. Finally, [2] shows that a reasonable myopic GSP bidding strategy converges to this equilibrium.

### 3. AD AUCTIONS WITH PARTICIPATION COSTS

Consider a GSP ad auction, and suppose there is a cost  $Z \geq 0$  to participate in the auction.  $Z$  can be interpreted as an advertiser's transaction cost required to enter the campaign into the ad platform. Elements of  $Z$  include creating an account, setting advertising parameters, monitoring effectiveness, adjusting bids, and paying bills.  $Z$  is not transferred to the auctioneer. Therefore, a high platform's participation cost,  $Z$ , relative to its capacity level will cause an advertiser to forego use of the ad platform, even if the advertiser would otherwise find it profitable to use that platform. For example, if the following inequality holds

$$\frac{Z}{C} > \alpha_1 v, \quad (1)$$

then an advertiser with valuation  $v$  will never enter the auction since the maximum possible utility for such an advertiser occurs when he receives the first slot and pays 0.

In this section we analyze the equilibria of the following two-stage game: In the first stage, each bidder decides whether to participate in the auction, as a function of the number of other bidders that decide also to participate. In the second stage, all bidders that decide to participate submit a bid. We assume the value of each bidder is private information.

We model the auction as a two-stage game so that bidders need not commit to entering the auction immediately following the realization of their valuations. Instead, we permit entry/exit in the first stage. In the second stage, the bidders who enter face the same conditions as in [6] and [9], and the outcome coincides with the VCG outcome of the ad auction with these bidders.<sup>1</sup>

A strategy for bidder  $j$  is a function  $s_j : \{1, \dots, N\} \rightarrow \{0, 1\}$  where  $s_j(n) = 1$  means that bidder  $j$  enters the auction given that  $n$  other bidders also enter the auction, and  $s_j(n) = 0$  means  $j$  does not enter.

#### 3.1 Decision Rule for Advertisers

<sup>1</sup>In [1] the authors do not model participating costs, but rather let advertisers choose a single ad auction to participate in.

We denote by  $U(v, n)$  the expected utility (before considering entry cost  $Z$ ) of a bidder with value  $v$  who decides to participate in the auction, given that exactly  $n$  other bidders also participate. Thus, if bidder  $j$  knows that exactly  $n$  other bidders will participate, he will enter the auction if and only if

$$U(v, n) \geq Z. \quad (2)$$

After bidders decide whether to enter, a set of bidders  $B$  enter the auction, and their equilibrium bids coincide with the GSP equilibrium in [6] and [9].

The following lemma will be useful. It provides two monotonicity properties: the expected utility for a bidder who enters the auction is non-decreasing in the number of participants, and is increasing in the bidder's value.

LEMMA 3.1. *For any  $n < N$ , and any  $v$ :*

1.  $U(v, n) > U(v, n + 1)$ .
2.  $\partial U(v, n) / \partial v > 0$ .

PROOF. Let  $Q(v, n)$  and  $P(v, n)$  denote the expected click-through rate and expected payment for a bidder with value  $v$  that decides to enter the auction given that there are  $n - 1$  other bidders in the auction. Thus the expected utility of such a bidder is

$$U(v, n) = vQ(v, n) - P(v, n). \quad (3)$$

Note that

$$Q(v, n) = C \sum_{1 \leq k \leq n} \binom{n-1}{k} \alpha_k F(v)^{n-k} [1 - F(v)]^{k-1}. \quad (4)$$

Let  $X_n$  be a random variable distributed over  $\{\alpha_1, \dots, \alpha_N\}$  such that  $P(X_n = \alpha_k) = \binom{n-1}{k} p^{n-k} (1-p)^{k-1}$  where  $p = F(v)$ , and let  $Y_n \sim \text{Bin}(n-1, p)$ , i.e.  $P(Y_n = k) = \binom{n-1}{k} p^{n-k} (1-p)^{k-1}$ . Note that  $P(X_n = \alpha_k) = P(Y_n = k)$ . For every  $m > 1$ , define  $\mu(m, n) = (\alpha_m - \alpha_{m-1}) P(Y_n \geq m-1)$  and let  $\mu(1, n) = \alpha_1 P(Y_n \geq 0)$ . Observe that  $Q(v, n) = CE[X_n] = C \sum_{m \geq 1} \mu(m, n)$ . Note that  $\mu(1, n) = \alpha_1$ . By assumption,  $\alpha_{k+1} - \alpha_k \leq 0$ . Moreover,  $P(Y_n \geq k)$  is decreasing with  $n$ , since more trials with the same probability of success will lead to more successes. Therefore, for all  $m > 1$ , every  $\mu(m, n)$  is non-increasing with  $n$ , implying that  $Q(v, n) > Q(v, n + 1)$ . By [8],

$$U(v, n) = \int_0^v Q(x, n) dx. \quad (5)$$

Therefore  $U(v, n) > U(v, n + 1)$ .

By (5) the derivative of  $U(v, n)$  with respect to  $v$  is equal to  $Q(v, n)$ , which completes the proof since  $\partial Q(v, n) / \partial v > 0$ .  $\square$

### 3.2 Equilibrium Analysis

We now define an equilibrium of the two-stage game.

DEFINITION 3.2. *A profile of strategies  $(s_1, \dots, s_N)$  is in equilibrium in the two-stage game if there exists an  $n$  such that  $|\{j \in \mathcal{N} : s_j(n-1) = 1\}| = n$  and for every  $j$  with  $s_j(n-1) = 0$ ,*

$$U(v_j, n) \geq Z \quad (6)$$

and for every other  $j$ ,

$$U(v_j, n+1) < Z. \quad (7)$$

Bidders choose to enter the auction according to inequality (2). Thus, in steady-state, the endogenous number of entrants  $n^*$  is some threshold value such that any additional entrant would force a bidder to exit. Equivalently, an equilibrium in this game is a set of participants  $B$  such that each bidder  $j \in B$  prefers to enter the auction, given that  $|B \setminus \{j\}|$  are also participating; and every other bidder prefers not to participate in these circumstances. A set of participants  $B$  is in equilibrium if  $|B| = n$ , for every bidder  $j \in B$  inequality (6) is satisfied, and for every other bidder inequality (7) is satisfied.

### 3.3 Equilibrium Existence

In this section we show that an equilibrium always exists by construction. The idea is to add bidders one by one in decreasing order of their valuations into the set of participating bidders  $B$  as long as their expected utility does not exceed the participation costs. As we shall see later this equilibrium is not unique.

THEOREM 3.3. *There exists an  $n^* \leq N$  such that the set of bidders with the  $n^*$  largest valuations will participate in the auction in equilibrium.*

PROOF. By part 1 of Lemma 3.1 every bidder  $j$  has a threshold  $n_j = n(v_j) \leq N$  such that

$$\begin{cases} U(v_j, n) \geq Z & \text{for every } 1 \leq n \leq n_j; \\ U(v_j, n) < Z & \text{for every } n < n_j \leq N. \end{cases} \quad (8)$$

Moreover, by part 2 of Lemma 3.1, if  $v \geq v'$  then for every  $n \geq 1$ :

$$U(v, n) \geq U(v', n). \quad (9)$$

In particular, for  $n = n(v')$ , we have  $U(v, n(v')) \geq U(v', n(v')) \geq Z$ . By definition  $n(v)$  is the largest  $n$  such that  $U(v, n) \geq Z$ , implying that  $n(v) \geq n(v')$ . Thus:

$$n(v_{(1)}) \geq n(v_{(2)}) \geq \dots \geq n(v_{(N)}). \quad (10)$$

Define  $n^* \in \{0, \dots, N\}$  to be the largest integer such that:

$$n(v_{(n^*)}) \geq n^* - 1, \quad (11)$$

and let  $B = \{g(1), \dots, g(n^*)\}$ , i.e.  $B$  is the set of  $n^*$  highest bidders. By (10) and the definition of  $n^*$ , we obtain that for every bidder  $j \in B$ ,  $U(v_j, n^*) > Z$  and for every  $j \notin B$ ,  $U(v_j, n^*) \leq Z$ , which completes the proof.  $\square$

### 3.4 Equilibrium Uniqueness

The equilibrium constructed in the proof of Theorem 3.3 has the property that if a bidder with value  $v$  participates in the auction, then every bidder with value larger than  $v$  also participates in the auction. We refer to such an equilibrium as a *monotone equilibrium*. The proof of Theorem 3.3 showed uniqueness of monotone equilibria assuming ties do not exist. For simplicity we will assume throughout the entire paper that there are no ties. Formally,

THEOREM 3.4. *An ad auction with participation costs has a unique monotone equilibrium.*

However, there also exist non-monotone equilibria. Intuitively, such an equilibrium can occur when an advertiser with a low valuation that participates, causing an advertiser with a higher valuation to decline to participate. The following example presents a non-monotone equilibrium:

EXAMPLE 1. *Consider an environment with two bidders and one slot where  $v_1 = 0.5, v_2 = 0.4$ , and  $\alpha_1 = 1, C = 1$ . Suppose  $F \sim U[0, 1]$  and  $Z = 0.3$ . Each bidder will participate in the auction if it is the sole bidder because  $U(v, 0) = v > 0.3$  for  $v \in \{v_1, v_2\}$ . However  $U(v_1, 1) = -0.5 * 0 + 0.5 * 0.25 < 0.3$  and  $U(v_2, 1) = -0.5 * 0 + 0.5 * 0.2 < 0.3$ . Thus  $n(v_1) = n(v_2) = 0$ , implying that there are two equilibria, one in which only bidder 1 participates in the auction, and one in which only bidder 2 participates. But  $v_2 < v_1$ , so the latter equilibrium is non-monotone.*

Examples with more bidders and more slots can be constructed in a similar fashion.

### 3.5 Advertiser Size and Multi-homing

Inequalities (1) and (2) offer a testable implication: Large advertisers multihome because they can spread participation cost  $Z$  across a large volume of ad purchases, whereas small advertisers find the participation costs too large to justify signing up with smaller platforms.

We test these claims with data from multiple services that track and preserve advertising at multiple ad platforms. We report normalized advertiser sizes based on independent observations by two different ad monitoring services.<sup>2</sup> Based on data from a first data collection service, Table 1 compares the size (impression count, normalized with maximum value set to 1,000) of advertisers that use one, two, or all three of the ad platforms we examine. Based on data from a second service, Table 2 measures the size of multi-homing and

<sup>2</sup>By agreement with our data sources, we do not report their names.

non-multihoming advertisers by other metrics: Proportion of sites achieving an Alexa ranking (requiring that a site be among roughly 25 million most popular sites on the web), average rank (bottom-coding unranked sites as 40 million), as well as average “reach” (number of users visiting the site) and average page views. (Reach and page-views are reported per thousand users.) Figure 1 plots the distribution of reach by multi-homing and non-multihoming advertisers.

By each metric, these tables are consistent with inequalities (2) and (1) and our model of advertisers’ participation costs. In particular, the advertisers that purchase ads from all three platforms are strikingly larger than the advertisers that purchase ads only from one or two platforms: They buy more ad impressions (Table 1) and are more likely to be ranked by Alexa, achieve a lower average rank (i.e. greater traffic), larger reach, and more page-views (Table 2) Meanwhile, the advertisers who choose to use only Google are the smallest by far – further confirming that small advertisers tend not to multi-home. Figure 1 confirms that triply-multihoming advertisers have pointwise larger reach than double-multihoming advertisers which in turn have pointwise larger reach than single-multihoming advertisers.

### 3.6 Comparative Statics

In this section, we examine comparative statics on the equilibrium number of entrants in a single auction. In the following proposition, we identify the minimum capacity increment (decrement) for the increase (decrease) of the number of participants. We define  $n^*(C)$  to be the number of participants in the monotone equilibrium when the capacity is  $C$ .

**PROPOSITION 3.5.** *Fix  $C$  to be the capacity, let  $\bar{C} = C + \Delta C$  where  $\Delta C > 0$  and let  $n^* = n^*(C)$ . Then  $n^*(\bar{C}) > n^*$  if and only if:*

$$\Delta C \geq \frac{CZ}{U(v_{(n^*+1)}, n^*+1)} - C. \quad (12)$$

*Let  $\bar{C} = C + \Delta C$  where  $\Delta C < 0$ . Then  $n^* < n^*(\bar{C})$  if and only if:*

$$-C < \Delta C < \frac{CZ}{U(v_{(n^*)}, n^*)} - C. \quad (13)$$

**PROOF.** In an equilibrium with  $n^*$  entrants,  $U(v_{(j)}, n^*) \geq Z$  for each  $j \leq n^*$  and  $U(v_{(j)}, n^*+1) < Z$  for each  $j > n^*$ . If  $\Delta C \geq \frac{CZ}{U(v_{(n^*+1)}, n^*+1)} - C$ , then  $U(v_{(n^*+1)}, n^*+1) \geq \frac{CZ}{C+\Delta C}$ , implying that  $n^*$  increases by at least 1 from  $C$  to  $\bar{C}$ . The proof for the second part is analogous.  $\square$

The model also indicates an ad platform’s preferences as to capacity and participation cost. It can be shown that more capacity  $C$  always increases revenue, both for the direct effect (more clicks to sell) and the indirect effect (weakly more

advertisers bidding). Also, lowering participation cost  $Z$  weakly increases an ad platform’s revenue, again by weakly increasing the number of participating advertisers. Formally,

**PROPOSITION 3.6.**

1. *The revenue of an ad platform weakly increases with the capacity.*
2. *The revenue of an ad platform weakly decreases with the participation cost.*

**PROOF.** Since the equilibrium is as in the VCG outcome the revenue given  $n^*$  is:

$$C \sum_{k=1}^{n^*} \sum_{j=k}^{n^*} (\alpha_j - \alpha_{j+1}) v_{(j)}.$$

If  $C$  increases then  $n^*$  weakly increases implying part 1. If  $Z$  decreases  $n^*$  weakly increases implying part 2.  $\square$

## 4. COMPETING AUCTIONS

In this section, we study bidders’ welfare when a join occurs between auctions with specified capacity and technology.

### 4.1 Joining Auctions

In this section, we establish a concept of “joining” auctions such that their available positions are pooled, and all bidders participating in one auction automatically participate in the other also. What happens to bidder welfare and ad platform revenue if two ad auctions are joined? These questions take on special relevance in light of a July 2009 proposal for a partnership between Microsoft and Yahoo, as well as a June 2008 proposed partnership between Google and Yahoo (ultimately aborted after antitrust regulators raised concerns).

We now model a multi-auction environment. Auctions vary in their capacity and in their position-dependent click-through rates. Each bidder pays a cost of entry  $Z$  for each auction entered, and each bidder may enter one or more auction(s). We begin with two definitions.

**DEFINITION 4.1.** *The set of click-through rates have the property of diminishing differences whenever  $\alpha_k - \alpha_{k+1} \geq \alpha_{k+1} - \alpha_{k+2}$  for each  $k \leq K$ .*

**DEFINITION 4.2.** *We say that auction auction  $i$  and auction  $i'$  join to form auction  $\tilde{i}$  if auction  $\tilde{i}$ ’s capacity  $C_{\tilde{i}}$  is such that*

$$\max\{C_i, C_{i'}\} \leq C_{\tilde{i}} \leq C_i + C_{i'} \quad (14)$$

*and  $\tilde{i}$ ’s click-through rates are*

$$\alpha_{\tilde{i}k} = \max\{\alpha_{ik}, \alpha_{i'k}\} \quad (15)$$

*where  $\alpha_{ik}$  ( $\alpha_{i'k}$ ) is the click-through rate of slot  $k$  in auction  $i$  ( $i'$ ).*

Taking the capacities of each auction as the size of the set of consumers of the engine, inequality (14) is equivalent to the condition that bounds  $C_{\tilde{i}}$ . Because some consumers use multiple search engines, we allow for overlap of capacity between two ad platforms – meaning a joined ad platform might have less capacity than the sum of capacities of its contributors. Furthermore, we assume that no consumer of either engine is lost upon join.

When auctions join, what click-through rates result? We envision ad platforms choosing the best components of each contributor, which implies click-through rates given by the stronger of the joining platforms. Hence the approach in (15).

To explore circumstances in which a joined auction can either positively or negatively affect overall bidder welfare, we introduce a notion of a “uniformly stronger” auction. This property reflects the extreme case in which the resulting auction does not yield any gain to bidders. In fact, in this circumstance, the joined auction may reduce some bidders’ welfare by increasing prices and/or displacing some participants.

**DEFINITION 4.3.** *Auction  $i$  is uniformly stronger than auction  $i'$  if and only if  $C_i \geq C_{i'}$ , and  $\alpha_{ik} \geq \alpha_{i'k}$  for each  $k$ . We denote this by  $A_i \geq_{us} A_{i'}$ .*

**LEMMA 4.4.** *Let  $B_i$  and  $B_{i'}$  denote the set of participants in a monotone equilibrium in auctions  $i$  and  $i'$  respectively. If  $A_i \geq_{us} A_{i'}$  then  $B_i \supseteq B_{i'}$ .*

**PROOF.** To prove the lemma, it is sufficient to show that

$$U_i(v, n) \geq U_{i'}(v, n) \quad \forall v, n. \quad (16)$$

By (5) and (4), each bidder’s expected utility is strictly increasing in  $C$  and the click-through rates. Since  $A_i \geq_{us} A_{i'}$ , we obtain (16).  $\square$

## 4.2 Joining Auctions to Make All Bidders Better Off

In the following theorem, we provide a condition in which a joined auction offers a sufficient improvement in capacity and technology to make every bidder weakly better off ex post.

**THEOREM 4.5.** *Suppose auctions  $i$  and  $i'$  join to form auction  $\tilde{i}$ . Let  $C_i \geq C_{i'}$  and  $\alpha_{i'k} \geq \alpha_{ik}$  for each  $k \leq K$ . Then every bidder is weakly better off if:*

$$C_{\tilde{i}} \geq \frac{C_{i'}\alpha_{i'k} + C_i\alpha_{ik}}{(N-k)\alpha_{i'k+1} - (N-k-1)\alpha_{i'k}} \quad \forall k \leq K \quad (17)$$

**PROOF.** Suppose auction  $i$  and  $i'$  join to form  $\tilde{i}$ . By Lemma 4.4,  $B_{\tilde{i}} \supseteq B_i$  and  $B_{\tilde{i}} \supseteq B_{i'}$ .

Suppose that  $n_i^* \geq n_{i'}^*$ . (The proof for the other direction is similar.)

Consider bidders who do not receive any position in either auction before the join. These are bidders ranked in position  $k > n_i^*$ . Because such bidders already achieve utility of 0, the joined auction  $\tilde{i}$  cannot make such bidders worse off.

Consider a bidder who wins positions in both auctions before the join. We introduce new notation to characterize ex post utility: denote by  $u_l(v, k)$  the utility of a bidder with value  $v$  who wins position  $k$  in auction  $l$ . Under VCG, we have:

$$u_l(v, k) = C\alpha_k v - C \sum_{n_i^* > j \geq k} (\alpha_j - \alpha_{j+1})v_{(j+1)} \quad (18)$$

where  $\alpha_k = 0$  for  $k \geq K$ .

Prior to the join, a bidder with valuation  $v$  who wins in both auctions must win position  $k \leq n_{i'}^*$  in auction  $i'$ . Moreover, such a bidder would win the same position and receive utility  $u_i(v, k) + u_{i'}(v, k)$ .

Using assumption (17) we get:

$$C_{\tilde{i}}\alpha_{\tilde{i}k}v - C_i\alpha_{ik}v - C_{i'}\alpha_{i'k}v \geq \quad (19)$$

$$C_{\tilde{i}}(N-k)(\alpha_{\tilde{i}k} - \alpha_{\tilde{i}k+1})v \geq \quad (20)$$

$$C_{\tilde{i}} \sum_{n_i^* > j \geq k} (\alpha_{\tilde{i}j} - \alpha_{\tilde{i}j+1})v_{(j+1)} \geq \quad (21)$$

$$C_{\tilde{i}} \sum_{n_i^* > j \geq k} (\alpha_{\tilde{i}j} - \alpha_{\tilde{i}j+1})v_{(j+1)} - S \quad (22)$$

where  $S = C_{i'} \sum (\alpha_{i'j} - \alpha_{i'j+1})v_{(j+1)} + C_i \sum (\alpha_{ij} - \alpha_{ij+1})v_{(j+1)}$ .

The last inequality (22) is equivalent to:

$$C_{\tilde{i}}\alpha_{\tilde{i}k}v - C_{\tilde{i}} \sum (\alpha_{\tilde{i}j} - \alpha_{\tilde{i}j+1})v_{(j+1)} \geq$$

$$C_i\alpha_{ik}v - C_i \sum (\alpha_{ij} - \alpha_{ij+1})v_{(j+1)} +$$

$$C_{i'}\alpha_{i'k}v - C_{i'} \sum (\alpha_{i'j} - \alpha_{i'j+1})v_{(j+1)}$$

which is equivalent to:

$$u_{\tilde{i}}(v, k) \geq u_i(v, k) + u_{i'}(v, k) \quad (23)$$

Therefore bidders who win in both auctions are better off after the join.

Consider a bidder who wins position  $k$  in auction  $i$  with  $n_i^* \geq k > n_{i'}^*$ . Such a bidder with valuations  $v$  will have ex post utility of  $u_i(v, k)$ . Since we have already shown (23), such a bidder, who wins a single position, is also better off after the join.  $\square$

The theorem’s conditions stipulate intuitive requirements for bidders to gain from a joined ad auction: the resulting

click-through rate and auction capacity must be sufficiently improved relative to the offerings of the ad auctions when separate. First, the auction with fewer bidders must add value to the join through a point-wise larger click-through rate. Second, there must be minimal overlap between the two auctions, so that  $C_{\tilde{i}}$  is sufficiently larger than both  $C_i$  and  $C_{i'}$ . It is necessary for  $C_{\tilde{i}}$  to be sufficiently large so that bidders in auction  $i$  (who already face higher prices due to more bidders in  $i$ ) gain sufficiently from joining the two auctions. Note for example that if both auctions have the same click-through rates then  $C_{\tilde{i}}$  is the sum of the capacities.

### 4.3 Example with Exponential Click-through Rates

Suppose  $\alpha_{ik} = \beta_i^k$  for each  $k \leq K$  where  $\beta_i < 1$ . Note that exponential click-through rates obey both monotonic ordering of  $\alpha_{ik}$  and the *diminishing differences* property. Following the framework in Theorem 4.5, we set  $\beta_{\tilde{i}} = \beta_{i'} > \beta_i$ . Condition (17) then becomes:

$$C_{\tilde{i}} \geq \frac{C_{i'}\beta_{i'}^k + C_i\beta_i^k}{(N-k)\beta_{i'}^{k+1} - (N-k-1)\beta_{i'}^k} \quad \forall k \leq K$$

Simplifying and rearranging:

$$C_{\tilde{i}} - C_{i'} - C_i \frac{\beta_i^k}{\beta_{i'}^k} \geq C_{\tilde{i}}(1 - \beta_{i'})(N - k) \quad (24)$$

Because  $\beta_i < \beta_{i'}$ , we know:

$$C_{\tilde{i}} - C_{i'} - C_i \frac{\beta_i}{\beta_{i'}} \leq C_{\tilde{i}} - C_{i'} - C_i \frac{\beta_i^k}{\beta_{i'}^k}$$

and

$$C_{\tilde{i}}(1 - \beta_{i'})(N - 1) \geq C_{\tilde{i}}(1 - \beta_{i'})(N - k).$$

Thus the generalized restrictions in equation (24) become:

$$C_{\tilde{i}} - C_{i'} - C_i \frac{\beta_i}{\beta_{i'}} \geq C_{\tilde{i}}(1 - \beta_{i'})(N - 1) \quad (25)$$

since such condition implies the required restrictions for each  $k \leq K$ .

By the subadditive property of joined auction  $\tilde{i}$  and equation (25):

$$C_{i'} + C_i \geq C_{\tilde{i}} \geq C_{\tilde{i}}(1 - \beta_{i'})(N - 1) + C_{i'} + C_i \frac{\beta_i}{\beta_{i'}}$$

Satisfying that condition requires:

$$C_{i'} + C_i \geq C_{\tilde{i}}(1 - \beta_{i'})(N - 1) + C_{i'} + C_i \frac{\beta_i}{\beta_{i'}}$$

Rearranging:

$$\frac{C_i}{C_{\tilde{i}}} \geq (N - 1) \frac{1 - \beta_{i'}}{1 - \frac{\beta_i}{\beta_{i'}}} \quad (26)$$

Since  $C_i \leq C_{\tilde{i}}$  and  $N \geq 2$  (so that the RHS of (26) is not trivially 0), inequality (26) requires:

$$1 - \frac{\beta_i}{\beta_{i'}} > 1 - \beta_{i'}$$

Rearranging yields the requirement:

$$\beta_{i'} > \sqrt{\beta_i} \quad (27)$$

Condition (27) requires that the auction with superior technology have  $\beta_{i'}$  sufficiently larger than  $\beta_i$ . Furthermore, if  $N$  is very large, then  $\beta_{i'}$  must approach 1 in order to satisfy the requirement in (26). Note that these are necessary conditions, but not sufficient conditions.

### 4.4 Joining Auctions that Make Some Bidders Worse Off

In other circumstances, joining two auctions can make bidders worse off. For example, if one of the auctions is uniformly stronger than the other and the resulting capacity remains equal to its original capacity (i.e. the uniformly weaker auction represents a subset of consumers of the stronger auction), then joining the auctions will make some bidders weakly worse-off.

The following theorem identifies sufficient conditions that make bidders worse off:

**THEOREM 4.6.** *Suppose  $A_i \geq_{us} A_{i'}$ , and auctions  $i$  and  $i'$  join following Definition 4.2. If  $C_{\tilde{i}} = C_i$ , then any bidder that wins in both auctions is worse off.*

**PROOF.** The assumption implies that  $\alpha_{\tilde{i}k} = \alpha_{ik} \geq \alpha_{i'k}$  for each  $k \leq K$  and  $C_{\tilde{i}} = C_i$ . Then the joined auction will be identical to auction  $i$ , and thus  $u_{\tilde{i}}(v, k) = u_i(v, k)$  for any  $v$  and  $k$ . Prior to the join, if a bidder wins position  $k \leq n_i^*$  (and thus the bidder receives a placement in both auctions), then his total ex post pre-merger utility of  $u_i(v, k) + u_{i'}(v, k)$  is greater than his post-merger  $u_{\tilde{i}}(v, k)$ .  $\square$

## 5. POLICY IMPLICATIONS AND FUTURE WORK

Joining ad platforms can attract substantial regulatory attention: In November 2008, the Department of Justice planned to file antitrust charges to stop the proposed Google-Yahoo transaction. More recently, in September 2009, the Department of Justice sought additional information from Microsoft and Yahoo about their proposed partnership. At first glance it might seem paradoxical to claim that the Google-Yahoo transaction is undesirable, for advertisers and for the economy as a whole, while the Microsoft-Yahoo transaction offers net benefits. But our analysis suggests that that conclusion is entirely possible. In particular, by creating a joined ad platform of larger size than Microsoft or Yahoo

alone, the transaction lets advertisers spread participation costs over a larger purchase – making it worth the while of small to midsize advertisers to sign up with the joined Microsoft-Yahoo platform.

Our results also inform an understanding of ad platforms’ participation costs. The basic intuition is clear: Ad platforms want to lower participation costs – making signup easy and eliminating unnecessary barriers so customers (advertisers) can sign up. What can stand in the way? Turning to observed market practices, we are struck by one ad platform that takes steps to increase *other* platforms’ participation costs: Google’s AdWords API restrictions make it unnecessarily difficult for an advertiser to copy, export, and/or synchronize campaign data from Google onto other ad platforms [4]. Our model does not specifically speak to why an ad platform would want to hinder a competing ad platform. But in practice the reason is clear: Preventing a competing platform from attracting advertisers reduces the quality of that competing platform (fewer ads yielding an inferior match with users’ searches), cuts that competing platform’s revenue (impeding future investment), and generally hinders that competing platform’s efforts at growth. Sure enough, Google’s tactics seem to have those effects; certainly competing platforms struggle to attract and retain advertisers [5]. Our analysis shows why this interference can be so effective in hindering competing platforms’ growth – depriving competitors of necessary scale.

Talk of “competing” ad auctions rings more true in some countries than others. In the United States, the largest ad platform, Google, enjoys a market share beyond 70%. But in many countries, Google’s market share is significantly higher; in Spain, Germany, and the Netherlands, Google’s share exceeds 90%. Once a market tips that far, competitors can offer advertisers only a small volume of traffic, making it particularly difficult for other ad platforms to regain any position at all, and making participation costs particularly weighty. Indeed, despite ongoing efforts from Microsoft and Yahoo, in most countries Google’s share has only risen. In future work, we seek to explore the price effects and other implications of Google’s exceptional and still-increasing market share.

Our model assumes an exogenous capacity for each ad platform. But in practice, the total volume of user searches is finite, and search engines compete to attract users (e.g. through quality of organic search results, free bundled services, toolbar installations, awareness-building advertising, and cashback). In a further extension, we seek to model competition for user attention, e.g. through a fixed total  $\bar{C}$  such that increasing one search engine’s  $C_i$  requires reducing some competitor’s  $C_{i'}$ .

We view our contribution as threefold. First, whereas standard models of online advertising take advertisers’ participation as exogenous, we explicitly model an advertiser’s decision to use or ignore a given ad platform. Second, we offer a model of participation costs influencing advertisers’ choice of ad platforms, and we provide empirical support for our model. Third, we analyze the prospect of joining auctions to mitigate participation costs, and we characterize when such joins do and do not increase welfare.

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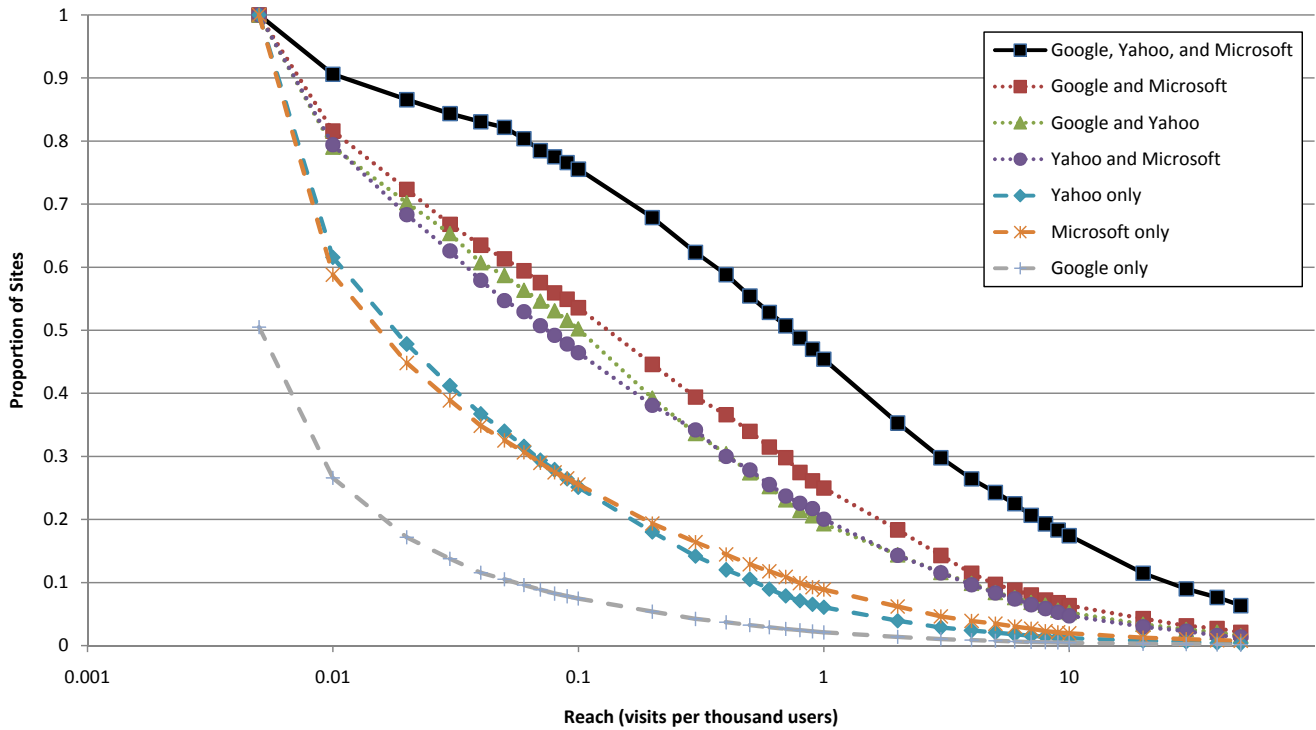
	Normalized Impression Count	Normalized Advertiser Count
Google only	17.34	1000.00
Yahoo only	26.01	338.65
Microsoft only	80.92	84.21
Google and Yahoo	663.58	209.26
Google and Microsoft	78.03	24.61
Yahoo and Microsoft	*	*
Google, Yahoo, and Microsoft	1000.00	65.76

**Table 1: Advertiser Size and Multi-homing Status (Source 1)**

\* - Our data source did not identify any advertisers using Yahoo and Microsoft but not Google.

	Proportion Ranked	Average Rank	Average Reach	Average Page-Views
Google only	0.696	17,428,974	4.19	0.22
Yahoo only	0.735	14,784,742	5.20	0.40
Microsoft only	0.705	15,838,598	4.40	0.41
Goole and Yahoo	0.862	8,305,741	17.11	1.18
Google and Microsoft	0.888	7,234,154	4.70	0.26
Yahoo and Microsoft	0.871	8,325,335	3.07	0.15
Google, Yahoo, and Microsoft	0.940	3,803,684	62.94	5.64

**Table 2: Advertiser Size and Multi-homing Status (Source 2)**



**Figure 1: Advertiser Reach and Multi-homing Status (Source 2)**